Goal Programming

Ryan McGuinness

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Problem Descripiton

The Emax Corporation has developed three new products, and a decision must be made on which mix of products are to be released. Primary consideration is given to total profit, workforce stability, and achieving an increase in the company's overall earnings next year from the $75 million achieved this year. In particular, we are asked to solve the following linear programming model:

Maximize $Z = P - 6C - 3D$, where
P = total (discounted) profit over the life of the new products,
C = change (in either direction) in the current level of employment, and
D = decrease in next year's earnings from the current year's.

Factor	Product 1	Product 2	Product 3	Goal	Units
Total Profit	20	15	25	Maximize	Millions of Dollars
Employment Level	6	4	5	$= 50$	Hundreds of Employees
Earnings Next Year	8	7	5	≥ 75	Millions of Dollars

Solving the Goal Programming Model

Defining Variables

Letting x_i represent the production rate of product i, we define the auxiliary variables as $y_1 = 6x_1 + 4x_2 + 5x_3 - 50 = y_1^+ - y_1^-$ and $y_2 = 8x_1 + 7x_2 + 5x_3 - 75 = y_2^+ - y_2^-$. Then, y_1^+ and y_1^- represents the amount over and under the employment level goal, respectively, and y_2^+ and y_2^- represents the amount over and under the earnings goal over next year, respectively. Additionally, we can express the total profit variable P as $P = 20x_1 + 15x_2 + 25x_3$.

Formulating the LP Model

We must now rewrite the objective function Z in terms of the above variables. Considering $C = y_1^+ + y_1^-$ and $D = y_2^-$ we have $Z = P - 6C - 3D = 20x_1 + 15x_2 + 25x_3 - 6y_1^+ - 6y_1^- - 3y_2^-$.

The full LP Model is as follows:
Maximize $Z = 20x_1 + 15x_2 + 25x_3 - 6y_1^+ - 6y_1^- - 3y_2^-$ subject to
$6x_1 + 4x_2 + 5x_3 - (y_1^+ - y_1^-) = 50$,
$8x_1 + 7x_2 + 5x_3 - (y_2^+ - y_2^-) = 75$,
$x_j \geq 0$, $y_i^+ \geq 0$, and $y_i^- \geq 0$.

Solving the LP Model

The model described above is represented in the file `Emax.lp`. We will use the `lpSolveAPI` package to read the model and solve it.

```
library(lpSolveAPI)
gp <- read.lp("Emax.lp")

# Solving the model
solve(gp)
```

```
## [1] 0
```

```
gp
```

```
## Model name:
##              x1    x2    x3   y1p   y1m   y2p   y2m
## Maximize     20    15    25    -6    -6     0    -3
## R1            6     4     5    -1     1     0     0  =  50
## R2            8     7     5     0     0    -1     1  =  75
## Kind        Std   Std   Std   Std   Std   Std   Std
## Type       Real  Real  Real  Real  Real  Real  Real
## Upper       Inf   Inf   Inf   Inf   Inf   Inf   Inf
## Lower         0     0     0     0     0     0     0
```

We use the `get.objective` and `get.variables` functions to find the maximum value of the objective function and the values of the decision variables. The first three variables in the `Emax.lp` model correspond to the decision variables x_j.

```
# Finding the objective function value
get.objective(gp)
```

```
## [1] 225
```

```
# Finding the values of the variables
# In the order: x1 x2 x3 y1+ y1- y2+ y2-
get.variables(gp)
```

```
## [1]  0  0 15 25  0  0  0
```

The LP model output tells us that the objective function is maximized if products 1 and 2 are ignored and only product 3 is produced at a rate of 15 units per period, despite the requirement of 2500 new employees.

Assessing Results

The solution of ignoring 2 of the 3 products in favor of increasing the number of employees by 50% appears absurd. However, as the problem is stated, this is the solution which optimizes the objective function. When increasing the penalty of the change in the level employment by just 1, the solved model will no longer call for an increase in the workforce and will call for the production of product 2. Although the problem as it is asked

Prioritizing Goals

If we instead consider the goals of workforce stability and an increase in next year's earnings as a higher priority than total profit over the life of the new products, we instead get the following LP model:
$Maximize\ Z = 20x_1 + 15x_2 + 25x_3 - 6My_1^+ - 6My_1^- - 3My_2^-$ subject to
$6x_1 + 4x_2 + 5x_3 - (y_1^+ - y_1^-) = 50,$
$8x_1 + 7x_2 + 5x_3 - (y_2^+ - y_2^-) = 75,$
$x_j \geq 0$, $y_i^+ \geq 0$, and $y_i^- \geq 0$.

Solving the LP Model

The model above is represented in `Emax_prioritized.lp`. As before, we use the `lpSolveAPI` package to solve this model.

```
gp_preempt <- read.lp("Emax_prioritized.lp")

# Solving the model
solve(gp_preempt)
```

```
## [1] 0
```

```
gp_preempt
```

```
## Model name:
##               x1     x2     x3    y1p    y1m    y2p    y2m
## Maximize      20     15     25  -6000  -6000      0  -3000
## R1             6      4      5     -1      1      0      0  =  50
## R2             8      7      5      0      0     -1      1  =  75
## Kind         Std    Std    Std    Std    Std    Std    Std
## Type        Real   Real   Real   Real   Real   Real   Real
## Upper        Inf    Inf    Inf    Inf    Inf    Inf    Inf
## Lower          0      0      0      0      0      0      0
```

```
# Finding the objective function value
get.objective(gp_preempt)
```

```
## [1] 208.3333
```

```
# Finding the values of the variables
# In the order: x1 x2 x3 y1+ y1- y2+ y2-
get.variables(gp_preempt)
```

```
## [1] 0.000000 8.333333 3.333333 0.000000 0.000000 0.000000 0.000000
```

This time, the model tells us to produce products 2 and 3 at the production rates of 8.33 and 3.33, respectively, and does not require any change in the level of employment, while the objective function is comparable to the function without prioritizing the employment and earnings goals.

Final Comments

While the first LP model above maximizes the objective function given by the problem, the solution of increasing the number of employees by 50% would very likely be considered infeasible. In this case, we examined the model where the goals of workforce stability and increasing earnings are given higher priority over profit over the life of the new products, which resulted in what ought to be a feasible solution with nearly the same objective function value as before. It is that solution where workforce stability is prioritized that management would likely be much happier with.