Debt Accumulation and Default in Low-Income Countries*

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Abstract

This paper studies the accumulation of external public debt in low-income countries (LICs) after receiving debt relief from multilateral lenders. While LICs who received debt relief from the International Monetary Fund in the early 2000s generally lowered their external debt in the initial years of relief, many experienced fast resurgence in debt after borrowing limits were lifted in 2006. Using a difference-in-differences model, we find countries that benefited from the relaxation are more likely to experience a significant increase in their debt-to-GDP ratio. We quantitatively evaluate the effects of debt limit relaxation using a model of sovereign default with two types of debt: subsidized loans from multilateral institutions and non-concessional loans from the private market. The model is calibrated using data from Mozambique prior to its default in 2016, and we find that an impatient government is necessary to match the data on debt accumulation and investment. In the calibrated model, lifting limits on non-concessional borrowing results in a 17.1 percent loss in households' consumption-equivalent welfare. Furthermore, the welfare benefits of relaxing borrowing limits with a counter-factually patient government are relatively small.

Keywords: debt crisis · low-income countries · concessional lending · debt relief

JEL Classification: F34 · F35 · F41 · H63 · O11

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1 Introduction

Many low-income countries (LICs) have historically relied on foreign aid and concessional loans from multilateral development institutions such as the World Bank and the International Monetary Fund (IMF) to finance their expenditures. Beginning in the late 1980s, many LICs accumulated unsustainable levels of debt. This prompted multilateral institutions to introduce debt relief programs in the early 2000s, such as the Highly Indebted Poor Countries Initiative (HIPC) and the Multilateral Debt Relief Initiative (MDRI).[1] Their goal was to provide grants and subsidized low-interest loans to reduce debt repayments to sustainable levels.

The debt relief initiatives initially achieved their goal of reducing debt burdens in LICs. To participate in these programs, the recipient country had to accept a strict limit on its borrowing from private lenders. In response to concerns that these limits on borrowing were overly restrictive to the governments of LICs, the policy of having strict debt limits was relaxed jointly by the IMF and the World Bank in 2009 (IMF, 2009b). In subsequent years, many countries participating in the debt relief programs experienced a resurgence in their borrowing to unsustainable levels. For example, as shown in Figure 1, Mozambique's external public debt-to-GDP ratio decreased from 79% in 2000 to 22% in 2006, but then increased back to 77% by 2016. In 2016, Mozambique defaulted on its infamous "Tuna bonds", bonds issued by Ematum, a government-backed agency in charge of promoting Mozambique's fishing industry. This made Mozambique the first African country to default on dollar bonds since Côte d'Ivoire in 2011 (Wallace, 2017). Since then, the Republic of Congo, Zambia, and Ghana have all defaulted on their external public debt.

This paper studies how relaxing limits on non-concessional borrowing affects debt accumulation and default risk in LICs. First, this is examined empirically by estimating the effect of policy relaxation on the external debt-to-GDP ratio using a difference-in-differences estimation. With 31 LICs over the period of 2000 to 2016, it is found that countries that

[1] See Appendix A for more details on debt relief programs.

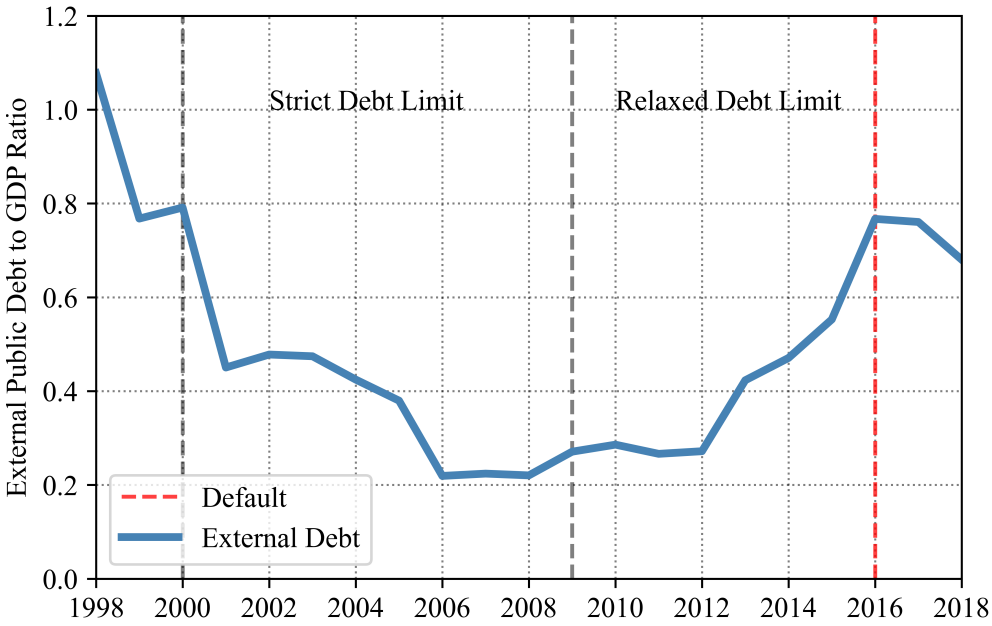


Figure 1: Mozambique's External Public Debt

benefited from policy relaxation are more likely to experience a significant increase in their external debt, on average reaching 54% of output between 2000 and 2016.

Motivated by this result, the policy is further evaluated quantitatively, using a small open economy model of sovereign default, in the spirit of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008), but extended to study LICs. Standard models of sovereign default mainly focus on developed and emerging economies. However, the debt composition in poor countries is different, since they rely heavily on concessional loans from multilateral development institutions (Koeda, 2008). In the model, two types of lenders are included to reflect LICs economies: multilateral institutions that lend at a subsidized interest rate, and private lenders who lend at the market rate.

The model builds on the work of Cole and Kehoe (2000) and Conesa and Kehoe (2017) by allowing for self-fulfilling debt crises, where the main risk of crisis comes from the loss of confidence from private lenders. If the government is impatient, having a discount factor lower than households and lenders, it borrows more than is socially optimal. The relaxation of strict debt limits implemented by the multilateral lenders created additional space to borrow. With little to no incentive to reduce debt, an impatient government continues to

accumulate it. The more impatient the government is, the more likely the country is to risk entering a debt crisis.

The model is calibrated using data from Mozambique prior to its default in 2016. We find that high impatience on behalf of the government is necessary to jointly match debt accumulation, servicing costs, and investment in the data. The calibrated model is then applied to examine the welfare effects of relaxing limits on private borrowing. Lifting limits on non-concessional borrowing in the model results in a very large loss in households' welfare, at 17.1 percent in consumption-equivalent terms. Furthermore, the welfare benefits of relaxing borrowing limits are relatively small in a version of the model where the government is as patient as households, at only 1.4 percent of consumption. This suggests the potential welfare losses from relaxing debt limits after debt relief are very large while the potential benefits are much smaller.

