



Binary Search

Efficient Searching Through Halves

Recap: Linear Search Complexity

- Best Case
 - target = **3**, List = [**3**, 6, 4, 9, 2]
 - $O(1)$ → Constant time



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 - target = **2 or ?**, List = [3, 6, 4, 9, **2**]



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- Average Case
 - target = 4, List = [3, 6, 4, 9, 2]



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From Linear to Binary Search: A Faster Way

- **Recap Linear Search:**

- Checks **each element** in a list one by one



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- **Inefficient** as the list grows larger



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- Imagine searching through **1 million** records



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- Let's explore this idea with a **number guessing game**



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- Imagine thinking of a number **from a list of 16 sorted numbers**



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- For **1 million items**, Binary Search takes around **20 steps at most**

- **Setting up the Challenge:**

- Let's explore this idea with a **number guessing game**
- Imagine thinking of a number **from a list of 16 sorted numbers**
- Can you guess it in no more than **4 attempts?**



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72



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Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. $\text{mid} = (0 + 15) // 2 = 7$



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Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

Is **12** == **35**? Yes, Greater, Less? **Less**

We are thinking of
12

Calculate middle index
mid = (first + last) // 2

1. mid = (0 + 15) // 2 = 7



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6
List	3	7	12	18	23	27	30

Is **12** == **35**? Yes, Greater, Less? **Less**

We are thinking of
12

Calculate middle index
mid = (first + last) // 2

1. mid = (0 + 15) // 2 = 7



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6
List	3	7	12	18	23	27	30

Is **12** == **35**? Yes, Greater, Less? **Less**

We are thinking of
12

Calculate middle index
mid = (first + last) // 2

1. mid = (0 + 15) // 2 = 7

2. mid = (0 + 6) // 2 = 3



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6		
List	3	7	12	18	23	27	30		

Is **12** == **35**? Yes, Greater, Less? **Less**

We are thinking of
12

Calculate middle index
mid = (first + last) // 2

1. mid = (0 + 15) // 2 = 7

2. mid = (0 + 6) // 2 = 3



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6		
List	3	7	12	18	23	27	30		

Is **12 == 18?** Yes, Greater, Less? **Less**

We are thinking of
12

Calculate middle index
mid = (first + last) // 2

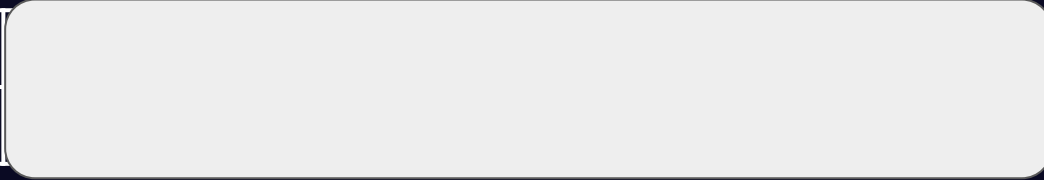
1. mid = (0 + 15) // 2 = 7

2. mid = (0 + 6) // 2 = 3



Binary Search: Finding a Number in Sorted List

Index	0	1	2
List	3	7	12



We are thinking of
12

Is **12** == **18**? Yes, Greater, Less? **Less**

Calculate middle index
mid = (first + last) // 2

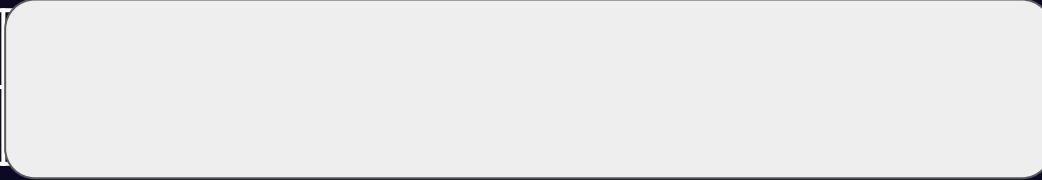
1. mid = (0 + 15) // 2 = 7

2. mid = (0 + 6) // 2 = 3



Binary Search: Finding a Number in Sorted List

Index	0	1	2
List	3	7	12



We are thinking of
12

Is **12** == **18**? Yes, Greater, Less? **Less**

Calculate middle index
mid = (first + last) // 2

1. mid = (0 + 15) // 2 = 7

2. mid = (0 + 6) // 2 = 3

3. mid = (0 + 2) // 2 = 1



Binary Search: Finding a Number in Sorted List

Index	0	1	2			
List	3	7	12			

Is **12** == **18**? Yes, Greater, Less? **Less**

We are thinking of
12

Calculate middle index

mid = (first + last) // 2

1. mid = (0 + 15) // 2 = 7

2. mid = (0 + 6) // 2 = 3

3. mid = (0 + 2) // 2 = 1



Binary Search: Finding a Number in Sorted List

Index	0	1	2			
List	3	7	12			

Is **12 == 7?** Yes, Greater, Less? **Greater**

We are thinking of
12

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 $\text{mid} = (\text{first} + \text{last}) // 2$

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Binary Search: Finding a Number in Sorted List

Index		2	
List		12	

Is **12** == **7**? Yes, Greater, Less? **Greater**

We are thinking of
12

Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. mid = $(0 + 15) // 2 = 7$

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Binary Search: Finding a Number in Sorted List

Index		2	
List		12	

Is **12** == **7**? Yes, Greater, Less? **Greater**

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12

Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. $\text{mid} = (0 + 15) // 2 = 7$

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Binary Search: Finding a Number in Sorted List

Index		2	
List		12	

Is **12** == **7**? Yes, Greater, Less? **Greater**

We are thinking of
12

Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. $\text{mid} = (0 + 15) // 2 = 7$

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Binary Search: Finding a Number in Sorted List

Index		2	
List		12	

Is **12 == 12?** Yes, Greater, Less? **Yes**

We are thinking of
12

Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. $\text{mid} = (0 + 15) // 2 = 7$

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Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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Input Size: **16**

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Input Size: **16**

Total Comparisons: **4**

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Linear Search: Comparisons **∞** Input Size

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1

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Linear Search: Comparisons **∞** Input Size

1**1**

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mid = (first + last) // 2
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Linear Search: Comparisons **∞** Input Size

1
10

1
10

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1
10
(n)

1
10
(n)

\Rightarrow **$O(n)$**



Binary Search: Finding a Number in Sorted List

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We are thinking of
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Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{ccc} 1 & 1 & \\ 10 & 10 & \\ \textcircled{n} & \textcircled{n} & \Rightarrow O(n) \end{array}$$

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 $\text{mid} = (\text{first} + \text{last}) // 2$

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Binary Search: Comparisons ? Input Size



Binary Search: Finding a Number in Sorted List

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Binary Search: Comparisons **?** Input Size
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Binary Search: Comparisons ? Input Size
4 **16**

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Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array}$$



Binary Search: Finding a Number in Sorted List

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Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. $\text{mid} = (0 + 15) // 2 = 7$

2. $\text{mid} = (0 + 6) // 2 = 3$

3. $\text{mid} = (0 + 2) // 2 = 1$

4. $\text{mid} = (2 + 2) // 2 = 2$

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. $\text{mid} = (0 + 15) // 2 = 7$

2. $\text{mid} = (0 + 6) // 2 = 3$

3. $\text{mid} = (0 + 2) // 2 = 1$

4. $\text{mid} = (2 + 2) // 2 = 2$

What is n?

