Quantitative Reasoning & Interview Preparation

Informal Notes for Problems on Probability, Estimation, and Financial Intuition

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Contents

1 Introduction

This document is a curated portfolio of algorithmic solutions. Each selected problem includes:

- a brief problem summary,
- the key idea,
- an algorithm outline,
- a correctness sketch,
- time and space complexity,
- a reference C++ implementation.

The focus is on clarity, correctness, and reasoning.

2 How This Book Is Written

This book is intended to mirror how problems are discussed in quantitative development technical interviews and presented by a candidate on a whiteboard.

The emphasis is on:

- stating assumptions explicitly,
- building intuition before formalism,
- using diagrams where helpful,
- sanity-checking results,
- (optionally) using pseudocode for simulation-style verification.

This is **not** a formal mathematics text. Proofs are informal, notation is kept light, and equations are used only when they improve understanding.

Each entry is a *final explanation*, not a transcript of exploratory work.

3 Probability

3.1 Coin Flips: Simulation vs Expectation

Problem

Flip a fair coin n times. Let X be the number of heads. What is $\mathbb{E}[X]$? How could we sanity-check the result via simulation?

Assumptions

- The coin is fair: $P(\text{Heads}) = P(\text{Tails}) = \frac{1}{2}$.
- Flips are independent.

Intuition

Each flip contributes either 1 head or 0 heads. On average, a single flip contributes $\frac{1}{2}$ head, so across n flips we expect about $\frac{n}{2}$ heads.

Reasoning

Write $X = \sum_{i=1}^{n} I_i$, where $I_i = 1$ if flip i is heads and 0 otherwise.
    Then:

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[I_i]$$

and since $\mathbb{E}[I_i] = P(I_i = 1) = \frac{1}{2}$ for a fair coin:

$$\mathbb{E}[X] = n \cdot \frac{1}{2} = \frac{n}{2}.$$

Procedure (Simulation Pseudocode)

Algorithm 1: Monte Carlo sanity check for $\mathbb{E}[X]$

    Input: n flips per trial, T trials
    Output: Estimated mean number of heads
    sumHeads $\leftarrow 0$;
    for $t \leftarrow 1$ **to** T **do**
        heads $\leftarrow 0$;
        for $i \leftarrow 1$ **to** n **do**
            sample $U \sim \text{Uniform}(0,1)$;
            if $U < 1/2$ **then**
                heads $\leftarrow$ **heads** $+1$;
        sumHeads $\leftarrow$ **sumHeads** $+$**heads**;
    return sumHeads$/T$;

Sanity Check

- For $n = 1$, $\mathbb{E}[X] = 0.5$ makes sense.

- For large n, the estimate should concentrate near $n/2$.

- If you plot the histogram of heads across trials, it should look roughly binomial.

Takeaway

For n independent fair coin flips, the expected number of heads is:

$$\mathbb{E}[X] = \frac{n}{2}.$$

Simulation provides a quick sanity check without changing the underlying reasoning.

4 Estimation

4.1 Coin Flips: Simulation vs Expectation

Problem

Flip a fair coin n times. Let X be the number of heads. What is $\mathbb{E}[X]$? How could we sanity-check the result via simulation?

Assumptions

- The coin is fair: $P(\text{Heads}) = P(\text{Tails}) = \frac{1}{2}$.

- Flips are independent.

Intuition

Each flip contributes either 1 head or 0 heads. On average, a single flip contributes $\frac{1}{2}$ head, so across n flips we expect about $\frac{n}{2}$ heads.

Reasoning

Write $X = \sum_{i=1}^{n} I_i$, where $I_i = 1$ if flip i is heads and 0 otherwise.
    Then:

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[I_i]$$

and since $\mathbb{E}[I_i] = P(I_i = 1) = \frac{1}{2}$ for a fair coin:

$$\mathbb{E}[X] = n \cdot \tfrac{1}{2} = \tfrac{n}{2}.$$

Procedure (Simulation Pseudocode)

Algorithm 2: Monte Carlo sanity check for $\mathbb{E}[X]$

Input: n flips per trial, T trials
Output: Estimated mean number of heads
sumHeads $\leftarrow 0$;
for $t \leftarrow 1$ **to** T **do**
    heads $\leftarrow 0$;
    for $i \leftarrow 1$ **to** n **do**
        sample $U \sim \text{Uniform}(0, 1)$;
        if $U < 1/2$ **then**
            heads $\leftarrow$ **heads** $+1$;
    sumHeads $\leftarrow$ **sumHeads** $+$**heads**;
return sumHeads$/T$;

Sanity Check

- For $n = 1$, $\mathbb{E}[X] = 0.5$ makes sense.

