vanderpol

January 30, 2019

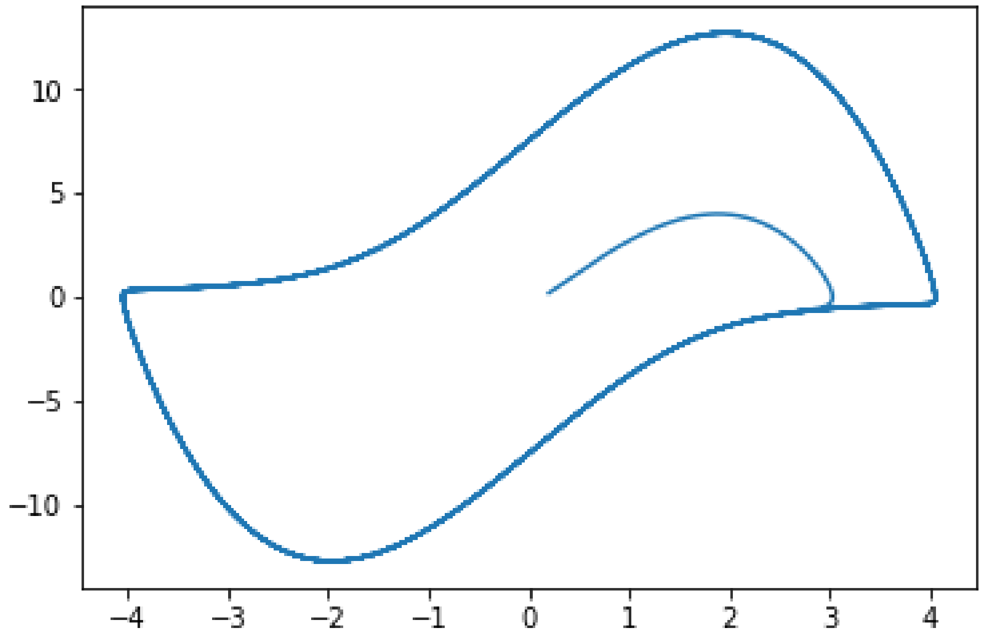
1 Training ANNs to learn the Van Der Pol equation

Lets create and run a system per the 1993 paper "Identification of Continuous-Time Dynamical Systems: Neural Network Based Algorithms and Parallel Implementation" which can be found at https://arxiv.org/abs/comp-gas/9305001

Lets see what Python shows for the Van Der Pol equation.

In [53]:
```
#Settings used in https://arxiv.org/pdf/comp-gas/9305001.pdf
mu = 1.
delta = 4.0
omega = 1.0

#Settings used in https://arxiv.org/pdf/comp-gas/9305001.pdf
#nSamples=500
nSamples=10000

#mu = 0.2
#delta = 1.0
#omega = 1.0
```

In [54]:
```
# %load viewPredict.py
import numpy as np
from pylab import *
from scipy.integrate import odeint

def van_der_pol_oscillator_deriv(x, t):
    nx0 = x[1]
    nx1 = -mu * (x[0] ** 2.0 - delta) * x[1] - omega * x[0]
    res = np.array([nx0, nx1])
    return res

ts = np.linspace(0.0, 50.0, nSamples)

xs = odeint(van_der_pol_oscillator_deriv, [0.2, 0.2], ts)
plt.plot(xs[:,0], xs[:,1])

plt.show()
```