Related Literature This paper adds to the large body of literature on sovereign debt and default.[2] More specifically, this paper is closely related to the literature on self-fulfilling debt crises. Cole and Kehoe (2000) was the first to provide a theory of self-fulfilling debt crises. They show a country can experience a debt crisis if international lenders stop lending, even when it would be optimal for the government to repay its debt if lenders continued to buy bonds. This implies there is an interval of debt holdings where a debt crisis can occur with positive probability depending on whether lenders coordinate to stop lending. More recent examples of papers from this literature include Conesa and Kehoe (2017), Ayres, Navarro, Nicolini and Teles (2018), Bocola and Dovis (2019), Lorenzoni and Werning (2019), Stangebye (2020), Aguiar, Chatterjee, Cole and Stangebye (2022), and Ayres, Navarro, Nicolini and Teles (2024). This paper contributes to the self-fulfilling debt crisis literature by studying a model with two types of debt: concessional loans from multilateral institutions, and non-subsidized loans from international lenders. Additionally, the model allows for impatience by the government, adjustment costs on investment, and occasionally

[2] Aguiar and Amador (2014) provides a detailed review of the literature on sovereign debt.

binding borrowing constraints. These frictions allow us to account for institutional features of LICs that affect their susceptibility to debt crises.

Existing papers have studied debt accumulation under impatient governments. Chatterjee and Eyigungor (2019) study a model where the impatience of the government arises endogenously from political economy constraints. Aguiar, Amador and Fourakis (2020) studies the welfare effects of having a government with a different discount factor than households. Similar to the results of our welfare analysis, they find an impatient government can create large welfare losses and autarky is sometimes preferable from the perspective of households. Samano (2021) studies an economy with a patient central bank and an impatient government. They find that impatience makes the government accumulate more debt than what is socially optimal, and the central bank accumulates reserves to undo government over-borrowing.

This paper also draws on the literature on the interaction of debt accumulation and investment in capital. Park (2017) adds endogenous capital accumulation into an otherwise standard quantitative sovereign default model and shows that, conditional on a level of debt, default incentives are U-shaped in the capital stock. Gordon and Guerrón-Quintana (2018) also studies the effects of capital on default incentives. They find that capital has nontrivial effects on the incentive to default, but a larger capital stock almost always reduces default risk in equilibrium. Esquivel (2024) studies how externalities from decentralized capital investment impedes the ability of a benevolent government to make optimal borrowing and default decisions. Song and Mihalache (2025) study a model with decentralized capital investment, long-term public debt, and an impatient government. They find that even when decentralized investment creates externalities, autarky may be preferable to households relative to the impatient government's optimal policy. This paper also studies a model where the risk of default is decreasing in the stock of capital. A large motivation for relaxing limits on borrowing is to allow LICs to use debt to finance investment based on the belief that investment will prevent future crises. Similar to the results of Song and Mihalache

(2025), this paper finds that the benefits from allowing an impatient government to finance investment are much smaller than the welfare losses from higher default risk.

Finally, this paper is related to the literature on debt relief policies and debt sustainability in LICs. In Easterly (2002), the author studies the paradox of debt related to heavily indebted poor countries (HIPCs). HIPCs became heavily indebted after two decades of debt relief efforts. He shows that impatience can lead to over-borrowing. Koeda (2008) investigates optimal debt policy for LICs through a concessional lending problem. The paper finds that LICs tend to accumulate a large amount of concessional debt to smooth consumption rather than for investment purposes. Having access to subsidized loans, the country becomes forever aid dependent. Reinhart and Trebesch (2016) study the dynamics of debt relief and its aftermath. They find that the economic landscape of debtor countries improves significantly after debt relief operations, but only if these involve debt write-offs. Softer forms of debt relief, such as maturity extensions and interest rate reductions, are not generally followed by higher economic growth or improved credit ratings (Reinhart and Trebesch, 2016). Songwe and Awiti (2021) report that at least 48% of African countries had a debt ratio above 70% in 2019. They argue that the speed of debt accumulation matters and that monitoring the speed of debt could alter the path of debt to more sustainable levels. Ndulu and O'Connell (2021) mention that public debt levels in sub-Saharan Africa rose fast in the wake of the 2008 financial crisis, and many countries are now classified by the World Bank and the IMF as at high risk of debt distress.

Outline The rest of the paper is organized as follows. Section 2 describes the relaxation policy and documents that countries that benefited from the intervention see a significant increase in their external debt-to-GDP ratio. Section 3 describes the quantitative model and Section 4 presents the results of the quantitative model and performs welfare analysis on the relaxation policy. Finally, Section 5 concludes.

2 Motivation

This section presents empirical evidence regarding the interaction between the relaxation of the debt limit policy and external debt. Subsection 2.1 describes the new debt limits policy implemented jointly by the IMF and the World Bank. Subsection 2.2 outlines the data sources used and demonstrates that LICs that benefited from the relaxation policy seem more likely to experience a significant increase in their external debt.

2.1 External Debt Limits Policy

Following the late 1980s and early 1090s sovereign debt crises in SSA countries, Paris Club members and multilateral institutions implemented the HIPC and MDRI debt relief programs. Debt relief was conditioned on sound economic management and poverty reduction strategies (Coulibaly et al., 2019). The debt relief programs were initially successful and led to a decrease in debt to output ratio, which in turn resulted in an increase in resources for the recipient country. However, the higher level of resources created a risk for free-riding from non-concessional lenders.[3] To minimize the free-rider risk, the IMF and World Bank programs typically included limits on external debt holdings. In 2006, a non-concessional borrowing policy (NCBP) was implemented, by the International Development Association (IDA) of the World Bank Group, to ensure the long-term debt sustainability of countries who participated in the HIPC and MDRI debt relief programs. A loan is considered concessional if it is offered with a grant element of at least 35%. The NCBP required the external debt contracted or guaranteed by the official sector to include minimum concessionality requirements, typically of 35%. Generally, this would mean a restriction of non-concessional external borrowing (NCB), also known as the zero NCB ceiling.[4] However, if a country expected to borrow only on concessional terms violated this rule, then penalties would be

[3]The World Bank's International Development Association (IDA) defines free-riding as situations in which debt relief or grants could potentially cross-subsidize lenders that offer non-concessional loans to recipient countries.

[4]In some cases, exceptions on the zero NCB ceiling would be made that would allow for looser limits. See IMF and World Bank (2006).

implemented as part of enforcing debt sustainability. These penalties differed depending on the country's level of debt risk. Countries facing a high risk of debt distress typically experienced a reduction in grant volumes, while those at low risk faced stricter loan conditions (World Bank, 2006).

The policy's main criticism was that the concessionality requirements were overly constraining to the LICs. As a result, the IMF implemented new guidelines in late 2009, followed by the introduction of more flexibility in the NCBP. The new policy goal was to take into account the broad range of situations that the LICs were facing. Meaning that if a country was at a high risk of being in debt distress, it should have tighter concessionality requirements than a country with a low risk of being in debt distress. LICs public financial management (PFM) capacity was also a factor: the higher a country's PFM capacity, the more likely it would be able to implement and benefit from looser concessionality requirements. Under this new framework, concessionality requirements were divided into groups so that higher capacity countries had more options, while lower capacity countries continued with the existing concessionality requirements.[5] By the end of June 2012, 24 of the 33 LICs with IMF-supported programs qualified for the more flexible borrowing options under the revised policy. Additionally, around half of the countries receiving support from the Poverty Reduction and Growth Trust (PRGT) were allowed to borrow some funds on non-concessional terms, an increase from approximately one-third in January 2009 (IMF, 2012b).

