Introduction to Input-Output Analysis
Lecture 1: Foundations

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Delft University of Technology
Academic Year 2017-2018, Quarter 4

Outline

Today

- Introduction
- The IO table
- The IO model
- Given $\boldsymbol{Z}$ and $\boldsymbol{x}$, find $\boldsymbol{A}$ and $\boldsymbol{L}$
- Computer demonstration
- Info about assessment

Next lecture

- Closed IO model
- Leontief multipliers

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- Office hours by appointment

Materials

- Syllabus available on Brightspace
- Textbook: Miller & Blair (2009) “Input-Output Analysis”
- Slides soon available on Brightspace

If final demand for Dutch agricultural products were to increase by 500 million euro…

- How much additional output would the Dutch economy have to produce to meet this new demand?
- What is the **total output requirement** of 500 million euro final demand for agricultural products?
- How much additional energy would the Dutch economy use to meet this new demand?
- What is the **total energy requirement** of 500 million euro final demand for agricultural products?

Input-output analysis a.k.a. interindustry analysis

- Focus is on interdependence of industries
- Industries are both producers and consumers
- As consumers, they purchase goods (and services) for use as intermediate input in production
- As producers, they sell their output to other industries for use as intermediate input and to end-users for final consumption

- **Input-output model** is constructed from observed data for a particular geographic region
- **Input-output table** summarizes that data

The basic structure of an IO table

$C + I + G$

		Buying sectors (producers as consumers)				
		Agriculture	Industry	Services	Final demand	Total outputs
Selling sectors	Agriculture	\$20	\$20	\$10	\$50	\$100
	Industry	\$30	\$50	\$20	\$100	\$200
	Services	\$10	\$10	\$20	\$60	\$100
	Primary inputs	\$40	\$120	\$50	\$0	
	Total inputs	\$100	\$200	\$100		

Labor & capital

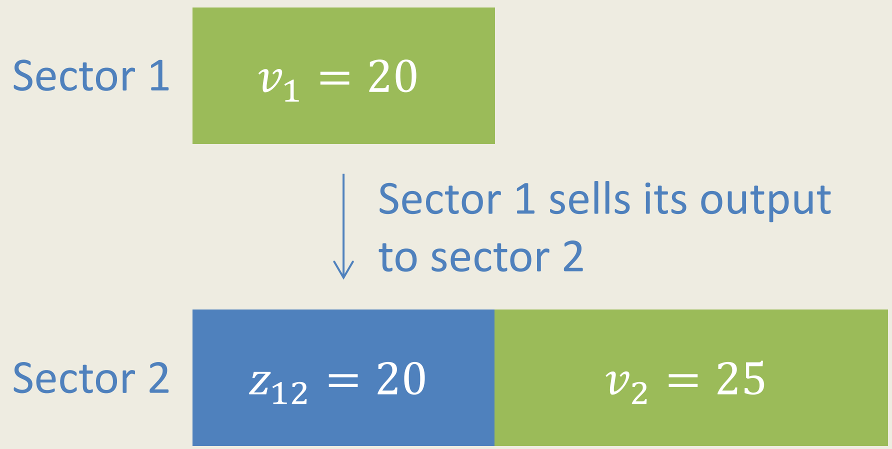
$$\begin{aligned} \text{Column sums} &= \text{Row sums} \\ \text{Total inputs} &= \text{Total outputs} \end{aligned}$$

1 Crop and animal production, hunting and related service activities
2 Forestry and logging
3 Fishing and aquaculture
4 Mining and quarrying
5 Manufacture of food products, beverages and tobacco products
6 Manufacture of textiles, wearing apparel and leather products
7 Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
8 Manufacture of paper and paper products
9 Printing and reproduction of recorded media
10 Manufacture of coke and refined petroleum products
11 Manufacture of chemicals and chemical products
12 Manufacture of basic pharmaceutical products and pharmaceutical preparations
13 Manufacture of rubber and plastic products
14 Manufacture of other non-metallic mineral products
15 Manufacture of basic metals
16 Manufacture of fabricated metal products, except machinery and equipment
17 Manufacture of computer, electronic and optical products
18 Manufacture of electrical equipment
19 Manufacture of machinery and equipment n.e.c.
20 Manufacture of motor vehicles, trailers and semi-trailers
21 Manufacture of other transport equipment
22 Manufacture of furniture; other manufacturing
23 Repair and installation of machinery and equipment
24 Electricity, gas, steam and air conditioning supply
25 Water collection, treatment and supply
26 Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
27 Construction
28 Wholesale and retail trade and repair of motor vehicles and motorcycles
29 Wholesale trade, except of motor vehicles and motorcycles
30 Retail trade, except of motor vehicles and motorcycles
31 Land transport and transport via pipelines
32 Water transport
33 Air transport
34 Warehousing and support activities for transportation
35 Postal and courier activities
36 Accommodation and food service activities
37 Publishing activities
38 Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities
39 Telecommunications
40 Computer programming, consultancy and related activities; information service activities
41 Financial service activities, except insurance and pension funding
42 Insurance, reinsurance and pension funding, except compulsory social security
43 Activities auxiliary to financial services and insurance activities
44 Real estate activities (excluding imputed rent)
45 Imputed rents of owner-occupied dwellings
46 Legal and accounting activities; activities of head offices; management consultancy activities
47 Architectural and engineering activities; technical testing and analysis
48 Scientific research and development
49 Advertising and market research
50 Other professional, scientific and technical activities; veterinary activities
51 Rental and leasing activities
52 Employment activities
53 Travel agency, tour operator reservation service and related activities
54 Security and investigation activities; services to buildings and landscape activities; office administrative, office support and other business support activities
55 Public administration and defence; compulsory social security
56 Education
57 Human health activities
58 Social work activities
59 Creative, arts and entertainment activities; libraries, archives, museums and other cultural activities; gambling and betting activities
60 Sports activities and amusement and recreation activities
61 Activities of membership organisations
62 Repair of computers and personal and household goods
63 Other personal service activities
64 Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use
65 Activities of extra-territorial organisations and bodies

