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CMPT 431 Distributed Systems

Fall 2019

Reasoning Correctness

https://www.cs.sfu.ca/~keval/teaching/cmpt431/fall19/

Instructor: Keval Vora

Concurrent Programs

- How to design concurrent programs?
- What does concurrency mean when ordering is important?
  - E.g., concurrent queues, concurrent stacks
- How to verify whether concurrent solution is correct?
  - Naïve: Check for all possible results. Infeasible.

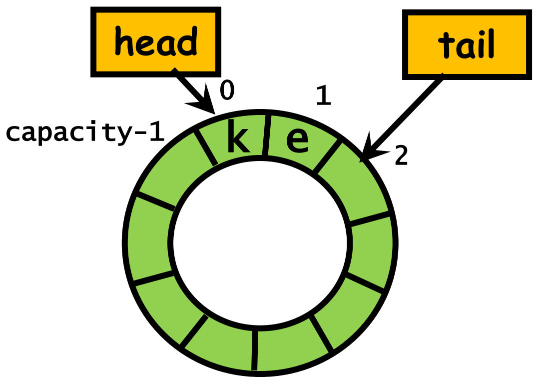
- Properties that can help in:
  - developing concurrent solutions
  - analyzing concurrent solutions
  - provide correctness and progress guarantees

Reading

R

- [AMP] Chapter 3
  - Upto 3.7
- [Paper] Linearizability: A Correctness Condition for Concurrent Objects: https://cs.brown.edu/~mph/HerlihyW90/p463-herlihy.pdf
  - Upto section 3
- [Paper] How to Make a Multiprocessor Computer That Correctly Executes Multiprocess Programs: https://www.microsoft.com/en-us/research/uploads/prod/2016/12/How-to-Make-a-Multiprocessor-Computer-That-Correctly-Executes-Multiprocess-Programs.pdf

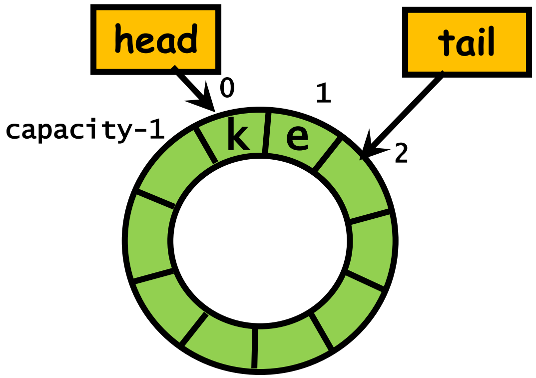
Single-Enqueuer/Single-Dequeuer Queue



- Sequential code is available

```
class Queue<T> {
  volatile int head = 0, tail = 0;
  T[] items;
  public Queue(int capacity) {        {
    items = (T[])new Object[capacity];
    head = 0; tail = 0;
  }
  public void enq(T x) throws FullException {
    if (tail - head == items.length)
      throw new FullException();
    items[tail % items.length] = x;
    tail++;
  }
  public T deq() throws EmptyException {
    if (tail - head == 0)
      throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  }
}
```

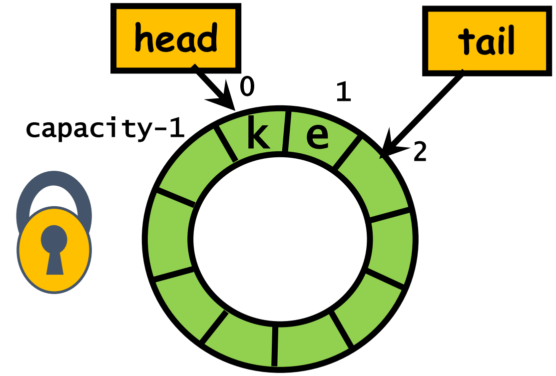
Single-Enqueuer/Single-Dequeuer Queue




```
class WaitFreeQueue<T> {
  volatile int head = 0, tail = 0;
  T[] items;
  public WaitFreeQueue(int capacity) {
    items = (T[])new Object[capacity];
    head = 0; tail = 0;
  }
  public void enq(T x) throws FullException {
    if (tail - head == items.length)
      throw new FullException();
    items[tail % items.length] = x;
    tail++;
  }
  public T deq() throws EmptyException {
    if (tail - head == 0)
      throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  }
}
```

- Concurrent code is same!
  - How is it correct?
  - enq() & deq() can happen at same time?

