

**SUPPLEMENTAL:**  
**USING MULTILEVEL MODELS TO ESTIMATE VARIATION IN**  
**ACHÉ HUNTING RETURNS AS A FUNCTION OF FAILURE**  
**RATE, PACKAGE SIZE, AGE, AND INDIVIDUAL**

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## 1. MODEL FITTING

The accompanying model fitting code presents Stan source for all of the models presented in the main text and here in the supplemental. In order to use this code, users will need to also install `rstan` from <http://mc-stan.org>.

The R package `glmer2stan` provides a less technical interface for defining and sampling from Stan models. We did not originally use this package to fit the models in this paper, but rather wrote the package to assist others in fitting similar models to their own data. This package is available from McElreath's website, currently at <http://xcelab.net/rm/>. Users who are familiar with `glmer` formula notation will find it easy to define multiple-outcome multilevel models using `glmer2stan`. The package can also fit single-outcome multilevel models with gaussian, binomial, poisson, ordered logic, and gamma outcome distributions. Some aspects of the models in this paper, such as missing data imputation, are not available through `glmer2stan`.

To provide an example of `glmer2stan` notation, once you have it installed, along with `rstan`, this code in Figure 1 will load the Ache data (included with `glmer2stan`) and fit the `Vary2` model.

## 2. CONVERGENCE

Convergence for the Stan model runs was very rapid, as can be seen in the trace plots (Figure 2). Eight varying slopes are shown here, to illustrate rapid convergence and efficiency of the Stan chains. Orange and blue traces are for independent chains, initialized at dispersed starting values. The gray shaded region in each trace indicates the adaptation, or burn-in, region. Corresponding JAGS chains end up in the same region, but take much longer to converge and have high autocorrelation.

```

library(glmer2stan)

# load hunting data
data(Ache)

# make squared and cubed age variables
Ache$age.s2 <- Ache$age.s^2
Ache$age.s3 <- Ache$age.s^3

m_Vary_2 <- glmer2stan(
  kg.meat ~ (1|hunter.id) + age.s + age.s2 + age.s3 ,
  data=Ache , family='ziggamma' , sample=TRUE , calcDIC=TRUE )

```

FIGURE 1. Code to fit model Vary<sub>2</sub> using glmer2stan.

### 3. COMPUTING DIC

We computed DIC directly from the MCMC samples, using the definitions:

$$p_D = \bar{D} - \hat{D},$$

$$\text{DIC} = \bar{D} + p_D = \hat{D} + 2p_D,$$

where  $\bar{D}$  is the average deviance across MCMC samples,  $\hat{D}$  is the deviance at the average parameter values (averaged across MCMC samples), and  $p_D$  is the penalty corresponding to the effective number of parameters.

The R scripts for performing these calculations are available from the corresponding author. The dev trace monitor in our JAGS and Stan code provides a computed deviance for each set of parameter samples, making computation of  $\bar{D}$  a simple matter of computing the expectation of the dev samples. Computing  $\hat{D}$  requires just computing the expected values of the parameters and then passing them to a zero-inflated gamma density function, included in our R code.

### 4. COMPUTING POSTERIOR MODEL WEIGHTS

By analogy to Akaike weights, we computed posterior model weight using DIC. These weights are estimates of the posterior probability of each model family and provide a useful way to both compare and average model predictions. Each weight for a model  $i$  in a set of  $m$  models is defined as:

$$w_i = \frac{\exp(-0.5\text{DIC}_i)}{\sum_{j=1}^m \exp(-0.5\text{DIC}_j)},$$

where  $\text{DIC}_i$  is the DIC of the  $i$ th model.

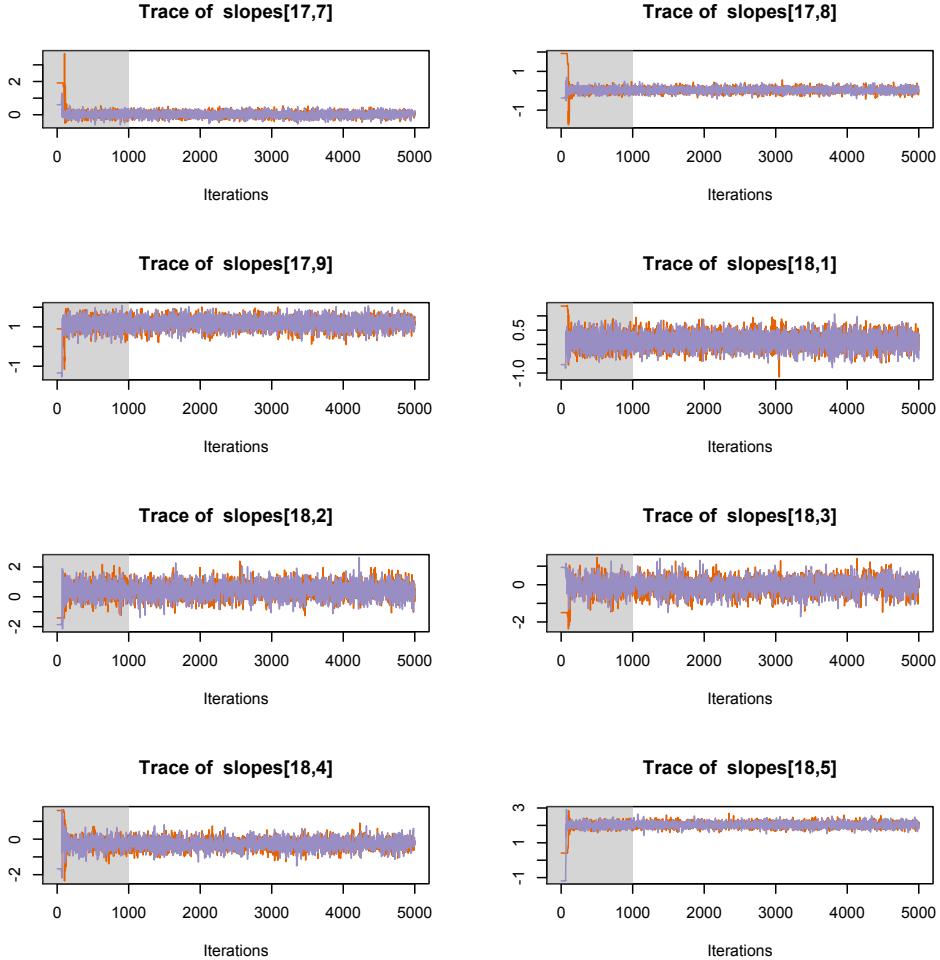


FIGURE 2. Trace plots of MCMC samples produced by Stan.

## 5. PARAMETER ESTIMATES

Table 1 shows the parameter estimates for the six models not presented in the main text. Same as the main text, “ $Vary_x$ ” denotes  $x$  varying effects on age, “D” denotes Julian date as a covariate, and “h” denotes hunt duration (imputed).

## 6. ALTERNATIVE AGE FUNCTION

The cubic model of age is flexible and easy to estimate. However, it allows for unlikely estimates, such as hunters who get better until death. In order to be sure that our results are not sensitive to that kind of mis-specification, we also estimated a different age model, one that forces individual hunters to get worse as age moves from an estimated peak age. The function is, shown here for the

TABLE 1. Parameter sample expectations and standard deviations (in parentheses). Variance components reported as variances, with standard error of the variance.

	Fixed	Vary <sub>2</sub>	Vary <sub>9</sub>	Vary <sub>9D</sub>	Vary <sub>9h</sub>	Vary <sub>9Dh</sub>
<b>Zeros</b>						
Intercept	-0.15(0.02)	0.06(0.08)	0.09(0.10)	0.07(0.09)	1.1(0.17)	1.0(0.18)
Age	0.11(0.03)	0.33(0.07)	0.16(0.11)	0.02(0.12)	0.16(0.11)	0.03(0.12)
Age <sup>2</sup>	0.23(0.02)	0.31(0.03)	0.14(0.10)	0.20(0.09)	0.15(0.10)	0.22(0.09)
Age <sup>3</sup>	-0.06(0.01)	-0.05(0.02)	-0.12(0.06)	-0.09(0.06)	-0.12(0.06)	-0.08(0.06)
Date				0.13(0.05)		0.13(0.05)
Hours					-0.14(0.02)	-0.14(0.02)
<b>Non-zeros</b>						
Intercept	1.9(0.01)	1.9(0.03)	1.9(0.04)	1.9(0.04)	1.8(0.06)	1.8(0.06)
Age	-0.05(0.01)	-0.06(0.02)	-0.06(0.05)	-0.05(0.05)	-0.05(0.05)	-0.05(0.05)
Age <sup>2</sup>	-0.04(0.01)	-0.05(0.01)	-0.02(0.05)	-0.02(0.05)	-0.02(0.05)	-0.03(0.05)
Age <sup>3</sup>	0.03(0.01)	0.03(0.01)	0.05(0.03)	0.04(0.03)	0.05(0.03)	0.04(0.03)
Date				0.01(0.02)		0.01(0.02)
Hours					0.004(0.005)	0.004(0.006)
log $\theta$	-1.1(0.02)	-1.1(0.02)	-0.74(0.10)	-0.76(0.10)	-0.74(0.10)	-0.75(0.10)
<b>Variance</b>						
<b>Zeros</b>						
Intercept		0.43(0.09)	0.46(0.12)	0.35(0.10)	0.47(0.13)	0.38(0.10)
Age			0.36(0.12)	0.30(0.11)	0.36(0.13)	0.32(0.11)
Age <sup>2</sup>			0.20(0.08)	0.15(0.07)	0.20(0.08)	0.16(0.07)
Age <sup>3</sup>			0.09(0.03)	0.09(0.03)	0.10(0.03)	0.09(0.03)
<b>Non-zeros</b>						
Intercept		0.03(0.01)	0.08(0.02)	0.08(0.02)	0.08(0.02)	0.08(0.02)
Age			0.06(0.02)	0.06(0.02)	0.06(0.02)	0.06(0.02)
Age <sup>2</sup>			0.05(0.01)	0.05(0.01)	0.05(0.01)	0.05(0.01)
Age <sup>3</sup>			0.03(0.01)	0.03(0.01)	0.03(0.01)	0.03(0.01)
log $\theta$			0.88(0.08)	0.82(0.16)	0.82(0.16)	0.82(0.15)

mean of the non-zeros:

$$\log \mu_{ij} = s_{j5} + I(A_{ij})(A_{ij} - s_{j6})^2,$$

where  $A_{ij}$  is the standardized age for case  $i$  and hunter  $j$ , and  $I(A_{ij})$  is a function that returns  $s_{j7}$  when  $A_{ij}$  is less than  $s_{j6}$  and otherwise returns  $s_{j8}$ . Essentially, this function defines an asymmetric parabola, with a center at  $s_{j6}$  and rate of increase/decrease to left of  $s_{j7}$  and to the right of  $s_{j8}$ . This function is much less flexible than a standard cubic polynomial, but it also satisfies the desiderata of having interpretable parameters and reducing wild swings in path. This is because changes in either rate parameter do not necessarily influence the other rate, unlike in a cubic polynomial.

Figure 3 shows the average and individual hunter functions, estimated with this model. The general facts remain unchanged: more variation in failures than package size, more variation late in age for failures, little drop off in package size with age. However, since this age function is less flexible than the cubic used in

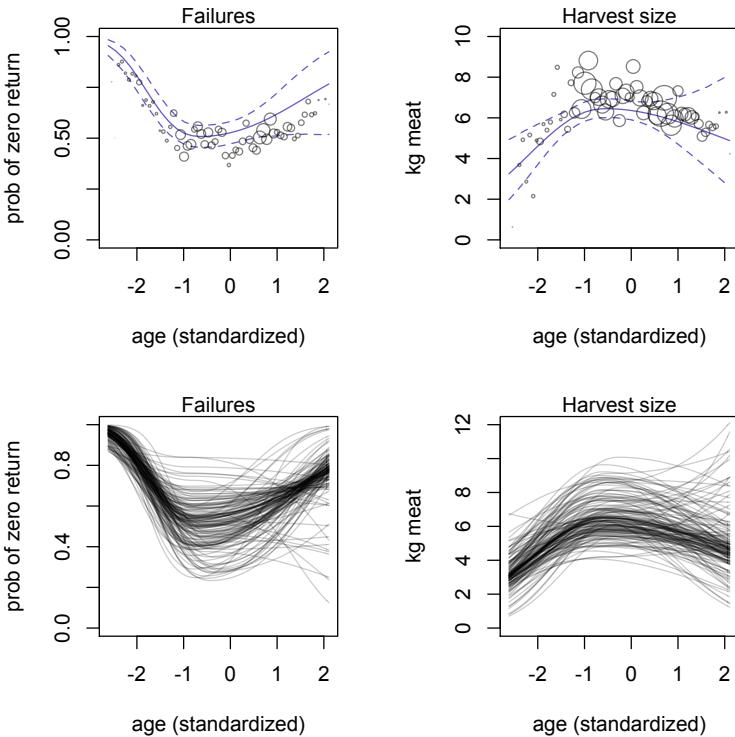


FIGURE 3. Average and hunter-specific predictions for the alternative age model.

the main text, there are fewer wild swings at late ages. As a result of the reduced flexibility, the deviance of this model is 50195, which is greater than the 50006 of the Vary<sub>9hDv</sub> model in the main text. However, the DIC of the alternative model is only 54129, which is actually better than the 54258 of the best model in the main text. One way to interpret this result is that the alternative age model fits the sample worse, because it is less flexible than the cubic polynomial. However, it is expected to generalize better, because its reduced flexibility results in smaller standard errors and less over fitting risk.

## 7. COMPARATIVE PREDICTIONS FROM ALL MODELS

Figures 4 and 5 that follow show the predictions from all six models not presented in the main text. We condense both mean predictions (black curves) and individual hunter predictions (blue curves) into the same plots. Note that most of the qualitative inferences about hunter variation appear in model Vary<sub>2</sub>, with failures exhibiting much more variation than harvests. Allowing individual

age functions to vary in the Vary<sub>9</sub> models does improve fit, but without changing the gross inferences about the population.

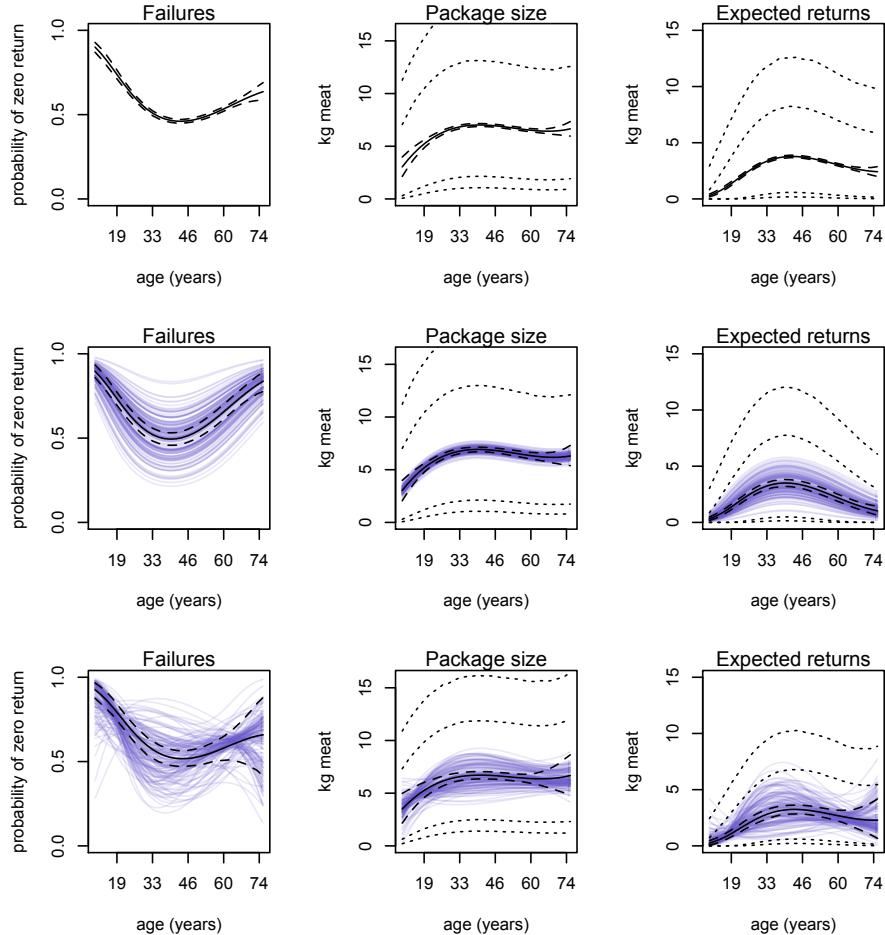


FIGURE 4. Predictions for models Fixed (top row), Vary<sub>2</sub> (middle), and Vary<sub>9</sub> (bottom). Black curves are means (solid), 95% HPDI's of mean (dashed), and 80% and 95% confidence intervals of predictions (dotted). Blue curves are individual hunter mean predictions. Note lack of blue curves in top row, because the Fixed model has no varying effects, effectively assuming that all hunters are identical in expectation.

