Hw 5

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11/7

$y = 2x+1 \quad X \sim exp(\alpha)$

1. ~~here~~

$$f_x(x) = \begin{cases} xe^{-xx} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$f_x(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

a.

$$P(y \leq y) = P(2x+1 \leq y) = F_y(y)$$

$$= P\left(x \leq \frac{y-1}{2}\right)$$

$$= F_x\left(\frac{y-1}{2}\right)$$

$$F_y(x) = \begin{cases} 1 - e^{-\lambda\left(\frac{y-1}{2}\right)} & y \geq 1 \\ 0 & y < 1 \end{cases}$$

$$f_y(y) = \frac{d}{dx} F_y(y)$$

$$f_y(y) = \begin{cases} \lambda/2\, e^{-\lambda\left(\frac{y-1}{2}\right)} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

b.

$$f_y(y) = f_x(l^{-1}(y)) \left|\frac{dx}{dy}\right|$$

$$x = l^{-1}(y) = \frac{y-1}{2} \qquad \frac{dx}{dy} = \frac{1}{2}$$

$$f_y(y) = \begin{cases} \lambda e^{-\lambda\left(\frac{x-1}{2}\right)} \cdot \frac{1}{2} & y \geq 1 \\ 0 & x < 1 \end{cases}$$

$$F_{X}(x) = \begin{cases} 1 - e^{-\lambda\left(\frac{x-1}{2}\right)} & y \geq 1 \\ 0 & x < 1 \end{cases}$$

2. a.

$F_Y(y) = P(Y \le y)$

$P\left(\frac{20}{X} \le y\right)$

$1 - P\left(\frac{20}{y} \le X\right)$

$1 - F_X\left(\frac{20}{y}\right)$

$1 - \frac{20-4y}{3y}$

$\boxed{F_Y(y) = \frac{7}{3} - \frac{20}{3y}}$

$\boxed{f_Y = \frac{20}{3y^2} \; ; \; 20/7 \le x \le 5}$

b. $f_X(x(y)) \left| \frac{dx}{dy} \right| = \frac{1}{3} \times \frac{20}{y^2}$

$\boxed{f_X(y) = \frac{20}{3y^2} \; ; \; 20/7 \le x \le 5}$

$\int_{20/7}^{x} \frac{20}{3y^2} \, dy = \frac{20}{3} \left[-\frac{1}{y}\right]_{20/7}^{x}$

$\boxed{F_Y(x) = -\frac{20}{3y} + \frac{7}{3}}$

3. a. $e^t \cdot P(x=1) + e^{2t} P(x=2) + e^{3t} \cdot P(x=3$

$\frac{e^t}{2} + \frac{e^{2t}}{4} + \frac{e^{3t}}{4}$

b. $\frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 3$

$\frac{1}{2} + \frac{2}{4} + \frac{3}{4} = \frac{7}{4}$

c. $1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{4} + 3^2 \cdot \frac{1}{4}$

$\frac{1}{2} + \frac{4}{4} + \frac{9}{4} = \frac{15}{4}$

d. $1^3 \cdot \frac{1}{2} + 2^3 \cdot \frac{1}{4} + 3^3 \cdot \frac{1}{4}$

$\frac{2}{4} + \frac{8}{4} + \frac{27}{4} = \frac{37}{4}$

4. $E(x^n) \begin{cases} 0 & n \text{ odd} \\ 2^{-n/2} \cdot \frac{n!}{(n/2)!} & n \text{ even} \end{cases}$

a. $E(x^3) = 0$

$E(x^4) = 2^{-4/2} \cdot \frac{4!}{2!}$

$\frac{1}{4} \cdot 12 = \boxed{3}$

b. $E(x+x) = E(x) + E(x) = 0 + 0 = 0$

$var = 1 + 1 = 2$

$d(x) = \frac{1}{\sqrt{2\pi \cdot 2}} \cdot e^{-\frac{1}{2 \times 2} y^2}$

5. $E(x^k) = \int_0^\infty x^k \lambda e^{-\lambda x}\, dx$

$\lambda \int_0^\infty x^{(k+1)-1} e^{-\lambda x}\, d\lambda$

$\lambda \dfrac{\Gamma(k+1)}{\lambda^{k+1}}$ $\qquad \dfrac{k!}{\lambda^k}$

$E(x^3) = \dfrac{3!}{\lambda^3} = \boxed{\dfrac{6}{\lambda^3}}$

$E(x^4) = \dfrac{4!}{\lambda^4} = \boxed{\dfrac{24}{\lambda^4}}$

$E(x^5) = \dfrac{5!}{\lambda^5} = \boxed{\dfrac{120}{\lambda^5}}$

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7. a. $M_N(s) = e^{2(e^x - 1)}$

$= \sum_{i=1}^{N} x_i$

$M_X(s) = \dfrac{e^{s-1}}{s}$

$M_Y(s) = E[e^{sY}]$

$E[e^{s(\sum_{i=1}^{N} x_i)}]$

$E_N[E(e^{s(x_1+x_2 \dots x_n)})|n]$

$E_N[(\frac{e^{s}-1}{s})^n]$

$e^{\lambda(\frac{e^{x}-1}{s}-1)}$

b. $\frac{d}{ds} M_Y(s)|_{s=0}$

$\frac{d}{ds} M_X(s)|_{s=0}$ $\quad X \lambda e^{\lambda(M_X(s)-1)}$

$\dfrac{\lambda}{2}$