Lock-based Queue



Easy to reason correctness:
no two operations happen at the **`same time'**

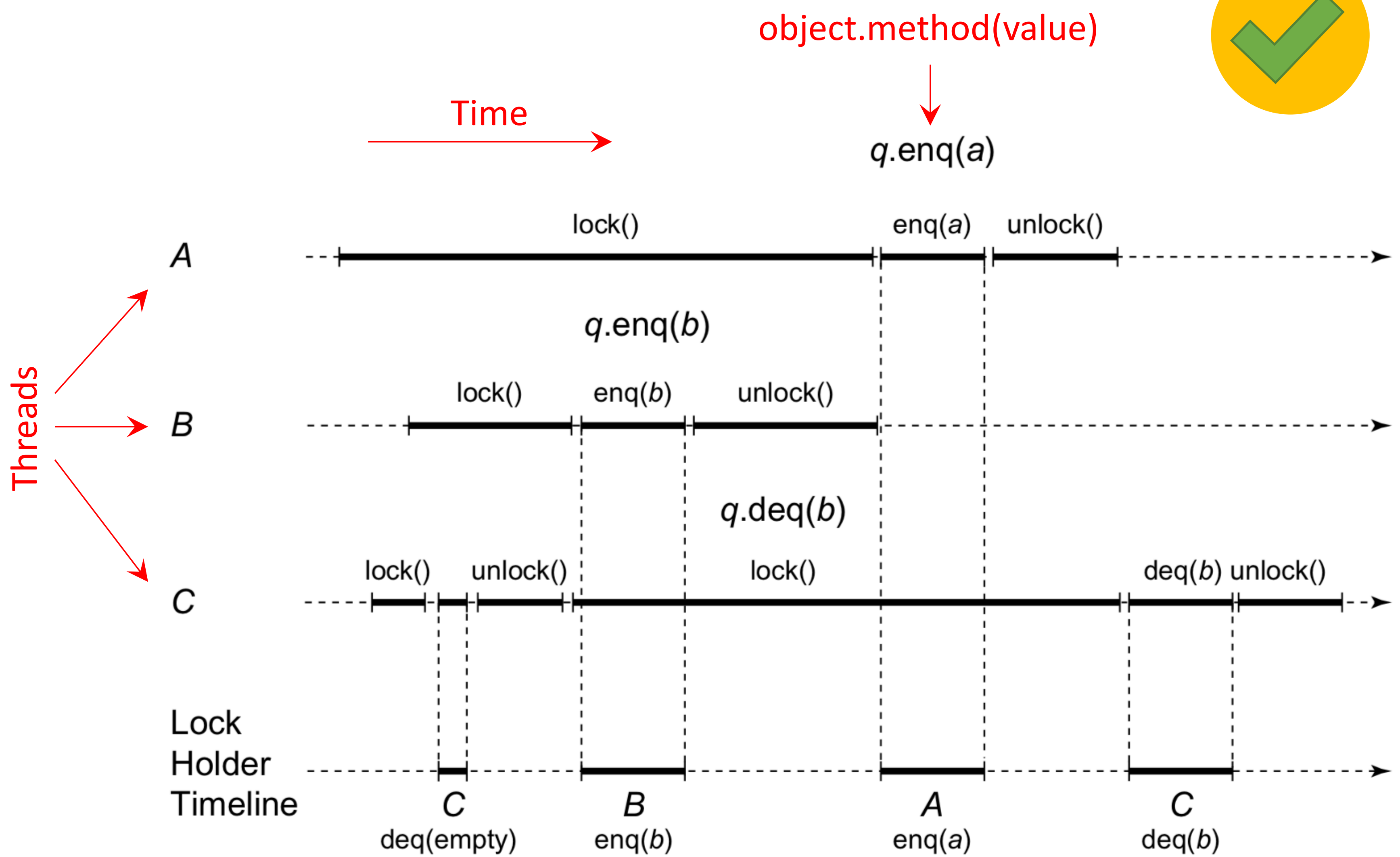
```
class LockBasedQueue<T> {
  int head, tail;
  T[] items;
  Lock lock;
  public LockBasedQueue(int capacity) {
    head = 0; tail = 0;
    lock = new ReentrantLock();
    items = (T[])new Object[capacity];
  }
  public void enq(T x) throws FullException {
    lock.lock();
    try {
      if (tail - head == items.length)
        throw new FullException();
      items[tail % items.length] = x;
       tail++;
    }  finally {
      lock.unlock();
    }
  }
  public T deq() throws EmptyException {
    lock.lock();
    try {
      if (tail == head)
        throw new EmptyException();
      T x = items[head % items.length];
      head++;
      return x;
    } finally {
      lock.unlock();
    }
  }
}
```

Operations & Execution Histories

- Operations defined based on the context
  - Queue example: enq() and deq() are primary operations
- Operations are ones that are performed by concurrent tasks, and that we want to analyze

- Plot Execution Histories on timelines to visually analyze when operations (can) occur
  - Execution Histories: finite sequence of operations