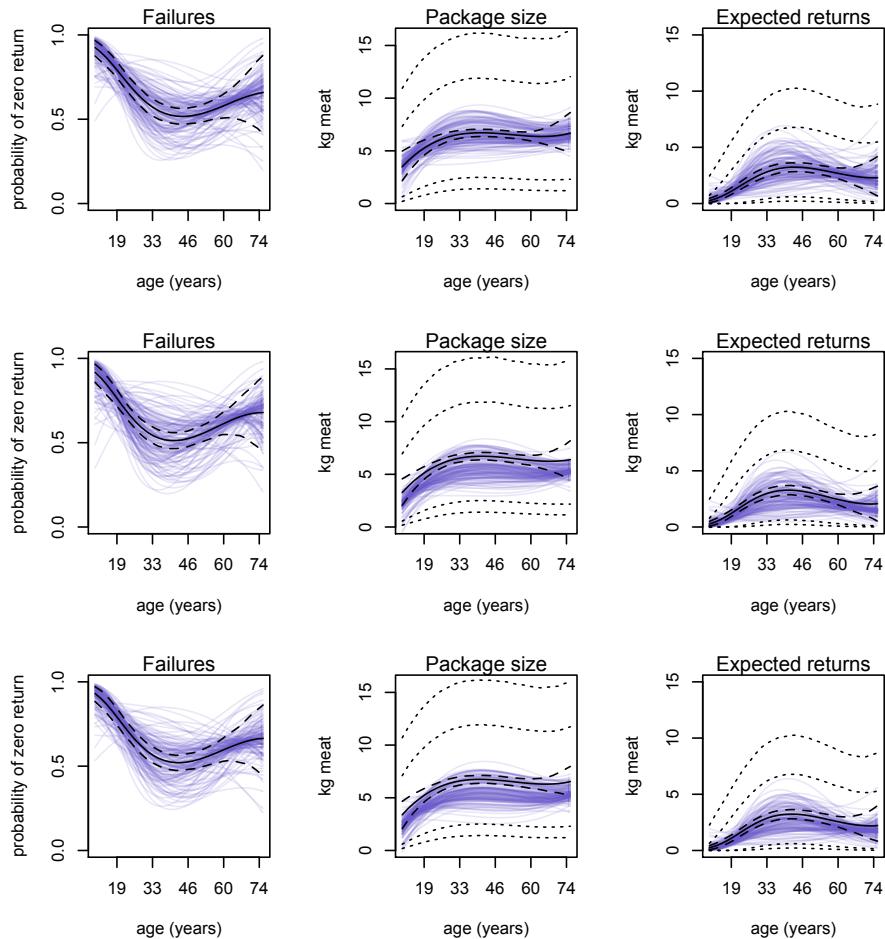


FIGURE 5. Predictions for models Vary<sub>9D</sub> (top row), Vary<sub>9h</sub> (middle), and Vary<sub>9Dh</sub> (bottom). Black curves are means (solid), 95% HPDI's of mean (dashed), and 80% and 95% confidence intervals of predictions (dotted). Blue curves are individual hunter mean predictions.

## 8. INDIVIDUAL HUNTER PREDICTIONS

Finally, in the following pages we show the predictions for each individual hunter in the sample, based upon samples from the  $\text{Vary}_{\text{hDv}}$  model. These plots show the high uncertainty associated with some aspects of individual hunter curves. Each row plots a single hunter, with ID number on the left. Hunter-specific sample sizes for both non-zeros and all trips are shown above the plots. The circles in each plot are the raw data for that hunter, and the gray horizontal lines show the empirical mean of the raw data. Dashed curves in these plots are 95% highest posterior density intervals.