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Calculate middle index
mid = (first + last) // 2

1. mid = (0 + 15) // 2 = 7

2. mid = (0 + 6) // 2 = 3

3. mid = (0 + 2) // 2 = 1

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What is n? \rightarrow **16**

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

1
10
n

1
10
n

$\Rightarrow O(n)$

Calculate middle index
mid = (first + last) // 2

1. **mid = (0 + 15) // 2 = 7**

2. **mid = (0 + 6) // 2 = 3**

3. **mid = (0 + 2) // 2 = 1**

4. **mid = (2 + 2) // 2 = 2**

What is n? \rightarrow **16**
Comparisons?

Binary Search: Comparisons ? Input Size

4
?

16
n

$\Rightarrow O(\lg n)$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

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What is n? \rightarrow **16**
Comparisons? \rightarrow **4**

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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What is n? $\rightarrow 16$
Comparisons? $\rightarrow 4$
What is $\lg n \rightarrow \lg 16$?

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

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Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index

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What is n ? $\rightarrow 16$

Comparisons? $\rightarrow 4$

What is $\lg n \rightarrow \lg 16$?

Must know base?



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

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Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

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3. $\text{mid} = (0 + 2) // 2 = 1$

4. $\text{mid} = (2 + 2) // 2 = 2$

What is n? $\rightarrow 16$

Comparisons? $\rightarrow 4$

What is $\lg n \rightarrow \lg 16$?

Must know base? $\rightarrow 2$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
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Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index
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What is n? \rightarrow **16**

Comparisons? \rightarrow **4**

What is $\lg n \rightarrow$ **$\lg 16$** ?

Must know base? \rightarrow **2**

What is $\lg n \rightarrow$ **$\lg_2 16$** ?



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index
mid = (first + last) // 2

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What is n? \rightarrow **16**

Comparisons? \rightarrow **4**

What is $\lg n \rightarrow \lg 16$?

Must know base? \rightarrow **2**

What is $\lg n \rightarrow \lg_2 16$? \Rightarrow

What **power of base** (2) will give **16**?



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index
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1. mid = (0 + 15) // 2 = 7

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What is n ? $\rightarrow 16$
Comparisons? $\rightarrow 4$
What is $\lg n \rightarrow \lg 16$?
Must know base? $\rightarrow 2$
What is $\lg n \rightarrow \lg_2 16$? \Rightarrow
What **power of base** (2)
will give **16**? $\Rightarrow 4$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{c} 1 \\ 10 \\ \textcircled{n} \end{array} \quad \begin{array}{c} 1 \\ 10 \\ \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

$$\begin{array}{c} 4 \\ \textcircled{?} \end{array} \quad \begin{array}{c} 16 \\ \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index
mid = (first + last) // 2

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What is n? $\rightarrow 16$
Comparisons? $\rightarrow 4$
What is $\lg n \rightarrow \lg 16$?
Must know base? $\rightarrow 2$
What is $\lg n \rightarrow \lg_2 16$? \Rightarrow
What **power of base** (2)
will give **16**? $\Rightarrow 4$, i.e. **$2^4 = 16$**



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index
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3. mid = (0 + 2) // 2 = 1

4. mid = (2 + 2) // 2 = 2

What is n ? $\rightarrow 16$
Comparisons? $\rightarrow 4$
What is $\lg n \rightarrow \lg 16$?
Must know base? $\rightarrow 2$
What is $\lg n \rightarrow \lg_2 16$? \Rightarrow
What **power of base** (2)
will give **16**? $\Rightarrow 4$, i.e. $2^4 = 16$

$\therefore \lg_2 n = \lg_2 16 = 4$



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index
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4. mid = (2 + 2) // 2 = 2

What is n? $\rightarrow 16$

Comparisons? $\rightarrow 4$

What is $\lg n \rightarrow \lg 16$?

Must know base? $\rightarrow 2$

What is $\lg n \rightarrow \lg_2 16$? \Rightarrow

What **power of base** (2)
will give **16**? $\Rightarrow 4$, i.e. **$2^4 = 16$**

$\therefore \lg_2 n = \lg_2 16 = 4$

log is also written as lg



Binary Search: Finding a Number in Sorted List

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
List	3	7	12	18	23	27	30	35	40	45	50	54	60	65	70	72

We are thinking of
12

Input Size: **16**

Total Comparisons: **4**

Linear Search: Comparisons \propto Input Size

$\lg n$ vs $\lg n + 1$

$$\begin{array}{cc} 1 & 1 \\ 10 & 10 \\ \textcircled{n} & \textcircled{n} \end{array} \Rightarrow O(n)$$

Binary Search: Comparisons ? Input Size

\log is also written as \lg

$$\begin{array}{cc} 4 & 16 \\ \textcircled{?} & \textcircled{n} \end{array} \Rightarrow O(\lg n)$$

Calculate middle index
 $\text{mid} = (\text{first} + \text{last}) // 2$

1. $\text{mid} = (0 + 15) // 2 = 7$

2. $\text{mid} = (0 + 6) // 2 = 3$

3. $\text{mid} = (0 + 2) // 2 = 1$

4. $\text{mid} = (2 + 2) // 2 = 2$

What is n ? $\rightarrow 16$

Comparisons? $\rightarrow 4$

What is $\lg n \rightarrow \lg 16$?

Must know base? $\rightarrow 2$

What is $\lg n \rightarrow \lg_2 16$? \Rightarrow

What **power of base** (2)
will give **16**? $\Rightarrow 4$, i.e. **$2^4 = 16$**

$\therefore \lg_2 n = \lg_2 16 = 4$



Linear vs. Binary Search:

Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 =$
32	32	$\lg_2 32 =$
64	64	$\lg_2 64 =$
...
1024	1024	$\lg_2 1024 =$
2048	2048	$\lg_2 2048 =$
...
1048576	1048576	$\lg_2 1048576 =$



Linear vs. Binary Search:

Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 =$
32	32	$\lg_2 32 =$
64	64	$\lg_2 64 =$
...
1024	1024	$\lg_2 1024 =$
2048	2048	$\lg_2 2048 =$
...
1048576	1048576	$\lg_2 1048576 =$



Linear vs. Binary Search:

Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 =$
2048	2048	$\lg_2 2048 =$
...
1048576	1048576	$\lg_2 1048576 =$



Linear vs. Binary Search:

Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 = 10$
2048	2048	$\lg_2 2048 = 11$
...
1048576	1048576	$\lg_2 1048576 =$



Linear vs. Binary Search:

