A Small State-Space Model of the Australian Economy

Shawn Chen-Yu Leu

Department of Economics & Finance

La Trobe University

Email: C.Leu@latrobe.edu.au

Jeffrey Sheen

Faculty of Economics & Business

University of Sydney

Email: jeffs@econ.usyd.edu.au

ABSTRACT

Using a small state space macroeconomic model, we apply maximum likelihood estimation and the Kalman filter to obtain joint estimates of the unobservable medium-run paths of potential output and its normal rate of growth, the natural rate of unemployment and the natural real interest rate. Our unobserved component analysis indicates that in 2006, the Australian economy was close to potential output, the normal rate of growth was low, unemployment was below the natural rate presaging inflationary pressures, and that the real rate of interest was significantly below its natural rate, suggesting that monetary policy was too expansionary.

Correspondence: Jeffrey Sheen
Faculty of Economics & Business
University of Sydney
Sydney, NSW 2006, Australia
Email: jeffs@econ.usyd.edu.au

12 September 2007 16:03

1. Introduction

A traditional view of business cycles is that they are short-run stochastic movements of real variables around their smoothed trend values. These smoothed trends or natural rates play an important role in macroeconomic models as benchmarks to compare with current values. Wicksell (1936) introduced the concept of the natural interest rate, and more recently, there has been a revival in the literature on the subject following Woodford's (2003) book. In contemporary terms, the natural rate of interest is the equilibrium real rate (sometimes called the neutral rate) that would arise if wages and prices were completely flexible, given current factors. Phelps (1967) and Friedman (1968) introduced the related idea of the natural rate of unemployment. More specifically, the tradeoff between inflation and unemployment is temporary, so that the actual unemployment rate converges to the natural rate, at which point the inflation rate remains constant. Thus this benchmark unemployment rate is also known as the non-accelerating inflation rate of unemployment (NAIRU). When the economy is anchored at the NAIRU, GDP must be at the natural level of output, which is sometimes called the level of potential output. A short-run output gap emerges if GDP deviates from the natural level of output. However this potential output need not be constant in the medium run—given productivity growth and factor accumulation, there will be a normal rate of growth of potential output in the medium run. As the horizon progresses to the long run, this growth rate becomes the steady-state growth rate of the economy. Short-run deviations from the natural (and normal) rates in the medium run can be explained by the presence of imperfect information (e.g. Lucas (1972)) or nominal rigidities. These deviations affect movements in aggregate demand and supply, which in turn stimulate adjustment processes to return the economy to the medium-run equilibrium.

These natural rate and level concepts are central to the conduct of monetary policy. An inflation-targeting central bank needs to assess the level of economic activity variables in relation to their natural values to judge the pressures on inflation relative to its target and on any other target variables. When output grows faster than normal and exceeds its potential value, the unemployment rate will fall short of the NAIRU, wage inflation will rise, the real interest rate will be below its natural level, and so there will be upward pressure on inflation. The central bank is likely to tighten monetary policy to steer inflation and output back to their target and natural values. The short-term interest rate rises from its current position until the medium-run equilibrium is restored, and monetary policy returns to its neutral stance.

Although these natural economic indicators provide useful information to economists and policymakers, they are unobservable by nature and must be inferred from the data. The objective of this paper is to estimate a multivariate, unobserved components (UC) model for the Australian economy that allows for the simultaneous estimation of the paths of potential output and its normal growth rate, the NAIRU and the natural real interest rate. The multivariate state space model comprises a dynamic IS equation of the output gap representing aggregate demand (AD), an expectations-augmented Phillips curve that represents aggregate supply (AS), an Okun's relation connecting cyclical movements of output to unemployment, and a first order condition from intertemporal optimization giving the medium run relationship between the real interest rate and output growth . The model is estimated using maximum likelihood over the period 1984Q1 to 2006Q4, extracting unobservable state variables with the Kalman filter. Inflation information from the AS relation is used to infer the output gap (defined as the difference between actual and potential output), which in turn through the dynamic IS equation is used to infer the real interest rate gap (defined as the difference between actual short-term

interest rate and its natural rate) and the natural real rate of interest. An Okun's equation relates the cyclical fluctuations in the product market (the output gap) to the cyclical fluctuations in the labour market (hence inferring the unemployment gap).

The paper proceeds as follows. In section 2, different univariate and multivariate measures of natural rates are discussed. This discussion motivates the multivariate model outlined in section 3. Section 4 describes the data, some econometric issues related to Kalman filter and presents the parameter estimates and the multivariate UC smoothed natural rates and their bootstrapped confidence intervals. Section 5 offers some concluding remarks.

2. Univariate and Multivariate Measures of Natural Rates

A widely used procedure to decompose macroeconomic variables (such as real output) into a trend (or potential output) and a cyclical (or the output gap) components is the Hodrick and Prescott (1997) filter.[1] The smoothness of the Hodrick-Prescott (or HP) stochastic trend depends on the input value of an *ad-hoc* smoothness parameter. If the value of the exogenous parameter is set to zero, the trend component and the actual series match each other; if the value of the parameter goes to infinity, the trend component approaches a linear deterministic trend. Baxter and King (1999) derived an estimate of the output cycle by passing the data through a filter that pre-specifies the relevant frequencies for the cycle and thus its persistence. Their approximate band-pass filter defines the cycle as having spectral power in the range between 6 and 32 quarters.

Pure statistical methods that simply 'let the data speak' do not include potentially useful information about the supply side of the economy and the business cycle contained in macroeconomic relationships such as the Phillips curve, Okun's law, and other

---

[1] The original paper appeared in 1980 as a Carnegie-Mellon discussion paper, and was eventually published unchanged in 1997.

indicators such as output capacity utilization.[2] Laxton and Tetlow (1992) proposed a multivariate extension to the univariate HP filter by conditioning the computation of the level of potential output on additional economic relationships. Boone *et al*. (2000) applied a multivariate HP filter to estimate the level of potential output for twenty one OECD countries. To estimate Australian potential output, de Brouwer (1998) incorporated information from inflation, unemployment and capacity utilization. Gruen *et al.* (2005) conditioned their estimates on the Phillips curve using real-time output data.

Another class of models—known as the unobserved components (UC) model—offers two advantages over the multivariate HP filter: (1) it permits a more complex system of dynamics; and (2) estimation is relatively more straightforward with the structural parameters estimated by maximum likelihood and the unobserved variables (or natural rates) derived by the Kalman filter.[3]

Within the UC framework, several papers attempted to estimate natural rates using different macroeconomic relationships. Clark (1989) and Kuttner (1991) used an Okun's equation, which defines the level of observable unemployment as a function of the unobservable output gap, to derive the level of U.S. potential output.[4] In Kuttner (1994), the level of potential output is extracted from information contained in the Phillips curve so that the economy maintains a constant level of inflation. The constant-inflation natural unemployment rate (or the NAIRU) is provided by King *et al*. (1995), Staiger *et al*. (1996) and Laubach (2001) using a similar framework.

Instead of conducting partial analysis on potential output or the NAIRU, it is possible to estimate a system of equations that features the Phillips curve, which imposes

---

[2] While there are many other univariate filters which may have better properties, our focus is on the gains from multivariate extensions.

[3] The multivariate HP filter implemented by Laxton and Tetlow (1992) is a two-step procedure. First, the economic relationships are separately estimated. The regression residuals are inserted into the multivariate HP minimization problem to estimate the unobservable variable. This two-step procedure is repeated with several iterations until convergence is achieved. See Boone (2000) for more details.