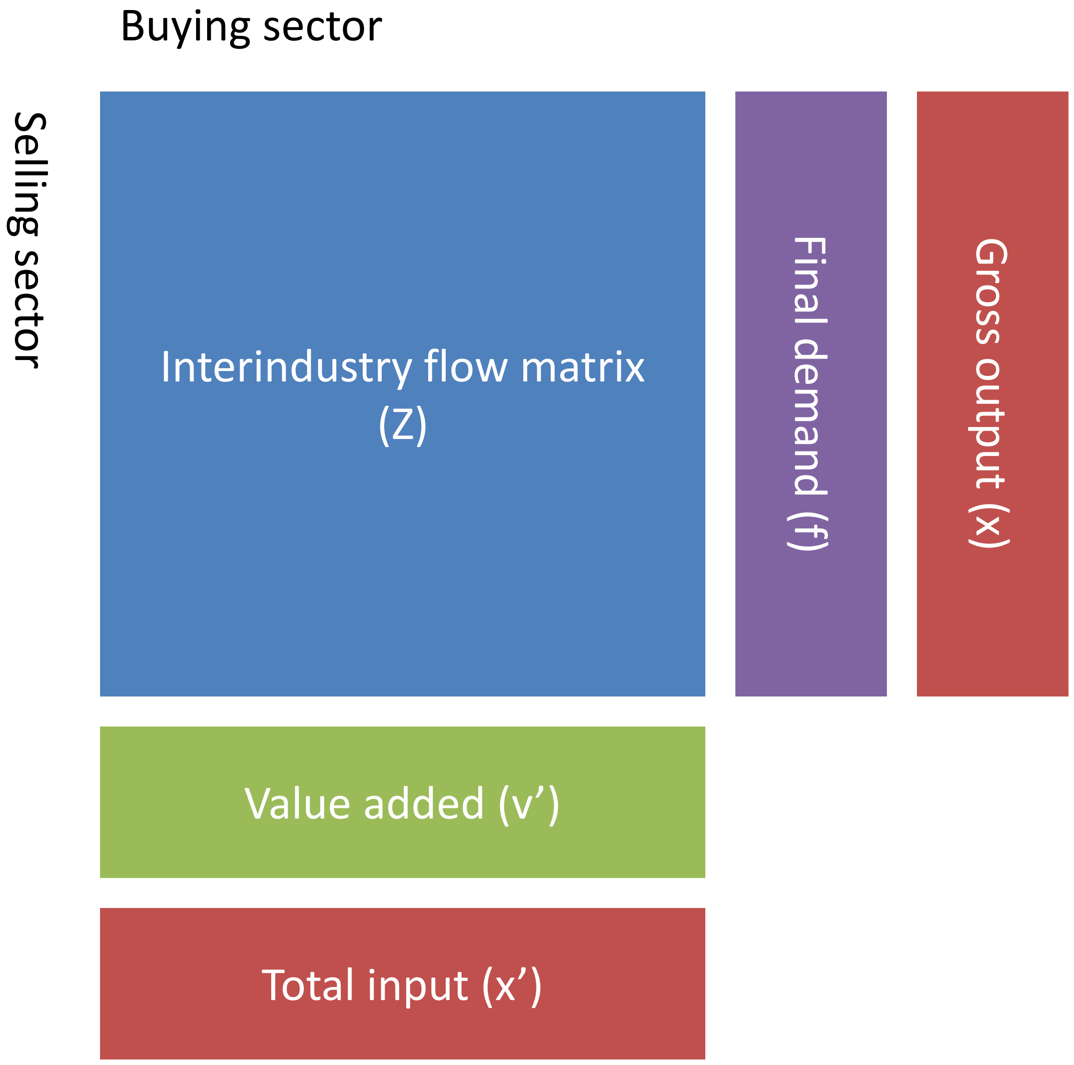
List of industries in [input-output tables published by Eurostat]

