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# CMPT 431 Distributed Systems

Fall 2019

## Concurrent Data Structures

<https://www.cs.sfu.ca/~keval/teaching/cmpt431/fall19/>

Instructor: Keval Vora

# Reading



- [AMP] Chapter 10 & 11
- [Paper] Simple, Fast, and Practical Non-Blocking and Blocking Concurrent Queue Algorithms:

[https://www.cs.rochester.edu/~scott/papers/1996\\_PODC\\_queues.pdf](https://www.cs.rochester.edu/~scott/papers/1996_PODC_queues.pdf)

Slides based on book material

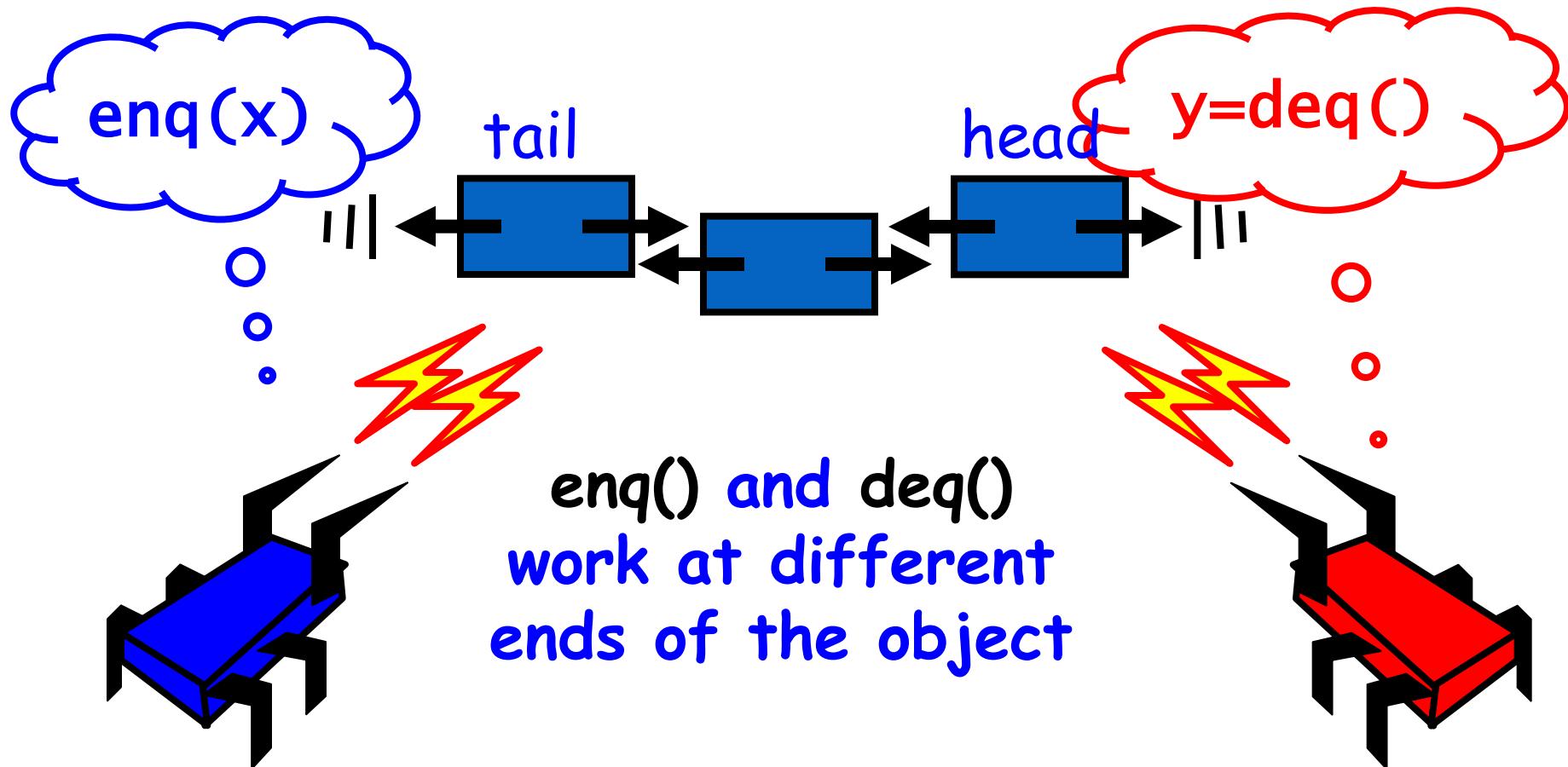
# Queues

- First-in-first-out (FIFO) order
- `enq()` & `deq()`
- Bounded v/s Unbounded
  - Capacity
- Blocking (Partial) v/s Non-Blocking (Total)
  - Removal from empty queue or addition to queue that's full
  - Blocking: caller waits until state changes
  - Non-Blocking: method throws exception

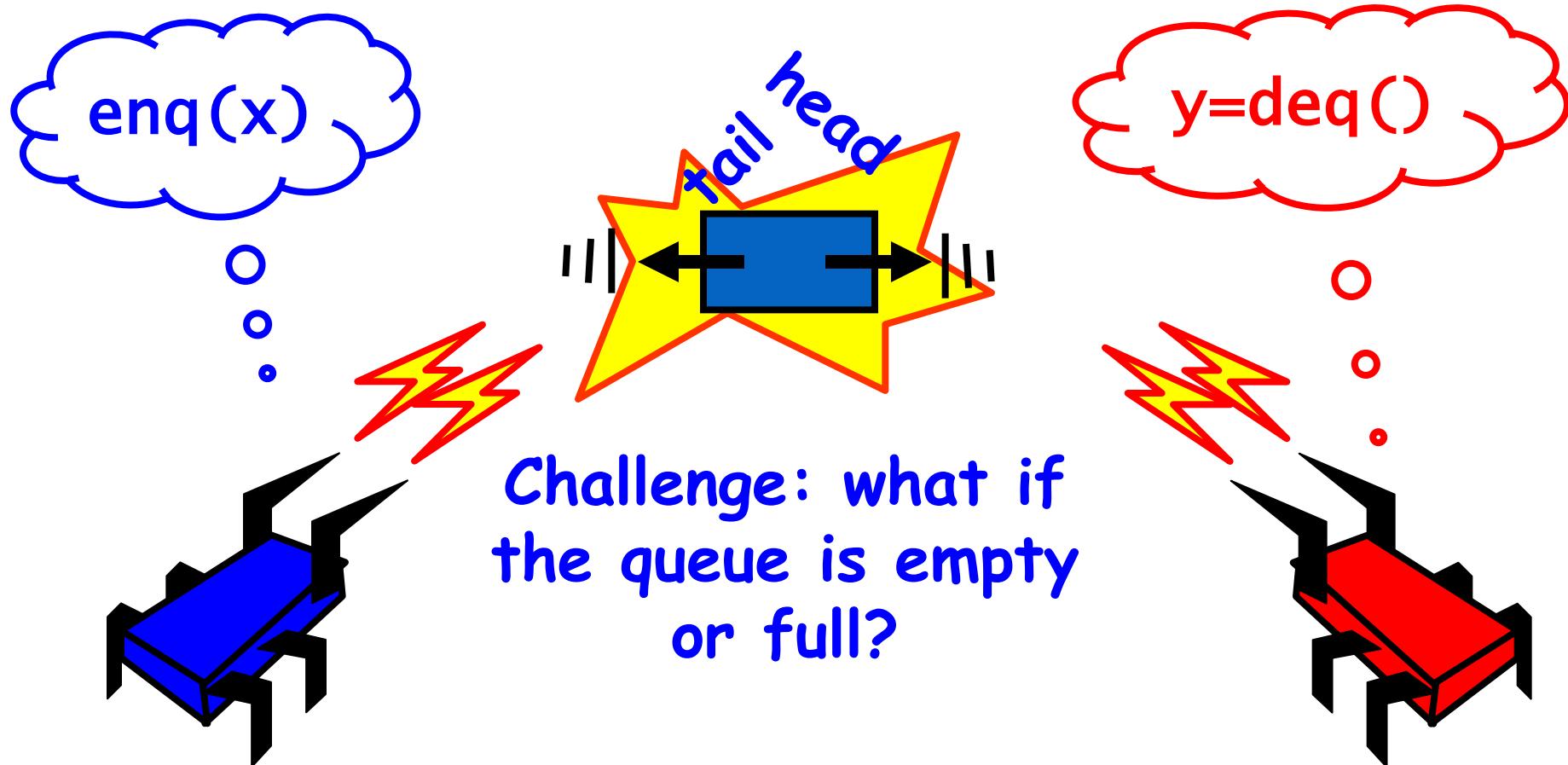
# We will look at ...

- Bounded, Blocking, Lock-based Queue
- Unbounded, Non-Blocking, Lock-free Queue
- ABA problem

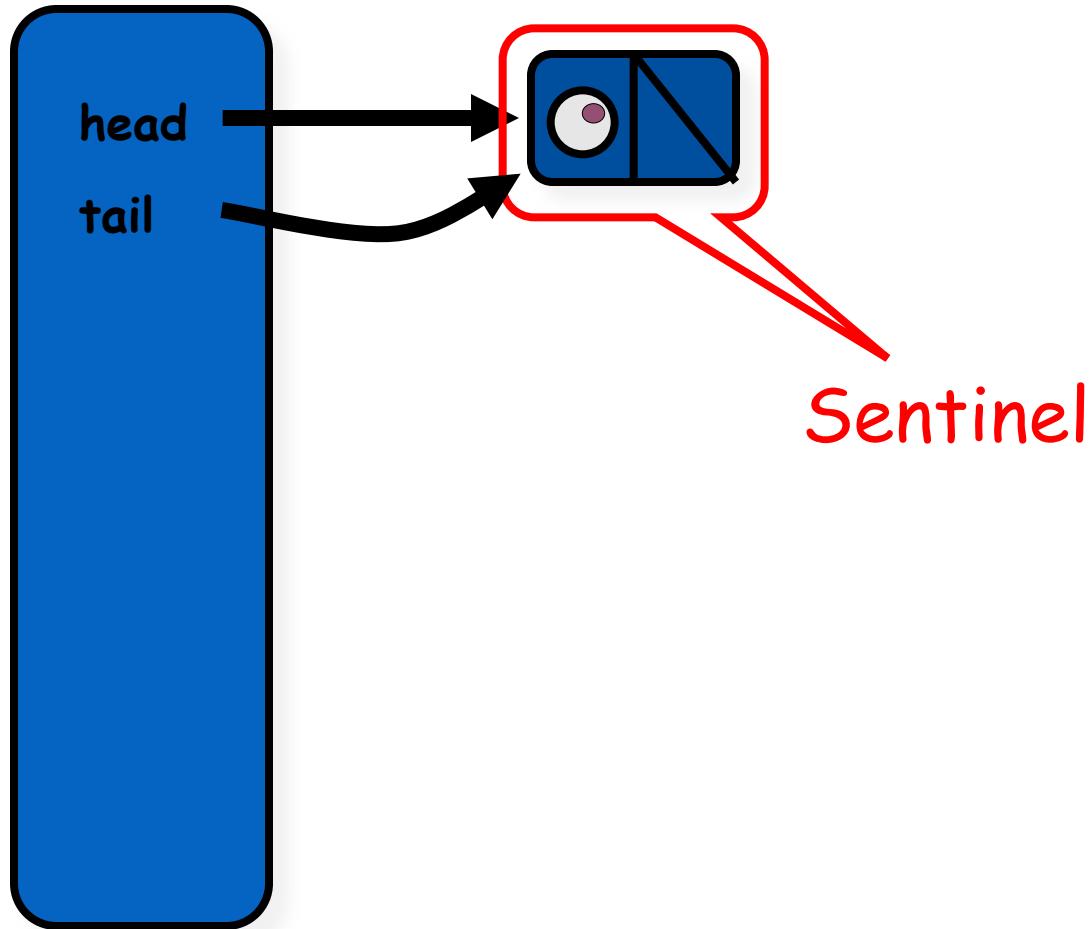
# Queue: Concurrency



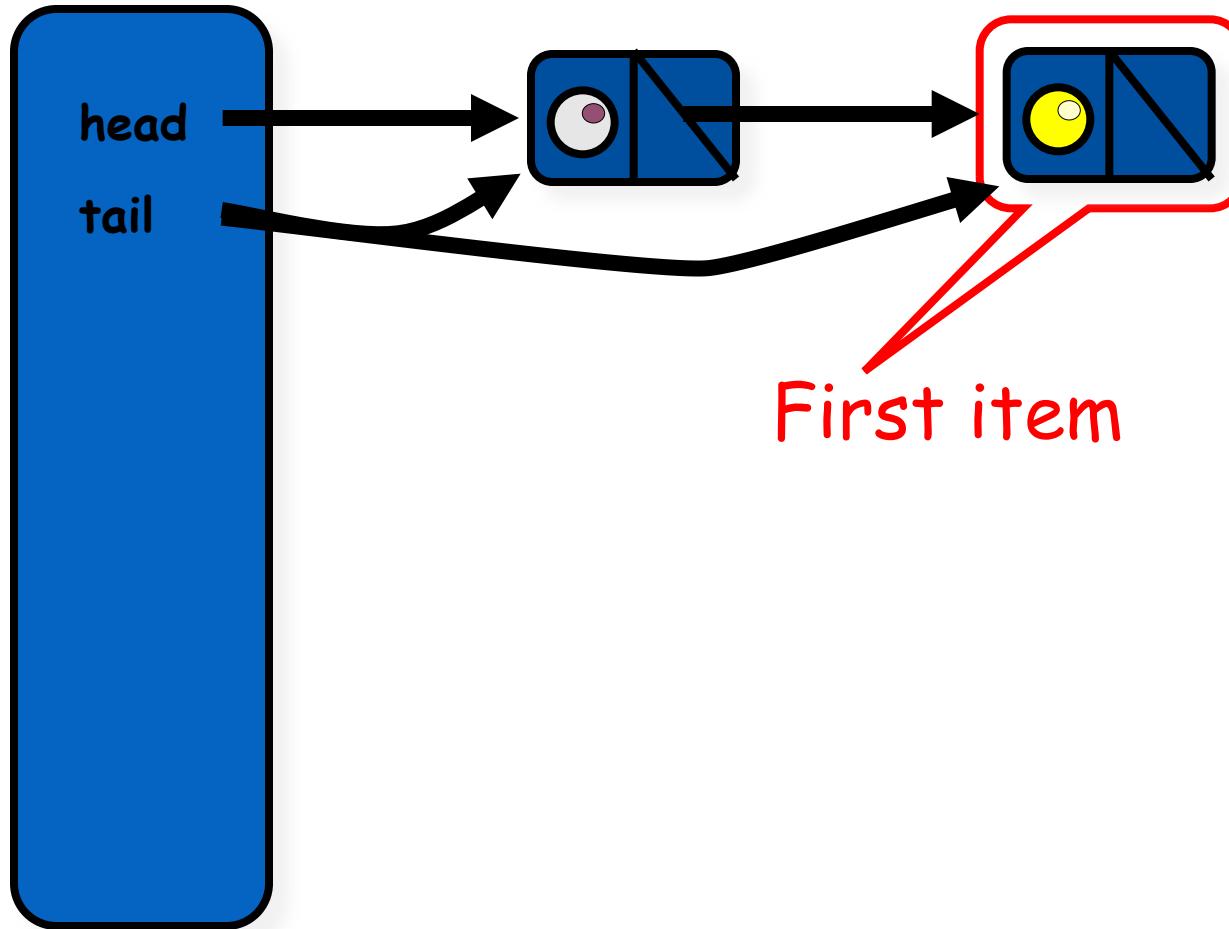
# Concurrency



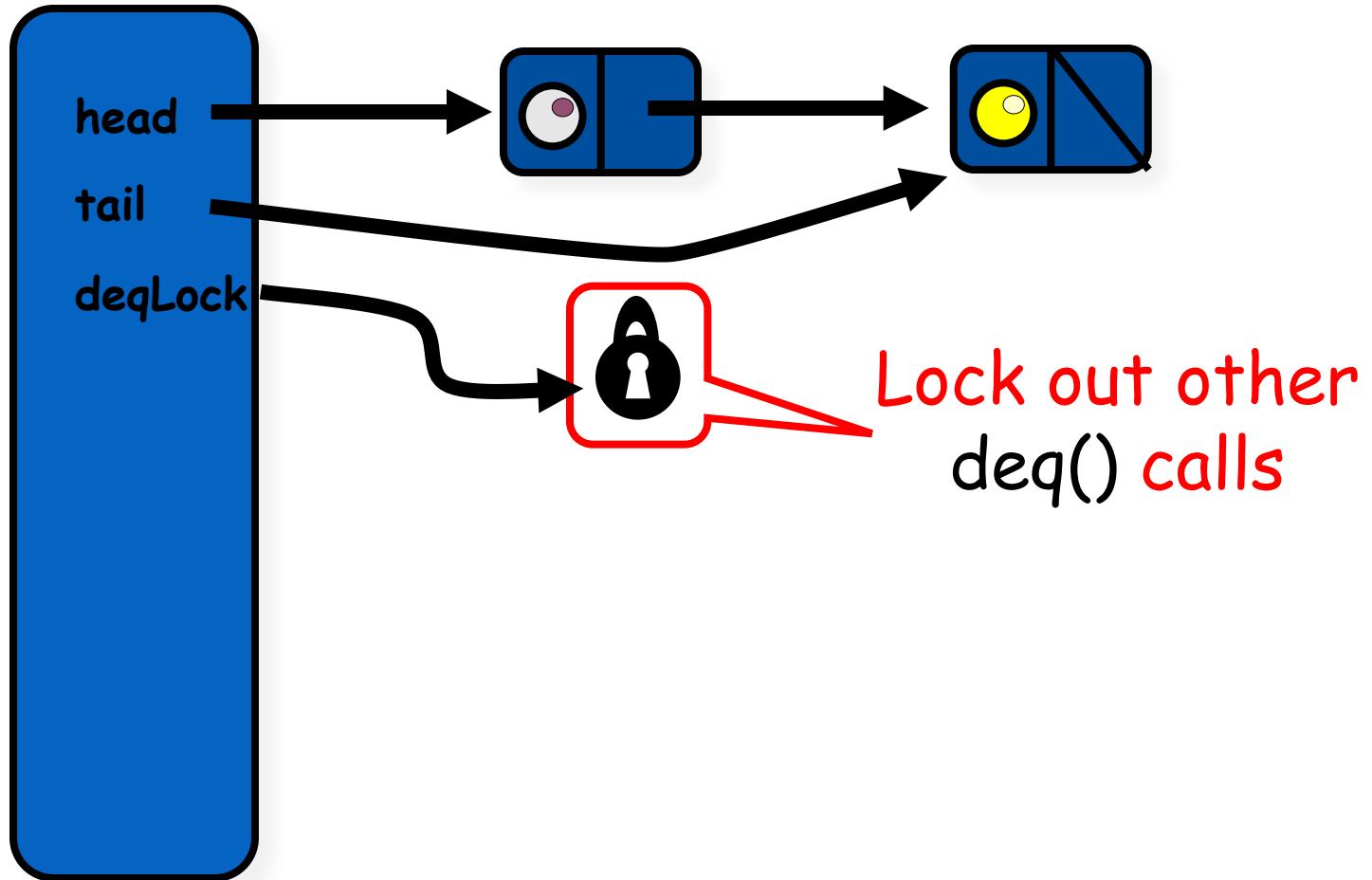
# Bounded Queue



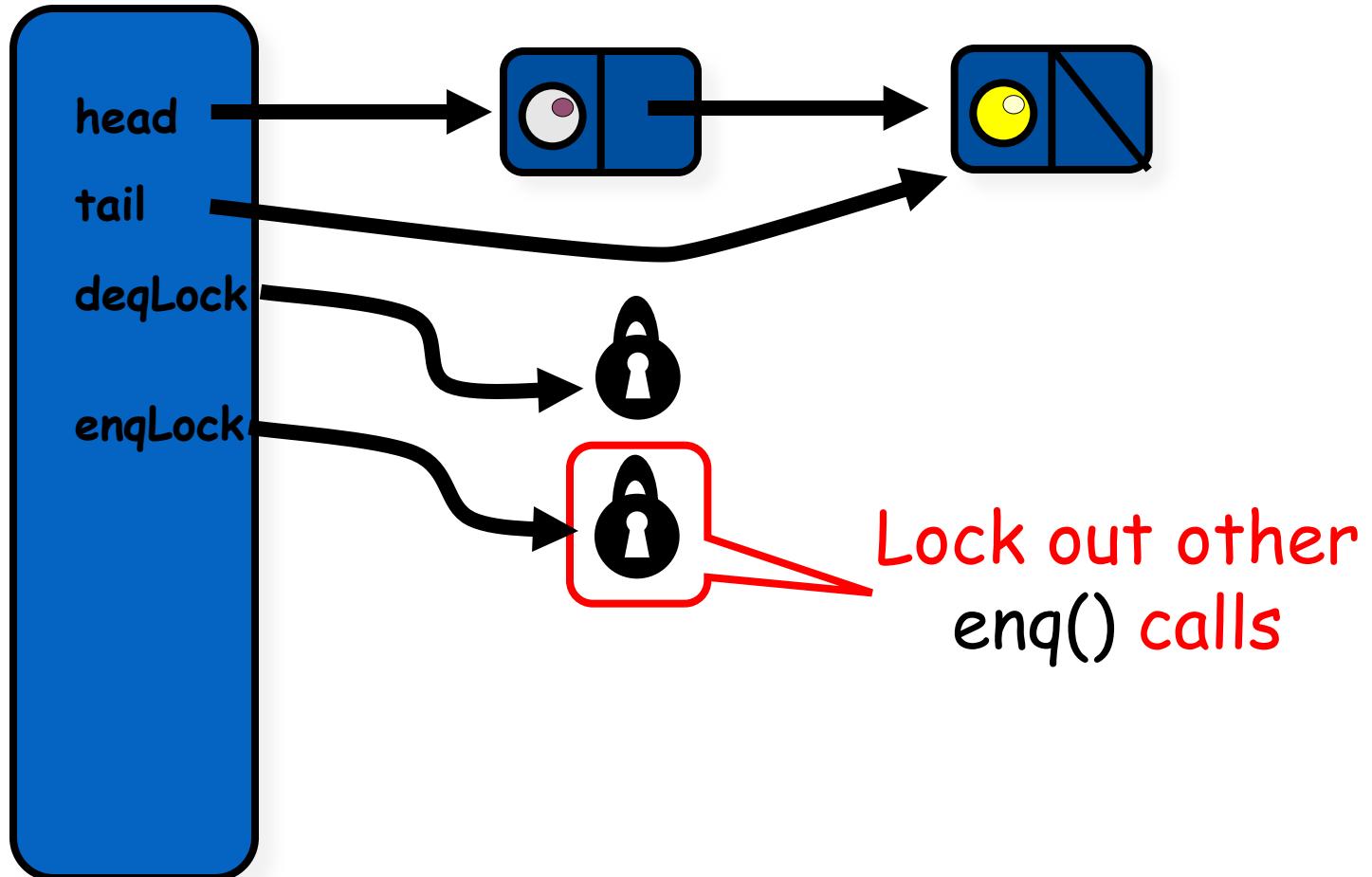
# Bounded Queue



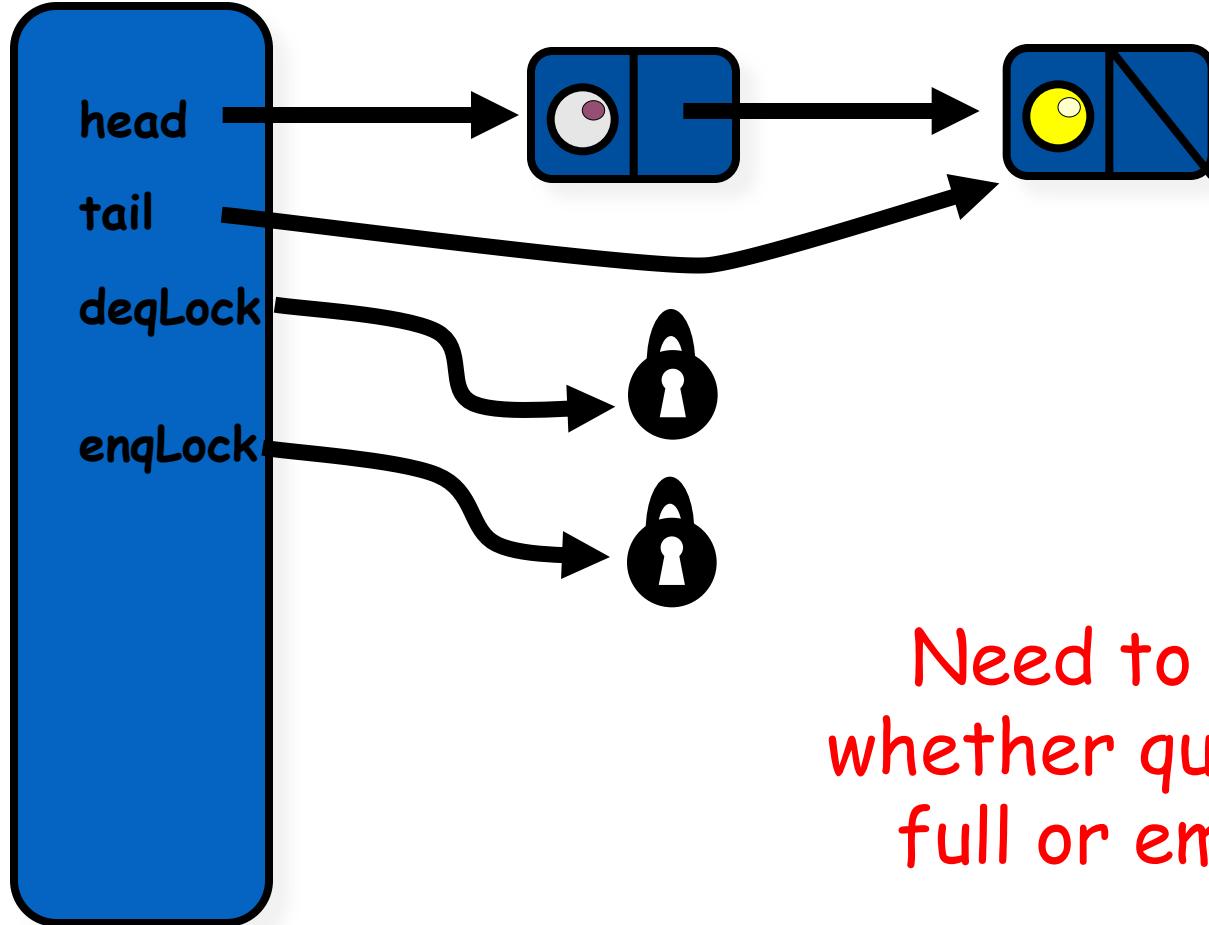
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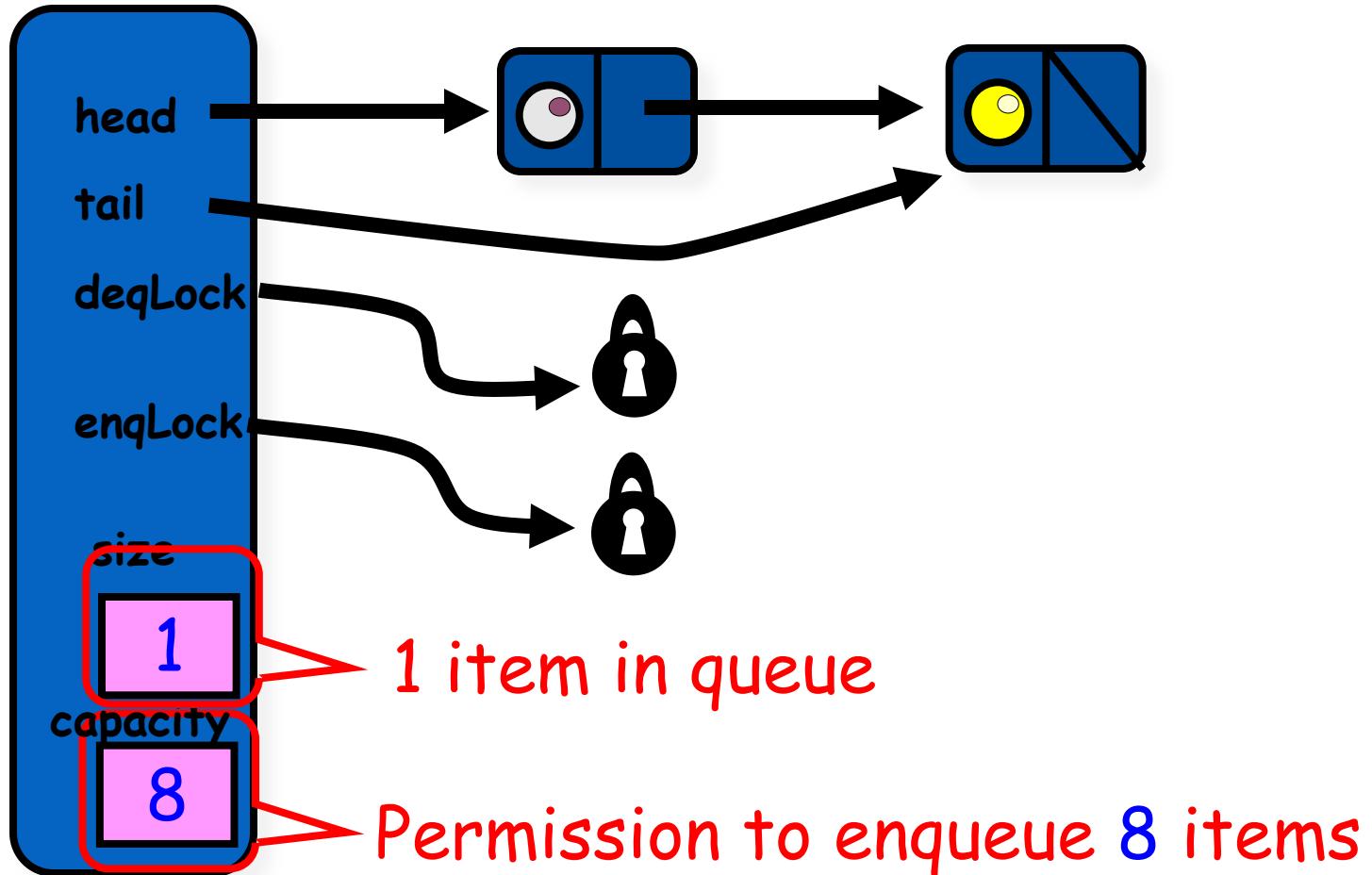
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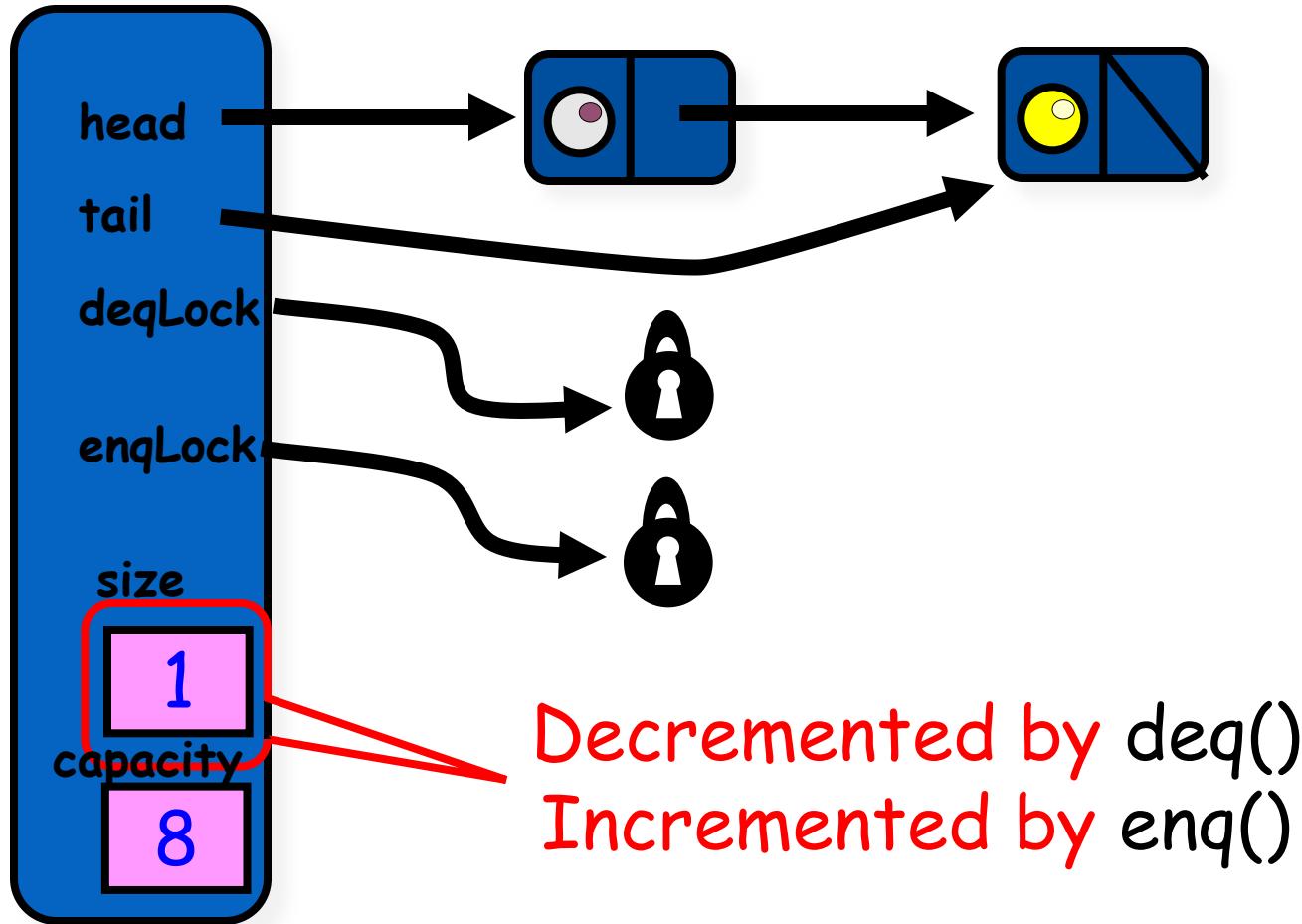
# Bounded Queue



