

## Introduction

Information Retrieval (IR) is finding material of unstructured nature that satisfies an information need from within large collections. Unstructured data typically refers to free text to query for some keywords or concepts.

The boolean retrieval model is used to respond to a query composed as a boolean expression, that is using AND, OR and NOT operators to join the query term. The first idea to maintain the relation between terms and documents containing them could be to use a matrix, but in practice this data structures turns out to be huge and sparse.

To solve this issue we make use of an inverted index where for each term  $t$  we must store a list of all documents that contain  $t$ , each of this identified by a unique id. The set of terms is called the dictionary, while the documents are called posting and consequently a list is called a postings list. The AND operation can now be implemented by intersecting the postings list, doing this in growing order is one of the possible optimizations to speedup the computation.

## Crawling

Web crawling is the process by which we gather pages from the Web graph to index them and support a search engine. It is possible to recognize different features that a web crawler should implement:

- Quality, it should be biased toward fetching “useful” pages first.
- Efficiency, it should avoid duplication, or near duplication, of the content crawled.
- Netiquette, there exist implicit and explicit policies regulating the access to a server by a crawler.
- Freshness, the content crawled should ideally reflect an updated image of the web.

## Architecture

The crawler begins with one or more URLs, that constitute a seed set, inside its URLs frontier. Continuously it picks a URL from the frontier, then fetches the corresponding web page. The fetched page is then parsed, to extract both the text and the links from the page. The extracted text is fed to a text-indexer, while the extracted links are then added to the URL frontier, which at all times consists of URLs whose corresponding pages have yet to be fetched by the crawler.

In the architecture of a crawler we can distinguish many different modules, all of these are executed by different, and possibly multiple, threads in a parallel and distributed computing fashion.

- Link extractor

```

while ( PageRepository not empty )
  pop page from PageRepository
  check for near duplication
  extract links
  push links in PriorityQueue
  • Downloader

while ( AssignedRepository not empty )
  pop URL from AssignedRepository
  download and archive page
  push page in PageRepository
  • Crawler manager

while ( PriorityQueue not empty )
  pop URLs from PriorityQueue
  foreach URL
    check if recently crawled
    preprocess URL (DNS, robots, etc)
    push URL in AssignedRepository

```

## Mercator

To guarantee the desired features of quality and politeness, it is mandatory for a crawler to implement some priority policies in the extraction of the URLs to be parsed. In the Mercator architecture this is done by using two set of FIFO queues: the front queues  $F$  and the back queues  $B$ .

After being extracted, a new URL is analyzed with some heuristic able to determine its priority value  $i$ , that is an integer between 1 and  $|F|$ . After this the value is assigned to the  $i$ -th front queue.

The URLs are then extracted from the front queues in a way biased by the priority of the queue, after this each one of the extracted URLs is pushed into one back queue according to its host. Each of the  $B$  back queues contains URLs from a single host, the mapping from hosts to back queues is maintained by an auxiliary table  $T$ . If one of the back queues gets empty a new host is immediately reassigned.

The URLs from the back queues are then extracted and inserted in a min-heap. This data structure contains one and only one element for each queue and it's ordered by the timestamps  $t_e$ , that contains the first viable moment to repeat a request to a certain host. The crawler main loop extracts the root of the min-heap, waits for the indicated time  $t_e$  to expire and then adds the URL to the `AssignedRepository`.

## Bloom Filter

To check if a page has been parsed or downloaded before a first control can be done on the URL, obviously, given the size of the web graph, storing all the crawled URLs in a dictionary is not a viable option.

A Bloom Filter (1970) is a probabilistic data structure used to check the elements of a set, it ensures a positive result if an element is in the set, but also could provide false positives. A binary array of size  $m$  is created, and then a family of  $k$  hash functions is used to map an object to a position in the array. To check if a certain element  $u$  is in the array is equivalent to the proposition  $\bigwedge_{i=1}^k h_i(u)$ .

Under the assumption of simple uniform hashing, the probability that a fixed position in the array contains zero is  $p = (1 - \frac{1}{m})^{kn} \approx e^{-kn/m}$ , and so the probability of a false positive  $\epsilon$  is equal to:

$$\begin{aligned} P(\forall i. B[h_i(u)] \neq 0) &= \\ P(B[h_i(u)] \neq 0)^k &= \\ (1 - P(B[h_i(u)] = 0))^k &= \\ (1 - e^{-kn/m})^k \end{aligned}$$

Fixed the size of the array  $m$  and the number of elements in the set  $n$ , from this formula we can derive an optimal value for the number of hash functions  $k_* = \ln 2 \frac{m}{n}$ , that implies an error rate  $\epsilon_* = (0.6185)^{\frac{m}{n}}$ .