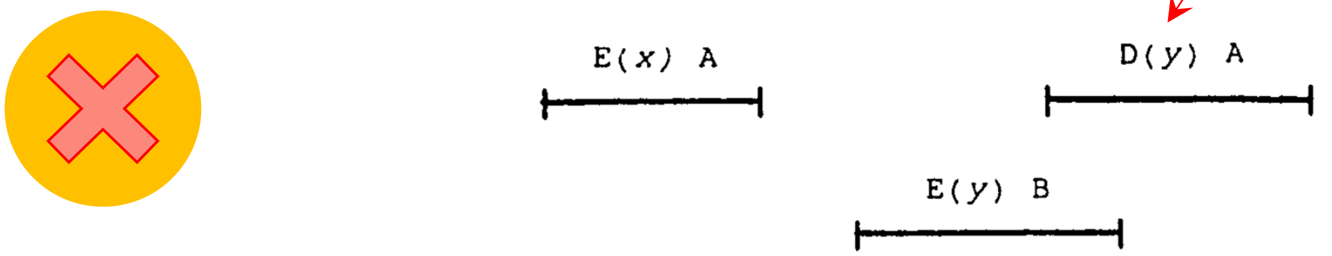
Execution Histories

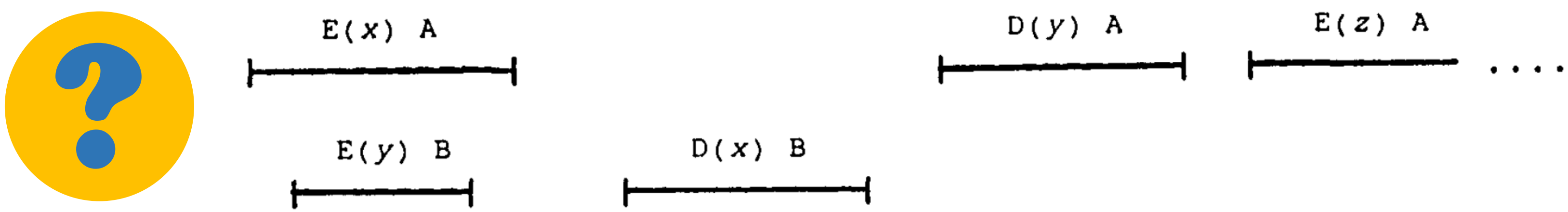


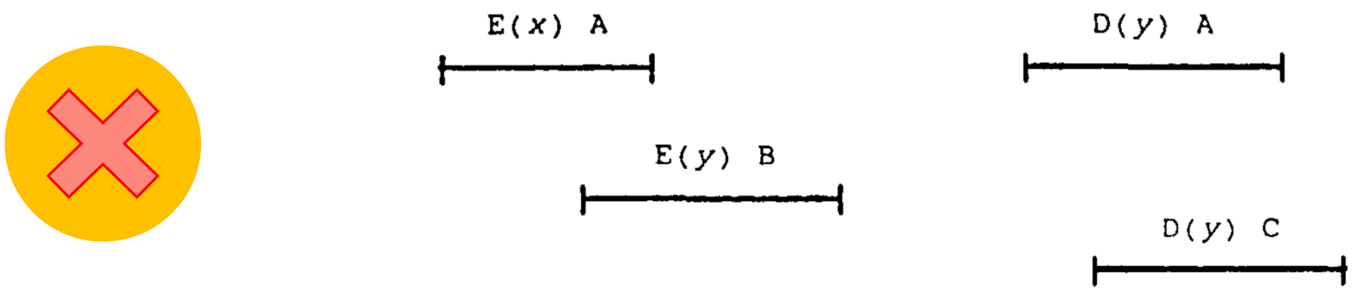
Execution Histories

operation(value) Thread

E = Enqueue
D = Dequeue







Correctness Conditions

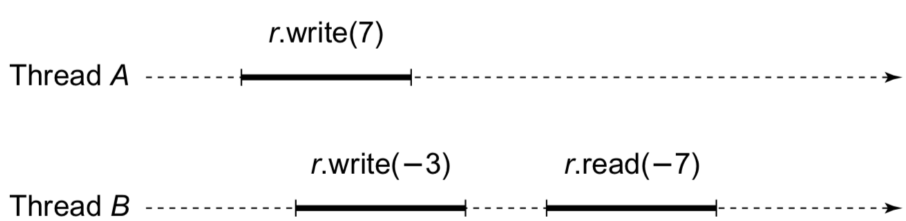
- Defining the set of allowable histories
- “allowable” → Depends on the context
- Degree of concurrency directly affected
  - Allow more histories → higher concurrency

- Sequential Consistency
- Linearizability
- (Serializability)

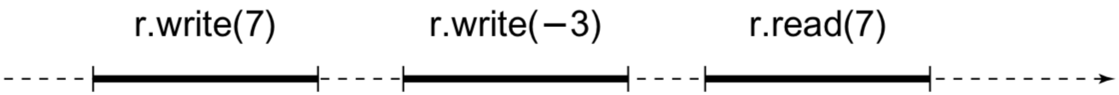
Sequential Executions

- Sequential methods have clear specifications in form of preconditions and postconditions
- With concurrency, multiple threads might invoke methods at the same time, making it difficult to reason about preconditions and postconditions in "isolation"
  - E.g., what does it mean for two overlapping enqueue operations? Which element will be dequeued first?
- Solution: show equivalence of histories