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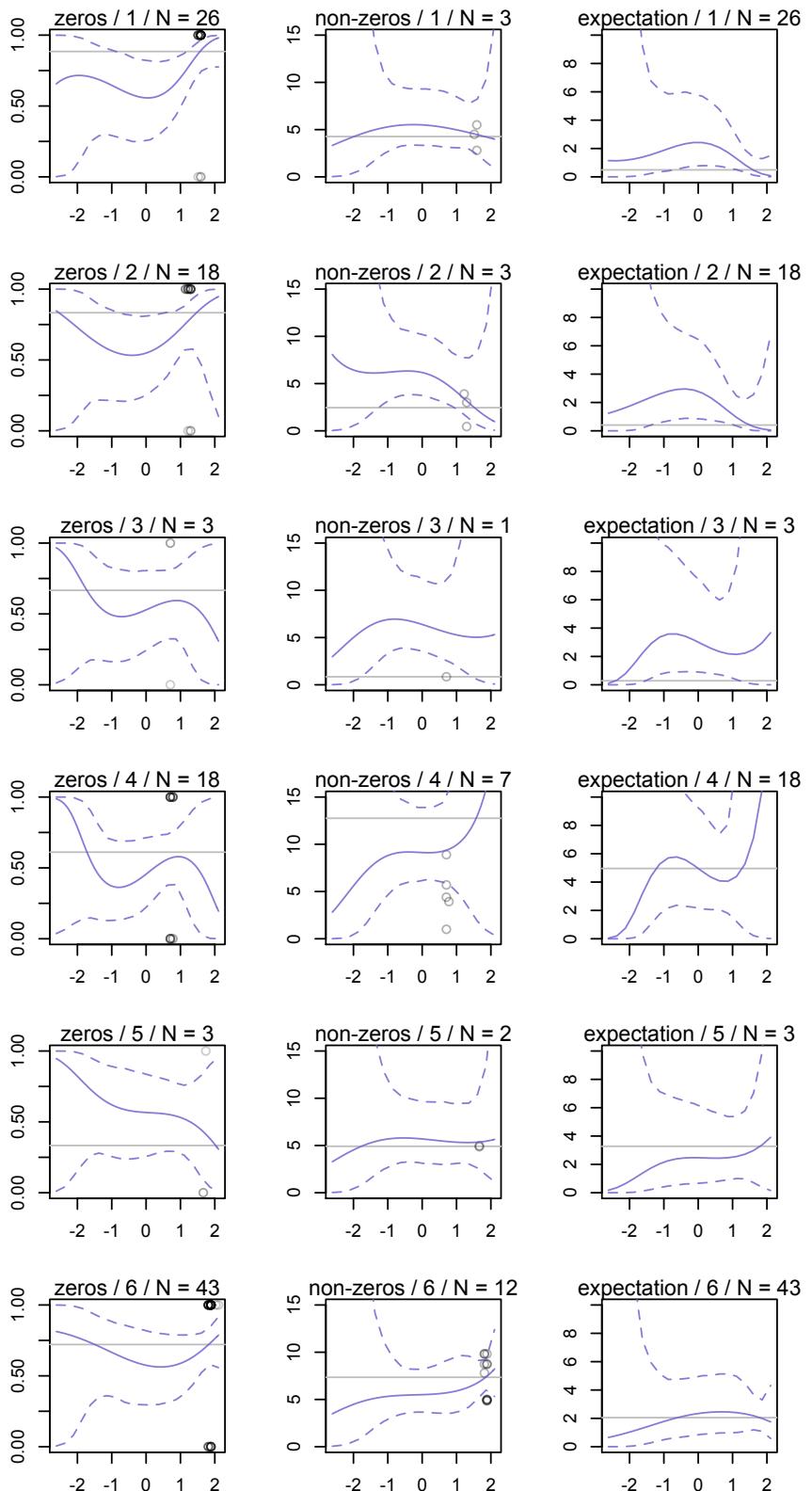
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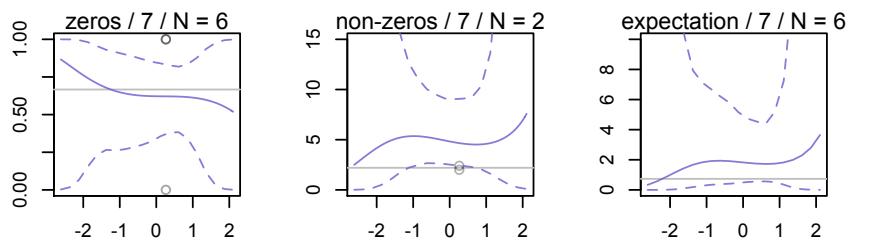
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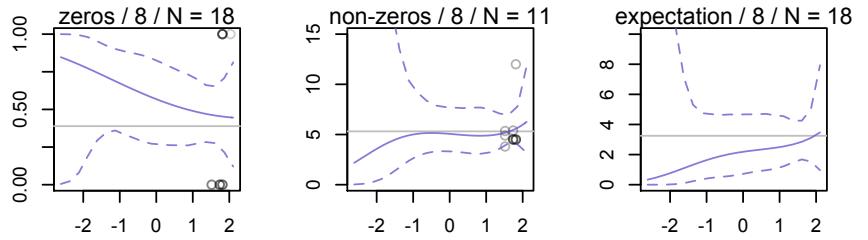
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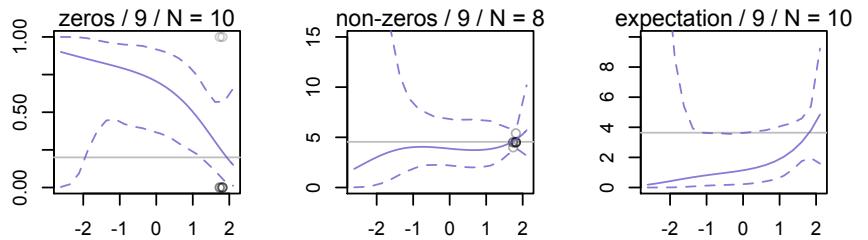
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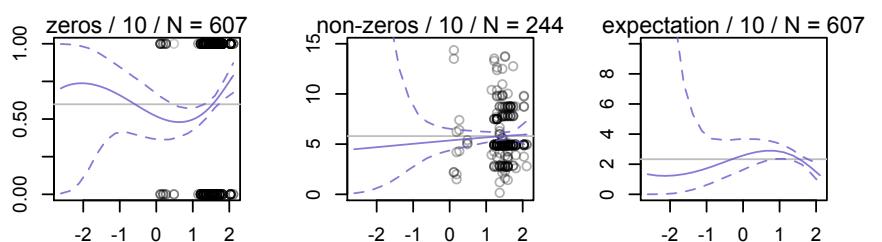
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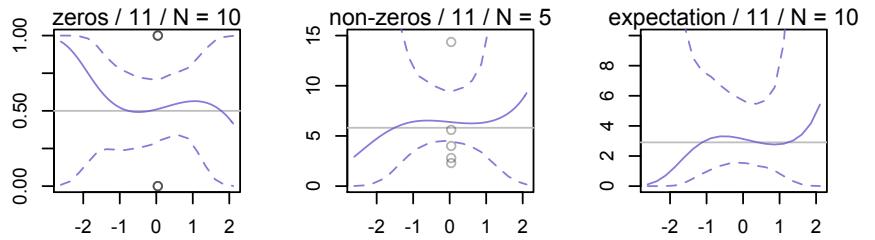
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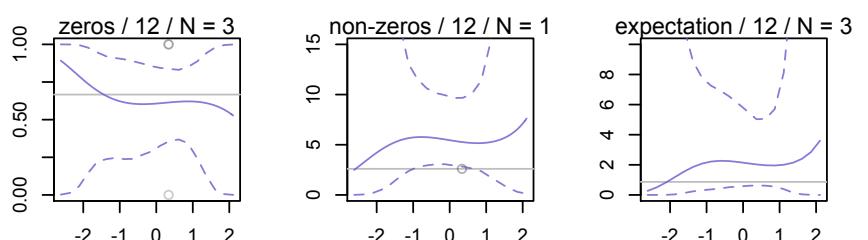
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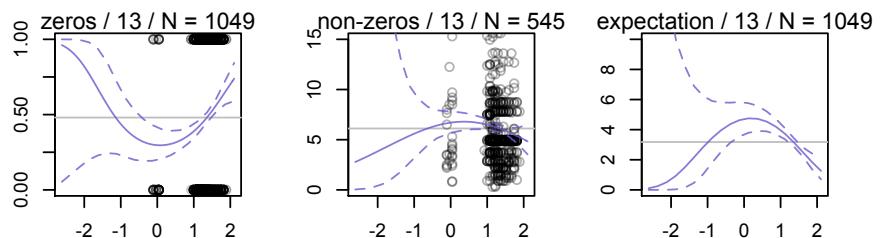
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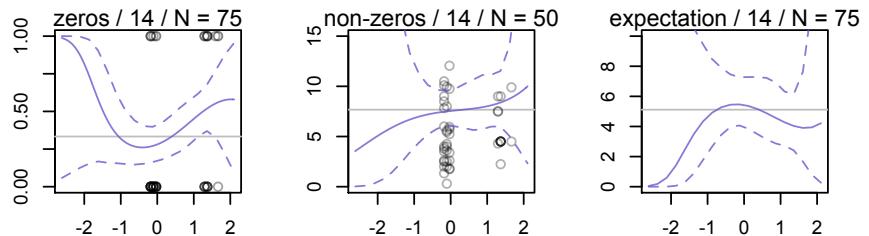
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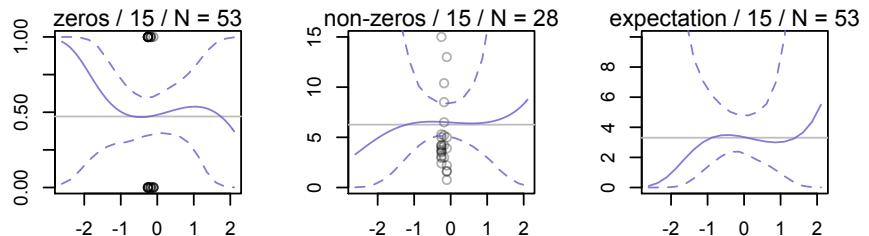
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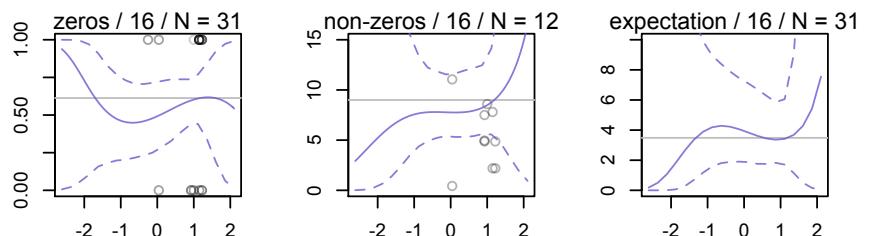
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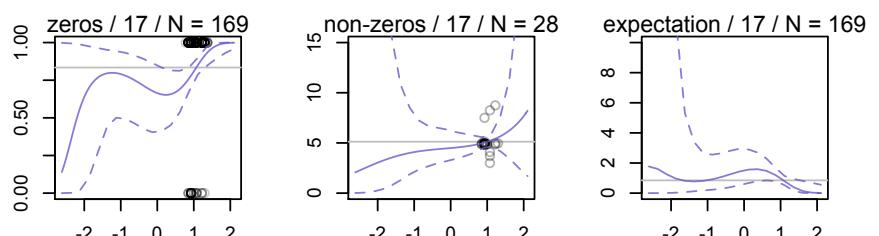
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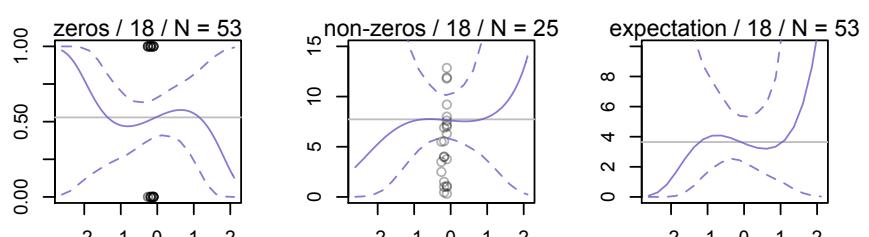
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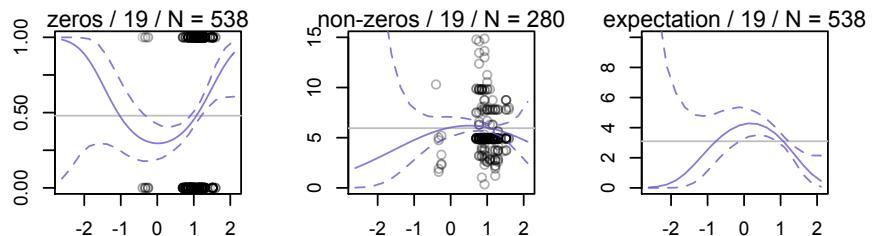
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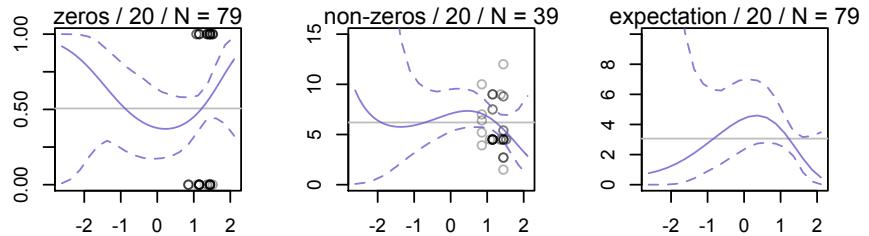
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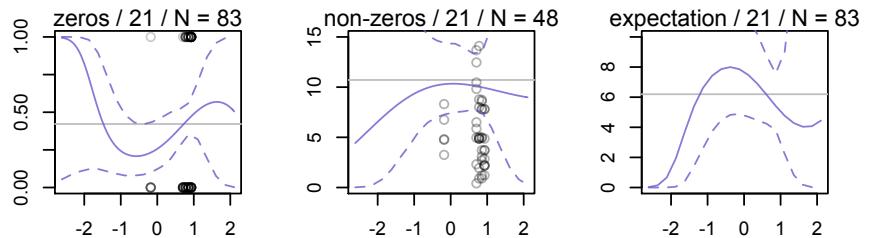
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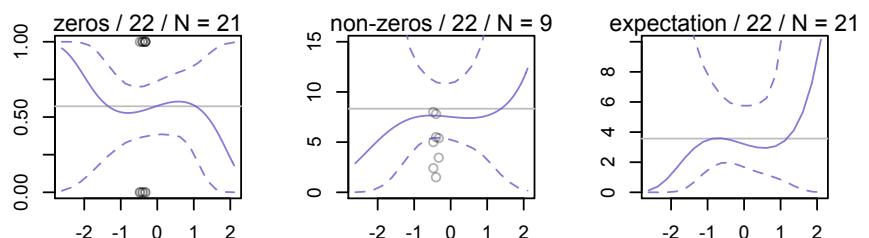
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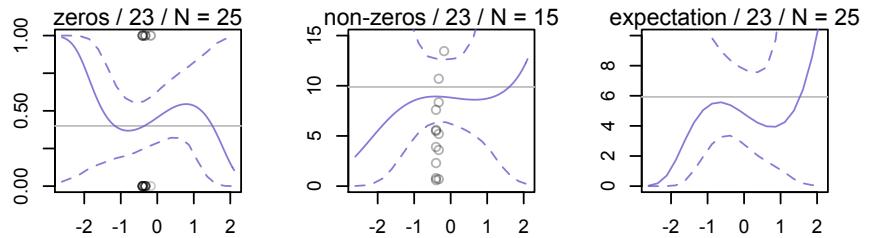
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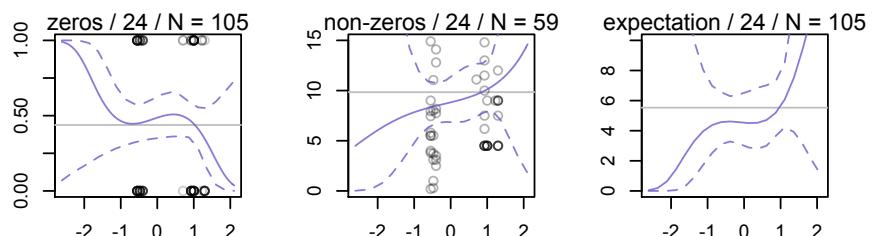
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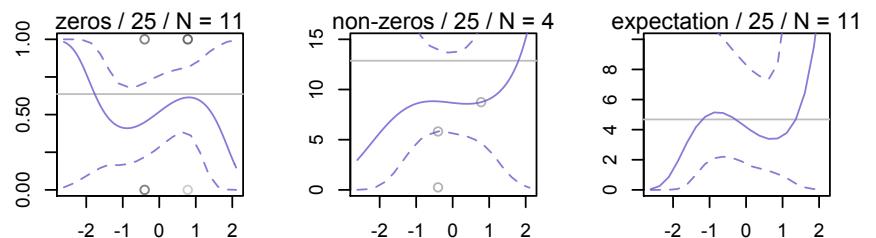
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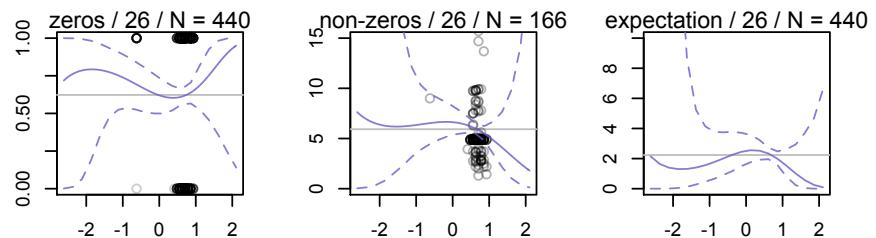
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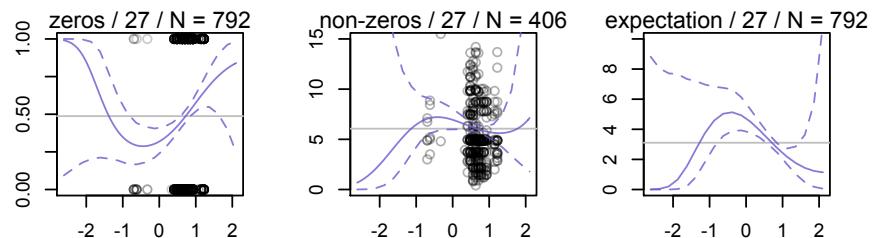
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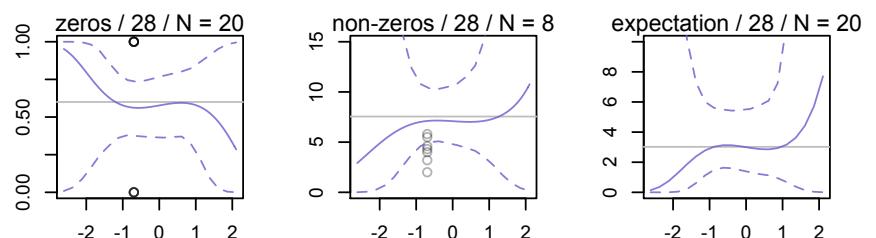
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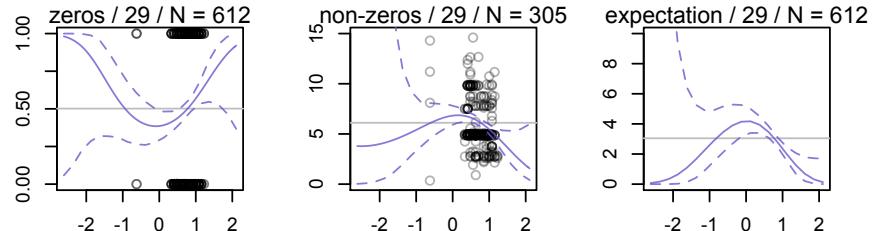