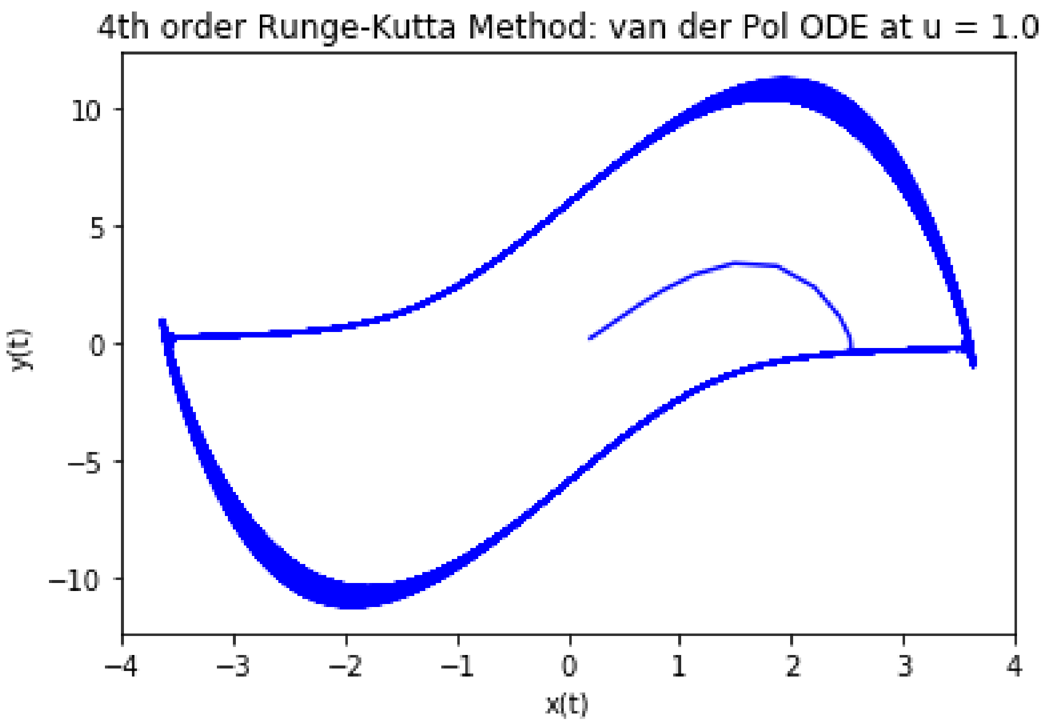


We want to train using a 4th order Runga-Kutta integrator like the following which generates the file train.dat. Note we change the Van Der Pol paramters to be those in the paper.

In [55]:
```
# RK2D.py: Plot out time series of integration steps of a 2D ODE
#      to illustrate the fourth-order Runge-Kutta method.
#
# For a 2D ODE
#     dx/dt = f(x,y)
#     dy/dt = g(x,y)
# See RKTwoD() below for how the fourth-order Rungle-Kutta method integrates.
#

# Import plotting routines
from pylab import *
import numpy as np
import struct

# The van der Pol 2D ODE
def van_der_pol_oscillator_deriv(x, t):
    nx0 = x[1]
    nx1 = -mu * (x[0] ** 2.0 - delta) * x[1] - omega * x[0]
    res = np.array([nx0, nx1])
    return res

# 2D Fourth-Order Runge-Kutta Integrator
def RKTwoD(x, f, t):
```

```
    x = np.array(x)
    k1 = dt * f(x,t)
    k2 = dt * f(x + x / 2.0,t)
    k3 = dt * f(x + k2 / 2.0,t)
    k4 = dt * f(x + k3,t)
    x = x + ( k1 + 2.0 * k2 + 2.0 * k3 + k4 ) / 6.0
    return x

# Simulation parameters
# Integration time step
dt = 0.1

# Time
t  = [ 0.0]

# The main loop that generates the orbit, storing the states
xs= np.empty((nSamples+1,2))
xs[0]=[0.2,0.2]

for i in range(0,nSamples):
  # at each time step calculate new x(t) and y(t)
  xs[i+1]=(RKTwoD(xs[i],van_der_pol_oscillator_deriv,dt))
  t.append(t[i] + dt)

#convert this to binary
fn = open('train.dat','wb')
fn.write(bytearray(struct.pack('i',int(2))))
fn.write(bytearray(struct.pack('i',int(2))))
fn.write(bytearray(struct.pack('i',int(nSamples))))

for i in range(0,nSamples):
    # write input
    fn.write(bytearray(struct.pack('f',xs[i][0])))
    fn.write(bytearray(struct.pack('f',xs[i][1])))
    #write output
    fn.write(bytearray(struct.pack('f',xs[i+1][0])))
    fn.write(bytearray(struct.pack('f',xs[i+1][1])))
fn.close()

#for i in range(0,nSamples):
 #   print(xs[i],xs[i+1])


# Setup the parametric plot
xlabel('x(t)') # set x-axis label
ylabel('y(t)') # set y-axis label
title('4th order Runge-Kutta Method: van der Pol ODE at u = ' + str(mu)) # set plot t
#axis('equal')
```

```
# Plot the trajectory in the phase plane
plot(xs[:,0],xs[:,1],'b')
show()
```



We train the neural network with the following. The maximum runtime is 60 seconds, but it only takes around 40 seconds to run to completion. The file param.dat is deleted below so we start from random initial conditions.