Under the new policy, based on its debt management history, Mozambique was assigned a hard cap of 1.5 billion US dollars on non-concessional borrowing. Between 2013 and 2014, the country breached the NCBP through three major non-concessional loans contracted via state-owned enterprises, each backed by sovereign guarantees. The first was *EMATUM* in 2013, amounting to $850 million, issued as a Eurobond and structured by VTB Capital and Credit Suisse. This loan was not disclosed to Parliament or the IMF. In 2014, *ProIndicus*

[5]See IMF (2009a) "Changing Patterns in Low-Income Country Financing and Implications for Fund Policies on External Financing and Debt".

contracted a commercial loan of $622 million, followed by *Mozambique Asset Management*, which secured another commercial loan amounting to $535 million. Together, these loans totaled over $2 billion, significantly exceeding the $1.5 billion ceiling, and were deemed non-concessional. As a result, the World Bank imposed a two-part penalty on Mozambique's IDA assistance in 2017. First, the grants were converted into standard loans. Second, Mozambique's total allocation was cut by 20%, reducing the overall amount of funding it received (World Bank, 2021).

As more countries gained access to non-concessional borrowing, their external debt portfolios also evolved. The external debt composition of sub-Saharan African countries has changed since the era before the debt relief programs. While concessional loans still make up large share, non-concessional debt has increased relative to concessional debt. The share of private lending to African countries has risen from 9% of external debt in 2000 to 17 percent in 2017 (Coulibaly et al., 2019). The composition of external creditors has also changed over time, moving from traditional bilateral creditors in the Paris Club and international development institutions to non-traditional bilateral and commercial creditors, such as Eurobond investors (IMF, 2023). Since 2006, 16 sub-Saharan African nations have been active in issuing Eurobonds in global financial markets. Additionally, a growing portion of the region's external financing has been obtained on non-concessional terms (Coulibaly et al., 2019). Between 2000 and 2017, private creditors' share of African countries' external debt nearly doubled, from 9% to 17%, while the proportions held by multilateral and bilateral sources declined (World Bank, 2018).

Accessing international markets offers several advantages for issuing countries, including the ability to quickly raise large amounts of funds. However, issuing Eurobonds can also raise refinancing risks, especially when the amounts are substantial and debt management frameworks are weak (World Bank, 2018). Furthermore, loans from international markets usually come with maturities of 5 to 10 years, which are not well suited for infrastructure projects that take much longer to generate returns (Coulibaly et al., 2019). According to

Faria and Tolosa (2007), countries in Sub-Saharan Africa with a high share of short and medium term, non-concessional debt face especially high rollover risk, as they depend on private lenders who are more likely to withdraw funding during economic downturns.

2.2 Empirical Motivation

We use annual data over the period between 2000 and 2016 for 31 low-income countries that benefited from the HIPC program in the early 2000s. The data (527 observations) are obtained from several databases: the World Bank's International Debt Statistics (external debt, external multilateral debt, external bilateral debt, external debt from private creditors), the World Bank's World Development Indicators (current GDP). We use Bloomberg to get yearly prices for bonds issued between 2010 and 2018 for Mozambique. To identify countries that benefited from the debt limit relaxation policy, we use the IMF (2012a) policy paper.[6] To get the list of African countries that issued bonds between 2006 and 2014, we use Mecagni et al. (2014), which examines the rise in international sovereign bonds issued by SSA economies and countries' central banks.

Table 1 lists the countries included in the analysis. Countries that issued Eurobonds after 2006 are classified as the treated group. Using the relaxation of the NCBP as the treatment variable proved challenging, as it is often difficult to determine whether a waiver under a given NCBP case represented a temporary exemption or a permanent change in status from a zero to a non-zero ceiling. Nevertheless, if a country had a zero ceiling in 2006 and subsequently issued a Eurobond, it can be inferred that its status changed to a non-zero ceiling. For example, Uganda did not issue Eurobonds but has been under a non zero ceiling policy since 2009.

Figure 2 shows the debt dynamics for both groups between 2000 and 2016. Notably, the differences are striking. Following the debt relief (HIPC and MDRI) window from 2000 to 2006, the debt ratio decreased at a fast pace in both groups. From 2008 to 2011, there

[6] See "2011 Review of Conditionality" - Table 3. PRGT-Eligible Countries: External Debt Concessionality Requirements Under The New Policy

Control *(Strict debt limit)*	**Treated** *(Relaxed debt limit)*
Benin	–
Burkina Faso	–
Burundi	–
Cameroon	–
Central African Republic	–
Chad	–
Comoros	–
Congo Democratic Republic	–
–	Côte d'Ivoire (2007)
–	Ethiopia (2011)
–	Ghana (2007)
Guinea	–
Guinea-Bissau	–
Haiti	–
–	Honduras (2013)
Liberia	–
Madagascar	–
Malawi	–
Mali	–
Mauritania	–
–	Mozambique (2013)
Nicaragua	–
Niger	–
–	Rwanda (2013)
Sao Tome and Principe	–
–	Senegal (2011)
Sierra Leone	–
–	Tanzania (2013)
Togo	–
–	Uganda (2009)
–	Zambia (2012)

Table 1: Sample of Countries

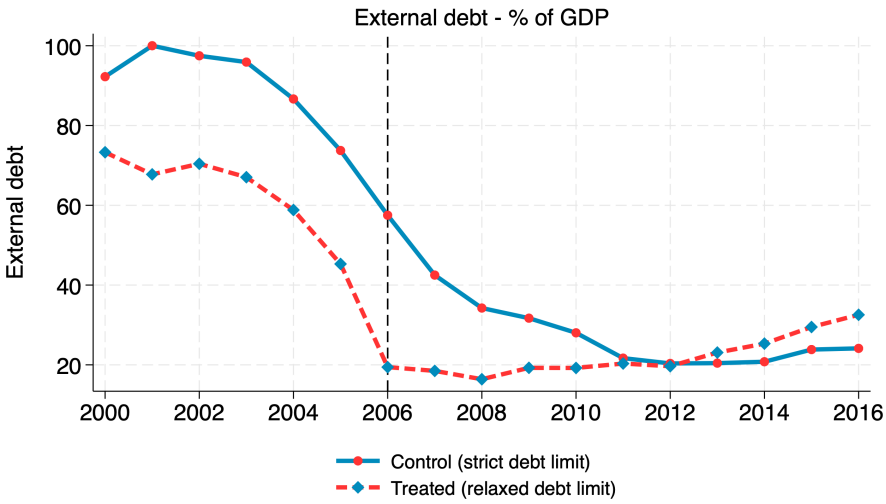


Figure 2: External Debt Dynamics

seemed to be a shift, with the treatment group accumulating debt at a faster pace than the control group. However, the former still held less debt on average than the control group. From 2011 onward, the average debt has been higher in treated countries as compared to the control group.

Table 2 displays the debt composition of countries that benefited from the IMF debt limit policy as well as that of countries with strict debt limit. The table shows external debt, non-subsidized debt and subsidized debt in GDP percent between the two groups, for the years 2003, 2004 and 2005, prior to the relaxation policy implementation. Countries under a strict debt limit seem to hold, on average, more debt when compared to countries that benefited from the policy. However, the difference is not statistically significant (column (3)).

To evaluate the IMF policy, we first estimate a difference-in-differences (DID) regression, as in Reinhart and Trebesch (2016). We examine the impact of the relaxation policy on external debt and we use the relaxation policy as the treatment. Using a two-way fixed effect model, the following equation is estimated using the linear regression:

$$log\left(Debt_{it}/GDP_{it}\right) = \beta_0 + \beta_1 D_{it} + \alpha_i + \eta_t + \epsilon_{it} \tag{1}$$

Year	Variable	**Control***	**Treated***	*p-value*
2003	External debt	95.90	67.05	*0.20*
	Non-subsidized debt	34.32	21.21	*0.23*
	Subsidized debt	61.58	45.84	*0.33*
2004	External debt	86.65	58.82	*0.18*
	Non-subsidized debt	28.03	17.24	*0.27*
	Subsidized debt	58.61	42.58	*0.26*
2005	External debt	73.74	45.29	*0.09*
	Non-subsidized debt	24.03	11.72	*0.14*
	Subsidized debt	49.71	33.57	*0.16*
	Number of countries	21	10	

Notes: This table presents descriptive statistics for the sample of analysis. Data come from the World Bank's International Debt Statistics and the World Bank's World Development Indicators. *Control = strict debt limit, Treated = relaxed debt limit - Eurobonds year of issuance.

