*Median Filtering

4	3	2	1
3	1	2	4
5	1	6	2
2	3	5	6

zero padding →

0	0	0	0	0	0
0	4	3	2	1	0
0	3	1	2	4	0
0	5	1	6	2	0
0	2	3	5	6	0
0	0	0	0	0	0

[0,0,0,0,0,1,3,3,4] → median = 0

Median Filter →

0	2	1	0
1	3	2	2
1	3	3	2
0	2	2	0

def:按照 filter 大小

eg: 3×3 找 sort 後的中值

並且將其替換為 median 的值

依旁邊 padding →

4	4	3	2	1	1
4	4	3	2	1	1
3	3	1	2	4	4
5	5	1	6	2	2
2	2	3	5	6	6
2	2	3	5	6	6

Median Filter →

3	3	2	2
3	3	2	2
3	3	3	4
2	3	5	6

要留意是否要 padding

① zero padding (只補 0)

② Repeat ... (依旁邊是多少補多少)

*Adjacency

0 1 1
0 1 0 ⇒ 4-Adjacency
0 0 1

0 1 1
0 1 0 ⇒ M-Adjacency
0 0 1

0 1 1
0 1 0 ⇒ 8-Adjacency
0 0 1

① 8連接包含了4、M連接

② M連接要符合 $N_4(p) \cap N_4($ 沒有來自V的元素

△詳見 2.11, 2.15

Sobel & Prewitt Filter

Sobel:

x方向 $\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ gy y方向 gx $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Prewitt:

x方向 gy $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

y方向 gx $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$M(x,y) = \sqrt[n]{|gx|^n + |gy|^n}$ ★ n通常設1 ⇒ $M(x,y) = |gx| + |gy|$

套用 Sobel & Prewitt

5	7	2
6	8	9
5	3	20

Image 的某塊

Sobel: gx: -5-12-5+2+18+20 = 18; gy: -5-14-2+5+6+20 = 10 → *

Prewitt: gx: -16+31 = 15; gy: -14+28 = 14 → >9

2.25 (1),(2) 皆要証明 a. separable; (b) symmetric

(1)

$r(x,y,u,v) = e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$

(a) separable: $r(x,y,u,v) = e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$

$= e^{-j2\pi(\frac{ux}{M})} e^{-j2\pi(\frac{vy}{N})}$

$= r_1(x,u) r_2(y,v)$

(b) symmetric: ∵ $r_1(x,y)$ is functionally equal to $r_2(x,y)$ ∴ $r_1(x,u) r_2(y,v) = r_1(x,u) r_2(y,v)$

(2)

$s(x,y,u,v) = \frac{1}{MN} e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$

(a) separable: $s(x,y,u,v) = \frac{1}{MN} e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$

$= \frac{1}{MN} e^{j2\pi(\frac{ux}{M})} e^{j2\pi(\frac{vy}{N})} = \frac{1}{MN} s_1(x,u) s_2(y,v)$

(b) symmetric: ∵ $s_1(x,y)$ is functionally equal to $s_2(x,y)$

∴ $\frac{1}{MN} s_1(x,u) s_2(y,v) = \frac{1}{MN} s_1(x,u) s_2(y,v)$

2.10 單位轉換 有 2 fields $\frac{1}{60} \times 2 = \frac{1}{30} s$

$2000 \times 1125 \times 24 \times 2 \times 3600 / \frac{1}{30}$

pixel 數量 RGB 2hrs 秒數

11664×10^{13} bits

6.5 Middle Column of the Image → midgray

$0.5R + G + 0.5B = 0.5(R+G+B) + 0.5G$

看起來是綠色，但須 midgray 的部分又其強度↑

2.15 (a) $(N_4^{(4,4,5)})$ 有多個路徑 (b) (6,4,6)

2.11 (a) 非 4-Adjacency 但是 M, 8-Adjacency

2.11 (b) 2.15

拉普拉斯公式: ① 有可能是 $\frac{\partial^2 f}{\partial x^2}$ ...