Comparing Steps with Input Growth

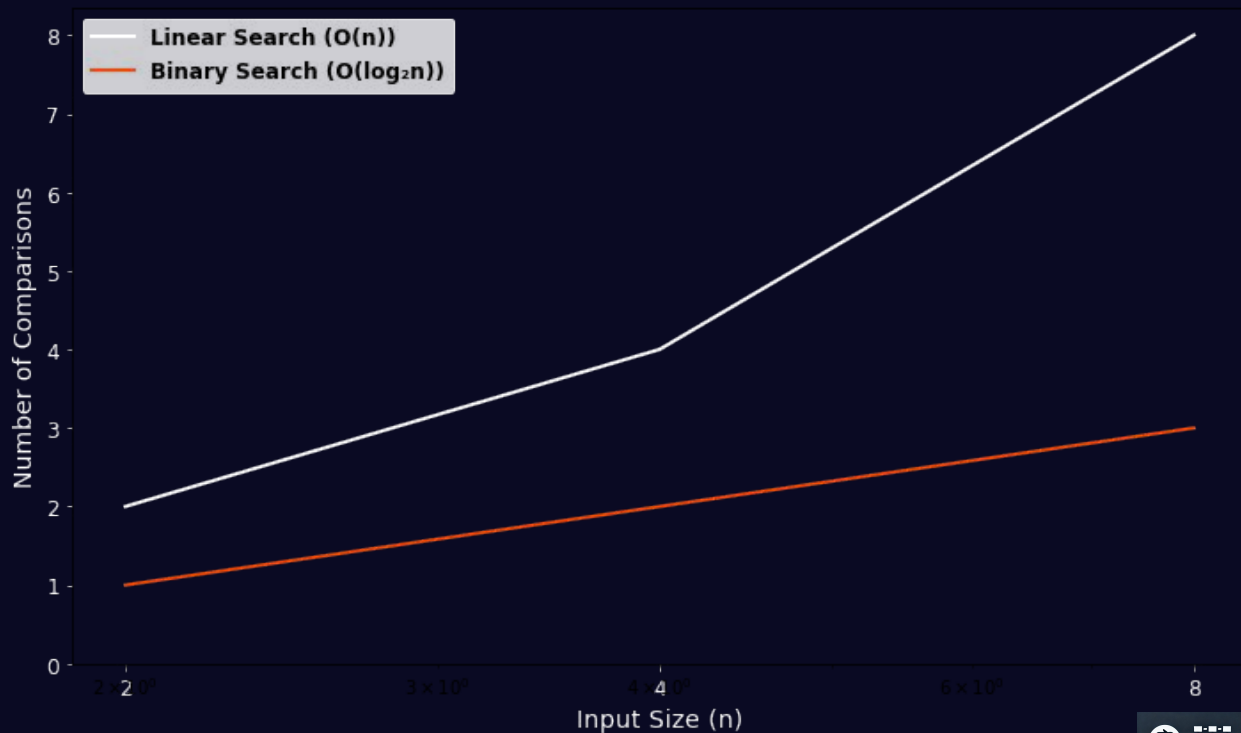
Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 = 10$
2048	2048	$\lg_2 2048 = 11$
...
1048576	1048576	$\lg_2 1048576 = 20$



Linear vs. Binary Search:

Comparing Steps with Input Growth

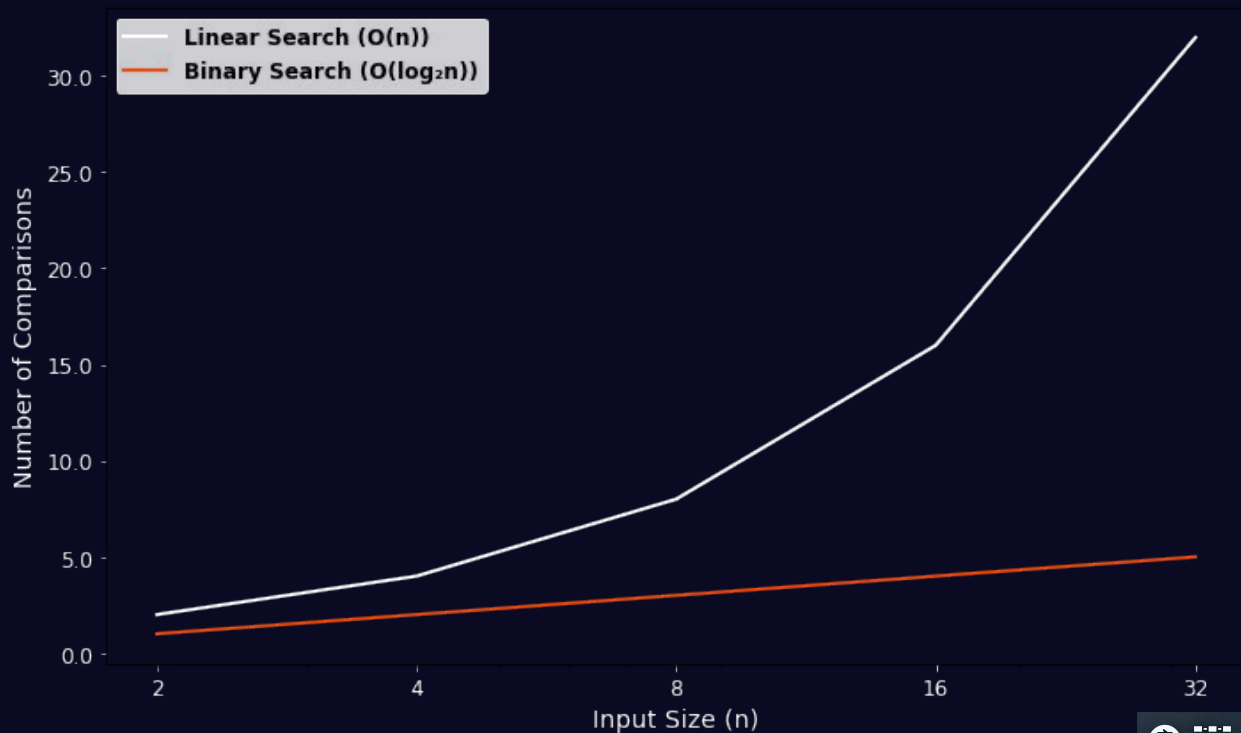
Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 = 10$
2048	2048	$\lg_2 2048 = 11$
...
1048576	1048576	$\lg_2 1048576 = 20$



Linear vs. Binary Search:

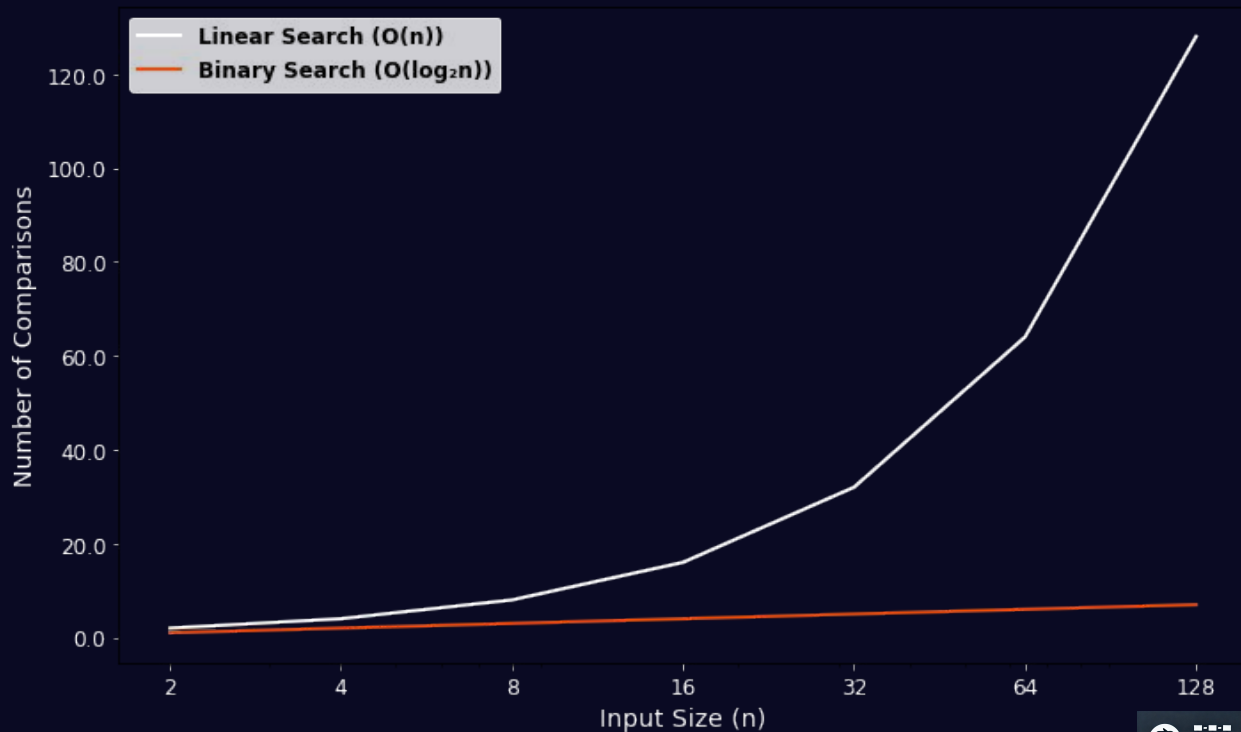
Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 = 10$
2048	2048	$\lg_2 2048 = 11$
...
1048576	1048576	$\lg_2 1048576 = 20$