## Various applications

The distributed computation of set intersection between two sets  $A$  and  $B$  can be done by computing the Bloom Filter of the first set, then computing  $Q$  as the set of all the  $b \in B$  elements that are present according to the bloom filter and then by intersecting  $A$  and  $Q$ , where the advantage is given by the fact  $|Q| < |B|$ . The bit cost of the transmission of the Bloom Filter of  $A$  and  $Q$  is  $\Theta(m_A) + (|A \cap B| + \epsilon|B|) \log |U|$ , so it is less expensive than sending the whole  $A$  set at a cost of  $|A| \log |U|$ .

The distributed computation of approximate set difference can be derived by the previous algorithm, when  $A$  can correctly compute the difference via  $A - B = A - Q$ , and  $B$  can obtain an approximation via  $B - A \approx B - Q$ .

Another possible approach to compute the set difference is using Patricia Trees derived from  $A$  and  $B$ . Comparing each node in a top-down fashion, if the node are equals the visit backtracks, otherwise it proceeds to all children, when a leaf is reached then the corresponding element of  $B$  is declared to be in  $B - A$ . This solution is obviously unfeasible in practice because of the cost of replicating the subsets in a node, but this could be avoided by using the same algorithm on a Merkle Tree, that stores instead the hash of each subset in the corresponding node. It's possible to improve again the space occupancy by computing a Bloom

Filter of the nodes of the Merkle Tree  $MT_A$ , and then visiting top-down the  $MT_B$  tree each node is checked against  $BF(MT_A)$ . This improvement comes at the cost of possible false positives during the matching of the nodes between the two trees. Assuming  $m_A = \Theta(|A| \log \log |B|)$  and the optimal  $k_A = \Theta(\log \log |B|)$ , the bit cost of the transmission is  $O(|A| \log \log |B|)$ , that is the bit size of the Bloom Filter.

### Spectral Bloom Filter

A Spectral Bloom Filter is a variation of a standard BF that makes use of an integer array instead of a binary one, where each position of the array counts the number of occurrences of an object  $s \in S$  in a multiset  $M$ . The space usage is slightly larger, but in a constant order, and opens to new possible use cases like aggregate and iceberg queries. The error probability is the same of the standard bloom filter, so  $\epsilon \approx (1 - p)^k$ .

The insertion and deletion operations are implemented by simply incrementing, or decrementing, each counter derived by the application of each hash function to the element. The query result is instead given by the minimum of all the counters, this minimum selection allows to select the counter where the minimum number of collision have occurred.

It can be noticed that if the minimum of all the counters relative to an object is repeated multiple times, it is less likely that the same item is subject to a false positive error. This situation, called recurring minimum, can be used to improve the overall performances of the SBF. We can operate then with two separate SBF, where the second one is smaller and is used to store single minimum elements.

#### INSERTION

```
insert x in SBF1
if x single minimum in SBF1:
    if x in SBF2:
        insert x in SBF2
    else:
        set counters of x in SBF2 as the min value of x in SBF1
```

#### DELETION

```
delete x from SBF1
if x single minimum in SBF1:
    if x in SBF2:
        delete x from SBF2
```

#### LOOKUP

```
if x recurring minimum in SBF1:
    return min x in SBF1
else if x in SBF2:
    return min x in SBF2
```

```

else
    return min x in SBF1

```

## Parallel Crawling

The web is too big to be crawled by a single crawler, so the work should be divided avoiding duplication of work. Using static assignment is difficult to load balance the URLs assigned to a crawler, also the situation of fault-tolerance where one downloader could be removed or created in a dynamic way makes the static assignment prone to errors.

A possible solution is the use of the consistent hashing technique. Given two<sup>1</sup> hashing functions  $h_s : \text{Server} \rightarrow m$  and  $h_u : \text{URL} \rightarrow m$ , we use an orientated circular mapping where the items are dynamically partitioned in arcs between the servers and, assuming clockwise orientation, each server needs to communicate only with its successor in case of mutation in the topography.

In average each server has assigned  $\frac{\# \text{servers}}{\# \text{items}}$  URLs, it's possible to prove that this happens with high probability.

## Locality-Sensitive Hashing

In data analysis a frequent issue is, given a set  $S$  of items, each one with  $d$  features, to find the largest group of similar items. The similarity is a function that, taken the features of two items, returns a value in the interval  $[0, 1]$ .

The brute-force approach is not useful because of the practically infinite number of possible groups, even limiting the group size to a constant  $L$  requires an incredible computational power. Introducing a certain level of approximation it's possible to consider a clustering algorithm like the famous machine learning algorithm K-means.

The LSH technique proposes instead to generate a fingerprint for every item, and then to transform the similarity between items into the equality of fingerprints. This approach is randomized and correct with high probability, also it guarantees local access to data reducing the number of I/O operations needed.