[4] In Kuttner (1991), the Okun's equation relates the change in unemployment to the output gap.

a constant-inflation restriction on the path of potential output or the NAIRU, and incorporating the covariation restrictions on cyclical output and cyclical unemployment through the Okun's relation. Some examples of papers that model the mutual dependency of output and unemployment include Apel and Jansson (1999a, 1999b) and Benes and N'Diaye (2004).

In addition to an Okun's relation that represents the linkages between output and unemployment, movements from the real interest rate relative to its natural rate can be embedded in the IS relation for the ouput gap describing product market equilibrium. The incorporation of this extra channel is likely to enhance the estimation of the cyclical paths of unobservable variables in the economy.

The natural real interest rate is likely to vary over time, and in an intertemporally optimal setting will be determined by factors such as underlying productivity growth and the rate of time preference. For example, Laubach and Williams (2003) found substantial variations in the natural interest rate over the past four decades in the U.S. They also suggested that there is an approximate one-for-one relationship between natural rate variation and changes in the growth rate of potential GDP.

3. A Multivariate Model of Unobserved Components

Both output and unemployment are decomposed in (1) and (2) into a stochastic trend component and the stochastic cyclical variations around this trend. The trend components are taken to be the level of potential output and the natural rate of unemployment (or the NAIRU) that are associated with the medium-run equilibrium when prices and wages have fully adjusted to shocks. When demand or supply shocks occur, there will be deviations from the trend component values in the short run because of nominal rigidities, and these are defined as the output and unemployment gap measures.

$$y_t = y_t^* + \tilde{y}_t \tag{1}$$

$$u_t = u_t^* + \tilde{u}_t \tag{2}$$

In (1) and (2), y_t is the log of real GDP, y_t^* is the log of potential GDP and $\tilde{y}_t$ denotes the output gap; u_t is the unemployment rate, u_t^* is the NAIRU value, and $\tilde{u}_t$ represents the unemployment gap. Note that all variables are potentially time-dependent.

Following Rudebusch and Svensson (1999), the aggregate demand side of the economy is described by a reduced-form IS equation as in (3):

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \left(r_{t-8} - r_{t-8}^* \right) + a_3 \Delta LTOT_{t-4} + a_4 \Delta \tilde{y}_t^{G7} + \varepsilon_t^{\tilde{y}} \tag{3}$$

A stationary AR(1) process is specified for the dynamic evolution of the output gap ($\tilde{y}_t$).[5] As in Laubach and Williams (2003), a real interest rate gap $(r_{t-j} - r_{t-j}^*)$ is included in the output gap equation. After preliminary OLS estimations using general to specific tests, we found that the 8th lag (2 years) of the real interest rate gap should be included. Stone *et al*. (2005) similarly found it necessary to include lags 1 to 7 of the same variable to uncover the effect of a change in monetary policy on the output gap. To capture the positive effect that a rising terms of trade has on the output gap, the fourth lag of the quarterly change of the (logged) terms of trade ($\Delta LTOT_{t-4}$) is included. Given Australia is a small open economy, the contemporaneous quarterly change in the G7 output gap ($\Delta \tilde{y}_t^{G7}$) is included to explain the influence of foreign economic activity on domestic output. In the medium run, the output gap converges to zero as do the real interest rate gap, the G7 output gap, and changes in the terms of trade.

---

[5] Watson (1986) and Clark (1987) for the U.S. economy and de Brouwer (1998) for the Australian economy, adopted an AR(2) process for the output gap. Our estimates rejected it in favour of an AR(1) process for Australia

The aggregate supply side of the economy is represented by an expectations-augmented Phillips curve (4):

$$\pi_t = b_1 \pi_{t-1} + b_2 \pi_{t-2} + b_3 \tilde{y}_{t-1} + b_4 \tilde{\pi}_{t-1}^{imp} + b_5 \tilde{\pi}_t^e + \varepsilon_t^\pi \tag{4}$$

where inflation expectations are assumed to be driven by backward- and forward-looking processes. One group of economic agents and firms relies only on past inflation information to condition their inflation forecasts, hence the inclusion of lagged inflation terms (π_{t-1} and π_{t-2}). This helps account for the persistence in inflation dynamics. Another group are forward-looking and so we employ consumer survey expected inflation as advocated by Roberts (1997). The influence of excess demand on inflation is captured by the lagged output gap, which reflects the nominal inertia of price responses to economic activity. The pass-through effect on to domestic inflation of import prices represents a supply factor, which enters the equation with a lag. More specifically, consumer survey inflation expectations and import price inflation are constructed to be zero in the medium run. The variable $\tilde{\pi}_t^e$ is the excess of consumer inflation expectations over lagged year-ended inflation, i.e. $\tilde{\pi}_t^e = \pi_t^e - \pi_{t-1}$, while $\tilde{\pi}_t^{imp}$ is the excess of import price inflation over lagged year-ended inflation, $\tilde{\pi}_t^{imp} = \pi_t^{imp} - \pi_{t-1}$ (see Gruen *et al.* (2005)). Equation (4) yields a vertical Phillips curve in the medium run, with output anchored at its potential level, actual equal to expected inflation ,and zero supply factors. This implies the constraint, $b_1 + b_2 = 1$.

The connection between the unemployment gap ($\tilde{u}_t$) and the output gap is represented by an Okun equation (5):

$$\tilde{u}_t = c_1 \tilde{u}_{t-1} + c_2 \tilde{u}_{t-2} + c_3 \tilde{y}_t + \varepsilon_t^{\tilde{u}} \tag{5}$$

Some degree of persistence in the dynamics of the unemployment gap is captured by an AR(2) process.

Equations (6) through (10) describe the laws of motion of the unobservable trends in the model. Potential output is modelled by (6) as a local linear trend, where the drift term μ_{t-1} representing the trend growth rate is a random walk process (7):[6]

$$y_t^* = y_{t-1}^* + \mu_{t-1} + \varepsilon_t^{y*} \qquad (6)$$

$$\mu_t = \mu_{t-1} + \varepsilon_t^{\mu} \qquad (7)$$

Gruen *et al.* (2005) found large shifts in the trend growth rate for the Australian economy since 1960. To incorporate this feature of the Australian economy, the local linear trend specification implies that potential output grows at the time-varying normal growth rate when all shocks dissipate in the medium run, i.e. $\Delta y_t^* = \mu_{t-1}$.[7]

From household intertemporal utility maximization evaluated in the medium run,[8] we posit a relationship that links the evolution of the natural interest rate to movements in the trend growth rate of output (see Laubach and Williams (2003)). This is shown in (8) where r_t^* is the natural rate of interest, d captures the inverse of the elasticity of

---

[6] Since the drift term is assumed to be I(1), this implies that potential output and log real GDP are I(2). This hypothesis is typically rejected by an ADF test. However, Stock and Watson (1998) pointed out that the test statistic tends to have high probability of type I error in falsely rejecting the true null when the innovation to the growth rate component has small variance.

[7] Alternatively the trend growth equation (7) can be modelled as an autoregressive process that makes y^* and y difference-stationary. However, the sum of the autoregressive parameters obtained during preliminary estimations suggest that it is almost identical to unity and hence highly persistent.