Table 2: Debt Composition of Countries with Relaxed or Strict Debt Limit Before 2006

where $Debt_{it}/GDP_{it}$ is the debt-to-GDP ratio for country i , D_{it} is an interaction term that equals one for the treated group, countries benefiting from the policy after 2006, and zero for the control group. α_i and η_t are respectively country and time fixed effects. Time t is from 2000 to 2016, while ϵ_{it} is the random error term. The coefficient of interest is β_1, as it captures the impact of the relaxation policy. For example, if the relaxation of the debt limit for a country leads to an increase in debt ratio, then $\beta_1 > 0$. The country fixed effects, α_i, take into account country time-invariant heterogeneity. The time fixed effects, η_t, capture a common time trend.

As is the case with all difference-in-differences analysis, parallel trends are assumed, meaning that the only difference before and after the policy between the two groups is coming from the relaxation of debt limit. Therefore, if countries did not experience a relaxation of their debt limit, the trend on debt-to-GDP should have been similar in both groups. In this light, we use an event study to look at common trends before and after treatment. We run the following regression:

$$log\left(Debt_{it}/GDP_{it}\right) = \rho_0 + \sum_{k=2005}^{2000} \rho_k treat_{ik} + \sum_{k=2006}^{2013} \rho_k treat_{ik} + \alpha_i + \eta_t + \epsilon_{it} \qquad (2)$$

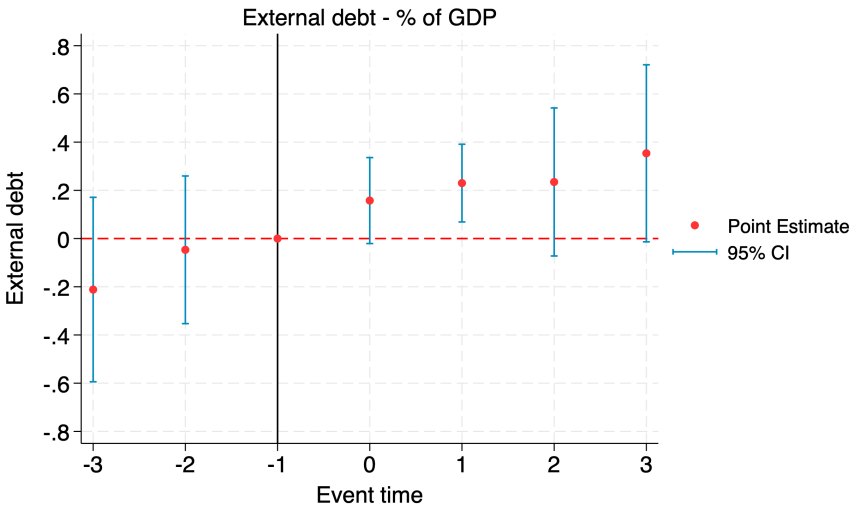


Figure 3: Event Study

where $Debt_{it}/GDP_{it}$ is the debt-to-GDP ratio for country i, $treat_{ik}$ are dummy variables for the time relative to treatment. As in equation (1) α_i and η_t are respectively country and time fixed effects, while ϵ_{it} is the random error term.

The results of the event study, depicted in Figure 3, show the point estimates and confidence intervals of the regression (2) for each time period before and after the treatment period. On the horizontal axis we plot the time dummies. For instance, if treatment happened in 2010 and the observation's period in the data is also 2010, then $t = 0$. If instead of 2009, the treatment occurred in 2011, then $t = 1$. The vertical line is a 95% confidence interval, the red horizontal line is set at 0 and the point of reference is at $t = -1$. Standard errors are clustered at the country level. Except for $t = -3$, the pre-treatment coefficients (ρ_k) are close to zero and are all statistically insignificant.

Table 3 reports the results of equation (1). Columns (1), (2) and (3) show the results of the linear regression with log of external debt-to-GDP, log of non-subsidized debt-to-GDP and log of subsidized debt-to-GDP as outcome variables, respectively. The analysis shows that, countries that benefited from the policy relaxation are more likely to experience an increase in their log external debt-to-GDP ratio (column (1)) by 0.43 or 54%, on average, between 2006 and 2016. Looking at columns (2) and (3) we can see the increase is coming

Outcome variables	External Debt	Non-subsidized debt	Subsidized Debt
	(1)	(2)	(3)
Post 06 x Treated group	**0.43*** **	**0.76*** **	0.17
	(0.17)	(0.25)	(0.19)
Intercept	4.38***	3.26***	3.80***
	(0.07)	(0.08)	(0.08)
Observations	526	285	285
R^2	0.69	0.58	0.65
Number of countries	31	31	31

Standard errors clustered at the country level in parentheses: $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

Table 3: Impact of Relaxation Policy on Log of Debt-to-GDP Ratio

from non-subsidized loans with a significant increase of 0.76, or 114%. Subsidized loans also increase but the results are not statistically significant. This is a drastic change compared to the late 80s and early 90s when the bulk of debt was mainly subsidized. It is important to notice that these are preliminary results and there might be omitted variables that may be creating an estimation bias for the coefficients.

3 Model

This section describes the structural model used to quantitatively study debt accumulation in low-income countries. The model is an extension of standard models from the self-fulfilling debt crisis literature, in particular Cole and Kehoe (2000) and Conesa and Kehoe (2017). To study low-income countries, we build on these existing frameworks to include both concessional loans from multilateral institutions and non-subsidized loans from international lenders. Additionally, the model allows for impatience by the government, adjustment costs on investment, and occasionally binding borrowing constraints. These frictions allow us to quantitatively account for institutional features of low-income countries that determine their susceptibility to debt crises.

Environment We study a small open economy with with an exogenous international

risk-free rate, r. The economy produces consumption goods each period using capital, k, according to the production function $Af(k)$, where A is the total factor productivity of the economy. There are three agents: the government, a representative multilateral institution, and a continuum of international lenders. For tractability, we allow the government to choose the level of consumption on behalf of households and abstract from households' individual consumption-investment decisions, as is common in the literature on sovereign debt risk.[7] Time is discrete and the government discounts the future at rate β. Following Aguiar, Amador and Fourakis (2020), the government's discount factor is not necessarily equal to the discount factor of international lenders. The government is said to be impatient if its discount factor is lower than the discount factor of international lenders implied by the risk-free rate, i.e., $\beta < 1/(1+r)$.

If the government wants to consume and invest at a higher level than its output, it must acquire loans from either international lenders or the multilateral institution. Loans from international lenders, B_n, are available at price $q_n \leq 1/(1+r)$, which is endogenously determined to clear the market. Loans from the multilateral institution, B_s, are subsidized and available at price $q_s > 1/(1+r)$, consistent with lower interest rates offered by multilateral institutions on concessional loans. Subsidized debt contracts are assumed to be fully enforceable, so the government cannot default on loans from the multilateral institution. However, the government is allowed to default on its non-subsidized debt, in which case it suffers a permanent reduction in productivity and a loss of the subsidized rate on its balance of subsidized debt.

