Introduction to Astronomy and Astrophysics — Spring 2010

Homework #3 – Solution

1. (a) 1" (1°/3600") (π radians /180°) = (500 x 10^{-9}m) / d ➔ d = 0.1 m = 10 cm
(b) 1" (1°/3600") (π radians /180°) = (21 x 10^{-2} m)/d ➔ d = 4.3 x 10^4 m = 43 km
Extra info: To achieve high resolution at radio wavelengths, we need to use interferometers rather than one absolutely enormous mirror. However, since the atmospheric "seeing" limits visible wavelength seeing to of order 1", Earth-based radio telescopes can achieve better resolution than can optical telescopes – we need only separate the antennae by more than 50 km.

2. (a) Using Wien's Displacement Law, the wavelength of maximum emission is λ_{max} = (2898 μm K)/T. Thus, T = (2898 μm K)/(123 x 10^{-3} μm) = 23,600 K.
(b) F = σT^4 = (5.67 x 10^{-8} W/m^{2}K^4) (23,600 K)^4 = 1.76 x 10^{10} W/m^2
L = F A = F 4πR^2 = 1.76 x 10^{10} W/m^2 (4π) (1.6 x 10^9 m)^2 = 5.66 x 10^{29} W
(c) E= hν = hc/λ = (6.63 x 10^{-34} J s)(3 x 10^8 m/s)/(123 x 10^{-9} m) = 1.62 x 10^{-18} J
(d) 5.66 x 10^{29} W/ 1.62 x 10^{-18} J = 3.5 x 10^{47} photons/sec

3. (a) ρ = m/(4π r^3 / 3) ➔ r^3 = 3 m / (4π ρ) = 3 (1 kg) / (4π 1000 kg/m^3) ➔ r = 0.062 m
(b) Energy per time emitted: L = A F = (4π r^2) (σ T^4)
Energy per time absorbed: L = A F = (π r^2) ($L_\odot$/4π R^2), where $L_\odot$ is the luminosity of the Sun, and R is the distance from the blob to the Sun.
Setting these luminosities equal, we have:
(4π r^2) (σ T^4) = (π r^2) ($L_\odot$/4π R^2) ➔
R^2 = $L_\odot$/(16π σ T^4)= 3.90 x 10^{26} W / (16π 5.67 x 10^{-8} W/m^2 K^{-4} [273 K]^4)
R = 1.6 x 10^{11} m = 1.6 x 10^8 km ≈ 1.0 AU

4. (a) 1/λ= 10.96776 μm^{-1} (1/2^2 – 1/3^2) => λ= 0.65647 μm = 6564.7 Å.
(b) λ = 6564.7 Å [1 + (200 km/s)/(3x10^5 km/s)] = 6569.1 Å
The hint challenge can be answered by making an Excel spreadsheet that calculates wavelengths for both equations and then calculates their fractional differences. I made a spreadsheet with v/c in the first column. The second column is λ/λo = 1 + A1. The third column is the relativistic λ/λo = SQRT((1+A1)/(1-A1)). The fourth column is fractional difference =B1/C1 – 1. Using this spreadsheet, I find that there is a one percent error when v/c = 0.14.

*5.

$$B_\nu = \frac{d\lambda}{d\nu} B_\lambda$$

$$\lambda = \frac{c}{\nu} \Rightarrow \frac{d\lambda}{d\nu} = -\frac{c}{\nu^2}$$

$$B_\nu = \frac{c}{\nu^2}\frac{2hc^2}{(c/\nu)^5}\frac{1}{(e^{hc/(ckT/\nu)}-1)} = \frac{2h\nu^3}{c^2}\frac{1}{(e^{h\nu/kT}-1)}$$

The negative sign in dλ/dν comes from the fact that increasing wavelength gives you decreasing frequency, so if you want to integrate in frequency from low frequency to high frequency (the usual way) you have to switch the minus sign to a plus sign.