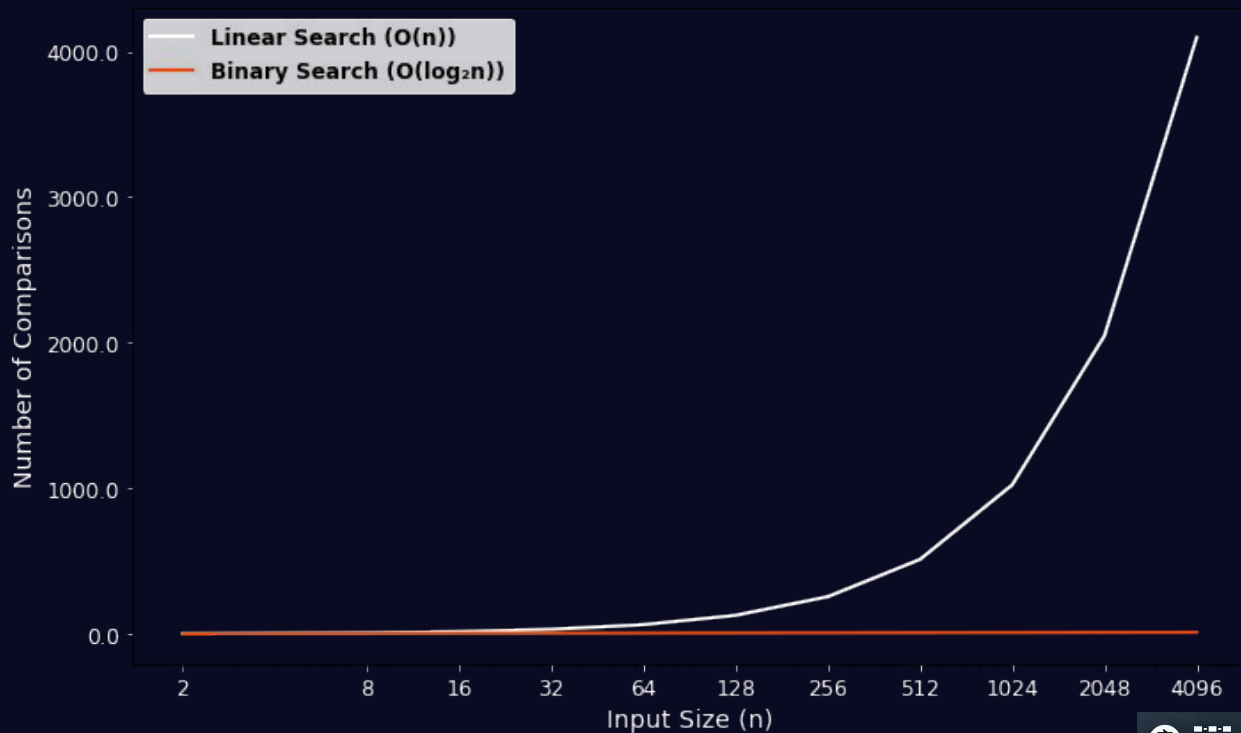
Linear vs. Binary Search: Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 = 10$
2048	2048	$\lg_2 2048 = 11$
...
1048576	1048576	$\lg_2 1048576 = 20$



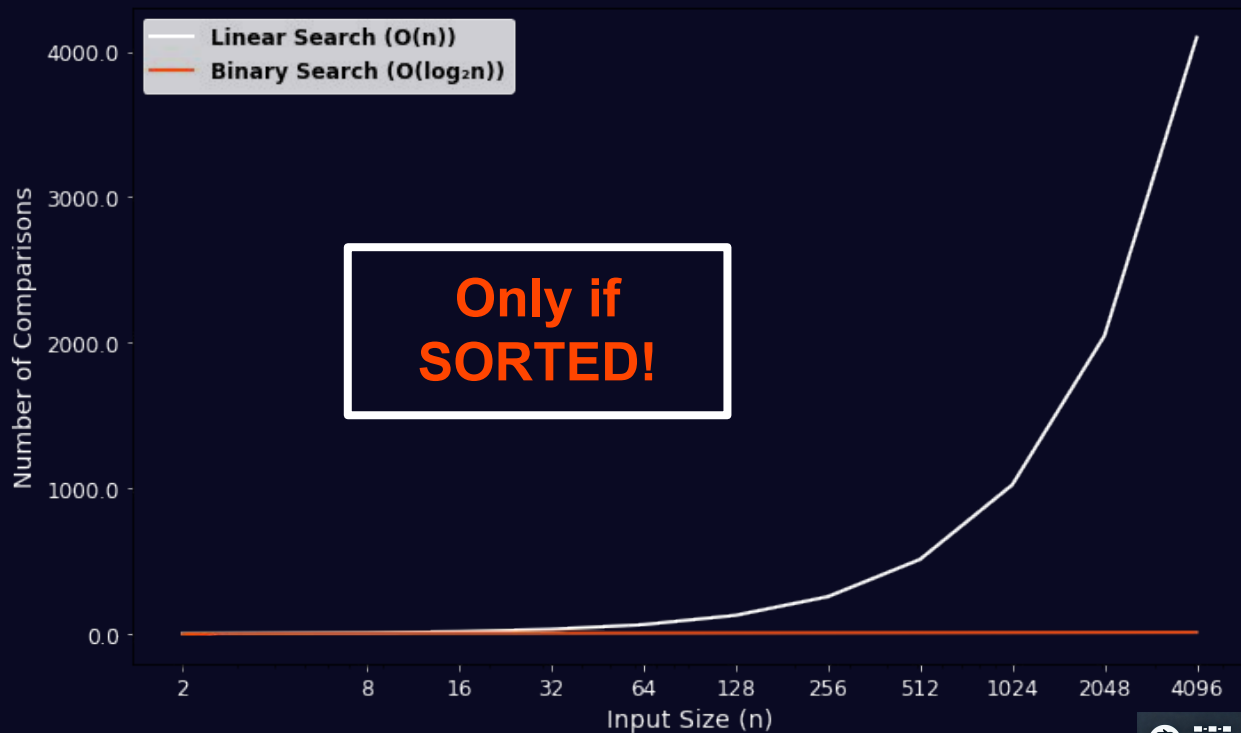
Linear vs. Binary Search: Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 = 10$
2048	2048	$\lg_2 2048 = 11$
...
1048576	1048576	$\lg_2 1048576 = 20$



Linear vs. Binary Search: Comparing Steps with Input Growth

Input Size (n)	Linear Search	Binary Search
8	8	$\lg_2 8 = 3$
16	16	$\lg_2 16 = 4$
32	32	$\lg_2 32 = 5$
64	64	$\lg_2 64 = 6$
...
1024	1024	$\lg_2 1024 = 10$
2048	2048	$\lg_2 2048 = 11$
...
1048576	1048576	$\lg_2 1048576 = 20$



