RELATIVE VALUATION METHODS IN CORPORATE FINANCE

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1. Introduction

- There are two broad approaches to valuation:

  1. Discounted Cash Flow (DCF) Valuation:
     - Measure the level and the riskiness of cash flows
     - Estimate how the market "would" assign a discount or premium to those risks
  2. Relative Valuation:
     - Find a portfolio of traded assets that replicate the cash flows
     - Use the market prices of those assets for valuation

- Important: We are after one number, the Present Value. When properly applied, all methods should result in the same answer

Limitations of DCF Valuation

- DCF requires making several assumptions, so there is a lot of room for error
- Cash flow projections:
  - Even with substantial effort (e.g., market research), very noisy estimates
  - Can be biased / optimistic (think of division managers competing for resources)
- Estimating cost of capital:
  - What is the right model for the risk-return trade-off? (e.g., problems with C.A.P.M.)
  - Forward-looking estimates (do you know the current market risk premium?)
  - Comps are often imperfect
- More generally: We are trying to approximate the market value of an asset. Markets aggregate a lot of information; not an easy task for a project analyst

Relative (or Derivative, or No-Arbitrage) Valuation

- Basic idea:
  - The objective of valuation is to figure out a market value
  - But it is difficult to turn what the market does into a formula (see previous slide)
  - One strategy is to "free-ride" on market valuation: Find traded assets with similar cash flows to the asset or project we are valuing

- More specifically: Find a portfolio of *traded assets* that replicates the cash flows of the asset to be valued
  - This is called the "Tracking" (or "Replicating") Portfolio
  - The market price of the tracking portfolio is the PV we are after
  - Market-based approach to valuation

A Motivating Example: A project has the following risk-free cash flows over the next two years:

Cash Flows:	t=0	t=1	t=2
	–400	70	530

We have the following information on two bonds:

Bond A: Pays $10 coupon in year 1 and matures in year 2, when it pays $110. The bond trades at $105 today

Bond B: Pays $20 coupon in year 1 and matures in year 2, when it pays $100. The bond trades at $106 today

Find the NPV of the project by forming its tracking portfolio

Answer:

Tracking Portfolio:

– Consider X units of Bond A and Y units of Bond B so that the bond portfolio replicates the project's future cash flows:

Year 1: $X \times \$10 + Y \times \$20 = \$70$

Year 2: $X \times \$110 + Y \times \$100 = \$530$

– Solving the above equations simultaneously gives $X = 3$ and $Y = 2$

That is, the tracking portfolio consists of

buying 3 units of Bond A

buying 2 units of Bond B

– Next, find the current market price of the tracking portfolio:

$3 \times \$105 + 2 \times \$106 = \$527$

This is by definition the PV of the project's future cash flows

– Finally, compute the NPV of the project:

NPV = PV – Cost = \$527 – \$400 = \$127

Remarks:

- Notice that by design, the tracking portfolio has the exact same cash flows as the project
- The project creates value for shareholders by delivering those cash flows at a lower cost ($400) than they could obtain in the market ($527)
- Aside: Using the above method, can you back out zero-coupon interest rates (i.e., one-year and two-year risk-free rates)? [try it as an exercise]

Relationship to "arbitrage" and "hedging":

- Suppose you want to invest in the project, but do not want to experience the cash flow fluctuations that will result from it. What can you do?
- Invest in the project, and simultaneously invest in the "**arbitrage portfolio**", which is the **exact opposite** of the tracking portfolio:

Cash flows	t=0	t=1	t=2
▪ Invest in project	–400	70	530
▪ Short-sell 3 units of bond A	+3×105	–3×10	–3×110
▪ Short-sell 2 units of Bond B	+2×106	–2×20	–2×100
Total cash flow	127	0	0

arbitrage profit

zero net CFs in years 1 and 2

- Perfectly hedged position in years 1 and 2. Akin to "selling the project"

Replication and Mispricing:

- Replication methods rely on market prices
- What if you believe that the market prices are wrong? Should you still use them?

Example continued:

- You believe that the interest rates will soon be much higher than what market participants expect
- As a result, you think the bonds are overvalued: in your opinion, Bond A is worth $96 and Bond B is worth $98
- Should you change your valuation of the project?