[8] For an infinite horizon representative consumer model, where period utility is $U_t = \sum_{s=1}^{\infty} \beta^{s-t} C_s^{1-1/\sigma}$ with σ being the intertemporal elasticity of substitution, and β is the rate of time preference, the first order condition is $\dfrac{\mu_{c,t}}{\sigma} = \ln \beta + \ln(1+r_t)$ where $\mu_{c,t}$ is the growth rate of consumption. We do not insert the implied restrictions from this intertemporal optimizing condition into the short-run product market equilibrium condition (3) on the grounds that many households are unable to optimize in the short run. However in the medium run, most will find a way to approach their optimal consumption trajectory. In the medium-run equilibrium of the open economy, the current account to GDP ratio will be constant, as will be the consumption to GDP ratio, i.e. $\mu_{c,t} = \mu_t$. Therefore, at low interest rates, the medium-run relation between the real interest rate and output growth is approximated by $r_t^* = \dfrac{\mu_t}{\sigma} - \ln \beta$.

intertemporal substitution, and z_t is a random walk process as in (9) that represents other possible determinants such as the rate of time preference and risk premia.[9]

$$r_t^* = d\mu_t + z_t \tag{8}$$

$$z_t = z_{t-1} + \varepsilon_t^z \tag{9}$$

Lastly, the natural rate of unemployment follows a pure random walk:

$$u_t^* = u_{t-1}^* + \varepsilon_t^{u*} \tag{10}$$

To complete the description of the multivariate UC model, we assume that all innovations $\varepsilon = (\varepsilon_t^{\tilde{y}}, \varepsilon_t^{\pi}, \varepsilon_t^{\tilde{u}}, \varepsilon_t^{y*}, \varepsilon_t^{\mu}, \varepsilon_t^{z}, \varepsilon_t^{u*})'$ are i.i.d. normally distributed with zero mean and finite variances. In addition, they are serially and contemporaneously uncorrelated with each other.

4. Data and Empirical Results

4.1 Data

The quarterly data span starts from 1984:1 to 2006:4. All data unless otherwise specified were obtained from the ABS. y_t and u_t are the Australian real GDP and unemployment rate. Domestic inflation is calculated as the year-ended change in the log of the Treasury underlying CPI which is spliced in September 1999 to the headline CPI. The same procedure is applied to compute the import inflation rate which is based on the log of the import chain price index. The real interest rate is the nominal cash rate less the inflation rate, i.e. $r_t = i_t - \pi_t$. Estimates of the G7 output gap are extracted from the OECD database. Quarterly changes in the log of the terms of trade index are used since they offer higher

---

[9] Laubach and Williams (2003) considered an AR(2) process, and Garnier and Wilhelmsen (2005) an AR(1). However, these alternative specifications did not generate economically sensible results for the Australian economy.

explanatory power in preliminary OLS estimation. Inflation expectations are consumers'

inflation expectations measured by the Melbourne Institute as the median expected

inflation rate for the year ahead.

4.2 Estimation Issues

Before proceeding with estimation, the multivariate UC model is cast in the state-space

form (see the appendix). Parameters are estimated by maximum likelihood and the natural

rates, or state variables, are simultaneously extracted using the Kalman filter. The Kalman

filter is a recursive algorithm that sequentially updates a linear projection of a dynamic

system. In each period the Kalman filter provides the (one-sided) optimal predictions of

the natural rates for that period conditional on information available up to and including

the current period. Once the filtered natural rates are obtained, it is possible to 'smooth

over' the natural estimates that are conditioned on information from the full sample;

therefore the smoothed natural rates can be thought of as two-sided estimates. There are

two important estimation issues related to the Kalman filter that need to be resolved:

namely the choice of the initial values of the state vector and covariance matrix and the

estimation of the innovation variances.

To set the initial values for the state vector, the gap measures are assumed to be

stationary, and so a value of zero is assigned to them. For the natural rate measures, the

initial value is set to the value of the first observation of the associated variable. The

dynamics of the multivariate UC model are non-stationary because the trend equations are

specified to be random walks. Therefore we follow the usual practice of assigning diffuse

priors to the diagonal elements of the initial state covariance matrix.[10]

---

[10] We found it necessary to begin the recursion with a diffuse prior of zero to tie the estimated trends to a path that would run through the data. The result is that the filtered and smoothed estimates are very close to the first observation of the variable.

It is common in the literature to choose a value for the signal-to-noise ratio and impose it in the maximum likelihood estimation. One example in our model is the ratio of potential output innovation to trend growth innovation, $\lambda = \sigma_{y^*} / \sigma_{\mu}$. Because these two unobserved variables are non-stationary, their cumulated variance goes to infinity, and so their maximum likelihood estimates (MLE) have a point mass at zero even though their true values are greater than zero. Stock (1994, section 4) discussed this so-called 'pile-up' problem which prevents the efficient estimation of the innovation of non-stationary state variable. Laubach and Williams (2003) found that the MLEs of σ_{μ} and σ_z are zero, and so they applied the median-unbiased estimation procedure developed by Stock and Watson (1998). The first step is to obtain the median-unbiased estimates of the signal-to-noise ratio. In the second stage, the ratio is imposed in the system estimation. Laubach and Williams (2003) and Garnier and Wilhelmsen (2005) followed this approach to estimate potential output and the natural real interest rate for the U.S. and the Euro zone respectively.

In recognition of the pile-up problem, Messonier and Renne (2004) argued that it becomes difficult to pin down a sensible path of the natural real interest rate, because r^* is an unobserved process that is linked to two other unobserved processes, μ and z. Instead, they followed the approach in King *et al*. (1995), Staiger *et al*. (1996), and Laubach (2001) in fixing the signal-to-noise ratio at particular values and testing them statistically in reference to a baseline model.

We offer another perspective to the need for fixing the values of the unconditional variances of innovations through the point of view of parameter identification. By first-differencing the potential output equation (6) and the trend growth equation (7), we get:

$$\Delta y_t^* = \mu_{t-1} + \varepsilon_t^{y^*} \tag{11}$$

$$\Delta \mu_t = \varepsilon_t^{\mu} \tag{12}$$

where according to (11) and (12) the two structural parameters to be identified are σ_{y*}^2

and σ_μ^2.

Lag (12) by one period to obtain:

$$\mu_{t-1} = \frac{\varepsilon_{t-1}^\mu}{\Delta} \qquad (13)$$

Substitute (13) into (12) yields:

$$\Delta y_t^* = \frac{\varepsilon_{t-1}^\mu}{\Delta} + \varepsilon_t^{y*}$$

or $\qquad (14)$

$$\Delta^2 y_t^* = \varepsilon_{t-1}^\mu + \varepsilon_t^{y*} - \varepsilon_{t-1}^{y*}$$

The autocovariance functions of the reduced-form equation $\Delta^2 y_t^*$ are

$$\gamma(0) = \sigma_\mu^2 + 2\sigma_{y*}^2$$
$$\gamma(1) = -\sigma_{y*}^2 \qquad (15)$$
$$\gamma(\tau) = 0 \quad \text{for } \tau \geq 2$$

where $\gamma(\tau)$ is the τ-th order autocovariance function. Given (15), we have two reduced

form parameters to map to two structural parameters. Hence the order condition is

satisfied to uniquely identify the structural parameters σ_{y*}^2 and σ_μ^2.

Now consider the r^* and z equations (8) and (9). Substitute out μ_t and z_t with

$\mu_t = \varepsilon_t^\mu / \Delta$ and $z_t = \varepsilon_t^z / \Delta$ to yield the following reduced form equation of $\Delta^2 r_t^*$:

$$\Delta^2 r_t^* = d\varepsilon_t^\mu + \varepsilon_t^z \qquad (16)$$

We need to identify from (16) the structural parameters d and σ_z^2 with σ_μ^2 already

identified previously in (15). The autocovariance functions of $\Delta^2 r_t^*$ are

$$\gamma(0) = d^2\sigma_\mu^2 + \sigma_z^2$$
$$\gamma(\tau) = 0 \quad \text{for } \tau \geq 1 \qquad (17)$$

Therefore in this case we have an under-identification problem as there is only one

reduced-form parameter available to link to the two structural parameters d and σ_z^2.