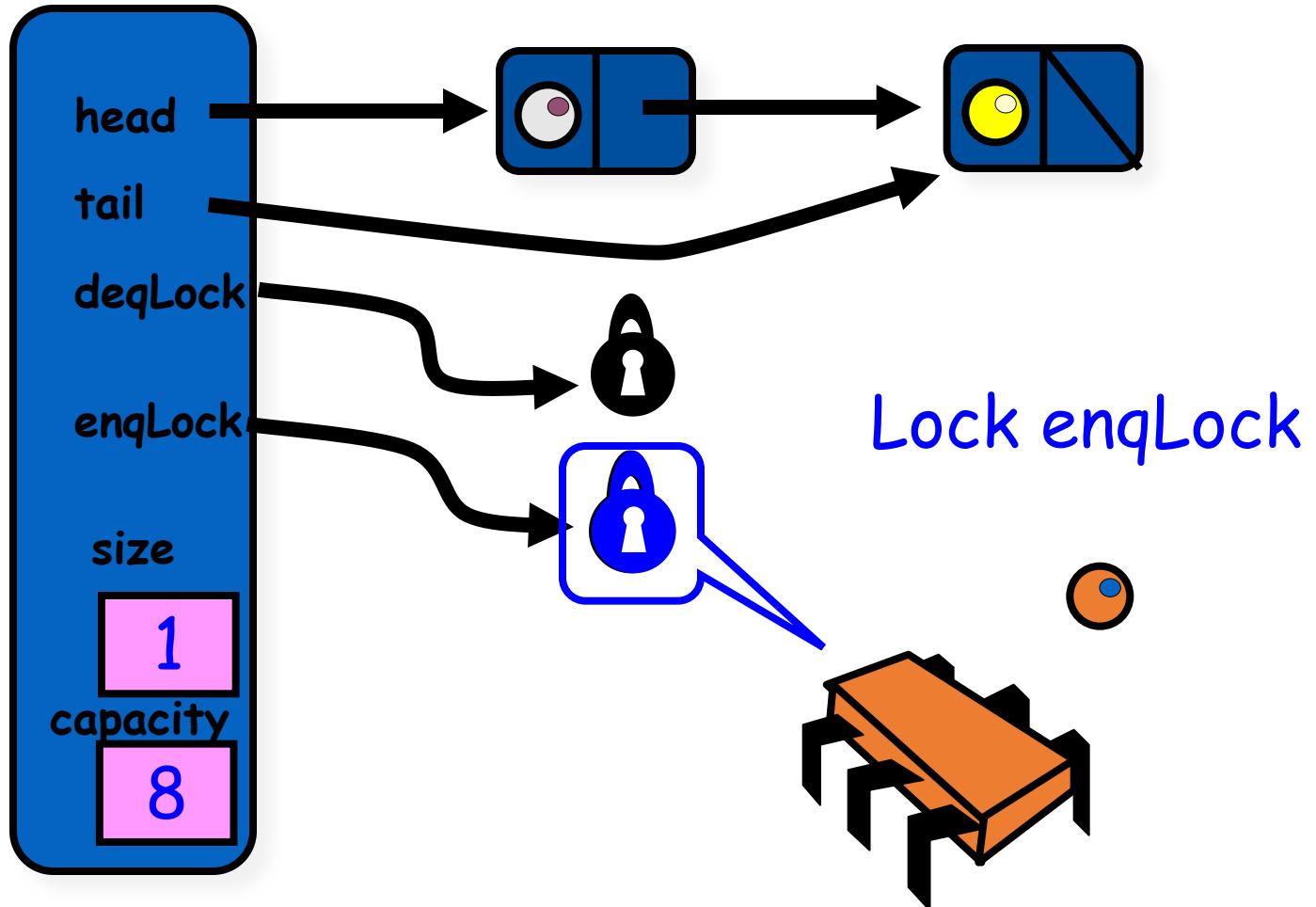
# Bounded Queue



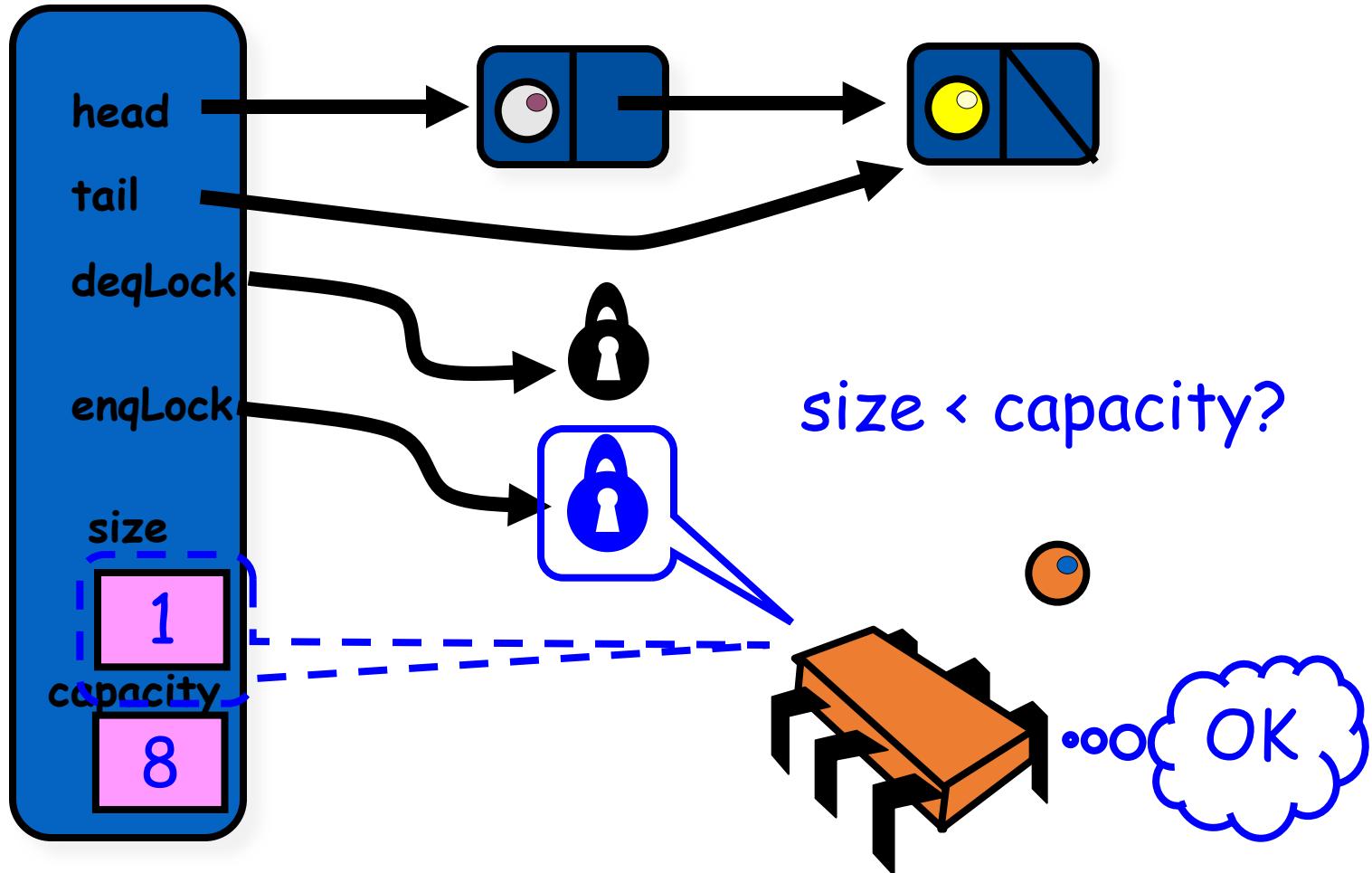
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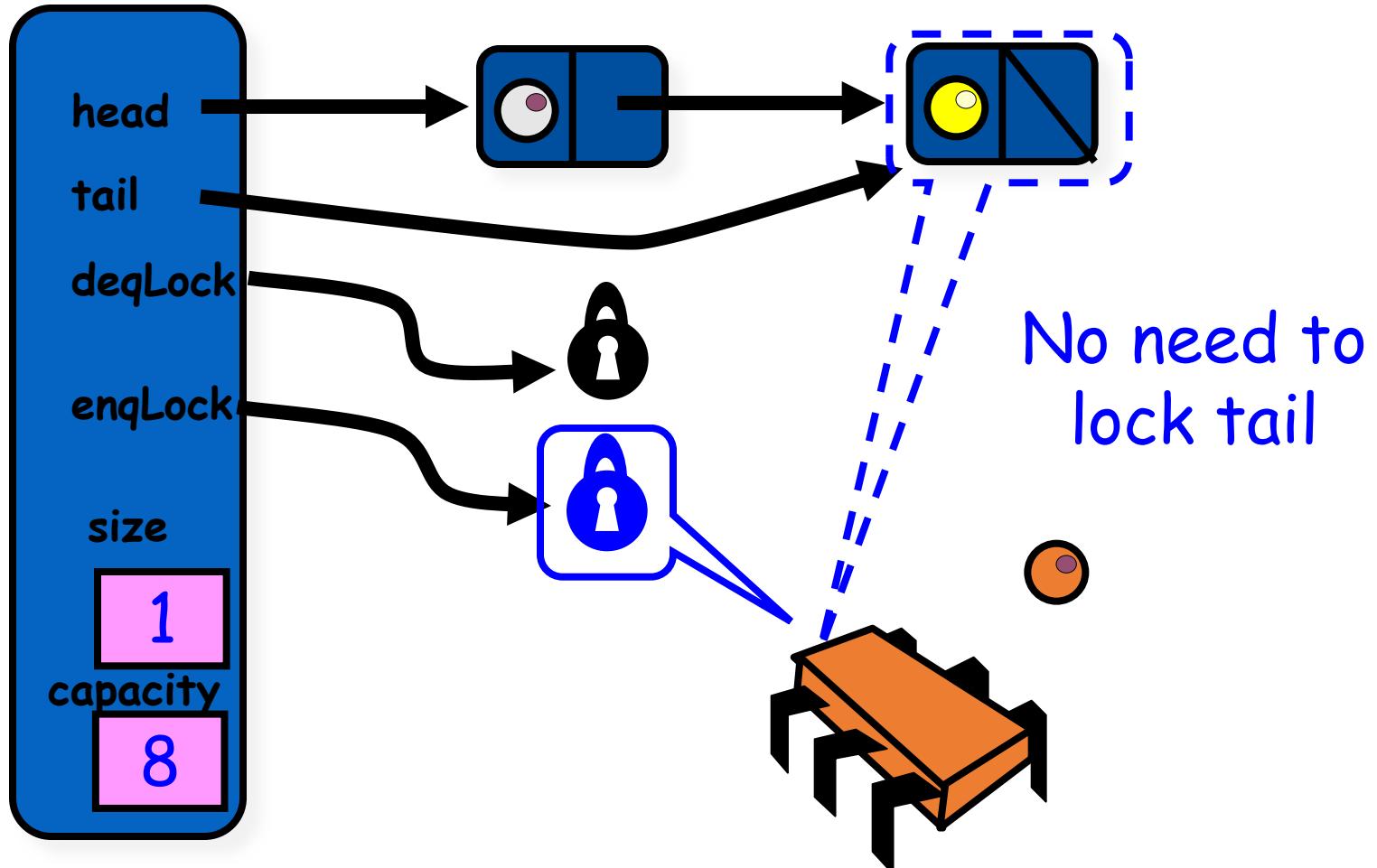
# Bounded Queue: Enqueuer



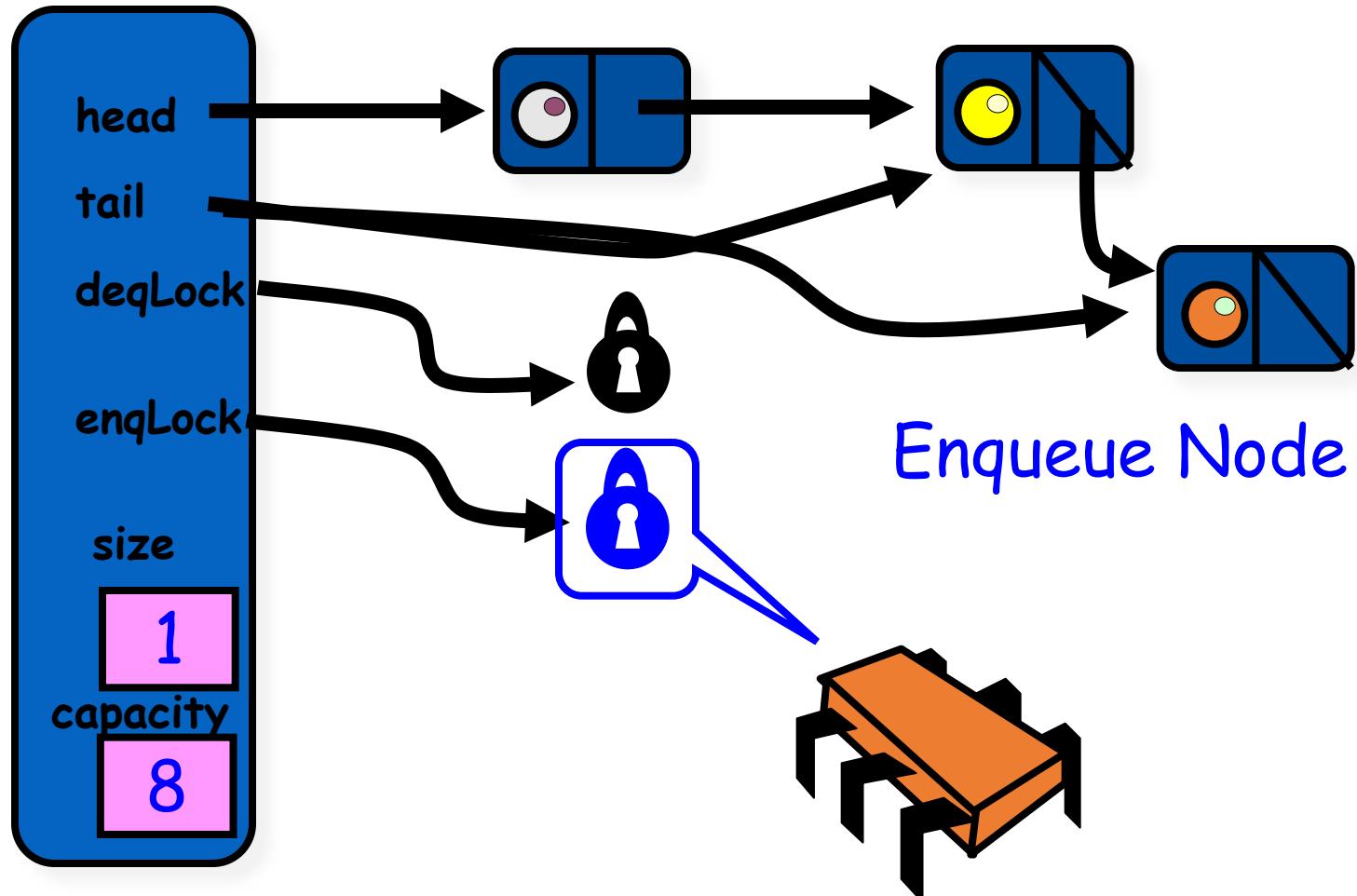
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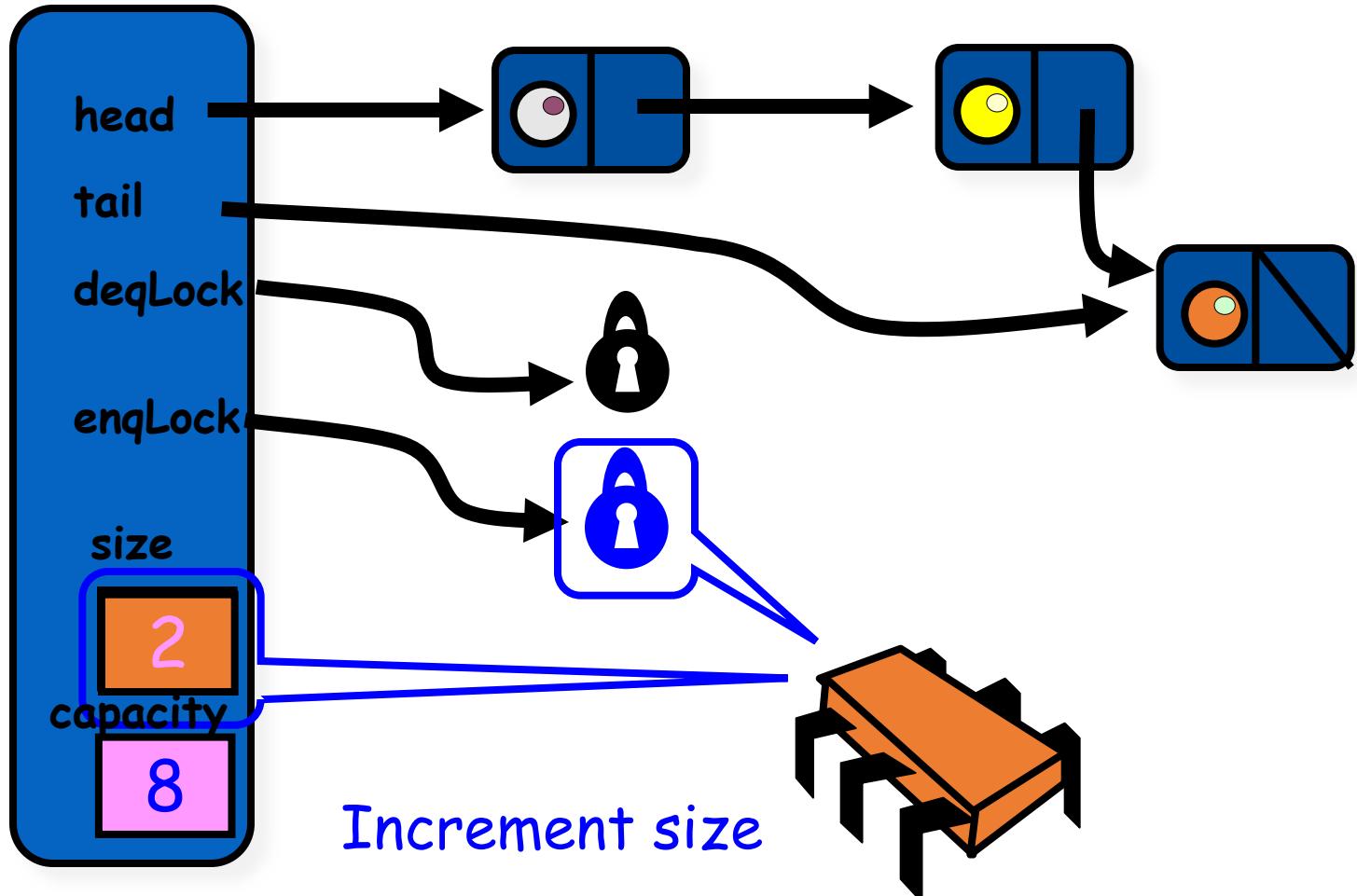
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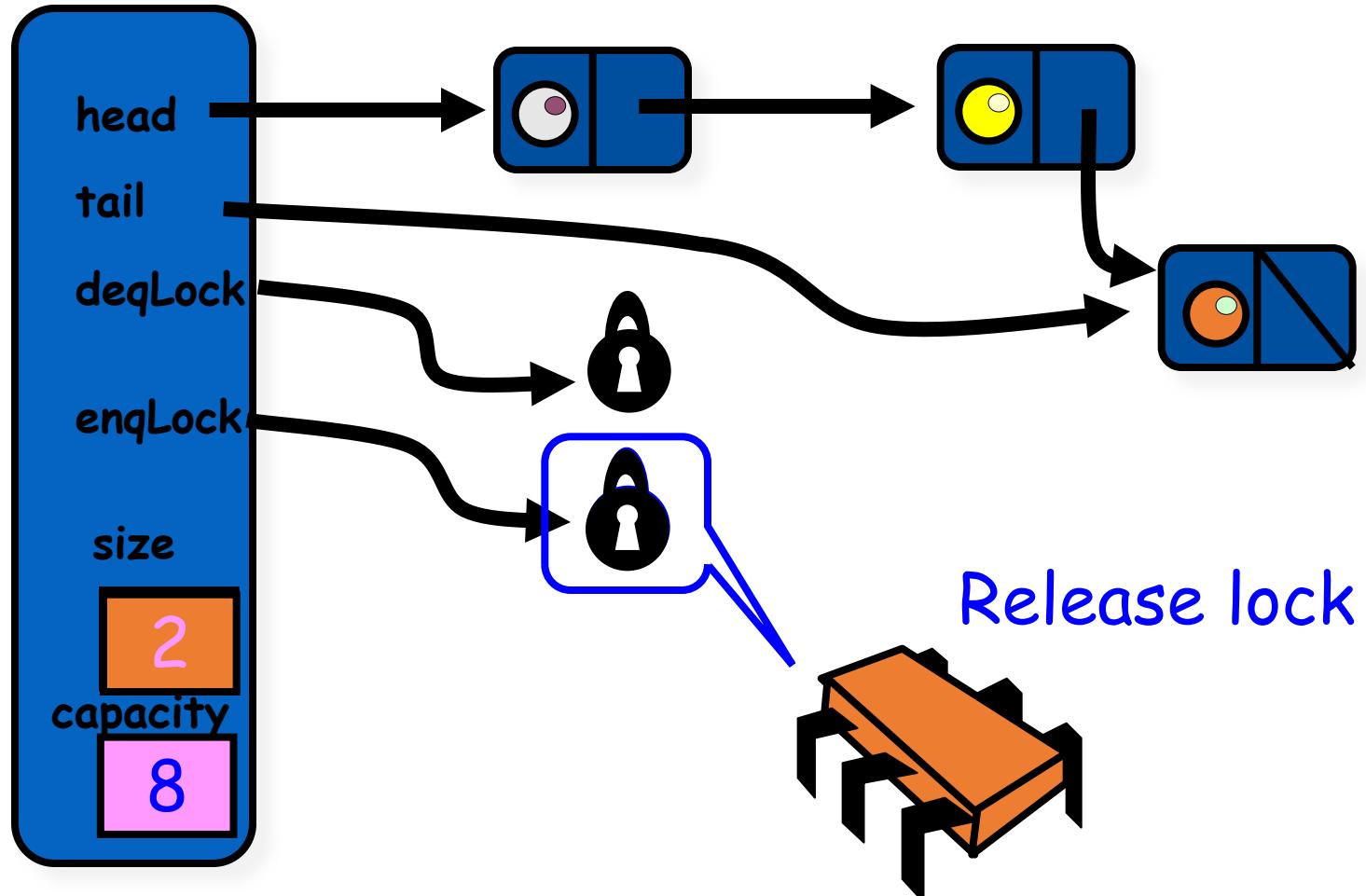
# Bounded Queue: Enqueuer