$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$

DFT

DFT → 連續函

6	8	9
5	3	20

Image 的I區塊

$g_y = -5 - 14 - 2 + 5 + 6 + 20 = 10$

Prewitt: $g_x = -16 + 31 = 15$, $g_y = -14 + 28 = 14$ > 9

2-10 單位轉換：有 2 fields: $\frac{1}{60} \times 2 = \frac{1}{30}s$

$2000 \times 1125 \times 24 \times 2 \times 3600 / \frac{1}{30}$ (pixel 數量, RGB, 2hrs 秒數) $= 11664 \times 10^{13}$ bits

Middle column of the image: $0.5R + G + 0.5B = 0.5(R+G+B) + 0.5G$

看起來是綠色，但消 mid gray 成分，使其強度↑

(4,4,5) (6,4,6)

m,8-Adjacency

尤拉公式：※θ 有可能是 $\frac{2\pi ux}{N}$ 這種型式！

$e^{j\theta} = \cos\theta + j\sin\theta$

$\cos\theta = [e^{j\theta} + e^{-j\theta}]/2$

$\sin\theta = [e^{j\theta} - e^{-j\theta}]/2j$

而 $\mathrm{sinc}(m) = \frac{\sin(\pi m)}{\pi m}$ 日後會用到

$\nabla^2 f(x,y) = \frac{d^2 f(x,y)}{dx^2} + \frac{d^2 f(x,y)}{dy^2}$

※ 1-DFT:

$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$ where $j = i \Rightarrow \sqrt{-1}$

而 $f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$ → 反傅立葉

※ 2-DFT → 連續型

$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx\,dy$

而 $f(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} du\,dv$

2-DFT (離散型) 及反傅立葉

$F(u,v) = \frac{1}{MN}\sum_{0}^{M-1}\sum_{0}^{N-1} f(x,y) e^{-j2\pi\left[\frac{ux}{M} + \frac{vy}{N}\right]}$

其中 $u = 0,1,2,3,\ldots,M-1$; $v = 0,1,2,3,\ldots,N-1$

$\Rightarrow f(x,y) = \sum_{0}^{M-1}\sum_{0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

其中 $x = 0,1,2,3,\ldots,M-1$; $y = 0,1,2,3,\ldots,N-1$

(2,3) pure blue: 0000FF cell(7,8)

4.26

(a)

$\nabla^2 f(t,z) = \frac{d^2 f(t,z)}{dt^2} + \frac{d^2 f(t,z)}{dz^2}$

又 $\mathfrak{F}[\nabla^2 f(t,z)] = \mathfrak{F}\left[\frac{d^2 f(t,z)}{dt^2}\right] + \mathfrak{F}\left[\frac{d^2 f(t,z)}{dz^2}\right]$

$= (j2\pi u)^2 F(u,v) + (j2\pi v)^2 F(u,v)$

$= -4\pi^2(u^2+v^2) F(u,v)$

$G(u,v) = H(u,v) F(u,v)$ 當 $H(u,v) = -4\pi^2(u^2+v^2)$

b)

$H(u,v) = -4\pi^2(u^2+v^2)$

for $u = 0,1,2,\ldots,M-1$

$v = 0,1,2,\ldots,N-1$

補充 RGB(_,_,_)

Black(0,0,0) Cyan(0,1,1)

White(1,1,1) Magenta(1,0,1)

yellow(1,1,0)

當 $H(u,v) = -4\pi^2(u^2+v^2)$ 進行中心化 transform

$H(u,v)$ 可寫成 $-4\pi^2\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]$

$\nabla^2 f(x,y) \Leftrightarrow -4\pi^2\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right] F(u,v)$ #

(c) Laplacian Filter is isotropic ⇒ 有旋轉不變性 可以用在「邊緣」強化，的 Laplacian Mask

eg: A -8 in the center so that its response is 0 in areas of constant intensity

4.27 說明/証明 lowpass filter

(a)

$g(x,y) = \frac{1}{4}[f(x,y+1) + f(x,y-1) + f(x-1,y) + f(x+1,y)]$

$G(u,v) = \frac{1}{4}\left[e^{j2\pi v/N} + e^{-j2\pi v/N} + e^{-j2\pi u/M} + e^{j2\pi u/M}\right] F(u,v)$

$= H(u,v) F(u,v)$ where $H(u,v) = \frac{1}{2}\left[\cos\left(\frac{2\pi u}{M}\right) + \cos\left(\frac{2\pi v}{N}\right)\right]$