In relation to previous studies that fix the unconditional variances of state variable innovations through the signal-to-noise ratio, we find the problem arises here with σ_z^2 and not σ_μ^2. Since the parameter d approximates the inverse of the intertemporal elasticity of substitution, we calibrate σ_z to 14 different values and discard those that do not generate significant estimates of d.[11]

4.3 Results

The calibrated values of $\sigma_z = 0.4$ to 0.8 yield significant estimates of d ranging from 4.40 to 4.91. This approximates to a range for the intertemporal elasticity of substitution (σ) between 0.20 and 0.23. We deem these to be reasonable estimates as Ogaki and Reinhart (1998) obtained estimates between 0.27 and 0.77 and Barsky *et al*. (1997) using micro-data came up with an estimate of 0.18.

Table 1 displays the parameter estimates of the multivariate UC model. Kalman smoothed estimates of potential output and its normal growth rate, the NAIRU and the natural real interest rate as well as the related gap measures are shown in Figures 1 to 7.

All of the estimated coefficients have the expected sign. The sum of the autoregressive parameter estimates of the IS equation (3) and the Okun equation (5) are each less than one, which is necessary for the stationary dynamics of the output gap and the unemployment gap.

In all of the figures, the UC natural rates are the smoothed estimates associated with $\sigma_z = 0.8$ as it yields the highest log likelihood. The multivariate unobserved components (MUC) measure (in blue) is compared to a univariate measure (in red) derived from the band-pass filter (BP). The shaded space above and below the multivariate measures represent 95% confidence intervals. These were obtained through bootstrapping by

---

[11] These are $\sigma_z = 0.005, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 3.0$.
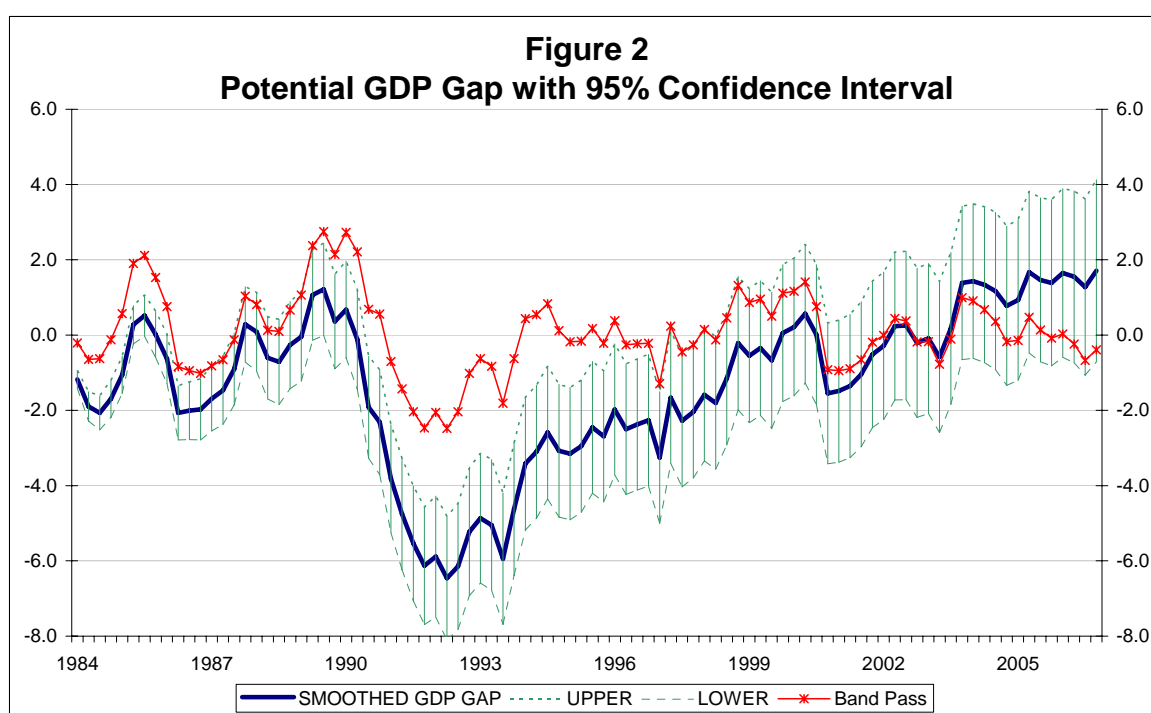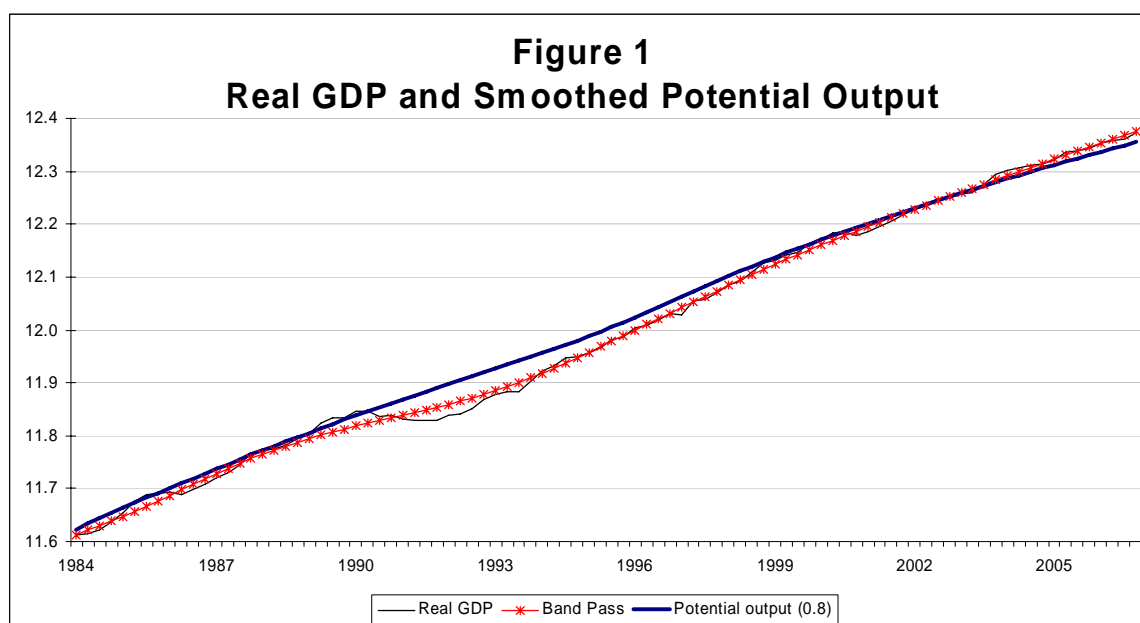
running 1500 conditional simulations to compute the second moment of the Kalman smooth states using Gibbs sampling. The resulting upper and lower bands represent the effects of both parameter and filter uncertainty

In Figures 1 and 2, the MUC pattern of potential output indicates a brief period of expansion at the end of the 1980s. This is followed by a period of excess capacity covering much of the 1990s. In comparison to the BP potential output measure, the MUC measures suggest that the Australian economy headed into contraction in 1990Q2, earlier by two quarters. In addition, the MUC contraction is steeper and more persistent, attaining its trough in 1992Q2 at -6.5% as opposed to the BP at -2.5%. Towards the end of the sample period, however, the MUC output gap disagrees with the BP measure, suggesting that the economy was in a period of growing excess demand to 1.7% (though the 95% confidence interval just includes 0). This shows the merits of conditioning the path of potential output by incorporating information from the aggregate supply side through the Phillips curve and Okun's law, and from the aggregate demand side through the dynamic IS curve with intertemporal optimization .