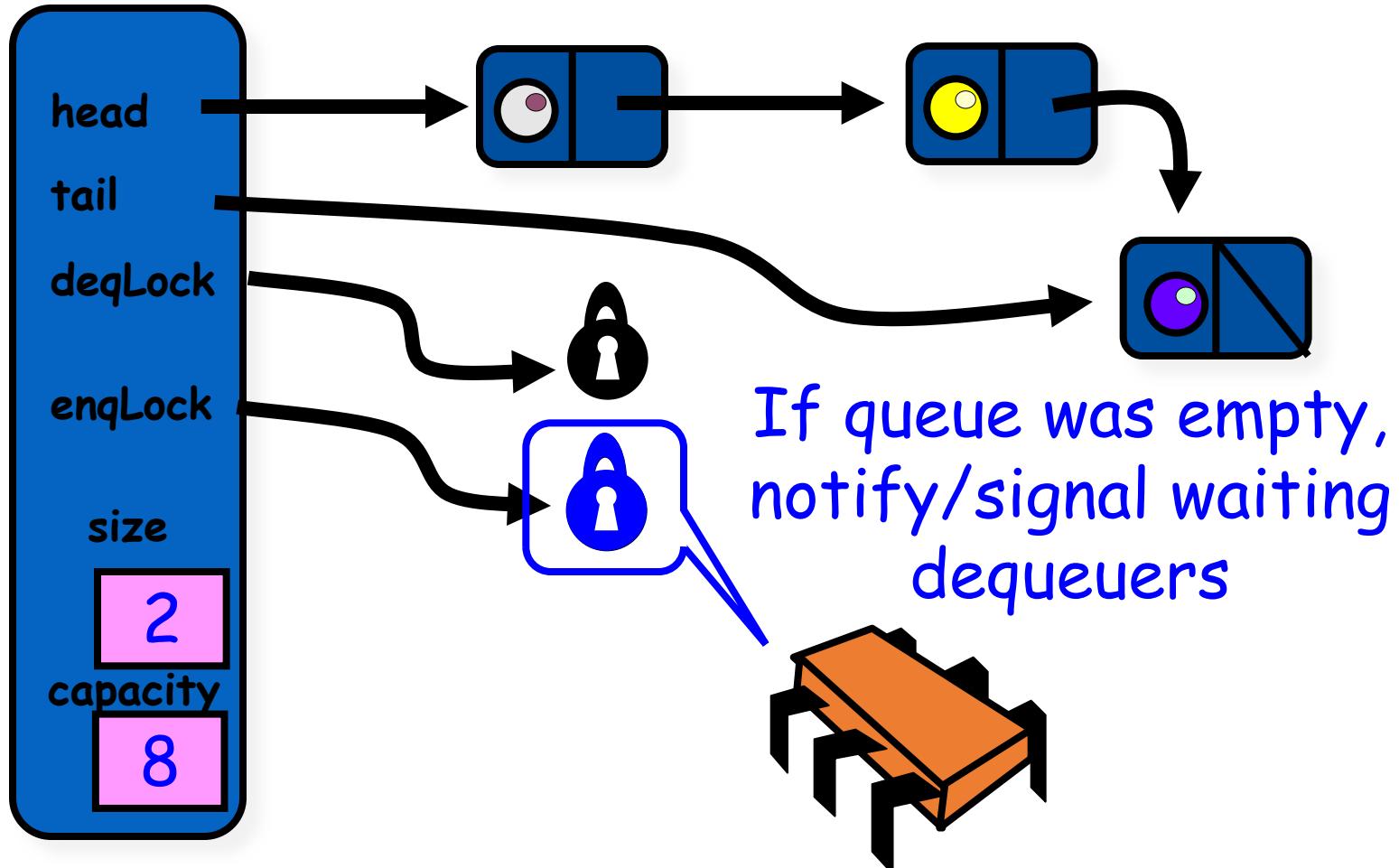
# Bounded Queue: Enqueuer



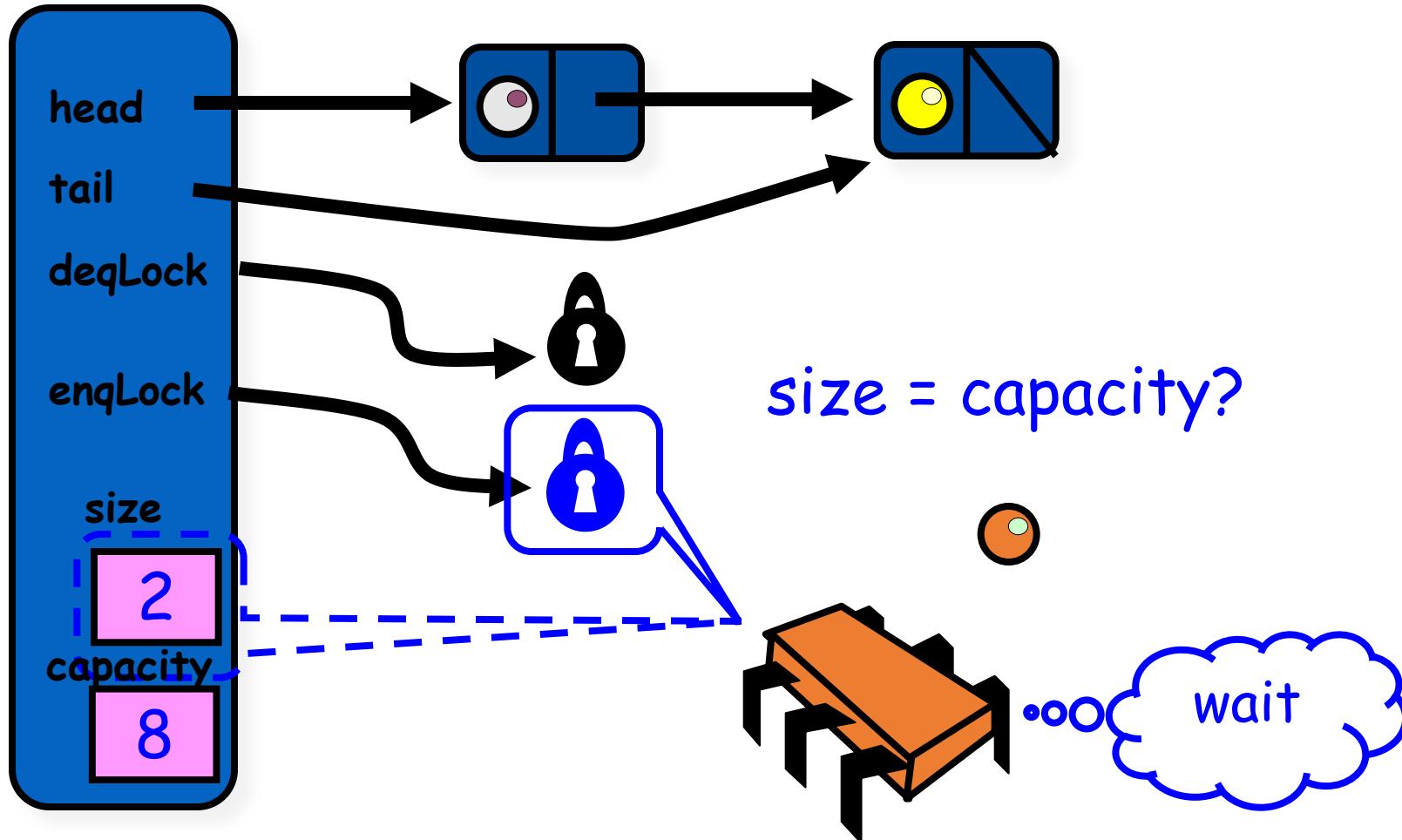
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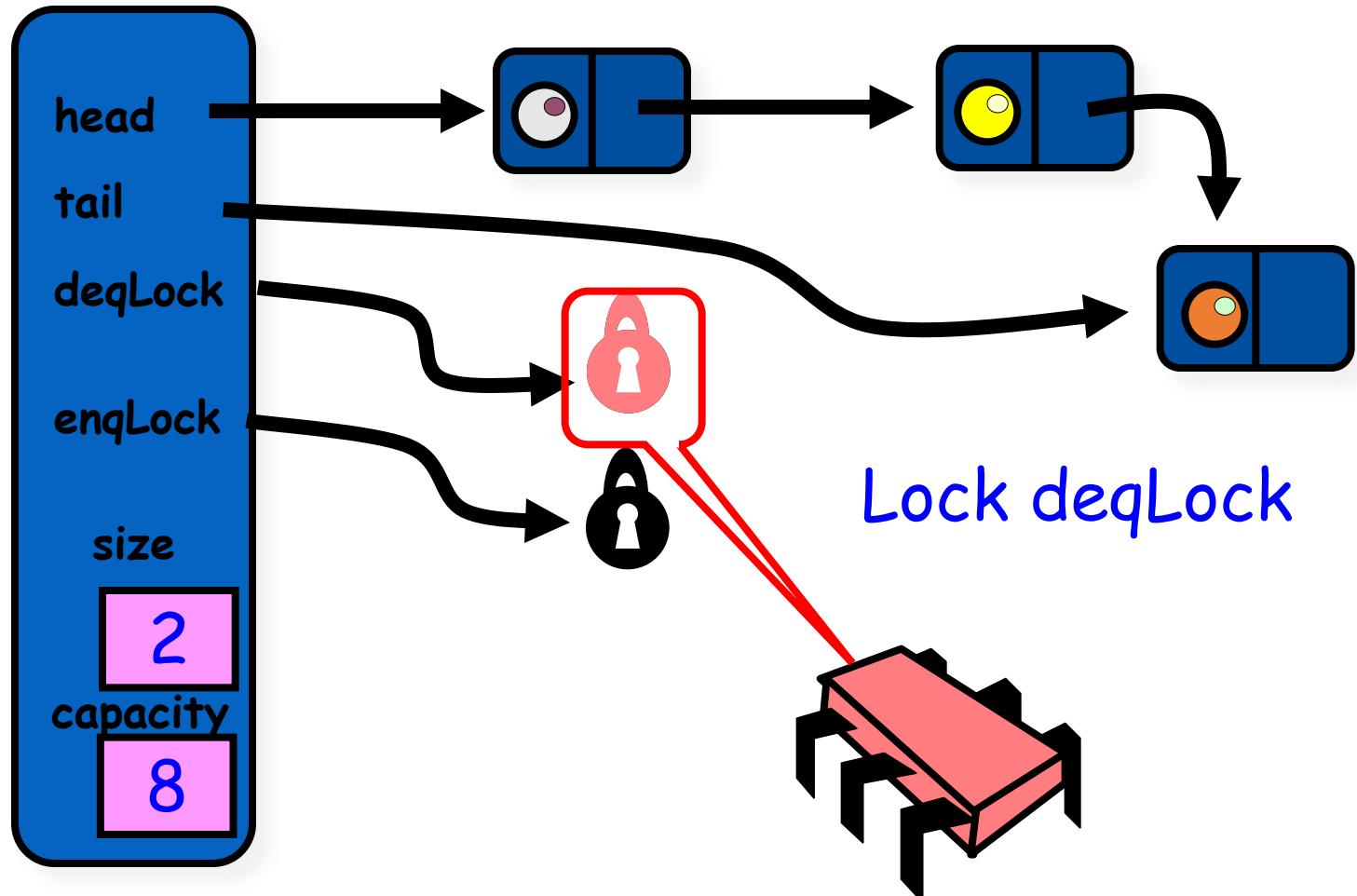
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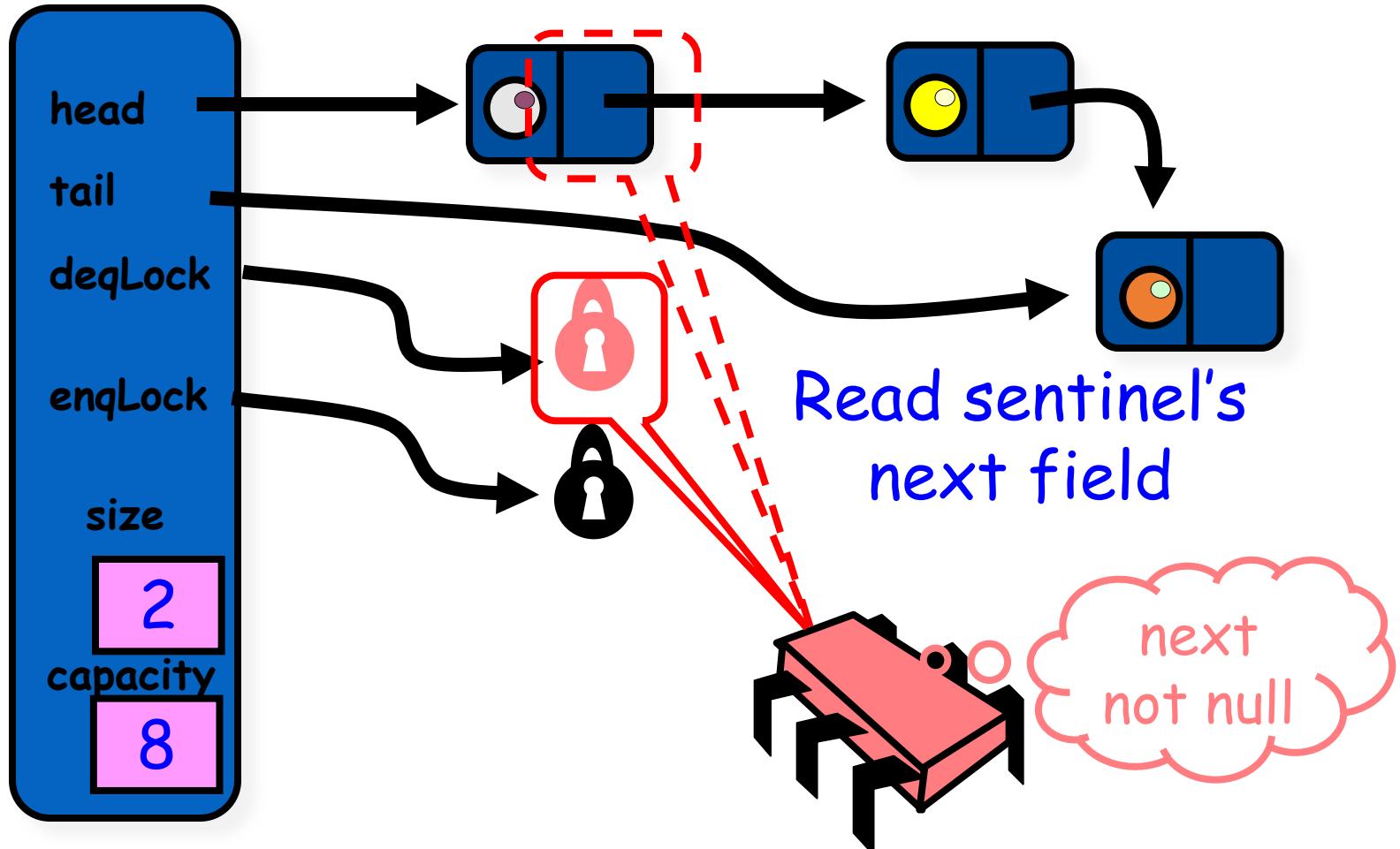
# Bounded Queue: Enqueuer