Gross output vs value added

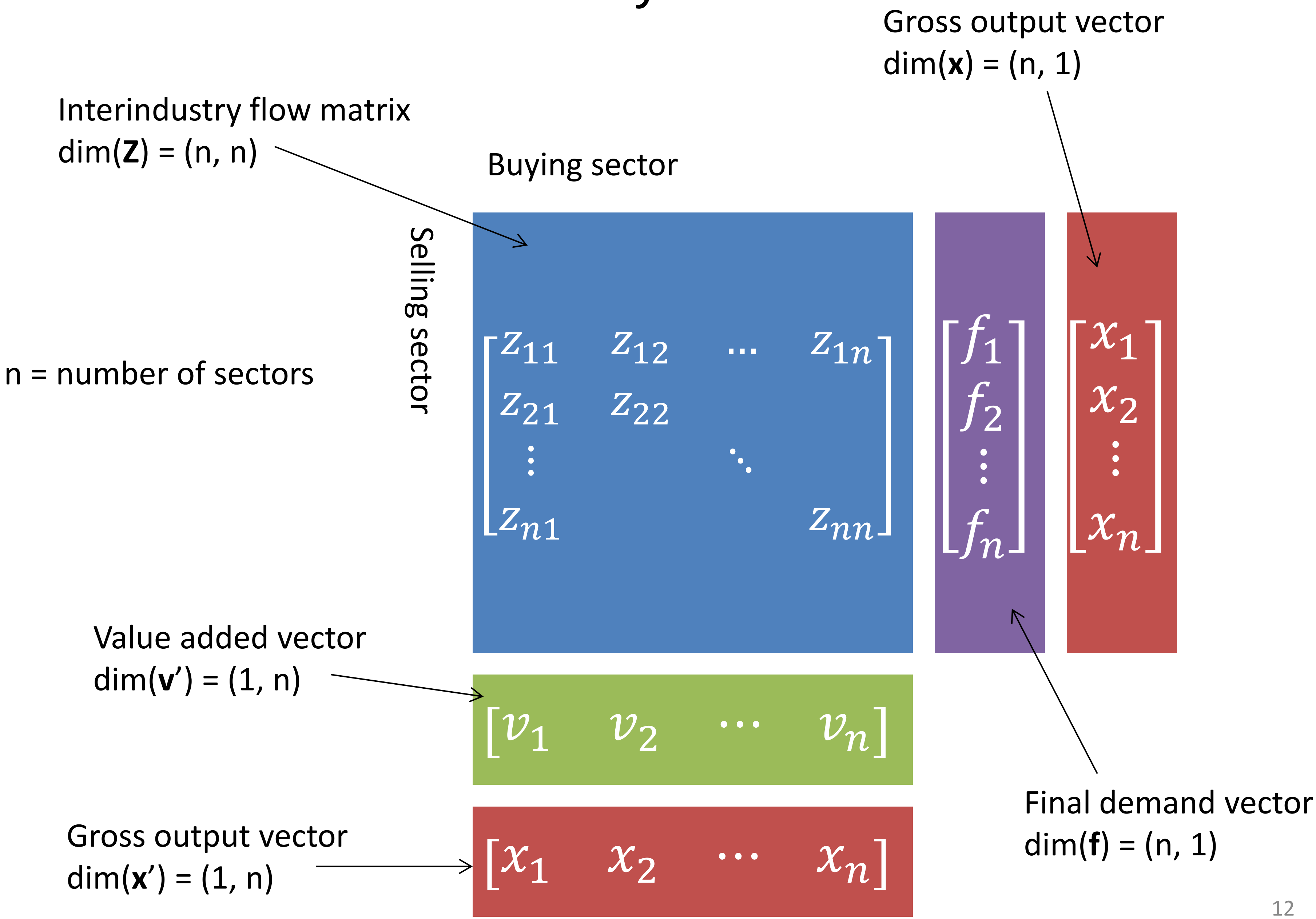
- Sector 1 buys only primary inputs
- Sector 2 buys intermediate inputs from sector 1, and also primary inputs
- In sector 1: $v_1 = x_1$
  value added = gross output
- In sector 2: $v_2 < x_1 = z_{12} + v_2$
  value added < gross output
- Summing gross output across sectors involves double-counting ($v_1 = z_{12}$ is counted twice): $x = x_1 + x_2 = v_1 + z_{12} + v_2$



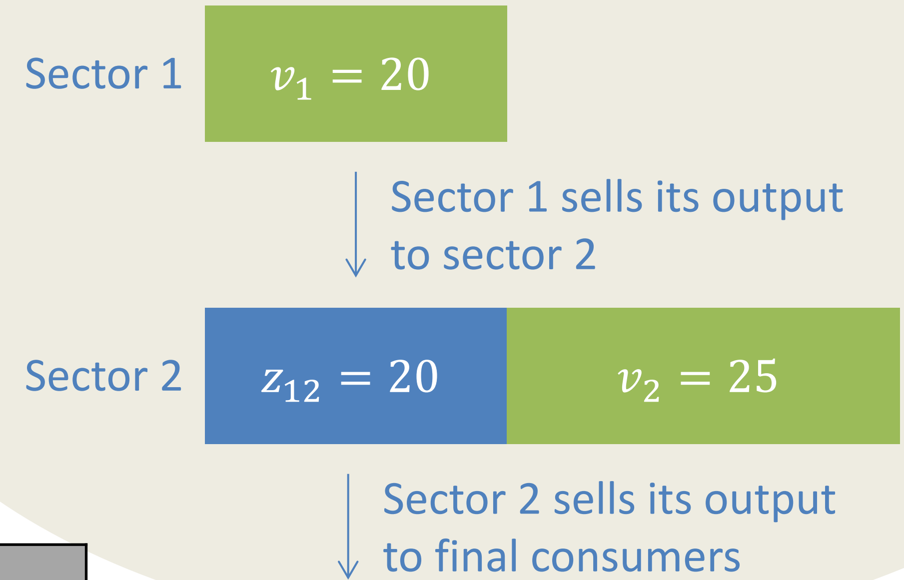
IO table of closed economy



IO table of closed economy



Fill in the numbers in the table (3min)