**Table 1: Estimation Results of the Multivariate UC Model ((1) through (10))**

Parameters	$\sigma_z=0.4$	$\sigma_z=0.6$	$\sigma_z=0.8$
a_1	0.726	0.678	0.640
	(0.000)	(0.000)	(0.000)
a_2	-0.145	-0.160	-0.168
	(0.005)	(0.001)	(0.000)
a_3	0.092	0.092	0.090
	(0.006)	(0.005)	(0.009)
a_4	0.879	0.865	0.839
	(0.000)	(0.000)	(0.000)
b_1	1.197	1.198	1.200
	(0.000)	(0.000)	(0.000)
b_2	-0.197	-0.198	-0.200
	(0.036)	(0.031)	(0.082)
b_3	0.057	0.054	0.051
	(0.007)	(0.004)	(0.008)
b_4	0.025	0.025	0.025
	(0.000)	(0.000)	(0.000)
b_5	0.068	0.070	0.070
	(0.011)	(0.006)	(0.011)
c_1	1.333	1.340	1.349
	(0.000)	(0.000)	(0.000)
c_2	-0.522	-0.527	-0.535
	(0.000)	(0.000)	(0.000)
c_3	-0.118	-0.116	-0.114
	(0.000)	(0.000)	(0.000)
d	4.914	4.826	4.401
	(0.008)	(0.007)	(0.015)
Standard Errors			
$\sigma_{\tilde{y}}$	0.510	0.489	0.470
	(0.000)	(0.000)	(0.000)
σ_{π}	0.362	0.362	0.362
	(0.000)	(0.000)	(0.000)
$\sigma_{\tilde{u}}$	0.064	0.065	0.065
	(0.003)	(0.002)	(0.001)
σ_{y*}	0.000	0.000	0.000
	(0.999)	(0.999)	(0.999)
σ_{μ}	0.056	0.055	0.054
	(0.000)	(0.000)	(0.000)
σ_{u*}	0.157	0.155	0.153
	(0.000)	(0.000)	(0.000)
Log likelihood	-88.11	-87.20	-86.43

Note: p-values are given in parentheses.

Figure 1
Real GDP and Smoothed Potential Output

Real GDP — Band Pass ✳ Potential output (0.8)

Figure 2
**Potential GDP Gap with 95% Confidence Interval**

SMOOTHED GDP GAP ····· UPPER ‒ ‒ LOWER ✳ Band Pass

We show in figure 3 that the MUC measure of the NAIRU fell throughout the sample period in general, except for the temporary and minor pickup around the 1990-91 recession. Interestingly, the MUC NAIRU declined fastest in the 1980s, which suggests

that was the substantive decade of labour market reform. Unlike the BP filter which essentially plots the trend line through the unemployment data, the multivariate NAIRU was much lower for most of the time. At the end of 1996, the NAIRU was down to 5.6% (with a 95% confidence interval of {4.2%,7.1%}). Even though the unemployment rate has been on a downward trend since the early 1990s, the result suggests that the accompanying slower decline in the NAIRU has buffered the Australian economy from inflationary pressure. Mirroring the estimates of the output gap, the MUC unemployment gap values in Figure 4 show that the slack conditions in the labour market persisted for much of the 1990s, despite the BP measure suggesting tightness after 1994. At the end of 2006, unlike the BP filter, the MUC measure indicates that the labour market had become increasingly tight (though the 95% confidence interval did just include 0).

Figure 3
Unemployment: NAIRU & 95% Confidence Interval

Figure 4
**Smoothed Unemployment Gap & 95% Confidence Interval**
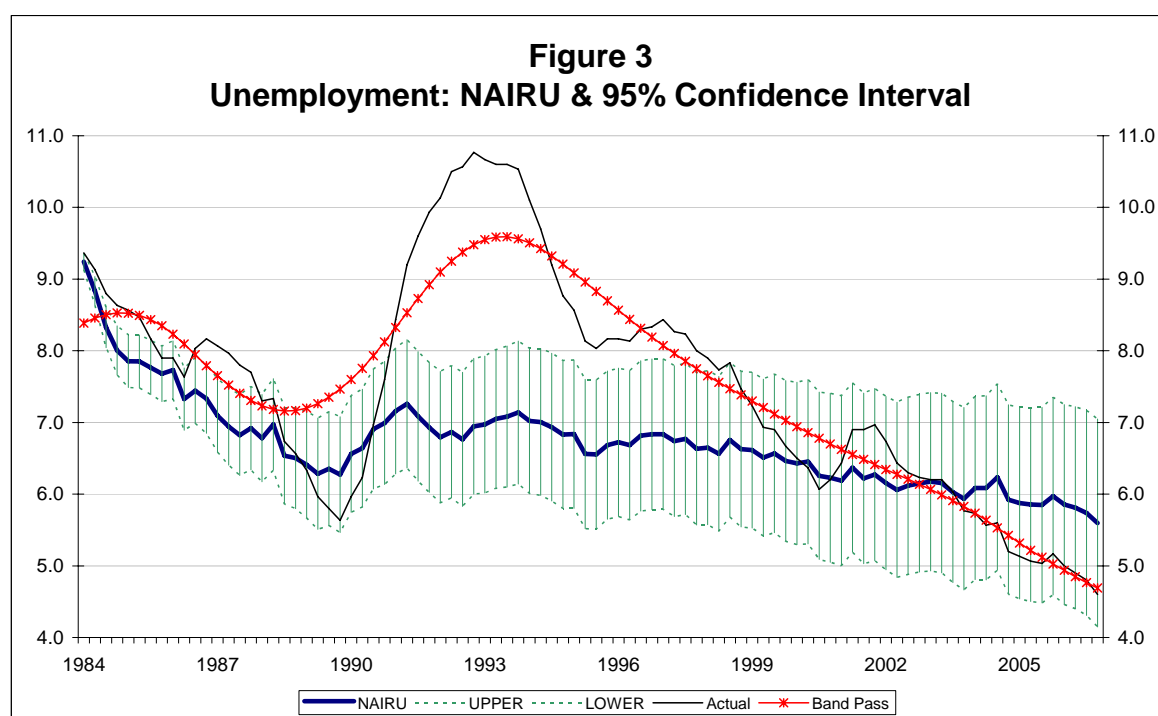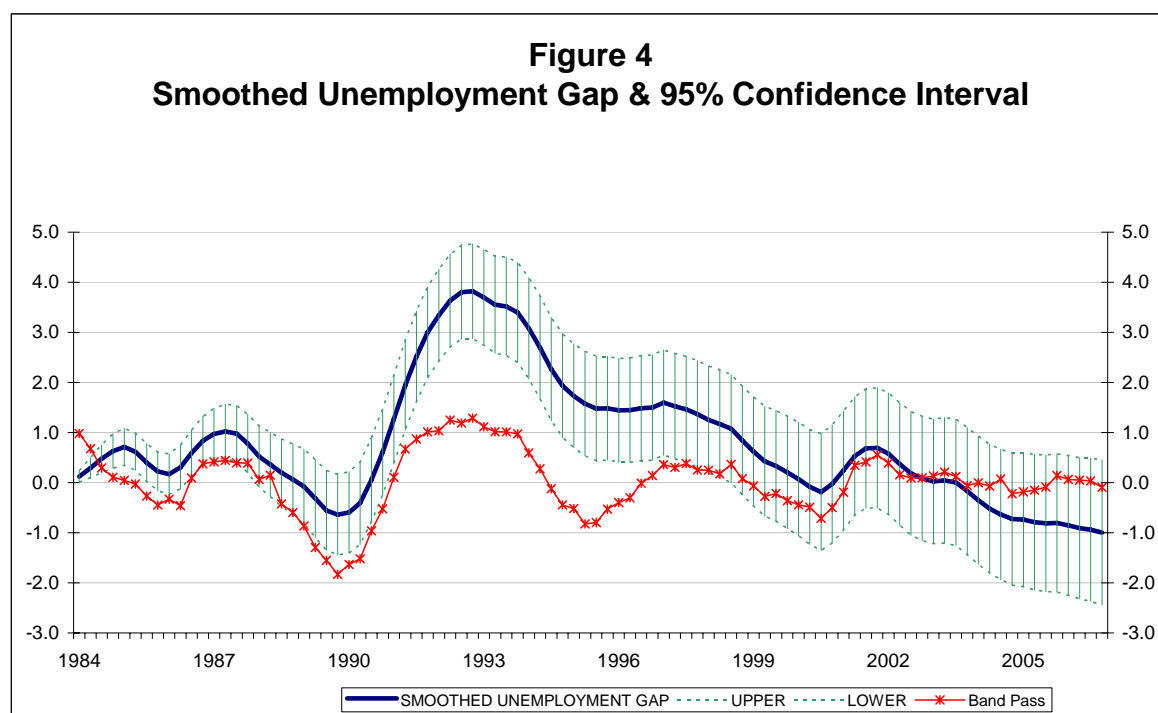
The BP and MUC filter provide contrasting perspectives on the real interest rate, and by implication on the monetary policy stance over the sample period. As seen in Figures 5 and 6, the univariate BP estimates suggest that the natural real interest rate was significantly higher than the MUC estimates until 1999, but the roles reverse after 1999. The BP real interest rate gap measure indicates that monetary policy became expansionary from 1987 after the stock market crash and for about three years after the recession in 1991, but was only modestly restrictive (+4.9%) between the two expansionary phases. Given the depth of the recession, this is an unsatisfactory result. On the other hand, the MUC measure suggests that monetary policy was highly contractionary (peaking at +13.6% beginning 1990), only being reversed into expansion after 1996. During the severe monetary policy contraction in 1989-90, the actual real cash rate went up dramatically, and the BP natural real rate measure followed it up to a degree in its economic blindness. By contrast, in 1989-90, the MUC real rate fell, which is what