Method calls should appear to happen in a one-at-a-time sequential order



Sequential Consistency



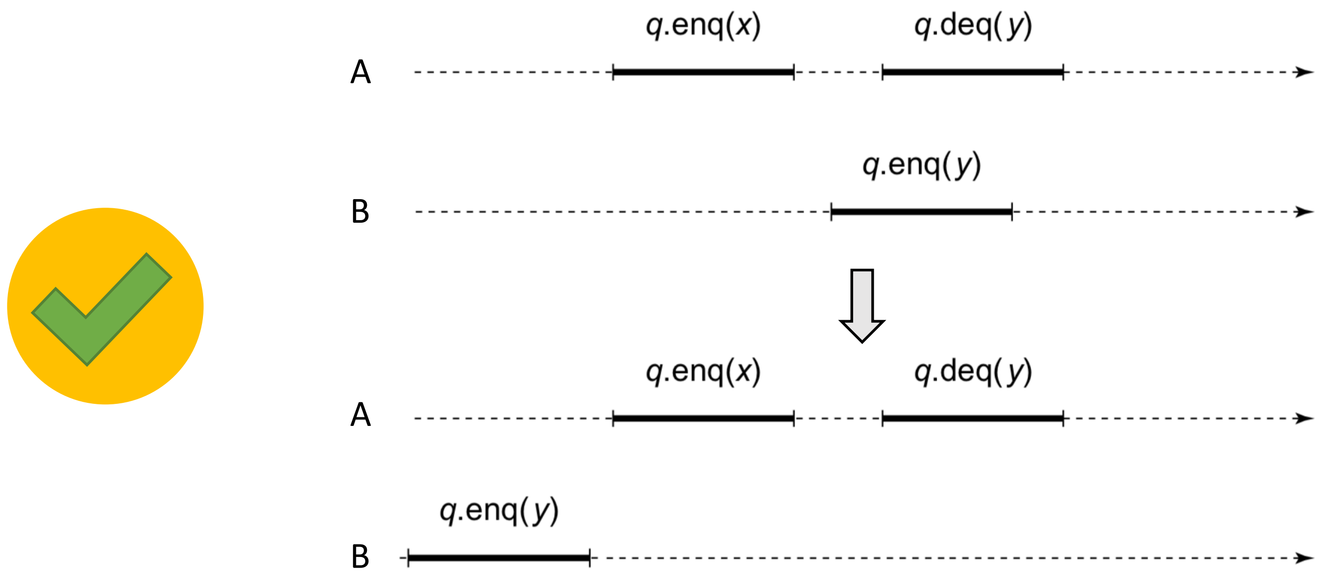
- [P1] Method calls should appear to happen in a one-at-a-time sequential order
- [P2] Method calls should appear to take effect in program order (i.e., order defined by single thread's code)
  - Purely sequential computations behave the way we would expect

- To verify, reorder method calls sequentially so that:
  - they are consistent with program order
  - meet the object's sequential specification

How to Make a Multiprocessor Computer That Correctly Executes Multiprocess Programs, Leslie Lamport, IEEE TC 1979

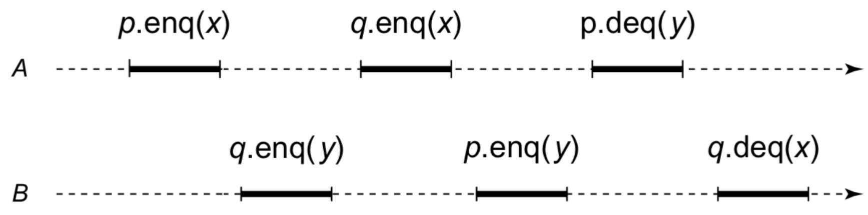
Sequential Consistency

- Counter-intuitive because it seems to violate FIFO



- Note: concurrent operations are allowed
  - as long as the two properties are satisfied

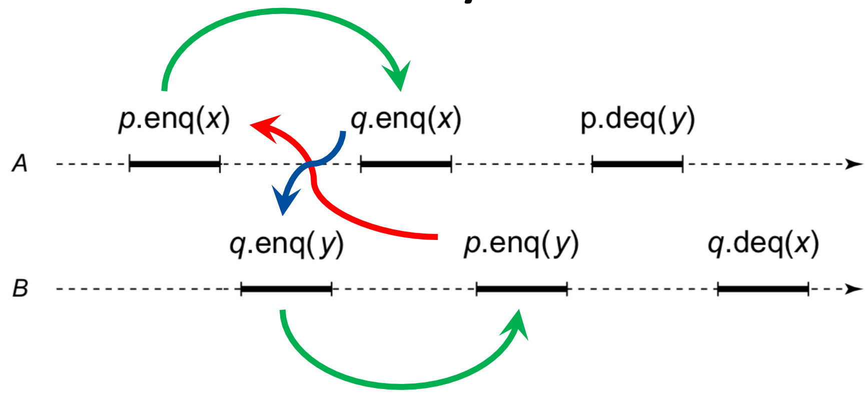
Sequential Consistency: Compositional?