		Buying sectors			
		$j = 1$	$j = 2$	Final demand	Total outputs
Selling sectors	$i = 1$				
	$i = 2$				
	Primary inputs			0	
	Total inputs				

Sector 1 $v_1 = 20$

Sector 1 sells its output to sector 2

Sector 2 $z_{12} = 20$ $v_2 = 25$

$$\sum_i z_{ij} + v_j = x_j$$

$$\boldsymbol{Z} = \begin{bmatrix} 0 & 20 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} 0 \\ 45 \end{bmatrix}$$

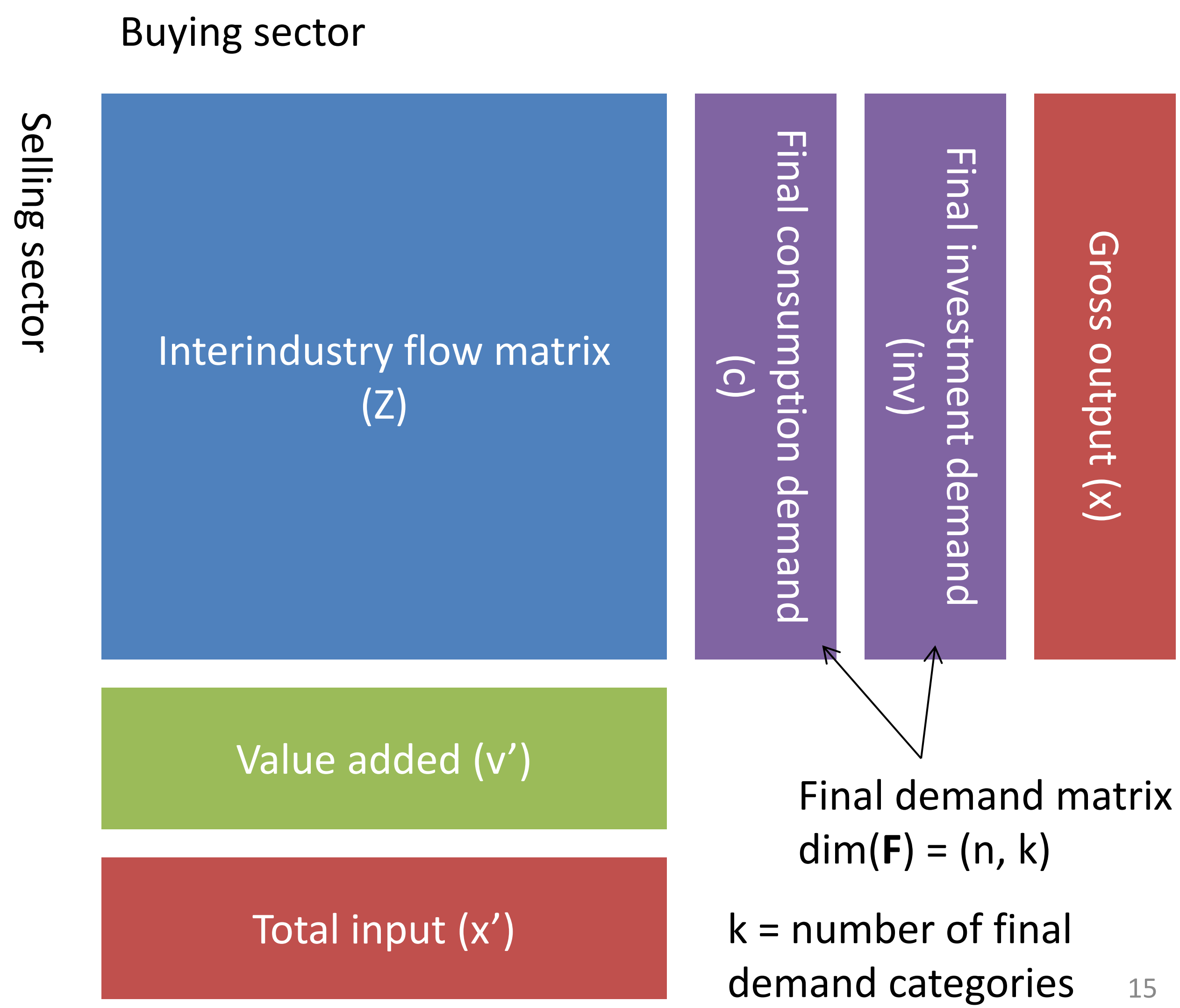
		Buying sectors			
		$j = 1$	$j = 2$	Final demand	Total outputs
Selling sectors	$i = 1$	\$0	\$20	\$0	\$20
	$i = 2$	\$0	\$0	\$45	\$45
	Primary inputs	\$20	\$25	\$0	
	Total inputs	\$20	\$45		