Answer: It depends (on the circumstances)

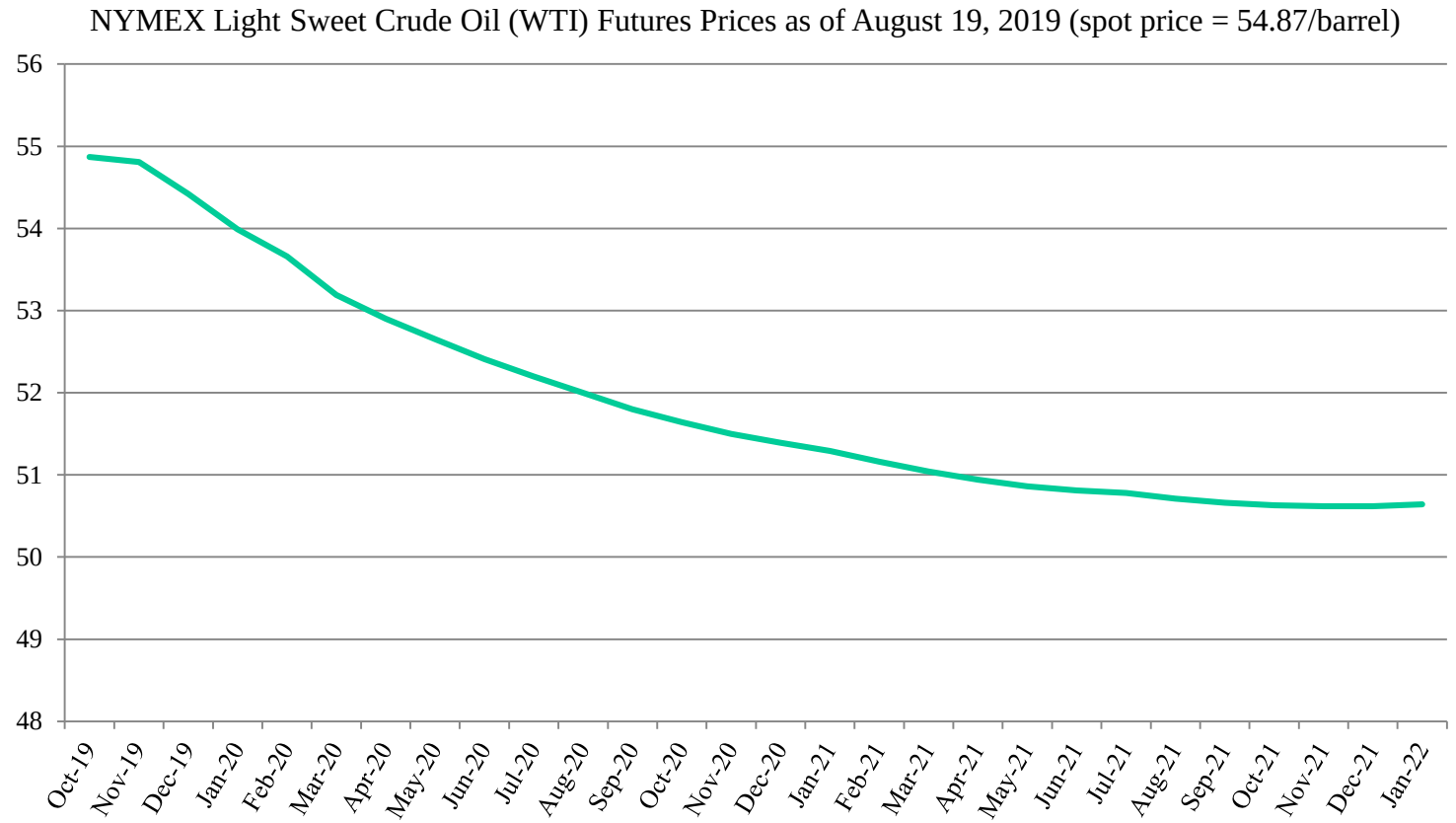
- Suppose you can hedge risks by taking the opposite position in financial markets, as in the example
  - Then your valuation is still valid: "Lock in" the project NPV by investing in it and hedging its cash flows by taking the arbitrage position
  - In practice, many contexts where risks can be hedged using derivatives (e.g., swap contracts)
  - Also, recall that hedging is similar to selling the project's future cash flows. May be feasible in some industries (e.g., drilling projects)

- If you cannot hedge or sell, are market prices still relevant? Useful?