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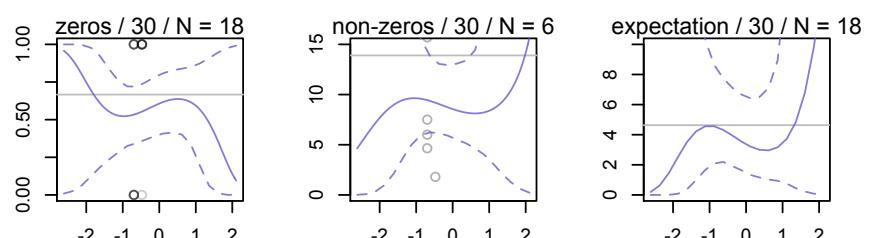
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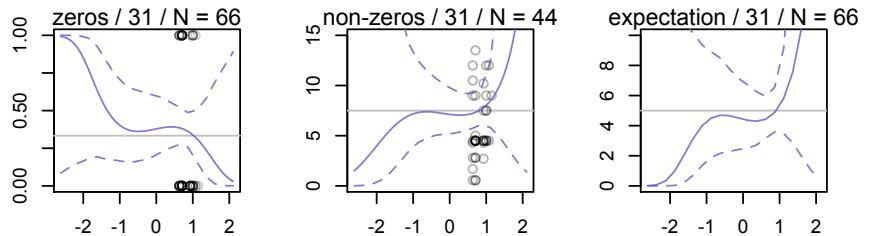
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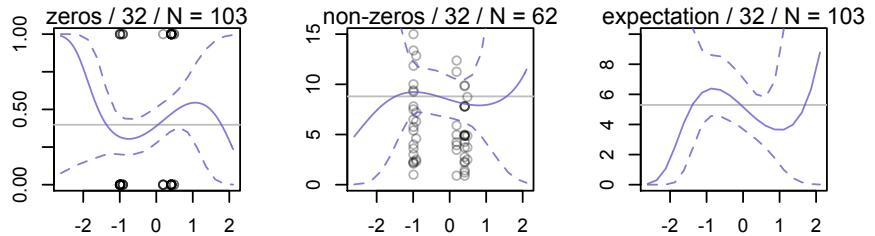
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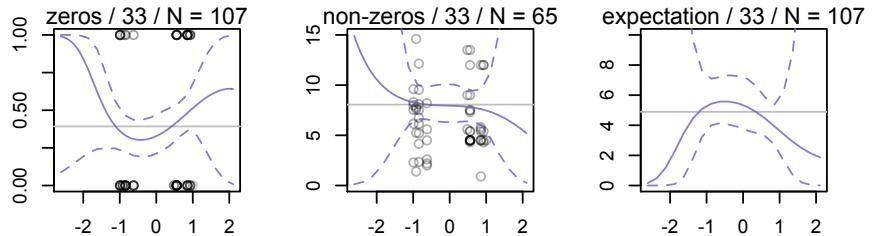
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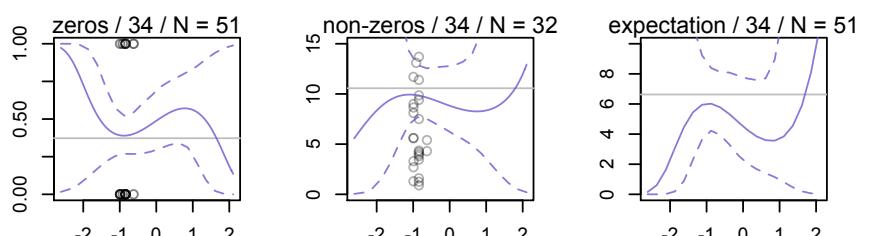
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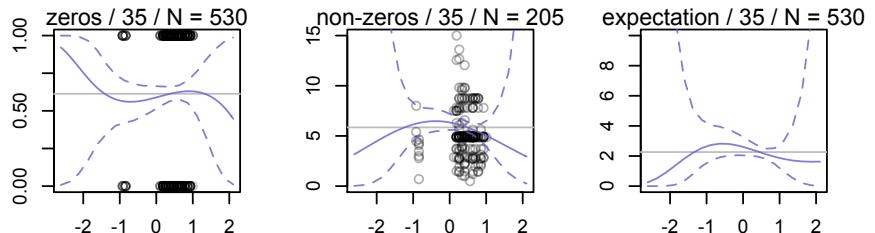
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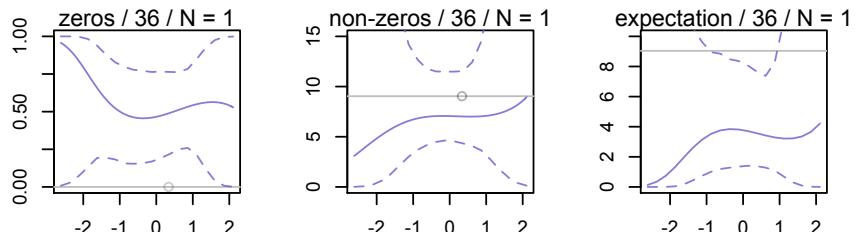
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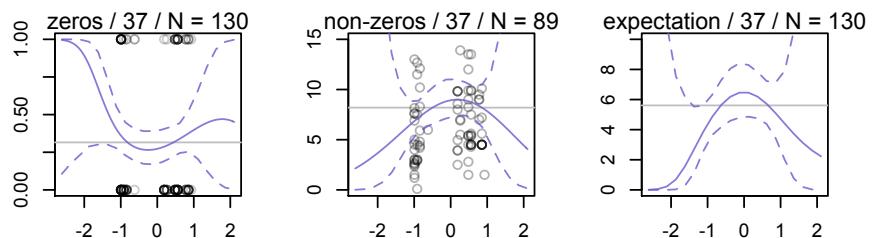
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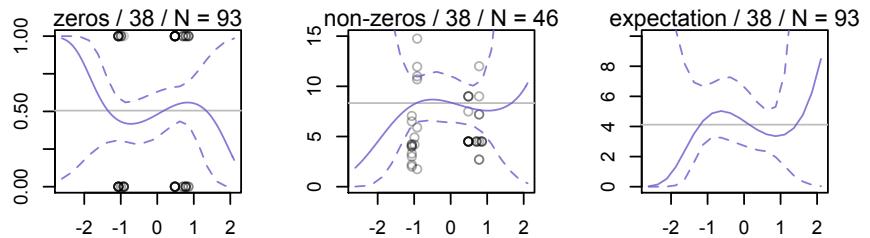
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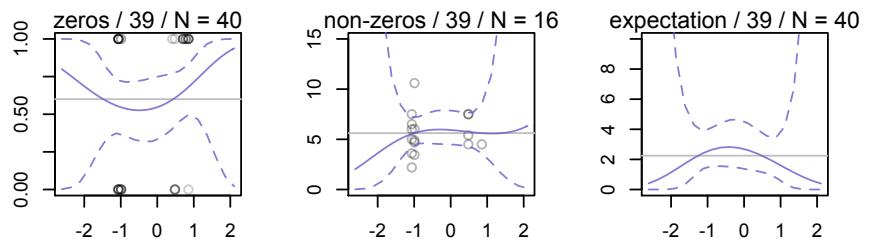
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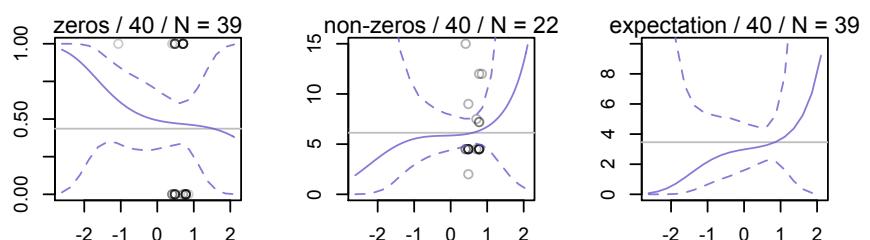
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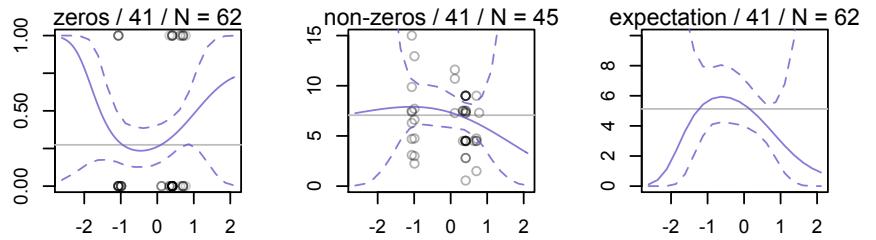
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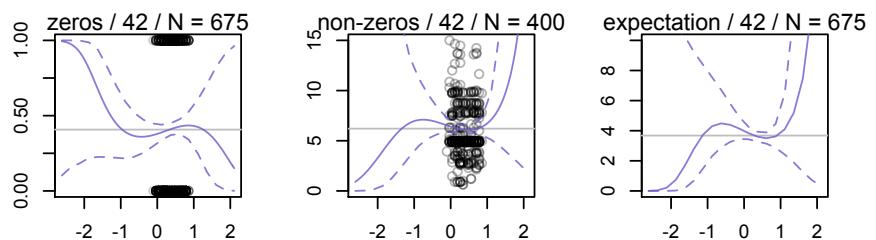
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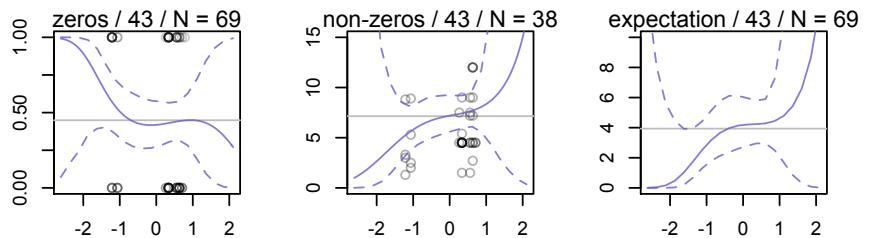
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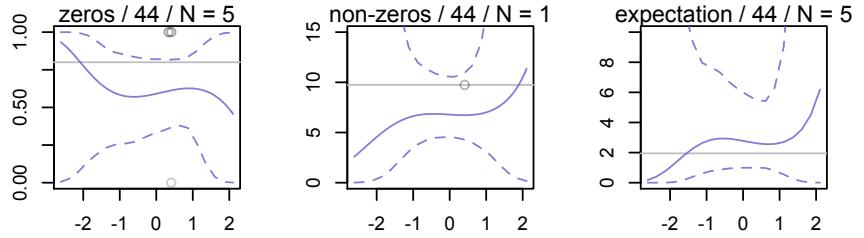
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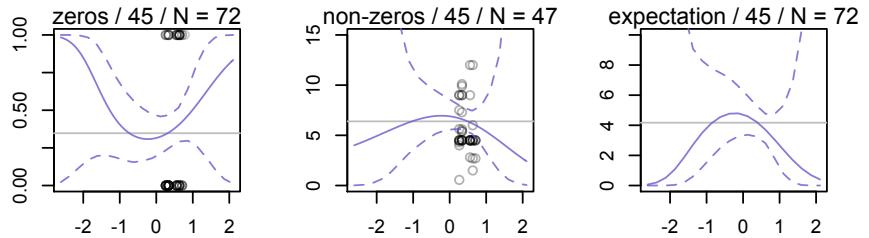
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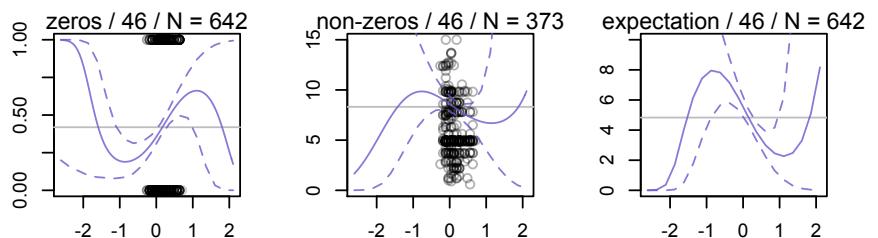
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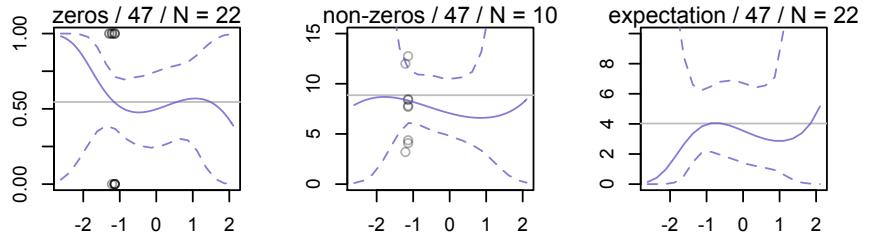
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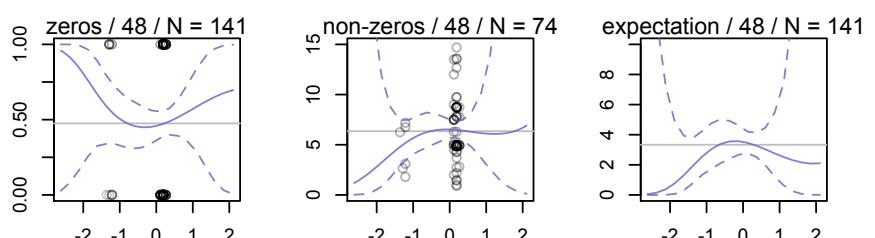
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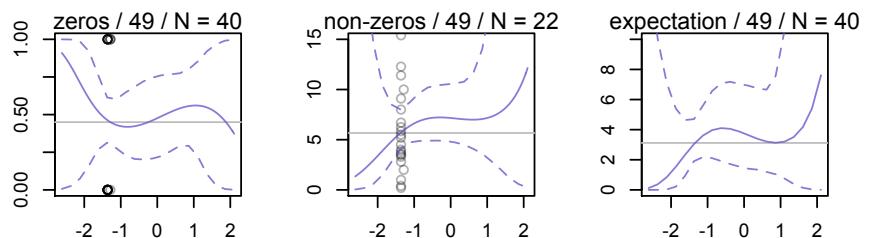
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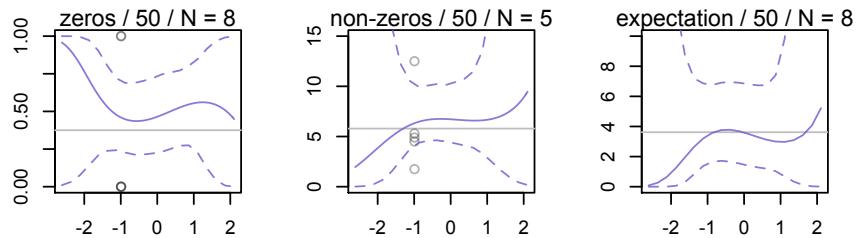
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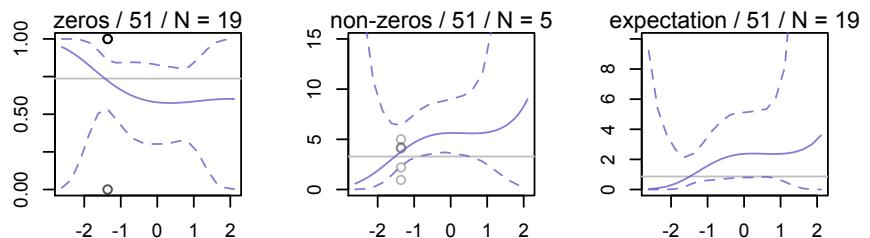
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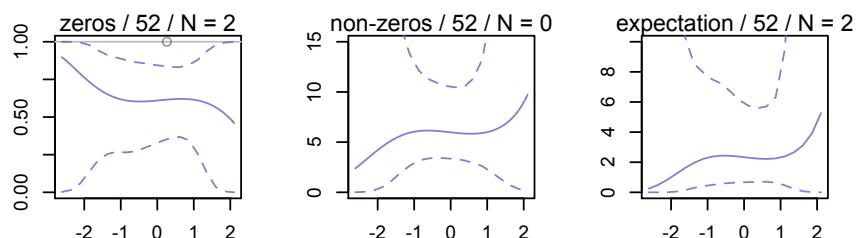
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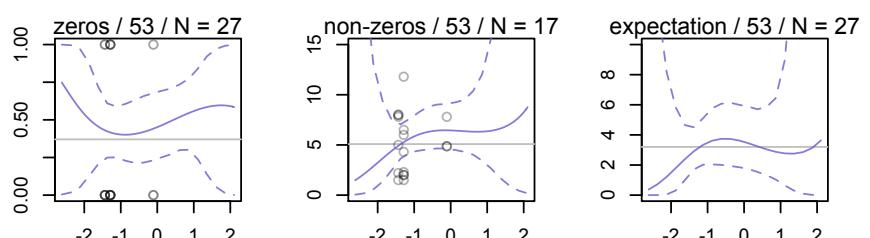