The following shows the basic configuration for this network

```
In [56]: f=open("FcnOfInterest_config.h","w")
         f.write("#define GFCN Tanh_G\n")
         f.write('#define RK4_H 0.1\n')
         f.write('#define PREDFCN rhs\n')
         f.write('#define EXPLICIT_RK4\n')
         f.close()
```

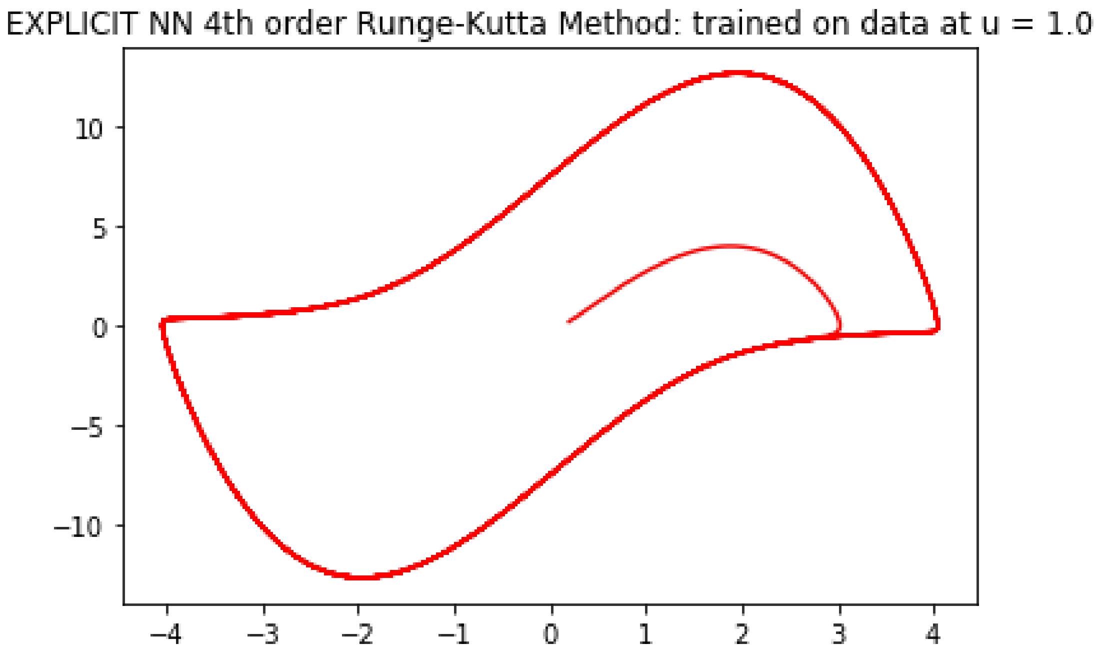
```
In [57]: import subprocess
         output = subprocess.getoutput(['make -j'])
         output = subprocess.getoutput(['sh BUILD.python.module.sh'])

         output = subprocess.getoutput(['rm -f param.dat'])
         output = subprocess.getoutput(['./nloptTrain.x -p param.dat -d train.dat -t 60'])
         print(output)
```

```
training data in: train.dat
using and/or writing params to: param.dat
OMP_NUM_THREADS 24
nInput 2 nOutput 2 nExamples 10000 in datafile (train.dat)
*******************
Objective Function: Least Means Squared
Function of Interest: EXPLICIT RK4 twolayer 2x5x5x2
Citation: https://arxiv.org/pdf/comp-gas/9305001.pdf
RK4_H=0.100000 with G() tanh()
Number params 57
Max Runtime is 60 seconds
Using NLOPT_LD_LBFGS
        Optimization Time 1.69746
        nlopt failed! ret -1
RUNTIME Info (10000 examples)
        DataLoadtime 0.00581213 seconds
        AveObjTime 0.00111849, countObjFunc 117, totalObjTime 0.129744
        Estimated Flops myFunc 217,average GFlop/s 1.94012 nFuncCalls 117
        Estimated maximum GFlop/s 3.95687, minimum GFLop/s 0.345928
        AveGradTime 0.0133307, nGradCalls 117, totalGradTime 1.5597
        nFuncCalls/nGradCalls 1.00
```

This shows we trained with the EXPLICIT RK4 implementation described in the paper. Let's look at the output by calling the C++ prediction function using the model parameters from the training run. We can then integrate to see if we learned the Van Der Pol equation.

