

STA 471 – Regression Analysis
Homework #2

September 21st, 2023

Richard McCormick
RLM443

1. For the data given in **Problem F**, on **Page 99**...

- a. Test to determine whether there is a linear relationship between X and Y at $\alpha = 0.05$. Be sure to state the null and alternative hypotheses, test statistic and its observed value, and conclusion.

$$\begin{array}{lll} \sum X = 63.6 & \sum Y = 62 & \sum XY = 339.1 \\ \sum X^2 = 339.18 & \sum Y^2 = 390 & \sum xy = 10.5 \\ \sum x^2 = 2.10 & \sum y = 69.67 & s_{xy} = 10.5 \\ \bar{x} = 5.3 & \bar{y} = 5.17 & s_{xx} = 339.18 - \frac{63.6^2}{12} = 2.1 \\ & & s_{yy} = 390 - \frac{62^2}{12} = 69.67 \\ & & b_1 = \frac{s_{xy}}{s_{xx}} = \frac{10.5}{2.1} = 5 \end{array}$$

$$\textcircled{1} H_0 \rightarrow \beta_1 = 0 \quad H_A \rightarrow \beta_1 \neq 0 \quad S = 0.4369$$

$$\textcircled{2} T_{obs}^2 = \frac{b_1^2}{S^2 / s_{xx}} = \frac{5^2}{0.4369^2 / 2.1} = 275.0395$$

$$t_{obs} = \sqrt{275.0395} = 16.58$$

$$\textcircled{3} t_{10} = 2.228$$

$\textcircled{4}$ Since $t_{obs}^{(1)} > t_{10} = 2.228$, we reject the null hypothesis at the $\alpha = 0.05$ level of significance and conclude there is a significant linear relationship between X & Y.

- b. Calculate a 95% confidence interval for β_1 and interpret the confidence interval. What is your conclusion based on the confidence interval?

$$t_{10} = 2.228, \quad b_1 = 5, \quad s = 0.4369, \quad \sum_{xx} = 2.1$$

$$CI: 5 \pm 2.228 \left(\frac{0.4369}{\sqrt{2.1}} \right)$$

$$CI: (4.328, 5.671)$$

We are 95% confident that β_1 is between 4.328 and 5.671. Because our observed value of $b_1 = 5$, we can conclude that it is a reasonable value.