```julia
using JuMP, HiGHS

println("HOMEWORK 4:\n")

###Question 1###
println("Question 1\n")
println("b.\n")

# Model
model = Model(HiGHS.Optimizer)

# Decision Variables
@variable(model, x_LA_D >= 0)  # LA to Dallas
@variable(model, x_LA_H >= 0)  # LA to Houston
@variable(model, x_SD_D >= 0)  # SD to Dallas
@variable(model, x_SD_H >= 0)  # SD to Houston
@variable(model, x_D_NY >= 0)  # Dallas to New York
@variable(model, x_D_CH >= 0)  # Dallas to Chicago
@variable(model, x_H_NY >= 0)  # Houston to New York
@variable(model, x_H_CH >= 0)  # Houston to Chicago

# Objective
@objective(model, Min,
    300 * x_LA_D + 110 * x_LA_H +
    420 * x_SD_D + 100 * x_SD_H +
    (450 + 700) * x_D_NY + (550 + 700) * x_D_CH +
    (470 + 900) * x_H_NY + (530 + 900) * x_H_CH)

# Constraints
# Supply constraints
@constraint(model, x_LA_D + x_LA_H <= 4)  # Los Angeles can supply up
to 400,000 barrels
@constraint(model, x_SD_D + x_SD_H <= 5)  # San Diego can supply up to
500,000 barrels

# Demand constraints
@constraint(model, x_D_NY + x_H_NY >= 3)  # New York requires at least
300,000 barrels
@constraint(model, x_D_CH + x_H_CH >= 4)  # Chicago requires at least
400,000 barrels

# Refinery output constraints
@constraint(model, x_D_NY + x_D_CH <= x_LA_D + x_SD_D)  # Dallas
output
@constraint(model, x_H_NY + x_H_CH <= x_LA_H + x_SD_H)  # Houston
output

# Solve
optimize!(model)
```

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# Results
println("Objective value: ", objective_value(model))
println("x_LA_D = ", value(x_LA_D))
println("x_LA_H = ", value(x_LA_H))
println("x_SD_D = ", value(x_SD_D))
println("x_SD_H = ", value(x_SD_H))
println("x_D_NY = ", value(x_D_NY))
println("x_D_CH = ", value(x_D_CH))
println("x_H_NY = ", value(x_H_NY))
println("x_H_CH = ", value(x_H_CH))

###Question 2###
println("Question 2\n")

# Model
model = Model(HiGHS.Optimizer)

# Decision Variables
@variable(model, x_1 >= 0)
@variable(model, x_2 >= 0)

# Objective
@objective(model, Max, x_1 + x_2)

# Constraints
@constraint(model, x_1 + 2x_2 <= 6)
@constraint(model, -x_1 + x_2 >= 6)

# Solve the model
#optimize!(model)

# Output the maximization results
#println("Objective value for the maximization is: ",
objective_value(model))
#println(" x_1 is ", value(x_1))
#println(" x_2 is ", value(x_2))

# Objective
@objective(model, Min, x_1 + x_2)

# Constraints
#optimize!(model)

# Output the  minimization results
#println("Objective value for the minimization is: ",
objective_value(model))
#println(" x_1 is ", value(x_1))
#println(" x_2 is ", value(x_2))

println("There is no feasible region so the problem is infeasible.")
```

```julia
###Question 3###
println("Question 3\n")

# Model
model = Model(HiGHS.Optimizer)

# Decision Variables
@variable(model, x_1 >= 0)
@variable(model, x_2 >= 0)

# Objective
@objective(model, Max, 4x_1 + x_2)

# Constraints
@constraint(model, 8x_1 + 2x_2 <= 16)
@constraint(model, x_1 + 2x_2 <= 10)

# Solve the model
optimize!(model)

# Output the maximization results
println("Objective value for the maximization is: ",
objective_value(model))
println(" x_1 is ", value(x_1))
println(" x_2 is ", value(x_2))

# Objective
@objective(model, Min, 4x_1 + x_2)

# Constraints
optimize!(model)

# Output the  minimization results
println("Objective value for the minimization is: ",
objective_value(model))
println(" x_1 is ", value(x_1))
println(" x_2 is ", value(x_2))

###Question 4###
println("Question 4\n")

# Model
model = Model(HiGHS.Optimizer)

# Decision Variables
@variable(model, x_1 >= 0)
@variable(model, x_2 >= 0)

# Objective
@objective(model, Max, -x_1 + 4x_2)
```

```
# Constraints
@constraint(model, x_1 - x_2 <= 6)
@constraint(model, 2x_1 + 3x_2 >= 3)

# Solve the model
#optimize!(model)

# Output the maximization results
#println("Objective value for the maximization is: ",
objective_value(model))
#println(" x_1 is ", value(x_1))
#println(" x_2 is ", value(x_2))
println("The maximization problem is unbounded.")