- For large n, the estimate should concentrate near $n/2$.

- If you plot the histogram of heads across trials, it should look roughly binomial.

Takeaway

For n independent fair coin flips, the expected number of heads is:

$$\mathbb{E}[X] = \tfrac{n}{2}.$$

Simulation provides a quick sanity check without changing the underlying reasoning.

5 Random Processes

5.1 Coin Flips: Simulation vs Expectation

Problem

Flip a fair coin n times. Let X be the number of heads. What is $\mathbb{E}[X]$? How could we sanity-check the result via simulation?

Assumptions

- The coin is fair: $P(\text{Heads}) = P(\text{Tails}) = \frac{1}{2}$.

- Flips are independent.

Intuition

Each flip contributes either 1 head or 0 heads. On average, a single flip contributes $\frac{1}{2}$ head, so across n flips we expect about $\frac{n}{2}$ heads.

Reasoning

Write $X = \sum_{i=1}^{n} I_i$, where $I_i = 1$ if flip i is heads and 0 otherwise.

Then:

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[I_i]$$

and since $\mathbb{E}[I_i] = P(I_i = 1) = \frac{1}{2}$ for a fair coin:

$$\mathbb{E}[X] = n \cdot \tfrac{1}{2} = \tfrac{n}{2}.$$

Procedure (Simulation Pseudocode)

Algorithm 3: Monte Carlo sanity check for $\mathbb{E}[X]$

Input: n flips per trial, T trials
Output: Estimated mean number of heads
sumHeads $\leftarrow 0$;
for $t \leftarrow 1$ **to** T **do**
    heads $\leftarrow 0$;
    for $i \leftarrow 1$ **to** n **do**
        sample $U \sim \text{Uniform}(0,1)$;
        if $U < 1/2$ **then**
            heads $\leftarrow$ **heads** $+1$;
    sumHeads $\leftarrow$ **sumHeads** $+$**heads**;
return sumHeads$/T$;

Sanity Check

- For $n = 1$, $\mathbb{E}[X] = 0.5$ makes sense.

- For large n, the estimate should concentrate near $n/2$.

- If you plot the histogram of heads across trials, it should look roughly binomial.

Takeaway

For n independent fair coin flips, the expected number of heads is:

$$\mathbb{E}[X] = \tfrac{n}{2}.$$

Simulation provides a quick sanity check without changing the underlying reasoning.

6 Financial Intuition

6.1 Coin Flips: Simulation vs Expectation

Problem

Flip a fair coin n times. Let X be the number of heads. What is $\mathbb{E}[X]$? How could we sanity-check the result via simulation?

Assumptions

- The coin is fair: $P(\text{Heads}) = P(\text{Tails}) = \frac{1}{2}$.

- Flips are independent.

Intuition

Each flip contributes either 1 head or 0 heads. On average, a single flip contributes $\frac{1}{2}$ head, so across n flips we expect about $\frac{n}{2}$ heads.

Reasoning

Write $X = \sum_{i=1}^{n} I_i$, where $I_i = 1$ if flip i is heads and 0 otherwise.
    Then:

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[I_i]$$

and since $\mathbb{E}[I_i] = P(I_i = 1) = \frac{1}{2}$ for a fair coin:

$$\mathbb{E}[X] = n \cdot \frac{1}{2} = \frac{n}{2}.$$

Procedure (Simulation Pseudocode)

Algorithm 4: Monte Carlo sanity check for $\mathbb{E}[X]$

Input: n flips per trial, T trials
Output: Estimated mean number of heads
sumHeads $\leftarrow 0$;
for $t \leftarrow 1$ **to** T **do**
    heads $\leftarrow 0$;
    for $i \leftarrow 1$ **to** n **do**
        sample $U \sim \text{Uniform}(0,1)$;
        if $U < 1/2$ **then**
            heads $\leftarrow$ **heads** $+1$;

    sumHeads $\leftarrow$ **sumHeads** $+$**heads**;
return sumHeads$/T$;

Sanity Check

- For $n = 1$, $\mathbb{E}[X] = 0.5$ makes sense.

- For large n, the estimate should concentrate near $n/2$.

- If you plot the histogram of heads across trials, it should look roughly binomial.

