CS 394R, HW5 FS 2019 (45 points) **Name** _____

1. (10 points) Let X be exponential and let $Y = 2X + 1$ For every $x \in (0, \infty]$

   (a) (5 points)
   Find $F_y(y)$ and $f_Y(y)$ using the CDF

   (b) (5 points)
   Find $F_y(y)$ and $f_Y(y)$ using the PDF

2. (10 points) Let X be uniform on $[4, 7]$ and let $Y = \frac{20}{x}$ For every $x \in [4, 7]$
   Find $F_y(y)$ and $f_Y(y)$

   (a) (5 points)
   Find $F_y(y)$ and $f_Y(y)$ using the CDF

   (b) (5 points)
   Find $F_y(y)$ and $f_Y(y)$ using the PDF

3. (5 points)
   Let X be a random variable that takes on the values 1, 2, and 3, with the following probabilities.
   $$P(X = 1) = \frac{1}{2}, \quad P(X = 2) = \frac{1}{4}, \quad P(X = 3) = \frac{1}{4}.$$
   (a) (2 points) Find the transform associated with X.

   (b) (1 points) Calculate $E[X]$

   (c) (1 points) Calculate $E[X^2]$

   (d) (1 points) Calculate $E[X^3]$

4. (5 points)
   Use the transform associated with the standard Normal random variable X to find the following:

   (a) (2 points) Calculate $E[X^3]$ and $E[X^4]$.

   (b) (3 points) Calculate the PDF of $X + X$.

5. (5 points)
Use the transform associated with the Exponential random variable to find the third, fourth, and fifth moments.

6. (5 points total)

   (a) (3 points) Find the PDF of the continuous random variable X associated with the transform

$$M(s) = \frac{1}{3} \cdot \frac{2}{2-s} + \frac{2}{3} \cdot \frac{3}{3-s}$$

   (b) (2 points) Find the fourth and fifth moments of the above PDF.

7. (5 points)
At a certain time, the number of of people that enter an elevator is a Poisson random variable with parameter λ. The weight of each person is independent of every other person's weight, and is uniformly distributed between 100 and 200 lbs. Let X_i, be the fraction of 100 by which the ith person exceeds 100 lbs, e.g., if the 7th person weighs 175 pounds, then $X_7 = 0.75$. Let Y be the sum of the X.

   (a) (3 points) Find the transform associated with Y

   (b) (2 points) Use the transform to find the expected value of Y