是 frequency Domain 的 Filter Transform Function

(b) 為 prove 是 lowpass filter

$H(u,v) = \frac{1}{2}\left[\cos\left(2\pi\left[u-\frac{M}{2}\right]\right)/M\right) + \cos\left(2\pi\left[v-\frac{N}{2}\right]\right)/N\right)\right]$

當 $u = 0 \to M-1$ 時；$\cos\left(2\pi\left[u-\frac{M}{2}\right]\right)/M$ 從 -1 開始

且 $u = \frac{M}{2}$ 時 = 1 (峰值)；$u = M$ 時 = -1

因此，filter 的振幅隨距離中心化 filter 原點的距離↓ 此為低通 filter 的特性。

4.28 說明/証明 highpass filter

(a)

$g(x,y) = f(x+1,y) - f(x,y) + f(x,y+1) - f(x,y)$

又 $G(u,v) = F(u,v) e^{j2\pi u/M} - F(u,v) + F(u,v) e^{j2\pi v/N} - F(u,v)$

$= \left[e^{j2\pi u/M} - 1\right] F(u,v) + \left[e^{j2\pi v/N} - 1\right] F(u,v)$

$= H(u,v) F(u,v)$

其中 $H(u,v) = \left[\left(e^{j2\pi u/M} - 1\right) + \left(e^{j2\pi v/N} - 1\right)\right]$

$= 2j\left[\sin(\pi u/M) e^{j\pi u/M} + \sin(\pi v/N) e^{j\pi v/N}\right]$

(b) 為證明是 highpass filter 可把 $H(u,v)$ 作成

$H(u,v) = 2j\left[\sin\left(\pi\left[u-\frac{M}{2}\right]/M\right) e^{j\pi u/M} + \sin\left(\pi\left[u-\frac{N}{2}\right]/N\right) e^{j\pi v/N}\right]$

在 filter 的中心 $\left(u = \frac{M}{2}\right)$, function value = 0

當 u,v↑ filter 值↓ 並且在 $u = M-1$; $v = N-1$ 時，會接近 -4 的極限值 而会是負的是因取導數時順序引起的，若採用 $f(x,y) - f(x+1,y)$ 和 $f(x,y) - f(x,y+1)$ 差分，就趨於 ...

而重點是 dc 項有被消掉，且 high frequency 也有通過

5.16 有 $h(x-\alpha, y-\beta) = e^{-[(x-\alpha)^2+(y-\beta)^2]}$

$g(x,y) = \iint_{-\infty}^{\infty} f(\alpha,\beta)\, h(x-\alpha, y-\beta)\, d\alpha\, d\beta$

$g(x,y) = \iint_{-\infty}^{\infty} \delta(\alpha-a)\, e^{-[(x-\alpha)^2+(y-\beta)^2]}\, d\alpha\, d\beta$

$\Rightarrow \iint_{-\infty}^{\infty} \delta(\alpha-a)\, e^{-[(x-\alpha)^2]}\, e^{-[(y-\beta)^2]}\, d\alpha\, d\beta$

$= \int_{-\infty}^{\infty} \delta(\alpha-a)\, e^{-[(x-\alpha)^2]}\, d\alpha \int_{-\infty}^{\infty} e^{-[(y-\beta)^2]}\, d\beta$

$= e^{-[(x-a)^2]} \int_{-\infty}^{\infty} e^{-[(y-\beta)^2]}\, d\beta$

where we used the fact that the integral of the impulse is nonzero only when $\alpha = a$

$\int_{-\infty}^{\infty} e^{-[(y-\beta)^2]}\, d\beta = \int_{-\infty}^{\infty} e^{-[(\beta-y)^2]}\, d\beta$

which is in the form of a constant times a Gaussian Density with variance $= \frac{1}{2} \Rightarrow \sigma = \sqrt{1/2}$

$e^{-(\beta-y)^2} = \sqrt{2\pi\times\frac{1}{2}}\left[\frac{1}{\sqrt{2\pi\times\frac{1}{2}}}\, e^{(-\frac{1}{2})\left[\frac{(\beta-y)^2}{(\frac{1}{2})}\right]}\right]$

the integral from minus to plus infinity of the quantity inside the brackets is 1 $\Rightarrow g(x,y)$:

$g(x,y) = \sqrt{\pi}\times e^{-[(x-a)^2]}$ # which is blurred version of the original image