- Is the sub-history for p sequentially consistent?
  - Reorder so that p.enq(y) comes before p.enq(x)
- Is the sub-history for q sequentially consistent?
  - Reorder so that q.enq(x) comes before q.enq(y)

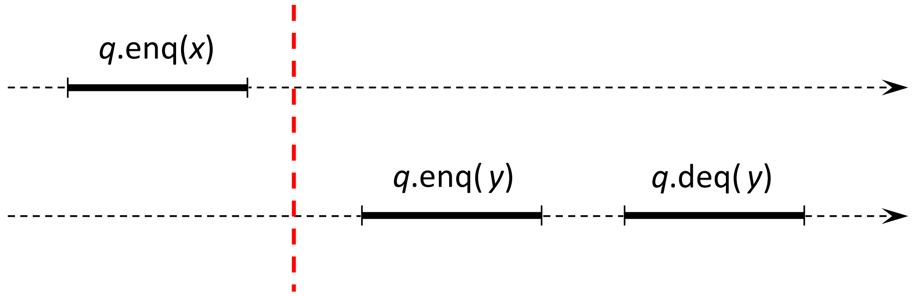
- Is the entire history sequentially consistent?
  - Need to explicitly check because SC is <u>not compositional</u>

Sequential Consistency: Not Compositional



- Is the entire history sequentially consistent?
  - p.deq(y) suggests: p.enq(y) → p.enq(x) [red arrow]
  - q.deq(x) suggests: q.enq(x) → q.enq(y) [blue arrow]
  - Program order [green arrows]
    q.enq(y) → p.enq(y) and p.enq(x) → q.enq(x)
  - Above orderings form a cycle (i.e., cannot be sequential)

Capturing Real-Time Precedence

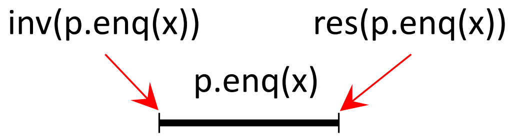


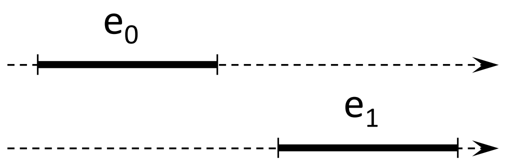
- Above history is sequentially consistent
- The real-time precedence ordering: q.enq(x) → q.enq(y)
- Intuition
  - Updates should become visible
  - deq operation should return x and not y
- Sequential consistency fails to capture real-time progress

Linearizability

- Captures real-time precedence ordering
- Informally: Operations should take effect instantaneously between its invocation and response

- Definitions:
  - inv(e): invocation of event e
  - res(e): response of event e
  - $e_0 <_H e_1$: res(e_0) precedes inv(e_1) in H
- $<_H$ is a partial order
  - Operations unrelated by $<_H$ are said to be concurrent





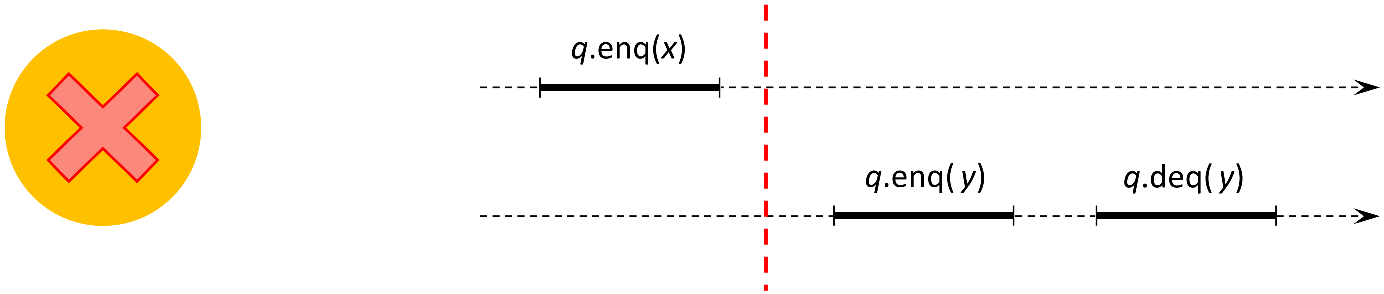
Linearizability: A Correctness Condition for Concurrent Objects, M. Herlihy and J. Wing, 1990

