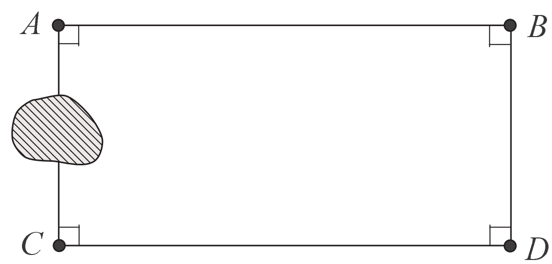
Gravitation & Cosmology — ASTR-4240
General Relativity — PHYS-4240

Class 11
Classical Tests of General Relativity II



Exercise (35 pts)

The figure above[1] describes the following experiment which could be carried out in principle. It shows a region of space where the gravitational field is only appreciable inside the cross-hatched region. Observers are in spaceships outside the field at points A, B, C, and D. They maneuver their ships until all four interior angles of the figure are exactly ninety degrees.

1. (10 pts) — Are coordinate time ("t time") and measured time the same or different for these observers. Explain briefly.

2. (5 pts) — Observer A sends a light signal to B which takes coordinate time Δt_{AB}. Similarly, C sends a light signal to D which takes coordinate time Δt_{CD}. How are Δt_{AB} and Δt_{CD} related? If they are the same say so. If not, give an inequality.

3. (5 pts) — Based on your answer to part 2, what can the observers conclude about the lengths of sides AB and CD?

4. (5 pts) — Now the observers measure the analogous quantities Δt_{AC} and Δt_{BD}. How are they related?

5. (5 pts) — Based on your answer to part 4, what can the observers conclude about the lengths of sides AC and BD?

[1]essentially identical to Fig. 4.10 on p. 201 of Ohanian & Ruffini.

6. (5 pts) — In at least one of the questions above, you should find that the times are different for light trips on parallel sides of the figure. One way to explain this would be to say that the speed of light is different on the two sides. Is this a viable explanation?

Solution

1. — They are the same. Let dt_{m} and dt be the measured and coordinate time differences between two nearby events. In the (t, x, y, z) coordinate system we have been using they are related by

$$dt_{\mathrm{m}} = \sqrt{g_{00}}\, dt, \tag{1}$$

where

$$g_{00} = 1 - \frac{r_{\mathrm{S}}}{r}. \tag{2}$$

Outside the field ($r \to \infty$) we have $g_{00} = 1$ so $dt_{\mathrm{m}} = dt$.

2. — They are equal.

3. — The lengths are equal.

4. — Because of light retardation, $\Delta t_{AC} > \Delta t_{BD}$.

5. — AC is longer than BD.

6. — No. Consider a continuous string of freely-falling observer along leg AC of the rectangle. There is no gravitational field in any of their frames (Equivalence Principle) so each of them will conclude that the local speed of light is c. It follows that the speed of light is c everywhere along AC.