5.17 有 T_1, T_2 相關

$x_0(t) = \begin{cases} at/T_1, & 0 \le t \le T_1 \\ a, & T_1 < t \le T_1+T_2 \end{cases}$ $\quad y_0(t) = \begin{cases} 0, & 0 \le t \le T_1 \\ \frac{b(t-T_1)}{T_2}, & T_1 \le t \le T_2 \end{cases}$

$H(u,v) = \int_0^{T_1} e^{-j2\pi[uat/T_1]}\, dt + \int_{T_1}^{T_1+T_2} e^{-j2\pi[ua + \frac{vb(t-T_1)}{T_2}]}\, dt$

$= \frac{T_1}{\pi ua}\sin(\pi ua)\, e^{-j\pi ua} + e^{-j2\pi ua}\int_{T_1}^{T_1+T_2} e^{-j2\pi vb(t-T_1)/T_2}\, dt$

$= \frac{T_1}{\pi ua}\sin(\pi ua)\, e^{-j\pi ua} + e^{-j2\pi ua}\int_0^{T_2} e^{-j2\pi vb\tau/T_2}\, d\tau$ ⇒其中 $\tau = t - T_1$ 代入式子

$= \frac{T_1}{\pi ua}\sin(\pi ua)\, e^{-j\pi ua} + e^{-j2\pi ua}\frac{T_2}{\pi vb}\sin(\pi vb)\, e^{-j\pi vb}$

而 $g(x,y) = \mathfrak{F}^{-1}[H(u,v)F(u,v)]$ → 其中 $F(u,v)$ 是原圖去作 Fourier Transform

5.21 pf.

$\nabla^2(s(x,y)) = \left[\frac{d^2 s(x,y)}{dx^2} + \frac{d^2 s(x,y)}{dy^2}\right] = \left(\frac{x^2+y^2-2\sigma^2}{\sigma^4}\right)\cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$

$\because H(u,v) = \mathfrak{F}[h(x,y)] = \mathfrak{F}[\nabla^2(s(x,y))]$

$F(u,v) = \mathfrak{F}(s(x,y)) = \mathfrak{F}\left[e^{-\frac{x^2+y^2}{2\sigma^2}}\right]$

而 又 $\because \mathfrak{F}[\nabla^2 s(x,y)] = -4\pi^2(u^2+v^2)F(u,v)$

又 $\because \mathfrak{F}\left[e^{-\frac{x^2+y^2}{2\sigma^2}}\right] = 2\pi\sigma^2 e^{-2\pi^2\sigma^2(u^2+v^2)}$ 代入

再將其代入 $H(u,v) = -4\pi^2(u^2+v^2)F(u,v)$

5.22

by Wiener Filter Eq. & 5.21 result

$|H(u,v)|^2 = \left|\left[-8\pi^3(u^2+v^2)\sigma^2 \cdot e^{-2\pi^2\sigma^2(u^2+v^2)}\right]\right|^2$

$= 64\pi^6\sigma^4(u^2+v^2)^2 \times e^{-4\pi^2\sigma^2(u^2+v^2)}$

代入Eq

$H_W(u,v) = \left[\frac{8\pi^3\sigma^2(u^2+v^2)\, e^{-2\pi^2\sigma^2(u^2+v^2)}}{\left[64\pi^6\sigma^4(u^2+v^2)^2\, e^{-4\pi^2\sigma^2(u^2+v^2)}\right]+K}\right]$

欲証 $H(u,v) = -8\pi^3\sigma^2(u^2+v^2)\, e^{-2\pi^2\sigma^2(u^2+v^2)}$

$H(u,v) = \left[-4\pi^2(u^2+v^2)\right]\cdot\left[2\pi\sigma^2 e^{-2\pi^2\sigma^2(u^2+v^2)}\right]$

$H(u,v) = -8\pi^3(u^2+v^2)\times\sigma^2\times e^{-2\pi^2\sigma^2(u^2+v^2)}$ #

5.22之前情提要 Wiener Filter Eq:

$H_W(u,v) = \left[\frac{1}{H(u,v)}\times\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\right]$

調与色彩修正.