Given the hamming distance  $D(p, q)$  between two binary vectors  $p$  and  $q$ , we define the similarity  $s$  as the probability, given an index  $i$ , that  $p[i] = q[i]$ , and this is equal to  $s = (1 - \frac{D(p, q)}{d})$ .

Now consider a set  $I$  of  $k$  random selected integers in the interval  $\{1, \dots, d\}$ , we call  $h_I(p)$  the projection of  $p$  into the  $I$  positions. The probability that two fixed projections of a pair of vectors are equal is  $P(h_I(p) = h_I(q)) = s^k$ .

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<sup>1</sup>The slides of the course mention only one function, also in some of the exercises the same function is used for both objects.

Using  $L$  different projections we can state that  $p$  is similar to  $q$  with high probability if  $\exists i. h_{I_i}(p) = h_{I_i}(q)$ . The probability of this event is:

$$\begin{aligned} P(p \text{ matches } q) &= \\ P(\exists i. h_{I_i}(p) = h_{I_i}(q)) &= \\ 1 - P(\forall i. h_{I_i}(p) \neq h_{I_i}(q)) &= \\ 1 - P(h_{I_i}(p) \neq h_{I_i}(q))^L &= \\ 1 - (1 - s^k)^L \end{aligned}$$

So strictly dependent on the actual similarity  $s$ , between  $p$  and  $q$ . It's possible to notice that while the  $k$  value reduces the false positives, the  $L$  reduces the false negatives.

In the practice of grouping similar items this technique is applied by generating  $L$  sets  $I_i$ , then computing for each item in the set its sketch, that is the  $L$ -ple containing all the  $h_{I_i}$  projections. Generating a graph of items where each node has an edge with any node with at least one equal projection in the sketch, permit to define the groups as the connected components in the graph. Given that this can be implemented via only scan and sort primitives, the number of IO operations to do this is  $\tilde{O}(\frac{n}{b})$ .

In the case of online queries instead, it's possible to create  $L$  hash tables where each table has  $2^k$  elements, similar elements to the queried one are the elements that collide with it.

Comparing LSH with K-means we could use this resume:

Algorithm	Optimality	Cost	Cost per iteration	Number of cluster
LSH	Global with high probability	Short sketch comparison	Sort $ S $ items	Not needed
K-nn	Locally optima	D-features comparison	$K \times S \times d$	It's a parameter

## Document duplication

The Web contains multiple copies of the same content that search engines try to avoid indexing, to keep down storage and processing overheads. Theoretically any hash function could be used to store a fingerprint of a web document to be compared for each new document, obviously there are some efficiency and space constraints that makes some solutions more interesting than others.

The rolling hash technique described by Karp-Rabin fingerprints is commonly

used. Given a prime number  $p$ , the fingerprint of an  $m$ -bit string  $A$  is  $f(A) = A \bmod p$ , it's possible to easily compute any shift of  $A$ . The probability of a collision between any pair  $A$  and  $B$  is equivalent to the probability that  $p$  divides  $A - B$ , that is practically zero.

This simplistic approach fails to capture a crucial and widespread phenomenon on the web: near duplication. In many cases, the contents of one web page are identical to those of another except for a few characters.

## Shingling

Given the set of all the possible  $q$ -grams of a document and their fingerprints called shingles, the shingling technique is used to reduce the near-duplicate document detection problem to intersection of the set of shingles of two distinct documents. We declare that page  $A$  and  $B$  are near duplicated if the intersection of  $S_A$  and  $S_B$  is large according to an arbitrary measure, like the Jaccard similarity defined as follows:

$$J(S_A, S_B) = \frac{|S_A \cap S_B|}{|S_A \cup S_B|}$$

This process requires a big amount of space, and the full cost of computing the intersection over the whole shingling sets.

## Min-hashing

A possible way to approximate the Jaccard similarity between two sets is using  $L$  random permutations to generate a sketch given by the minimum of the set in each considered permutation.

$$\langle \min \pi_1(S_A), \dots, \min \pi_L(S_A) \rangle$$

To share the same minimum value in the same permutation  $\pi_i$ , the minimum must have been taken from the intersection between all the possible values, so it's immediate that  $p = P(\min \pi_i(S_A) = \min \pi_i(S_B)) = J(S_A, S_B)$ .

We are now able to approximate the Jaccard similarity of the two sets by counting the number of equal components between two sketches and normalizing it via  $L$ , this is sound because of this observation:

$$\frac{\mathbb{E}(\text{\#equal components})}{L} = \frac{L * p}{L} = J(S_A, S_B)$$

The space occupancy of this technique can be reduced at the cost of introducing approximate results by projecting each sketch into a set of  $k$  integers for  $L'$  times, and then group them by equal component as in LSH.

## Cosine distance

Another possible way to compute the sketch of a vector is using the cosine distance. Given  $L$  random lines, we can compute the  $i$ -th element of the sketch as the result of the hash function  $h_i(p) = \text{sign}(r_i \cdot p)$ , where  $r_i$  is the  $i$ -th random line.