19

economic insight would suggest. Since actual and normal output and consumption growth

fell in that recession, the medium run real interest rate had to follow suit to a degree to

maintain intertemporal balance. Since 1999, the MUC real rate suggests monetary policy

has stayed relatively stimulative with the natural real interest rate converging on 4.3% at

the end of 2006. However the 95% confidence interval is wide {-3.5%,12.2%} indicating

the imprecision in obtaining estimates of the unobserved real interest rates. This result

underscores the caution exhibited by policymakers when making monetary policy

decisions.

Figure 5
Natural Real Interest Rate & 95% Confidence Interval

Figure 6
Real Interest Rate Gap and 95% Confidence Interval
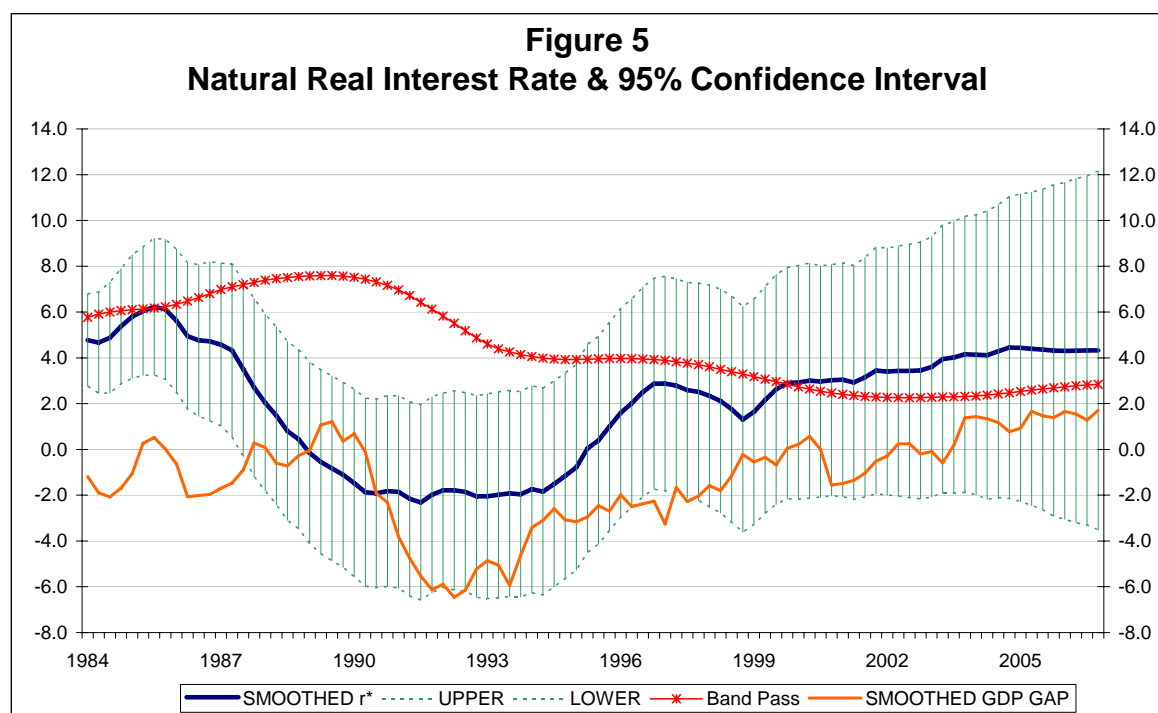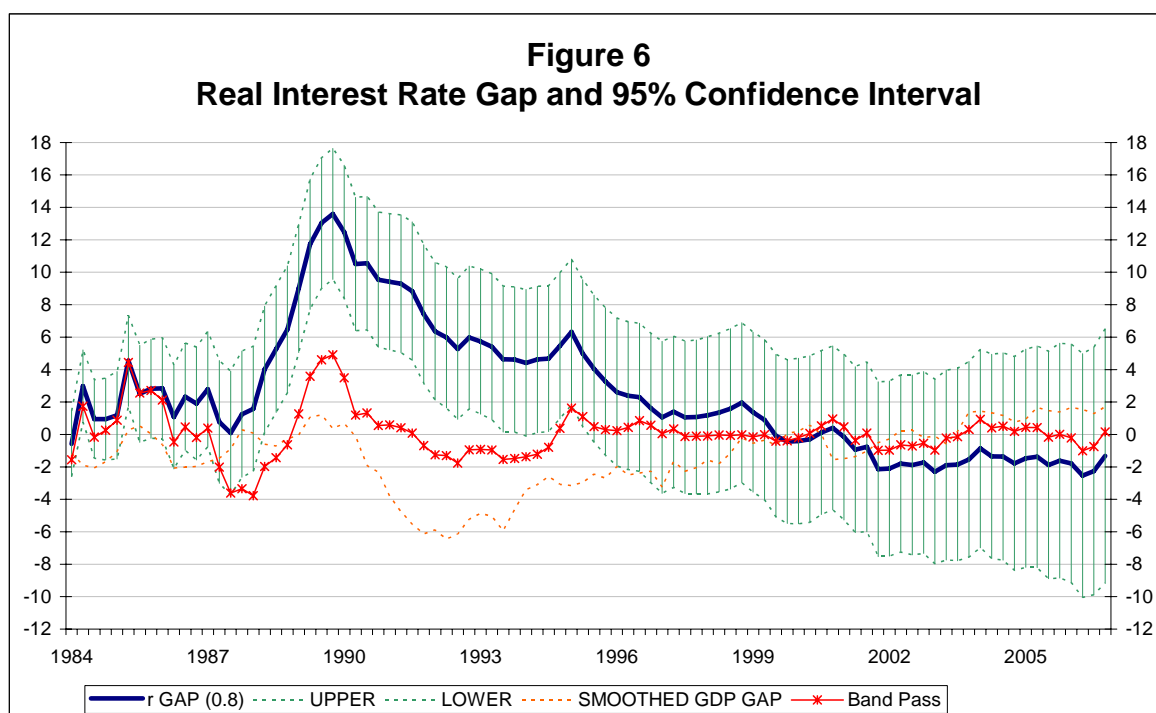
Finally, the estimates of the underlying normal quarterly growth rate of the economy are shown in Figure 7. The BP estimates exaggerate the movements and appears smoother than the MUC ones. The MUC estimates show a general fall in normal growth rates, which began well before the onset of the 1990-1 recession, but turned at the bottom of that recession, rising until the end of 1996. The normal rate of growth has actually declined significantly since 1996, despite output growth being consistently stable and positive. At the end of 2006, the normal growth rate was 2.4%, surprisingly lower than what it had been in the 1991 trough (2.80%). However the 95% confidence interval was wide, and again this is a recommendation for added caution in policy design.