$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.588 & 0.179 & 0.183 \\ 0.29 & 0.606 & 0.105 \\ 0 & 0.068 & 1.021 \end{bmatrix}\begin{bmatrix} R \\ G \\ B \end{bmatrix}$

$X_W = 0.588 + 0.179 + 0.183 = 0.95$

$Y_W \doteq 1.01$. $Z_W = 1.1$ #

cyan $\Rightarrow (0,1,1)$ → $X = 0.179 + 0.183$, $Y = 0.606 + 0.105$, $Z = 0.068 + 1.021$

此類推

$(q) = \begin{cases} \sqrt[3]{q} & q > 0.008856. \\ 7.787q + \frac{16}{116}; & q \le 0.008856 \end{cases}$

$116\times h\left(\frac{Y}{Y_W}\right) - 16$

$500\left[h\left(\frac{X}{X_W}\right) - h\left(\frac{Y}{Y_W}\right)\right]$

RGB → HSI

$H = \begin{cases} \theta & 若\ B \le G \\ 360 - \theta & 若\ B > G \end{cases}$

$\theta = \cos^{-1}\left\{\frac{\frac{1}{2}[(R-G)+(R-B)]}{\sqrt{(R-G)^2+(R-B)(G-B)}}\right\}$

$S = 1 - \frac{3}{\ldots}[\min(R, G, B)]$

6.12.

Color	R	G	B	H	S	I	Mono H	×S	×I
Black	0	0	0	–	0	0	–	–	0
Red	1	0	0	0	1	0.33	0	255	85
yellow	1	1	0	0.17	1	0.67	43	255	170
green	0	1	0	0.33	1	0.33	85	255	85
cyan	0	1	1	0.5	1	0.67	128	255	170
blue	0	0	1	0.67	1	0.33	170	255	85
Magenta	1	0	1	0.83	1	0.67	213	255	170
white	1	1	1	–	0	1	–	0	255

HSI → RGB

① $B = I(1-S)$

$R = I\left[1 + \frac{S\cos H}{\ldots}\right]$

② $120° \sim 240°$

$H = H - 120°$

$R = I(1-S)$

6.23

	$\frac{X}{X_W}$	$\frac{Y}{Y_W}$	$\frac{Z}{Z_W}$	$h(\frac{X}{X_W})$	$h(\frac{Y}{Y_W})$	$h(\frac{Z}{Z_W})$	L^*	a^*	b^*
ref	1	1	1	1	1	1	100	0	0
black	0	0	0	0.14	0.14	0.14	0	0	0
red	0.62	0.29	0	0.85	0.66	0.14	83	95	105
yellow	0.81	0.90	0.06	0.93	0.96	0.40	92	−16	113
green	0.19	0.61	0.06	0.57	0.85	0.40	51	−136	90
Cyan	0.38	0.71	1	0.73	0.89	1	68	−84	−22
Blue	0.19	0.11	0.94	0.58	0.47	0.98	51	53	−101
Magenta	0.81	0.4	0.94	0.93	0.73	0.98	92	100	−49
white	1	1	1	1	1	1	100	0	0
gray	0.5	0.5	0.5	0.79	0.79	0.79	16	0	0

③ $240° \sim 360°$

$H = H - 240°$

$G = I(1-S)$

$7.787q + \frac{16}{116}$; $q \leq 0.008856$

$$H = \begin{cases} \theta & 若 B \leq G \\ 360 - \theta & 若 B > G \end{cases}$$

Magenta	1	0	1	0.83	1	0.18	213	255	170
white	1	1	1	–	0	1	–	0	255

Blue	0.17	0.11	0.94	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Magenta	0.81	0.4	0.94	0.93	0.73	0.98	92	100	-49
white	1	1	1	1	1	1	100	0	0
gray	0.5	0.5	0.5	0.79	0.79	0.79	16	0	0

$L^* = 116 \times h\left(\frac{Y}{Y_W}\right) - 16$

$a^* = 500\left[h\left(\frac{X}{X_W}\right) - h\left(\frac{Y}{Y_W}\right)\right]$

$b^* = 200\left[h\left(\frac{Y}{Y_W}\right) - h\left(\frac{Z}{Z_W}\right)\right]$

$\theta = \cos^{-1}\left\{\frac{\frac{1}{2}[(R-G)+(R-B)]}{\sqrt{(R-G)^2+(R-B)(G-B)}}\right\}$

$S = 1 - \frac{3}{R+G+B}[\min(R,G,B)]$

$I = \frac{1}{3}(R+G+B)$

HSI → RGB

① $B = I(1-S)$

$R = I\left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$

$G = 3I - (R+B)$

② $120^\circ \sim 240^\circ$

$H = H - 120^\circ$

$R = I(1-S)$

$G = I\left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$

$B = 3I - (R+G)$

③ $240^\circ \sim 360^\circ$

$H = H - 240^\circ$

$G = I(1-S)$

$B = I \times \left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$

$R = 3I - (G+B)$

讓我過!!!

