Activity #3 – Dark Matter Name ~~K~~ Answer Key

In this activity we will discuss why we think there is dark matter in the Universe. First, we will study the solar system. Then, we will use our understanding of the solar system to interpret information about other galaxies.

Solar system data

	Mass	Mean dist. from Sun	Orbital period
Mercury	3.3×10^{23} kg	0.39 AU	87.96 days
Venus	4.9×10^{24} kg	0.72 AU	224.7 days
Earth	6.0×10^{24} kg	1.00 AU	365.3 days
Mars	6.4×10^{23} kg	1.52 AU	687.0 days
Jupiter	1.9×10^{27} kg	5.20 AU	4333 days
Saturn	6.7×10^{26} kg	9.54 AU	10,759 days
Uranus	1.5×10^{25} kg	19.2 AU	30,685 days
Neptune	1.0×10^{26} kg	30.1 AU	60,188 days
Pluto	1×10^{22} kg	39.4 AU	90,700 days
Comet	8×10^{12} kg	20,000 AU	
Sun	2×10^{30} kg	--------	

There are thought to be about 10^{14} comets in the solar system.

1. What is the total mass of everything in the solar system, except the Sun? Show your work.

Adding the masses of the planets together (it doesn't matter if you include Pluto or not), plus 10^{14} times the mass of a comet, the total mass is $\boxed{3.5 \times 10^{27}\text{ kg}}$

This is less than twice the mass of Jupiter.

2. What fraction of the mass of the solar system is the Sun? Show your work.

$$\frac{2 \times 10^{30}\text{ kg}}{2 \times 10^{30}\text{ kg} + 3.5 \times 10^{27}\text{ kg}} = \boxed{99.8\%}$$

3. For each planet, assume the orbit is circular (a pretty good guess), and compute its speed. Include units. Remember, velocity = distance/time and for a circle, circumference =(2π)(radius).

