**Gravitation & Cosmology — ASTR-4240**
**General Relativity — PHYS-4961**

**Class 3**
**Tidal Forces**

## Exercise (30 pts)

On July 7, 1992 Comet Shoemaker Levy 9 broke apart as it passed about $96,000\,\mathrm{km}$ from the center of Jupiter. Assume that, prior to breakup, the comet was a spherical mass of radius $R = 0.5\,\mathrm{km}$ moving only under the gravitational influence of Jupiter. The mass of Jupiter is $M_\mathrm{J} = 5.69 \times 10^{28}\,\mathrm{g}$. Neglect the gravitational effects of the Sun and other bodies. Assume that the comet was held together entirely by its own gravity.

**1. (10 pts)** — Consider a chunk of rock on the side of the comet facing Jupiter. In the frame of the comet, what are the effective forces on the rock? Just describe them in words.

**2. (10 pts)** — Denote the mass of the rock by $m'$ and the mass of the comet by $M$. Write down an expression for the critical distance from Jupiter, $r_\mathrm{crit}$, where the rock is pulled away from the comet.

**3. (10 pts)** — Using your answer from Part 2, estimate the mass and density of the comet.

## Solution

**1.** — In a freely-falling reference frame attached to the comet, the only forces on the rock are the gravitational force by the comet and the tidal force by Jupiter.

**2.** — The rock is barely held onto the comet when the two forces balance:

$$\frac{GMm'}{R^2} = 2R \cdot \frac{GM_\mathrm{J}m'}{r_\mathrm{crit}^3}.$$

Solving for $r_\mathrm{crit}$ gives the criterion for breakup:

$$r_\mathrm{crit} = R\left(\frac{2M_\mathrm{J}}{M}\right)^{1/3}.$$

**3.** — Solve the last expression for $M$ to find

$$M = 2M_\mathrm{J}(R/r_\mathrm{crit})^3.$$

Plugging in the data gives $\boxed{M = 1.6 \times 10^{13}\,\mathrm{g}.}$ The density is $\boxed{\rho = 0.03\,\mathrm{g\,cm^{-3}}.}$