STATISTICAL RETHINKING 2026
HOMEWORK B03 SOLUTIONS

The statistical model is similar to the model from last week, but now with a multi-normal prior that contains all of the effects that vary by district. I will model children as an ordered monotonic effect, but this isn't a necessary part of the exercise.

```
library(rethinking)
data(bangladesh)
d <- bangladesh
dat <- list(
    C = d$use.contraception,
    D = as.integer(d$district),
    U = d$urban,
    A = standardize(d$age.centered),
    K = d$living.children )
dat$Kprior <- rep(2,3)

m2 <- ulam(
    alist(
        C ~ bernoulli(p),
        logit(p) <- a[D] + b[D]*U + bA*A +
                    bK[D]*sum( delta_j[1:K] ),

        # ordered monotonic kids
        vector[4]: delta_j <<- append_row( 0 , delta ),
        simplex[3]: delta ~ dirichlet( Kprior ),
        bA ~ normal(0,0.5),

        # non-centered varying effects for D and U
        transpars> vector[61]:a <<- abar[1] + v[,1],
        transpars> vector[61]:b <<- abar[2] + v[,2],
        transpars> vector[61]:bK <<- abar[3] + v[,3],
        transpars> matrix[61,3]:v <-
            compose_noncentered( sigma , L_Rho , Z ),

        # non-centered priors
        matrix[3,61]:Z ~ normal( 0 , 1 ),
        vector[3]:abar ~ normal(0,1),
        cholesky_factor_corr[3]:L_Rho ~ lkj_corr_cholesky( 4 ),
        vector[3]:sigma ~ exponential(1),

        # convert Cholesky to Corr matrix
```

```
        gq> matrix[3,3]:Rho <<- Chol_to_Corr(L_Rho)
    ) , data=dat , chains=4 , cores=4 )

precis(m2,depth=2)
```

```
delta[1]   0.74 0.08  0.61  0.85 1.00  2466.75  1731.44
delta[2]   0.16 0.08  0.05  0.29 1.00  2055.51  1251.46
delta[3]   0.10 0.05  0.03  0.20 1.00  2443.54  1419.38
bA        -0.25 0.07 -0.35 -0.15 1.01  2053.22  1422.49
abar[1]   -1.64 0.16 -1.89 -1.39 1.00  1214.65  1440.34
abar[2]    0.74 0.17  0.48  1.02 1.00  1358.22  1284.77
abar[3]    1.36 0.18  1.08  1.64 1.00  1596.27  1513.05
sigma[1]   0.61 0.14  0.41  0.85 1.00   659.24  1172.66
sigma[2]   0.72 0.21  0.38  1.05 1.01   515.79   411.42
sigma[3]   0.38 0.21  0.04  0.72 1.02   224.38   507.43
bK[1]      1.22 0.37  0.60  1.76 1.00  1333.10  1533.65
bK[2]      1.45 0.43  0.84  2.17 1.00  1743.85  1365.46
...
a[60]     -2.12 0.47 -2.89 -1.42 1.00  2780.43  1359.73
a[61]     -1.98 0.48 -2.74 -1.20 1.00  1179.28  1292.42
```

I abbreviated the vector output, but you get the idea. There isn't much to do with all this output, except confirm that the chains sampled well and that the individual mean effects and standard deviations are very similar to the previous model.

Let's look at the correlation matrix:

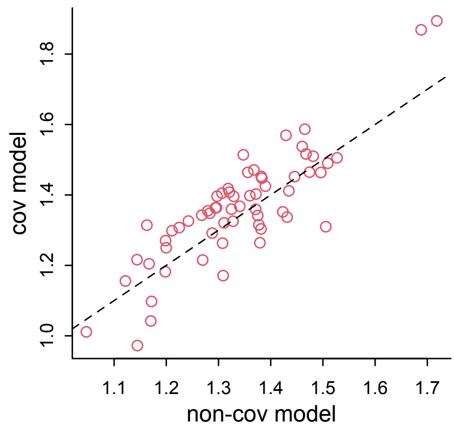
```
precis(m2,depth=3,pars="Rho")
```

```
           mean   sd  5.5% 94.5% rhat ess_bulk ess_tail
Rho[1,1]   1.00 0.00  1.00  1.00   NA       NA       NA
Rho[2,1]  -0.53 0.20 -0.79 -0.17 1.00   866.92   970.79
Rho[3,1]  -0.23 0.30 -0.67  0.30 1.00   696.90  1331.71
Rho[1,2]  -0.53 0.20 -0.79 -0.17 1.00   866.92   970.79
Rho[2,2]   1.00 0.00  1.00  1.00   NA       NA       NA
Rho[3,2]   0.10 0.29 -0.40  0.55 1.01  1178.82  1509.05
Rho[1,3]  -0.23 0.30 -0.67  0.30 1.00   696.90  1331.71
Rho[2,3]   0.10 0.29 -0.40  0.55 1.01  1178.82  1509.05
Rho[3,3]   1.00 0.00  1.00  1.00   NA       NA       NA
```

There is a negative correlation between intercepts [1] and urban slopes [2] as we found in lecture. But no strong and reliable correlations with [3], the kids effect. So maybe the pooling across effects won't impact the effect of kids as much as it does the effect of urban.

Now we want to compare the district-level estimates for bK to those from the non-covariance model.

```
post1 <- extract.samples(m1)
post2 <- extract.samples(m2)
plot( apply(post1$bK,2,mean) , apply(post2$bK,2,mean) ,
    xlab="non-cov model" , ylab="cov model" , col=2 )
abline(a=0,b=1,lty=2)
```



These are very similar. Note though the two extreme districts on the far right of the plot. They deviate more from the diagonal line (equality of the two models). This may be because extreme estimates get pooled more in any model, and in the new covariance model, there are more sources of pooling. But since the correlation matrix doesn't show any strong correlations for [3], most estimates are similar between the models.