# Objective
@objective(model, Min, -x_1 + 4x_2)

# Constraints
optimize!(model)

# Output the  minimization results
println("Objective value for the minimization is: ",
objective_value(model))
println(" x_1 is ", value(x_1))
println(" x_2 is ", value(x_2))

###Question 5###
println("Question 5\n")

# Model
model = Model(HiGHS.Optimizer)

# Decision Variables
@variable(model, x_1 >= 0)
@variable(model, x_2 >= 0)

# Objective
@objective(model, Max, 3x_1 + x_2)

# Constraints
@constraint(model, 2x_1 + x_2 <= 6)
@constraint(model, x_1 + 3x_2 <= 9)

# Solve the model
optimize!(model)

# Output the maximization results
println("Objective value for the maximization is: ",
objective_value(model))
```

```
println(" x_1 is ", value(x_1))
println(" x_2 is ", value(x_2))

# Objective
@objective(model, Min, 3x_1 + x_2)

# Constraints
optimize!(model)

# Output the  minimization results
println("Objective value for the minimization is: ",
objective_value(model))
println(" x_1 is ", value(x_1))
println(" x_2 is ", value(x_2))
```

HOMEWORK 4:

Question 1

b.

```
Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
Presolving model
6 rows, 8 cols, 16 nonzeros
6 rows, 8 cols, 16 nonzeros
Presolve : Reductions: rows 6(-0); columns 8(-0); elements 16(-0) -
Not reduced
Problem not reduced by presolve: solving the LP
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0     0.0000000000e+00 Pr: 2(7) 0s
          5     1.0470000000e+04 Pr: 0(0) 0s
Model   status      : Optimal
Simplex   iterations: 5
Objective value     :  1.0470000000e+04
HiGHS run time      :          0.00
Objective value: 10470.0
x_LA_D = 3.0
x_LA_H = 0.0
x_SD_D = 0.0
x_SD_H = 4.0
x_D_NY = 3.0
x_D_CH = 0.0
x_H_NY = 0.0
x_H_CH = 4.0
Question 2

There is no feasible region so the problem is infeasible.
Question 3
```

```
Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
Presolving model
2 rows, 2 cols, 4 nonzeros
2 rows, 2 cols, 4 nonzeros
Presolve : Reductions: rows 2(-0); columns 2(-0); elements 4(-0) - Not
reduced
Problem not reduced by presolve: solving the LP
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0    -1.4999972737e+00 Ph1: 2(4); Du: 2(1.5) 0s
          2     8.0000000000e+00 Pr: 0(0) 0s
Model   status      : Optimal
Simplex   iterations: 2
Objective value     :  8.0000000000e+00
HiGHS run time      :          0.00
Objective value for the maximization is: 8.0
 x_1 is 2.0
 x_2 is 0.0
Solving LP without presolve or with basis
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0    -1.0000013946e+00 Ph1: 1(1); Du: 1(1) 0s
          1     0.0000000000e+00 Pr: 0(0) 0s
Model   status      : Optimal
Simplex   iterations: 1
Objective value     :  0.0000000000e+00
HiGHS run time      :          0.00
Objective value for the minimization is: 0.0
 x_1 is 0.0
 x_2 is 0.0
Question 4

The maximization problem is unbounded.
Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
Presolving model
2 rows, 2 cols, 4 nonzeros
0 rows, 0 cols, 0 nonzeros
Presolve : Reductions: rows 0(-2); columns 0(-2); elements 0(-4) -
Reduced to empty
Solving the original LP from the solution after postsolve
Model   status      : Optimal
Objective value     : -6.0000000000e+00
HiGHS run time      :          0.00
Objective value for the minimization is: -6.0
 x_1 is 6.0
 x_2 is 0.0
Question 5

Running HiGHS 1.6.0: Copyright (c) 2023 HiGHS under MIT licence terms
```

```
Presolving model
2 rows, 2 cols, 4 nonzeros
2 rows, 2 cols, 4 nonzeros
Presolve : Reductions: rows 2(-0); columns 2(-0); elements 4(-0) - Not
reduced
Problem not reduced by presolve: solving the LP
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0    -3.9999952417e+00 Ph1: 2(7); Du: 2(4) 0s
          2     9.0000000000e+00 Pr: 0(0) 0s
Model   status      : Optimal
Simplex   iterations: 2
Objective value     :  9.0000000000e+00
HiGHS run time      :          0.00
Objective value for the maximization is: 9.0
 x_1 is 3.0
 x_2 is 0.0
Solving LP without presolve or with basis
Using EKK dual simplex solver - serial
  Iteration        Objective     Infeasibilities num(sum)
          0    -2.0000014690e+00 Ph1: 2(3); Du: 2(2) 0s
          1     0.0000000000e+00 Pr: 0(0) 0s
Model   status      : Optimal
Simplex   iterations: 1
Objective value     :  0.0000000000e+00
HiGHS run time      :          0.00
Objective value for the minimization is: 0.0
 x_1 is 0.0
 x_2 is 0.0
```