Since $q_s > q_n$ by assumption, the government will only accumulate non-subsidized debt once it reaches its borrowing limit on concessional loans, which is set to be a fraction of output $\bar{B}_s$. In general, this means the price of the subsidized bonds will be greater than the discount factor of the government. This implies that the government will choose to borrow on concessional terms until they hit the borrowing limit, i.e., $B_s' = \bar{B}_s Af(k)$. Even so, the effect

[7] For example, see Arellano (2008) and Conesa and Kehoe (2017).

of subsidized debt in the model is not trivial. Consistent with the actions of multilateral lenders after Mozambique's 2016 default, we assume that the government loses the lower rate on its subsidized loans if it defaults on its other debt obligations. This cost must be taken into consideration by the government when choosing whether to risk defaulting on its non-subsidized debt.

Private lenders are risk-neutral, so the market clearing price of non-subsidized debt is proportional to one minus the expected probability of default in the next period. The government will choose to default on its non-subsidized debt if the value of repaying its debt is less than the value of defaulting. Additionally, we allow for self-fulfilling debt crises where international lenders may choose to stop lending to the government even when the government's value of repaying is higher than the value of defaulting. As in Cole and Kehoe (2000), the government enters a *crisis zone* if it accumulates a level of debt where it would choose to default if international lenders suddenly refused to buy new bonds. In the crisis zone, lenders willingness to continue purchasing bonds from the government is determined by the realization of a sunspot variable, ζ, which is independently drawn at the end of each period from a standard uniform distribution.[8]

International lenders are risk-neutral and maximize their present discounted value of payments on bonds purchased from the government. Therefore, the market clearing price of non-subsidized debt is equal to the lenders' discount factor, adjusted by the probability of a self-fulfilling debt crisis occurring:

$$q_n(\mathbf{s}';\mathbf{s}) = \begin{cases} 1/(1+r) & ,B'_n \leq \hat{B}^c_n(\mathbf{s}';\mathbf{s}) \\ \left[1 - D_n(\mathbf{s}';\mathbf{s})\right]/(1+r) & ,B'_n \in (\hat{B}^c_n(\mathbf{s}';\mathbf{s}), \hat{B}^d_n(\mathbf{s}')] \\ 0 & ,\hat{B}^d_n(\mathbf{s}') < B'_n \end{cases} \tag{3}$$

[8]The specific cause of default in Mozambique and other low-income countries is beyond the scope of this paper. The self-fulfilling crisis framework provides a tractable way to model debt accumulation into the crisis zone without having to take a stand on which shocks realistically cause the government to default once it reaches high levels of debt. Therefore, the sunspot variable in this model can be understood as a stand-in for any shock that might precipitate a debt crisis, e.g., interest rates, productivity, or commodity prices.

where $D_n(\mathbf{s}';\mathbf{s})$ is the probability of a debt crisis occurring. Since there are no shocks other than the sunspot variable, the probability of a debt crisis is a deterministic function of the state of the economy and the government's choices. We specify the specific functional form of D_n in Section 4.1.

Timing The timing in each period is as follows:

1. The government chooses to repay or default on its existing non-subsidized debt B_n.
2. If no default occurs, the government chooses next period debt, B_n', given the price schedule offered by international lenders.
3. If default occurs, the government can no longer acquire loans from international lenders and they suffer a permanent loss in productivity equal to fraction Z of their potential output.
4. The government chooses its levels of consumption expenditures, g, and investment, x_k.
5. The sunspot variable, ζ, is realized. International lenders will refuse to purchase new bonds from the government in the next period if $\zeta < D_n(\mathbf{s}';\mathbf{s})$.

Government Problems Given the the state of the economy, $\mathbf{s} = (B_s, B_n, k)$, and prices, the government chooses its levels of both subsidized and non-subsidized debt, as well as investment, to maximize its value. In the case where the government chooses to repay its non-subsidized debt, it solves the following recursive problem:

$$V(\mathbf{s}) = \max_{B_s', B_n', x_k} \log g + \beta \left[(1 - D_n(\mathbf{s};\mathbf{s}')) \max\{V(\mathbf{s}'), V^{def}(\mathbf{s}')\} + D_n(\mathbf{s};\mathbf{s}') V^{def}(\mathbf{s}') \right], \tag{4}$$

subject to its budget constraint,

$$g = Af(k) + q_s B_s' + q_n(\mathbf{s}';\mathbf{s}) B_n' - B_s - B_n - x_k,$$

the law of motion for investment,

$$k' = (1-\delta)k + x_k - \chi(k, x_k), \tag{5}$$

and the borrowing limits on both types of debt,

$$B_s' \leq \bar{B}_s A f(k) \text{ and } B_n' \leq \bar{B}_n A f(k). \tag{6}$$

As in Gordon and Guerrón-Quintana (2018), we include an adjustment cost for investment, which is represented by the term $\chi(k, x_k)$ in equation (5). This makes it costly for the government to change its capital stock from the current level. This cost is necessary to generate the hump-shaped increase in investment in the data following the relaxation of the strict limit on non-concessional debt.

In the case where the government defaults on its non-subsidized debt, it becomes excluded from international debt markets and loses the subsidized rate on its debt held by the multilateral institution. The value of defaulting is given by the solution to the following problem:

$$V^{def}(\mathbf{s}) = \max_{B_s', x_k} \log g + \beta V^{def}(\mathbf{s}'), \tag{7}$$

subject to its budget constraint,

$$g = (1 - Z) A f(k) + \frac{B_s'}{1 + r} - B_s - x_k,$$

the law of motion for investment, (5), and the constraint on subsidized borrowing, (6). The price on subsidized debt is now equal to $1/(1+r)$, reflecting that the multilateral institution has revoked the subsidized rate on its loans.

Crisis and Default Regions We call the *crisis zone* the levels of debt where the government is at a positive risk of experiencing a debt crisis. To solve for the lower boundary of the crisis zone, we need to define the value of being excluded from international debt markets. Let V^{aut} be the value of autarky, where the government is not allowed to borrow from international lenders,

$$V^{aut}(\mathbf{s}) = \max_{B_s', x_k} \log g + \beta V^{aut}(\mathbf{s}'), \tag{8}$$

subject to its budget constraint,

$$g = Af(k) + q_s B'_s - B_s - x_k,$$

the law of motion for investment, (5), and the constraint on subsidized borrowing, (6). The boundary of the crisis zone is defined as the values of non-subsidized debt such that the government is indifferent between defaulting and repaying its debt in full before entering autarky. Define the government's expenditures if they repay as

$$g^c(B'_s, k'; B_s, B_n, k) = Af(k) + q_s B'_s - B_s - B_n - x_k$$

The boundary of the crisis zone can then be defined as the correspondence $\hat{B}^c_n(\mathbf{s}';\mathbf{s})$ such that

$$V^{def}(\mathbf{s}) = \log g^c\Big(B'_s, k'; B_s, \hat{B}^c_n(\mathbf{s}';\mathbf{s}), k\Big) + \beta V^{aut}(\mathbf{s}').$$

given the definitions of V^{def} and V^{aut} in equations (7) and (8), respectively.

We call the *default region* the area of the state space where the government defaults with probability one because its value of defaulting is greater than its value of repaying regardless of whether international lenders will continue lending, i.e., $V^{def}(\mathbf{s}) > V(\mathbf{s})$.