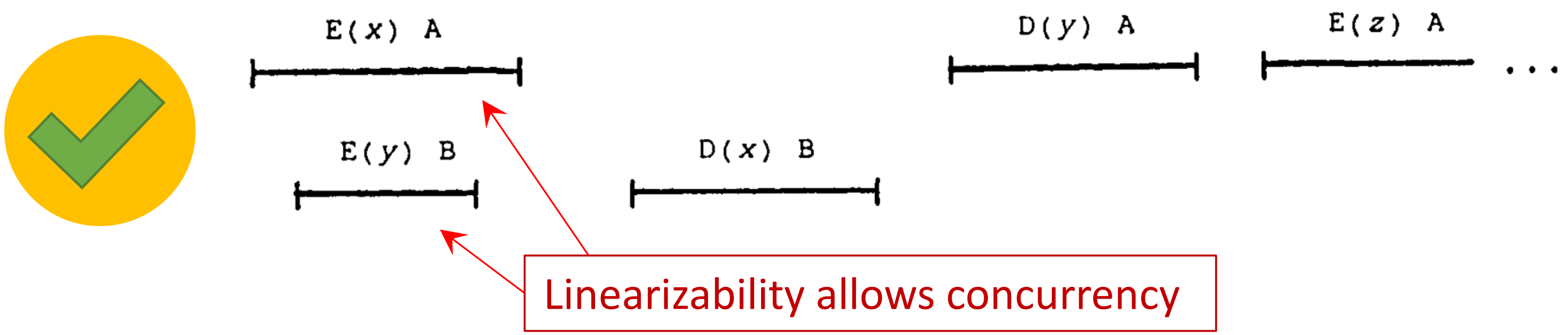
Linearizability

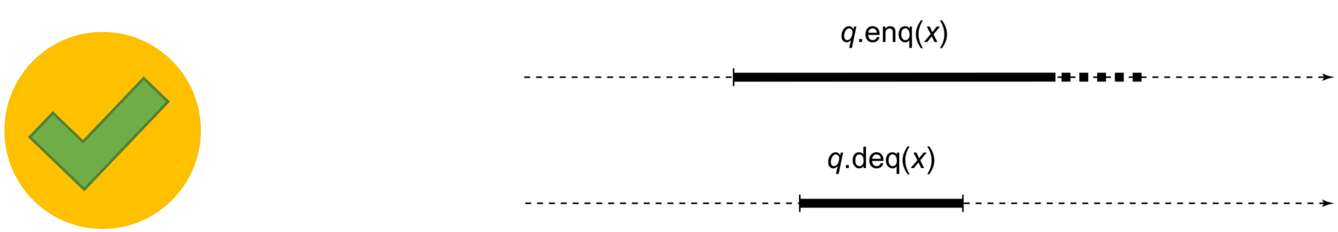
- [P1] Equivalent to some legal sequential history S
  (informally same as [P1] for SC)
- [P2] $<_H \subseteq <_S$
  (informally, real-time precedence ordering should be preserved)

- Note: equivalence between complete(H’) and S
  - Read text/paper for formalisms

