Introduction to Digital Image Processing HW #3 (Due 05/02/2024)

(Gonzalez 3rd edition)

1. Problem 4.14 (10%)

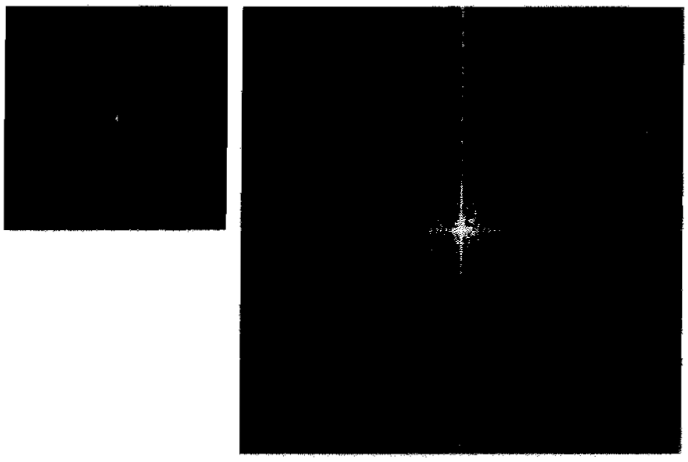
   Prove that both the 2-D continuous and discrete Fourier transforms are linear operations (see Section 2.6.2 for a definition of linearity).

2. Problem 4.15 (10%)

   You are given a "canned" program that computes the 2-D, DFT pair. However, it is not known in which of the two equations the $1/MN$ term is included or if it was split as two constants $1/\sqrt{MN}$ in front of both the forward and inverse transforms. How can you find where the term(s) is (are) included if this information is not available in the documentation?

3. Problem 4.22 (10%)

   The two Fourier spectra shown are of the same image. The spectrum on the left corresponds to the original image, and the spectrum on the right was obtained after the image was padded with zeros. Explain the significant increase in signal strength along the vertical and horizontal axes of the spectrum shown on the right.



4. Problem 4.23 (10%)

   You know from Table 4.2 that the dc term, $F(0, 0)$, of a DFT is proportional to the average value of its corresponding spatial image. Assume that the image is of size $M \times N$. Suppose that you pad the image with zeros to size $P \times Q$, where P and Q are given in Eqs. (4.6-31) and (4.6-32). Let $F_p(0, 0)$ denote the dc term of the DFT of the padded function.

   (a) What is the ratio of the average values of the original and padded images?

   (b) Is $F_p(0, 0) = F(0, 0)$? Support your answer mathematically.

5. Problem 4.26 (10%)

(a) Show that the Laplacian of a continuous function $f(t, z)$ of continuous variables t and z satisfies the following Fourier transform pair [see Eq. (3.6-3) for a definition of the Laplacian]:

$$\nabla^2 f(t, z) \Leftrightarrow -4\pi^2(\mu^2 + \nu^2)F(\mu, \nu)$$

[*Hint:* Study entry 12 in Table 4.3 and see Problem 4.25(d).]

(b) The preceding closed form expression is valid only for continuous variables. However, it can be the basis for implementing the Laplacian in the discrete frequency domain using the $M \times N$ filter

$$H(u, v) = -4\pi^2(\mu^2 + \nu^2)$$

for $u = 0, 1, 2, \ldots, M - 1$ and $v = 0, 1, 2, \ldots, N - 1$. Explain how you would implement this filter.

(c) As you saw in Example 4.20, the Laplacian result in the frequency domain was similar to the result of using a spatial mask with a center coefficient equal to -8. Explain the reason why the frequency domain result was not similar instead to the result of using a spatial mask with a center coefficient of -4. See Section 3.6.2 regarding the Laplacian in the spatial domain.

6. Problem 4.27 (10%)

Consider a 3×3 spatial mask that averages the 4 closest neighbors of a point (x, y), but excludes the point itself from the average.

(a) Find the equivalent filter, $H(u, v)$, in the frequency domain.

(b) Show that your result is a lowpass filter.

7. Problem 4.28 (10%)

Based on Eq.(3.6-1), one approach for approximating a discrete derivative in 2-D is based on computing differences of the form $f(x + 1, y) - f(x, y)$ and $f(x, y + 1) - f(x, y)$.

(a) Find the equivalent filter, $H(u, v)$, in the frequency domain.

(b) Show that your result is a highpass filter.

8. **Fourier Spectrum and Average Value** (10%)
   (a) Download Fig. 4.41(a) from the course web site and compute its (centered) Fourier spectrum.
   (b) Display the spectrum.
   (c) Use your result in (a) to compute the average value of the image.

9. **Edge Detection Combined with Smoothing and Thresholding (10%)**
   (a) Extend the program from HW #2 p.9(a) to compute the Sobel gradient using the masks in Fig. 3.41 (d) & (e). Your program should implement Eq. (3.6-12), and

have the option of outputting a binary image by comparing each gradient point against a specified threshold, T.

(b) Download Fig. 2.35(c) from the course web site. By combining smoothing with a 3 x 3 mask from HW #2 p.9(a) and your program from (a), process Fig. 2.35(c) and produce a binary image that isolates (segments) the large blood vessel in the center of the image. This will require repeated trials of smoothing and choices of T. Looking at the histogram (HW #2 p.8) of the gradient image before it is thresholded will help you select a value for T. Please also specify the final T value you used.

10. **Highpass Filtering Combined with Thresholding (10%)**

(a) Implement the Gaussian highpass filter of Eq. (4.9-4). You must be able to specify the size, M x N, of the resulting 2D function. In addition, you must be able to specify the location of the center of the Gaussian function.

(b) Download Fig. 4.57(a) from the course web site and use your program from (a) combined with thresholding to approximate the results in Fig. 4.57 (Note that you will be using a Gaussian instead of a Butterworth filter).