Definition of Recursive Equilibrium Given the state of the economy, $\mathbf{s} = (B_s, B_n, k)$, a recursive equilibrium is defined by 1) the value functions $V(\mathbf{s})$, $V^{def}(\mathbf{s})$, 2) the policy functions $B'_s(\mathbf{s})$, $B'_n(\mathbf{s})$, $x_k(\mathbf{s})$, $(B^{def}_s)'(\mathbf{s})$, $x^{def}_k(\mathbf{s})$, 3) the supply of non-subsidized loans offered by the lenders, $b'_n(\mathbf{s})$, and 4) the price schedule for non-subsidized debt, $q_n(\mathbf{s}';\mathbf{s})$, such that:

1. Given prices and constraints, the value and policy functions solve the government's problems described in equations (4) and (7)

2. The market for non-subsidized debt clears: $b'_n(\mathbf{s}) = B'_n(\mathbf{s})$

4 Quantitative Analysis

The model is solved numerically using standard value function iteration to evaluate its quantitative predictions for the accumulation of non-subsidized debt and investment following a relaxation of strict limits on non-concessional borrowing.[9] Section 4.1 explains the specific functional forms used in the model, how the parameters of the model are chosen and the calibration strategy. Then, Section 4.2 demonstrates the optimal choices of the government and how it accumulates debt in the model. Finally, in Section 4.3, the calibrated model is used to perform welfare analysis in order to evaluate the welfare cost of relaxing the strict debt limits policy.

4.1 Parameter Selection

Functional Forms Following Gordon and Guerrón-Quintana (2018), the adjustment cost of investment is specified as a quadratic function of investment that is equal to zero if the capital stock is unchanged from the previous period:

$$\chi(k, x_k) = \bar{\chi}\left(\frac{x_k}{\delta k} - 1\right)^2 \delta k.$$

The value of $\bar{\chi}$ is calibrated in the model to match the level of investment in the data following the relaxation of strict limits on borrowing. As in Koeda (2008), the production function is specified as a decreasing returns to scale function, $f(k) = k^{\alpha}$. Under this assumption, there is a unique optimal level of capital, and the implied return to investment decreases as the capital stock increases to the optimum.

The probability of a debt crisis occurring, $D_n(\mathbf{s}'; \mathbf{s})$, is assumed to be increasing in the level of debt the government accumulates within the crisis zone. In particular, following Hördahl and Tristani (2013), a linear-quadratic function is used to specify the relationship

[9] See Appendix B for more details on the computation of the model.

between debt accumulation and default risk. The probability of default is given by

$$D_n(\mathbf{s}';\mathbf{s}) = \begin{cases} 0 & ,B'_n < \hat{B}^c_n(\mathbf{s}';\mathbf{s}) \\ \epsilon[B'_n - \hat{B}^c_n(\mathbf{s}';\mathbf{s})]^2 & ,B'_n \in (\hat{B}^c_n(\mathbf{s}';\mathbf{s}), \hat{B}^d_n(\mathbf{s}')] \\ 1 & ,\hat{B}^d_n(\mathbf{s}') < B'_n \end{cases} \tag{9}$$

Under this assumption, the probability of default is zero for levels of debt outside the crisis and default regions, but then increases inside the crisis zone as the level of non-subsidized debt becomes further from the boundary.

Calibration The benchmark model is calibrated using data on external debt and investment for Mozambique between 2010 and 2018. Data on external debt holdings was taken from the World Bank's International Debt Statistics, while data on investment and output was taken from the World Bank's World Development Indicators. Parameter values for the model are summarized in Table 4. Some parameters are calibrated inside the model, while others are taken from other studies of sovereign debt accumulation with similar quantitative frameworks. Calibrated parameters are chosen so that a selection of moments in the data exactly coincide with simulated data from the model. A period in the model corresponds to one year in the data.

The average subsidized interest rate is set to match the official concessional lending practice of 0.75%, following Koeda (2008). The risk-free interest rate, r, is set to 2% to match the yield on risk-free bonds, following Conesa and Kehoe (2017). The default penalty, Z, is set to 0.05, which is standard in the literature.[10] We initiate the model with an initial capital stock normalized to be equal to one and with binding limits on borrowing that are calibrated to be consistent with debt holdings prior to 2012. We begin the simulation in 2012 as this was the first year where debt purchases or investment increased noticeably relative

[10]For example, this value is used by Alonso-Ortiz, Colla and Da-Rocha (2017), Conesa and Kehoe (2017), and Koeda (2008)

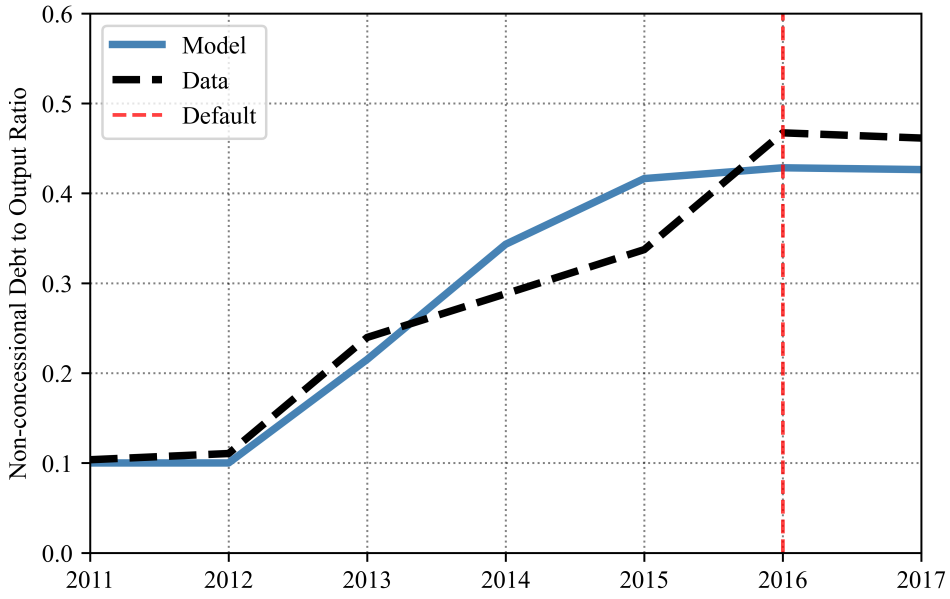
Description	Parameter	Value	Source/Target
International Risk-free Rate	r	2.0%	Conesa and Kehoe (2017)
Concessional Debt Price	$1-1/q_s$	0.75%	Koeda (2008)
Capital Depreciation Rate	δ	0.1	Koeda (2008)
Capital Share of Output	α	0.33	Koeda (2008)
Default Output Penalty	Z	0.05	Standard
Initial Capital Stock	k_0	1.0	Normalization
Concessional Debt Limit	$\bar{B}_s$	0.2	Concessional Debt/GDP, 2010-2016
Non-Concessional (NC) Debt Limit	$\bar{B}_n$	0.60	NC Debt/GDP, 2012-2014 (0.27)
Probability of Debt Crisis	ϵ	0.90	NC Debt/GDP, 2014-2017 (0.39)
Discount Factor	β	0.845	NC Debt Servicing, 2012-2014 (4.8%)
Initial NC Debt	$B_{n,0}$	0.1	NC Debt/GDP, 2010-2012
Productivity	A	3.0	Investment/GDP, 2012-2014 (0.38)
Investment Adjustment Cost	$\bar{\chi}$	0.08	Investment/GDP, 2014-2017 (0.26)