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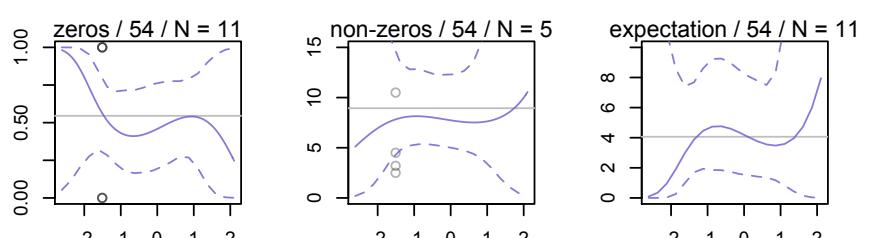
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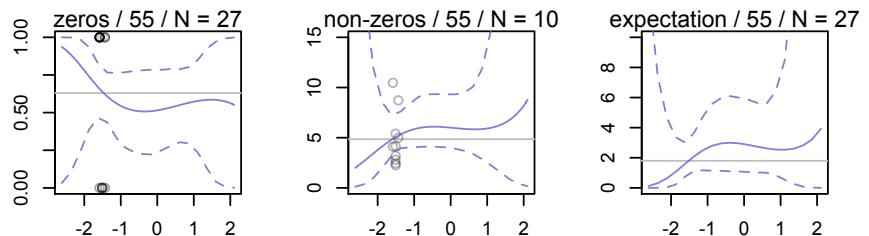
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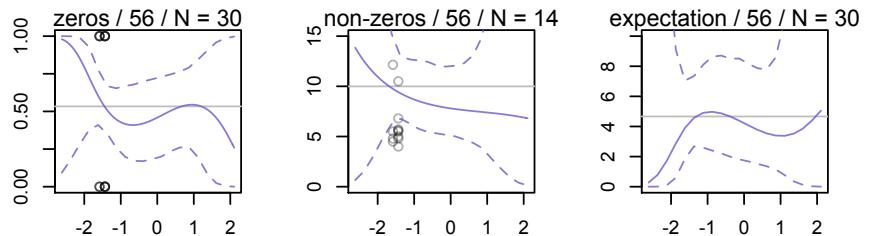
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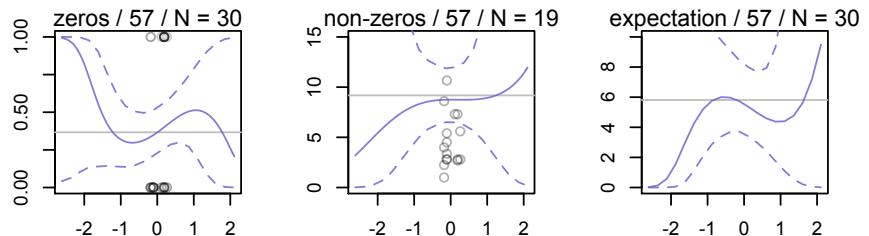
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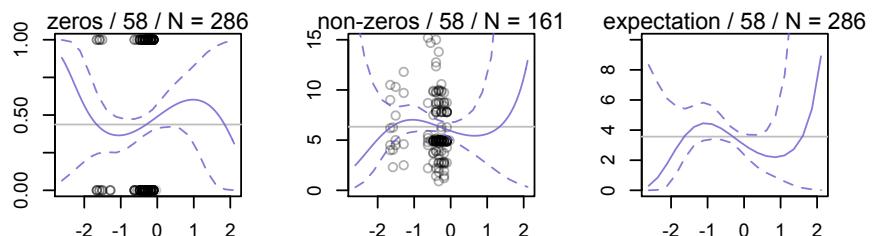
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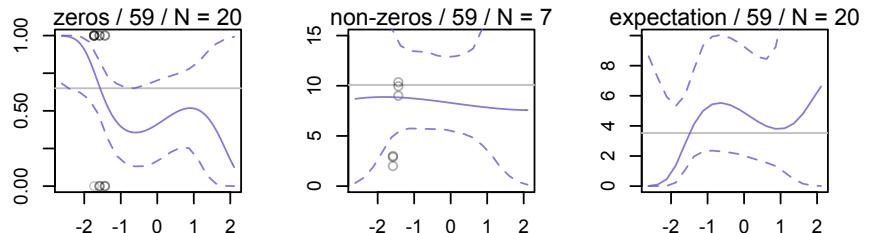
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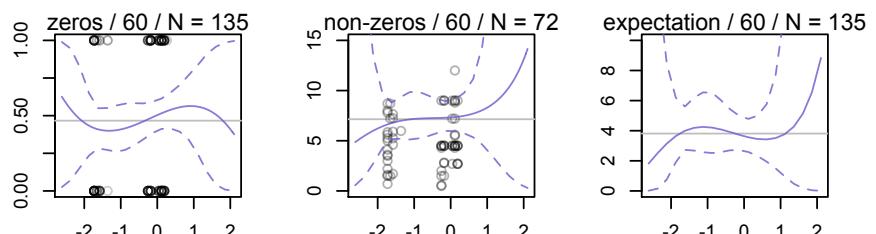
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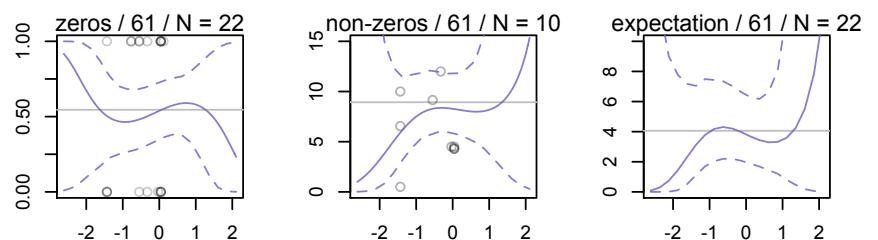
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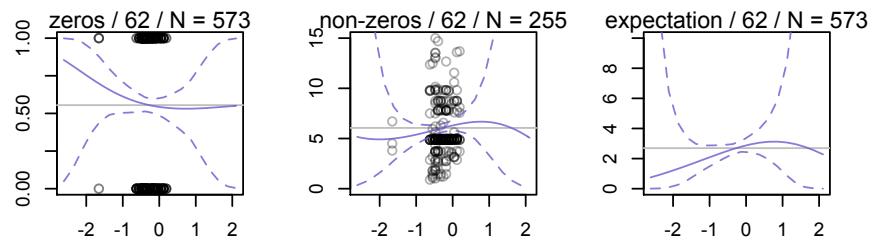
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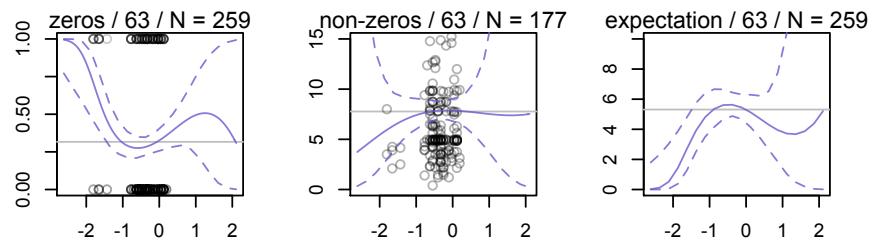
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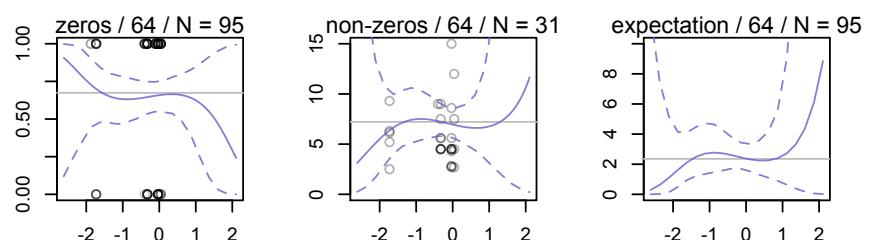
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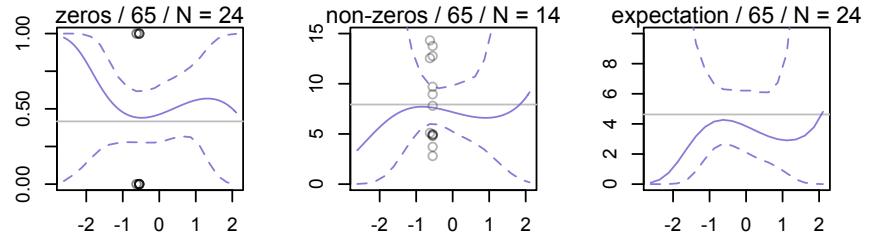
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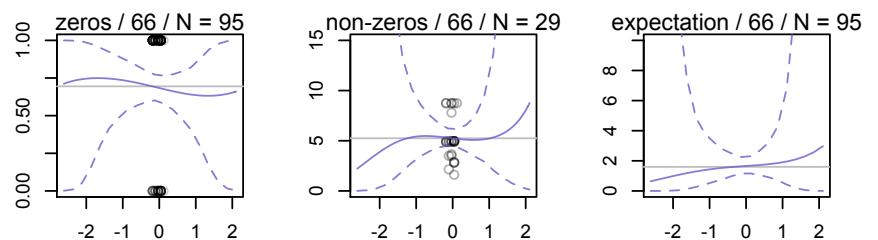
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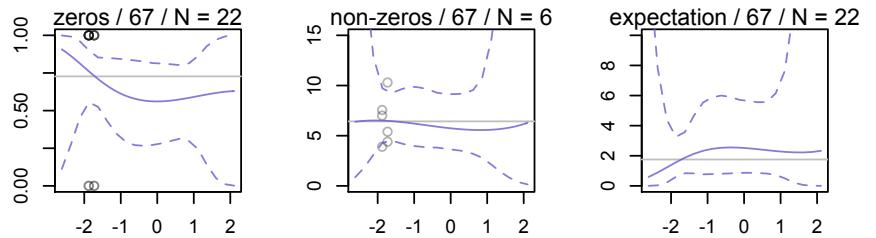
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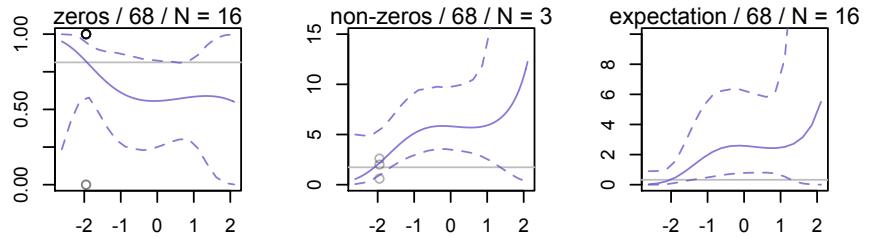
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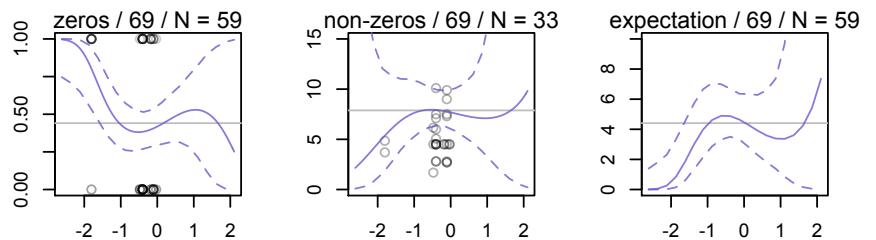
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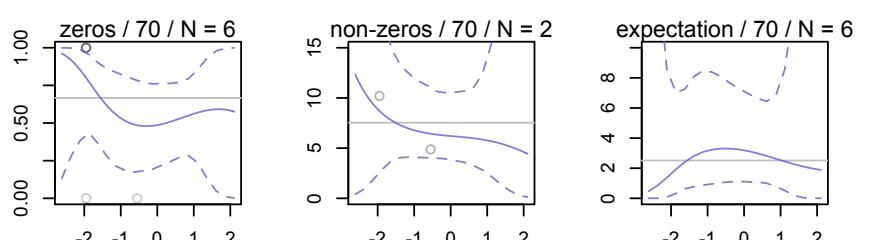
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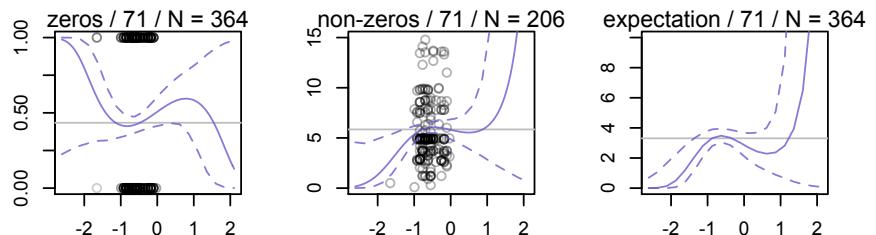
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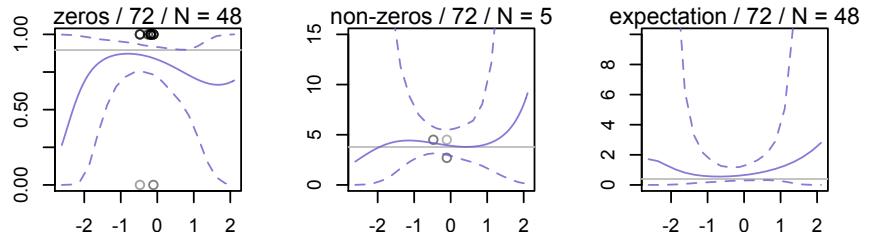
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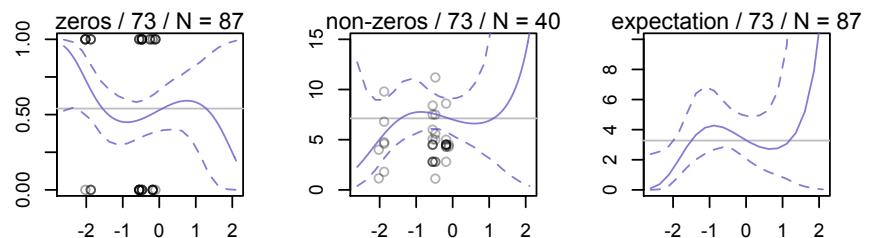
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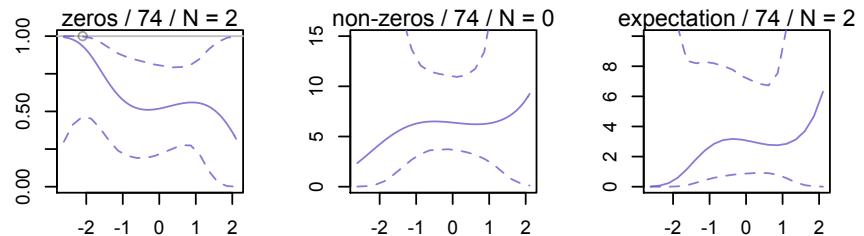
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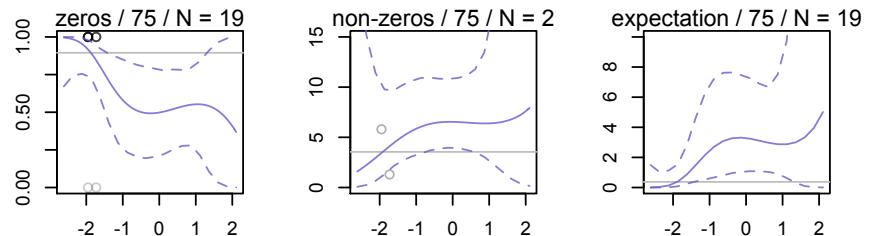
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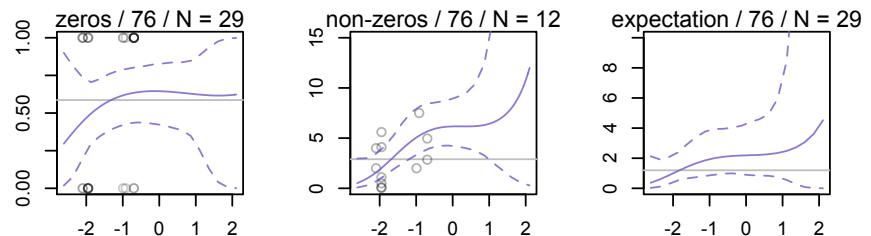