$$\sum_j z_{ij} + f_i = x_i$$

$$\boldsymbol{x} = \begin{bmatrix} 20 \\ 45 \end{bmatrix}$$

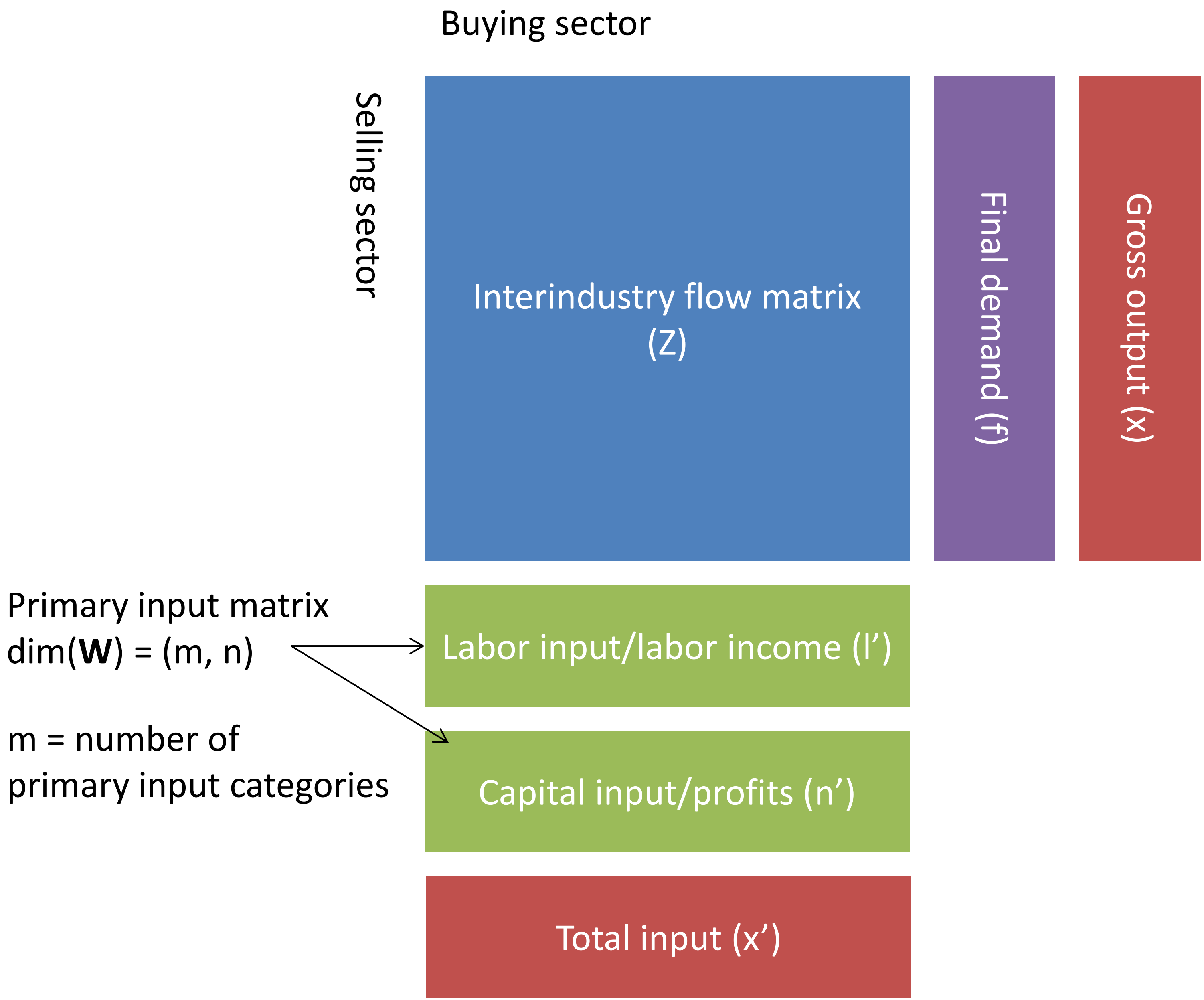
$$\boldsymbol{v}' = [20 \quad 25]$$

IO table with multiple final demand categories

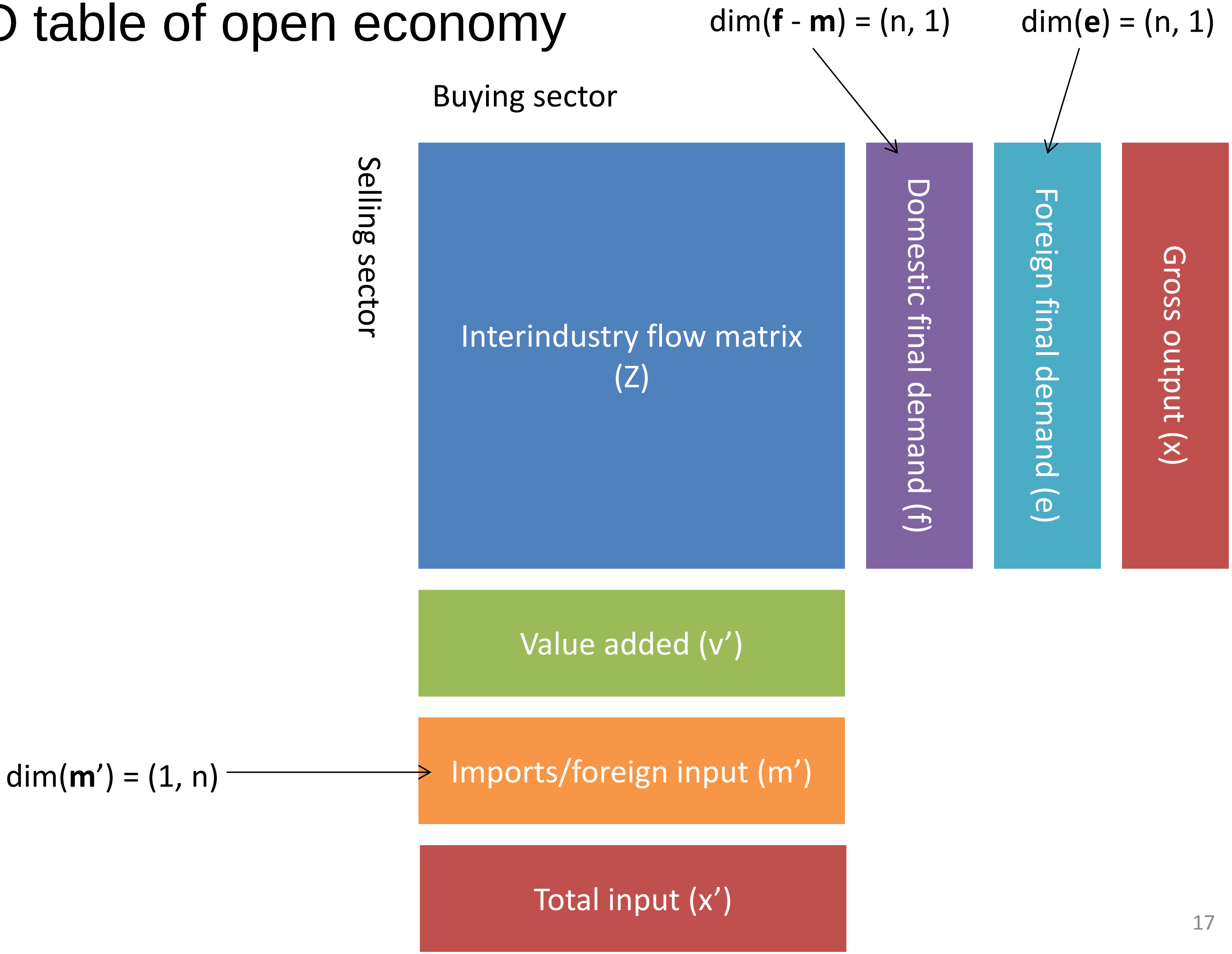


k = number of final demand categories

IO table with two primary inputs/factors



IO table of open economy



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IO model assumes that relationships between output volumes and input volumes are stable

- Example: Dutch bicycle industry uses 15 tons raw steel (input) to produce 1000 bicycles (output) per year
- Stable relationship between input and output implies

$$\frac{15 \text{ tons steel}}{1000 \text{ bicycles}} = const.$$

- Bicycle production also requires plastic and other intermediate inputs

$$\frac{1 \text{ ton plastic}}{1000 \text{ bicycles}} = const.$$

- Model treats input-output ratios as **technical coefficients**

$$a_{i=1} = \frac{15 \text{ tons steel}}{1000 \text{ bicycles}} \text{ and } a_{i=2} = \frac{1 \text{ ton plastic}}{1000 \text{ bicycles}}$$

Technical coefficients reflect physical relationships

- IO tables report values (e.g. dollars)