In [58]:
```python
# Import plotting routines
from pylab import *
import numpy as np
import struct
from farbopt import PyPredFcn

from scipy.integrate import odeint

def van_der_pol_oscillator_deriv(x, t):
    nx0 = x[1]
    nx1 = -mu * (x[0] ** 2.0 - delta) * x[1] - omega * x[0]
    res = np.array([nx0, nx1])
    return res

ts = np.linspace(0.0, 50.0, nSamples)

xs = odeint(van_der_pol_oscillator_deriv, [0.2, 0.2], ts)

title('EXPLICIT NN 4th order Runge-Kutta Method: trained on data at u = ' + str(mu))

plt.plot(xs[:,0], xs[:,1],'r')
```

```
#plt.gca().set_aspect('equal')
#plt.savefig('vanderpol_oscillator.png')
plt.show()
```



2 Implicit implementation

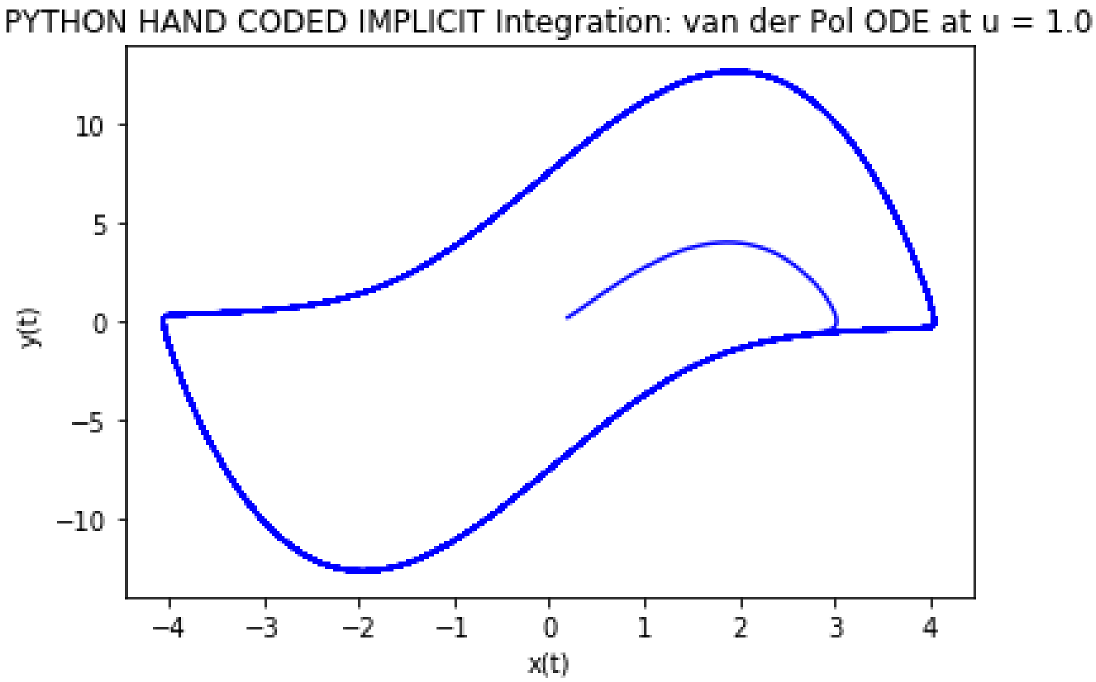
Looks good! Now lets try the implicit integrator using ANN recursion. See paper for more details

```
In [89]: # Simulation parameters
         # Integration time step
         # The number of time steps to integrate over
         #nSamples = 10000
         dt_step = 0.02
         nRecurIters=int(dt/dt_step) # we iterate to the previous dt
         writeImplicitData=1 # keep the previous data set

         def ImplicitMethod(x,f,t):
             x = np.array(x)
             Yn_1 = np.array(f(x,t))
             t1 = np.array(f(x,t))
             for i in range(nRecurIters):
                 Yn_1 = x + dt_step/2. * (t1 + np.array(f(Yn_1,t)))
             return Yn_1
```

```
In [90]: #Implicit integrator
```

```python
# Import plotting routines
from pylab import *
import numpy as np
import struct

# The van der Pol 2D ODE
def van_der_pol_oscillator_deriv(x, t):
    nx0 = x[1]
    nx1 = -mu * (x[0] ** 2.0 - delta) * x[1] - omega * x[0]
    res = np.array([nx0, nx1])
    return res


# Time
t  = [ 0.0]

# The main loop that generates the orbit, storing the states
xs= np.empty((nSamples+1,2))
xs[0]=[0.2,0.2]

for i in range(0,nSamples):
  # at each time step calculate new x(t) and y(t)
  xs[i+1]=(ImplicitMethod(xs[i],van_der_pol_oscillator_deriv,dt))
  t.append(t[i] + dt)

if(writeImplicitData != 0):
    fn = open('train.dat','wb')
    fn.write(bytearray(struct.pack('i',int(2))))
    fn.write(bytearray(struct.pack('i',int(2))))
    fn.write(bytearray(struct.pack('i',int(nSamples))))

    for i in range(0,nSamples):
        # write input
        fn.write(bytearray(struct.pack('f',xs[i][0])))
        fn.write(bytearray(struct.pack('f',xs[i][1])))
        #write output
        fn.write(bytearray(struct.pack('f',xs[i+1][0])))
        fn.write(bytearray(struct.pack('f',xs[i+1][1])))
    fn.close()

#for i in range(0,nSamples):
 #   print(xs[i],xs[i+1])

#
# Setup the parametric plot
xlabel('x(t)') # set x-axis label
ylabel('y(t)') # set y-axis label
title('PYTHON HAND CODED IMPLICIT Integration: van der Pol ODE at u = ' + str(mu)) #
```

```
#axis('equal')

# Plot the trajectory in the phase plane
plot(xs[:,0],xs[:,1],'b')
show()
```