Two sketches share the same value iff. the line  $r$  doesn't lie in the angle  $\alpha$  between the original vectors of the sketches, so the probability of this event is  $P(h_i(p) = h_i(q)) = \frac{\pi - \alpha}{\pi} = 1 - \frac{\alpha}{\pi}$ .

By dividing the number of not shared components between two sketches, divided by the number of components  $L$  we obtain an approximation of  $\cos(\alpha)$  for small  $\alpha$  angles.

## Compressed storage of the web-graph

The directed graph representing the web has three peculiar characteristics:

- Skewed distribution, the probability that a node has  $x$  links follows the power law  $\frac{1}{x^\alpha}$ , where experimentally  $\alpha \approx 2.1$ .
- Locality, usually most of the hyperlinks point to URLs in the same host.
- Similarity, if two URLs are close in lexicographic order, then they tend to share many hyperlinks

Permuting the host, reversing the dot order, it's possible to create a sequence of adjacency lists that uses the locality and the similarity properties to generate contiguous areas. This situation can be exploited to reduce space occupancy using the copy-list technique. In this technique each list has associated:

- Out-degree, the number of links exiting the page.
- A reference list, in respect to there is the compression.
- A copy list, that is a bit-string of the same length of the out-degree of the reference.
- Extra nodes, that stores the links not shared with the reference.

The bits in the copy list can be compressed using run-length encoding in a copy-block. This is done by storing the first bit on the list and then the length of each region of consecutive equal bits minus one, the integer representing the last region of the list could be dropped because the number of bits is constrained by the out-degree.

## Index construction

In sorting a list of strings, the indirect list containing pointers breaks locality at an higher level, when the number of elements is huge this becomes a considerable problem.



## Blocked sort-based indexing

To construct an inverted index the collection of documents is scanned to generate the term-docID pairs, then the pairs are sorted by term and the docIDs are grouped in the posting list. For small collections, all this can be done in memory, the problem requires more attention in a scenario where large collections have to be indexed. To make index construction more efficient, we represent unique terms as termIDs, this requires a first scan to compile the vocabulary, and a second one to construct the inverted index.

With main memory insufficient, we need to use an external sorting algorithm like multi-way MergeSort. This algorithm sorts  $N$  items with a main memory of size  $M$  and disk-pages of size  $B$ , the first pass is to produce  $\frac{N}{M}$  sorted runs, and then merge them for a total cost of  $O(\frac{N}{B} \log_{\frac{M}{B}} \frac{N}{M})$  IO operations.

## Single-pass in-memory indexing

Blocked sort-based indexing has excellent scaling properties, but it needs a data structure for mapping terms to termIDs. For very large collection, this data structure does not fit into memory. A more scalable alternative is SPIMI, that using terms writes each block's dictionary to disk, and then starts a new dictionary for the next block.

The tokens in the document are analyzed one by one, and the relative posting list is filled using a doubling algorithm. When the memory is full the block is written to the disk, and a new one is started. The overall algorithm is faster because it doesn't require sorting nor the creation of a term-termID vocabulary.

## Distributed indexing

When the index dimension becomes too big the only solution is to use distributed indexing algorithms for index construction. Two obvious alternative index implementations are partitioning by terms and partitioning by documents.

Partitioning by terms means that each node of the cluster contains a partition of the terms and all of their posting lists. A query is routed to the nodes corresponding to its query terms. In principle, this allows greater concurrency, but in practice this behaviour turns out to be non-trivial for multiword queries and load-balancing.

A more common implementation is to partition by documents, each node contains the index for a subset of all documents. Each query is distributed to all nodes, with the results from various nodes being merged before presentation to the user. This partitioning simplifies the communication, but requires to contact all the nodes to compute any global operation.

## Dynamic indexing

When dealing with dynamic collections new approaches are needed to correctly index them. One simple solution, called auxiliary index, consist in using multiple indexes, a main one and other smallest where to insert newly arrived documents and periodically re-index all the collection into one main index; also deletion can be handled in this situation by invalidating a bit vector. Storing each postings list as a separate file, then the merges imply consists of extending each postings list of the main index by the corresponding postings list of the auxiliary index, this is unfeasible because the difficulties for a file system in handling a big number of files. This consolidation process is costly.

A better solution is provided by logarithmic merge, where a series of exponentially increasing indexes are allocated in the disk and in memory an index is present larger as the smallest on disk. If an index  $I_i$  becomes too big its content is merged with the successive  $I_{i+1}$ , and so on if after merging also this index becomes too big, this maintains the invariant about the fact that each one of the indexes on the disk is either empty or full.

Each text participates to no more than  $\log \frac{C}{M}$  mergings because at each merge the text moves to a next index and they are at most  $\log \frac{C}{M}$ , where  $C$  is the total size of the collection.