- Value is price times physical quantity $x = p \cdot q$
- Redefining the physical unit of measurement…

$$\begin{aligned} x^{\$} &= p^{\text{per ton}} \cdot q^{\text{in tons}} \\ &= \frac{p^{\text{per ton}}}{1000} \cdot q^{\text{in tons}} \cdot 1000 \\ &= p^{\text{per kg}} \cdot q^{\text{in kg}} \end{aligned}$$

… does not change the dollar value

- Defining the physical unit to be the quantity that one dollar buys is equivalent to assuming all prices equal one:

$$\begin{aligned} x^{\$} &= p^{\text{per old unit}} \cdot q^{\text{in old unit}} \\ &= \frac{p^{\text{per old unit}}}{p^{\text{per old unit}}} \cdot q^{\text{in old unit}} \cdot p^{\text{per old unit}} \\ &= 1 \cdot q^{\text{in new unit}} \end{aligned}$$

Relative price changes pose a challenge to the interpretation of technical coefficients

- $a_{12} = \frac{z_{12}}{x_2} = \frac{p_1 \cdot q_{12}}{p_2 \cdot q_2}$
- Input price inflation blows up technical coefficient, $\frac{\partial a_{12}}{\partial p_1} > 0$
- Output price inflation shrinks technical coefficient, $\frac{\partial a_{12}}{\partial p_2} < 0$
- If price data is available, no problem, simply adjust the technical coefficients