Linearizability

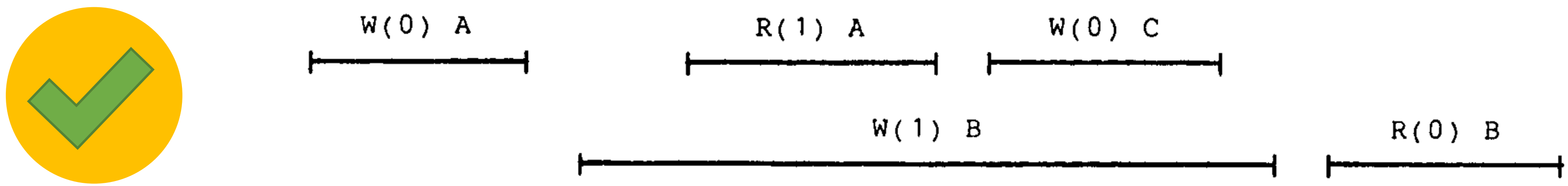


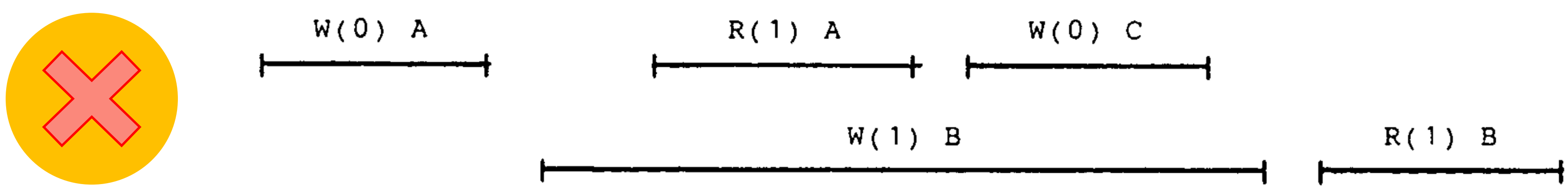


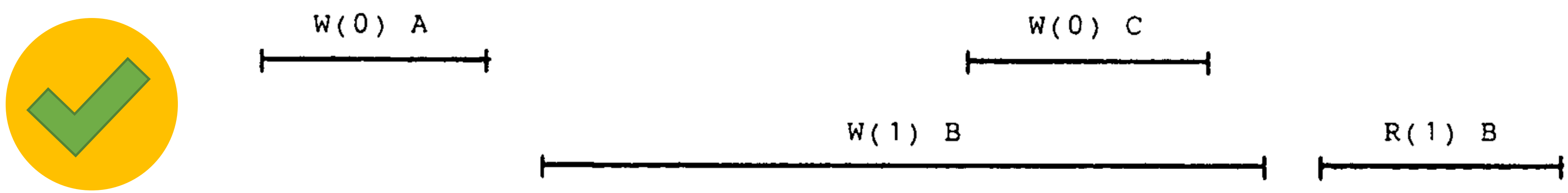


Linearizability

- Read-Write variable







Linearization Points

- Linearization points are points where method takes effect
- Useful to verify and design concurrent solutions

- For lock-based implementations, the critical section is its linearization point
- For implementations without any lock, linearization point is typically a single step where effects of method call become available to other method calls

Linearization Points

- For implementations without any lock, linearization point is typically a single step where effects of method call become available to other method calls
- enq(): if full, when exception gets thrown
  otherwise, when tail gets updated
- deq(): if empty, when exception gets thrown
  otherwise, when head is updated

```
public void enq(T x) throws FullException {
  if (tail - head == items.length)
    throw new FullException();
  items[tail % items.length] = x;
  tail++;
}
```

```
public T deq() throws EmptyException {
  if (tail - head == 0)
    throw new EmptyException();
  T x = items[head % items.length];
  head++;
  return x;
}
```

Linearizability

- Every linearizable execution is sequentially consistent, but not vice versa
- Linearizability is composable
  - Formally, H is linearizable if, and only if, for each object x, H|x is linearizable
  - where H|x means sub-history of H containing operations over x
  - Proof in text/paper

Serializability

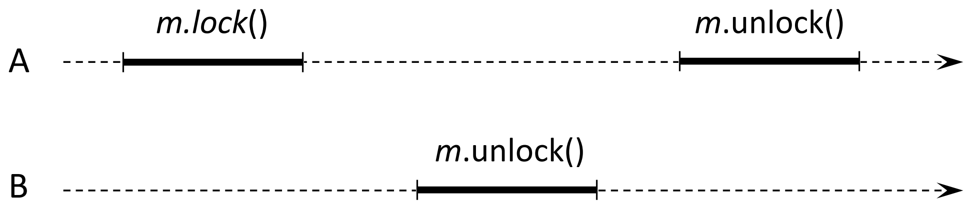
- Often considered in database transactions
- History is serializable if it is equivalent to a serial execution

- Similar to Linearizability, except doesn't capture real-time

- Data-centric perspective
  - Preserve semantics of transactions
  - Don't care about program order (or threads)

Serializabiltiy: https://en.wikipedia.org/wiki/Serializability

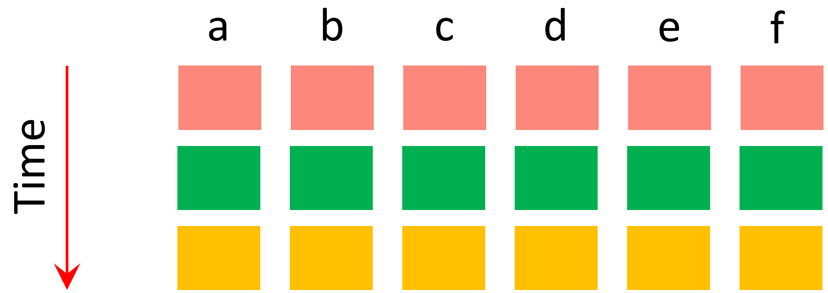
Serializability



- Intuitively incorrect
  - lock is held by both threads at the same time
- Serializable
  - B's m.unlock() can move before A's m.lock()
- Not linearizable – Why?
- Typically:
  - Linearizability useful for components like data structures
  - Serializability useful for complex components like transactions