## Compression of documents

In a modern search engine the raw documents are needed for various applications, one of the more common is the dynamic extraction of a snippet depending on the query. The most important tradeoff in data compression is given by the inverse proportionality of the compression rate and the decompression speed. Recently many technologies have been developed, among the others we recall Snappy and Brotli by Google and LZFS by Apple.

### LZ77

The LZ77 algorithm, used by gzip, compresses data by exploiting repeated substrings in a document. Given a document it generates a set of triplets of the form  $\langle \text{distance, length, next-char} \rangle$  by scanning the document using a fixed size buffer window. So, at position  $i$  the algorithm checks if a substring  $A[i, k]$  is repeated at least once in the substring  $A[i - b, i - 1]$ , where  $b$  is the buffer size, if so it generates a triplet and skips to the next non compressed character. The decompression reverses this process, by applying the sorted triplets to the initially empty string  $\epsilon$ .

### Z-delta compression

This algorithm is used to compress a new file  $f_n$  by using another known file  $f_k$ . Reprising the idea of the LZ77 algorithm the file  $f_k$  and  $f_n$  are concatenated,

then by scanning from  $f_n$  the set of triplets  $f_d$  is generated, it's possible to prove that  $f_d$  it's an optimal coverage, containing the minimal number of triplets. Now it's possible to obtain  $f_n$  concatenating  $f_k$  to the empty string  $\epsilon$  and then by applying the triplets as in the LZ77 algorithm.

The Z-delta algorithm can also be used to compress a cluster of files by constructing a graph where all the files are represented by a node. Any directed edge  $\langle i, j \rangle$  is weighted by the number of triples needed to compress  $f_j$  using  $f_i$  as common knowledge. By adding a dummy node  $\epsilon$  an optimal compression of the cluster is found by computing a minimum directed spanning tree rooted in the dummy node. It should be noticed that the number of edges is quadratic in respect to the number of files, and it's costly to generate them. Since we are interested in using only edges between similar files, we can use locality sensitive hashing as an heuristic in the construction of the graph.

### **rsync**

This algorithm is used to synchronize the content of two files in a client-server scenario, let's suppose that the old one  $f_o$  is in the client, and the new one  $f_n$  is in the server and they're possibly very different, so we can't assume common knowledge between them. Given a block size  $b$  the client generates a non-overlapping sequence of blocks, then each of these is hashed and sent to the server. Once that the server has received the hashes it inserts them in a dictionary. After this it scans  $f_n$  with a window of size  $b$  and rolling hashes each buffer, if it doesn't match any of the ones in the dictionary the character in the first position of the buffer is sent to the client, otherwise it sends the index of the block.

### **zsync**

The zsync algorithm is useful to reduce the work load of the server and could be considered as a symmetrical approach to rsync. This time is the server that computes the  $\frac{n}{b}$  blocks, and then sends their hashes to the client. Via rolling hash the client looks for the received hashes in  $f_o$  and replies to the server with a bitmask of length  $\frac{n}{b}$  where  $B[i]$  is set to one iff. the block  $H_i$  is present in  $f_o$ . Given the bitmask the server is able to z-delta compress the block that the client doesn't have according to the ones it owns, enabling the client to construct the full file.

## **Document parsing**

After the documents are crawled their content has to be tokenized, that is generating a stream of tokens ("words") from a document. The tokenization is a critical module in a search engine, in OSS frameworks is common to leave the implementation to the user. After this the tokens are normalized by using one or more linguistic models, and once normalized they can be indexed. The first

issue in parsing a document is identifying features like the format, the language and the character set; all of these are classification problems dealt via heuristics.

## Tokenization

A token is an instance of a sequence of characters in some particular document, that are grouped together as a useful semantic unit for processing. Each such token is now a candidate for becoming an index entry, that is a term, after further processing.

The major question of the tokenization phase is what are the correct tokens to use? An easy answer like splitting by space, or by hyphens, isn't appropriate in the majority of the use cases. The language identification is so fundamental because of the different ways to separate words that could become terms.

## Stop words

A common practice was to don't index stop words, because they are frequent and meaningful. But the current trend is away from doing this, good compression techniques and good query optimization techniques enables the search engine to cheaply store stop words.

## Normalization

The normalization is a language-dependent process that transforms tokens into a canonical form, this is done usually by removing hyphens and periods. Another issue regards case-folding, a best practice is to minimize all the characters and leave the context to the other words. Other than syntactically we have to handle via a thesauri cases of synonyms and homonyms, historically this was done brute force using an handmade thesauri.

## Stemming and lemmatization

The stemmer is a module able to recognize variations of the same word, this is done by using root prefixes of the tokens. A possible stemmer can be built by using the Porter's algorithm.

Lemmatization is the process to reduce variant forms to a base form, for example the transformation of a conjugated verb to its infinite form.