Table 4: Summary of Model Parameters

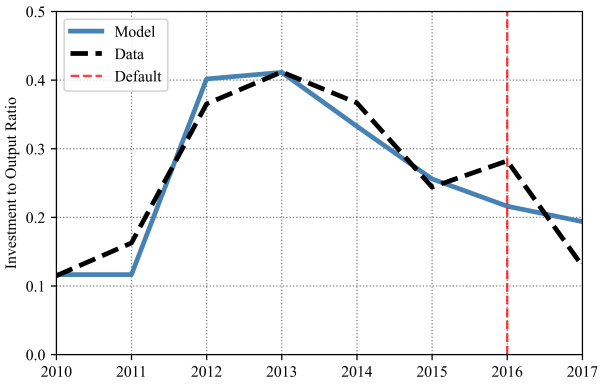
to their values under the strict debt limit policy.[11] The government's discount factor, β, the average default risk, ϵ, total factor productivity, A, and the adjustment cost for investment, $\bar{\chi}$, are jointly calibrated by matching 1) the average level of servicing on non-subsidized debt as a fraction of its debt balance, 2) average level of non-subsidized debt between 2014 and 2017 as a fraction of output, and 3) the average level of investment between 2012 and 2017. Notably, the calibration results in a discount factor equal to 0.845, which is substantially lower than the price of risk-free bonds: $1/(1+r) \approx 0.98$. Therefore, a highly impatient government is necessary to match the data in Mozambique in the years leading up to its default.

Model Fit Figure 4 compares simulated data from the model to data from Mozambique for the period between 2010 and 2018. Figure 4i compares the path of debt accumulation

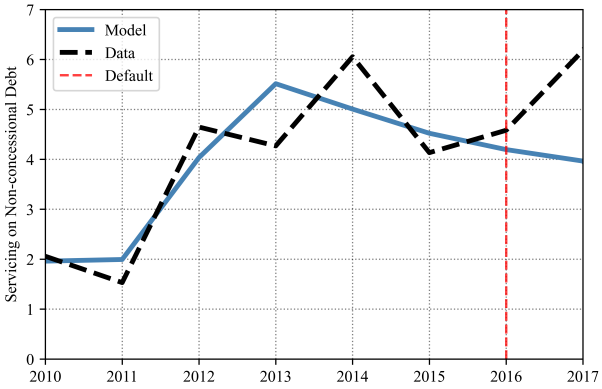
[11] We do not attempt to account for why there was virtually no debt accumulation in the two years immediately after the debt limit relaxation policy. We focus only on matching data after 2011, when sizable changes occurred in debt accumulation and investment.



(i) Non-concessional Debt-to-GDP Ratio



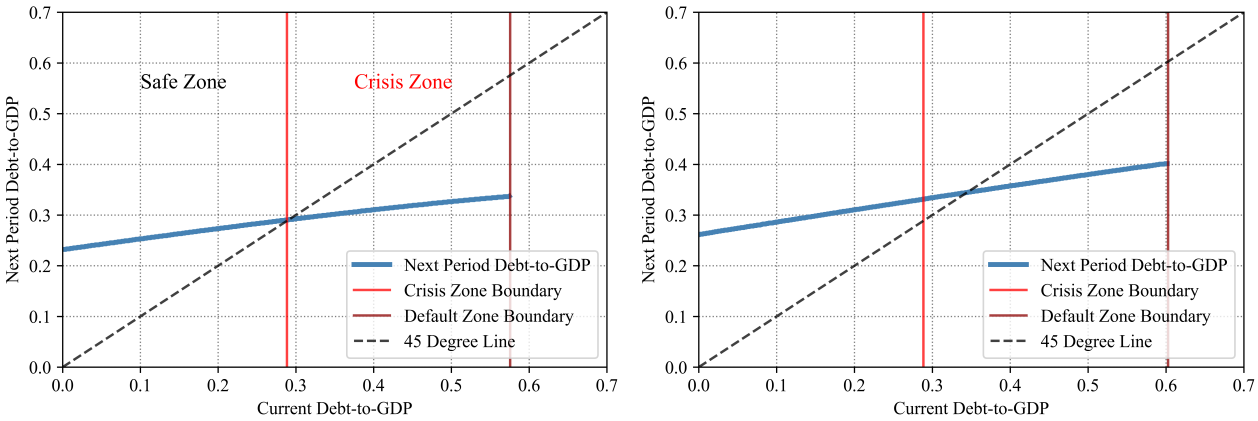
(ii) Investment-to-GDP Ratio



(iii) Servicing Costs on Non-Concessional Debt

Figure 4: Comparison of Data and Model Simulation for Mozambique, 2010 – 2017

from the simulated model (B_n), represented by the solid line, and the path of debt in the data, represented by the dashed line. The vertical line marks 2016, which is the year when Mozambique defaulted on its debt. Figures 4ii and 4iii display the same comparison for investment (x_k) and debt servicing costs ($1/q_n - 1$), respectively. The model fits well with the entire paths of its counterparts in the data even though it was calibrated only to match the average levels.



(i) Very High Default Probability ($\epsilon = 20$)

(ii) High Default Probability ($\epsilon = 2$)

(iii) Baseline Default Probability ($\epsilon = 0.9$)

(iv) Low Default Probability ($\epsilon = 0.2$)

Figure 5: Debt Accumulation Under Different Probability of Crisis

Notes: For simplicity, the policy functions in this figure are computed holding capital constant at its steady state value under autarky.

4.2 Debt Accumulation in the Model

Now, we describe in further detail how the government accumulates debt into the crisis zone. Figure 5 plots the policy functions for debt accumulation by the government for different probabilities of experiencing default in the crisis zone, as defined in (9). The boundary of the crisis zone is slightly less than 30 percent of output.[12] Below that threshold, the government is in the safe zone and it will never be optimal for the government to default, even when

[12]For ease of interpretation, the policy functions in Figure 5 are computed holding capital constant at its steady state value under autarky. These values will vary significantly with the capital stock and investment, but the qualitative behavior of debt accumulation will be similar.

international lenders stop lending. Above that cutoff, the government is in the crisis zone and a crisis will occur with positive probability. Further, above the crisis zone is the default zone, where defaulting becomes optimal regardless of whether or not lenders are available.

Since the government is impatient, the price of non-subsidized debt is always greater than its discount factor in the safe zone, so its optimal policy is to accumulate debt at least to the lower bound of the crisis zone. The amount of debt the government will choose to accumulate within the crisis zone depends on the probability of a crisis occurring. Figure 5i shows that the government will accumulate virtually no debt in the crisis zone if the probability of defaulting is very high. Meanwhile, when the probability of default is low and interest rates stay approximately constant throughout the crisis zone, Figure 5iv shows that the government will continue accumulating debt until it reaches the default zone. Figures 5ii and 5iii show the intermediate cases where the government accumulates past the lower bound of the crisis zone, but stops before reaching the default zone. This reflects the government's tradeoff between accumulating more debt and being charged higher rates by lenders. The government will continue accumulating debt until the risk-adjusted price for debt, as computed in (3), is equal to the government's discount factor. Therefore, an impatient government will choose to risk default as long as it believes the probability of defaulting is sufficiently low.