Figure 7
Normal GDP Growth Rate & 95% Confidence Interval
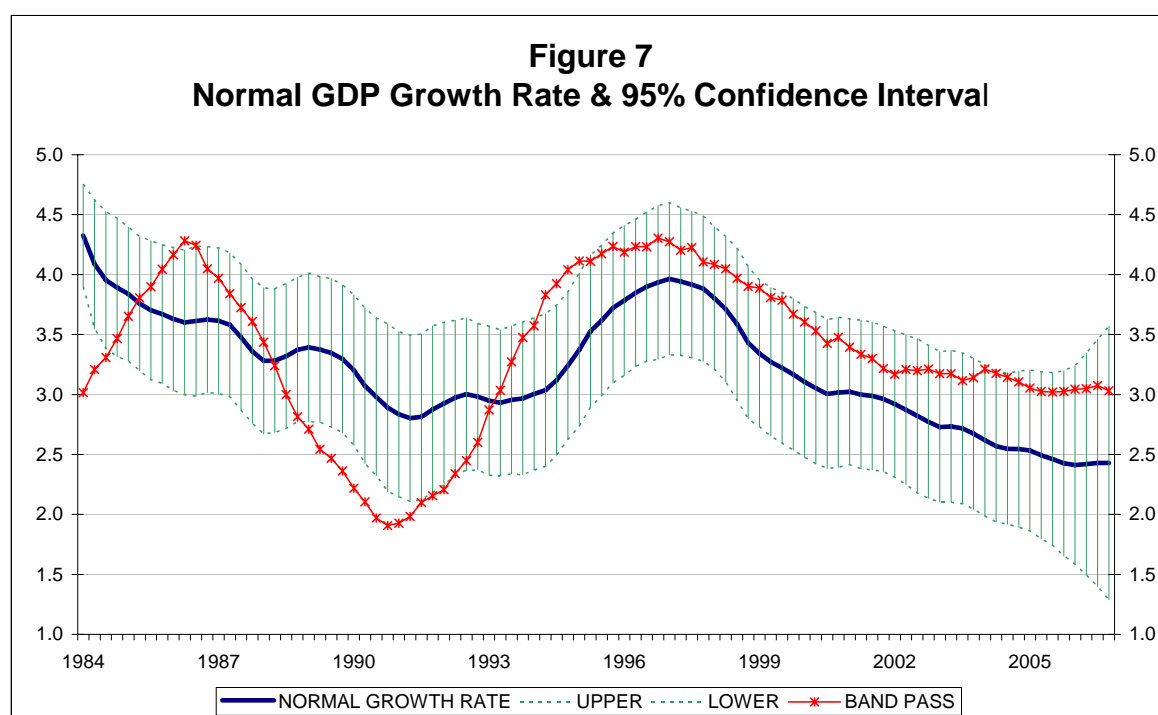
Concluding Comments

Natural rates and normal growth rates are medium-run benchmarks that permit a judgement about whether the actual rates are too high, too low or just right. We have jointly estimated the time paths of these unobservable benchmarks using maximum likelihood methods with the Kalman filter for Australian data from 1984 to 2006. We constructed a standard macroeconomic model for inflation, output and unemployment, and our parameter estimates were all significant with the expected signs. From the inferred natural rate of unemployment, the natural level of output (or potential output) and its normal growth rates, and the natural real rate of interest, we have been able to assess the state of the actual economy and comment on the stance of monetary policy over the two decades of the sample.

We find that our multivariate unobserved components model generates results that have far more economic significance than a univariate band-pass filter. We provide bootstrapped 95% confidence intervals for our estimates of the time paths of the unobservable natural rate variables. These intervals are typically wide, and this leads us to conclude that policy makers wisely practise caution when designing their monetary and fiscal policy responses.

At the end of 2006, we conclude that output was just above potential (+2%), that its normal growth rate was actually quite low at about 2.4% on an annualized basis, that unemployment was about a 1 percentage point below its natural rate of 5.6%, and that the real cash rate was actually 1.3 percentage points below its natural rate of 4.3%. This suggests that monetary policy was possibly still expansionary, but it is not clear that this could have any beneficial effect with output above potential, and with monetary policy expected to be neutral in regard to the normal rate of growth. Correcting the downward trend in the normal growth rate of GDP will require more than monetary and fiscal policy responses by government.

Appendix

The state space representation is consisted of a measurement equation:

$$\mathbf{w}_t = \mathbf{F}\boldsymbol{\xi}_t + \mathbf{v}_t \qquad (18)$$

and a state equation:

$$\boldsymbol{\xi}_t = \mathbf{G}\boldsymbol{\xi}_{t-1} + \mathbf{H}\mathbf{x}_t + \boldsymbol{\upsilon}_t \qquad (19)$$

where $\mathbf{w}_t$ is the vector of observable variables, $\boldsymbol{\xi}_t$ is the vector of state (or unobservable variables), and $\mathbf{x}_t$ is the vector of exogenous variables. $\mathbf{v}_t$ and $\boldsymbol{\upsilon}_t$ are white noise innovation vectors.

The measurement equations in matrix form:

$$
\begin{bmatrix} y_t \\ u_t \\ \pi_t \end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{y}_t \\ \pi_t \\ \pi_{t-1} \\ \tilde{u}_t \\ \tilde{u}_{t-1} \\ y_t^* \\ \mu_t \\ r_t^* \\ r_{t-1}^* \\ r_{t-2}^* \\ r_{t-3}^* \\ r_{t-4}^* \\ r_{t-5}^* \\ r_{t-6}^* \\ r_{t-7}^* \\ z_t \\ u_t^*
\end{bmatrix}
\qquad (20)
$$

