Homework 2

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STOR415: INTRODUCTION TO OPTIMIZATION
DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH
—————— **SPRING 2024** ——————

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HOMEWORK 2: LINEAR PROGRAMMING APPLICATIONS

- For this homework, you will need to submit both handwritten work and the output and code from Jupyter notebooks. Gradescope only allows the submission of a single PDF, however, you can combine multiple PDFs into one using free tools online or Adobe Acrobat, which all UNC students should have access to. When submitting this assignment, please assign pages to specific questions to make it easier to grade.
- For each coding problem, create an ipynb with exactly the same name as is required in the problem. In the Julia code, declare variables with the name given in the problem. Then, after solving the problem, in the last cell of your notebook print (or use @show) all the values of all of the variables in your optimization problem as well as the value of the objective function.
- Please comment add comments to your code describing the variables, constraints and objective function of your model.
- Ensure that your notebook runs properly before submitting it. In the main bar, perform **Clear Outputs of All Cells** then **Run All** to ensure that there are no errors.
- To generate a PDF of your notebook:
  - In the main bar, click **Export** (may be hidden behind a 3 dots dropdown menu)
  - Choose Export as PDF (may require additional extensions).
  - If Export as PDF fails or does not give proper output, export the file as HTML, open this HTML file in a web browser and save the HTML file as a PDF.
  - If you have issues with this, first open the folder containing your notebooks in VSCode (instead of just the notebook itself), then try exporting to PDF. This often fixes issues that may otherwise occur.
  - If you cannot get exporting to work in VSCode, you may use a Jupyter Notebook to PDF converter online.

Question 1. (**35 points**): The director of Awesome Tech startup needs to decide what salaries to offer its employees for the coming year. In order to keep the employees satisfied, she needs to satisfy the following constraints:

- Tom wants at least $30,000 or he will quit;
- Peter, Nina, and Samir each want to be paid at least $8000 more than Tom;

- Gary wants his salary to be at least as high as the combined salary of Tom and Peter;
- Linda wants her salary to be $500 more than Gary;
- The combined salary of Nina and Samir should be at least twice the combined salary of Tom and Peter;
- Bob's salary is at least as high as that of Peter and at least as high as that of Samir;
- The combined salary of Bob and Peter should be at least $75,000;
- Linda should not make more money than the combined salary of Bob and Tom.

a) Write an LP that will determine the salaries for the employees of Awesome Tech that satisfy each of these constraints while minimizing the total salary expenses.

b) Write an LP that will determine the salaries for the employees of Awesome Tech that satisfy each of these constraints while minimizing the salary of the highest paid employee.

c) Now, create a Jupyter notebook titled *salaries.ipynb* and implement your solution to part b. In the Julia code, declare variables for each person's salary as well as the variables needed to reformulate the min-max problem into a linear program. Model and solve this problem as a linear program. To check your solution, the optimal objective is 68,500.

Question 2. (**30 points**): Superior Roasters Coffee Company mixes specialty blends of coffee to seel to Big Cup Coffee, a small chain of coffee shops. The beans used in their specialty coffee are listed in the table below:

Bean Type	Cost Per Pound ($)	Available amount (pounds)
Columbian	1.00	550
Brazilian	0.85	450
Sumatran	1.55	650

Superior Roasters' products are:

- **Robust Joe** must consist of 60-75% Sumatran beans and at least 10% Columbian beans. Each pound of Robust Joe can be sold to Big Cup for $4.25.
- **Light Joe** must consist of 50-60% Brazilian beans and no more than 20% Sumatran beans. Each pound of Light Joe can be sold to Big Cup for $3.95.

a) Formulate an LP to maximize the total profit (revenue minus cost). Write down your mathematical model.

b) Now, create a Jupyter notebook named *coffee.ipynb* to implement and solve the model you developed in part A. In the Julia code, declare the following vectors of symbols:

types = [:Columbian, :Brazilian, :Sumatran]
blends = [:Robust, :Light]

and then declare a variable x which is indexed over types and blends. Create dictionaries indexed over either *types* or *blends* which represent the number of beans available, the purchase prices, and the coffee prices. Then, define all of the constraints and the objective function. An example constraint (assuming that x is indexed over [types,blends] and the JuMP model is called "coffeeblend"):

#Robust blend has at least 60 percent sumatran
*@constraint(coffeeblend, RSLowerBound, x[:Sumatran,:Robust] >= 0.6*sum(x[i,:Robust] for i in types))*

To check that you correctly implemented this problem, the maximum profit is $4902.5.

