# 1 Linear Algebra

## 1.1 Solution

1. Performing the indicated operations (Definition CVA, Definition CVSM), we obtain the vector equations

$$\begin{bmatrix}3\\2\end{bmatrix} = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\alpha+0\\0+\beta\end{bmatrix} = \begin{bmatrix}\alpha\\\beta\end{bmatrix} \tag{1}$$

   Since the entries of the vectors must be equal by Definition CVE we have $\alpha = 3$ and $\beta = 2$

2.

$$\begin{aligned}\vec{v}+\vec{w} &= (v_x+w_x, v_y+w_y) &(2)\\ &= (8\cos(290^0)+6\cos(60^0)),(8\sin(290^0)+6\sin(60^0)) &(3)\\ &\approx (2.74+3),(-7.52+5.2) &(4)\\ &\approx (5.74,-2.32) &(5)\end{aligned}$$

3. $\mathbf{A}+\mathbf{B}\begin{bmatrix}1&4&-3\\6&3&0\end{bmatrix}+\begin{bmatrix}3&2&1\\-2&-6&5\end{bmatrix}=\begin{bmatrix}1+3&4+2&-3+1\\6+(-2)&3+(-6)&0+5\end{bmatrix}=\begin{bmatrix}4&6&-2\\4&-3&5\end{bmatrix}$

4. $\alpha\,\mathbf{B}=\frac{1}{2}\begin{bmatrix}3&2&1\\-2&-6&5\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&1&\frac{1}{2}\\-1&-3&-\frac{5}{2}\end{bmatrix}$

5. $\begin{bmatrix}1&3&1\\0&1&0\\1&1&2\end{bmatrix}\times\begin{bmatrix}2&-5&-1\\0&1&0\\-1&2&1\end{bmatrix}=\begin{bmatrix}1\times2+3\times0+1\times(-1)&1\times(-5)+3\times1+1\times2&1\times(-1)+3\times0+1\times1\\0\times2+1\times0+0\times(-1)&0\times(-5)+1\times1+0\times2&0\times(-1)+1\times0+0\times1\\1\times2+1\times0+2\times(-1)&1\times(-5)+1\times1+2\times2&1\times(-1)+1\times0+2\times1\end{bmatrix}$
   $=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

6. Determinant of matrix $\mathbf{A}$
   $\begin{bmatrix}1&3&1\\0&1&0\\1&1&2\end{bmatrix}=1\times1\times2+3\times0\times1+1\times0\times1-1\times1\times1-1\times0\times1-2\times0\times3=1$

   Inverse of matrix $\mathbf{A}$
   $\begin{bmatrix}1&3&1\\0&1&0\\1&1&2\end{bmatrix}^{-1}=\begin{bmatrix}2&-5&-1\\0&1&0\\-1&2&1\end{bmatrix}$

7.

$$\mathcal{O}=a_1\begin{bmatrix}2&-1\\1&3\end{bmatrix}+a_2\begin{bmatrix}0&4\\-1&2\end{bmatrix}+a_3\begin{bmatrix}4&2\\1&3\end{bmatrix} \tag{6}$$

$$\begin{bmatrix}0&0\\0&0\end{bmatrix}=\begin{bmatrix}2a_1+4a_3&-a_1+4a_2+2a_3\\a_1-a_2+a_3&3a_1+2a_2+3a_3\end{bmatrix} \tag{7}$$

   By our definition of matrix equality (Definition ME) we arrive at a homogeneous system of linear equations

$$2a_1+4a_3=0 \tag{8}$$
$$-a_1+4a_2+2a_3=0 \tag{9}$$
$$a_1-a_2+a_3=0 \tag{10}$$
$$3a_1+2a_2+3a_3=0 \tag{11}$$

   The coefficient matrix of this system row-reduces to the matrix

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix} \tag{12}$$

   and from this we conclude that the only solution is $a_1=a_2=a_3=0$. Since the relation of linear dependence is trivial, the set $S$ is linearly independent

8. Answer $p_A(x) = -2 - 5x + x^2$

9. Answer $p_A(x) = -5 + 4x^2 - x^3$

10. if $\lambda = 2$ is an eigenvalue of $\mathbf{A}$, the matrix $A - 2I_4$ will be singular and its null space will be the eigenspace of $\mathbf{A}$. So we form this matrix and row reduce

$$A - 2I_4 = \begin{bmatrix} 16 & -15 & 33 & -15 \\ -4 & 6 & -6 & 6 \\ -9 & 9 & -18 & 9 \\ 5 & -6 & 9 & -6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

With two free variables, we know a basis of the null space(Theorem BNS) will contain two vectors. Thus the null space of $A - 2I_4$ has dimension two, and so the eigenspace of $\lambda = 2$ has dimension 2 also, $\gamma_A(2) = 2$