6.12 Hue is undefined when $R=G=B=0$ since $\theta = \cos^{-1}\left(\frac{0}{0}\right)$

In addition, saturation is undefined when $R=G=B=0$ $1 - 0 = 1$

△Histogram Equalization & Matching $[0, L-1] = [0, 7]$ ∴ $L = 8$

Assume that size ⇒ 64×64 pixels

此為 $r \to s$

r_k	n_k	$P_r(r_k) = n_k/MN$	s_k	
$r_0=0$	790	0.19	$7 \times 0.19 = 1.33$	→ 1
$r_1=1$	1023	0.25	$7 \times 0.25 + 1.33 = 3.08$	→ 3
$r_2=2$	850	0.21	$3.08 + 7 \times 0.21 = 4.55$	→ 5
$r_3=3$	656	0.16	$4.55 + 7 \times 0.16 = 5.67$	→ 6
$r_4=4$	329	0.08	$5.67 + 7 \times 0.08 = 6.23$	→ 6
$r_5=5$	245	0.06	$6.23 + 7 \times 0.06 = 6.65$	→ 7
$r_6=6$	122	0.03	$6.65 + 7 \times 0.03 = 6.86$	→ 7
$r_7=7$	81	0.02	$6.86 + 7 \times 0.02 = 7$	→ 7

$s \to z$ (Histogram Matching)

z_q	指定的 $p_z(z_q)$		Actual $p_z(z_q)$	
$z_0=0$	0		0	
$z_1=1$	0		0	
$z_2=2$	0		0	
$z_3=3$	0.15	1.05	0.19	1.33
$z_4=4$	0.2	2.45	0.25	$1.33 + 1.75 = 3.08$
$z_5=5$	0.3	4.55	0.21	$3.08 + 1.47 = 4.55$
$z_6=6$	0.2	5.95	0.24	$4.55 + 1.68 = 6.23$
$z_7=7$	0.15	7	0.11	$6.23 + 0.77 = 7$

轉換 function $G(z_q)$

z_q	$G(z_q)$
$z_0=0$	0
$z_1=1$	0
$z_2=2$	0
$z_3=3$	1
$z_4=4$	2
$z_5=5$	5
$z_6=6$	6
$z_7=7$	7

$s \to k$

s_k	→ z_q
1	3
3	4
5	5
6	6
7	7

補充 ↓ 我要畢業!!!

$\bar{f}(x,y) = \frac{1}{MN} \times \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) = \frac{1}{MN}F(0,0)$

$g(x,y) = f(x,y) + c \times [\nabla^2 f(x,y)]$

$g_{mask} = f(x,y) - \bar{f}(x,y)$

$g(x,y) = f(x,y) + k \times g_{mask}(x,y)$

$k = 1$ 鈍化

$k > 1$ highboost

$k < 1$ 去除鈍化

$F(u,v) = \Im[f(t,z)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z) e^{-j2\pi(ut+vz)} dt\,dz$

if $\Im[a_1 f_1(t,z) + a_2 f_2(t,z)] = a_1\Im[f_1(t,z)] + a_2\Im[f_2(t,z)]$

又 $\Im = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [a_1 f_1(t,z) + a_2 f_2(t,z)] \times e^{-j2\pi(ut+vz)} dt\,dz$

$= a_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(t,z) e^{-j2\pi(ut+vz)} dt\,dz$ → ~~$\Im[a_1 f_1(t,z)$~~

$+ a_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2(t,z) e^{-j2\pi(ut+vz)} dt\,dz$

$= a_1 \Im[f_1(t,z)] + a_2 \Im[f_2(t,z)]$ #

$\Im[a_1 f_1(x,y) + a_2 f_2(x,y)] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[a_1 f_1(x,y) + a_2 f_2(x,y)] e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$

$= a_1 \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f_1(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} + a_2 \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f_2(x,y)$ ~~

$= a_1 \Im[f_1(x,y)] + a_2 \Im[f_2(x,y)]$

I... transform [illegible]