Question 3. (35 points): A major shopping mall developer in Chapel Hill is selecting the tenants mix for its next facility. Stores of various product types i are being considered for arrangement into the new mall's different sectors j. Each sector will have at most area A (in thousands of square feet) and funds calculated using c_i (dollars per 1000 square foot) must be set aside for "finishing" of areas allocated to stores of type i. From prior experience, the developer can estimate the present worth p_{ij} of revenues from a type i store located in sector j and can further estimate the required floor space a_i (in thousands of square feet) for a type i store. The developer requires: (i) having between n_i and N_i stores of each type i; (ii) having a total area between f_i and F_i (thousands of square feet) devoted to stores of each type i; and, (iii) not exceeding a budget of b (dollars) for finishing costs. Assume that the number of each type of store does not need to be an integer. Use the sets S to denote the set of all types of stores and C to denote the set of all sectors.

a) Formulate a linear program to find an optimal tenant mix that will **maximize total present worth**.

b) Now, the shopping mall developer wishes to investigate equalizing the present worth in each sector. That is, the shopping mall developer wishes to **maximize the lowest present worth (profit) of any sector**. Modify the model from part (a) to handle this new objective. You only need to list your modifications, not the whole model.

——————— **The end** ———————

1. a) Tom $= X_1$, Peter $= X_2$, Nina $= X_3$, Samir $= X_4$, Gary $= X_5$
Linda $= X_6$, Bob $= X_7$

$$\min Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$$

s.t.

$$X_1 \geq 30{,}000$$

$$X_2 + X_3 + X_4 \geq X_1 + 8{,}000$$

$$X_5 \geq X_1 + X_2$$

$$X_6 = X_5 + 500$$

$$X_3 + X_4 \geq X_1 + X_2$$

$$X_7 \geq X_2$$

$$X_7 \geq X_4$$

$$X_2 + X_7 \geq 75{,}000$$

$$X_5 < X_1 + X_2$$

b) WLOG the same objective, variables, and constraints can be used w/ the addition of one variable $X_{highest}$ = Highest Salary that has the following additional constraints.

s.t.

$$X_{highest} \geq X_1$$

$$X_{highest} \geq X_2$$

$$\vdots$$

$$X_{highest} \geq X_7$$

2. a) Columbian $= X_{1j}$ Brazilian $= X_{2j}$ Sumatran $= X_{3j}$

Robust $= X_{i1}$ Light $= X_{i2}$

$$\max z = 4.25(X_{11} + X_{21} + X_{31}) + 3.95(X_{12} + X_{22} + X_{32})$$
$$- \left(1(X_{11} + X_{12}) + 0.85(X_{21} + X_{22}) + 1.55(X_{31} + X_{32})\right)$$

$$X_{31} \geq 0.6(X_{11} + X_{21} + X_{31})$$

s.t. $X_{31} \leq 0.75(X_{11} + X_{21} + X_{31})$

$$X_{11} \geq 0.1(X_{11} + X_{21} + X_{31})$$

$$X_{22} \geq 0.5(X_{12} + X_{22} + X_{32})$$

$$X_{22} \leq 0.6(X_{12} + X_{22} + X_{32})$$

$$X_{32} \leq 0.2(X_{12} + X_{22} + 3_{32})$$

$$X_{11} + X_{12} \leq 550$$

$$X_{21} + X_{22} \leq 450$$

$$X_{31} + X_{32} \leq 650$$

variables

3. a. X_{ij} = # of stores of type i in sector j

objective

$$\max z = \sum_{j \in C} \sum_{i \in S} p_{ij} x_{ij}$$

for each j:

$$\sum_{i \in S} a_i x_{ij} \leq A \qquad j = 1$$

for each i:

$$f_i \leq \sum_{j \in C} a_i x_{ij} \leq F_i$$