- In practice, how good are managers in predicting the markets (interest rates, commodities, currencies, etc.)?

2. Valuation Using Forward Prices

- Forward contract: An obligation to buy/sell a security or commodity at a pre-specified price at some specific future date
- Futures contract: Similar to a forward contract, but trades at an exchange



Forward and Futures Contracts Basics:

- Derivative contracts in zero net supply: For every buyer (the long side), there is a seller (the short side)
- **Traders:** Hedgers and speculators (both can be on either side of the market)
- **Trading:** Forwards are over-the-counter contracts. Futures are standardized contracts; they trade on organized exchanges that offer clearing house services
- **Initiation:** No cash payments by either side at contract initiation (except for fees and margin requirements):

*The price (i.e., economic value) of a forward / futures contract is **zero** at initiation*

- **After initiation:** Contract value fluctuates (can be positive or negative)
  - Futures are marked-to-market on a daily basis
- **Settlement:**
  - Typically in kind (some futures contracts allow for cash settlement)
  - However, liquid futures contracts can effectively be cash settled, by entering into an offsetting position just before maturity

Forward and Futures Contracts Basics:

- **Pricing:**
  - Forward prices are not exactly expected values of future spot prices
  - They also reflect risk premiums, which depend on traders' hedging needs
  - Example:
    - If most of the hedgers in the oil futures market are producers who want to sell in advance, the forward price will be lower than the expected price (i.e., speculators are compensated for bearing the risk of owning oil in the future)
    - If most hedgers are consumers of oil (e.g., airlines), the forward price will be higher than the expected price (i.e., speculators are compensated for bearing the risk of providing oil in the future)

- Valuation using forward prices is closely related to the concept of **certainty equivalents**
- Let's start with the certainty equivalent for an individual:
  - Consider a risky payoff A one year from now, which is either $50K or $150K with equal likelihood
  - You are offered the choice between A versus a risk-free payoff B, again a year from now
  - At what value of B would you be indifferent between A and B?
  - The amount at which you are indifferent is called the certainty equivalent of A
  - For an individual, the certainty equivalent reflects attitudes toward risk

Valuation Using Forward Prices

- Main Insight: The forward price of, say, a commodity, represents the *certainty equivalent* to the market of the future (risky) spot price
- If
  - ✓ The risk in project cash flows stems mainly from future price uncertainty of a particular input or output (e.g., a commodity)
  - ✓ There is a forward market for that input or output

  then forward prices can be used to find the PV of cash flows
- Procedure:

  1. Calculate $CE(\tilde{C})$, the certainty equivalent of the risky cash flow

  2. Discount at the risk-free rate to find the PV:

$$PV = \frac{CE(\tilde{C})}{1 + r_f}$$

- Comparison with DCF:

  In DCF, risk adjustment is made to the discount rate. In CE valuation, risk adjustment is implicit in the certainty equivalent cash flow

Example: Valuation of a Copper Mine

Your company is considering to purchase a copper mine for $120,000. The mine will produce 50,000 pounds of copper one year from now, 30,000 pounds of copper two years from now, and nothing afterward

Assume that:

- Extraction costs are $1.50 per pound
- Forward prices of copper are $F_1 = \$3/\text{pound}$ and $F_2 = \$2.80/\text{pound}$
- Interest rates are 2% for one-year bonds and 3% for two-year zero coupon bonds

a) Describe the *actual* and the *expected* cash flows of the mine

b) Describe the *certainty equivalent* cash flows

c) Value the mine

d) Find the tracking portfolio of the mine

Answer:

a) Actual cash flows: (unknown today, will be realized over the next two years)

$$\tilde{C}_1 = 50{,}000(\tilde{p}_1 - \$1.50) = 50{,}000\,\tilde{p}_1 - \$75{,}000$$

$$\tilde{C}_2 = 30{,}000(\tilde{p}_2 - \$1.50) = 30{,}000\,\tilde{p}_2 - \$45{,}000$$

Expected cash flows: With DCF, you would have to estimate <u>expected</u> copper prices in year 1 and year 2 to calculate these

$$E(\tilde{C}_1) = 50{,}000\,E(\tilde{p}_1) - \$75{,}000$$

$$E(\tilde{C}_2) = 30{,}000\,E(\tilde{p}_2) - \$45{,}000$$

b) Certainty Equivalent cash flows (*known* today)

$$CE(\tilde{C}_1) = 50{,}000 \times \$3 - \$75{,}000 = \$75{,}000$$

$$CE(\tilde{C}_2) = 30{,}000 \times \$2.80 - \$45{,}000 = \$39{,}000$$

c) Valuation:

