Homework 7

Richard L. McCormick

Northern Arizona University

STA471: Statistical Regression

Dr. Jin Wang

November 21st, 2023

1. Problem G: If we believe in the "origin-shift" criterion, is the model: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 + \beta_{22} X_2^2 + \beta_{122} X_1 X_2^2 + \varepsilon$ a "well-formulated" one?

$$Y = \beta_0 + \beta_1(z_1 + a_1) + \beta_2(z_2 + a_2) + \beta_{12}(z_1 + a_1)(z_2 + a_2) + \beta_{22}(z_2 + a_2)^2 + \beta_{122}(z_1 + a_2)(z_2 + a_2)^2$$

$$= \beta_0 + \beta_1 a_1 + \beta_1 z_1 + \beta_2 a_2 + \beta_2 z_2 + \beta_{12}(z_1 z_2 + z_1 a_2 + z_2 a_1 + a_1 a_2) + \beta_{22}(z_2^2 + 2 z_2 a_2 + a_2^2) + \beta_{122}(z_1 + a_2)(z_2^2 + 2 z_2 a_2 + a_2^2)$$

$$(z_1 z_2^2 + z_1 2 z_2 a_2 + z_1 a_2^2 + a_2 z_2^2 + 2 z_2 a_2^2 + a_2^3)$$

$$\alpha_0 = (\beta_0 + \beta_1 a_1 + \beta_2 a_2 + \beta_{12} a_1 a_2 + \beta_{22} a_2^2 + \beta_{122} a_2^3)$$

$$\alpha_1 = (\beta_1 + \beta_{12} a_2 + \beta_{122} a_2^2)$$

$$\alpha_2 = (\beta_2 + \beta_{12} a_1 + \beta_{22} \cdot 2 a_2 + \beta_{122}(a_2 + 2 a_2^2))$$

$$\alpha_{12} = (\beta_{12} + \beta_{122} 2 a_2)$$

$$\alpha_{22} = (\beta_{22} + \beta_{122} a_2)$$

$$\alpha_{122} = (\beta_{122})$$

$$= \alpha_0 + \alpha_1 z_1 + \alpha_2 z_2 + \alpha_{12} z_1 z_2 + \alpha_{22} z_2^2 + \alpha_{122} z_1 z_2^2$$

The model is well formulated.

2. Problem H: The data shown below, which relate to a study of the quantity of vitamin B_2 in turnip green, are taken from the "Annual progress report on the soils-weather project, 1948," by J. T. Wakeley, University of North Carolina (Raleigh) Institute of Statistics Mimeo Series 19 (1949). The variables are:

X_1 = radiation in relative gram calories per minute during the preceding half day of sunlight (coded by dividing by 100),

X_2 = average soil moisture tension (coded by dividing by 100),

X_3 = air temperature in degrees Fahrenheit (coded by dividing by 10),

y = milligrams of vitamin B_2 per gram of turnip green.

These data were used by R. L. Anderson and T. A. Bancroft in *Statistical Theory in Research*, McGraw-Hill, New York, 1959, on p. 192, to fit the model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_1 X_2 + \varepsilon$$

Develop a suitable fitted equation using these data and compare its form with the form of the one fitted by Anderson and Bancroft.