$$V = \frac{d}{t} = \frac{2\pi r}{\text{period}}$$

Mercury 0.028 AU/day

Venus 0.020 AU/day

Earth 0.017 AU/day

Mars 0.014 AU/day

Jupiter 0.0075 AU/day

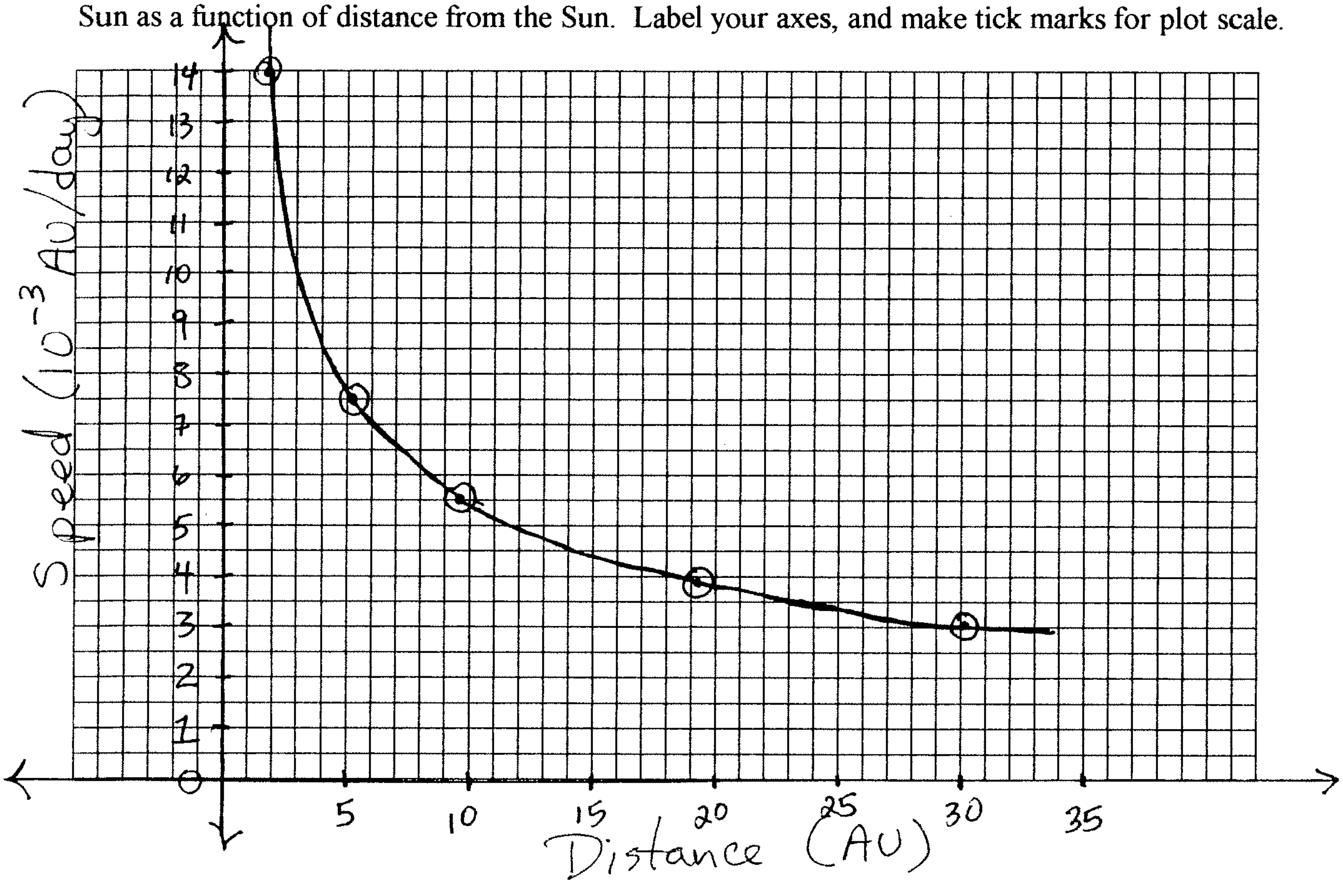
Saturn 0.0056 AU/day

Uranus 0.0039 AU/day

Neptune 0.0031 AU/day

Pluto 0.0027 AU/day

4. For Mars, Jupiter, Saturn, Uranus, and Neptune, plot the speed the planet moves around the Sun as a function of distance from the Sun. Label your axes, and make tick marks for plot scale.



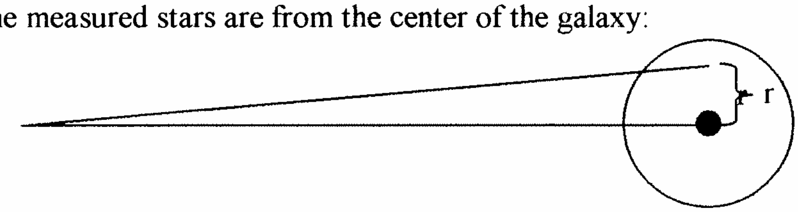
How do we measure rotation curves in other galaxies?

It is easiest to measure rotation curves for "edge-on" galaxies. Instead of seeing a circular galaxy with spiral arms, in an edge-on galaxy we look along the plane of the galaxy. For example, and edge-on galaxy might look like this:

Now think about the fact that the stars in the galaxy are rotating around the center. That means that the stars on one side of the galaxy are coming towards us, and the stars on the other side are going away. Well, actually, the whole galaxy is probably moving away from us, so the stars on one side will have a smaller Doppler shift than the galaxy center, and the stars on the other side will have a larger Doppler shift than the galaxy center.

With a spectrograph, we can look at the light in each little piece of the galaxy along its length. For each piece, we spread the light out in wavelength to produce a spectrum. Each spectrum is the combined spectrum of a group of stars in that part of the galaxy. The group of stars will be moving together and thus give a common Doppler shift of the spectral lines. From this Doppler shift, one can determine the velocity towards or away from us. Since we are very far away, the orbital velocity is approximately the difference between the measured Doppler shift velocity at some location and the Doppler shift velocity of the center of the galaxy.

From the image of the galaxy, one can look at the angle between the center of the galaxy and each measured group of stars. If we also know the distance to the galaxy, we can figure out what distance, r, the measured stars are from the center of the galaxy:



Since we know know both the radius from the galaxy center and the velocity of the stars for a number of places in the galaxy, we can plot a rotation curve for the galaxy.

