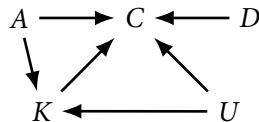


STATISTICAL RETHINKING 2026

B02 SOLUTIONS

1. Here's the DAG from lecture, for reference:



To estimate the effect of children K on C , we need to close the backdoors through A and U . So we need a model that includes A and U , in addition to K .

Now how we include A , K , U and D is up to us. I start with the correlated varying effects model from the lecture, which includes U and D . I will use an ordered monotonic effect for K . This is a technique from Chapter 12, section 12.4, page 391. There are only the values 1 to 4 observed. So we only need 4 parameters. To let the effect of K vary by district, I will like the leading coefficient vary by district. This means the maximum effect varies by district, but the relative increments for each kid are the same across districts. (If you didn't bother with the ordered monotonic effect, that's fine. I just want to show you how to do it.)

It is tempting to include A in the same way, but we only have it as a centered value, so we don't know the actual ages (although it isn't too hard to reconstruct them from the relative values).

Here's my model. I will use a non-centered parameterization, because that's likely to sample better.

```

library(rethinking)
data(bangladesh)
d <- bangladesh
dat <- list(
  C = d$use.contraception,
  D = as.integer(d$district),
  U = d$urban,
  A = standardize(d$age.centered),
  K = d$living.children )
dat$Kprior <- rep(2,3)

m1 <- ulam(
  alist(
    C ~ bernoulli(p),
    logit(p) <- a[D] + bU[D]*U + bA*A +
      bK[D]*sum( delta_j[1:K] ),
    a ~ normal(0, 1),
    bU ~ normal(0, 1),
    bA ~ normal(0, 1),
    bK ~ normal(0, 1),
    delta_j ~ dcauchy(0, 1)
  )
)
  
```

```

# ordered monotonic kids
vector[4]: delta_j <- append_row( 0 , delta ),
simplex[3]: delta ~ dirichlet( Kprior ),
bA ~ normal(0,0.5),

# centered varying effects for D and U and K
transpars> vector[61]:a <- a_bar + z_a*sigma_a,
transpars> vector[61]:bU <- bU_bar + z_bU*sigma_bU,
transpars> vector[61]:bK <- bK_bar + z_bK*sigma_bK,

# non-centered priors
vector[61]:z_a ~ normal(0,1),
vector[61]:z_bU ~ normal(0,1),
vector[61]:z_bK ~ normal(0,1),
c(a_bar,bU_bar,bK_bar) ~ normal(0,1),
c(sigma_a,sigma_bU,sigma_bK) ~ exponential(1)
) , data=dat , chains=4 , cores=4 )

precis(m1,depth=1)

```

	mean	sd	5.5%	94.5%	rhat	ess_bulk	ess_tail
bA	-0.25	0.07	-0.36	-0.15	1.01	2318.12	1574.92
bK_bar	1.34	0.17	1.07	1.61	1.00	1620.83	1265.32
bU_bar	0.70	0.15	0.47	0.94	1.00	1843.55	1464.07
a_bar	-1.62	0.15	-1.86	-1.40	1.00	1408.18	1623.99
sigma_bK	0.30	0.16	0.04	0.57	1.01	329.72	519.70
sigma_bU	0.51	0.23	0.14	0.89	1.00	375.60	487.58
sigma_a	0.46	0.11	0.29	0.63	1.02	370.89	327.74

The average effect bK of kids is positive: more kids, more contraception. We can inspect the increments for each child by looking at the delta parameters:

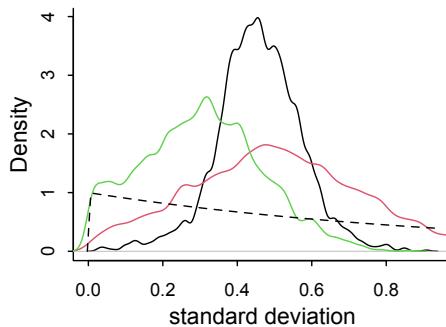
```
precis(m1,depth=2,pars="delta_j")
```

	mean	sd	5.5%	94.5%	rhat	ess_bulk	ess_tail
delta[1]	0.73	0.08	0.60	0.86	1	2654.77	1729.73
delta[2]	0.17	0.08	0.05	0.30	1	2305.58	1341.75
delta[3]	0.10	0.06	0.03	0.20	1	2111.03	1232.68

Remember that $\text{delta}[1]$ is the first non-zero increment, so it's the effect of the second child. The first child (the lowest observed value) is absorbed into the intercept. The second child has a large positive effect. The next two much less.

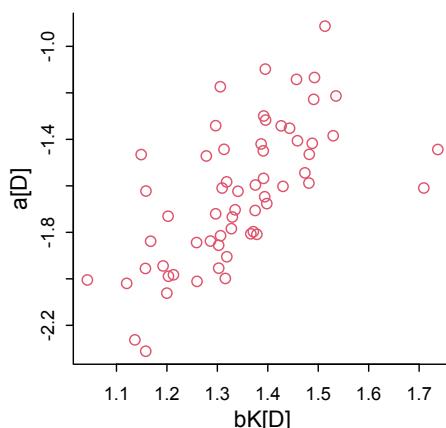
The districts do vary in the maximum effect. Let's look at the three sigma parameters and compare them to the prior:

```
post <- extract.samples(m1)
dens( post$sigma_a , xlab="standard deviation" )
dens( post$sigma_bU , col=2 , add=TRUE )
dens( post$sigma_bK , col=3 , add=TRUE )
curve( dexp(x,1) , add=TRUE , lty=2 )
```



The dashed curve is the prior, the black is the intercepts, the red is the urban effects, and the green is the kids effects. So there is less variation among districts in the kids effects than the urban effects, but both are quite uncertain. Maybe there is some pattern of interest in the posterior means. Let's try plotting each bK against the intercept for each district:

```
plot( apply(post$bK,2,mean) , apply(post$a,2,mean) , col=2 ,
      xlab="bK[D]" , ylab="a[D]" )
```



The is a strong positive correlation. Districts with largest base rates of contraception also have larger effects of surviving children. Next week you can build a model that estimates this correlation.