STA 471 – Regression Analysis
Homework #4

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We define:

A = $\begin{bmatrix} 4 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix}$, B = $\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 3 & 2 \end{bmatrix}$, C = $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$

Calculate the matrices below, or say it is impossible to do so, if it is impossible to.

1. B + C

It is impossible to perform this calculation, as B & C are different dimensions.

2. BB'

$$B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 3 & 2 \end{bmatrix} \qquad B^T = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

$$B \times B^T = \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} -1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(-1)+(1)(1), & (-1)(2)+(1)(3), & (-1)(3)+(1)(2) \\ (2)(-1)+(3)(1), & (2)(2)+(3)(3), & (2)(3)+(3)(2) \\ (3)(-1)+(2)(1), & (3)(2)+(2)(3), & (3)(3)+(2)(2) \end{bmatrix}$$

$$\boxed{B \times B^T = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 13 & 12 \\ -1 & 12 & 13 \end{bmatrix}}$$

3. A + B'B

This calculation is impossible. Matrix B is 3×2, and B^T is 2×3. Therefore, $B^T \times B$ results in a matrix of size 2×2.

Because Matrix A is 3×3, it cannot be added to $B^T \times B$.

4. BC

$$B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 3 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

$$B \times C = \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(4)+(1)(3) & (-1)(3)+(1)(2) \\ (2)(4)+(3)(3) & (2)(3)+(3)(2) \\ (3)(4)+(2)(3) & (3)(3)+(2)(2) \end{bmatrix}$$

$$\boxed{B \times C = \begin{bmatrix} -1 & 1 \\ 17 & 12 \\ 18 & 13 \end{bmatrix}}$$

5. $AA^{-1}BC$

$$A \times A^{-1} = I$$

$$I \times B = B$$

$$B \times C = \left.\begin{bmatrix} -1 & 1 \\ 17 & 12 \\ 18 & 13 \end{bmatrix}\right\} \text{From \#4}$$

$$\boxed{A \times A^{-1} \times B \times C = \begin{bmatrix} -1 & 1 \\ 17 & 12 \\ 18 & 13 \end{bmatrix}}$$

6. CB'

$$C \times B^{T} = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} -1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (4\times-1)+(3\times1) & (4)(2)+(3\times3) & (4\times3)+(3\times2) \\ (3\times-1)+(2\times1) & (3\times2)+(2\times3) & (3\times3)+(2\times2) \end{bmatrix}$$

$$\boxed{C \times B^{T} = \begin{bmatrix} -1 & 17 & 18 \\ -1 & 12 & 13 \end{bmatrix}}$$

7. CAB

It is impossible to do this calculation. Matrix C is 2×2, but Matrix A is 3×3. Therefore, the calculation cannot be performed.

8. BC^{-1}, where $C^{-1} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$

$$B \times C^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times-2)+(1\times3) & (-1\times3)+(1\times-4) \\ (2\times-2)+(3\times3) & (2\times3)+(3\times-4) \\ (3\times-2)+(2\times3) & (3\times3)+(2\times-4) \end{bmatrix}$$

$$\boxed{B \times C^{-1} = \begin{bmatrix} 5 & -7 \\ 5 & -6 \\ 0 & 1 \end{bmatrix}}$$

9. A^{-1}

$$A = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix} \qquad |A| = \left(\begin{array}{l} (4 \times 4 \times 2) + (0 \times 0 \times 3) + \\ (0 \times 0 \times 3) - (3 \times 4 \times 3) - \\ (0 \times 0 \times 2) - (0 \times 0 \times 4) \end{array} \right)$$

$$= (4 \times 4 \times 2) - (3 \times 4 \times 3)$$

$$= 32 - 36 \qquad \boxed{|A| = (-4)}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} \begin{vmatrix} 4 & 0 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 0 & 3 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 0 & 3 \\ 4 & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & 0 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 4 \\ 3 & 0 \end{vmatrix} & -\begin{vmatrix} 4 & 0 \\ 3 & 0 \end{vmatrix} & \begin{vmatrix} 4 & 0 \\ 0 & 4 \end{vmatrix} \end{bmatrix}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} (4 \times 2) - (0 \times 0) & -[(0 \times 2) - (0 \times 3)] & (0 \times 0) - (3 \times 4) \\ -[(0 \times 2) - (0 \times 3)] & (4 \times 2) - (3 \times 3) & -[(4 \times 0) - (3 \times 0)] \\ (0 \times 0) - (4 \times 3) & -[(4 \times 0) - (0 \times 3)] & (4 \times 4) - (0 \times 0) \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} \frac{8}{-4} & \frac{0}{-4} & \frac{-12}{-4} \\ \frac{0}{-4} & \frac{-1}{-4} & \frac{0}{-4} \\ \frac{-12}{-4} & \frac{0}{-4} & \frac{16}{-4} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 3 \\ 0 & 1/4 & 0 \\ 3 & 0 & -4 \end{bmatrix}}$$

10. $A'A(A')^{-1}A^{-1}$

A is a symmetric matrix.

$$A^T \times A = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (4\times4)+(0\times0)+(3\times3) & (4\times0)+(0\times4)+(3\times0) & (4\times3)+(0\times0)+(3\times2) \\ (0\times4)+(4\times0)+(0\times3) & (0\times0)+(4\times4)+(0\times0) & (0\times3)+(4\times0)+(0\times2) \\ (3\times4)+(0\times0)+(2\times3) & (3\times0)+(0\times4)+(2\times0) & (3\times3)+(0\times0)+(2\times2) \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 25 & 0 & 18 \\ 0 & 16 & 0 \\ 18 & 0 & 13 \end{bmatrix}$$

Note: Since A is symmetric, $(A^T)^{-1} \equiv A^{-1}$

$$A^T A (A^T)^{-1} = \begin{bmatrix} 25 & 0 & 18 \\ 0 & 16 & 0 \\ 18 & 0 & 13 \end{bmatrix} \times \begin{bmatrix} -2 & 0 & 3 \\ 0 & 1/4 & 0 \\ 3 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} (25\times-2)+(0\times0)+(18\times3) & (25\times0)+(0\times1/4)+(18\times0) & (25\times3)+(0\times0)+(18\times-4) \\ (0\times-2)+(16\times0)+(0\times3) & (0\times0)+(16\times1/4)+(0\times0) & (0\times3)+(16\times0)+(0\times-4) \\ (18\times-2)+(0\times0)+(13\times3) & (18\times0)+(0\times1/4)+(13\times0) & (18\times3)+(0\times0)+(13\times-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

$$A^T A (A^T)^{-1} = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix} = A$$

$$A^T A (A^T)^{-1} = A$$

So...

$$A^T A (A^T)^{-1} \cdot A^{-1} = A \times A^{-1}$$

$$A \times A^{-1} = I$$

$$\boxed{\Rightarrow A^T A (A^T)^{-1} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$