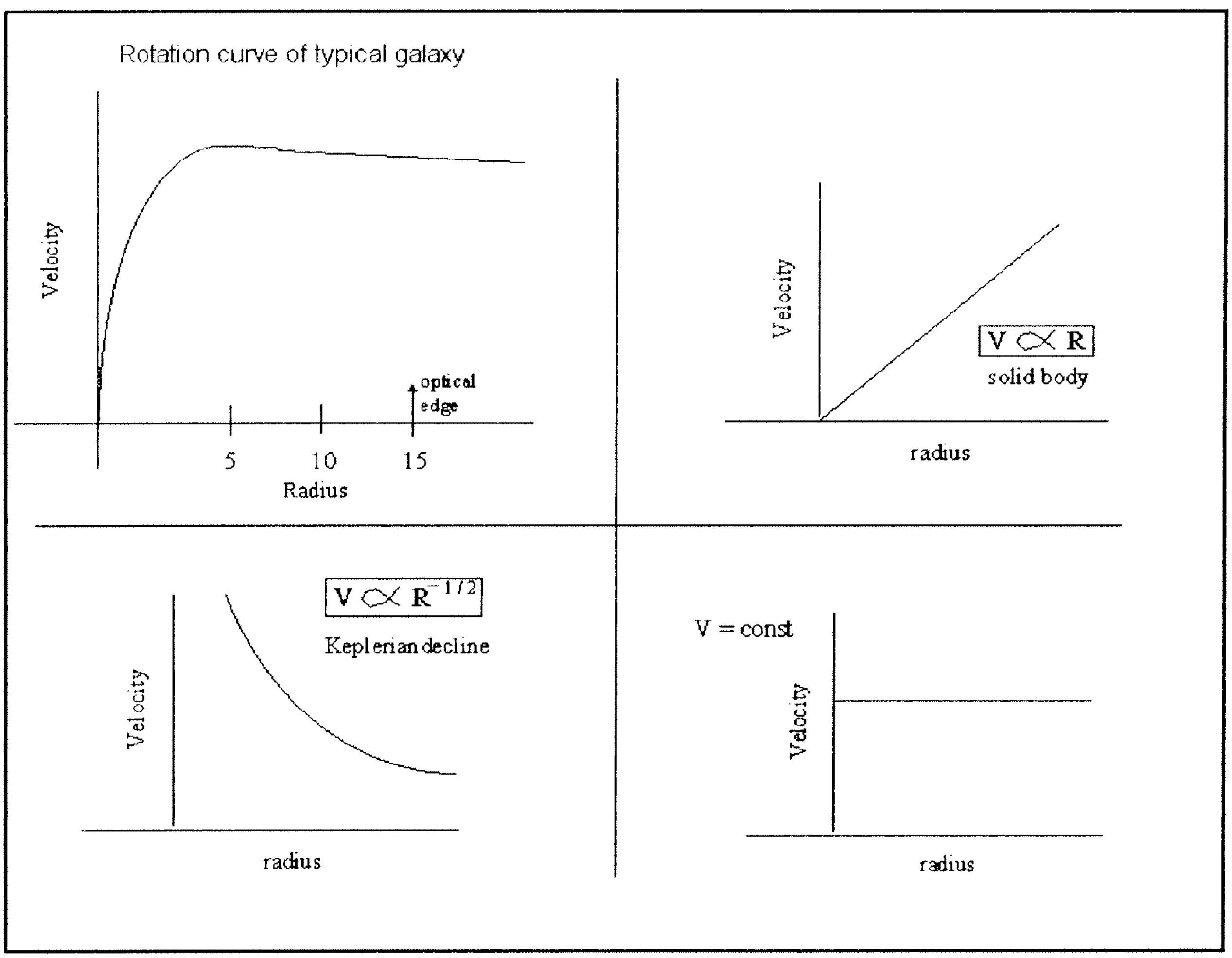
Rigid bodies (like wheels, for example) have rotation curves that look like the top right diagram below. The rigid body rotation curve does not depend on the mass distribution within the rigid body. The outer parts of a wheel have farther to go in one wheel rotation, so they move faster.

