Gravitation & Cosmology — ASTR-4240
General Relativity — PHYS-4240

Class 10
Classical Tests of General Relativity I

Exercise (20 pts)

The international time standard uses very precise atomic clocks in different laboratories around the world. However gravitational time dilation will cause atomic clocks at different altitudes to run at different rates. Do the timekeepers need to worry about this effect?

1. (10 pts) — Consider two identical atomic clocks, one located at sea level and the other at an altitude of 1600 m (=1 mile) above sea level. If the two clocks have the same reading at time $t = 0$, by how much will they differ at $t = 1\,\mathrm{year}$?

2. (10 pts) — Suppose the labs use cesium atomic clocks, which keep time accurate to about 1 part in 10^{14}. Do they need to correct for gravitational time dilation?

Solution

1. — Let τ_2 and τ_1 be the elapsed time recorded during one year by the clocks at sea level and 1600 m, respectively, and define $\Delta\tau \equiv \tau_2 - \tau_1$. Then

$$\frac{\Delta\tau}{\tau} = g\,\frac{\Delta r}{c^2}, \tag{1}$$

where Δr is the altitude difference. Setting $g = 9.8\,\mathrm{m\,s^{-2}}$, $\Delta r = 1600\,\mathrm{m}$, and $\tau = 1\,\mathrm{yr} = 3.15 \times 10^7\,\mathrm{s}$ gives

$$\Delta\tau_{\mathrm{GR}} \approx 5\,\mu\mathrm{s}. \tag{2}$$

2. — If the clocks are accurate to 1 part in 10^{14} they would typically differ by only

$$\Delta\tau_{\mathrm{ACC}} \approx 10^{-14}\,\mathrm{yr} = 0.3\,\mu\mathrm{s} \tag{3}$$

after one year in the absence of gravitational time dilation. Since $\Delta\tau_{\mathrm{GR}} \sim 10\Delta\tau_{\mathrm{ACC}}$, corrections for gravitational time dilation are extremely important.