Binary Search Algorithm

Algorithm BinarySearch



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Index	0	1	2	3	4	5	6	7
List	3	7	12	18	23	27	30	35

Target to Search: **13**



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

Index	0	1	2	3	4	5	6	7
List	3	7	12	18	23	27	30	35

Target to Search: **13**



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

Index	0	1	2	3	4	5	6	7
List	3	7	12	18	23	27	30	35

Target to Search: **13**

Expected Output: **-1**



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$

	low							high
	↓							↓
Index	0	1	2	3	4	5	6	7
List	3	7	12	18	23	27	30	35

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**



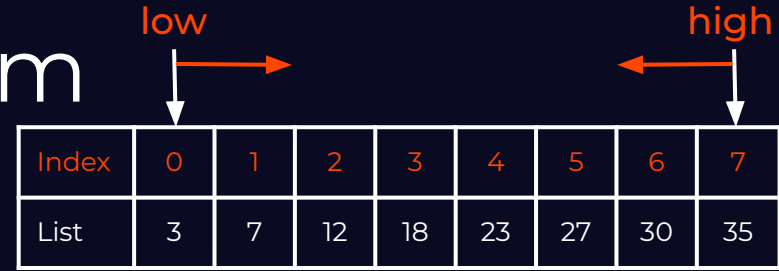
Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:



The diagram shows a sorted list with 8 elements. Above the list, a 'low' pointer is at index 0 and a 'high' pointer is at index 7. Arrows indicate the search range from index 0 to index 7.

Index	0	1	2	3	4	5	6	7
List	3	7	12	18	23	27	30	35

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:
 - a. Set $mid \leftarrow (low + high) // 2$

	low								high	
	→		←							
Index	0	1	2	3	4	5	6	7		
List	3	7	12	18	23	27	30	35		

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:
 - a. Set $mid \leftarrow (low + high) // 2$



The diagram shows a sorted list with 8 elements. Above the list, three orange arrows indicate the current search range: 'low' points to index 0, 'mid' points to index 3, and 'high' points to index 7. The element at index 3 (18) is highlighted with an orange border.

Index	0	1	2	3	4	5	6	7
List	3	7	12	18	23	27	30	35

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. $mid = (0 + 7) // 2 = 3$



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:
 - a. Set $mid \leftarrow (low + high) // 2$

	low			mid				high
	↓	→		↓			←	↓
Index	0	1	2	3	4	5	6	7
List	3	7	12	18	23	27	30	35

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Index	0	1	2	3	4	5	6	7
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1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$

2. While $low \leq high$ do:

a. Set $mid \leftarrow (low + high) // 2$

b. If $A[mid] = \text{target}$ then
return mid

Is 18 == 13?

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

	low			mid				high
	↓	→		↓			←	↓
Index	0	1	2	3	4	5	6	7
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c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

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Length of List: **8**

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Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

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	↓	→		↓			←	↓
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2. While $low \leq high$ do:

a. Set $mid \leftarrow (low + high) // 2$

b. If $A[mid] = \text{target}$ then

return mid

c. Else If $A[mid] > \text{target}$ then

$high \leftarrow mid - 1$

Is 18 > 13?, YES

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. **mid** = $(0 + 7) // 2 = \mathbf{3}$



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

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	↓	↓	
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List	3	7	12

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c. Else If $A[mid] > \text{target}$ then

$high \leftarrow mid - 1$

Is 18 > 13?, YES

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

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Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

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Binary Search Algorithm

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Output: **Index** of **target** in A, or **-1** if target is not found

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 $high \leftarrow mid - 1$

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Length of List: **8**

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Binary Search Algorithm

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Output: **Index** of **target** in A, or **-1** if target is not found

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a. Set $mid \leftarrow (low + high) // 2$

b. If $A[mid] = \text{target}$ then
return mid

c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

Is 7 == 13?

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$

2. mid = $(0 + 2) // 2 = \mathbf{1}$



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

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	↓	↓	↓	
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a. Set $mid \leftarrow (low + high) // 2$

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return mid

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 $high \leftarrow mid - 1$

Is 7 > 13?

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$

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Binary Search Algorithm

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c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. $mid = (0 + 7) // 2 = 3$

2. $mid = (0 + 2) // 2 = 1$

Is $7 < 13$?, **YES**



Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found

	low	mid	high	
	↓	↓	↓	
Index	0	1	2	
List	3	7	12	

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$

2. While $low \leq high$ do:

a. Set $mid \leftarrow (low + high) // 2$

b. If $A[mid] = \text{target}$ then
return mid

c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

d. Else
 $low \leftarrow mid + 1$

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$

2. mid = $(0 + 2) // 2 = \mathbf{1}$

Is $7 < 13$?, **YES**

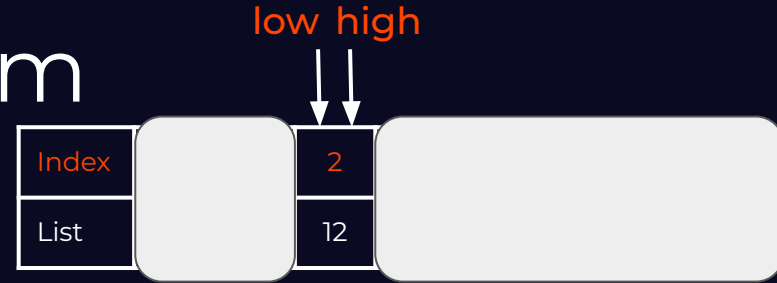


Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

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1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$

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b. If $A[mid] = \text{target}$ then
return mid

c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

d. Else
 $low \leftarrow mid + 1$

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. $mid = (0 + 7) // 2 = 3$

2. $mid = (0 + 2) // 2 = 1$

Is $7 < 13$?, **YES**

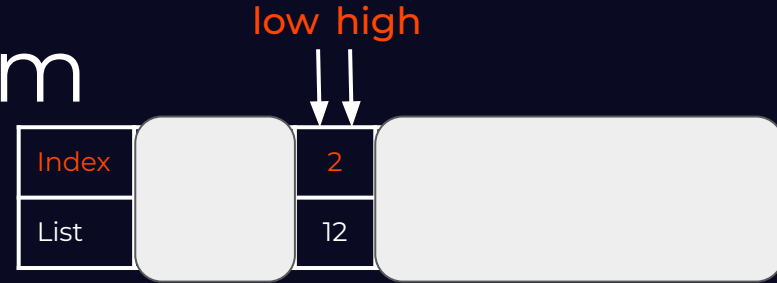


Binary Search Algorithm

Algorithm BinarySearch

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Output: **Index** of **target** in A, or **-1** if target is not found



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b. If $A[mid] = \text{target}$ then
return mid

c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

d. Else
 $low \leftarrow mid + 1$

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. $mid = (0 + 7) // 2 = 3$

2. $mid = (0 + 2) // 2 = 1$

3. $mid = (2 + 2) // 2 = 2$

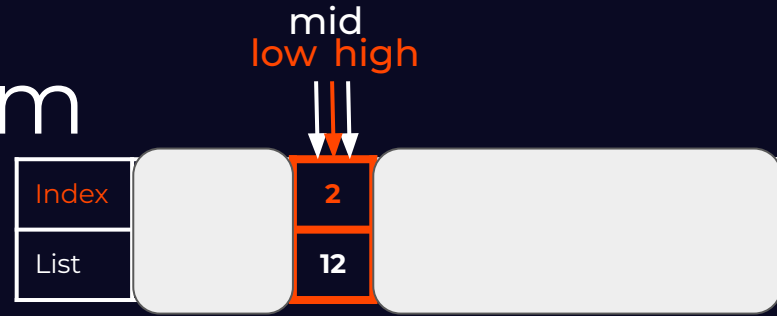


Binary Search Algorithm

Algorithm BinarySearch

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1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:
 - a. Set $mid \leftarrow (low + high) // 2$
 - b. If $A[mid] = \text{target}$ then
 $\text{return } mid$
 - c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$