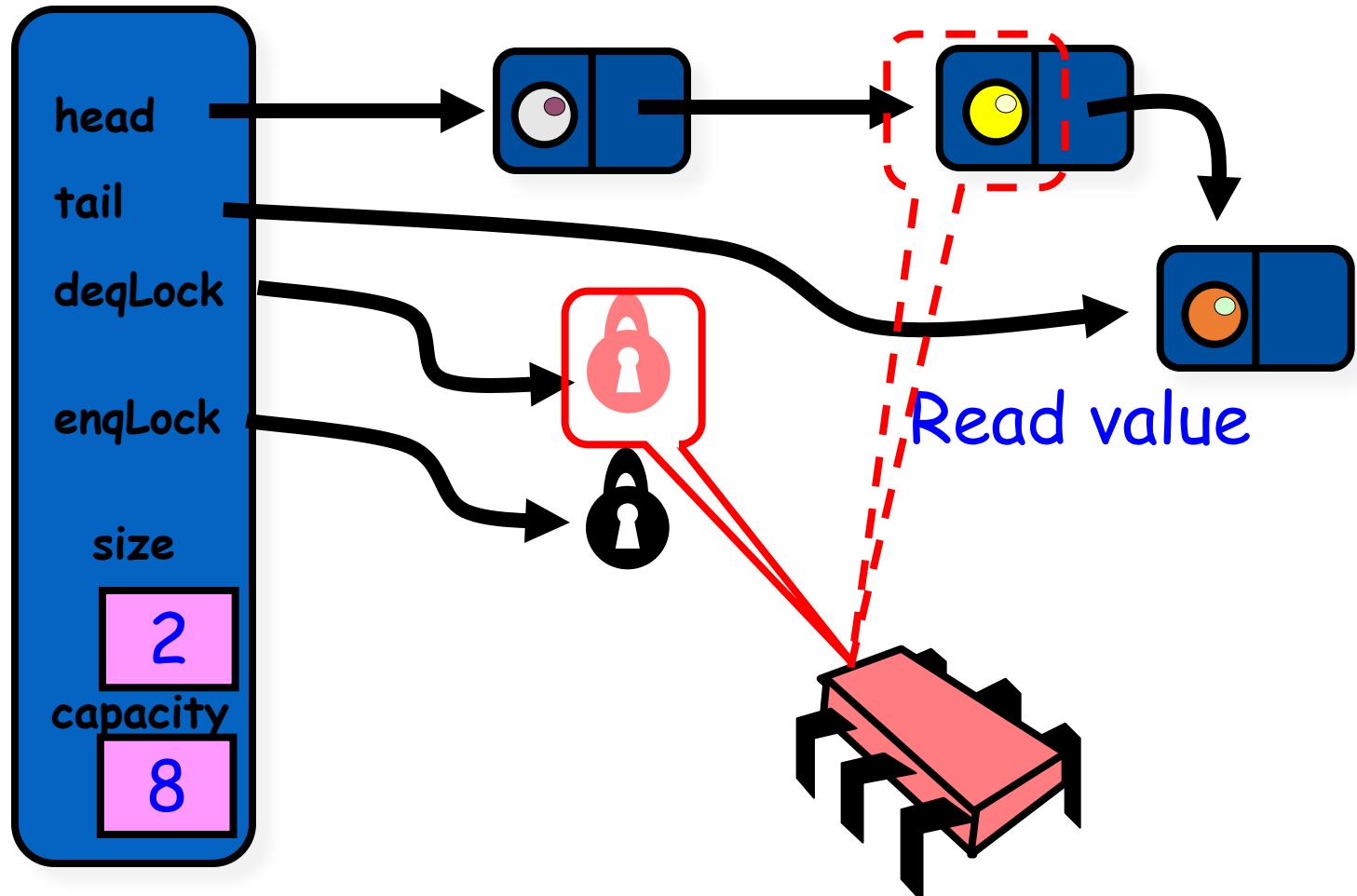
# Bounded Queue: Dequeuer



# Bounded Queue: Dequeuer

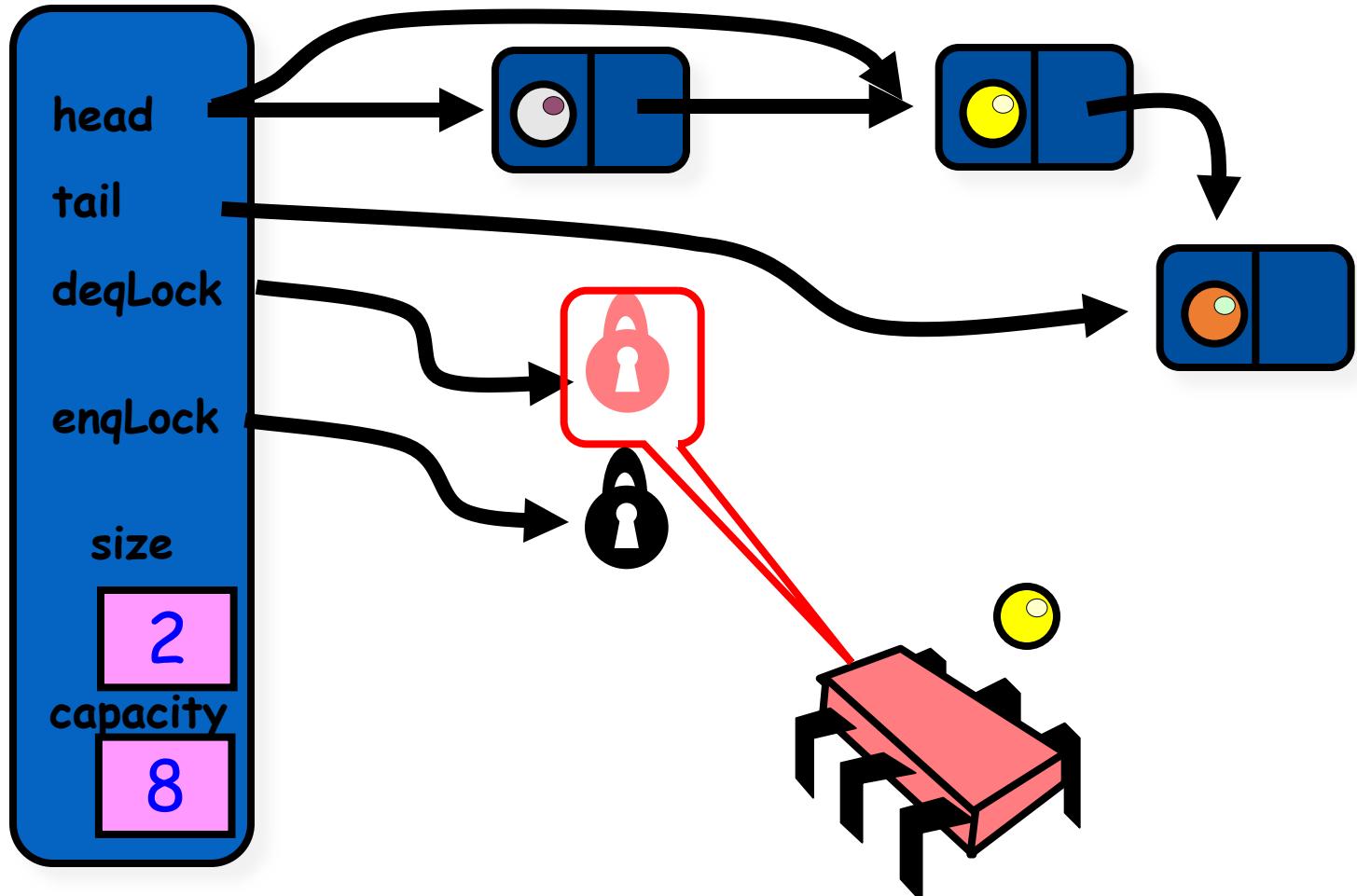


# Bounded Queue: Dequeuer

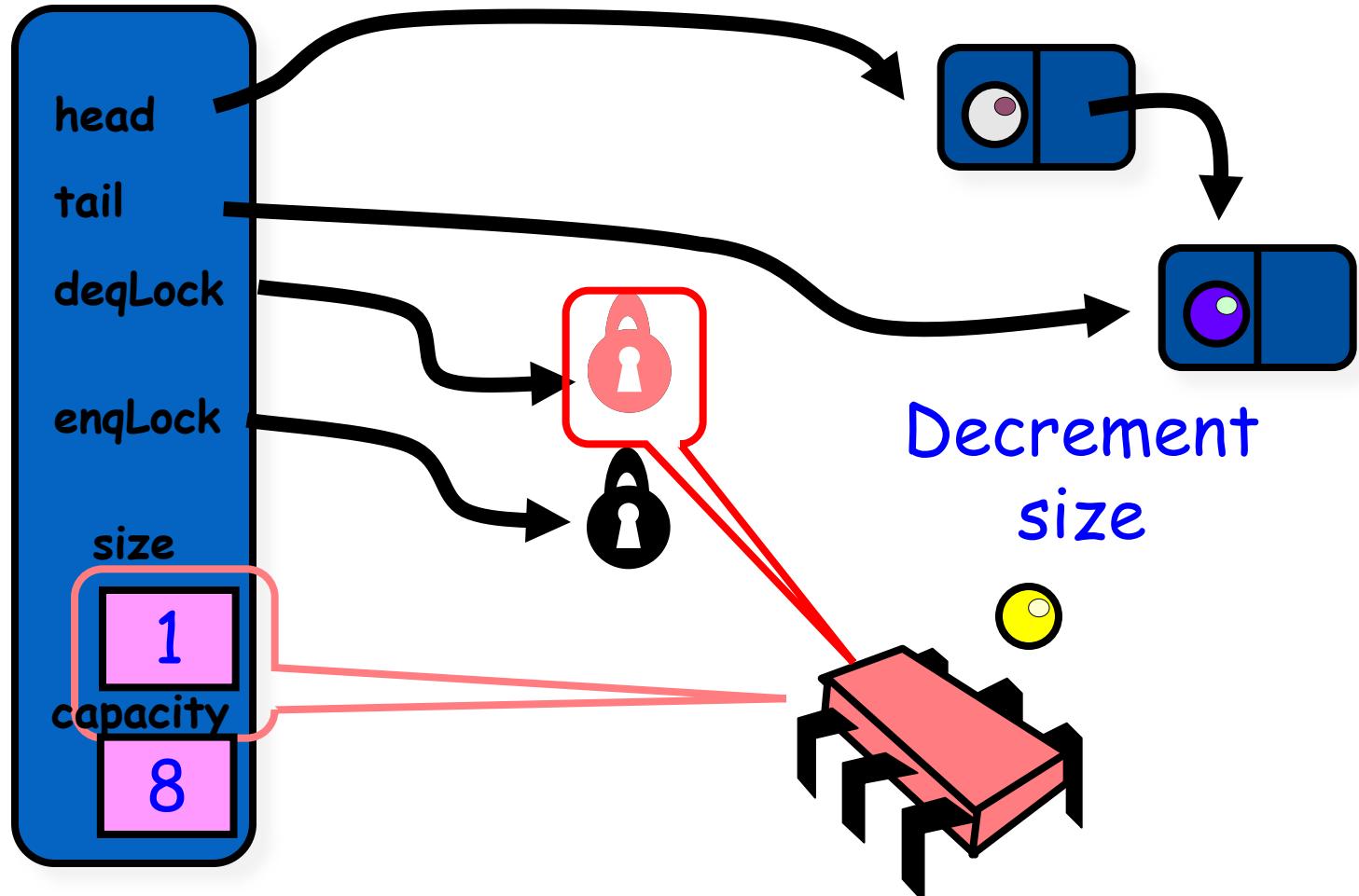


# Bounded Queue: Dequeuer

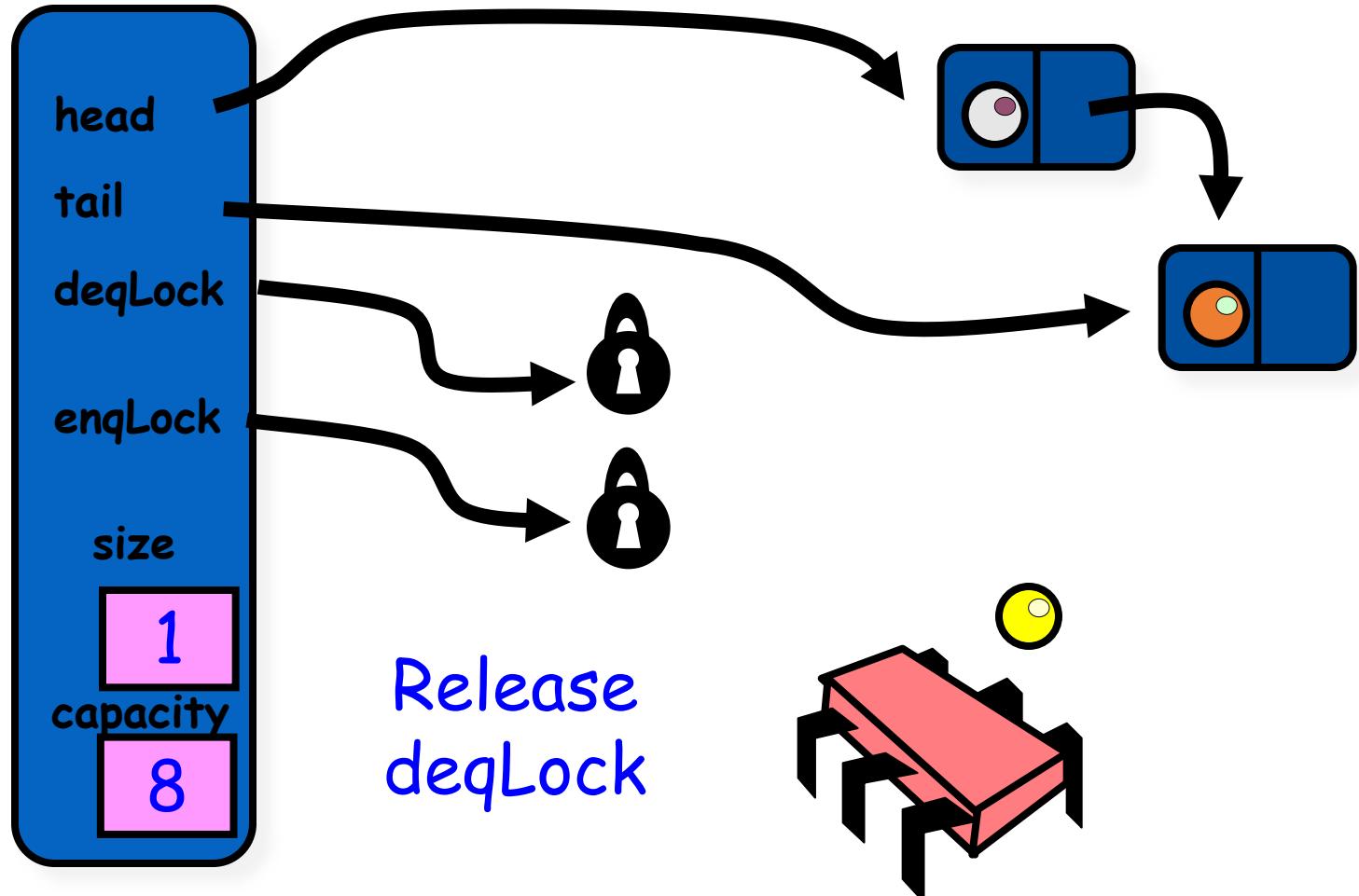
Make first Node  
new sentinel



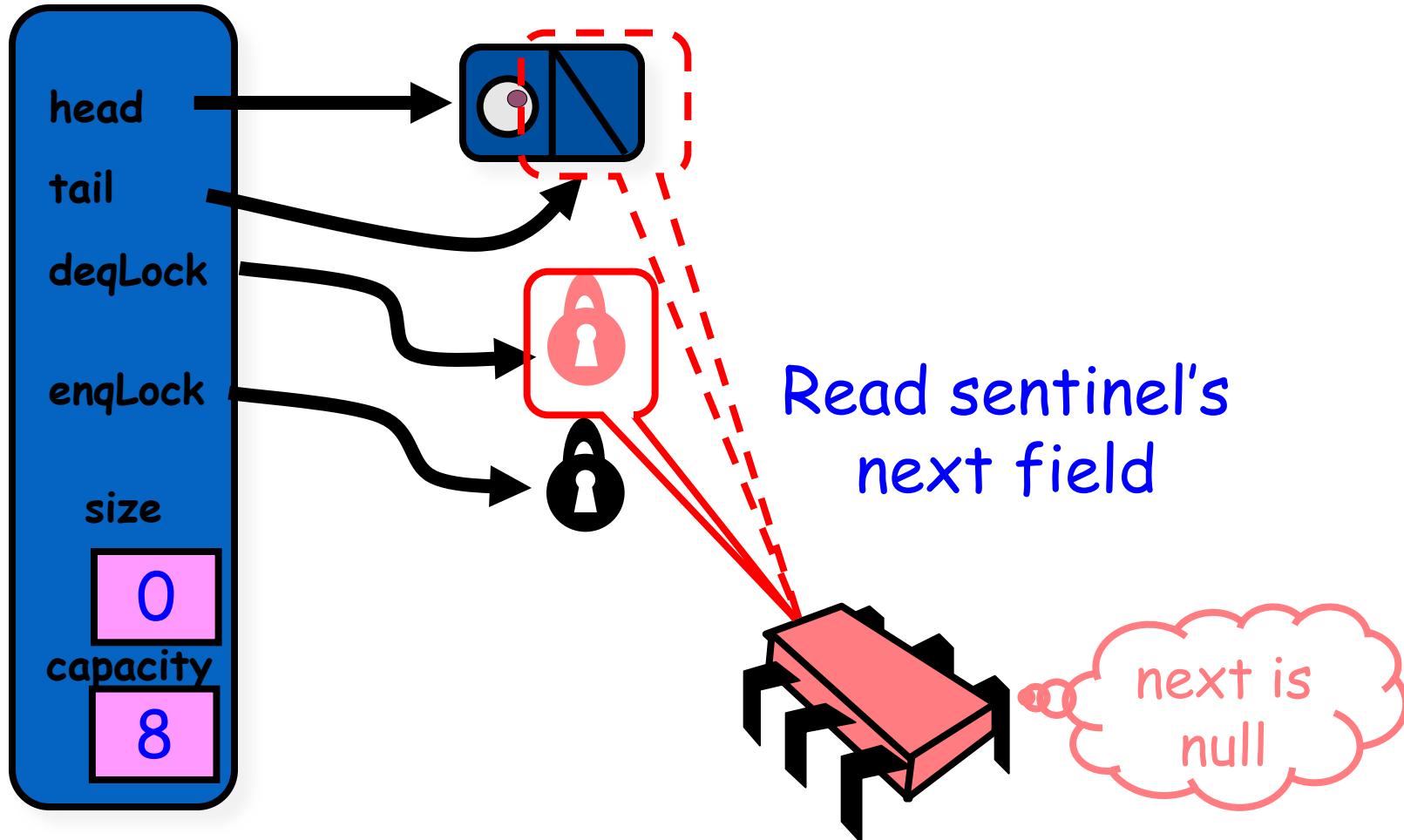
# Bounded Queue: Dequeuer



# Bounded Queue: Dequeuer



# Bounded Queue: Dequeuer



# Bounded Queue

```
1  public class BoundedQueue<T> {
2      ReentrantLock enqLock, deqLock;
3      Condition notEmptyCondition, notFullCondition;
4      AtomicInteger size;
5      Node head, tail;
6      int capacity;
7      public BoundedQueue(int _capacity) {
8          capacity = _capacity;
9          head = new Node(null);
10         tail = head;
11         size = new AtomicInteger(0);
12         enqLock = new ReentrantLock();
13         notFullCondition = enqLock.newCondition();
14         deqLock = new ReentrantLock();
15         notEmptyCondition = deqLock.newCondition();
16     }
```

# Bounded Queue

```
17  public void enq(T x) {  
18      boolean mustWakeDequeueuers = false;  
19      enqLock.lock();  
20      try {  
21          while (size.get() == capacity)  
22              notFullCondition.await();  
23          Node e = new Node(x);  
24          tail.next = e; tail = e;  
25          if (size.getAndIncrement() == 0)  
26              mustWakeDequeueuers = true;  
27      } finally {  
28          enqLock.unlock();  
29      }  
30      if (mustWakeDequeueuers) {  
31          dequeLock.lock();  
32          try {  
33              notEmptyCondition.signalAll();  
34          } finally {  
35              dequeLock.unlock();  
36          }  
37      }  
38  }
```

## Linearization point for enq()?

```
39  public T deq() {  
40      T result;  
41      boolean mustWakeEnqueueuers = true;  
42      dequeLock.lock();  
43      try {  
44          while (size.get() == 0)  
45              notEmptyCondition.await();  
46          result = head.next.value;  
47          head = head.next;  
48          if (size.getAndDecrement() == capacity) {  
49              mustWakeEnqueueuers = true;  
50          }  
51      } finally {  
52          dequeLock.unlock();  
53      }  
54      if (mustWakeEnqueueuers) {  
55          enqLock.lock();  
56          try {  
57              notFullCondition.signalAll();  
58          } finally {  
59              enqLock.unlock();  
60          }  
61      }  
62  }  
63  return result;
```

# Bounded Queue

```
17  public void enq(T x) {  
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19      enqLock.lock();  
20      try {  
21          while (size.get() == capacity)  
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23          Node e = new Node(x);  
24          tail.next = e; tail = e;  
25          if (size.getAndIncrement() == 0)  
26              mustWakeDequeueuers = true;  
27      } finally {  
28          enqLock.unlock();  
29      }  
30      if (mustWakeDequeueuers) {  
31          dequeLock.lock();  
32          try {  
33              notEmptyCondition.signalAll();  
34          } finally {  
35              dequeLock.unlock();  
36          }  
37      }  
38  }
```

**Logical addition**

```
39  public T deq() {  
40      T result;  
41      boolean mustWakeEnqueueuers = true;  
42      dequeLock.lock();  
43      try {  
44          while (size.get() == 0)  
45              notEmptyCondition.await();  
46          result = head.next.value;  
47          head = head.next;  
48          if (size.getAndDecrement() == capacity) {  
49              mustWakeEnqueueuers = true;  
50          }  
51      } finally {  
52          dequeLock.unlock();  
53      }  
54      if (mustWakeEnqueueuers) {  
55          enqLock.lock();  
56          try {  
57              notFullCondition.signalAll();  
58          } finally {  
59              enqLock.unlock();  
60          }  
61      }  
62  }  
63  return result;
```

# Bounded Queue

```
17  public void enq(T x) {  
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23          Node e = new Node(x);  
24          tail.next = e; tail = e;  
25          if (size.getAndIncrement() == 0)  
26              mustWakeDequeueuers = true;  
27      } finally {  
28          enqLock.unlock();  
29      }  
30      if (mustWakeDequeueuers) {  
31          dequeLock.lock();  
32          try {  
33              notEmptyCondition.signalAll();  
34          } finally {  
35              dequeLock.unlock();  
36          }  
37      }  
38  }
```

Dequeueuers won't see  
logical addition

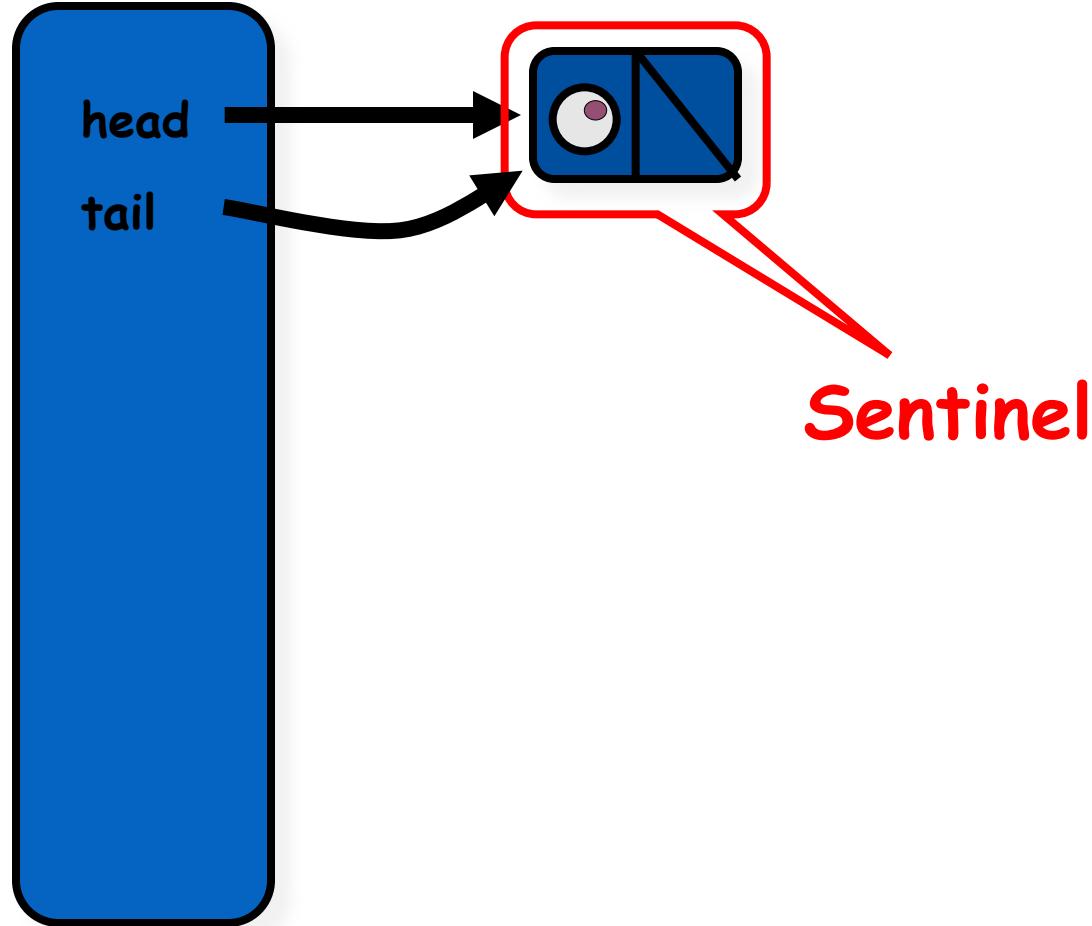
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52      }  
53      if (mustWakeEnqueueuers) {  
54          enqLock.lock();  
55          try {  
56              notFullCondition.signalAll();  
57          } finally {  
58              enqLock.unlock();  
59          }  
60      }  
61  }  
62  return result;  
63 }
```

# Unbounded (Total) Lock-Free Queue

- enq() always succeeds
- deq() throws exception (or returns false) on empty queue
- Rely on atomics (CAS) instead of locks

```
bool cas(pointer p, int old, int new) {  
    if (*p != old) { return false; }  
    *p ← new;  
    return true;  
}
```

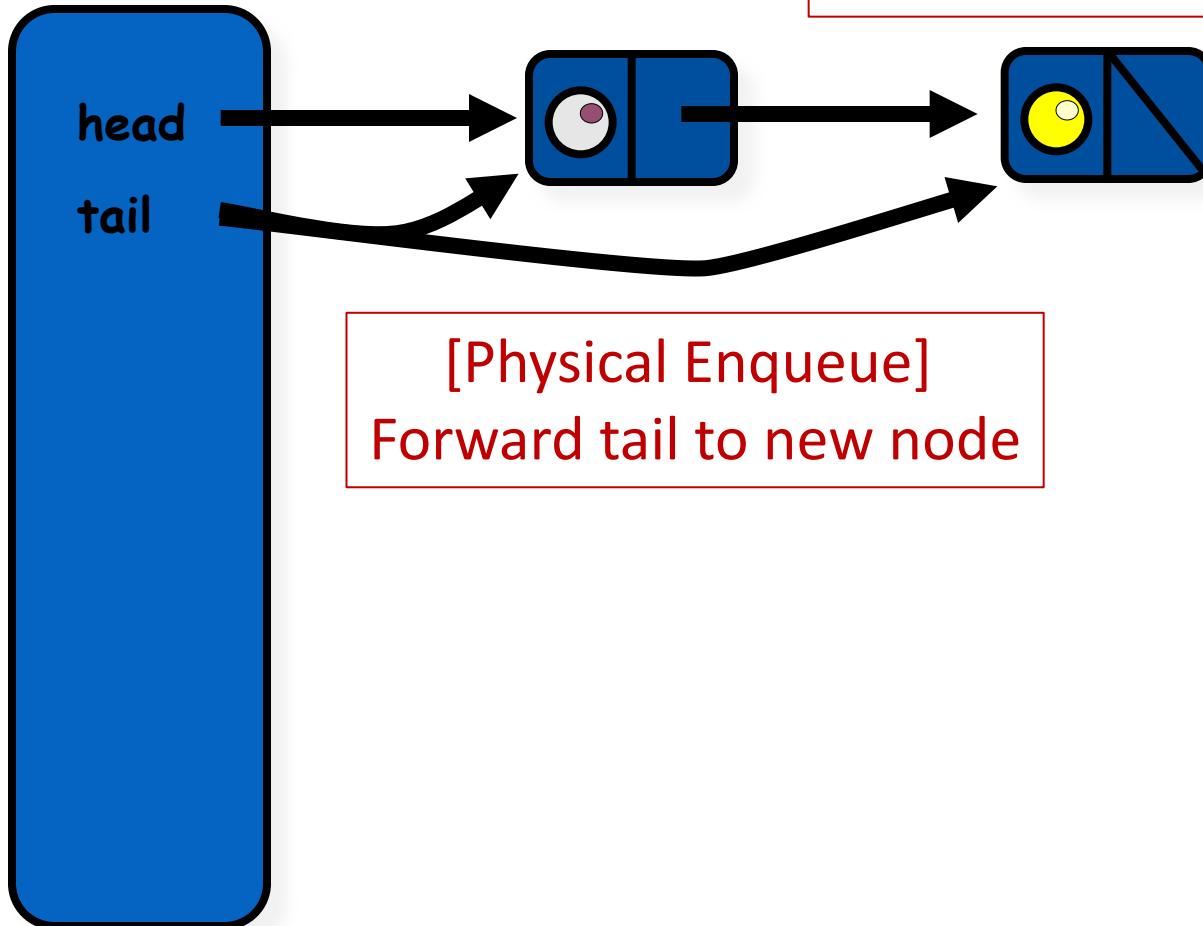
# Lock-Free Queue



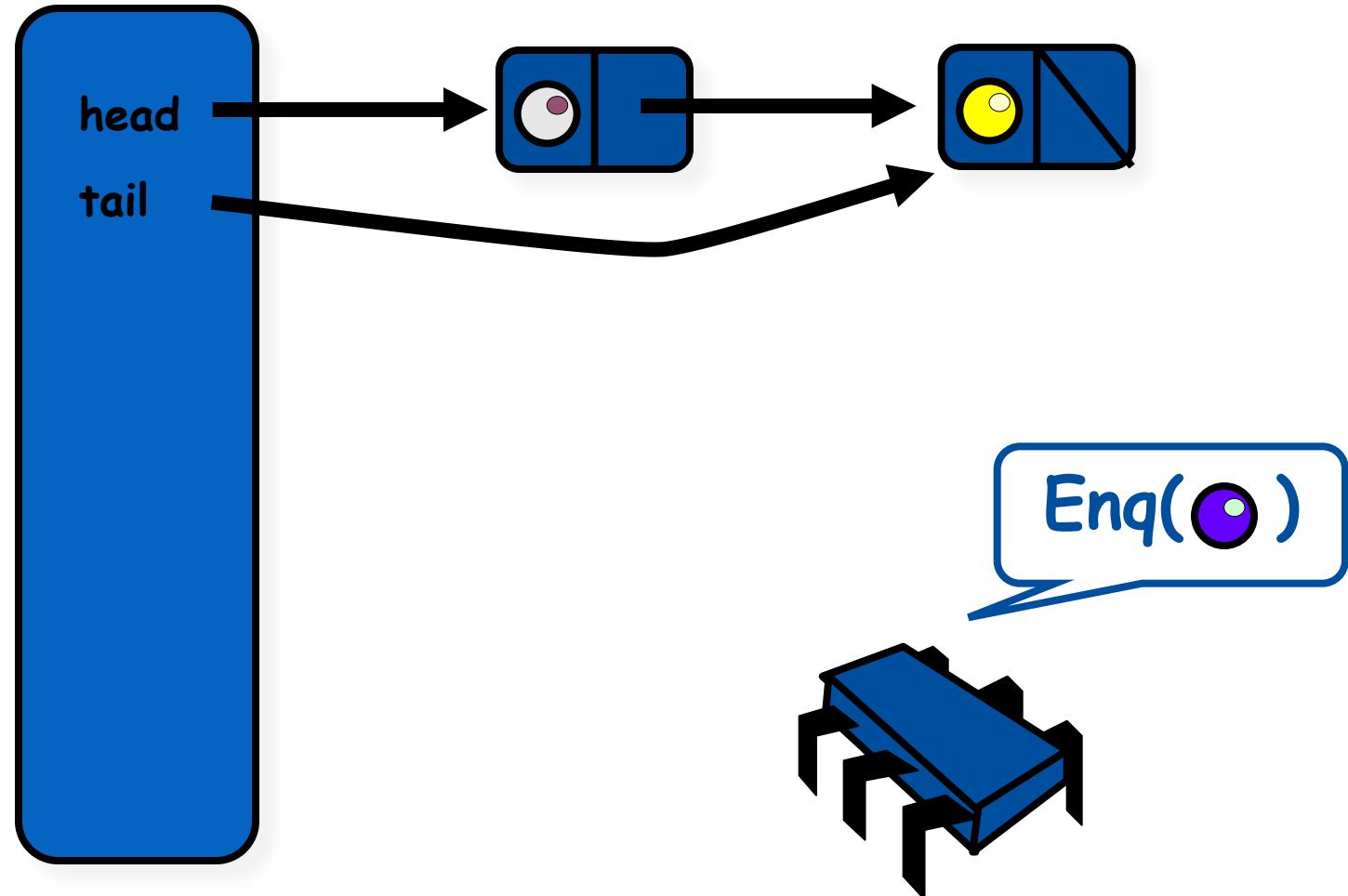
# Lock-Free Queue: Enqueue

Steps for Enqueue?

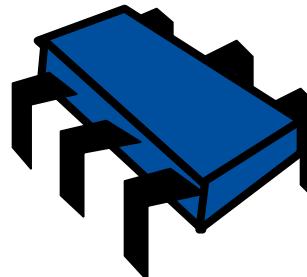
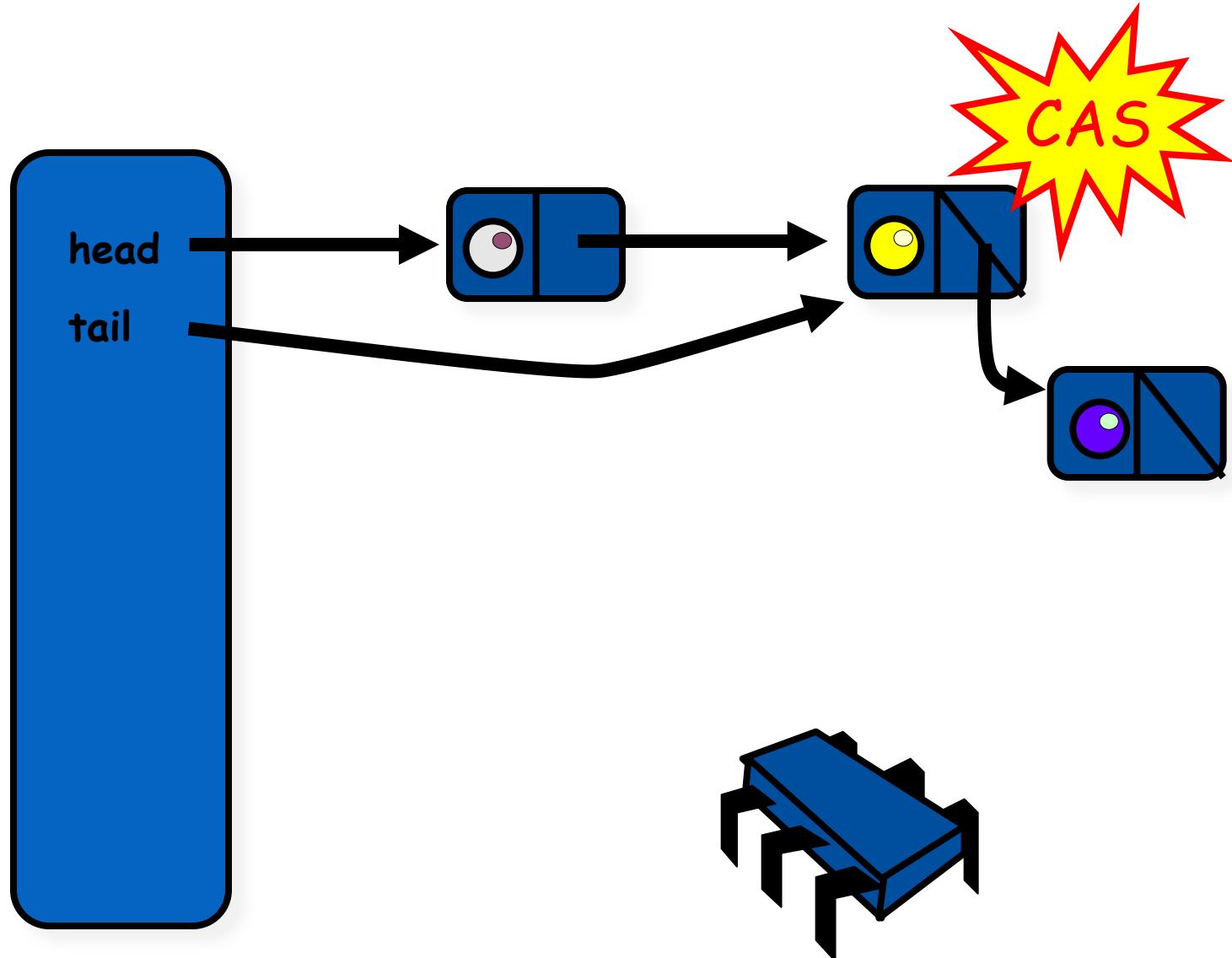
[Logical Enqueue]  
Point tail.next to new node



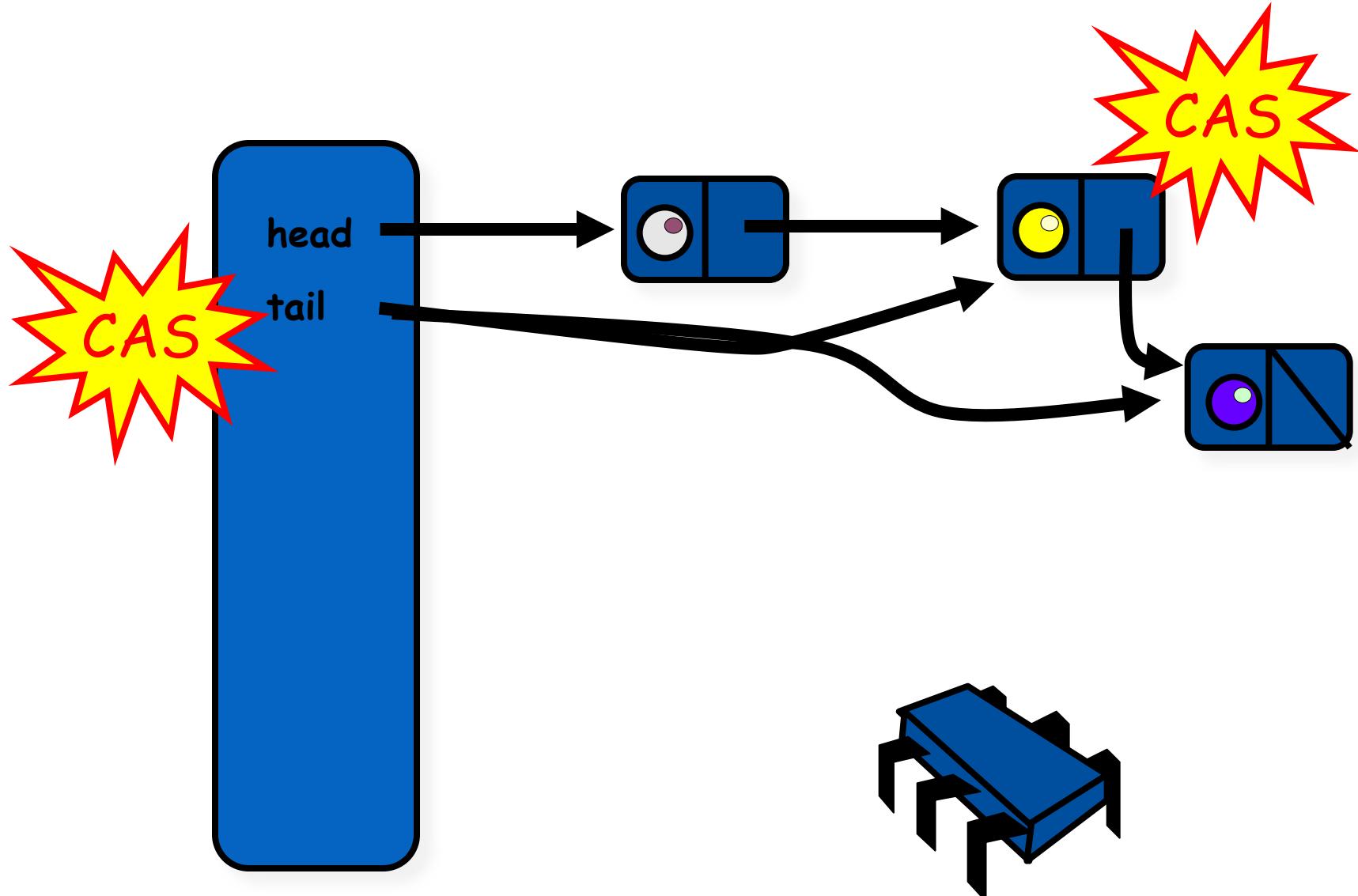
# Lock-Free Queue: Enqueue



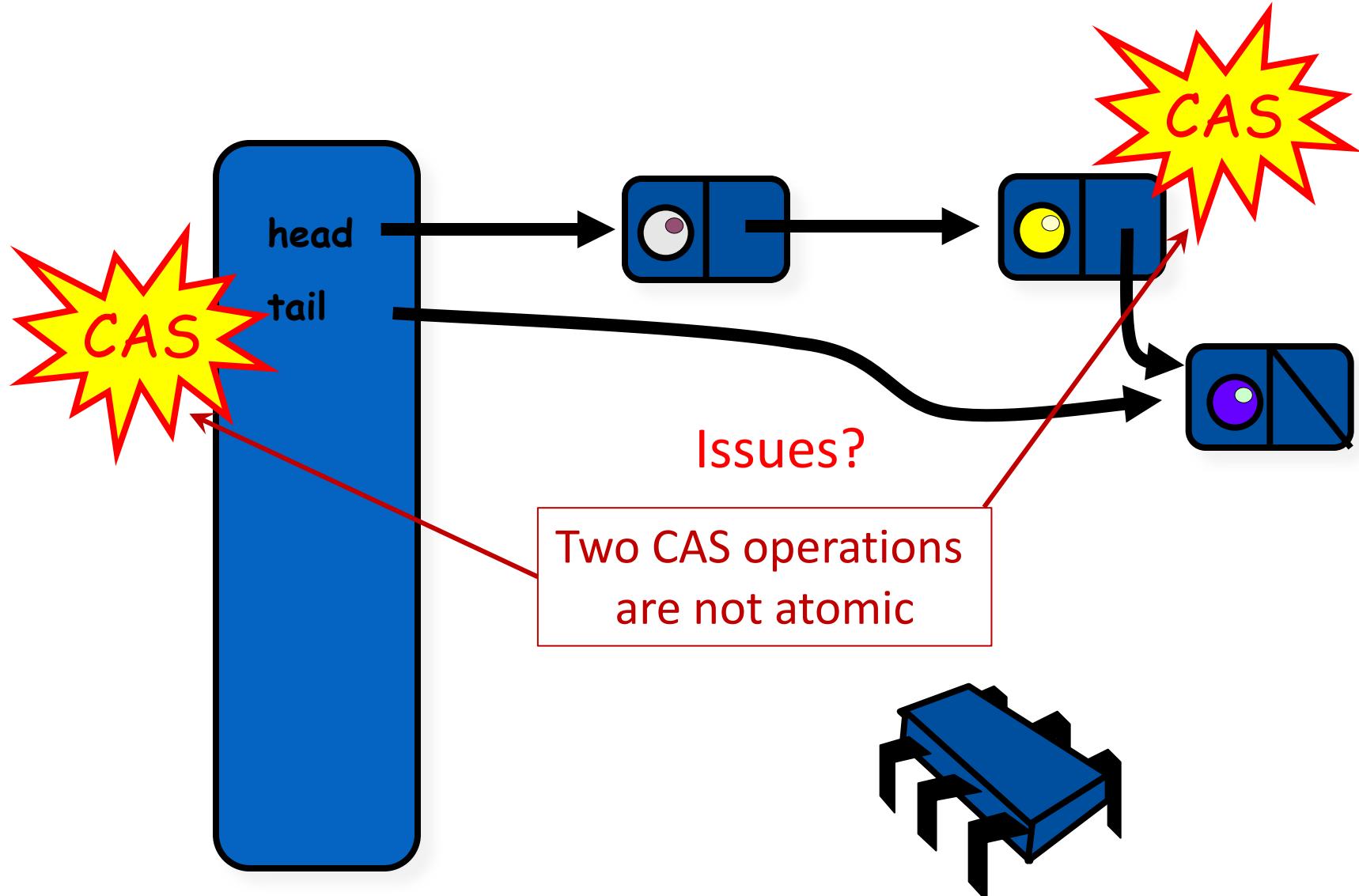
# Lock-Free Queue: Logical Enqueue



# Lock-Free Queue: Physical Enqueue



# Lock-Free Queue: Physical Enqueue



# Lock-Free Queue: Enqueue

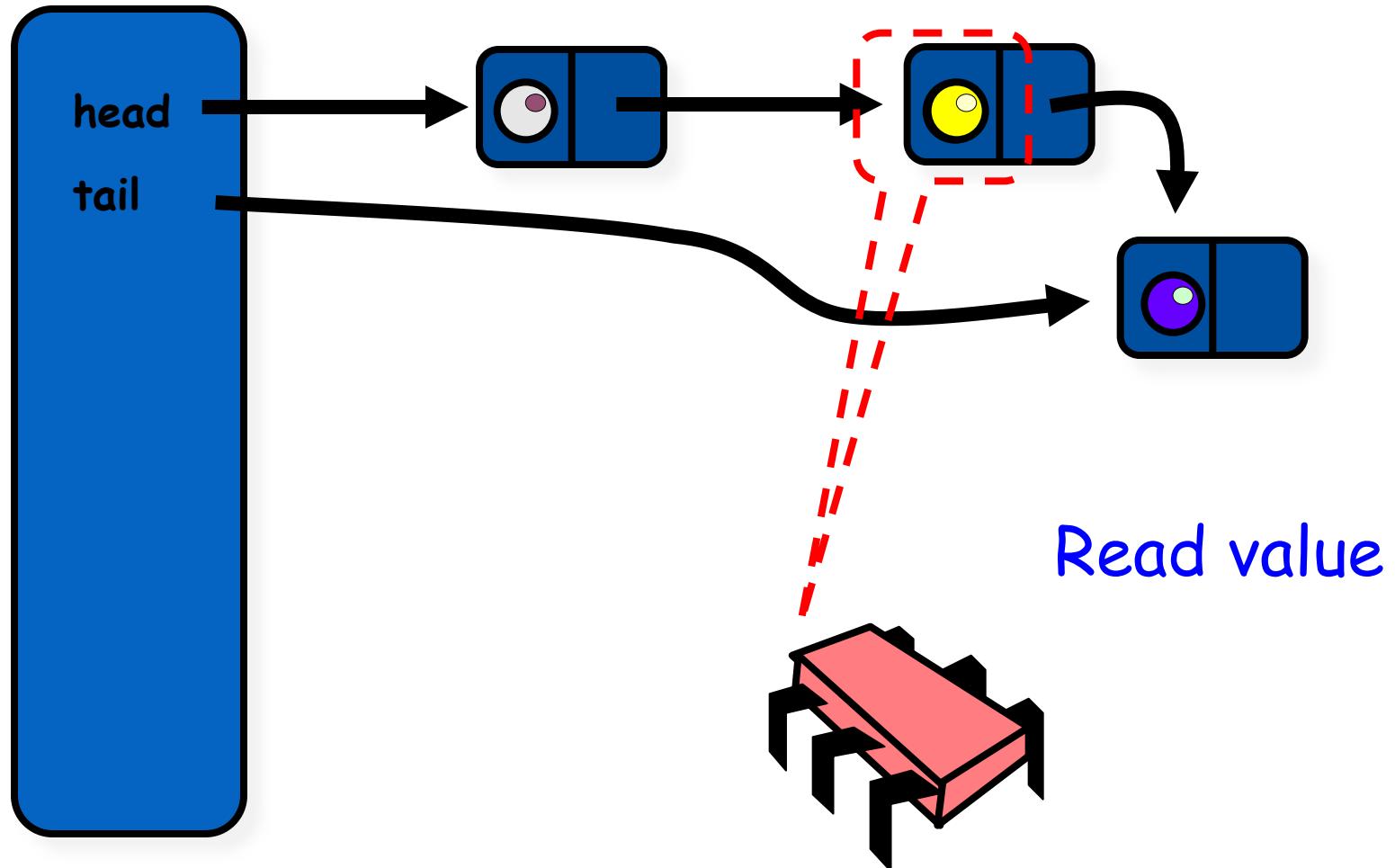
- Two CAS operations are not atomic
- When logical enqueue CAS fails
  - Restart the enqueue process
- When physical enqueue CAS fails
  - Ignore it – **Why?**
  - Similar to lock-free set implementation

# Lock-Free Queue

- The tail can refer to either:
  - Actual last node, or
  - Penultimate node
- What to do when we find a trailing tail?
- Stop and help fix it
  - CAS queue's tail field to tail.next

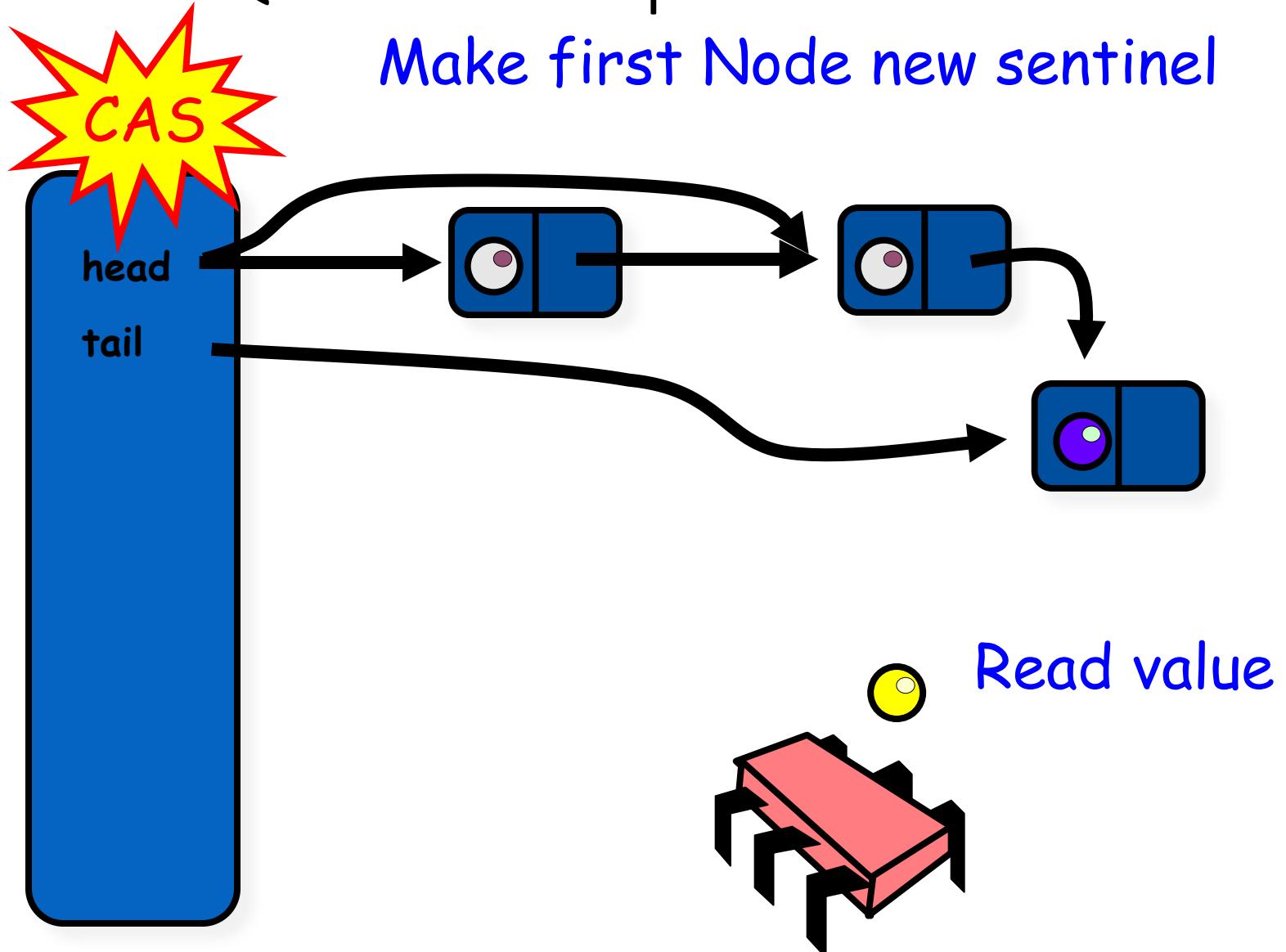
Can tail refer to  
any other node?

# Lock-Free Queue: Dequeuer



# Lock-Free Queue: Dequeuer

Make first Node new sentinel



# Unbounded (Total) Lock-Free Queue

