# CS425: Game Programming 1 Lecture 09: Particle System

October 3, 2018

Lecturer: Jyh-Ming Lien

## 1 Introduction

### 1.1 Data Structure

State of a particle  $s = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix}$ , where x and position and v is the velocity.

**State of a particle system** Given n particles in a particle system, we can represent the system as

$$S = \begin{bmatrix} s_1 \\ s_2 \\ \dots \\ s_n \end{bmatrix}$$

# 1.2 Physics Engine

Physics engine is usually used in game to simulate the motion of objects, such as particles. The following simple procedule is the core of a physics engine (using a particle system as an example).

- 1. Get states of particles (positions and velocities)
- 2. Get forces F applied to each particles
- 3. Compute derivatives from the forces F
- 4. Using ODE solvers to compute the new positions and velocities
- 5. Set the states back to the particles

#### 2 Forces

There are many different type of forces, such as gravity and friction. In general, when multiple forces are applied to a single particle, these forces are combined linearly (with weights) into a single force F (as the F in step 3 above). There are typically three types of forces:

- 1. Unary Forces:
  - Gravity
  - · Viscous drag

## 2. N-ary Forces:

- mass-spring system (usually used for modeling deformable objects)
- flocking system (coherence, alignment, separation)
- fluid dynamics (Navier-stroke equations)
- 3. Spatial related forces (varies based on the locations):
  - collision
  - wind
  - contact
  - user interaction

# 2.1 Viscous drag

This is similar to damping in spring system (See below). This is highly recommended for stability in simulating particles.

$$F = -k_d v$$
,

where  $k_d$  is a user defined constant and v is the linear velocity.

# 2.2 Hook's law spring

$$F = -\left(k_s(|\ell| - r) + k_d \frac{v\ell}{|\ell|}\right) \frac{\ell}{|\ell|},$$

where  $k_s$  and  $k_d$  are spring and damping constants,  $\ell$  is the vector pointing at the particle and parallel to the spring (i.e., a vector between two connected particles), r is the resting length of the spring, and, finally, v is the linear velocity of the particle.

# 2.3 Flocking

Flocking system is used to simulate the motion of a group of coherent entities, like a school of fishes and a flock of birds. The motion is governed by three simple local rules, whose forces are linearly combined to create the flocking force. For a given particle, its flocking force is defined as:

$$F = k_{co}F_{coherent} + k_{se}F_{separation} + k_{al}F_{alignment},$$

where  $k_{co}$ ,  $k_{se}$ ,  $k_{al}$  are user defined constants that can be used to influence the behavior of the flock. Each of these threes forces are defined as the following.

**Coherence**  $F_{coherent} = x - O_{nei}$ 

**Separation**  $F_{separation} = O_{nei} - x$ 

**Alignment**  $F_{alignment} = V_{nei}$ 

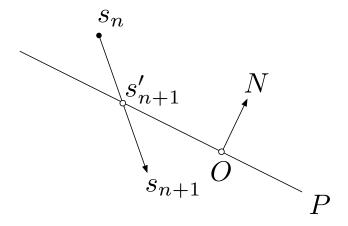
Here  $O_{nei}$  is the centroid of the positions of the neighboring particles and  $V_{nei}$  is the averaged velocity of of the neighboring particles. The neighboring particles of a given particle can be defined in many ways, such as particles within  $\delta$  distance from the given particle or k closest particles.

#### 2.4 User interaction

Allows user to exert force to particles. Idea is to attached a spring between the mouse pointer and particles.

#### 2.5 Collision

Given that we found the collision of a particle between state  $s_n$  and  $s_{n+1}$  with a plane  $P = \{N, O\}$ , we can find the exact collision state  $s'_{n+1}$  where the particle is on the colliding plane P. Here, N is the normal direction of P and O is a point on P. You can find  $s'_{n+1}$  using binary search since  $s_n$  is above (inside) and  $s_{n+1}$  is below (outside) the plane (or any 2D/3D objects).



**Collision response** To handle collision, we decompose the velocity of the particle at  $s'_{n+1}$  into two vectors:  $v_N$  and  $v_T$ , where  $v_N$  is parallel to N and  $v_T$  is perpendicular to N (i.e. parallel to the plane). The new velocity of the particle at  $s'_{n+1}$  now becomes

$$v'_{n+1} = v_T + k_r(-v_N) \;,$$

where  $k_r$  is called *restitution coefficient*, a property of the plane. We then continue to simulate the particle from  $s'_{n+1}$  with this new velocity.

**Constact** We say that the particle is in contact with the plane P if  $v_N = \emptyset$ . If an external force F is applied to push the particle into P a contact force is applied back as -F.

**Friction** Friction force is applied in the direction opposite to  $v_T$  and is proportional to the external force pushing the particle into P. Therefore,  $F = -k_f(-F_{ext}N)v_T$ , where  $k_f$  is friction constant and  $F_{ext}$  is an external force applied on the particle.