Takeaway

For n independent fair coin flips, the expected number of heads is:

$$\mathbb{E}[X] = \tfrac{n}{2}.$$

Simulation provides a quick sanity check without changing the underlying reasoning.

7 Systems Thinking

7.1 Coin Flips: Simulation vs Expectation

Problem

Flip a fair coin n times. Let X be the number of heads. What is $\mathbb{E}[X]$? How could we sanity-check the result via simulation?

Assumptions

- The coin is fair: $P(\text{Heads}) = P(\text{Tails}) = \tfrac{1}{2}$.

- Flips are independent.

Intuition

Each flip contributes either 1 head or 0 heads. On average, a single flip contributes $\tfrac{1}{2}$ head, so across n flips we expect about $\tfrac{n}{2}$ heads.

Reasoning

Write $X = \sum_{i=1}^{n} I_i$, where $I_i = 1$ if flip i is heads and 0 otherwise.
Then:

$$\mathbb{E}[X] = \sum_{i=1}^{n} \mathbb{E}[I_i]$$

and since $\mathbb{E}[I_i] = P(I_i = 1) = \tfrac{1}{2}$ for a fair coin:

$$\mathbb{E}[X] = n \cdot \tfrac{1}{2} = \tfrac{n}{2}.$$

Procedure (Simulation Pseudocode)

Algorithm 5: Monte Carlo sanity check for $\mathbb{E}[X]$

Input: n flips per trial, T trials
Output: Estimated mean number of heads
sumHeads $\leftarrow 0$;
for $t \leftarrow 1$ **to** T **do**
    heads $\leftarrow 0$;
    for $i \leftarrow 1$ **to** n **do**
        sample $U \sim \text{Uniform}(0, 1)$;
        if $U < 1/2$ **then**
            heads $\leftarrow$ **heads** $+1$;
    sumHeads $\leftarrow$ **sumHeads** $+$**heads**;
return sumHeads$/T$;

Sanity Check

- For $n = 1$, $\mathbb{E}[X] = 0.5$ makes sense.

- For large n, the estimate should concentrate near $n/2$.

- If you plot the histogram of heads across trials, it should look roughly binomial.

Takeaway

For n independent fair coin flips, the expected number of heads is:

$$\mathbb{E}[X] = \tfrac{n}{2}.$$

Simulation provides a quick sanity check without changing the underlying reasoning.

8 Language Considerations

8.1 Coin Flips: Simulation vs Expectation

Problem

Flip a fair coin n times. Let X be the number of heads. What is $\mathbb{E}[X]$? How could we sanity-check the result via simulation?

Assumptions

- The coin is fair: $P(\text{Heads}) = P(\text{Tails}) = \tfrac{1}{2}$.

- Flips are independent.

Intuition

Each flip contributes either 1 head or 0 heads. On average, a single flip contributes $\tfrac{1}{2}$ head, so across n flips we expect about $\tfrac{n}{2}$ heads.

Reasoning

Write $X = \sum_{i=1}^{n} I_i$, where $I_i = 1$ if flip i is heads and 0 otherwise.
   Then:

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and since $\mathbb{E}[I_i] = P(I_i = 1) = \tfrac{1}{2}$ for a fair coin:

$$\mathbb{E}[X] = n \cdot \tfrac{1}{2} = \tfrac{n}{2}.$$

Procedure (Simulation Pseudocode)

Algorithm 6: Monte Carlo sanity check for $\mathbb{E}[X]$

Input: n flips per trial, T trials
Output: Estimated mean number of heads
sumHeads $\leftarrow 0$;
for $t \leftarrow 1$ **to** T **do**
    heads $\leftarrow 0$;
    for $i \leftarrow 1$ **to** n **do**
        sample $U \sim \text{Uniform}(0,1)$;
        if $U < 1/2$ **then**
            heads $\leftarrow$ **heads** $+1$;

    sumHeads $\leftarrow$ **sumHeads** $+$**heads**;
return sumHeads$/T$;

Sanity Check

- For $n = 1$, $\mathbb{E}[X] = 0.5$ makes sense.

- For large n, the estimate should concentrate near $n/2$.

- If you plot the histogram of heads across trials, it should look roughly binomial.

Takeaway

For n independent fair coin flips, the expected number of heads is:

$$\mathbb{E}[X] = \tfrac{n}{2}.$$

Simulation provides a quick sanity check without changing the underlying reasoning.

9   References