```
public void enq(T value) {  
    Node node = new Node(value);  
    while (true) {  
        Node last = tail.get();  
        Node next = last.next.get();  
        if (last == tail.get()) {  
            if (next == null) {  
                if (last.next.compareAndSet(next, node)) {  
                    tail.compareAndSet(last, node);  
                    return;  
                }  
            } else {  
                tail.compareAndSet(last, next);  
            }  
        }  
    }  
}
```

```
public T deq() throws EmptyException {  
    while (true) {  
        Node first = head.get();  
        Node last = tail.get();  
        Node next = first.next.get();  
        if (first == head.get()) {  
            if (first == last) {  
                if (next == null) {  
                    throw new EmptyException();  
                }  
                tail.compareAndSet(last, next);  
            } else {  
                T value = next.value;  
                if (head.compareAndSet(first, next))  
                    return value;  
            }  
        }  
    }  
}
```

# Unbounded (Total) Lock-Free Queue

```
public void enq(T value) {  
    Node node = new Node(value);  
    while (true) {  
        Node last = tail.get();  
        Node next = last.next.get();  
        if (last == tail.get()) {  
            if (next == null) {  
                if (last.next.compareAndSet(next, node)) {  
                    tail.compareAndSet(last, node);  
                    return;  
                }  
            } else {  
                tail.compareAndSet(last, next);  
            }  
        }  
    }  
}
```

Logical enqueue

```
public T deq() throws EmptyException {  
    while (true) {  
        Node first = head.get();  
        Node last = tail.get();  
        Node next = first.next.get();  
        if (first == head.get()) {  
            if (first == last) {  
                if (next == null) {  
                    throw new EmptyException();  
                }  
                tail.compareAndSet(last, next);  
            } else {  
                T value = next.value;  
                if (head.compareAndSet(first, next))  
                    return value;  
            }  
        }  
    }  
}
```

# Unbounded (Total) Lock-Free Queue

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    Node node = new Node(value);  
    while (true) {  
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        if (last == tail.get()) {  
            if (next == null) {  
                if (last.next.compareAndSet(next, node)) {  
                    tail compareAndSet(last, node);  
                    return;  
                }  
            } else {  
                tail.compareAndSet(last, next);  
            }  
        }  
    }  
}
```

Physical enqueue

```
public T deq() throws EmptyException {  
    while (true) {  
        Node first = head.get();  
        Node last = tail.get();  
        Node next = first.next.get();  
        if (first == head.get()) {  
            if (first == last) {  
                if (next == null) {  
                    throw new EmptyException();  
                }  
                tail.compareAndSet(last, next);  
            } else {  
                T value = next.value;  
                if (head.compareAndSet(first, next))  
                    return value;  
            }  
        }  
    }  
}
```

# Unbounded (Total) Lock-Free Queue

```
public void enq(T value) {  
    Node node = new Node(value);  
    while (true) {  
        Node last = tail.get();  
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        if (last == tail.get()) {  
            if (next == null) {  
                if (last.next.compareAndSet(next, node)) {  
                    tail.compareAndSet(last, node);  
                    return;  
                }  
            } else {  
                tail.compareAndSet(last, next);  
            }  
        }  
    }  
}
```

Forward tail

```
public T deq() throws EmptyException {  
    while (true) {  
        Node first = head.get();  
        Node last = tail.get();  
        Node next = first.next.get();  
        if (first == head.get()) {  
            if (first == last) {  
                if (next == null) {  
                    throw new EmptyException();  
                }  
                tail.compareAndSet(last, next);  
            } else {  
                T value = next.value;  
                if (head.compareAndSet(first, next))  
                    return value;  
            }  
        }  
    }  
}
```

# Unbounded (Total) Lock-Free Queue

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public void enq(T value) {  
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```

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        }  
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```

Forward tail

# Unbounded (Total) Lock-Free Queue

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            if (next == null) {  
                if (last.next.compareAndSet(next, node)) {  
                    tail.compareAndSet(last, node);  
                    return;  
                }  
            } else {  
                tail.compareAndSet(last, next);  
            }  
        }  
    }  
}
```

Linearization Point?

```
public T deq() throws EmptyException {  
    while (true) {  
        Node first = head.get();  
        Node last = tail.get();  
        Node next = first.next.get();  
        if (first == head.get()) {  
            if (first == last) {  
                if (next == null) {  
                    throw new EmptyException();  
                }  
                tail.compareAndSet(last, next);  
            } else {  
                T value = next.value;  
                if (head.compareAndSet(first, next))  
                    return value;  
            }  
        }  
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}
```

# Unbounded (Total) Lock-Free Queue

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                if (last.next.compareAndSet(next, node)) {  
                    tail.compareAndSet(last, node);  
                    return;  
                }  
            } else {  
                tail.compareAndSet(last, next);  
            }  
        }  
    }  
}
```

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public T deq() throws EmptyException {  
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        Node next = first.next.get();  
        if (first == head.get()) {  
            if (first == last) {  
                if (next == null) {  
                    throw new EmptyException();  
                }  
                tail.compareAndSet(last, next);  
            } else {  
                T value = next.value;  
                if (head.compareAndSet(first, next))  
                    return value;  
            }  
        }  
    }  
}
```

Enqueuing thread OR helper thread  
succeeds on CAS to swing tail

# Unbounded (Total) Lock-Free Queue

```
public void enq(T value) {  
    Node node = new Node(value);  
    while (true) {  
        Node last = tail.get();  
        Node next = last.next.get();  
        if (last == tail.get()) {  
            if (next == null) {  
                if (last.next.compareAndSet(next, node)) {  
                    tail.compareAndSet(last, node);  
                    return;  
                }  
            } else {  
                tail.compareAndSet(last, next);  
            }  
        }  
    }  
}
```

```
public T deq() throws EmptyException {  
    while (true) {  
        Node first = head.get();  
        Node last = tail.get();  
        Node next = first.next.get();  
        if (first == head.get()) {  
            if (first == last) {  
                if (next == null) {  
                    throw new EmptyException();  
                }  
                tail.compareAndSet(last, next);  
            } else {  
                T value = next.value;  
                if (head.compareAndSet(first, next))  
                    return value;  
            }  
        }  
    }  
}
```

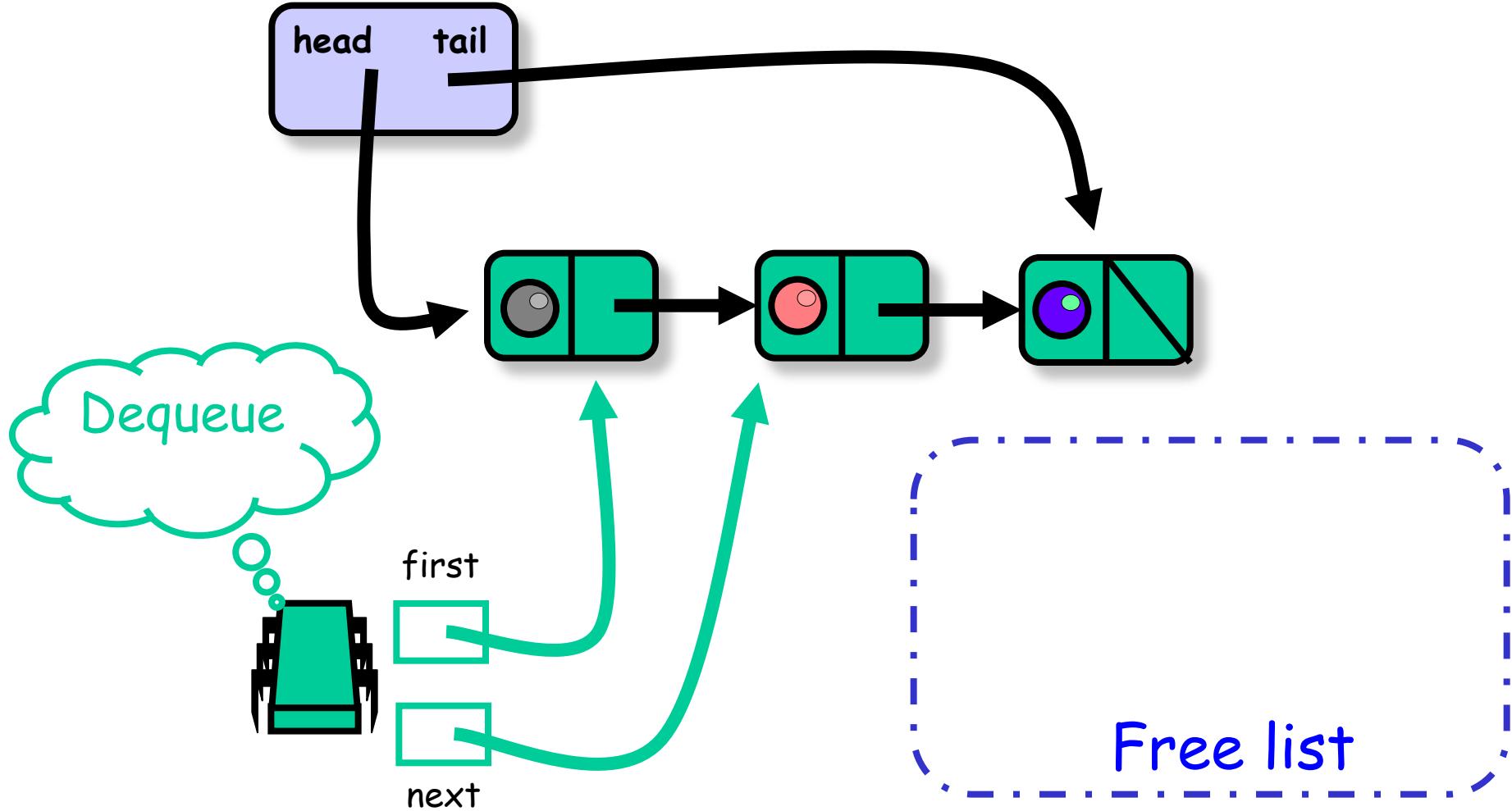
successful CAS to swing head

# Memory Management

- So far we relied on GC to recycle dequeued nodes
- Let's do our own memory management
- Simplest design
  - Each thread has its own private free list
  - Enqueue(): Allocate nodes for using thread's free list  
(if private list is empty, call malloc/new)
  - Dequeue(): Deallocated nodes go back to thread's free list

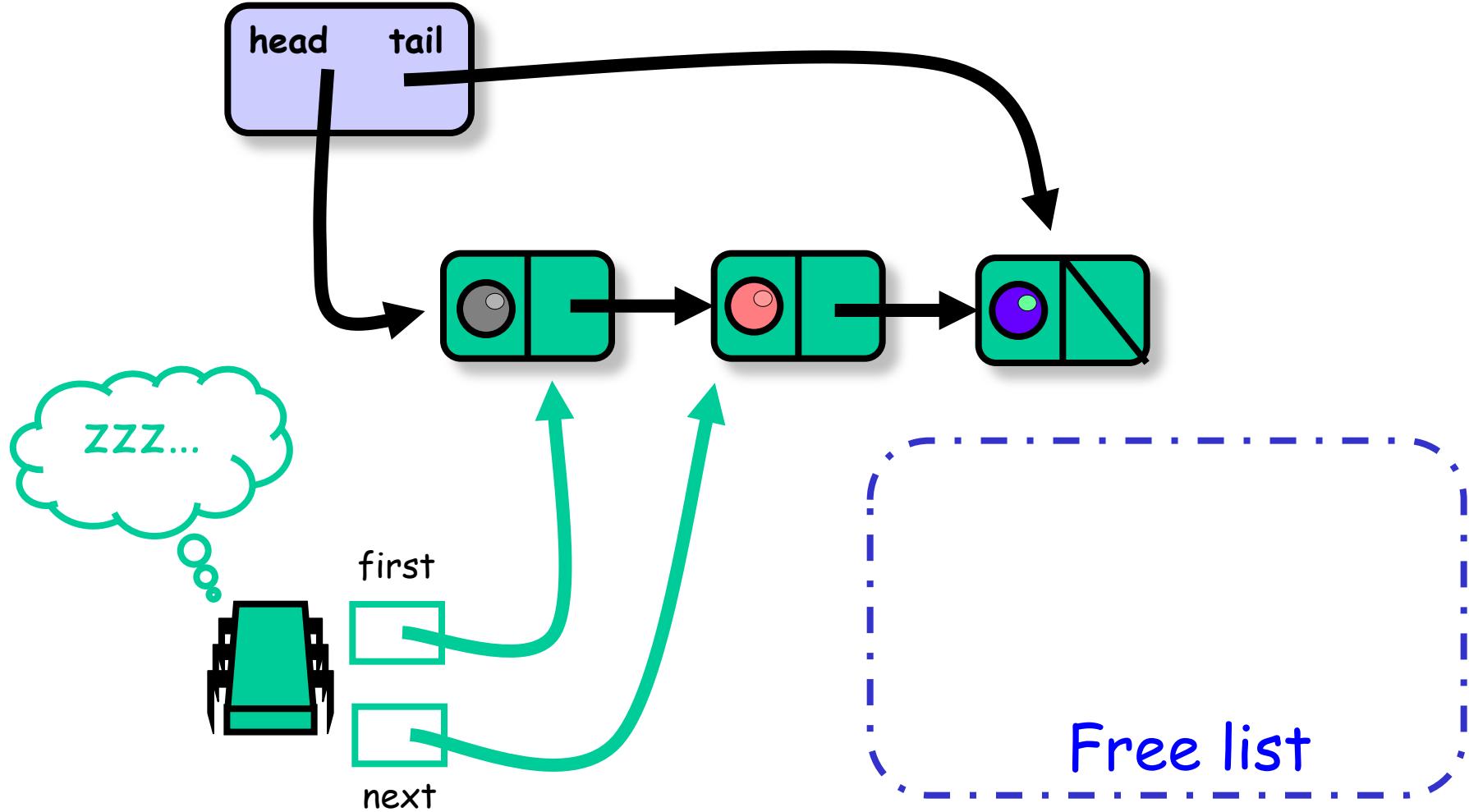
Want to redirect head  
from grey to red