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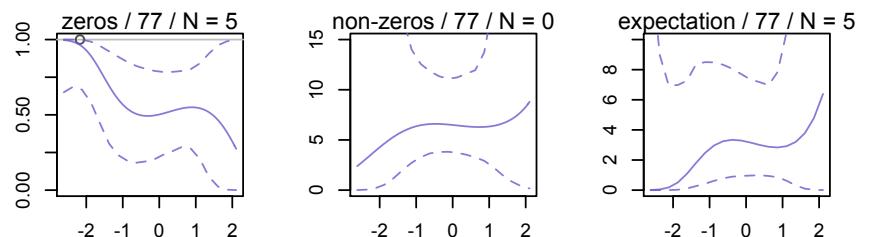
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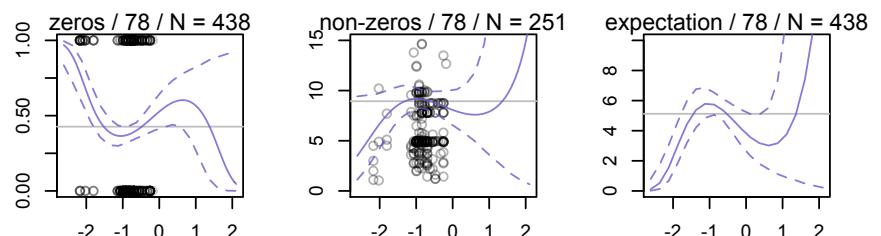
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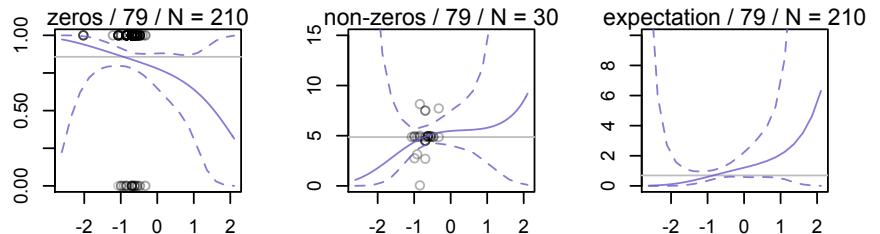
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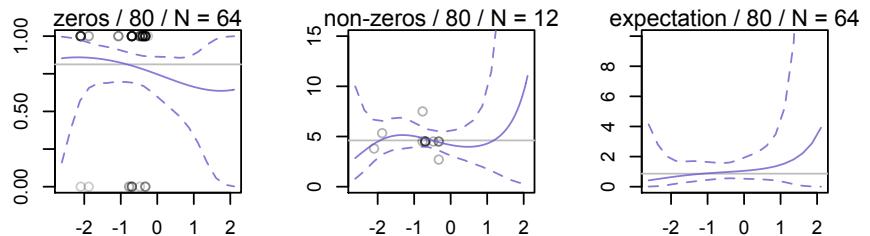
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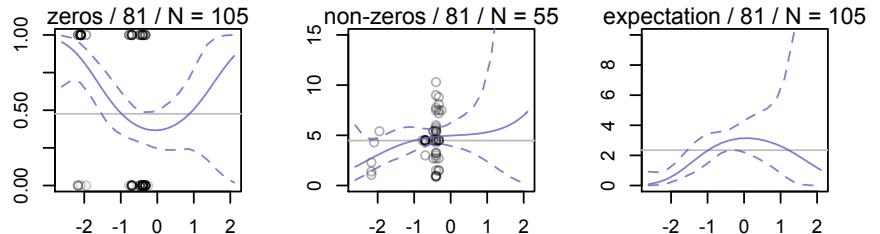
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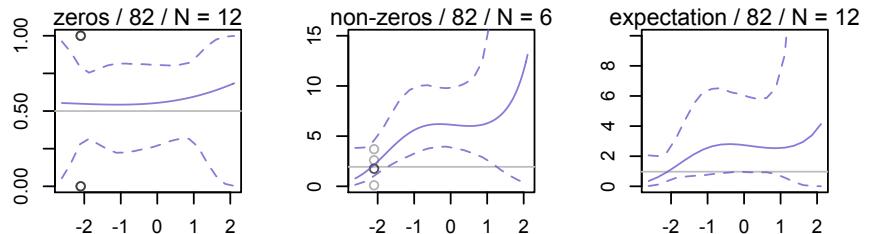
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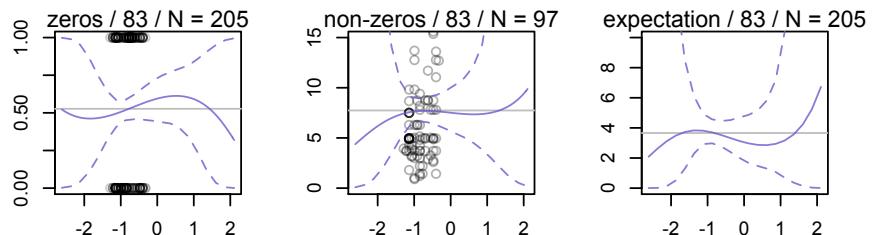
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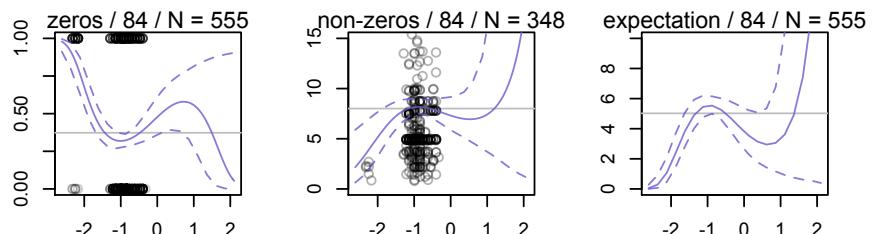
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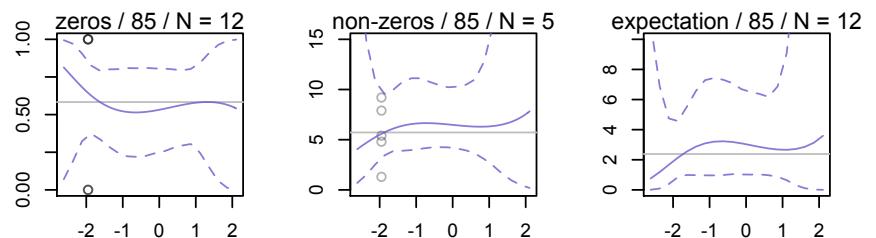
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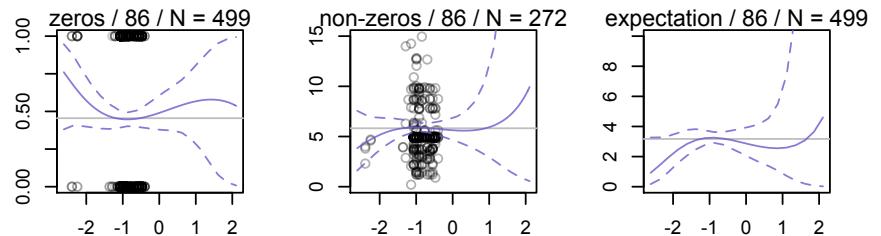
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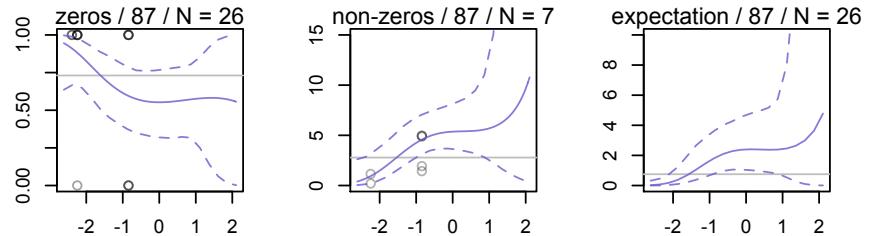
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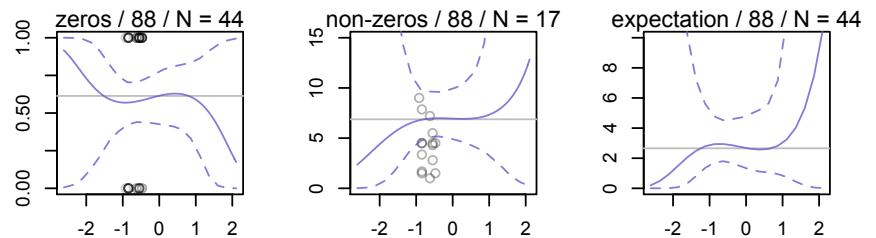
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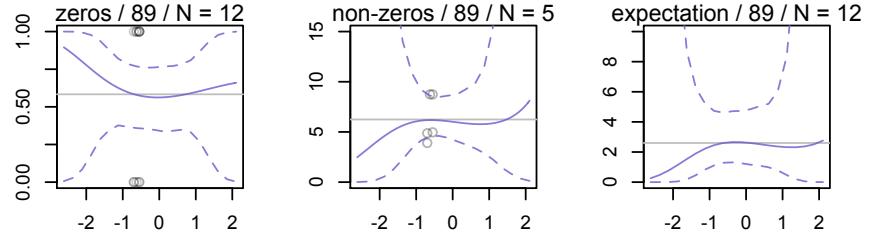
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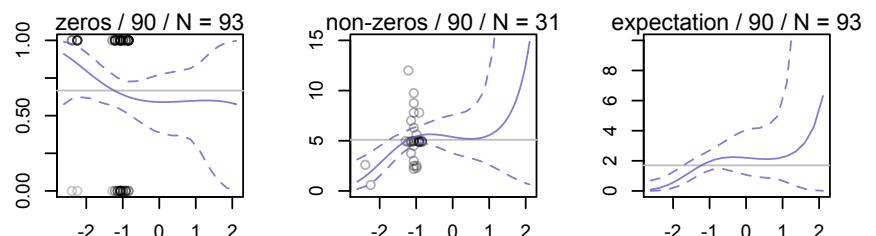
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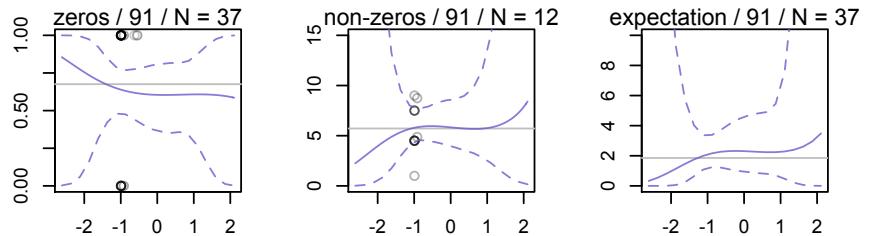
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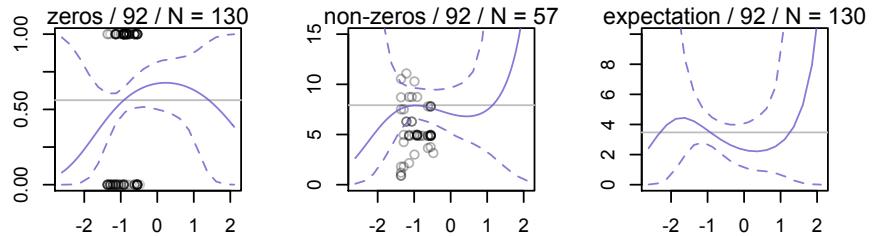
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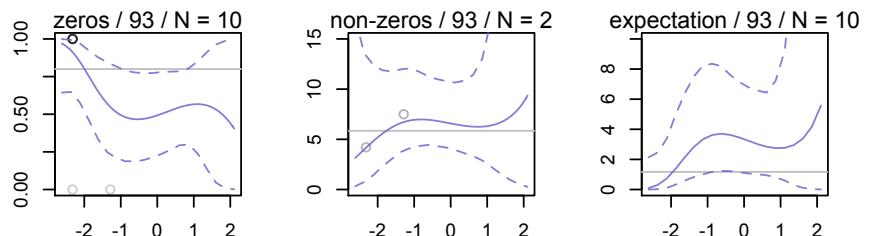
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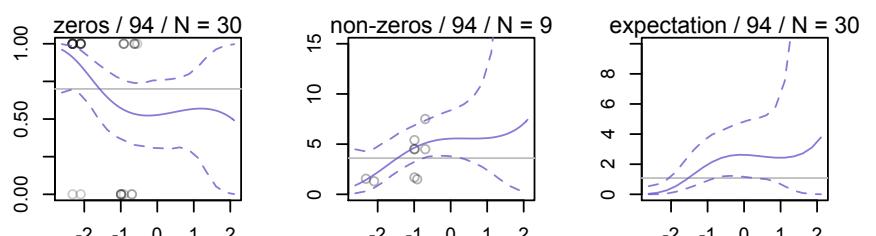
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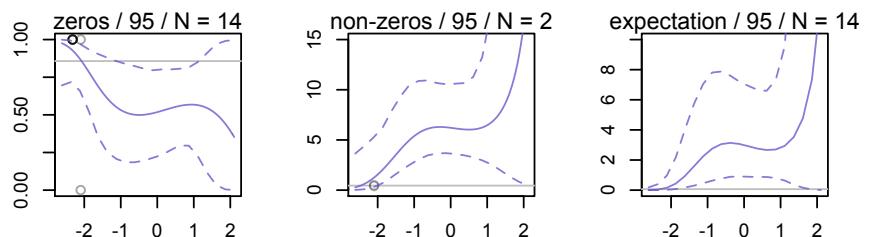
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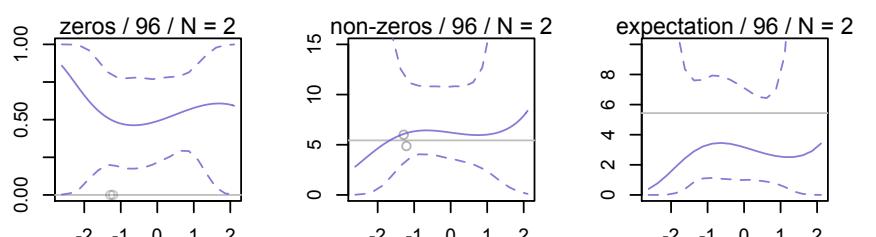
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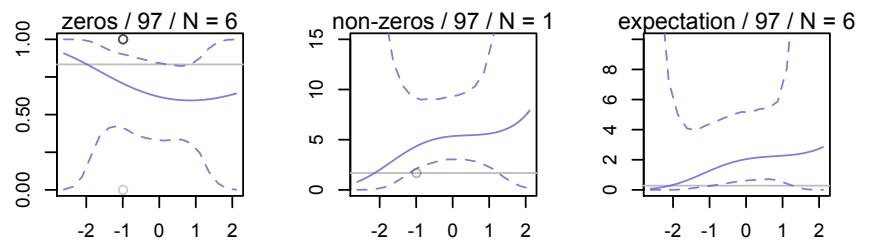
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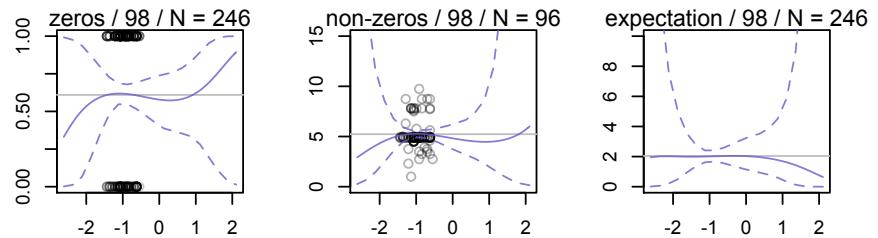
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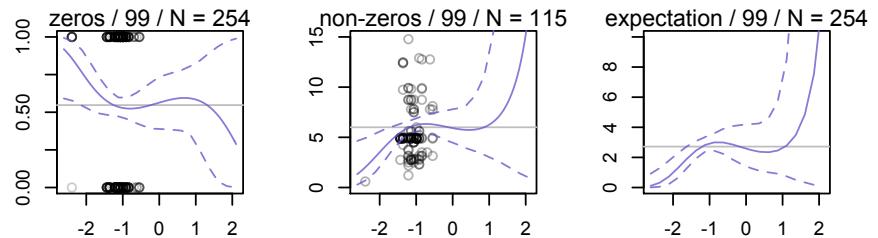
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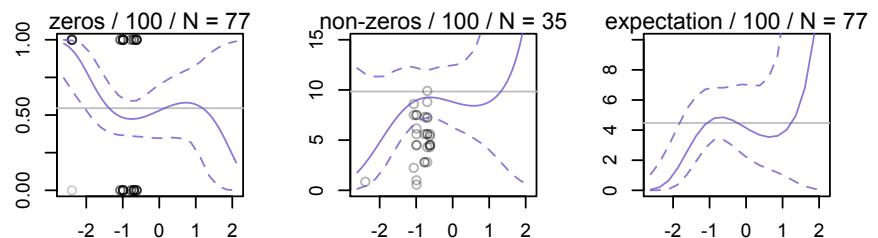