$\boldsymbol{A} = \boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-1}$ is the **technical coefficients matrix** or **direct requirements matrix**

$$x_1 = z_{11} + z_{12} + f_1$$
$$x_2 = z_{21} + z_{22} + f_2$$

$$\boldsymbol{x} = \boldsymbol{Z}\,\boldsymbol{i} + \boldsymbol{f}$$

$$x_1 = \frac{z_{11}}{x_1} \cdot x_1 + \frac{z_{12}}{x_2} \cdot x_2 + f_1$$
$$x_2 = \frac{z_{21}}{x_1} \cdot x_1 + \frac{z_{22}}{x_2} \cdot x_2 + f_2$$

$$\boldsymbol{x} = \underbrace{\boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-\mathbf{1}}}\ \underbrace{\widehat{\boldsymbol{x}}\,\boldsymbol{i}} + \boldsymbol{f}$$

$$x_1 = a_{11} \cdot x_1 + a_{12} \cdot x_2 + f_1$$
$$x_2 = a_{21} \cdot x_1 + a_{22} \cdot x_2 + f_2$$

$$\boldsymbol{x} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{f}$$

$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1}$ is the **Leontief inverse** or **total requirements matrix**

$$x_1 = z_{11} + z_{12} + f_1$$
$$x_2 = z_{21} + z_{22} + f_2$$

$$\boldsymbol{x} = \boldsymbol{Z}\,\boldsymbol{i} + \boldsymbol{f}$$

$$x_1 = \frac{z_{11}}{x_1} \cdot x_1 + \frac{z_{12}}{x_2} \cdot x_2 + f_1$$
$$x_2 = \frac{z_{21}}{x_1} \cdot x_1 + \frac{z_{22}}{x_2} \cdot x_2 + f_2$$

$$\boldsymbol{x} = \underbrace{\boldsymbol{Z}\,\hat{\boldsymbol{x}}^{-1}}\ \underbrace{\hat{\boldsymbol{x}}\,\boldsymbol{i}} + \boldsymbol{f}$$

$$x_1 = a_{11} \cdot x_1 + a_{12} \cdot x_2 + f_1$$
$$x_2 = a_{21} \cdot x_1 + a_{22} \cdot x_2 + f_2$$

$$\boldsymbol{x} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{f}$$

$$(\boldsymbol{I} - \boldsymbol{A})\,\boldsymbol{x} = \boldsymbol{f}$$
$$\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\,\boldsymbol{f}$$

$$\boldsymbol{A} = \boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-1}$$

$$\begin{bmatrix} \frac{z_{11}}{x_1} & \frac{z_{12}}{x_2} & \cdots & \frac{z_{1n}}{x_n} \\ \frac{z_{21}}{x_1} & \frac{z_{22}}{x_2} & & \frac{z_{2n}}{x_n} \\ \vdots & & \ddots & \\ \frac{z_{n1}}{x_1} & \cdots & & \frac{z_{nn}}{x_n} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & & \\ \vdots & & \ddots & \\ z_{n1} & & & z_{nn} \end{bmatrix} \begin{bmatrix} \frac{1}{x_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{x_2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{x_n} \end{bmatrix}$$

The IO model: $\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\,\boldsymbol{f}$

$$
\begin{array}{lcccccccc}
x_1 & = & l_{11}\cdot f_1 & + & l_{12}\cdot f_2 & + & \cdots & + & l_{1n}\cdot f_n \\
x_2 & = & l_{21}\cdot f_1 & + & l_{22}\cdot f_2 & + & \cdots & + & l_{2n}\cdot f_n \\
\vdots & & & & & & & & \\
x_n & = & l_{n1}\cdot f_1 & + & l_{n2}\cdot f_2 & + & \cdots & + & l_{nn}\cdot f_n
\end{array}
$$

$$
\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}
=
\begin{bmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ l_{21} & l_{22} & & \\ \vdots & & \ddots & \\ l_{n1} & & & l_{nn} \end{bmatrix}
\begin{bmatrix} \vdots \\ f_n \end{bmatrix}
$$

Typical element of $\boldsymbol{L}$ is
$l_{ij} = \partial x_i / \partial f_j$, the output effect
in sector i when final demand
for sector j increases by one unit

Assumptions of IO model

- Constant returns to scale
- No substitution between inputs/factors
- Final demand exogenous
- Prices constant
- Economy operates below full capacity

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1. One-sector pure-labor economy
2. Two-sector pure-labor economy
3. Corn economy
4. Two intermediate input-using sectors
5. Simplest case of intersectoral dependence

One-sector pure-labor economy (1)

$$\boldsymbol{Z} = z = 0$$
$$\boldsymbol{x} = x > 0$$

Find $\boldsymbol{A}$

$$\boldsymbol{A} = \boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-1} = \frac{z}{x} = a = 0$$

Find $\boldsymbol{L}$

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = (1 - a)^{-1}$$
$$= 1$$

$$x = L \cdot f = f$$

- To satisfy f units final demand, economy needs to produce f units gross output

Two-sector pure-labor economy (2)

$$\boldsymbol{Z} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \gg 0$$

Find $\boldsymbol{A}$

$$\boldsymbol{A} = \boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Find $\boldsymbol{L}$