```
if (head.compareAndSet(first, next))  
    return value;
```



Want to redirect head  
from grey to red

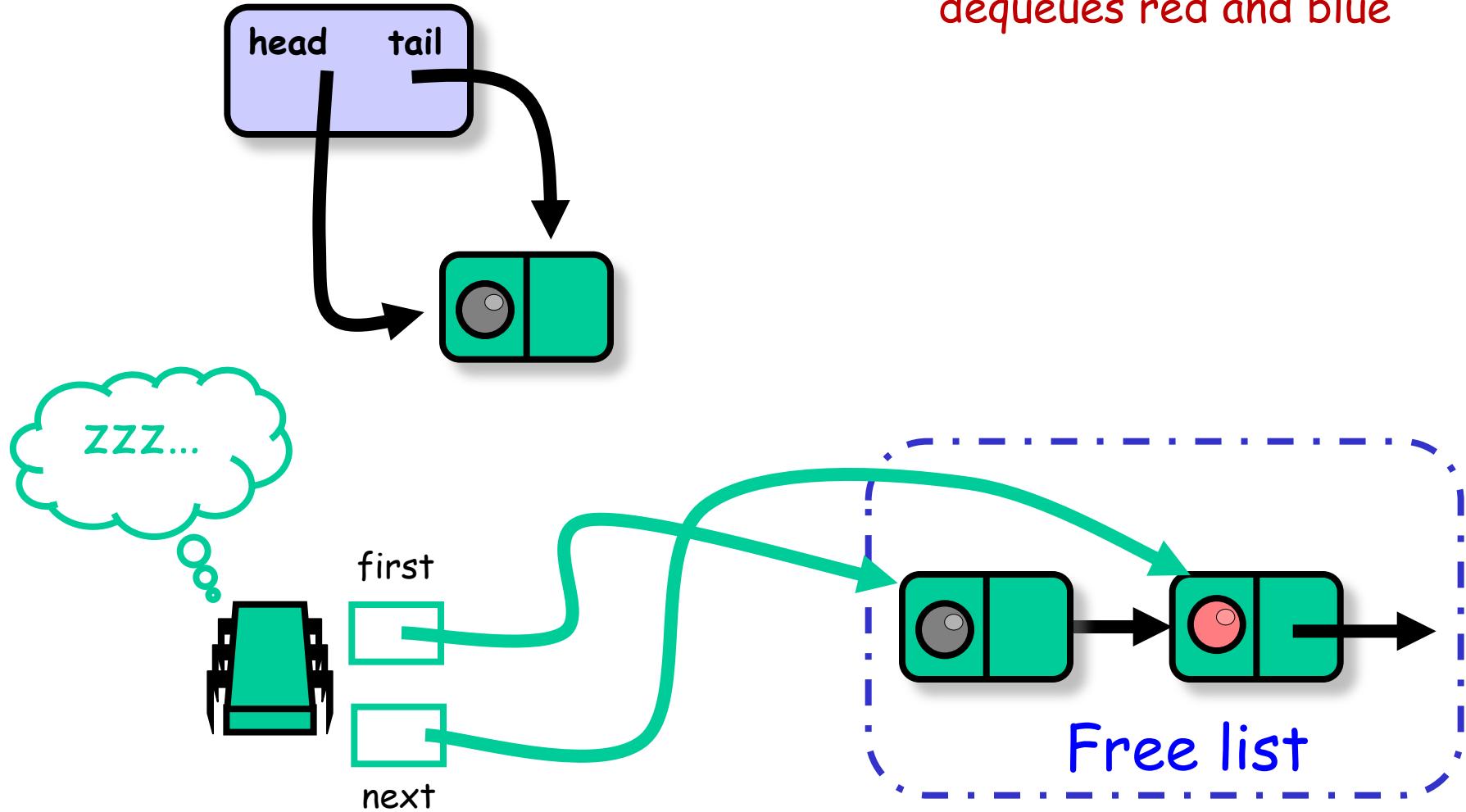
```
if (head.compareAndSet(first, next))  
    return value;
```



Want to redirect head  
from grey to red

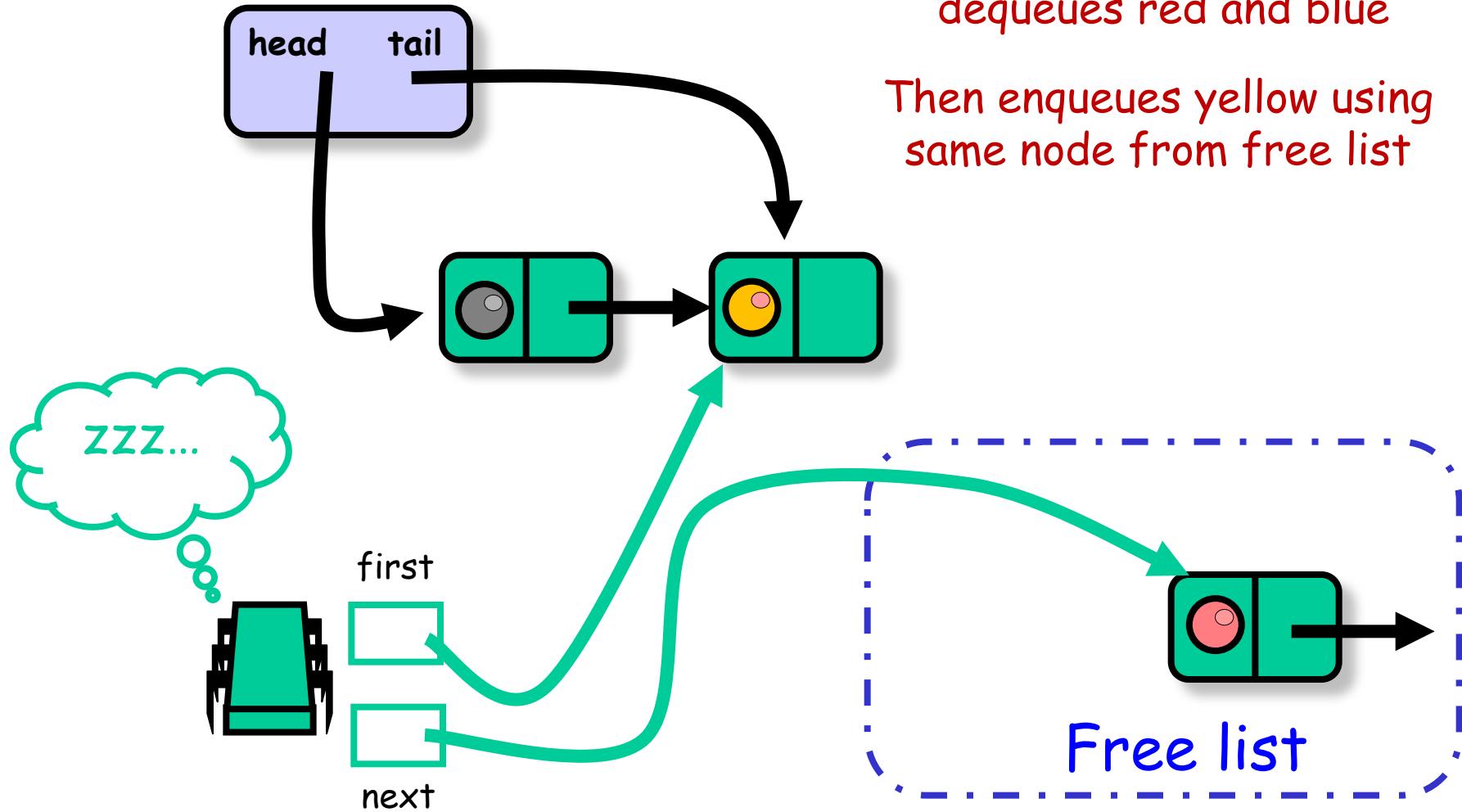
```
if (head.compareAndSet(first, next))  
    return value;
```

Another thread  
dequeues red and blue



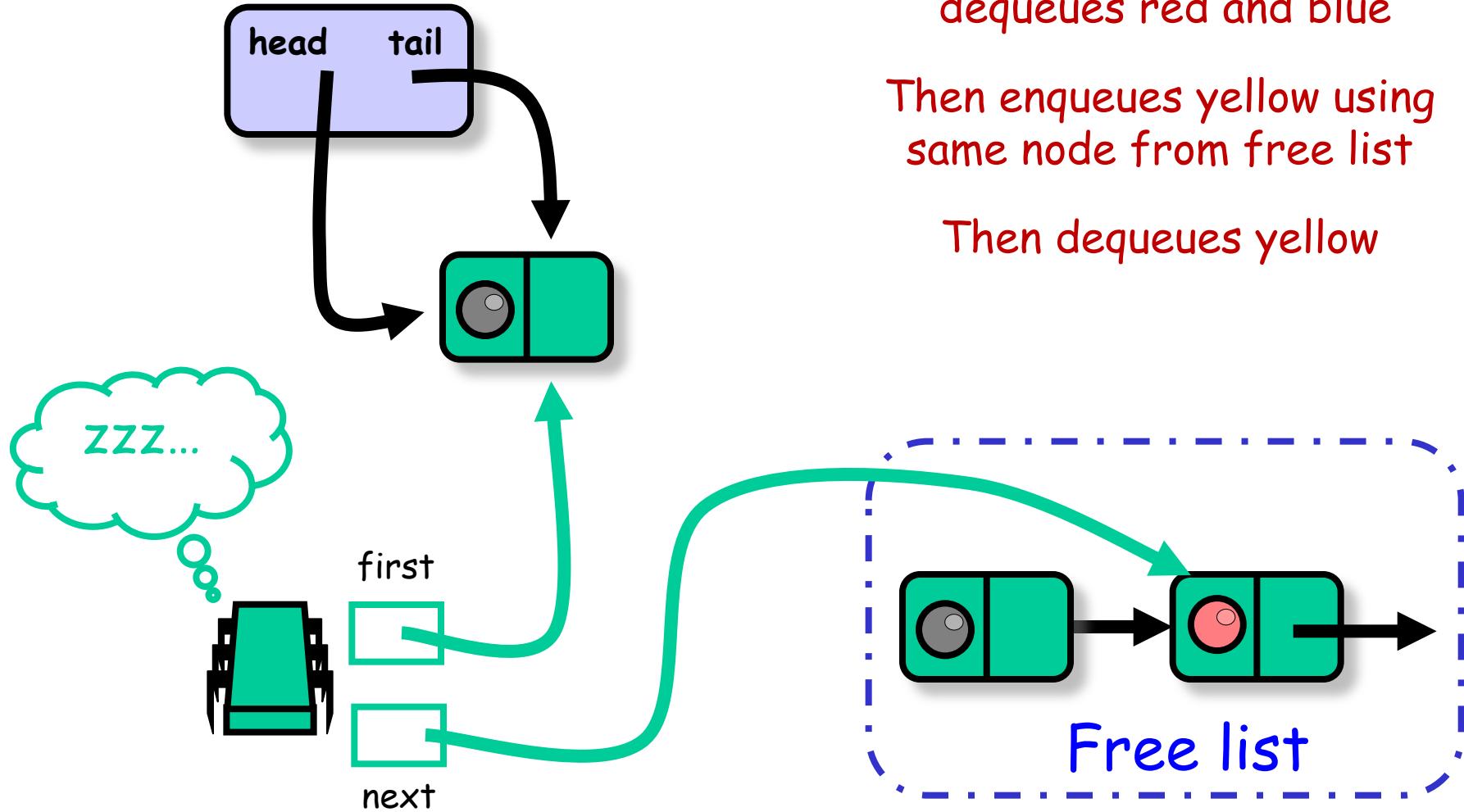
Want to redirect head  
from grey to red

```
if (head.compareAndSet(first, next))  
    return value;
```



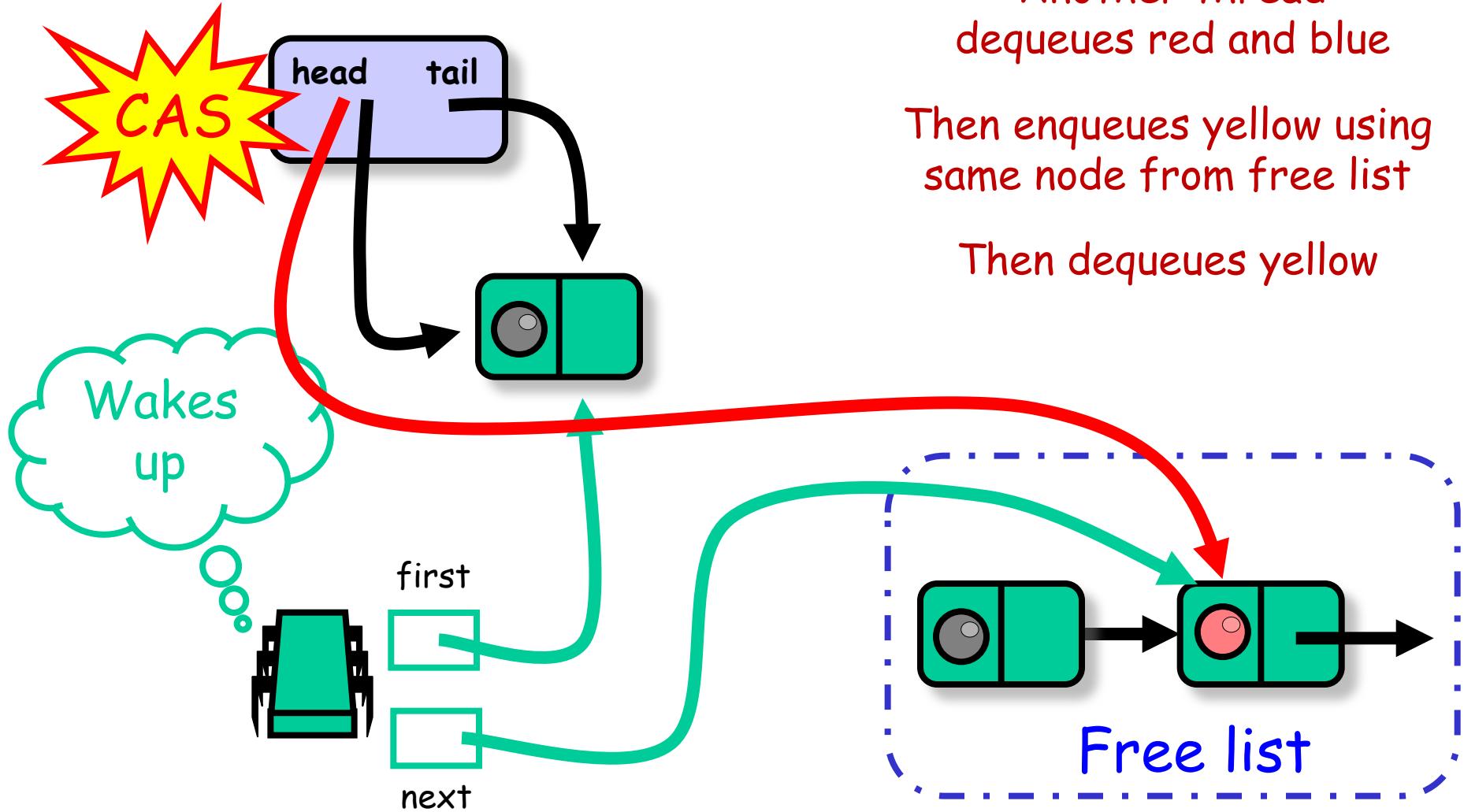
Want to redirect head  
from grey to red

```
if (head.compareAndSet(first, next))  
    return value;
```

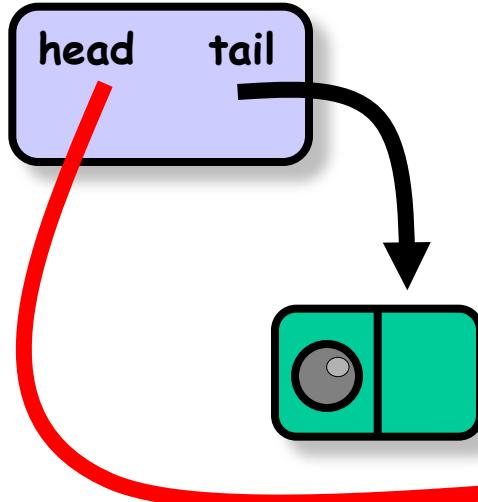


Want to redirect head  
from grey to red

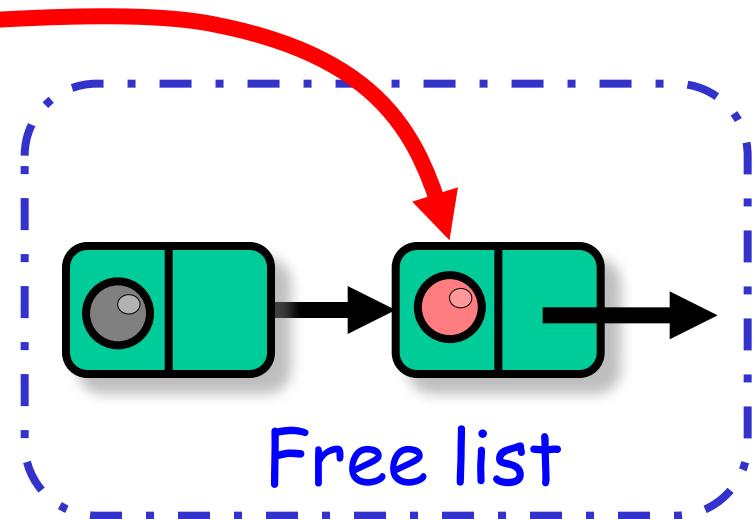
```
if (head.compareAndSet(first, next))  
    return value;
```



```
if (head.compareAndSet(first, next))  
    return value;
```

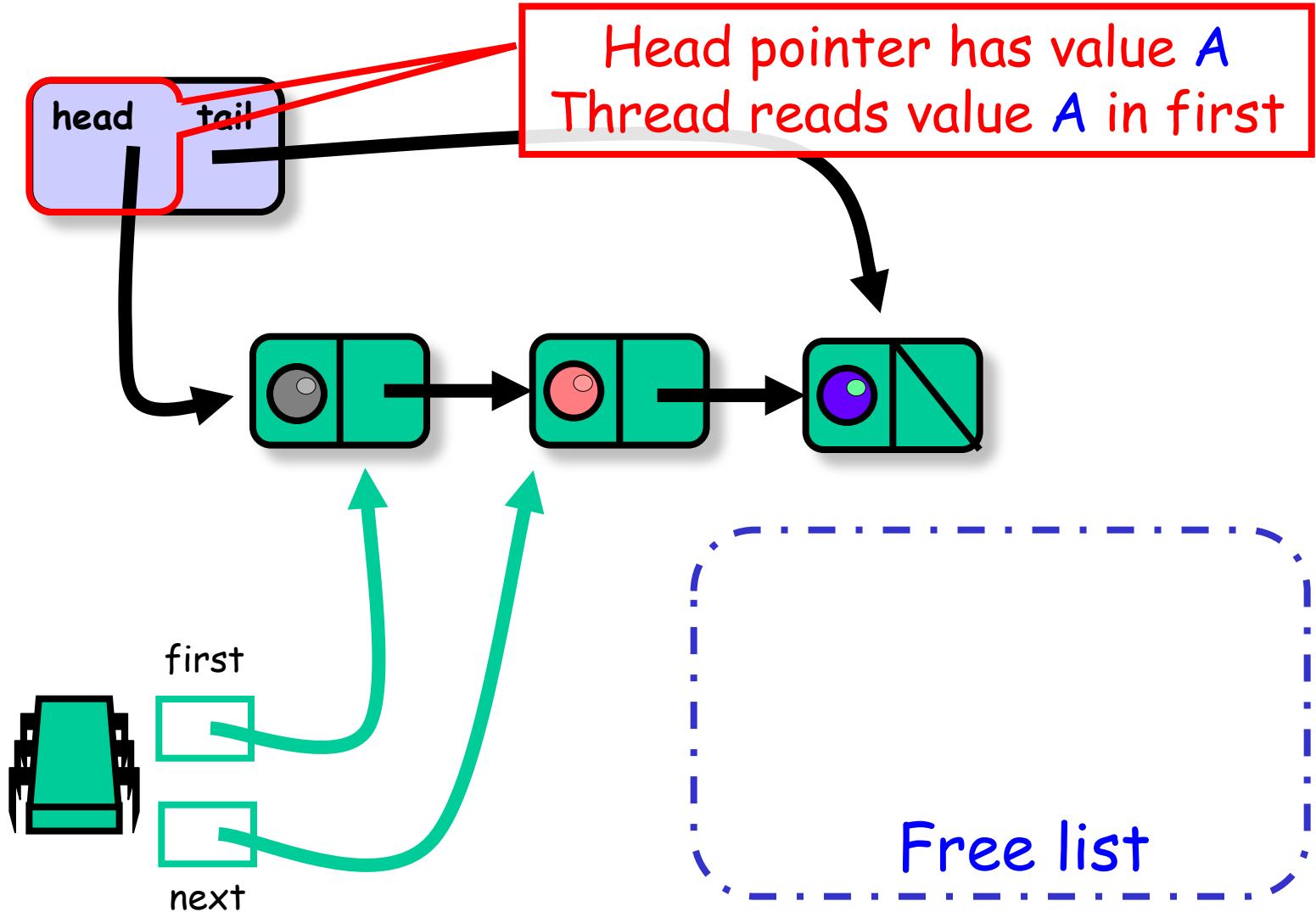


What went wrong?



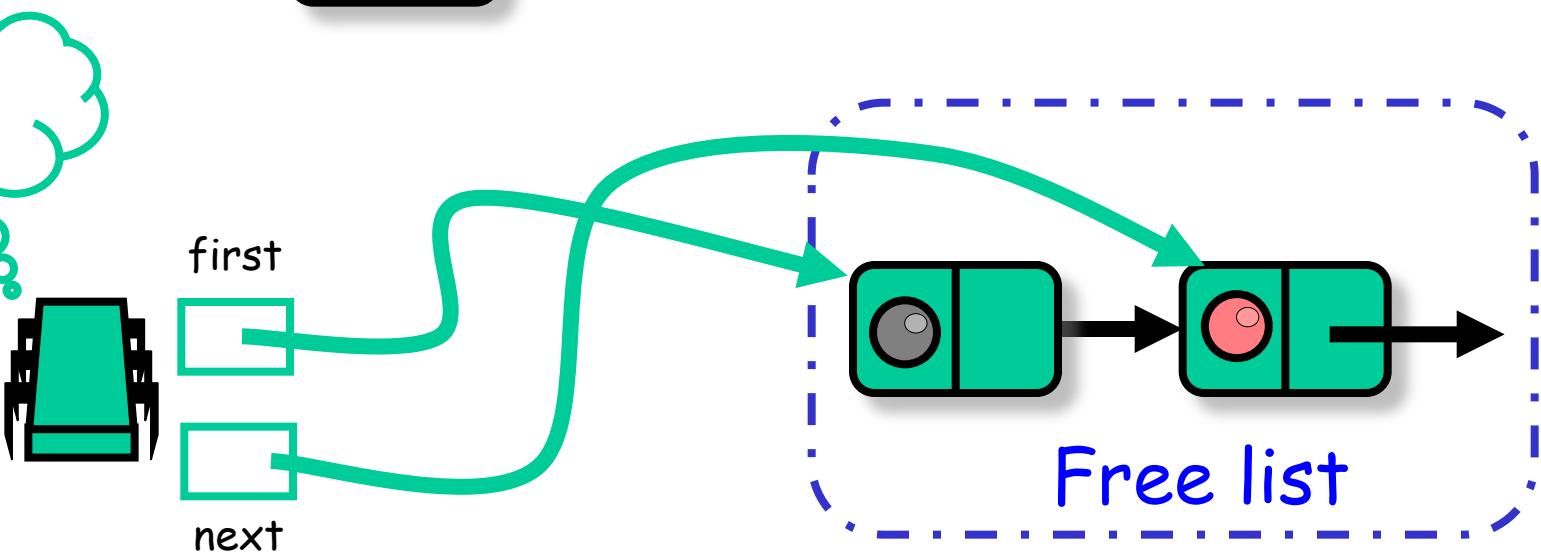
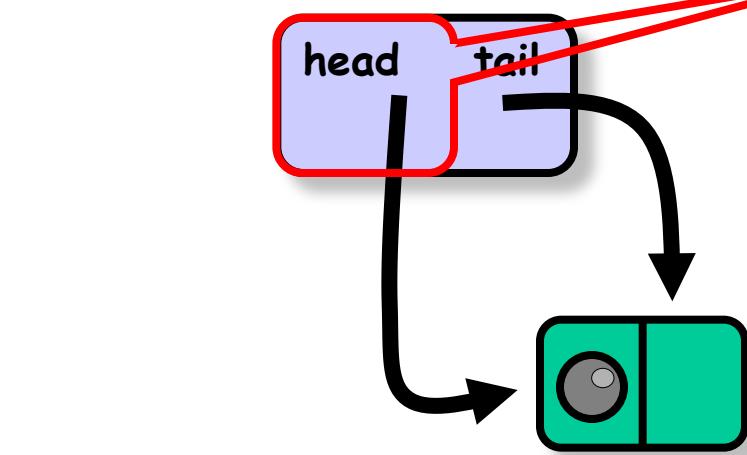
Free list

```
if (head.compareAndSet(first, next))  
    return value;
```



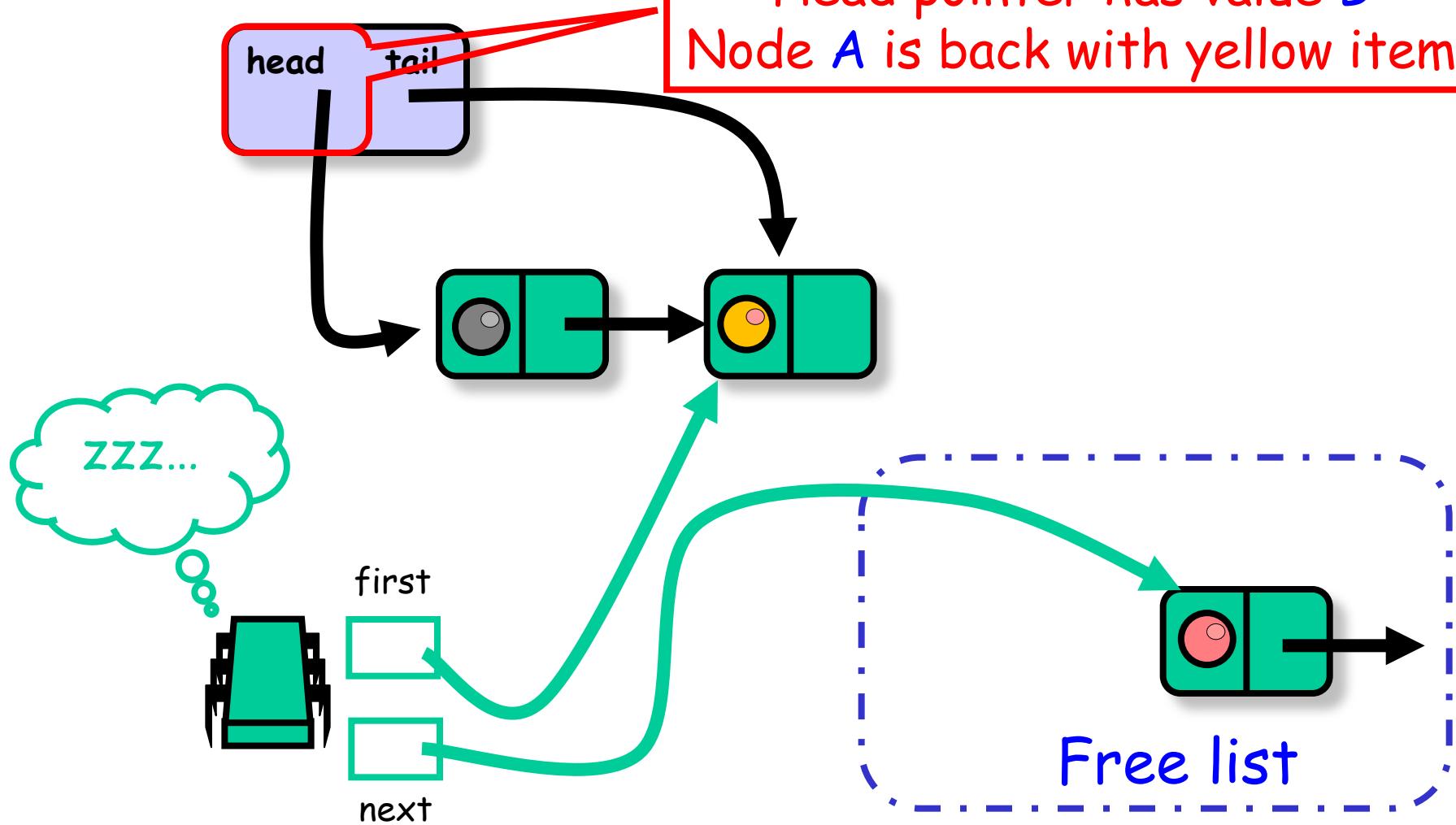
```
if (head.compareAndSet(first, next))  
    return value;
```

Head pointer has value B  
Node A is freed



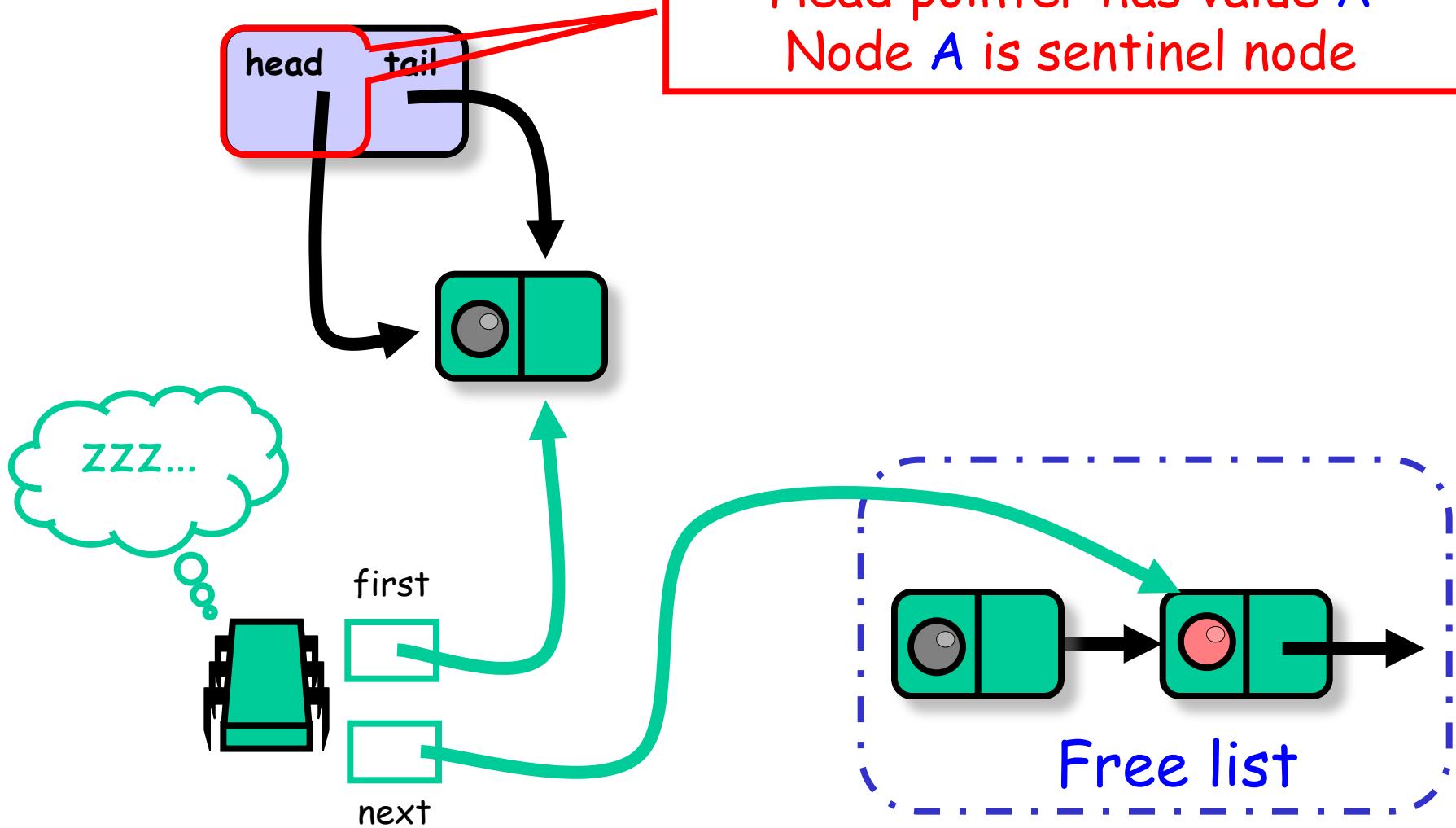
```
if (head.compareAndSet(first, next))  
    return value;
```

Head pointer has value B  
Node A is back with yellow item



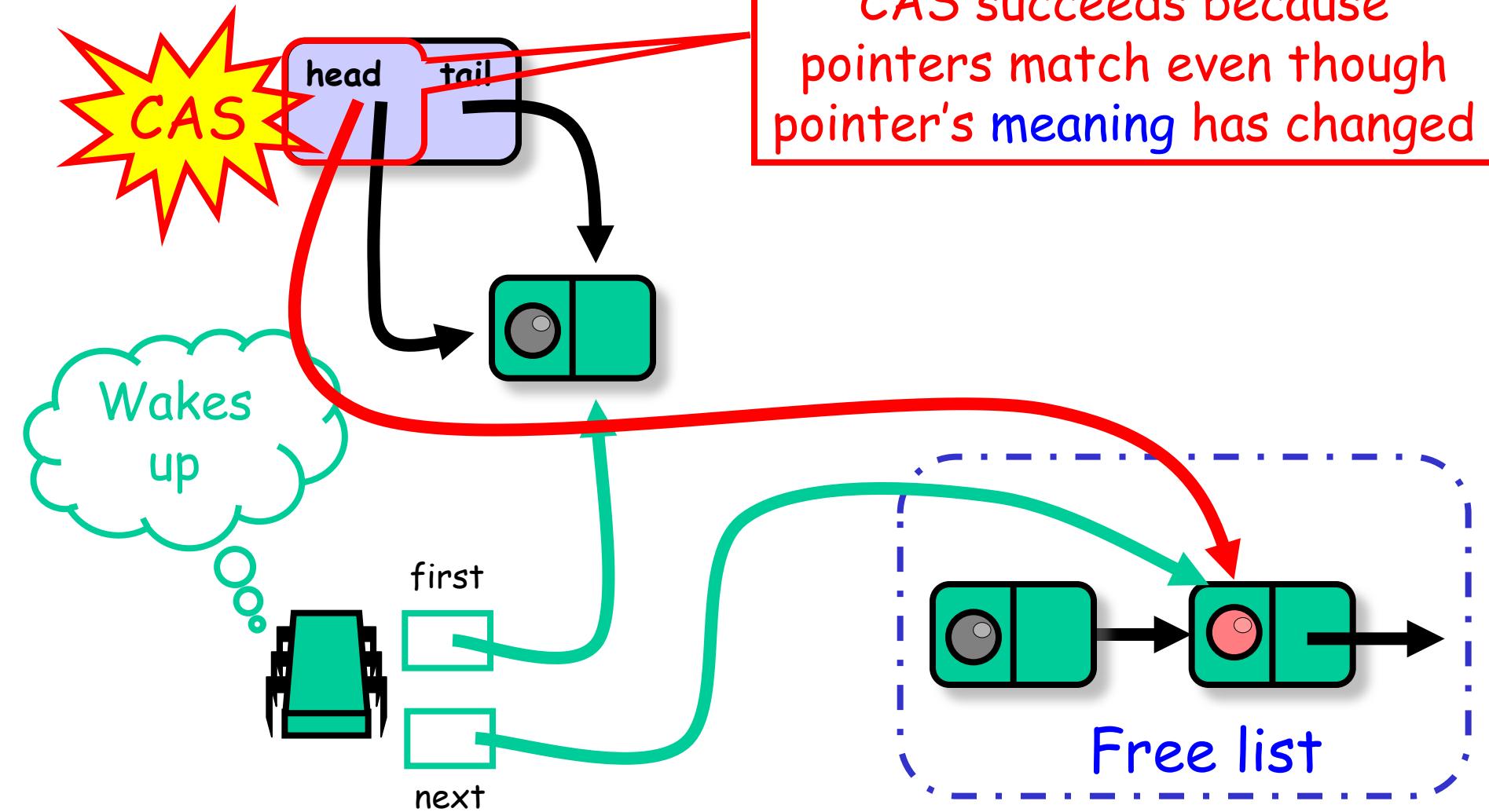
```
if (head.compareAndSet(first, next))  
    return value;
```

Head pointer has value A  
Node A is sentinel node