The value of the mine is the certainty equivalent cash flows discounted at the corresponding interest rates:

$$PV = \frac{75,000}{1+0.02} + \frac{39,000}{(1+0.03)^2} = \$110,290$$

Note that we discount at the risk-free rates

Therefore the value of the mine is $110,290

Since the purchase price is $120,000, the investment has a negative NPV of $110,290 – $120,000 = – $9,710

d) Convincing your skeptical boss:

- You take your valuation analysis to your boss and tell him about the negative NPV
- Your boss disagrees, saying that he thinks copper prices will be much higher than market's current expectations. He believes that copper will sell for $4/pound next year and $3.50/pound two years from now
- What should you do?
  - Can you convince him somehow that this is not a good investment?
  - Could this be a good investment if he is right?

Answer:

- This is a bad investment, even if your boss is correct in his expectation of higher copper prices
- Recall that our job is to beat the risk-return trade-off in the market
- Forward valuation suggests that the mine's cash flow stream can be replicated less expensively in the market
- Therefore, even if your boss is correct, buying copper in the market (using forward contracts) is a better investment than buying the mine
- Let's see this more directly by finding the **tracking portfolio** of the mine's cash flows

Forming the tracking portfolio:

- Using one-year forward contracts, buy 50,000 pounds of copper at price F_1
- Using two-year forward contracts, buy 30,000 pounds of copper at price F_2
- Invest X dollars in one-year bonds
- Invest Y dollars in two-year bonds

To find X and Y, match the tracking portfolio cash flows to the mine's cash flows:

At year 1:

$$\underbrace{50{,}000 \times (\tilde{p}_1 - \$3) + X(1+0.02)}$$

$$50{,}000\,\tilde{p}_1 - \$150{,}000 + X(1+0.02) = 50{,}000\,\tilde{p}_1 - \$75{,}000 \quad \Rightarrow X = \$73{,}529$$

At year 2:

$$\underbrace{30{,}000 \times (\tilde{p}_2 - \$2.80) + Y(1+0.03)^2}$$

$$30{,}000\,\tilde{p}_2 - \$84{,}000 + Y(1+0.03)^2 = 30{,}000\,\tilde{p}_2 - \$45{,}000 \quad \Rightarrow Y = \$36{,}761$$

What is the cost of the tracking portfolio?

- Buy 50,000 pounds of copper at price F_1 ⟶ costs $0 today
- Buy 30,000 pounds of copper at price F_2 ⟶ costs $0 today
- Invest X = $73,529 in one-year bonds ⟶ costs $73,529 today
- Invest Y = $36,761 in two-year bonds ⟶ costs $36,761 today

- The total cost of the tracking portfolio is 0 + 0 +73,529 + 36,761 = $110,290
- The tracking portfolio has the exact same cash flows as the mine by design, and costs less. Therefore, buying the mine is a bad (i.e., "dominated") investment
- If your boss thinks copper prices will increase, he should invest in the tracking portfolio

Summary

- Relative valuation: Value assets by "replication" using traded assets
- Advantages:
  - Market prices are forward-looking indicators of pricing of risk
  - Markets are good at aggregating a lot of information that is not otherwise readily available
  - Market-based valuation provides discipline, reduces the temptation and opportunities to reverse-engineer the valuation model to obtain a favored decision
- Disadvantages/limitations:
  - In most applications, it is difficult to find traded assets that closely track the cash flows of the asset to be valued
    - Most assets/projects have idiosyncratic characteristics that are hard to replicate
  - Relative valuation inherently assumes that other investors have done their homework. Sometimes there is reason to believe that they haven't (e.g., bubbles)
    - Can you imagine a world where everyone relies on relative valuation and nobody does DCF?

- Even in contexts where relative valuation is difficult to implement, it is a useful approach for formulating corporate strategy
  - Reminds managers that their job is to beat the risk-return trade-off investors face
  - Can learn from the market prices of similar but not identical assets (e.g., why is the value of a comparable firm so high or low?)
- In this note we took a first look at relative valuation philosophy and applications. Next we will analyze option pricing, where relative valuation is the norm