Target to Search: 13

Expected Output: **-1**

Length of List: 8

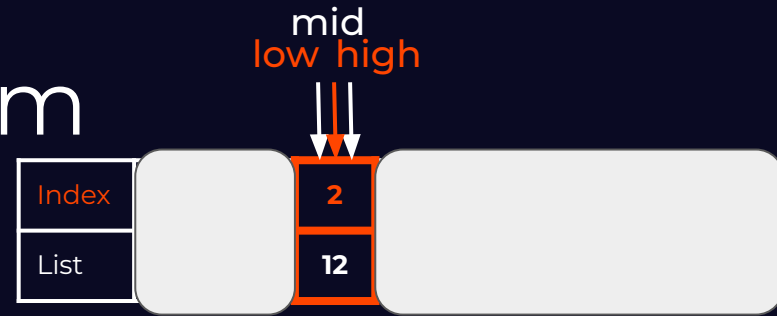
1. $\text{mid} = (0 + 7) // 2 = 3$
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Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

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 - b. If $A[mid] = \text{target}$ then
 $\text{return } mid$
 - c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$

Is $12 == 13$?

Target to Search: 13

Expected Output: **-1**

Length of List: 8

1. $\text{mid} = (0 + 7) // 2 = 3$
2. $\text{mid} = (0 + 2) // 2 = 1$
3. $\text{mid} = (2 + 2) // 2 = 2$

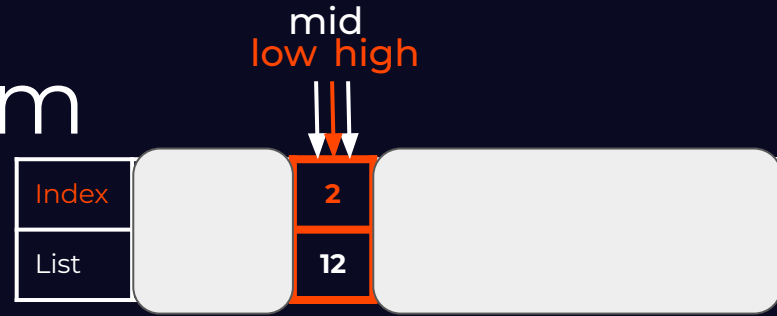


Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

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1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
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 - a. Set $mid \leftarrow (low + high) // 2$
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 - d. Else
 $low \leftarrow mid + 1$

Is $12 > 13$?

Target to Search: 13

Expected Output: **-1**

Length of List: 8

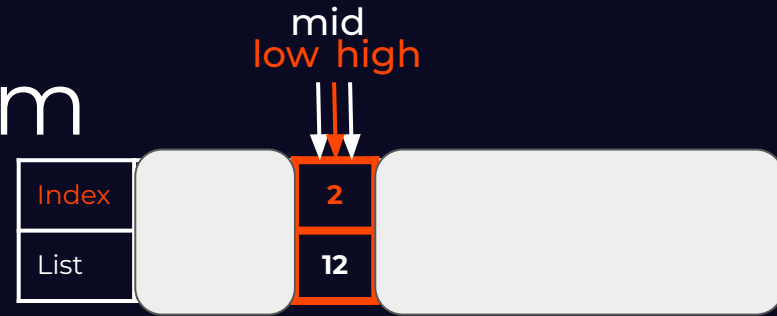
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Algorithm BinarySearch

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 $low \leftarrow mid + 1$

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. $mid = (0 + 7) // 2 = 3$

2. $mid = (0 + 2) // 2 = 1$

3. $mid = (2 + 2) // 2 = 2$

Is $12 < 13$?, **YES**



Binary Search Algorithm

Algorithm BinarySearch

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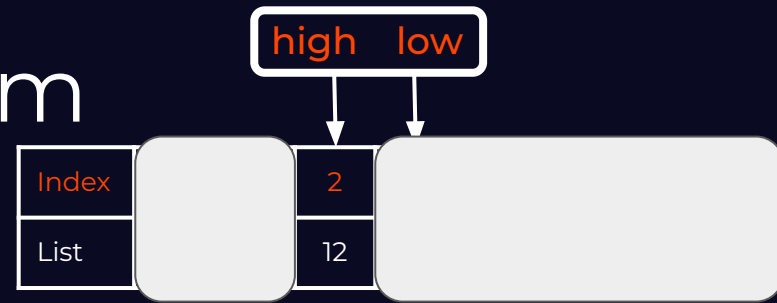
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d. Else
 $low \leftarrow mid + 1$

Is 12 < 13?, **YES**



Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$

2. mid = $(0 + 2) // 2 = \mathbf{1}$

3. mid = $(2 + 2) // 2 = \mathbf{2}$

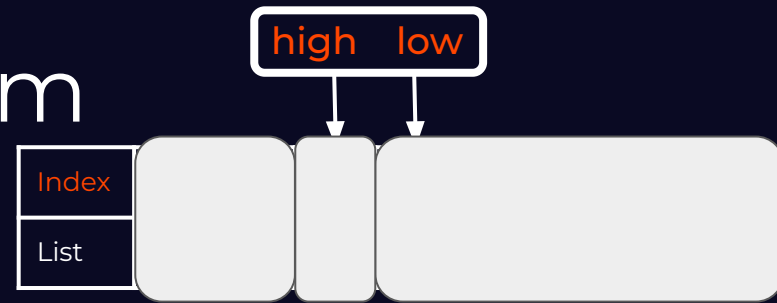


Binary Search Algorithm

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target** in A, or **-1** if target is not found



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return mid

c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

d. Else
 $low \leftarrow mid + 1$

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$

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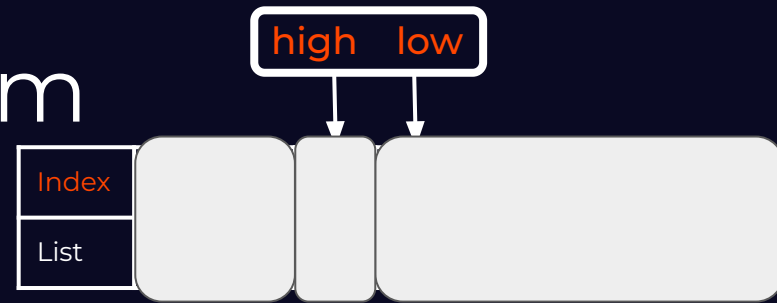


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return mid

c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$

d. Else
 $low \leftarrow mid + 1$

3. **Return -1** // Target not found

Target to Search: **13**

Expected Output: **-1**

Length of List: **8**

1. mid = $(0 + 7) // 2 = \mathbf{3}$

2. mid = $(0 + 2) // 2 = \mathbf{1}$

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Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: Index of **target**, or **-1** if not found

```
def binary_search(A, target):
```

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:
 - a. Set $mid \leftarrow (low + high) // 2$
 - b. If $A[mid] = \text{target}$ then
 $\text{return } mid$
 - c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found



Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

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 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found

```
def binary_search(A, target):
```

```
    low = 0
```

```
    high = len(A) - 1
```



Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: Index of **target**, or **-1** if not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
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 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found

```
def binary_search(A, target):
```

```
    low = 0
```

```
    high = len(A) - 1
```

```
    while low <= high:
```



Binary Search in Python

Algorithm BinarySearch

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Output: Index of **target**, or **-1** if not found

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 - c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found

```
def binary_search(A, target):
```

```
    low = 0
```

```
    high = len(A) - 1
```

```
    while low <= high:
```

```
        mid = (low + high) // 2
```



Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: Index of **target**, or **-1** if not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
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 - c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found

```
def binary_search(A, target):
```

```
    low = 0
    high = len(A) - 1
    while low <= high:
        mid = (low + high) // 2
        if A[mid] == target:
            return mid
```



Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: Index of **target**, or **-1** if not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:
 - a. Set $mid \leftarrow (low + high) // 2$
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 - c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found

```
def binary_search(A, target):
```

```
    low = 0
    high = len(A) - 1
    while low <= high:
        mid = (low + high) // 2
        if A[mid] == target:
            return mid
        elif A[mid] > target:
            high = mid - 1
```



Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: Index of **target**, or **-1** if not found

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 $low \leftarrow mid + 1$
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def binary_search(A, target):
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    while low <= high:
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            return mid
        elif A[mid] > target:
            high = mid - 1
        else:
            low = mid + 1
```



Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: Index of **target**, or **-1** if not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
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 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found

```
def binary_search(A, target):
```

```
    low = 0
    high = len(A) - 1
    while low <= high:
        mid = (low + high) // 2
        if A[mid] == target:
            return mid
        elif A[mid] > target:
            high = mid - 1
        else:
            low = mid + 1
    return -1
```



Binary Search in Python

Algorithm BinarySearch

Input: A sorted list **A**, a target value

Output: **Index** of **target**, or **-1** if not found

1. Initialize $low \leftarrow 0$, $high \leftarrow \text{length}(A) - 1$
2. While $low \leq high$ do:
 - a. Set $mid \leftarrow (low + high) // 2$
 - b. If $A[mid] = \text{target}$ then
 return mid
 - c. Else If $A[mid] > \text{target}$ then
 $high \leftarrow mid - 1$
 - d. Else
 $low \leftarrow mid + 1$
3. Return **-1** // Target not found

```
def binary_search(A, target):
```

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```

A = [3, 7, 12, 18, 23, 27, 30, 35]

target = 12



Binary Search in Python

Algorithm BinarySearch

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        else:
            low = mid + 1
    return -1
```

A = [3, 7, 12, 18, 23, 27, 30, 35]

target = 12

result = binary_search(A, target)



Binary Search in Python

Algorithm BinarySearch

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        else:
            low = mid + 1
    return -1
```

```
A = [3, 7, 12, 18, 23, 27, 30, 35]
```

```
target = 12
```

```
result = binary_search(A, target)
```

```
if result != -1:
```

```
    print(f'Target found at index {result}')
```



Binary Search in Python

Algorithm BinarySearch

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A = [3, 7, 12, 18, 23, 27, 30, 35]
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if result != -1:
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```
else:
```

```
    print('Target not found')
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Recursion: An Alternative to Iteration



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of
all positive integers from **1 to n**.



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

Mathematically: $n! = n \times (n - 1) \times (n - 2) \times \dots \times 1$



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Example: $4! = 4 \times 3 \times 2 \times 1 = 24$



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Example: $4! = 4 \times 3 \times 2 \times 1 = 24$

```
def iterative_factorial(n):
```



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Example: $4! = 4 \times 3 \times 2 \times 1 = 24$

```
def iterative_factorial(n):  
    result = 1
```



Recursion: An Alternative to Iteration

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Example: $4! = 4 \times 3 \times 2 \times 1 = 24$

```
def iterative_factorial(n):  
    result = 1  
    for i in range(1, n + 1):
```



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

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Example: $4! = 4 \times 3 \times 2 \times 1 = 24$

```
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```



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    for i in range(n, 1, -1):  
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Alternative:

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    return result
```

Alternative:

Factorial(**n**)

n * Factorial(**n-1**)



Recursion: An Alternative to Iteration

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Alternative:

Factorial(**n**)

$$n * \text{Factorial}(n-1)$$

(n-1) * Factorial(n-2)

...

(n-n-2) * Factorial(1)



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Alternative:

```
Factorial(n)  
    n * Factorial(n-1)  
    (n-1) * Factorial(n-2)  
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    (n-n-2) * Factorial(1)  
    return 1
```



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Factorial(n)  
    n * Factorial(n-1)  
    (n-1) * Factorial(n-2)  
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    (n-n-2) * Factorial(1)  
    return 1
```

Alternative:

```
Factorial(4)
```



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Alternative:

```
Factorial(n)  
    n * Factorial(n-1)  
    (n-1) * Factorial(n-2)  
    ...  
    (n-n-2) * Factorial(1)  
    return 1
```

Alternative:

```
Factorial(4)  
    4 * Factorial(3)
```



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    return result
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Alternative:

```
Factorial(n)  
    n * Factorial(n-1)  
    (n-1) * Factorial(n-2)  
    ...  
    (n-n-2) * Factorial(1)  
    return 1
```

Alternative:

```
Factorial(4)  
    4 * Factorial(3)  
    3 * Factorial(2)
```



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Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

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    (n-n-2) * Factorial(1)  
    return 1
```

Alternative:

```
Factorial(4)  
    4 * Factorial(3)  
    3 * Factorial(2)  
    2 * Factorial(1)
```



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```

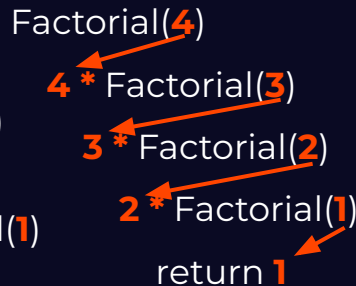
```
def iterative_factorial(n):  
    result = 1  
    for i in range(n, 1, -1):  
        result = result * i  
    return result
```

Alternative:

```
Factorial(n)  
    n * Factorial(n-1)  
    (n-1) * Factorial(n-2)  
    ...  
    (n-n-2) * Factorial(1)  
    return 1
```

Alternative:

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Factorial(4)  
    4 * Factorial(3)  
    3 * Factorial(2)  
    2 * Factorial(1)  
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```



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    return result
```

```
def iterative_factorial(n):
    result = 1
    for i in range(n, 1, -1):
        result = result * i
    return result
```

Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

(n-1) * Factorial(n-2)

• • •

(n-n-2) * Factorial(1)

```
return 1
```

Alternative:

Factorial(4)

4 * Factorial(3)

3 * Factorial(2)

2 * Factorial(1)

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return 1
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Recursive Case

Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

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Example: $4! = 4 \times 3 \times 2 \times 1 = 24$

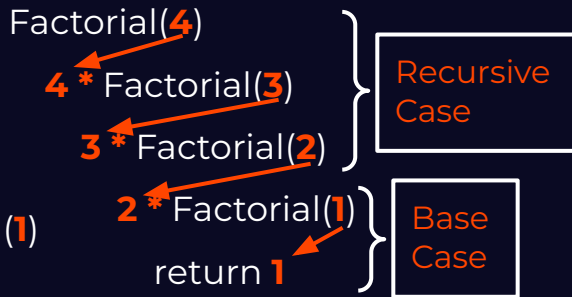
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def iterative_factorial(n):
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    return result
```

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def iterative_factorial(n):  
    result = 1  
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        result = result * i  
    return result
```