```
if (head.compareAndSet(first, next))  
    return value;
```

CAS succeeds because  
pointers match even though  
pointer's meaning has changed



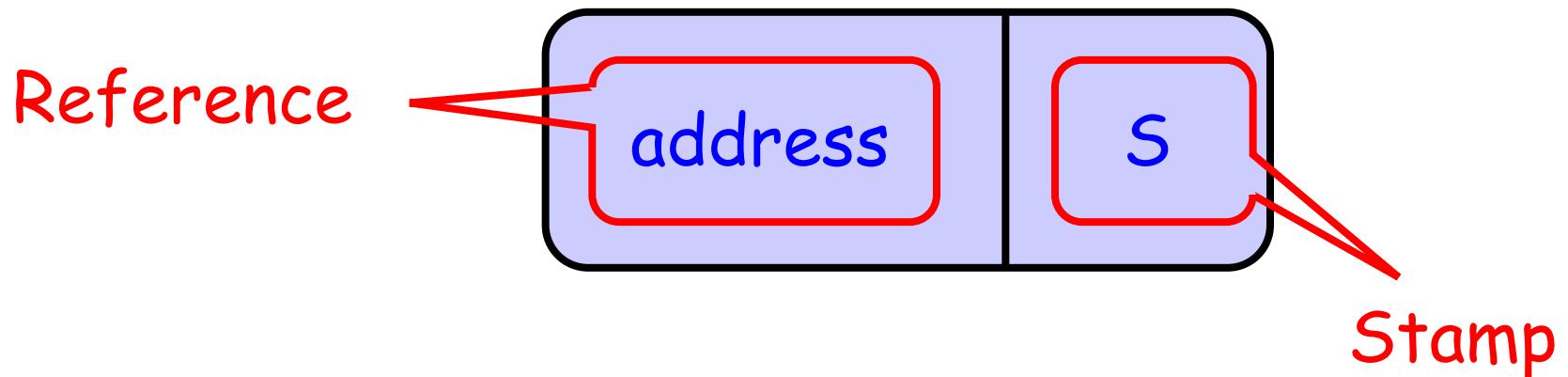
# ABA Problem

- head pointer went A - B - A
- Due to semantics of CAS
  - It looks at the value only
  - Advanced: Check out Load-link/Store-conditional

Load-link/Store-conditional, Wikipedia: <https://en.wikipedia.org/wiki/Load-link/store-conditional>

# Solution for ABA Problem

- Tag each pointer with a counter
- Pointer is unique over lifetime of node
- Pointer size v/s word size issues
  - Overflow?
- Java has `AtomicStampedReference` class



# Reading



- [Paper] Simple, Fast, and Practical Non-Blocking and Blocking Concurrent Queue Algorithms:  
[https://www.cs.rochester.edu/~scott/papers/1996\\_PODC\\_queues.pdf](https://www.cs.rochester.edu/~scott/papers/1996_PODC_queues.pdf)

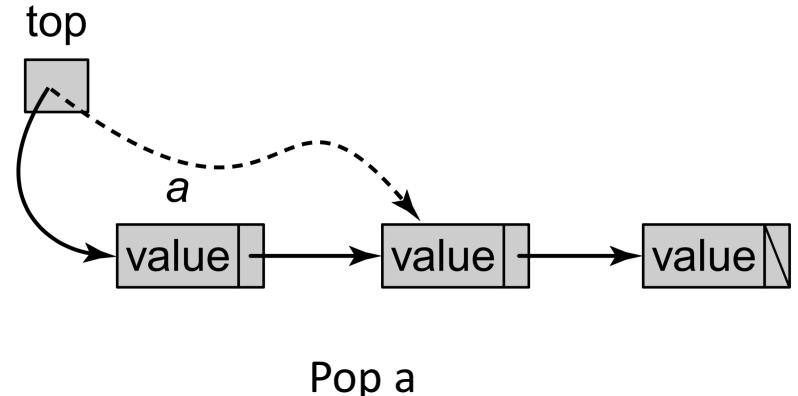
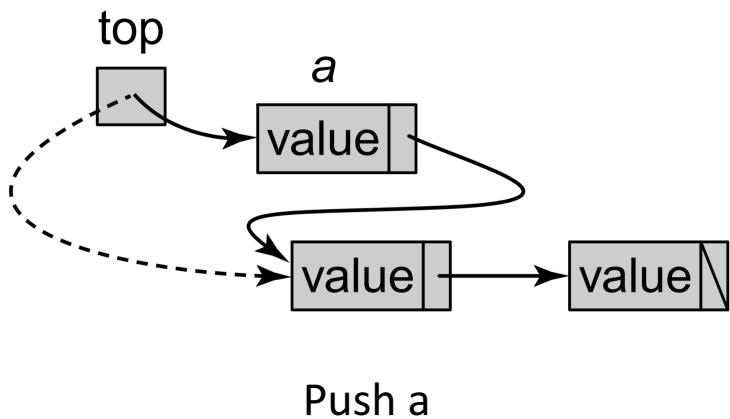
# Stacks

- Last-in-first-out (LIFO) order
- push() & pop()
- Appear inherently sequential?
- Little opportunity for concurrency?

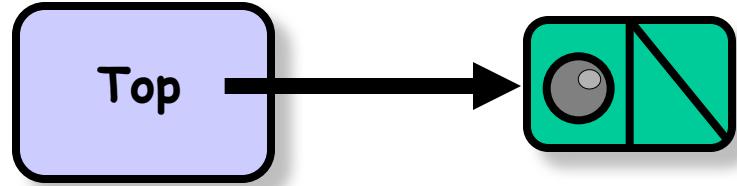
# Lock-Free Stack

- Implement using linked-list
- Unbounded Total Lock-Free Stack
  - Push and pop don't block

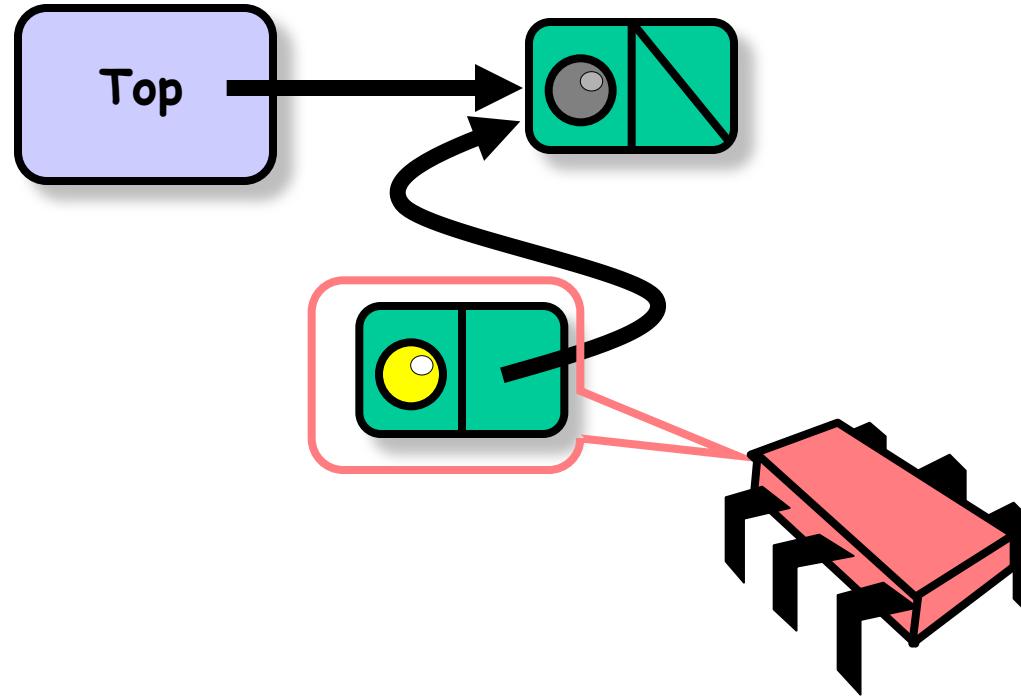
```
public class Node {  
    public T value;  
    public Node next;  
    public Node(T value) {  
        value = value;  
        next = null;  
    }  
}
```



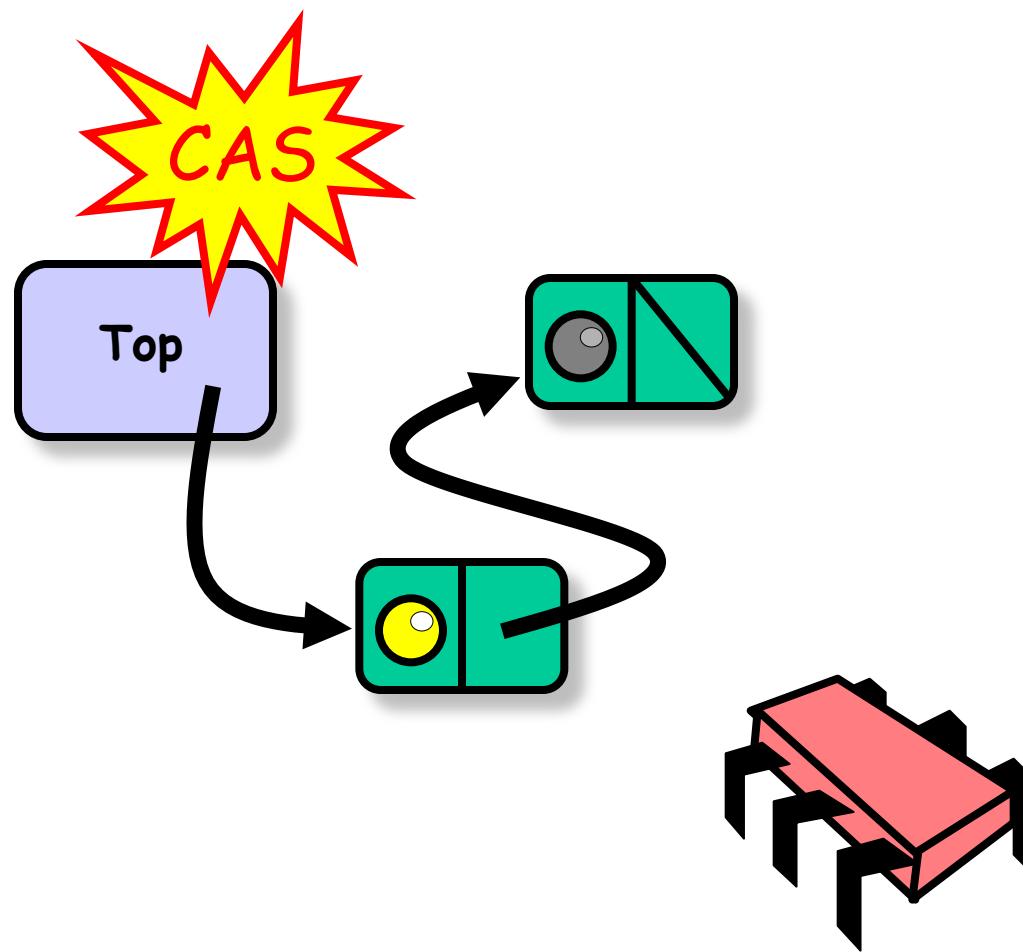
# Lock-Free Stack: Empty



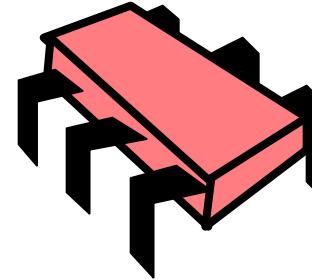
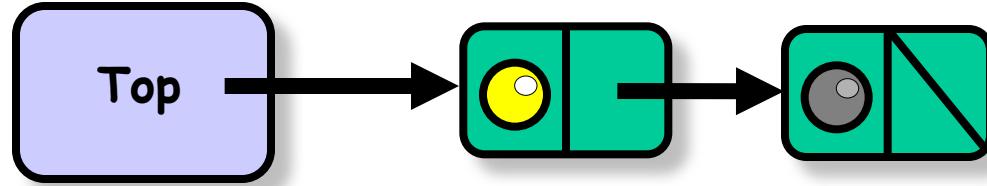
# Lock-Free Stack: Push



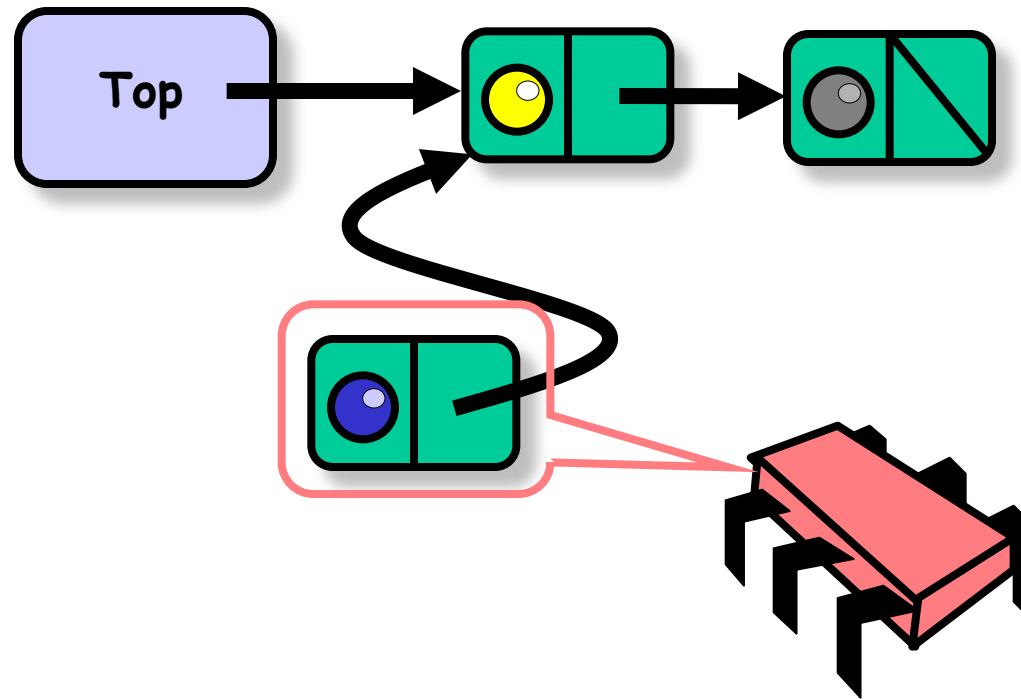
# Lock-Free Stack: Push



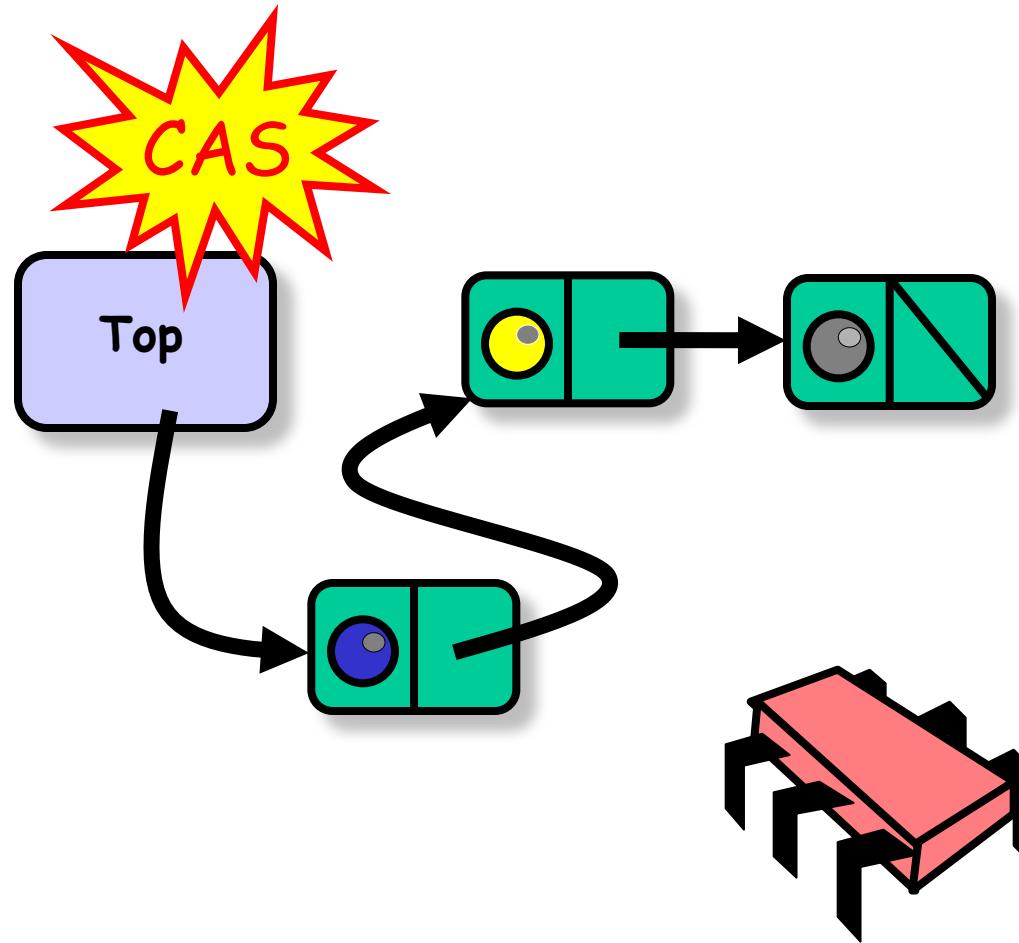
# Lock-Free Stack: Push



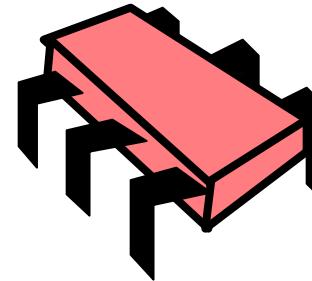
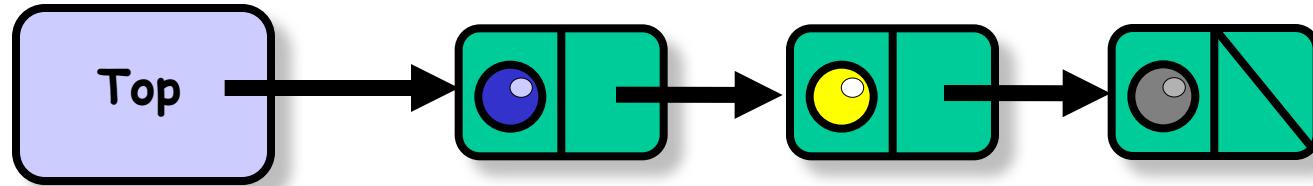
# Lock-Free Stack: Push



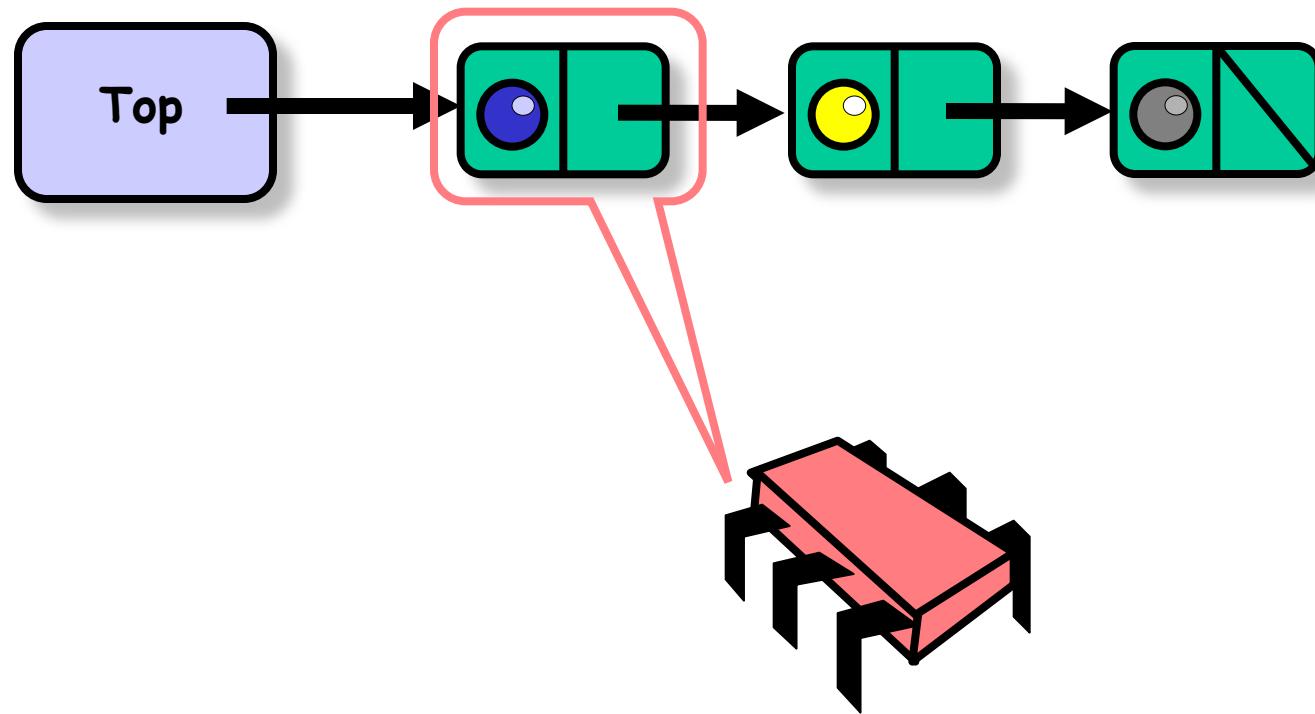
# Lock-Free Stack: Push



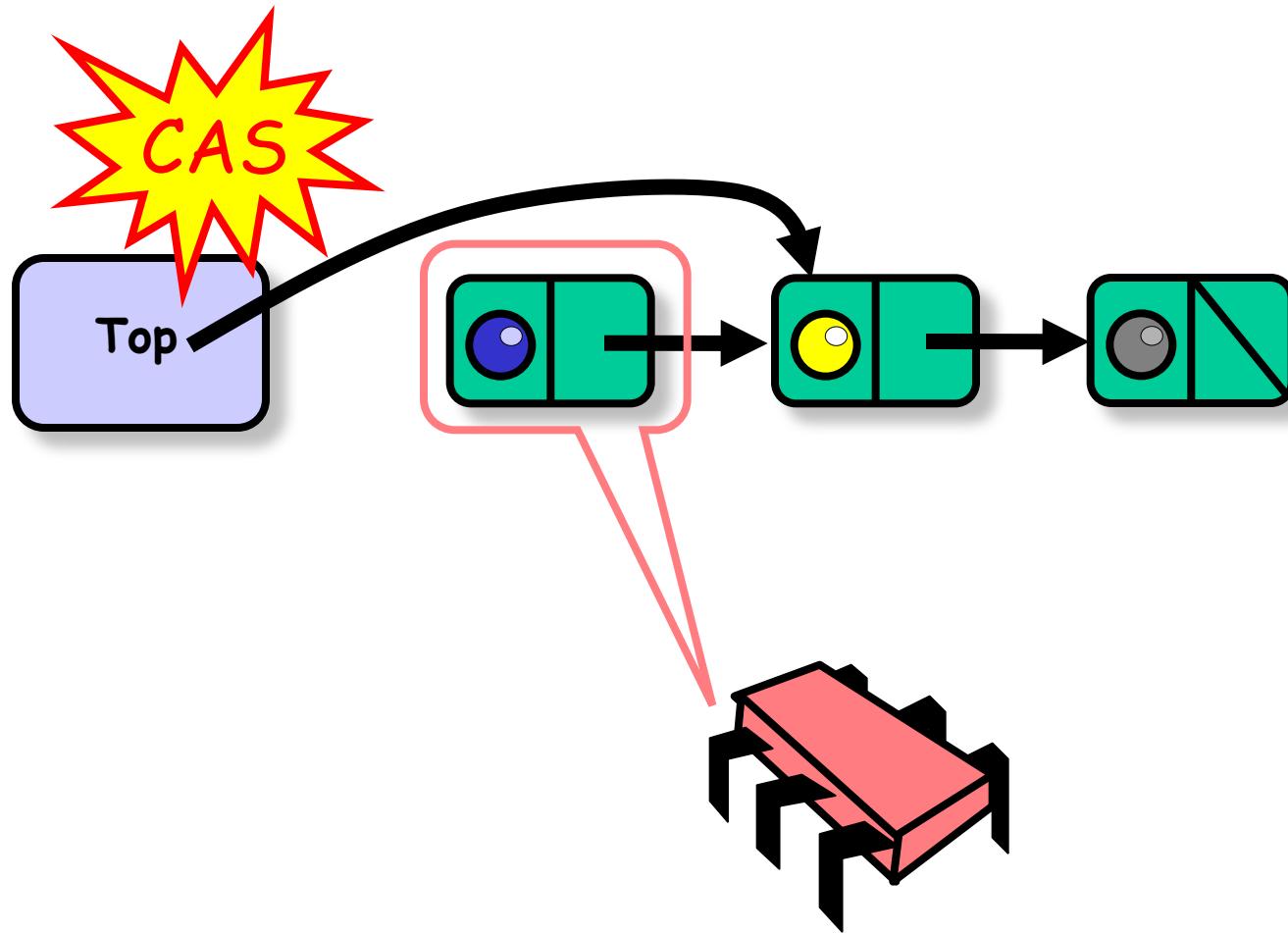
# Lock-Free Stack: Push



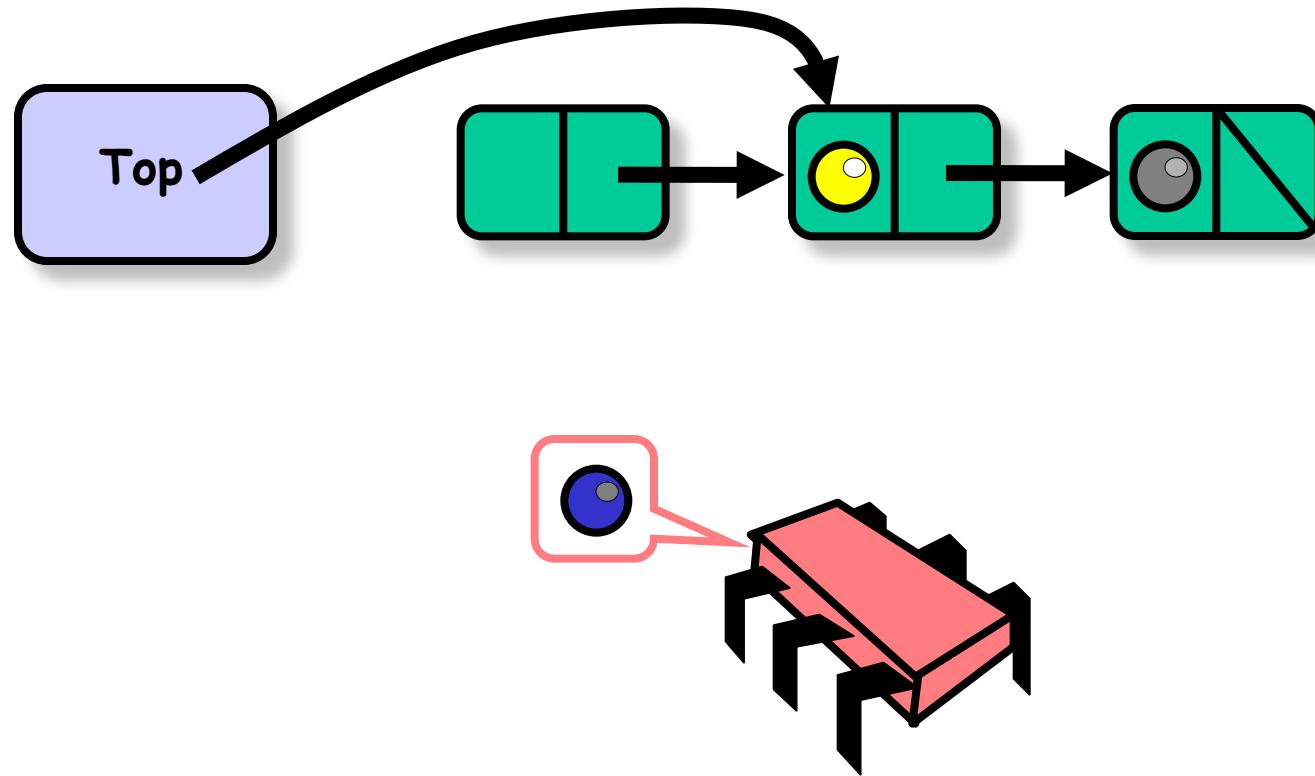
# Lock-Free Stack: Pop



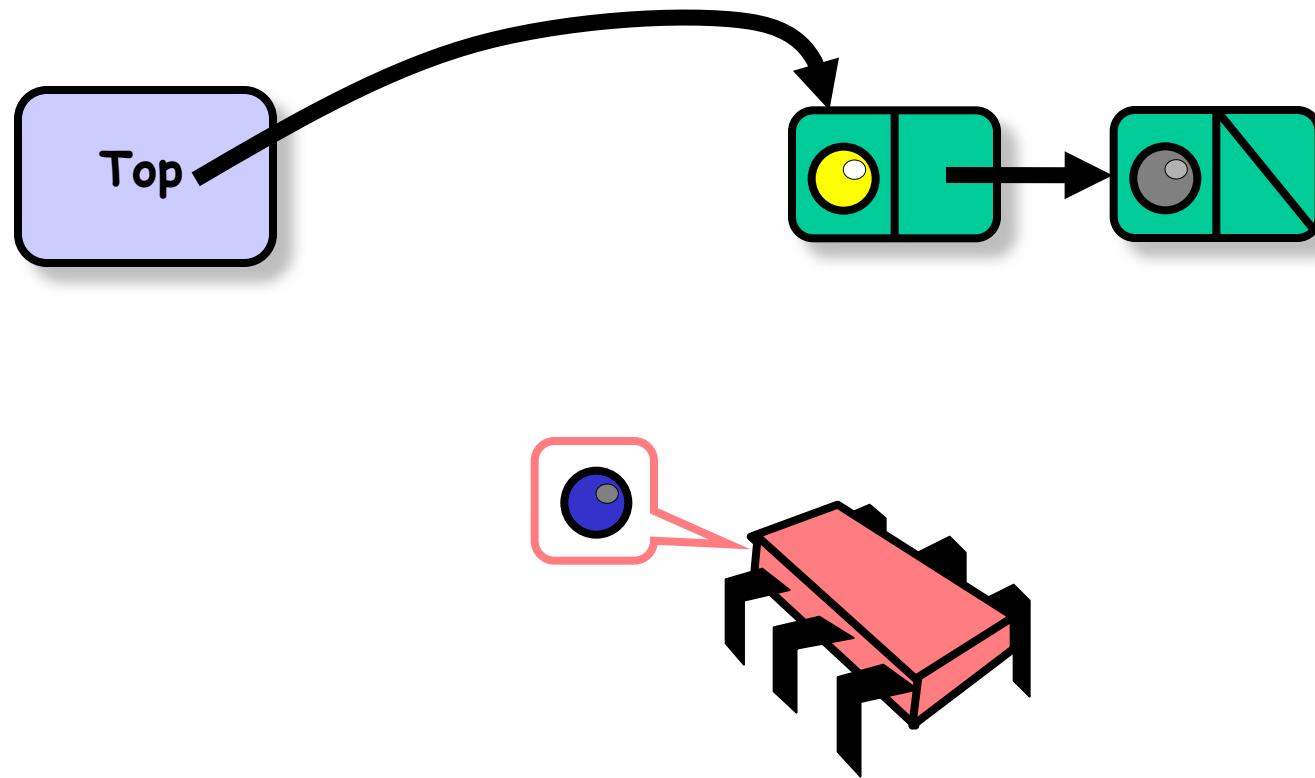
# Lock-Free Stack: Pop



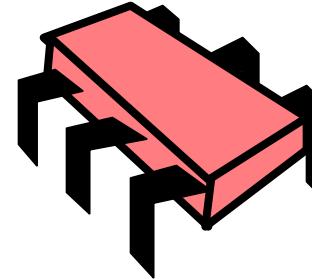
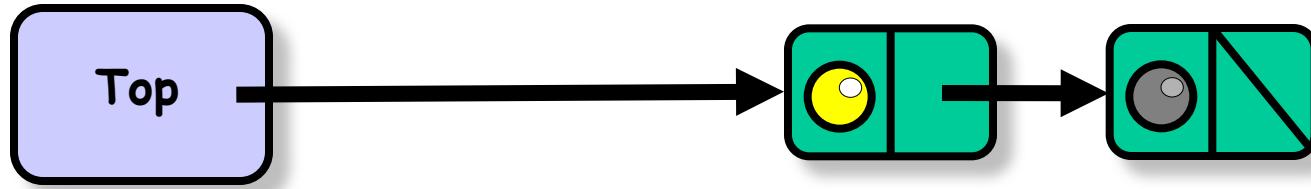
# Lock-Free Stack: Pop



# Lock-Free Stack: Pop



# Lock-Free Stack: Pop