Iterative Processing

- Values are computed repeatedly in a loop

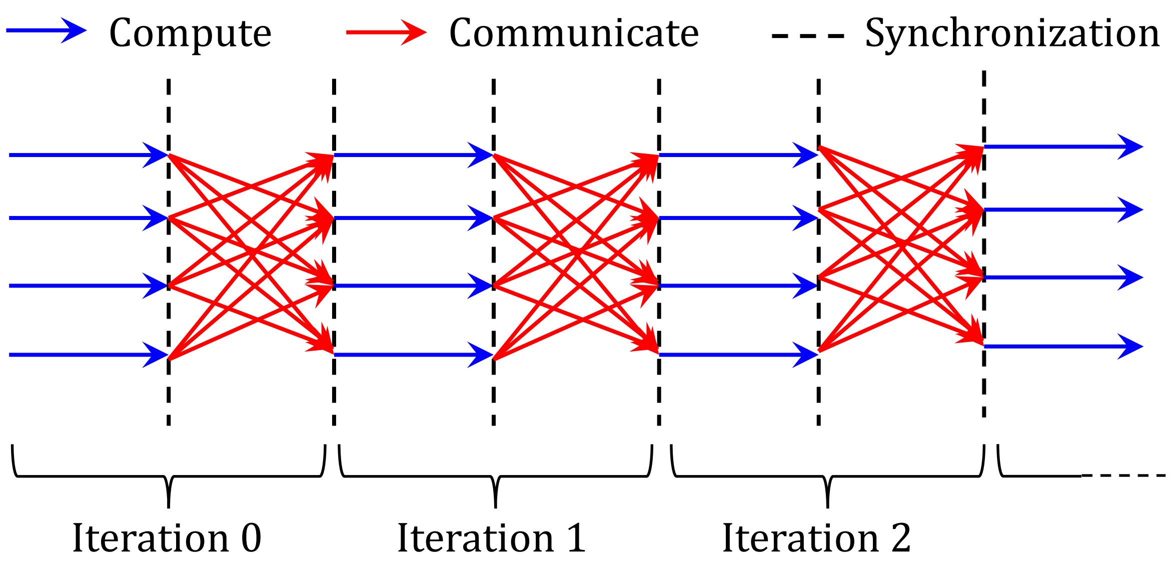


- Example:
  - $\text{pagerank[NEXT][v]} = 0.15 + 0.85 \times \sum_{\{u \in in(v)\}} \frac{\text{pagerank[CURR][u]}}{\text{degree(u)}}$
  - $\text{sssp[v]} = \min_{\{u \in in(v)\}}(\text{sssp[u]})$

- Processing semantics defined by algorithm's needs
- Dependencies defined based on value flows

Bulk-Synchronous Parallel Semantics

- Stems from Bulk Synchronous Parallel Model
  - Disjoint computation and communication phases separated by barriers

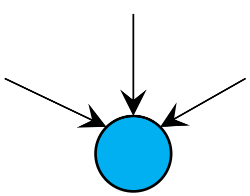


Valiant et al., A Bridging Model for Parallel Computation, CACM 1990.

Bulk-Synchronous Parallel Semantics

- Stems from Bulk Synchronous Parallel Model
  - Disjoint computation and communication phases separated by barriers
- Need to satisfy read-write dependencies
  - Values in iteration i are computed based on values from iteration i-1 only

- PageRank example:
  - pagerank[NEXT][v] = 0.15 + 0.85 × $\sum_{\{u \in in(v)\}} \frac{pagerank[CURR][u]}{degree(u)}$



$$\boxed{\forall (u, v) \in E, \;\; u^t \mapsto v^{t+1}}$$

Asynchronous Execution

- Values don't necessarily progress at same rate
- Recall SSSP:
  - $sssp[v] = min_{\{u \in in(v)\}}\ sssp[u]$
- Computation is often data-driven
- Correctness is typically coupled with algorithmic semantics
  - E.g., latest update must be eventually read
- Progress & Termination semantics
  - Livelock and deadlock conditions need to be verified
  - Termination conditions need to be defined

Progress Conditions

- Deal with performance instead of correctness
- Progress of a system/thread towards completion
- Blocking Implementations
  - Delays by one thread can prevent others from making progress
  - E.g., Lock-based queue example

- Non-Blocking Implementations
  - Delays by one thread do not prevent others from making progress
  - E.g., Single-enqueuer/single-dequeuer queue example

Non-Blocking Guarantee

- Wait-Freedom
  - Every operation finishes in finite number of steps
  - E.g., Single-enqueuer/single-dequeuer queue example
  - Strongest non-blocking guarantee, combining guaranteed system-wide throughput and starvation-freedom
  - Wait-free algorithms can be inefficient

- Lock-Freedom
  - Some operation finishes in finite number of steps
  - Allows individual threads to starve, but guarantees system-wide throughput
  - Useful especially when starvation is unlikely

Non-blocking algorithm: https://en.wikipedia.org/wiki/Non-blocking_algorithm

Reading (for Next Class)

R

- Introduction to Parallel Computing (Second Edition) by Ananth Grama, Anshul Gupta, George Karypis, Vipin Kumar
  - Chapter 3: Principles of Parallel Algorithm Design
  - Online Access: https://sfu-primo.hosted.exlibrisgroup.com/permalink/f/usv8m3/01SFUL_ALMA51188913690003611