1 Statistics

Solution

1. $\bar{X}$ has expected value 50 and standard deviation of $\sigma/\sqrt{n} = 4/\sqrt{10} = 1.26$

2. Using the formula
$$class \quad width \times frequency \quad density = frequency \tag{1}$$
gives the following table

Interval	Class width	Frequency density	Actual frequency
0-20	20	2	$2 \times 20 = 40$
20-30	10	3	$3 \times 10 = 30$
30-40	20	4	$4 \times 10 = 40$
40-60	20	3	$3 \times 20 = 60$
60-90	30	1	$1 \times 30 = 30$

$$(a) 40 + 30 + 40 = 110 \tag{2}$$
$$(b) 40 + 30 + 40 + 60 + 30 = 200 \tag{3}$$
$$\tag{4}$$

3. Imagine that the 12 pieces of fruit are numbered as $1, 2, \ldots, 12$. There are 12! possible orders tat which the 12 pieces of fruit can be taken out of the box. There are $7 \times 11!$ orders in which the last element is an apple. Hence the probability that the bowl will be empty after the last apple is taken from the box is equal to :
$$\frac{7 \times 11!}{12!} = \frac{7}{12} \tag{5}$$

4. Let A be the event that there is no ace among the five cards and B be the event that there is neither a king nor a queen among the five cards. The desired probability is given by
$$1 - P(A \cup B) = 1 - [P(A) + P(B) - P(AB)] \tag{6}$$
$$= 1 - \left[\frac{\binom{48}{5}}{\binom{52}{2}} + \frac{\binom{44}{5}}{\binom{52}{2}} - \frac{\binom{40}{5}}{\binom{52}{2}} \right] \tag{7}$$
$$= 0.1765 \tag{8}$$
$$\tag{9}$$

5. Label both the balls and the boxes as 1,2, and 3. Take an ordered sample space whose elements are all possible three-tuples (b_1, b_2, b_3), where b_i is the label of the box in which ball i is dropped. The sample space has 3^3 equally likely elements. Let A_i be the event that only box i is empty. The events A_1, A_2 and A_3 are mutually disjoint and have the same probability. Hence the desired probability is $P(A_1 \cup A_2 \cup A_3 = 3P(A_1)$. To find $P(A_1)$, note that the number of elements for which box 2(3) contains two balls and box 3(2) contains one ball is $\binom{3}{2} \times 3$. Hence the number of elements for which only box 1 is empty is $2 \times 3 = 6$. This gives $P(A_1) = \frac{6}{27}$ and so the probability that exactly one box will be empty is $\frac{2}{3}$

6. The p-value is defined as the smallest value of α for which the null hypothesis can be rejected.
   - if the p-value is less than or equal to α we reject the null hypothesis($p < \alpha$)
   - if the p-value is greater than α, we do not reject the null hypothesis ($p > \alpha$)

7. This is a problem that requires a t test for single samples. You have been given the fact that the population mean is 15. The sample mean is 17 with a standard dev. of 5.5.
Step One: Generally you should start by computing the st. dev. but that has been done so you can move on to computing the st. error. In this case st. error = 1.00
Step Two: Compute t using the sample mean, pop mean and st. error. In this case, t = 2.00
Step Three: Evaluate. The crit. value of t for a two tailed test is 2.045, for a one tailed test is 1.699. So, if you wrote a two tailed test you must accept the null. If you wrote a one tailed test you must reject the null and accept the alternative.

8. The pop. mean is given as 72 beats per minutes. The sample of 25 has an average of 69 with a standard dev. of 6.5. Step One: Again you need to solve for st. error. St. error = 1.30 Step Two: Solve for t test for single samples t = -2.31 Step Three: Evaluate. The critical value is 2.064. The computed value exceeds this value so there is a significant effect of the ind. var. of fitness.

9. Test statistics

$$Z = \frac{\bar{X} - 45}{4/\sqrt{10}} = 2.37 \tag{10}$$

$$\tag{11}$$

The rejection criterion at 5% is Z>1.96 or Z<-1.96, so we reject the null hypothesis. In the second test, the rejection criterion is Z>1.645 so we again reject the null.

10.

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k \tag{12}$$

$$H_a : \text{Not all means are equal} \tag{13}$$

$$\tag{14}$$

Suppose in reality that the null hypothesis is false. Does this mean that no two of the populations have the same mean? If not, what does it mean? If, in reality, the null hypothesis is false, this translates into "Not all of the means are the same" or equivalently "At least two of the means are not the same." (Notice that "at least two" applies to 2 or 3 or ... or k means.) This last statement in quotes is not equivalent to saying "No two of the populations have the same mean" since this is equivalent to saying, "All of the population means are different."