Looks good so let's train the network using the implicit integrator. Note this uses a form of recurrence connection. See paper for details.

```
In [91]: f=open("FcnOfInterest_config.h","w")
         f.write("#define GFCN Tanh_G\n")
         f.write('#define PREDFCN rhs\n')
         f.write('#define IMPLICIT_RK4\n')
         f.write(str('#define RK4_H ' + str(dt_step) +'\n'))
         f.write(str('#define RK4_RECURRENCE_LOOPS ' + str(nRecurIters) +'\n'))
         f.close()
```

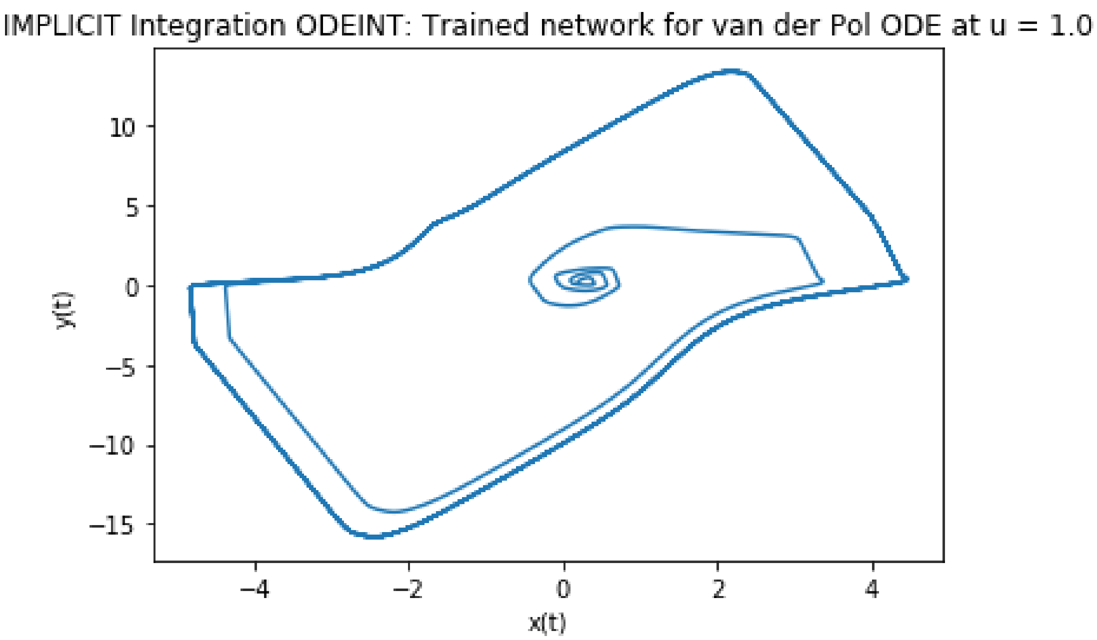
```
In [81]: import subprocess
         output = subprocess.getoutput(['make -j'])
         output = subprocess.getoutput(['sh BUILD.python.module.sh'])

         output = subprocess.getoutput(['rm -f param.dat'])
         print("------------------------------------------------------------------------------
         output = subprocess.getoutput(['./nloptTrain.x -p param.dat -d train.dat -t 60 --LN_B
         print(output)
         print("------------------------------------------------------------------------------
```

```
output = subprocess.getoutput(['./nloptTrain.x -p param.dat -d train.dat -t 60 --LD_TN
print(output)
```

```
----------------------------------------------------------------------------------------
training data in: train.dat
using and/or writing params to: param.dat
OMP_NUM_THREADS 24
nInput 2 nOutput 2 nExamples 10000 in datafile (train.dat)
*******************
Objective Function: Least Means Squared
Function of Interest: EXPLICIT RK4 twolayer 2x5x5x2
Citation: https://arxiv.org/pdf/comp-gas/9305001.pdf
RK4_H=0.020000 with G() tanh()
Number params 57
Max Runtime is 60 seconds
Using NLOPT_LN_BOBYQA
        Optimization Time 9.43433
        nlopt failed! ret -4
RUNTIME Info (10000 examples)
        DataLoadtime 0.0063119 seconds
        AveObjTime 0.000722267, countObjFunc 8829, totalObjTime 6.37617