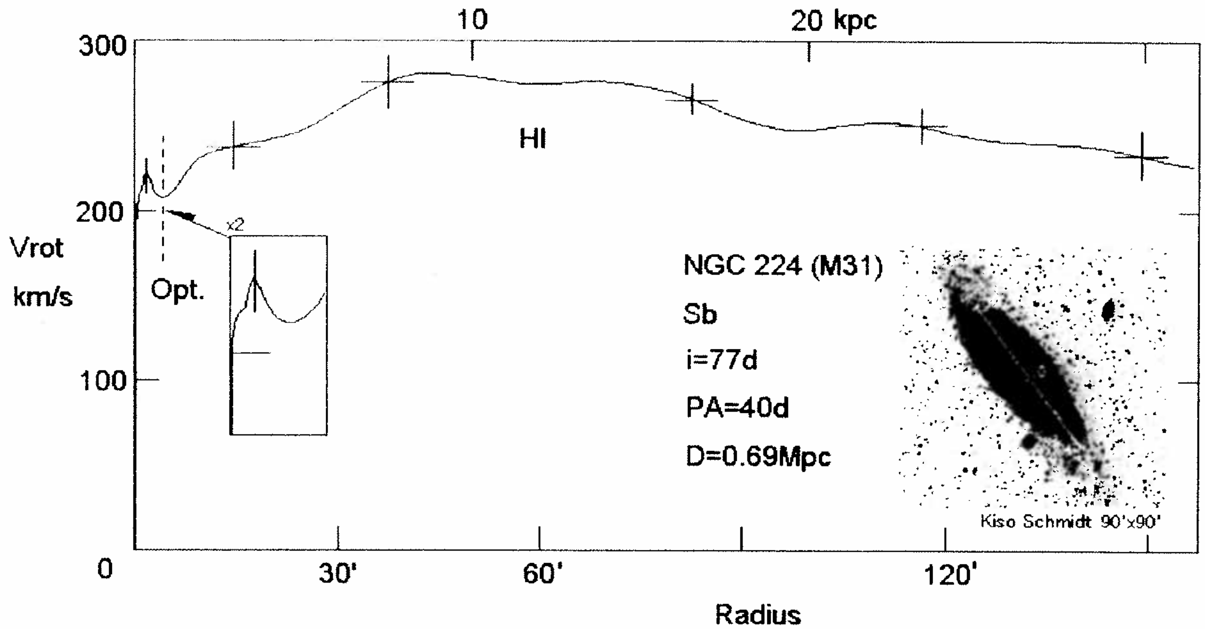
We distinguish that case from the case where we have little masses moving under the gravitational pull of the whole system. For example, planets and comets move freely under the gravitational pull of the whole solar system. Stars move freely under the gravitational pull of the whole system of stars in individual galaxies. Looking at rotation curves of gravitationally bound objects can tell you about the distribution of mass in the region the objects are in.

If all of the mass is concentrated in the center of the system, as in the solar system, we expect the velocity to be inversely proportional to the square root of the distance from the center of mass (like in the solar system). This case is shown in the bottom left panel below.

If the mass profile goes as $1/r^2$, then the speed of the particles is the same at all distances from the center. This case is shown in the bottom right panel below. With this mass profile, the total mass interior to radius "r" is proportional to "r."

In the top left panel, we show a typical galaxy rotation curve, and a typical galaxy light profile. Once the distance from the center is larger than the distance from the center of most of the mass, we expect the rotation curve to start falling, as if all of the mass were at the center.





5. The rotation curve for the Andromeda galaxy, along with an image of the Andromeda galaxy, is shown above. What is the radius of the light part of the Andromeda galaxy in arc minutes?

Some where between 30' and 50', depending on how you decide to measure it.

6. How you can tell whether most of the mass is within the radius calculated in question 6?

If most of the mass is within 50', then the rotation curve will fall off as $1/\sqrt{r}$ at larger radii (r) from the center.

7. From the plot, the velocities of all of the stars further than about 10 arcminutes from the center of the galaxy are about the same. What does that tell us about the relationship between a given radius from the galaxy center and the mass contained within a sphere of that radius?

The mass within radius "r" is proportional to that radius

8. Where is most of the mass in the galaxy?

Spread evenly over all distances $10' < r < 150'$ from the center, and could extend even further.