Integer Linear Programming Problem Using Branch and Bound

The Branch and Bound algorithm is a systematic method for solving optimization problems, particularly integer programming problems. The method works by dividing the feasible region into smaller subproblems (branching) and calculating bounds for each subproblem (bounding). Subproblems that cannot produce a better solution than the current best are discarded (pruning). The process continues recursively until all subproblems are solved or pruned.

Steps:

Initial Problem: Solve the relaxed problem to get an initial solution.

Branching: Create subproblems by adding new constraints.

(e.g., $x \leq k$ and $x \geq k+1, It\ is\ based\ on\ higher\ decimal\ value.)$

Bounding: Solve each subproblem to get bounds on the objective function.

Pruning: Discard subproblems that cannot yield a better solution than the current best.

Iterate: Continue branching and bounding until all subproblems are resolved.

The provided MATLAB code uses the Gurobi solver to solve a linear programming problem with the branch and bound method. The problem is defined as maximizing $Z = 5x_1 + 6x_2$ subject to constraints. The Gurobi solver is used to find the optimal solution for the relaxed problem (without integer constraints).

Initially, the relaxed problem (without integer constraints) is solved, yielding a non-integer solution $(x_1 = 3.2)$ and $(x_2 = 1.8)$ with $(\ Z\ =\ 26.8\ )$. Branching occurs based on the variable with the higher decimal value, (x_2), leading to two subproblems: $(x_2 \leq 1)$ and $(x_2 \geq 2)$. The branch$(x_2 \leq 1)$ provides an integer solution $(x_1 = 4)$ and $(x_2 = 1)$ with $(\ Z\ =\ 26\ )$. The branch $(x_2 \geq 2)$ further branches on (x_1), creating subproblems $(x_1 \leq 2)$ and $(x_1 \geq 3)$, but none yield better integer solutions than$(x_1 = 4)$and$(x_2 = 1)$. The optimal integer solution is thus $(x_1 = 4)$and$(x_2 = 1)$with$(Z = 26)$.

$$Max\ Z = 5x_1 + 6x_2$$
Subject to:
$$x_1 + x_2 \leq 5$$
$$3x_1 + 8x_2 \leq 24$$
$$x_1, x_2 \geq 0$$
$$\boldsymbol{x_1 = 3.2, x_2 = 1.8}$$
$$\boldsymbol{Z = 26.8}$$

$x_2 \geq 2$

$x_2 \leq 1$

$$Max\ Z = 5x_1 + 6x_2$$
Subject to:
$$x_1 + x_2 \leq 5$$
$$3x_1 + 8x_2 \leq 24$$
$$x_2 >= 2$$
$$x_1, x_2 \geq 0$$
$$\boldsymbol{x_1 = 2.67, x_2 = 2}$$
$$\boldsymbol{Z = 25.33}$$

$$Max\ Z = 5x_1 + 6x_2$$
Subject to:
$$x_1 + x_2 \leq 5$$
$$3x_1 + 8x_2 \leq 24$$
$$x_2 <= 1$$
$$x_1, x_2 \geq 0$$
$$\boldsymbol{x_1 = 4, x_2 = 1}$$
$$\boldsymbol{Z = 26}$$