        Estimated Flops myFunc 217,average GFlop/s 3.00443 nFuncCalls 8829
        Estimated maximum GFlop/s 4.22799, minimum GFLop/s 0.254943
----------------------------------------------------------------------------------------
training data in: train.dat
using and/or writing params to: param.dat
OMP_NUM_THREADS 24
nInput 2 nOutput 2 nExamples 10000 in datafile (train.dat)
*******************
Objective Function: Least Means Squared
Function of Interest: EXPLICIT RK4 twolayer 2x5x5x2
Citation: https://arxiv.org/pdf/comp-gas/9305001.pdf
RK4_H=0.020000 with G() tanh()
Number params 57
Max Runtime is 60 seconds
Using NLOPT_LD_TNEWTON_PRECOND_RESTART
        Optimization Time 60.0182
        found minimum 0.003990315414 ret 6
RUNTIME Info (10000 examples)
        DataLoadtime 0.00637955 seconds
        AveObjTime 0.000933198, countObjFunc 4281, totalObjTime 3.99409
        Estimated Flops myFunc 217,average GFlop/s 2.32534 nFuncCalls 4281
        Estimated maximum GFlop/s 4.29644, minimum GFLop/s 0.405688
        AveGradTime 0.0130796, nGradCalls 4281, totalGradTime 55.9939
        nFuncCalls/nGradCalls 1.00
```

In [93]:
```
# %load viewPredict.py
import numpy as np
```

```python
from pylab import *
from farbopt import PyPredFcn

from scipy.integrate import odeint

def predicted_rhs(x,t):
    return RHS.predict(x)

def van_der_pol_oscillator_deriv(x, t):
    nx0 = x[1]
    nx1 = -mu * (x[0] ** 2.0 - delta) * x[1] - omega * x[0]
    res = np.array([nx0, nx1])
    return res

ts = np.linspace(0.0, 50.0, nSamples)
# Setup the parametric plot
xlabel('x(t)') # set x-axis label
ylabel('y(t)') # set y-axis label
title('IMPLICIT Integration ODEINT: Trained network for van der Pol ODE at u = ' + st
RHS=PyPredFcn(b'param.dat')

xs = odeint(predicted_rhs, [0.2, 0.2], ts)
plt.plot(xs[:,0], xs[:,1])
plt.show()
```



In [ ]:

```
In [ ]:
```