The transition equations in matrix form:

$$
\begin{bmatrix}
\tilde{y}_t \\
\pi_t \\
\pi_{t-1} \\
\tilde{u}_t \\
\tilde{u}_{t-1} \\
y^*_t \\
\mu_t \\
r^*_t \\
r^*_{t-1} \\
r^*_{t-2} \\
r^*_{t-3} \\
r^*_{t-4} \\
r^*_{t-5} \\
r^*_{t-6} \\
r^*_{t-7} \\
z_t \\
u^*_t
\end{bmatrix}
=
\begin{bmatrix}
a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_2 & 0 & 0 \\
b_3 & b_1 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_3 a_1 & 0 & 0 & c_1 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_3 a_2 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\tilde{y}_{t-1} \\
\pi_{t-1} \\
\pi_{t-2} \\
\tilde{u}_{t-1} \\
\tilde{u}_{t-2} \\
y^*_{t-1} \\
\mu_{t-1} \\
r^*_{t-1} \\
r^*_{t-2} \\
r^*_{t-3} \\
r^*_{t-4} \\
r^*_{t-5} \\
r^*_{t-6} \\
r^*_{t-7} \\
r^*_{t-8} \\
z_{t-1} \\
u^*_{t-1}
\end{bmatrix}
+
\begin{bmatrix}
a_2 r_{t-8} + a_3 \Delta LTOT_{t-4} + a_4 \Delta \tilde{y}^{G7}_t \\
b_4 \tilde{\pi}^{imp}_{t-1} + b_5 \tilde{\pi}^e_t \\
0 \\
c_3 \left(a_2 r_{t-8} + a_3 \Delta LTOT_{t-4} + a_4 \Delta \tilde{y}^{G7}_t \right) \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
+
\begin{bmatrix}
\varepsilon^{\tilde{y}}_t \\
\varepsilon^{\pi}_t \\
0 \\
c_3 \varepsilon^{\tilde{y}}_t + \varepsilon^{\tilde{u}}_t \\
0 \\
\varepsilon^{y^*}_t \\
\varepsilon^{\mu}_t \\
d \varepsilon^{\mu}_t + \varepsilon^z_t \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\varepsilon^z_t \\
\varepsilon^{u^*}_t
\end{bmatrix}
\tag{21}
$$

The covariance matrix of the residuals of the transition equations is as follows:

$$
Q = \begin{bmatrix}
\sigma_{\bar{y}}^2 \\
0 & \sigma_{\pi}^2 \\
0 & 0 & 0 \\
c_3\sigma_{\bar{y}}^2 & 0 & 0 & c_3^2\sigma_{\bar{y}}^2 + \sigma_{\bar{u}}^2 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{y*}^2 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\mu}^2 \\
0 & 0 & 0 & 0 & 0 & 0 & d\sigma_{\mu}^2 & d\sigma_{\mu}^2 + \sigma_z^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_z^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_z^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{u*}^2
\end{bmatrix}
\qquad (22)
$$

References

Apel, M. and P. Jansson (1999a), "System Estimates of Potential Output and the NAIRU", *Empirical Economics*, 24, pp. 373-388.

Apel, M. and P Jansson (1999b), "A Theory-Consistent System Approach for Estimating Potential Output and the NAIRU", *Economics Letters*, 64, 271-275.

Ball, L. and N.G. Mankiw (2002), "The NAIRU in Theory and Practice", *NBER Working Paper 8940*.

Barsky, R., F. Juster, M. Kimball, and M. Shapiro (1997), "Preference Parameters and Behavioural Heterogeneity: An Experimental Approach in the Health and Retirement Study", *Quarterly Journal of Economics*, 112, 537-579.

Baxter, M. and R.G. King (1999), "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series", *Review of Economics and Statistics*, 81, 575-593.

Benes, J. and P. N'Diaye (2004), "A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to the Czech Republic", *IMF Working Paper WP/04/45*.

Boone, L. (2000), "Comparing Semi-Structural Methods to Estimate Unobserved Variables: The HPMV and Kalman Filters Approaches", *OECD Economics Department Working Papers No. 240*.

Boone, L., C. Giorno, M. Meacci, D. Rae, D. Turner, and P. Richardson (2000), "The Concept, Policy Use and Measurement of Structural Unemployment: Estimating a Time Varying NAIRU Across 21 OECD Countries", *OECD Economics Department Working Papers No. 250*.

de Brouwer, G. (1998), "Estimating Output Gaps", *RBA Research Discussion Paper 9809*.

Clark, P.K. (1987), "The Cyclical Component of U.S. Economic Activity", *Quarterly Journal of Economics*, 102, pp. 797-814.

Clark, P.K. (1989), "Trend Reversion in Real Output and Unemployment", *Journal of Econometrics*, 40, pp. 15-32.

Fair, R.C. (2005), "Natural Concepts in Macroeconomics", *Cowles Foundation Discussion Paper No. 1525, Yale University*.

Friedman, M. (1968), "The Role of Monetary Policy", *American Economic Review*, 58, pp.1-17.

Garnier, J. and B.R. Wilhelmsen (2005), "The Natural Real Interest Rate and the Output Gap in the Euro Area: A Joint Estimation", ECB Working Paper No. 56.

Gruen, D., T. Robinson and A. Stone (2005), "Output Gaps in Real Time: How Reliable Are They?", *Economic Record*, 81, pp. 6-18.

Hodrick, R.J. and E.C. Prescott (1997), "Postwar U.S. Business Cycles: An Empirical Investigation", *Journal of Money, Credit, and Banking*, 29, pp. 1-16.

King, R., J. Stock, and M. Watson (1995), "Temporal Instability of the Unemployment-Inflation Relationship", *Federal Reserve Bank of Chicago Economic Perspectives*, pp. 2-12.

Kuttner, K.N. (1991), "Using Noisy Indicators to Measure Potential Output", *Federal Reserve Bank of Chicago working paper 1991-14*.

Kuttner, K.N. (1994), "Estimating Potential Output As A Latent Variable", *Journal of Business and Economic Statistics*, 12, pp. 361-368.

Laubach, T. (2001), "Measuring the NAIRU: Evidence From Seven Economies", *Review of Economics and Statistics*, 83, pp. 21-231.

Laubach, T. and J.C. Williams (2003), "Measuring the Natural Rate of Interest", *Review of Economics and Statistics*, 85, pp. 1063-1070.

Laxton, D. and R. Tetlow (1992), "A Simple Multivariate Filter for the Measurement of Potential Output", *Bank of Canada Technical Report No. 59*.

Lucas, R.E. (1972), "Expectations and the Neutrality of Money", *Journal of Economic Theory*, 4, pp. 103-124.

Messonier, J.-S. and J.-P. (2004), "A Time-Varying Natural Rate of Interest for the Euro Zone", *mimeo*, Banque de France.

Okun, A.M. (1962), "Potential GNP: Its Measurement and Significance", in *Proceedings of the Business and Economics Section*, American Statistical Association, Washington D.C., pp. 98-104.

Ogaki, M. and C. Reinhart (1998), "Measuring Intertemporal Substitution: The Role of Durable Goods", *Journal of Political Economy*, 106, 1078-1098.

Phelps, E.S. (1968), "Phillips Curves, Expectations of Inflation and Optimal Unemployment Over Time", *Economica*, 34, pp. 254-281.

Rudebusch, G.D. and L.E.O. Svensson (1999), "Policy Rules for Inflation Targeting", in J.B. Taylor (ed.), *Monetary Policy Rules*, Chicago: University of Chicago Press, pp. 203-253.

Staiger, D., J. Stock and M. Watson (1996), "How Precise Are Estimates of the Natural Rate of Unemployment?", *NBER Working Paper 5477*.

Stock, J. (1994), "Unit Roots, Structural Breaks and Trends", in R. Engle and D. McFadden (eds.), *Handbook of Econometrics*, vol. 4, Amsterdam: Elsvier, pp. 2739-2841.

Stock, J. and M. Watson (1998), "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model", *Journal of the American Statistical Association*, 93, pp. 349-358.

Stone, A., Wheatley, T. and Louise Wilkinson (2005), "A Small Model of the Australian Macroeconomy: An Update", *RBA Research Discussion Paper 2005-11*.

Watson, M. (1986), "Univariate Detrending Methods with Stochastic Trends", *Journal of Monetary Economics*, 18, pp. 49-75.

Wicksell, K. (1936), *Interest and Prices*, London: Macmillian. Translation of the 1898 original edition.

Woodford, M. (2003), *Interest and Prices – Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton, New Jersey.