```

public class LockFreeStack<T> {
    AtomicReference<Node> top = new AtomicReference<Node>(null);
    static final int MIN_DELAY = ...;
    static final int MAX_DELAY = ...;
    Backoff backoff = new Backoff(MIN_DELAY, MAX_DELAY);

    protected boolean tryPush(Node node) {
        Node oldTop = top.get();
        node.next = oldTop;
        return(top.compareAndSet(oldTop, node));
    }

    public void push(T value) {
        Node node = new Node(value);
        while (true) {
            if (tryPush(node)) {
                return;
            } else {
                backoff.backoff();
            }
        }
    }
}

```

**Reduce contention**

```

protected Node tryPop() throws EmptyException {
    Node oldTop = top.get();
    if (oldTop == null) {
        throw new EmptyException();
    }
    Node newTop = oldTop.next;
    if (top.compareAndSet(oldTop, newTop)) {
        return oldTop;
    } else {
        return null;
    }
}

public T pop() throws EmptyException {
    while (true) {
        Node returnNode = tryPop();
        if (returnNode != null) {
            return returnNode.value;
        } else {
            backoff.backoff();
        }
    }
}

```

```

public class LockFreeStack<T> {
    AtomicReference<Node> top = new AtomicReference<Node>(null);
    static final int MIN_DELAY = ...;
    static final int MAX_DELAY = ...;
    Backoff backoff = new Backoff(MIN_DELAY, MAX_DELAY);

    protected boolean tryPush(Node node) {
        Node oldTop = top.get();
        node.next = oldTop;
        return top.compareAndSet(oldTop, node);
    }

    public void push(T value) {
        Node node = new Node(value);
        while (true) {
            if (tryPush(node)) {
                return;
            } else {
                backoff.backoff();
            }
        }
    }
}

```

Linearization Points?

```

protected Node tryPop() throws EmptyException {
    Node oldTop = top.get();
    if (oldTop == null) {
        throw new EmptyException();
    }
    Node newTop = oldTop.next;
    if (top.compareAndSet(oldTop, newTop)) {
        return oldTop;
    } else {
        return null;
    }
}

public T pop() throws EmptyException {
    while (true) {
        Node returnNode = tryPop();
        if (returnNode != null) {
            return returnNode.value;
        } else {
            backoff.backoff();
        }
    }
}

```

```

public class LockFreeStack<T> {
    AtomicReference<Node> top = new AtomicReference<Node>(null);
    static final int MIN_DELAY = ...;
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    Backoff backoff = new Backoff(MIN_DELAY, MAX_DELAY);
}

```

```

protected boolean tryPush(Node node) {
    Node oldTop = top.get();
    node.next = oldTop;
    return(top.compareAndSet(oldTop, node));
}

public void push(T value) {
    Node node = new Node(value);
    while (true) {
        if (tryPush(node)) {
            return;
        } else {
            backoff.backoff();
        }
    }
}

```

Linearization Points?

```

protected Node tryPop() throws EmptyException {
    Node oldTop = top.get();
    if (oldTop == null) {
        throw new EmptyException();
    }
    Node newTop = oldTop.next;
    if (top.compareAndSet(oldTop, newTop)) {
        return oldTop;
    } else {
        return null;
    }
}

public T pop() throws EmptyException {
    while (true) {
        Node returnNode = tryPop();
        if (returnNode != null) {
            return returnNode.value;
        } else {
            backoff.backoff();
        }
    }
}

```

Is this  
good enough?

```

public class LockFreeStack<T> {
    AtomicReference<Node> top = new AtomicReference<Node>(null);
    static final int MIN_DELAY = ...;
    static final int MAX_DELAY = ...;
    Backoff backoff = new Backoff(MIN_DELAY, MAX_DELAY);
}

```

```

protected boolean tryPush(Node node) {
    Node oldTop = top.get();
    node.next = oldTop;
    return(top.compareAndSet(oldTop, node));
}

public void push(T value) {
    Node node = new Node(value);
    while (true) {
        if (tryPush(node)) {
            return;
        } else {
            backoff.backoff();
        }
    }
}

```

Pushes and Pops  
happen serially

Is this  
good enough?

```

protected Node tryPop() throws EmptyException {
    Node oldTop = top.get();
    if (oldTop == null) {
        throw new EmptyException();
    }
    Node newTop = oldTop.next;
    if (top.compareAndSet(oldTop, newTop)) {
        return oldTop;
    } else {
        return null;
    }
}

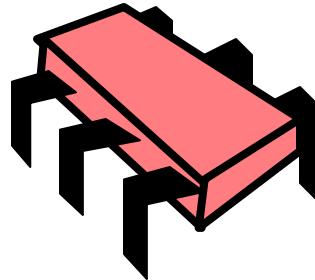
public T pop() throws EmptyException {
    while (true) {
        Node returnNode = tryPop();
        if (returnNode != null) {
            return returnNode.value;
        } else {
            backoff.backoff();
        }
    }
}

```

# Lock-Free Stack

- Operations happen serially
  - Push and pop fight for top
- Are stacks inherently sequential?
- How to enable concurrent operations?

# Observation



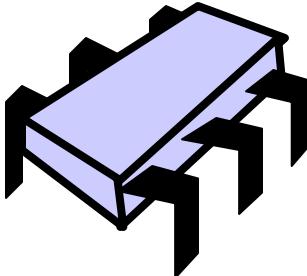
Push(○)

After an equal number  
of pushes and pops,  
stack stays the same

linearizable stack



Pop()

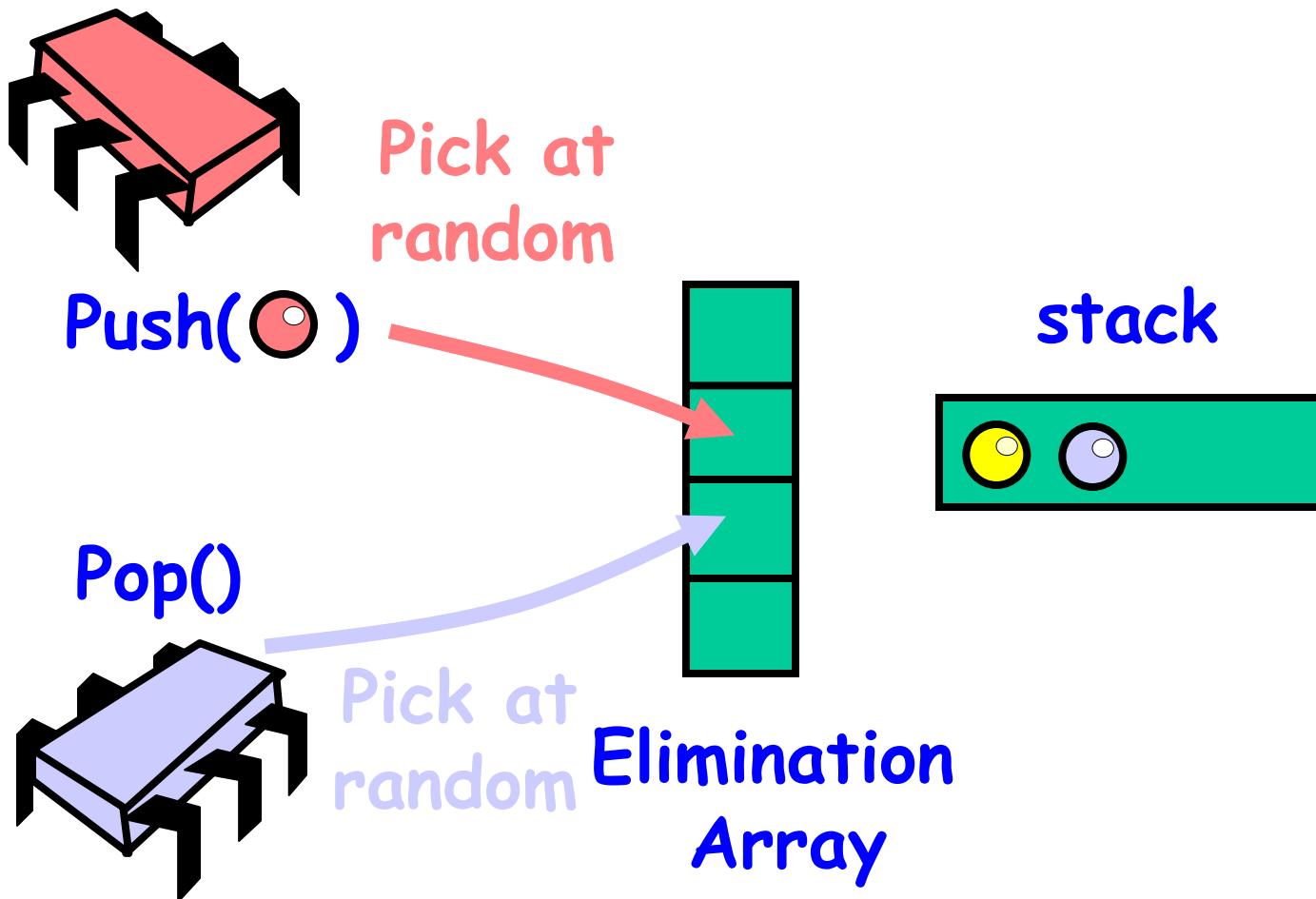


So if a push and a pop occurs  
concurrently, stack stays the same

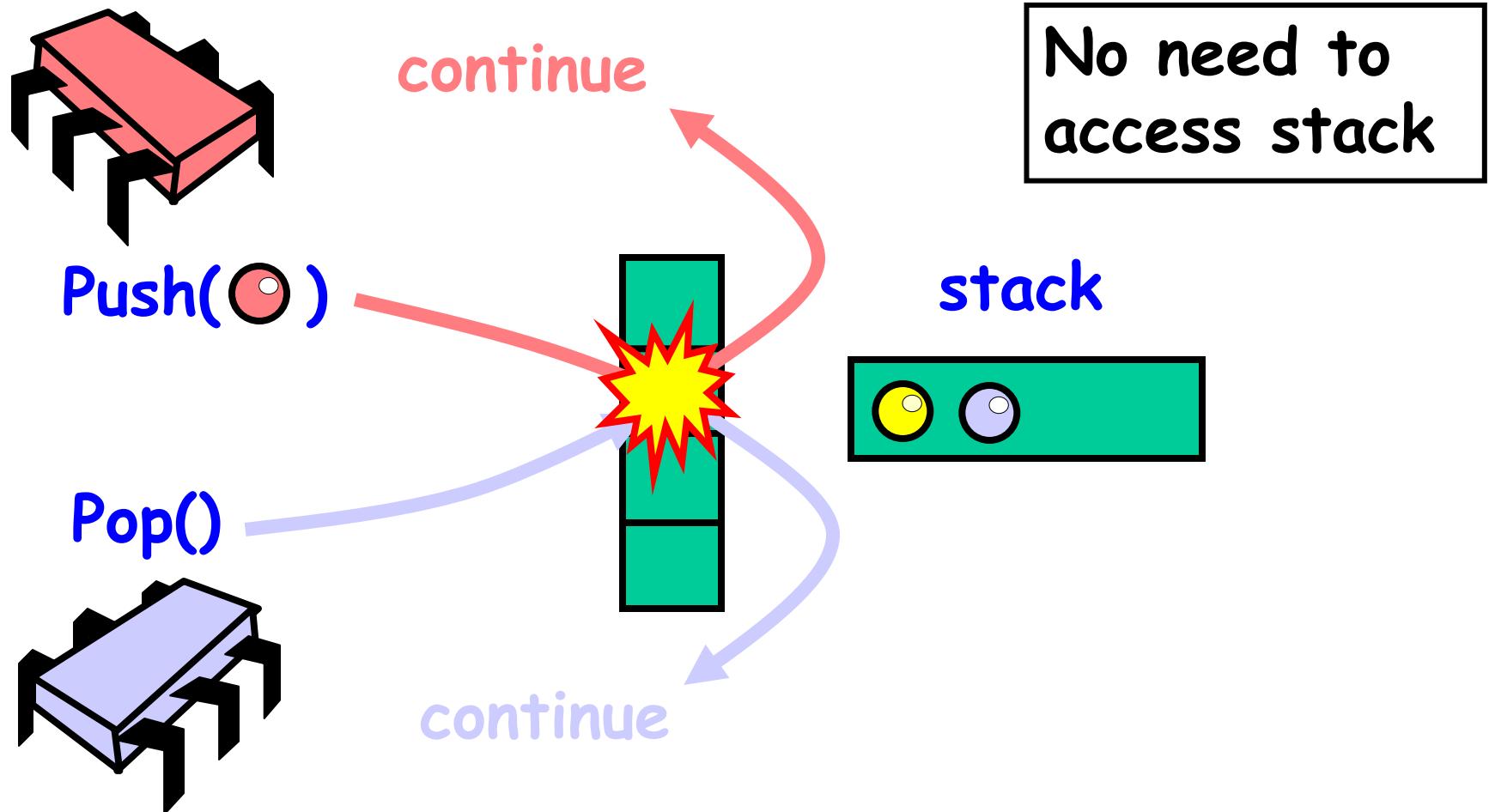
# Elimination Backoff

- Turn contention into parallelism
- Concurrent push and pop operations can be matched outside the linearizable stack

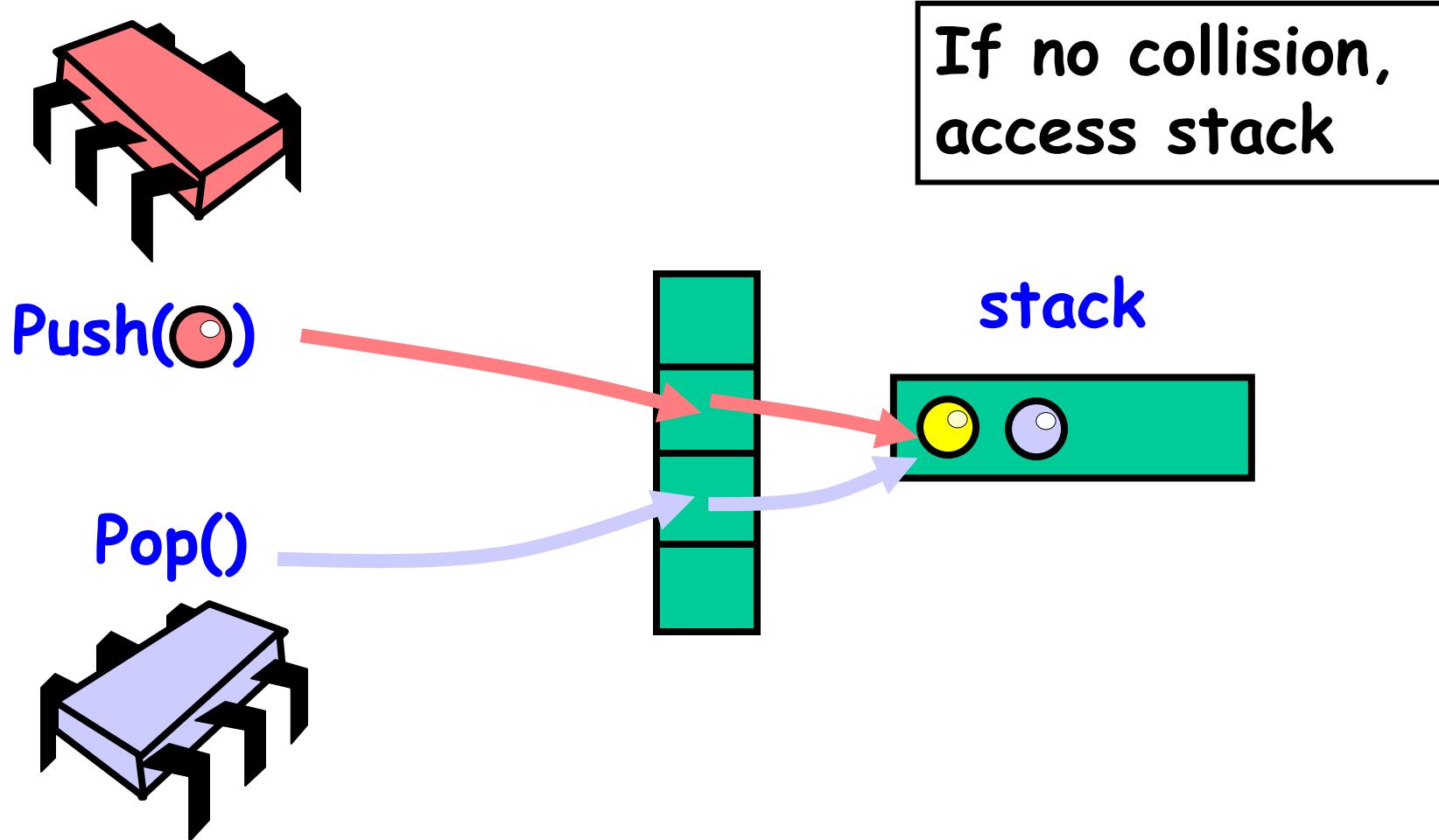
# Elimination Backoff



# Elimination Backoff: Collision



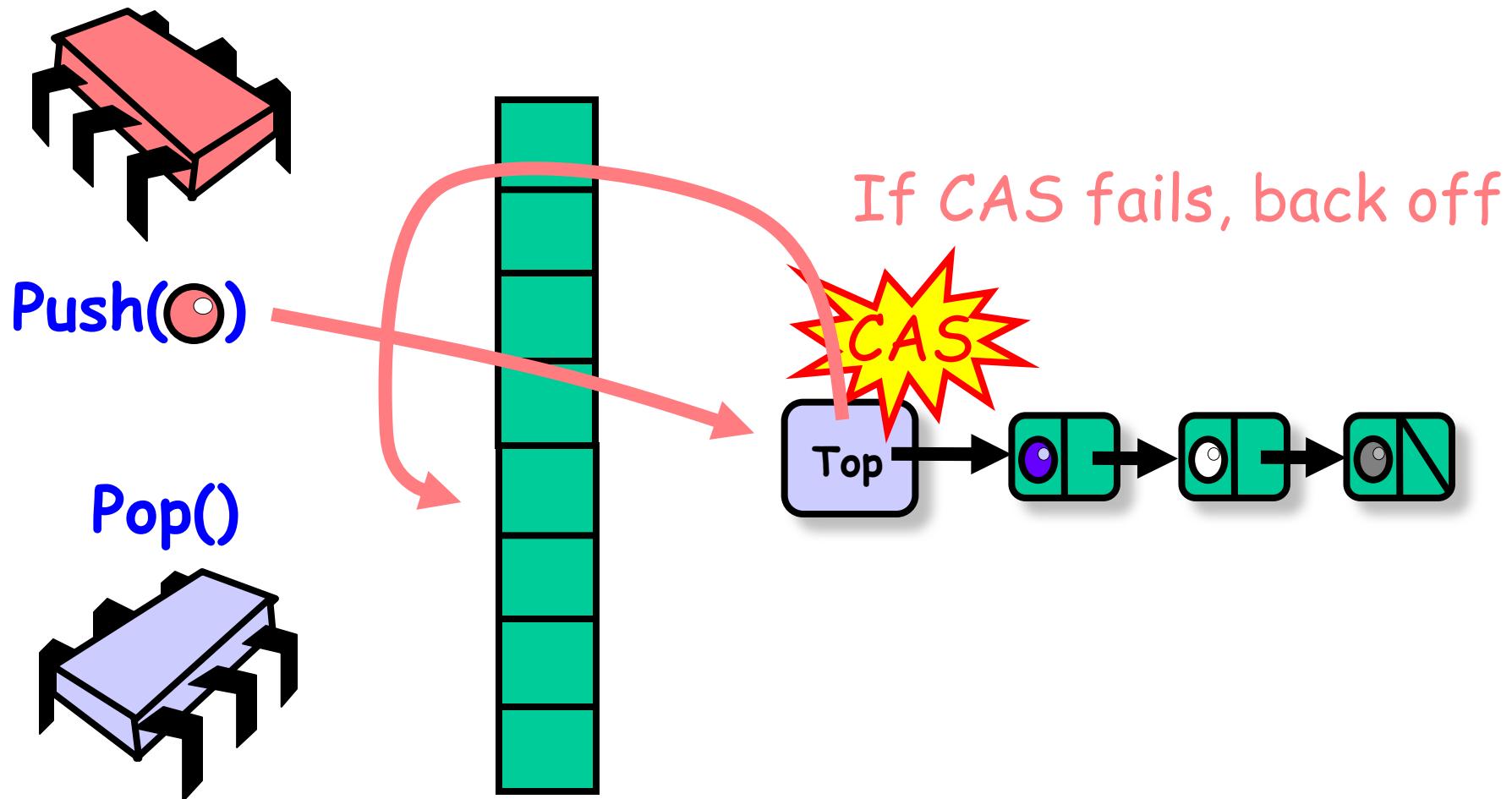
# Elimination Backoff: No Collision



# Elimination Backoff Stack

- Turn contention into parallelism
- Concurrent push and pop operations can be matched outside the linearizable stack
- Access Lock-free stack:
  - If uncontended, apply operation
  - If contended, back off to elimination array and attempt elimination

# Elimination Backoff Stack



# Elimination Array

Checkout code  
in Textbook!

- Every slot is in one of 3 states: EMPTY, WAITING, BUSY
- First thread finds an **EMPTY** slot:
  - Places its item and changes EMPTY to WAITING
  - Keeps on waiting for state to change (or until timeout)
- Second thread finds a **WAITING** slot:
  - Exchanges its item and changes WAITING to BUSY (and returns)
- First thread observes state change to **BUSY**:
  - Reads the item and changes state to EMPTY (and returns)
- Choices: Size of array, how to search for empty slot, etc.

# Elimination Backoff Stack

- How to reason whether this is linearizable?
- Un-eliminated calls:
  - Linearized as before
- Eliminated calls:
  - Concurrent operations
  - Linearize pop immediately after matching push  
(i.e., when they collide)
- Combination is a linearizable stack