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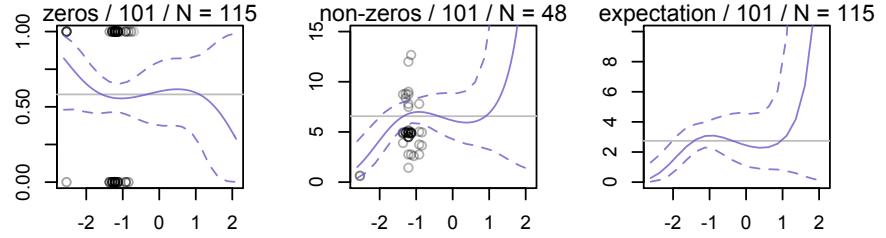
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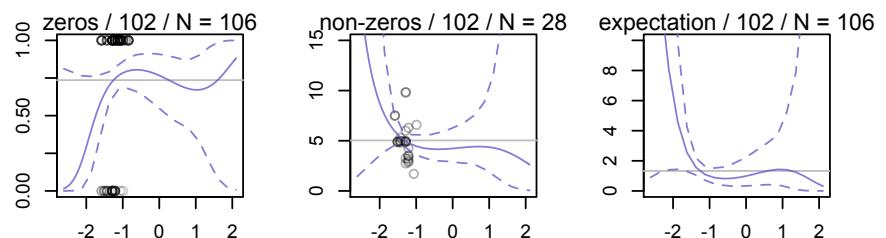
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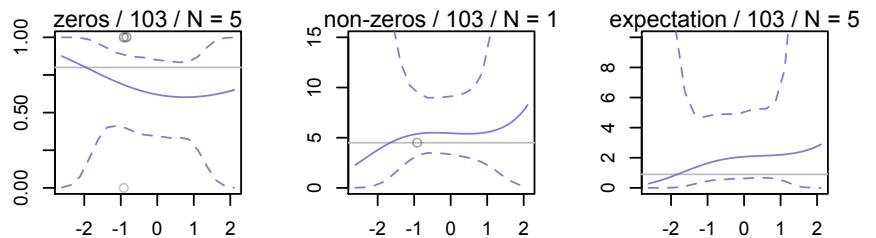
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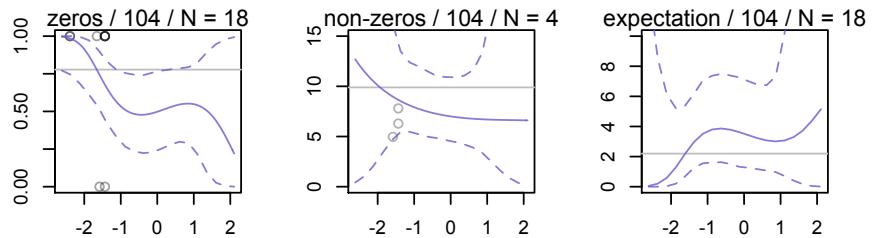
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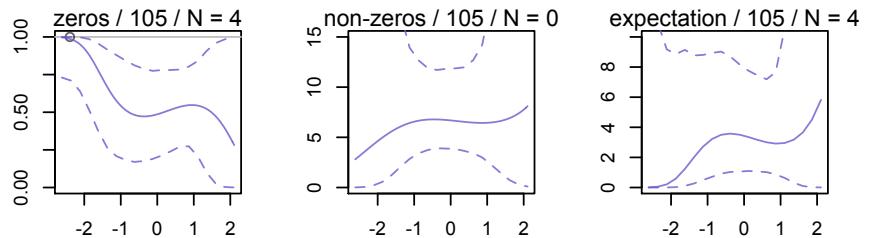
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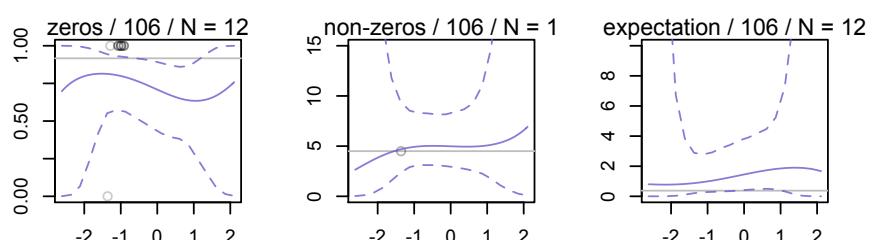
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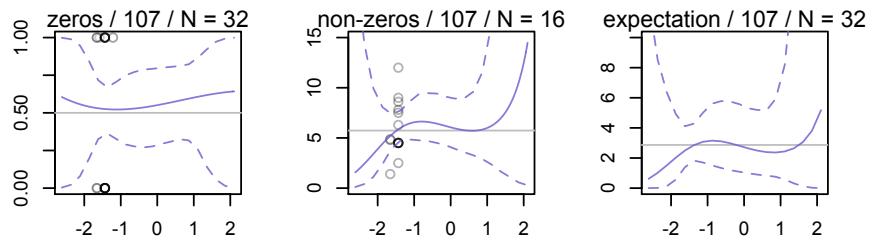
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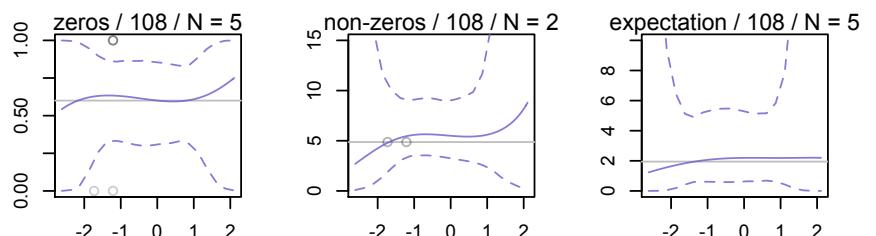
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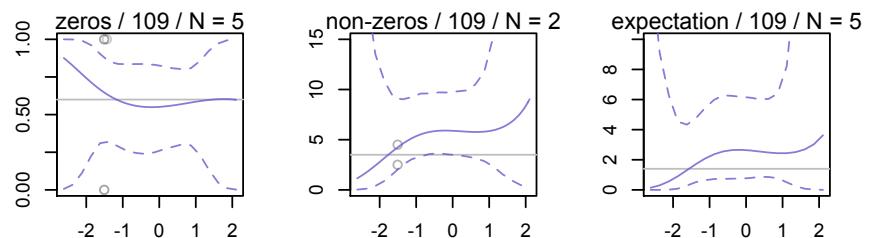
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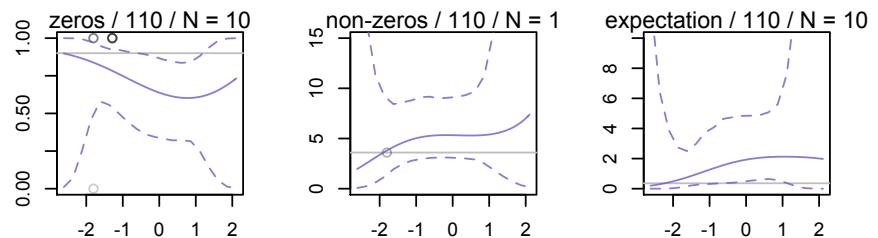
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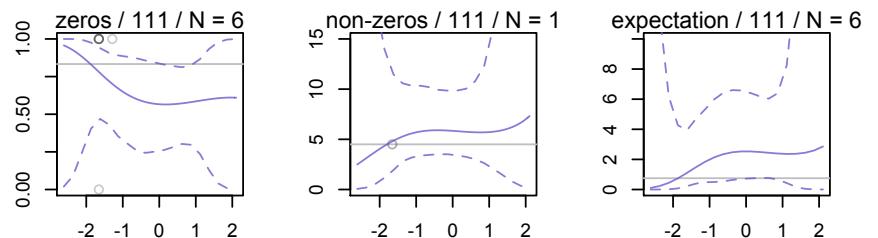
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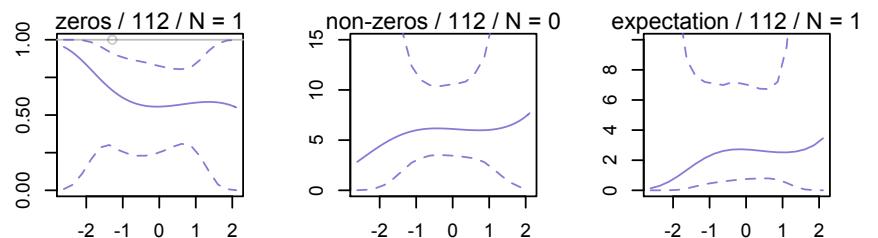
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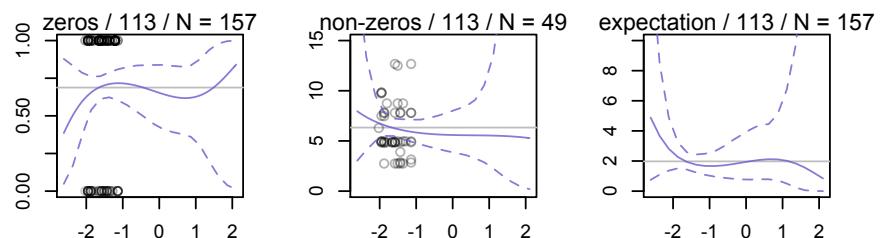
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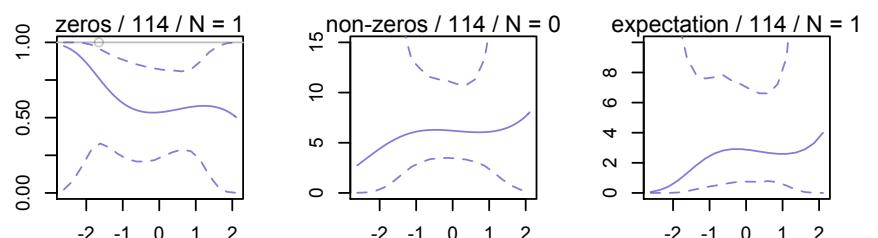
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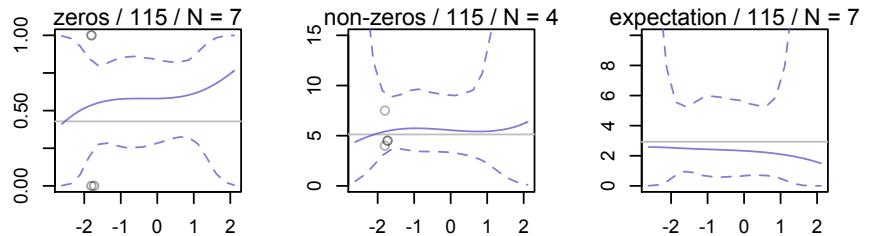
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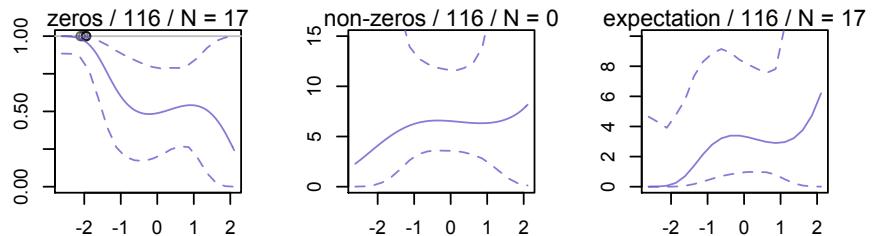
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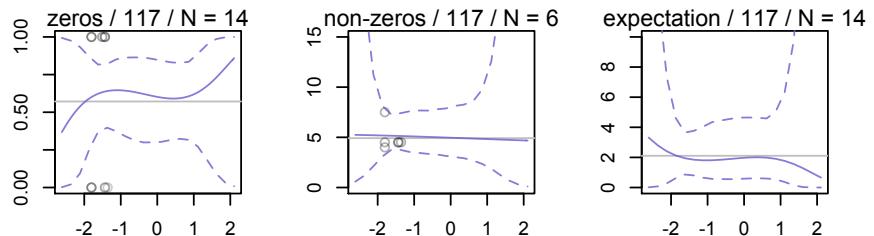
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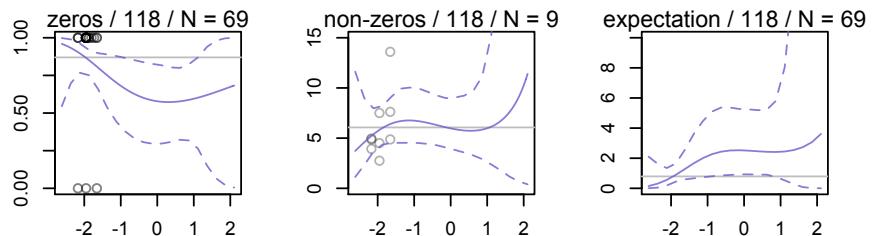
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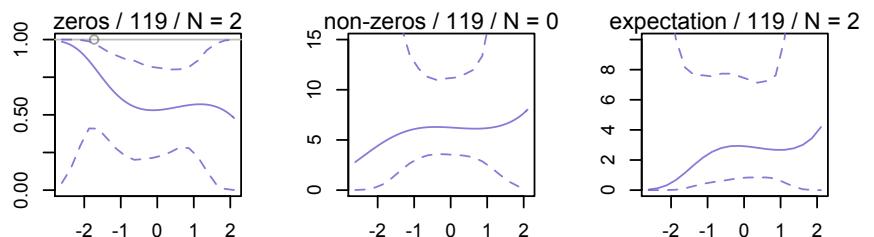
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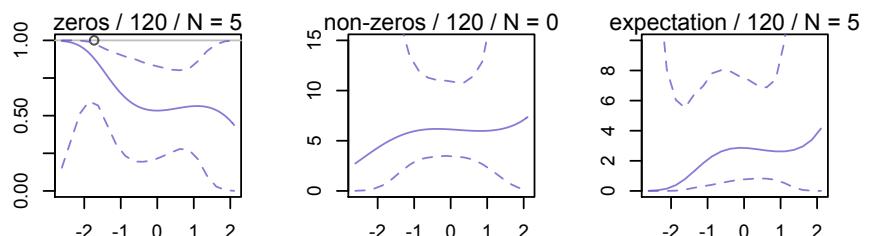
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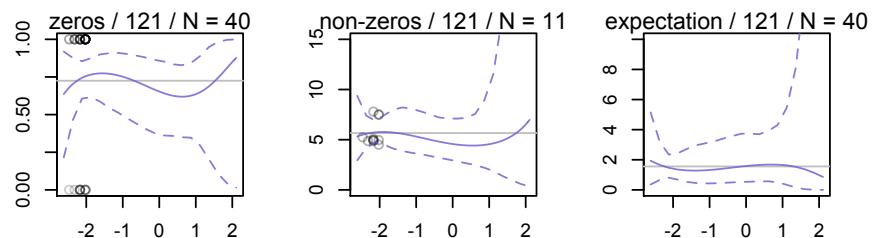
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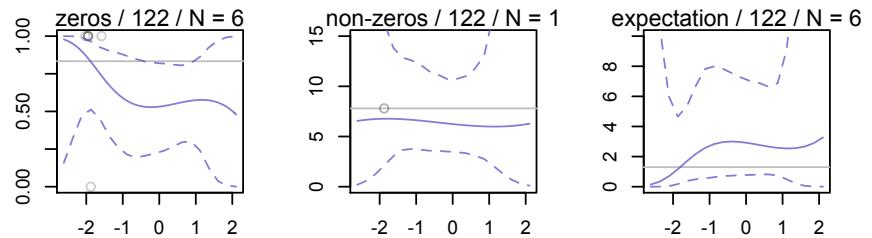
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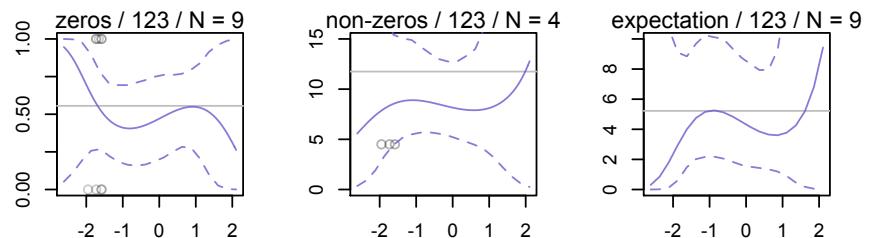
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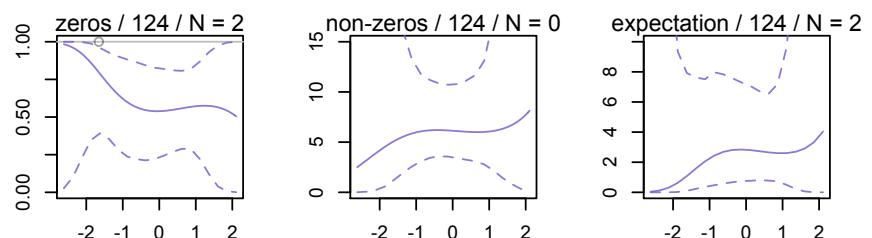