## Statistical properties of text

Tokens are not distributed uniformly, but they follow the so called Zipf law. The Zipf law states that few tokens are very frequent, a middle sized set has medium frequency and many are rare, formally:  $f_s(k) = \frac{c}{k^s}$ . This empirical law has been found true in all the known languages, most of the more frequent are stop words. The Zipf law is a power law, so its log-log plot is approximately a straight line.

Also the number of distinct tokens grows sublinearly according to the Heaps law  $n^\beta$  where  $\beta \approx 0.5 < 1$  and  $n$  is the number of total tokens. The interesting words from an IR point-of-view are, according to Luhn law, the ones with medium frequency.

## Keyword extraction

One interesting aspect in the process of keyword extraction is the analysis of the collocations. A collocation is a sequence of two or more words that correspond to some conventional way of saying where their constituent words are not substitutable or modifiable and can't fully infer the meaning of the collocation without the others.

Frequency sorting of all the adjacent pairs in a document is not useful because of the high frequency of prepositions, articles and other stop words. Better result can be obtained by using Part-of-Speech tagging, and by allowing only certain pairs to be ranked.

This solutions doesn't consider flexibility, often words are not adjacent to each other. A possible approach is computing the mean and the variance of the distance within a window of all the possible pairs of words. We can conclude that if the mean is large the collocation is not interesting, whilst if the mean is very small the pair should be treated as a collocation.

## Bi-grams

Pearson's chi-squared test ( $\chi^2$ ) is a statistical test applied to sets of categorical data to evaluate how likely it is that any observed difference between the sets arose by chance. It tests a null hypothesis stating that the frequency distribution of certain events observed in a sample is consistent with a particular theoretical distribution. The events considered must be mutually exclusive and have total probability 1.

In inferential statistics, the null hypothesis  $H_0$  is a general statement or default position that there is nothing new happening, like there is no association among groups, or no relationship between two measured phenomena.

In our scenario taking as a null-hypothesis that two words are independent, and so  $P(A \circ B) = P(A)P(B)$  it's possible to perform Pearson's chi-squared test to assess whether observations consisting of measures on two variables, expressed in a contingency table, are independent of each other

A contingency table  $O$ , that counts the occurrences of all the possible pairs fixed  $A$  and  $B$ , has to be computed. The so called degree of freedom is dependent on the size of the contingency matrix, as in  $df = (\text{rows} - 1)(\text{columns} - 1)$ .

<hr/>		
$w_1 = A \quad w_1 \neq A$		
<hr/>		
$w_1 = A \quad w_1 \neq A$		
<hr/>		
$w_2 = B$	$O_{11}$	$O_{12}$
$w_2 \neq B$	$O_{21}$	$O_{22}$
<hr/>		

Then we define  $E_{ij} = N * p_i * p_j$ , where  $p_i = \sum_j \frac{O_{ij}}{n}$  and  $p_j = \sum_i \frac{O_{ij}}{n}$ , and consequently to this:

$$\chi^2 = \sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

The  $\chi^2$  has to be compared against the critical value of the distribution, dependent of the degrees of freedom and a certain confidence level arbitrary chosen, if the value is smaller than the threshold the null hypothesis is plausible, and so it's not a good pair for a collocation.

## Rapid Automatic Keyword Extraction

The algorithm works on single, not much long, documents and provides fast and unsupervised keyword extraction exploiting the fact that keywords frequently contain multiple words but rarely contain punctuation or stop words.

Given a set of word delimiters, a set of phrase delimiters and a list of stop words, in the first phase the algorithm splits the document by words, then by phrases and then by stop words: the remaining words are considered candidate keywords

The ordered sequence of candidate keywords is scanned to compute the table of co-occurrences where the item  $C_{ij}$  counts the number of pairs that contains both the  $i$ -th and the  $j$ -th word. Given this table is possible to compute the frequency of a word as  $M_{ii}$ , and the degree of a word as the sum over its row  $\sum_j M_{ij}$ . We define the score of a word as the ratio  $\frac{deg(w)}{freq(w)}$ .

A set of adjoining keywords is a candidate if it appears at least twice in the document, if so the score for the new keyboard is the sum of its member keyword scores. Sorting by score enables the user to select the  $k$ -th most significative keywords.

## Search

### Prefix-string

Given a dictionary  $D$  of  $K$  strings, of total length  $N$ , store them in a way that we can efficiently support prefix searches for a patter  $P$  over them.

A trie is a useful data structure for p-search, but every implementation suffers of big space issues. Using a 2-level indexing solution can mitigate the problem, dividing the sorted elements in pages and constructing a trie on a sampling we can use the trie as a router. This solution requires typical only one IO operation to visit the trie, and also requires less space given that the trie is built over a subset of strings. A disadvantage to consider is the tradeoff in speed vs space given by the bucket size used for the sampling.