Alternative:

```
Factorial(n)
    n * Factorial(n-1)
    (n-1) * Factorial(n-2)
    ...
    (n-n-2) * Factorial(1)
    return 1
```

Alternative:



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

Mathematically: $n! = n \times (n - 1) \times (n - 2) \times \dots \times 1$

Example: $4! = 4 \times 3 \times 2 \times 1 = 24$

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    return result
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def iterative_factorial(n):  
    result = 1  
    for i in range(n, 1, -1):  
        result = result * i  
    return result
```

Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

(**n-1**) * Factorial(**n-2**)

...

(**n-n-2**) * Factorial(**1**)

return **1**

Alternative:

Factorial(**4**)

4 * Factorial(**3**)

3 * Factorial(**2**)

2 * Factorial(**1**)

return **1**



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

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Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

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...

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Alternative:

Factorial(**4**)

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3 * Factorial(**2**)

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    return result
```

Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

(**n-1**) * Factorial(**n-2**)

...

(**n-n-2**) * Factorial(**1**)

return **1**

Alternative:

Factorial(**4**)

4 * Factorial(**3**)

3 * Factorial(**2**)

2 * → **1 = 2**

~~return 1~~



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

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        result = result * i  
    return result
```

Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

(**n-1**) * Factorial(**n-2**)

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return **1**

Alternative:

Factorial(**4**)

4 * Factorial(**3**)

3 * Factorial(**2**)

2 * → **1 = 2**

~~return 1~~



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Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

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Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

(**n-1**) * Factorial(**n-2**)

...

(**n-n-2**) * Factorial(**1**)

return **1**

Alternative:

Factorial(**4**)

4 * Factorial(**3**)

3 *  → **2 = 6**

2 * Factorial(**1**) → **1 = 2**

~~return **1**~~



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

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Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

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return **1**

Alternative:

Factorial(**4**)

4 * Factorial(**3**)

3 *  → **2 = 6**

2 * Factorial(**1**) → **1 = 2**

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Alternative:

Factorial(**n**)

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(**n-1**) * Factorial(**n-2**)

...

(**n-n-2**) * Factorial(**1**)

return **1**

Alternative:

Factorial(**4**)

4 *  → **6 = 24**

3 * ~~Factorial(**2**)~~ → **2 = 6**

2 * ~~Factorial(**1**)~~ → **1 = 2**

~~return **1**~~



Recursion: An Alternative to Iteration

Factorial of a number **n** is the **product** of all positive integers from **1 to n**.

Mathematically: $n! = n \times (n - 1) \times (n - 2) \times \dots \times 1$

Example: $4! = 4 \times 3 \times 2 \times 1 = 24$

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def iterative_factorial(n):  
    result = 1  
    for i in range(1, n + 1):  
        result = result * i  
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```

Alternative:

Factorial(**n**)

n * Factorial(**n-1**)

(**n-1**) * Factorial(**n-2**)

...

(**n-n-2**) * Factorial(**1**)

return **1**

Alternative:

Factorial(**4**)

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Alternative:

4 * Factorial(**3**) → **24**

4 * Factorial(**3**) → **6** = **24**

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2 * Factorial(**1**) → **1** = **2**

return **1**



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Recursive
Case

Base
Case

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4 * Factorial(**3**)

3 * Factorial(**2**)

2 * Factorial(**1**)

return **1**

Recursive
Case

Base
Case

```
def recursive_factorial(n):
```

```
    if n == 1:
```

```
        return 1
```



Recursion: An Alternative to Iteration

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    ...  
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Alternative:

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Factorial(4)  
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Recursive Case

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def recursive_factorial(n):  
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Base Case

Recursive Case



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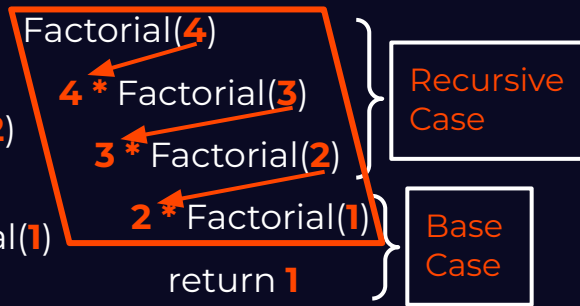
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    (n-1) * Factorial(n-2)  
    ...  
    (n-n-2) * Factorial(1)  
    return 1
```

Alternative:



```
def recursive_factorial(n):
```

```
    if n == 1: } Base Case  
        return 1  
    else:  
        return n * recursive_factorial(n - 1)  
} Recursive Case
```



Recursion: An Alternative to Iteration

- **Iteration:**
 - Most common approach to problem-solving.



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- Usually solves the problem by **redefining the original problem with smaller input size** (recursive case) until **base case** is met.
- Typically requires fewer lines of code and can make complex problems **easier to express**.
- Function calls and can lead to **stack overflow** if the **depth** of recursion is **too large**.



Converting Iterative Binary Search to Recursive

```
def binary_search(A, target):
```

```
    low = 0
```

```
    high = len(A) - 1
```

```
    while low <= high:
```

```
        mid = (low + high) // 2
```

```
        if A[mid] == target:
```

```
            return mid
```

```
        elif A[mid] > target:
```

```
            high = mid - 1
```

```
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            low = mid + 1
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    return -1
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```
A = [3, 7, 12, 18, 23, 27, 30, 35]
```

```
target = 12
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```
result = binary_search(A, target)
```

```
if result != -1:
```

```
    print(f"Target found at index {result}")
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```
def recursive_binary_search(A, target, low, high):
```

```
    if low > high: } Base Case 1
        return -1
```

```
    mid = (low + high) // 2
```

```
    if A[mid] == target: } Base Case 2
        return mid
```

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    while low <= high:  
        mid = (low + high) // 2
```

```
        if A[mid] == target:  
            return mid
```

```
        elif A[mid] > target:  
            high = mid - 1
```

```
        else:  
            low = mid + 1
```

```
    return -1
```

```
def recursive_binary_search(A, target, low, high):
```

```
    if low > high:  
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```

```
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    if A[mid] == target:  
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```
A = [3, 7, 12, 18, 23, 27, 30, 35]
```

```
target = 12
```

```
result = binary_search(A, target, 0, len(A)-1)
```

```
if result != -1:
```

```
    print(f'Target found at index {result}')
```

```
else:
```

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    print('Target not found')
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Converting Iterative Binary Search to Recursive

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def binary_search(A, target, low, high):
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```

#A[mid] < target:
(mistake in video)

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```
def recursive_binary_search(A, target, low, high):
```

```
    if low > high: } Base Case 1
        return -1
    mid = (low + high) // 2
    if A[mid] == target: } Base Case 2
        return mid
    elif A[mid] > target:
        return recursive_binary_search(A, target, __, __)
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```
def recursive_binary_search(A, target, low, high):
```

```
    if low > high:
```

Base Case 1

```
        return -1
```

```
    mid = (low + high) // 2
```

```
    if A[mid] == target:
```

Base Case 2

```
        return mid
```

```
    elif A[mid] > target:
```

```
        return recursive_binary_search(A, target, __, __)
```

```
    else:
```

```
        return recursive_binary_search(A, target, __, __)
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Recursive Cases

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Topic: **Sorting Algorithms**



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Focus: Exploring foundational sorting techniques and their importance in efficient data organization





Thank You for Watching!