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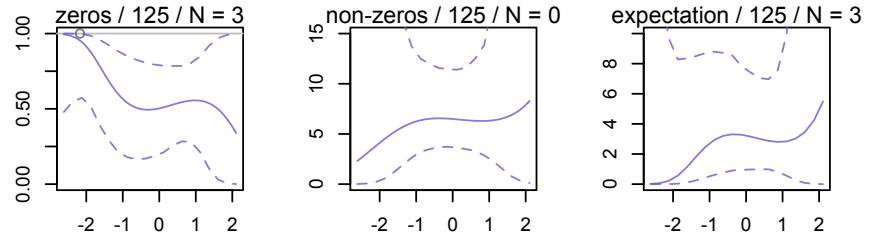
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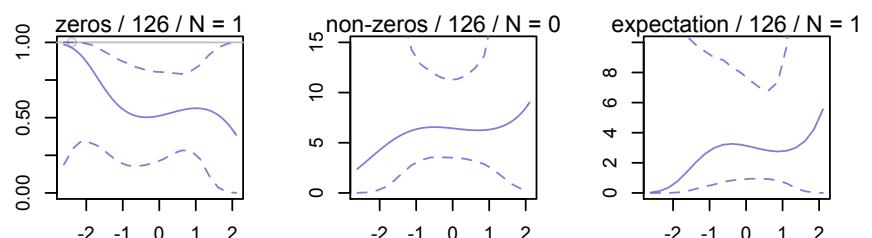
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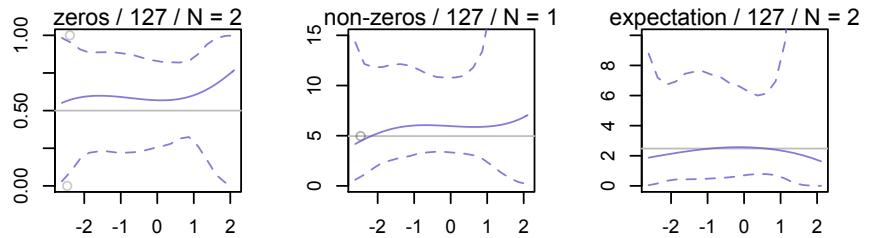
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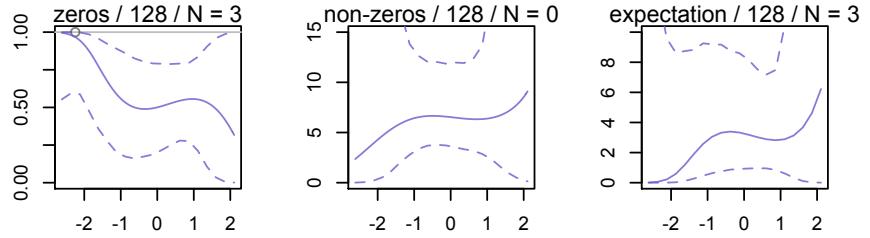
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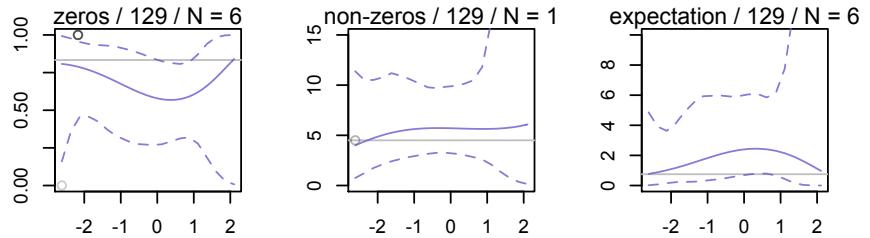
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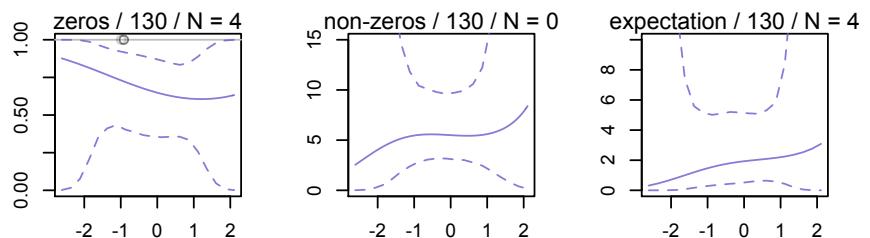
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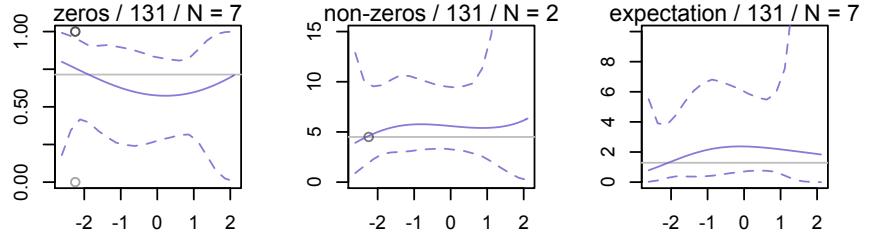
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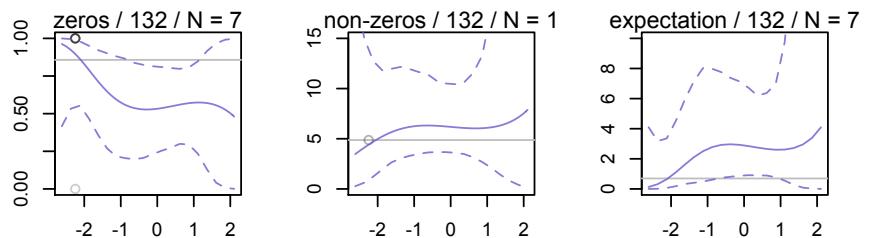
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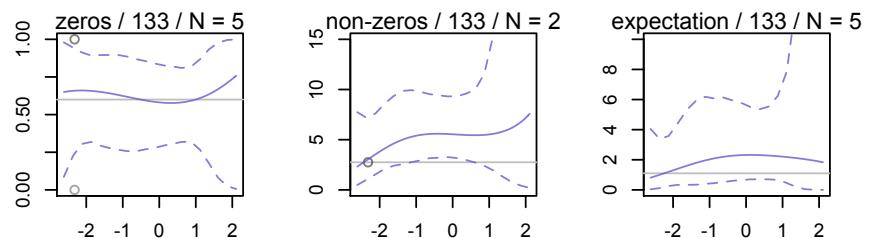
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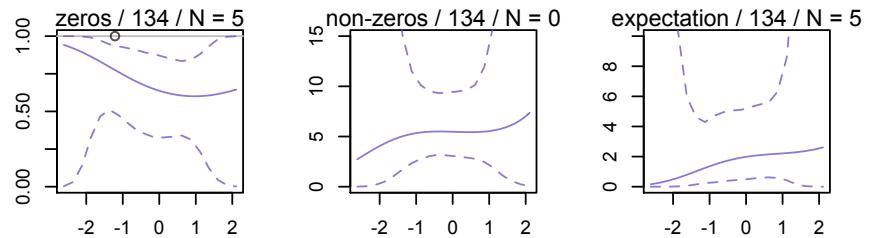
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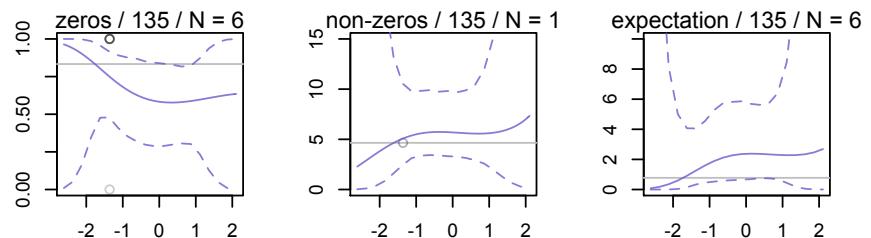
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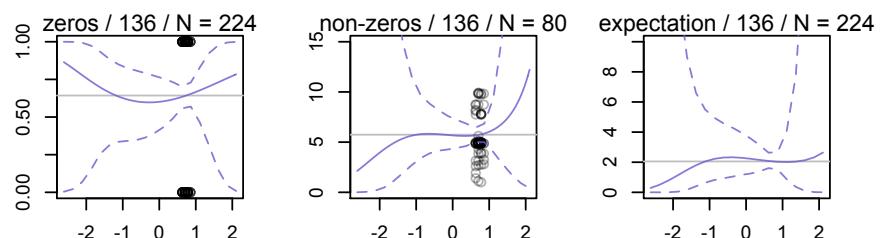
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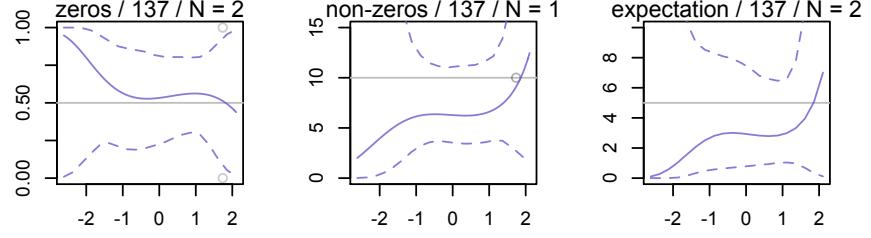
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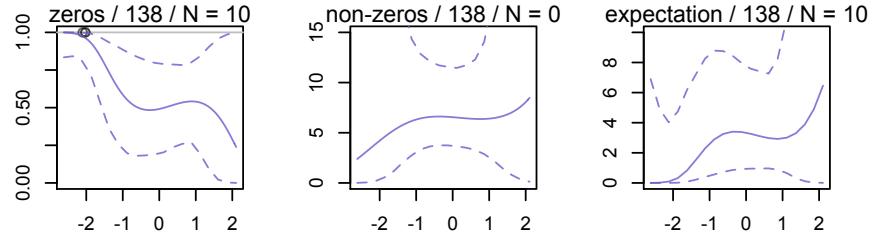
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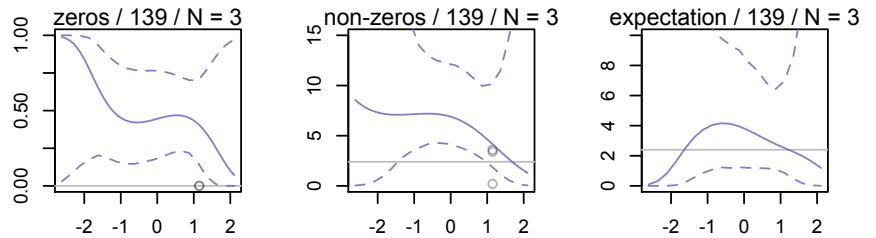
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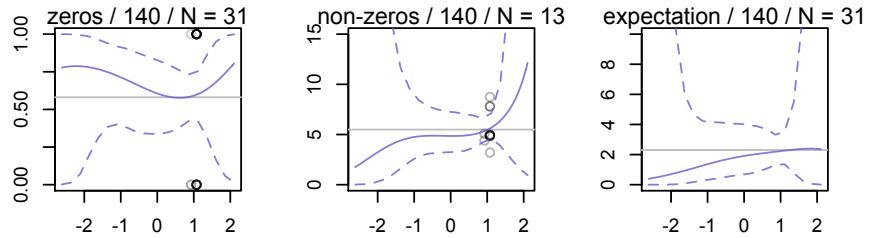
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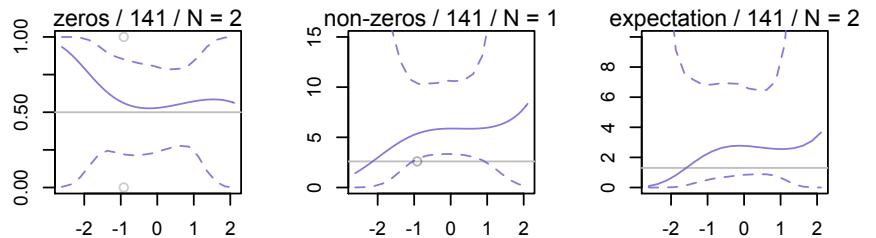
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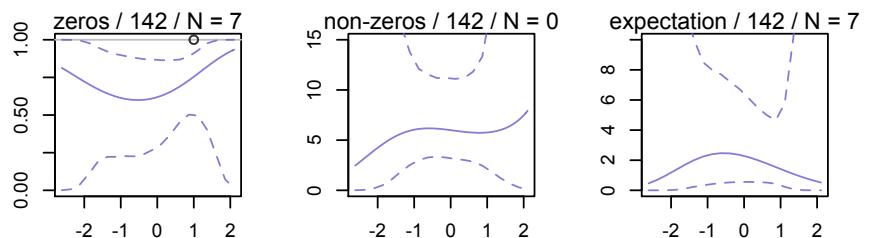
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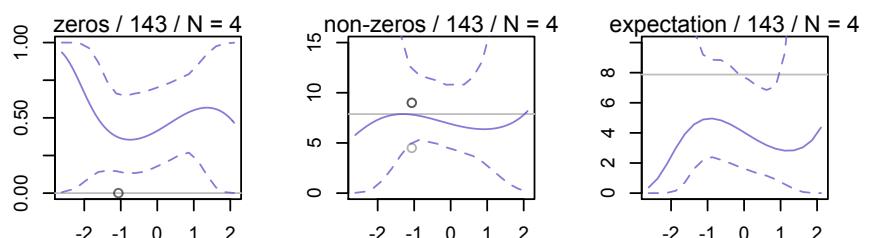
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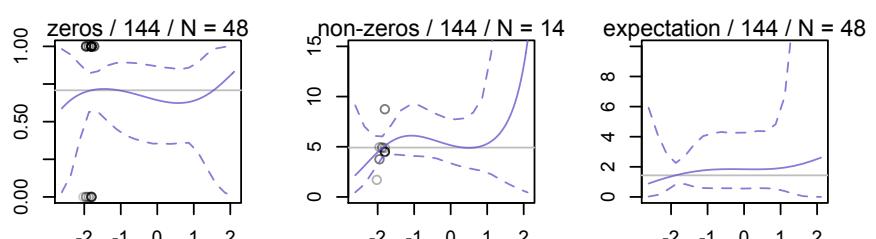
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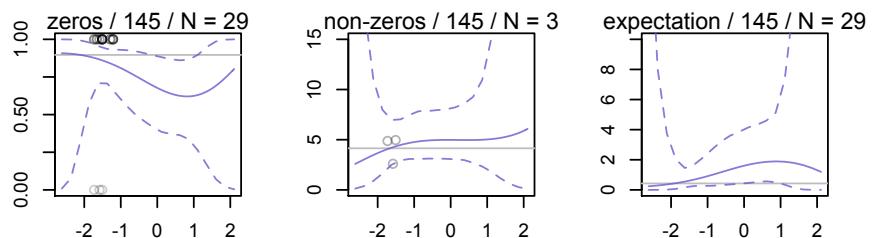
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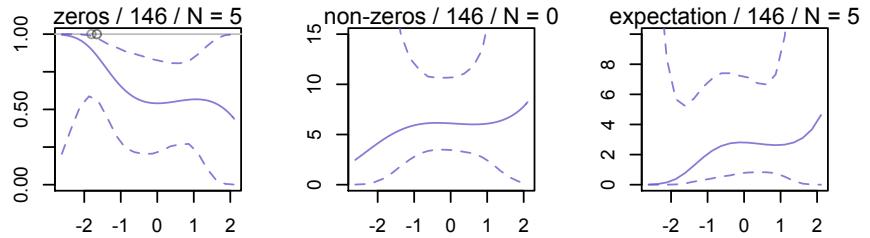
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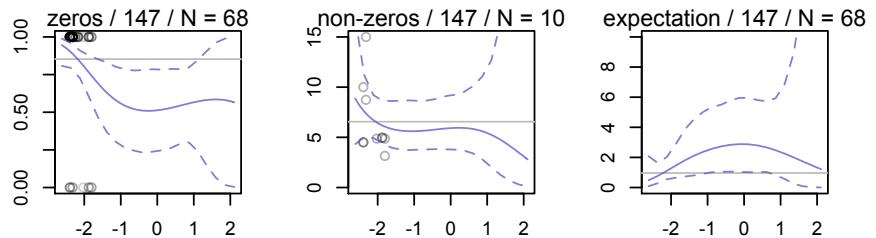
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