With front-coding it's possible to further reduce space usage. To compress a sorted list of strings, the prefix is substituted by one byte that indicates the number of prefix shared characters with the previous word. Obviously random access is not possible, but in our scenario the scanning of a bucket was already required for p-search in a 2-level indexing solution. If the strings don't use a termination character, the length of the string also has to be stored.

## Tolerant search

The area of spell correction can be used for two principal use cases: the correction of an indexed document and the correction of the user queries. Also this correction could occur by analyzing isolated words or by also considering the context of a sentence.

For the rest of the discussion we assume the presence of a lexicon, to correct isolated word in queries. Given a lexicon and a character sequence  $Q$ , return the words in the lexicon closest to  $Q$ . The definition of closeness can vary using different distance measures.

## Brute force

A brute force approach enumerates for each word in  $q$  in  $Q$ , all the possible  $\bar{q}$  words with edit distance  $d$  from  $q$ , if  $\bar{q}$  is in the dictionary then it's proposed as a suggestion to the user. The edit distance, or Levensthein distance, is computable via a dynamic programming algorithm where first the matrix  $E$  is constructed, and then the value  $E[n][m]$  is the edit distance of the two strings.

$$E_{ij} = \begin{cases} i & j = 0 \\ j & i = 0 \\ \min \begin{cases} E_{i-1,j-1} + W_l \\ E_{i,j-1} + W_i \\ E_{i-1,j} + W_d \end{cases} & i \neq 0 \wedge j \neq 0 \end{cases}$$

## One-error correction

The enumeration and the dynamic programming solutions are too costly, to improve the performance we have to assume the maximum edit distance as one. An useful approach is to transform the problem exploiting the fact that

enumeration deletions is easy, so given the dictionary of correct words  $D_1$  we can easily generate the dictionary  $D_2$  that contains  $\Theta(l|D_1|)$  words on average.

Now for each word  $q \in Q$  we can operate as follows:

- If  $q \in D_1$  the word is correct.
- For each  $\bar{q}$  given by dropping a character in  $q$ , if  $\bar{q} \in D_1$ , then  $q$  had one char more and  $\bar{q}$  has to be suggested.
- If  $q \in D_2$ , then  $q$  has one char less and so is not correct.
- For each  $\bar{q}$  given by dropping a character in  $q$ , if  $\bar{q} \in D_2$ , then  $q$  may contain a mismatch and so it's not correct.

This algorithm request linear time respect to the number of words  $q \in Q$ , but suffers of a space problem and could generate false matching results.

### Overlap distance

The overlap distance is an approximation of edit distance, used as an heuristic to generate a set of candidate words from a dictionary.

Assume that each word is anticipated by  $k - 1$  special character \$, this ensures that the number of k-grams will be equal to the length of the string that generated it. All the words in a dictionary have to be partitioned in k-grams, and then an inverted index is built over the k-grams.

A query  $Q$  generates  $|Q|$  k-grams, while each error affects  $k$  k-grams. So if  $Q$  and a given token have edit distance  $i$ , they have to share for sure at least  $|Q| - ki$  k-grams.

Given a query word  $Q$ , by splitting it in k-grams we are able to find the corresponding inverted lists, and so keep track of how many k-grams  $Q$  shares with each word in these lists. Fixed  $i$  it's now possible to check all the strings in the list, those that satisfy the filter condition are flagged as candidate strings.

### Wildcard queries

The wildcard queries  $\alpha\star$  have been already considered by prefix-search, but also the form  $\star\beta$  can be solved with the same techniques by constructing the data structures over the reverse of each term.

More interesting is the case of the form  $\alpha\star\beta$ . It's still possible to use the two previous approaches to p-search  $\alpha$  and s-search  $\beta$  and then intersecting the results. If the resulting lists are big the intersection could be expensive in terms of time.

The permuterm index proposes an approach that pays with space to reduce time complexity. Each term in a dictionary is indexed under all the possible rotation of the word, so for example the term **word** is indexed by **word\$,ord\$,rd\$,wo\$,d\$,wor\$,s\$,word**. Given a query in the form  $P\star Q$ , the search is reduced to a p-search of the rotated query  $Q\$P\star$ .



## Soundex

Under the name of soundex goes a class of heuristics to expand a query into phonetic equivalents. The soundex algorithm are language specific and mainly used for names; historically the first soundex was used to write down the names of the non-English immigrants in Coney Island.

## Query Processing

### Phrase queries (2.4)

A phrase query is a query where multiple words are considered as an atomic unit, to be able to support such queries, it is no longer sufficient for postings lists to be simply lists of documents that contain individual terms.