4.3 Welfare Analysis

The previous sections showed that lifting limits on non-concessional borrowing will result in the government choosing to accumulate debt into the crisis zone. Although defaulting produces large welfare losses, lifting debt limits provides liquidity the government can use to increase investment and smooth consumption. In this section, the calibrated model is used to quantitatively evaluate the net welfare effects of relaxing the strict debt limits policy. Welfare is computed from the perspective of households, who are assumed to have the same discount factor as international lenders. We compute the change in households' welfare

	Baseline ($\beta = 0.845$)	Patient ($\beta = \frac{1}{1+r} \approx 0.98$)
NC Debt to Output Ratio, 2016 (B_n)	42.8	53.6
Crisis Zone Lower Bound, 2016 ($\hat{B}_n^c$)	25.6	54.9
Default Zone Lower Bound, 2016 ($\hat{B}_n^d$)	57.0	92.9
Welfare Change for Households (γ)	-17.1%	1.4%

Table 5: Simulation of Impatient and Patient Government

from lifting debt limits in the calibrated economy, where the government is impatient. We also compute the change in welfare in a counter-factual version of the economy where the government is patient and has a discount factor equal to households.

The consumption-equivalent gain in welfare, γ, is computed as the percent change in consumption required to make households indifferent between lifting debt limits and staying under the strict debt limits policy. That is, the level of γ such that

$$E_0 \sum_{t=0}^{\infty} (1/(1+r))^t \log g_t = E_0 \sum_{t=0}^{\infty} (1/(1+r))^t \log[(1+\gamma) g_t^{aut}], \tag{10}$$

where g_t is the path of consumption in the baseline economy, and g_t^{aut} is the path of consumption under a counter-factual version of the economy where limits on non-subsidized borrowing were never relaxed, i.e., $\bar{B}_n A f(k_0) = B_{n,0}$. Then, to get welfare effects under a patient government, we compute the solution to the government's problem with β set to be equal to $1/(1+r)$, and recompute γ as in (10).

We simulate debt accumulation under both impatient and patient governments. The results of the simulations are summarized in Table 5. The boundaries of the crisis and default zones for a patient government are found to be much higher than for an impatient government. The patient government still accumulates a significant level of debt by 2016 since the marginal product of capital is higher than the interest on debt. However, unlike the impatient government, it does not enter the crisis zone. With an impatient government, relaxing debt limits leads to a very large decline in welfare, equal to 17.1 percent of consumption. Thus, the welfare losses from default strongly outweigh any benefits to households from allowing the government to borrow. Meanwhile, with a patient government there is a rela-

tively small increase in welfare, only 1.4 percent of consumption. This suggests that if multilateral institutions are unsure of the government's level of impatience, the potential welfare loss from relaxing the strict debt limits policy is much larger than any possible benefit.

5 Conclusion

This paper examines debt accumulation by recipient countries of debt relief programs both empirically and through the lens of a quantitative structural model of sovereign default. First, using a difference-in-differences estimation, with 31 low-income countries (LICs), we found that countries who benefited from policy relaxation were more likely to experience a significant increase in their non-subsidized debt. Then, we developed a model of sovereign debt with two types of creditors to reflect economies of LICs: multilateral institutions that lend at subsidized rates and private lenders that lend at non-subsidized rates. The model is calibrated using data from Mozambique prior to its default in 2016. An impatient government is necessary to jointly match debt accumulation, servicing costs, and investment in the data. Lifting limits on non-concessional borrowing in the calibrated model results in a very large loss in households' welfare, while the potential benefits of allowing a patient government to borrow are relatively small.

There are several important implications for policymakers. First, with LICs gaining access to the private market, more coordination between the government, the private lenders and the multilateral institutions should be implemented. Secondly, multilateral institutions should enforce more transparency, especially after large-scale debt relief programs such as the HIPC and MDRI. Regarding future guidelines on debt limit relaxation for LICs, perhaps a more gradual approach should be implemented in the future, especially for less diversified economies. Finally it is key to incentivize governments in those countries to implement sound policies.

Overall, this paper provides a tractable framework to study debt accumulation in low-

income nations with a government more impatient than households. Possible interesting extension of the paper would be to introduce long term debt, as in Conesa and Kehoe (2017) and Gordon and Guerrón-Quintana (2018), or allowing international lenders to be risk-averse, as in Lizarazo (2013).

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A The HIPC initiative and MDRI

Launched in 1996, the original Initiative for Heavily Indebted Poor Countries (HIPCs) marked the first time that multilateral, Paris Club, and other official bilateral and commercial creditors united in a joint effort to reduce the external debt of the world's most debt-laden poor countries to "sustainable levels" – that is, levels that allow these countries to service their debt through export earnings, aid, and private capital inflows without compromising long-term, poverty-reducing growth. Assistance under the HIPC initiative is limited to countries that have per capita incomes low enough (GNI per capita less than $2390) to qualify for World Bank and IMF concessional lending facilities, and that face unsustainable debt burdens even after traditional debt relief. The vast majority of beneficiary countries are in Africa.

The main critique of the original HIPC initiative was that the sustainability targets were set in light of the empirical work that had examined largely middle income countries (MICs) while LICs had less capacity to sustain external debt. In 1999, a review of the HIPC initiative was carried out by the World Bank and the IMF in broad consultation with civil society organizations and public officials. As a result, the international community agreed to enhance the HIPC initiative and committed to providing faster, broader, and deeper debt relief. With the qualifying thresholds being lowered, more countries were eligible for debt relief, while some were eligible for more debt relief. The country eligibility is determined as follows:

Decision Point - Stage 1: The HIPC needs to establish a three year (or less) track record of good macroeconomic performance. Once that goal is attained, the country is considered to have reached its "decision point". The country's eligibility and the amount of debt relief are then determined by the IMF and World Bank Boards. Debt relief and other assistance now begin flowing as soon as the decision point is reached, with the amount based on the country's immediate needs and capacity for channeling the funds to poverty-reducing purposes.

Decision Point - Stage 2: The HIPC needs to establish another track record by implementing the policies determined at the "decision point".

Completion Point: The international community commits to provide sufficient assistance by a particular date (the "completion point") in an amount that would enable the country to achieve debt sustainability. At the "completion point," the remainder of the full stock-of-debt reduction pledged is delivered.[13] As of February 2020, thirty six countries are at post-completion point.

The MDRI: In June 2005, the Group of 8 (G8) major industrial countries proposed that three multilateral institutions – the IMF, the International Development Association (IDA) of the World Bank, and the African Development Fund (AfDF) – cancel 100 percent of their debt claims on countries that had reached, or would eventually reach, the completion point

[13] https://www.imf.org/external/pubs/ft/exrp/debt/eng/index.htm

under the enhanced HIPC Initiative. The goal of the MDRI was to provide full debt relief to free up additional resources to help these countries reach the United Nations millennium development goals.[14]

Combined, the MDRI and HIPC initiative have provided around $99 billion in debt relief.

B Model Computation

The model described in Section 3 is solved numerically using standard value function iteration on discrete grids for debt and capital. Since the government will always accumulate subsidized debt up to the borrowing limit before taking out any non-concessional debt, it is a redundant state variable and the problem can be solved as a two state problem in non-subsidized debt and capital. Grids for non-subsidized debt and capital are created with 1000 grid points each. First, the values of defaulting and autarky are solved on the grid for capital, where the optimal choice for next period capital are computed using grid search. Then, the value of repaying and the price of non-subsidized debt are solved for every combination of capital and non-subsidized debt, where policy functions are computed using two-dimensional grid search. Using the policy functions, we simulate the economy on the discrete grids given initial conditions.

[14] https://www.imf.org/external/np/exr/facts/mdri.htm