

**Gravitation & Cosmology — ASTR-4240**  
**General Relativity — PHYS-4961**

**Class 4**  
**Spacetime Physics**

**Exercise (20 pts)**

- 1. (10 pts)** — Write out all of the components of the tensor

$$L^{\mu\nu} \equiv x^\mu p^\nu - x^\nu p^\mu, \quad (1)$$

where  $x^\mu$  and  $p^\mu$  are the position and 4-momentum of a point particle with rest mass  $m$  and ordinary velocity  $\mathbf{u}$ . Use ordinary units, i.e., do not set  $c = 1$ .

- 2. (10 pts)** — Do any of the components of  $L^{\mu\nu}$  describe a familiar physical quantity? Which ones and what is the quantity?

**Solution**

- 1.** The tensor has Cartesian components

$$L^{\mu\nu} = \begin{pmatrix} 0 & -\gamma mc(x - u_x t) & -\gamma mc(y - u_y t) & -\gamma mc(z - u_z t) \\ \gamma mc(x - u_x t) & 0 & \ell_z & -\ell_y \\ \gamma mc(y - u_y t) & -\ell_z & 0 & \ell_x \\ \gamma mc(z - u_z t) & \ell_y & -\ell_x & 0 \end{pmatrix}, \quad (2)$$

where

$$\boldsymbol{\ell} \equiv \mathbf{x} \times \mathbf{p}. \quad (3)$$

- 2.** Yes:  $\boldsymbol{\ell}$  is the orbital angular momentum of the particle.