One approach is to consider every pair of consecutive terms in a document as a phrase, to generate biwords. Each of these biwords is treated as a vocabulary term, and inserted as an entry in the dictionary. The query processing of biword is immediate, furthermore longer phrases can also be processed by breaking them down in overlapping pairs and using the AND operator. Without examining the documents it's not possible to verify the result of this boolean operation that can possibly cause false positive results. To optimize the results it's possible to use PoS tagging to construct an extended biword index.

Another approach consist in storing in the postings of a term, other than the documents is also stored the position in which that term occurs. Other than for phrase queries this approach can be used to solve free text queries, in which the results are biased according to the close proximity of each other.

The combination of these two schemes is often used in real word search engine.

Usually when dealing with phrase queries the search engine makes use of an iterative process where first tries to run the query as provided by the user, if the results are too many it tries to run multiple smaller queries and join them. If even in this case the set of results is too small the search engine must use other techniques, like vector space querying, to resolve the query.

### Zone indexes

Up to now a document has been considered as a sequence of terms, but this view can be enlarged by considering other features like the author, the title, the language and so on. This accessory features of a document are called its metadata.

A zone is a region of the document that can contain an arbitrary amount of text. Building and inverted index also on the zones allows the user to query on them. The information about the zones can be stored in the dictionary or in the postings.

## Optimization

### Tiered index

Caching can be useful to speedup query resolution, there are two possible and opposite approaches: to cache the query results, exploiting query locality, or to cache pages of posting lists, exploiting term locality.

Another possible solution consists in breaking postings up into a hierarchy of lists, sorted by importance. At query time only the top tier is used, unless it fails to provide a minimum number of documents, if so it recurses onto the lower tier.

### Skip pointers

In a skip list the number of skips is an important tradeoff, the more the skips the shorter the spans, it is most likely to skip but lots of comparisons are required to evaluate skip pointer. A simple heuristic for posting list of length  $L$  is to use  $\sqrt{L}$  evenly-spaced skip pointers.

## Posting list compression

Given that the postings are encoded as integers it's a good idea to encode small numbers in less space than large numbers.

The  $\gamma$ -code is a universal code for integers which uses a fixed model. Given an integer  $x$  and  $L$  the length of its optimal representation  $B(x)$ ,  $x$  is represented as a binary sequence composed of two parts: a sequence of  $L - 1$  zeros, followed by the binary representation  $B(x)$ . The decoding is easy, count the  $c$  consecutive number of zeros up to the first 1, then fetch the following  $c + 1$  characters and interpret the sequence as the integer  $x$ .

PForDelta code is a method that supports extremely fast decompression and achieves a small size in the compressed output whenever the values follow a Gaussian distribution. Fixed  $a, b > 0$ , all the integers in the interval  $[a, a + 2^b - 1]$  are encoded with  $b$  bits by translating them in the interval  $[0, 2^b - 1]$ , all the other integers are instead prefixed with  $b$  1 bits before a standard integer representation. The encoding does not occur in streaming, but instead the  $b$  bytes ones are used as an escape symbol for the explicit coded integers, stored in another partition. The value  $b$  is a trade-off, for bigger values there will be less elements in the "extra values" partition, but the "normal" partition size is increased.

In the  $t$ -nibble code, the binary representation  $B(x)$  is left-padded with zeros up to the minimum number of bits multiple of  $t - 1$ . Then groups of  $t$  bits are generated by dividing the intermediate representation in buckets of  $t - 1$  bits, prefixing with 1 the first bucket and with 0 all the other. Given a sequence of bits, the decompression only requires to scan shifting of  $t$  bits, concatenating the intermediate bits. The so called variable-byte code is a specialization of  $t$ -nibble, with  $t = 8$ .

The Elias-Fano code requires that the elements are strictly increasing. Fixed  $n$  the number of integers,  $m$  as the value of the maximum number plus one, we can compute  $z = \log_2 n$  and  $w = \log_2 \frac{m}{n}$ . For each integer represented in  $z + w$  bits the last  $w$  bits are concatenated in a list  $L$ , so  $|L| = nw = n \log \frac{m}{n}$ . The  $z$  remaining bits can possibly have  $2^z$  representations, we then construct another sequence  $H$  by counting the occurrences of each representation in a negative unary<sup>2</sup> representation, so  $|H| = 2^z + n$ . Overall the cost of the representation is  $|L| + |H| = n(2 + \log \frac{m}{n})$ , wasting only two bits per number respect to the optimal code.

There exist data structures able to decompress numbers in constant time. We propose instead the following algorithm to get the  $k$ -th number of the list:

- Set  $r$  as the number of 0 bits up to the  $k$ -th 1 bit in  $H$
- Get the  $k$ -th group of bits in  $L$
- The value of the integer is the concatenation of the  $r$ -th prefix representation and the group of bits found in  $L$ .

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<sup>2</sup>The negative unary representation of a number consist in the repetition a 1 per unit, terminated by a 0. That is 0, 10, 110, 1110, ...