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{x} = \boldsymbol{L} \cdot \boldsymbol{f} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

- To satisfy f_1 units final demand for sector 1's output:
  - Sector 1 needs to produce f_1 units gross output
  - Sector 2 needs to produce nothing
- By symmetry, effects of f_2 should be obvious

Corn economy (3)

$$\boldsymbol{Z} = z > 0$$
$$\boldsymbol{x} = x > 0$$
$$x > z \text{ (economy is viable)}$$

Find $\boldsymbol{A}$

$$\boldsymbol{A} = \boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-1} = \frac{z}{x} = a > 0$$

Find $\boldsymbol{L}$

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = (1 - a)^{-1} > 1$$

$$x = L \cdot f = (1 - a)^{-1} \cdot f$$

- To satisfy f units final demand, economy needs to produce $(1 - a)^{-1} \cdot f$ units gross output

Two intermediate input-using sectors (4)

$$\boldsymbol{Z} = \begin{bmatrix} z_{11} > 0 & 0 \\ 0 & z_{22} > 0 \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \gg 0$$

$$\text{(economy is viable)}$$

Find $\boldsymbol{A}$

$$\boldsymbol{A} = \boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-1}$$

$$= \begin{bmatrix} a_{11} = \dfrac{z_{11}}{x_1} & 0 \\ 0 & a_{22} = \dfrac{z_{22}}{x_2} \end{bmatrix}$$

Find $\boldsymbol{L}$

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1}$$

$$= \begin{bmatrix} 1 - a_{11} & 0 \\ 0 & 1 - a_{22} \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} (1 - a_{11})^{-1} > 1 & 0 \\ 0 & (1 - a_{22})^{-1} > 1 \end{bmatrix}$$

$$\boldsymbol{x} = \boldsymbol{L} \cdot \boldsymbol{f} = \begin{bmatrix} (1 - a_{11})^{-1} & 0 \\ 0 & (1 - a_{22})^{-1} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

- To satisfy f_1 units final demand for sector 1's output:
  - Sector 1 needs to produce $(1 - a_{11})^{-1} \cdot f_1$ units gross output
  - Sector 2 needs to produce nothing
- By symmetry, effects of f_2 should be obvious

Simplest case of intersectoral dependence (5)

$$\boldsymbol{Z} = \begin{bmatrix} 0 & z_{12} > 0 \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \gg 0$$

(economy is viable)

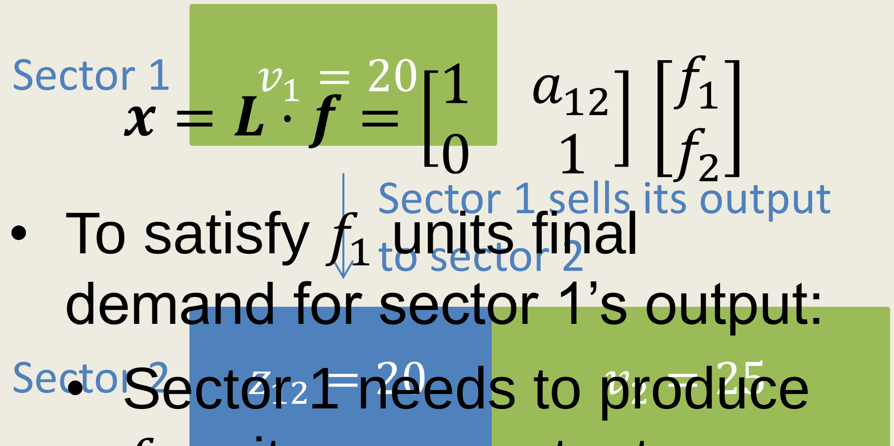
Find $\boldsymbol{A}$

$$\boldsymbol{A} = \boldsymbol{Z}\,\widehat{\boldsymbol{x}}^{-1} = \begin{bmatrix} 0 & a_{12} = \dfrac{z_{12}}{x_2} \\ 0 & 0 \end{bmatrix}$$

Find $\boldsymbol{L}$

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} 1 & -a_{12} \\ 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & a_{12} \\ 0 & 1 \end{bmatrix}$$



$$\boldsymbol{x} = \boldsymbol{L} \cdot \boldsymbol{f} = \begin{bmatrix} 1 & a_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

- To satisfy f_1 units final demand for sector 1's output:
  - Sector 1 needs to produce f_1 units gross output
  - Sector 2 needs to produce nothing
- To satisfy f_2 units final demand for sector 2's output:
  - Sector 1 needs to produce $a_{12} \cdot f_2$ units gross output
  - Sector 2 needs to produce f_2 units gross output

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Input-output tables

National

- OECD: http://www.oecd.org/trade/input-outputtables.htm
- Eurostat: http://ec.europa.eu/eurostat/web/esa-supply-use-input-tables

Multi-regional

- WIOD: http://www.wiod.org/database/wiots16
- OECD: http://www.oecd.org/sti/ind/inter-country-input-output-tables.htm
- Eora: http://www.worldmrio.com/

Assessment

- Practice assignment to be posted on Brightspace
- Graded assignment to be posted on Brightspace after the second lecture. Deadline a week later
- Final exam will include IO questions