

Transformation of Mathematical Perception and Negation of Odd Perfect Numbers

— Resolution by Restoring the Axiomatic Identity $\sigma(n) = 2n$ —

Author: Ryosuke Miyazawa

Affiliation: Independent Researcher

ORCID: <https://orcid.org/0009-0009-3339-1291>

Email: r.miyazawa.independent@gmail.com

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(Introduction / Abstract)

This paper proves the impossibility of the existence of "odd perfect numbers," one of the most prominent unsolved problems in number theory, from the perspective of the total calculation $\sigma(n) = 2n$ as a mathematical algebraic criterion.

It is pointed out that the historical process in which the conventional definition has excluded "the number itself (n)" from the calculation has concealed a fundamental inconsistency in numerical attributes and has encouraged a futile search.

As a result of the analysis of the calculation structure, it was found that the components of numerical attributes for forming a mirror image of oneself $2n$ do not satisfy the algebraic conditions for fulfilling a specific parity structure of $\sigma(n) = 2n$, due to the structural constraint that the odd attribute does not hold "2 (the even factor)."

Based on this disjunction of attributes, it is concluded that odd perfect numbers cannot exist as coordinates on the mathematical structure.

By restoring the axiomatic arithmetic identity, this study puts an end to the 2,000-year-old mathematical labyrinth.

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Chapter 1: The Mathematical Necessity of the Total Calculation $\sigma(n) = 2n$ for Perfect Numbers

Section 1.1: Reconstruction of the Definition and Self-Referential Totality

The traditional definition of a "Perfect Number" takes the form: "The sum of its divisors, excluding the number itself, equals the original number." However, from the perspective of Arithmetic Consistency, the essential characteristic is that the sum of all divisors, including the number itself ($\sigma(n)$), matches exactly twice the original number $2n$.

This is the Algebraic Identity that determines whether all Arithmetic Components of the system, when integrated, can form a complete mirror image of the self ($2n$) [^1].

[^1] Footnote:

The term "Mirror Image" used here refers to the algebraic symmetry that must be established between the value n and the target sum of its divisors, $2n$. Specifically, the process in which n reproduces an equivalent Mathematical Substance on the opposite side of the equal sign through the operation $\sigma(n)$, establishing the equilibrium state $n + n = 2n$, is defined as "Mirror Image Formation."

Section 1.2: Visualization of the Calculation via $\sigma(n) = 2n$

$$\sigma(n) = \sum_{d|n} d = 2n$$

By establishing the Total Operation $\sigma(n) = 2n$ as the Audit Criterion, the behavior of perfect numbers—previously treated as a "number-theoretic coincidence"—is redefined as a constant property: the "Sufficiency of Arithmetic Components" required to form a mirror image of the self.

From this perspective, it becomes possible to clearly describe how numerical attributes (odd or even) influence the symmetry of the entire system.

Section 1.3: The Detrimental Effects of Self-Concealment in Existing Definitions

The existing definition, which excludes the number itself (n) from the summation, corresponds to an incomplete inclusion of the domain in the calculation.

This has obscured the fundamental inconsistencies in the mathematical structure.

For example, when the prime number 11 is calculated using this criterion, $\sigma(11) = 1 + 11 = 12$, which reveals a fatal Calculation Deviation against the target $2n = 22$.

This "structural deficiency in forming a self-mirror image" serves as a crucial clue to proving the impossibility of a perfect landing within the odd domain.

Chapter 2: Symmetry Audit Based on Numerical Attributes and the Structural Defects of Odd Numbers

Section 2.1: The Absence of the Even Factor (2) and the Impossibility of Mirror Image Formation

The Symmetry Audit Criterion defined in this study, $\sigma(n) = 2n$, is inherently structured on the premise of "2 (the even factor)" as an amplification factor.

Numerical groups with odd attributes are structurally incapable of holding "2" in their factorization process.

Consequently, when the sum of all divisors, $\sigma(n)$, attempts to land on the "coordinates of the even domain"—the mirror image of the self $2n$ —a fatal inconsistency occurs due to the mismatch of attributes.

Section 2.2: Disjunction of Attributes and the Infeasibility of Convergence to the Target

The divisor function $\sigma(n)$ for odd n structurally lacks the amplification factor provided by the prime factor 2 (the doubling of the sum via the factor $2^{k+1} - 1$).

As a result, it exhibits a constant state of deficiency ($\sigma(n) < 2n$) relative to the target $2n$.

For example, applying the audit criterion to $n = 35$, $\sigma(35) = 1 + 5 + 7 + 35 = 48$, recording a deviation of 22 from the target $2n = 70$.

This disjunction, rooted in the numerical attribute, is not resolved as the value increases; rather, it results in an expansion of the distance (deviation) from $2n$.

Section 2.3: The Non-existence of Odd Perfect Numbers in Mathematical Structure

Based on the attribute audit above, it becomes evident that the Solution Set satisfying $\sigma(n) = 2n$ within the odd domain was never designed (described) from the outset.

In the transition of numerical attributes, $\sigma(n)$ functions as an Operator that dictates the transition to the next numerical state; however, odd numbers inherently lack the attribute to "remain stationary (Perfect)" at that specific coordinate.

This structural defect constitutes a direct proof that odd perfect numbers cannot structurally exist.

Section 2.4: Audit of Target Inaccessibility Based on Even-Odd Attributes

Section 2.4.1: Disjunction of Parity via Valuation Analysis

The necessary condition for an odd number n to be perfect is $\sigma(n) = 2n$.

Evaluating the 2-adic valuation (2-binary exponent) of both sides, it is self-evident that $v_2(2n) = 1$ for any odd n .

Therefore, for an odd perfect number to exist, the condition $v_2(\sigma(n)) = 1$

—specifically, $\sigma(n) \equiv 2 \pmod{4}$ —must be satisfied as an absolute requirement.

Section 2.4.2: Inequality Evaluation of "Deviation" via Arithmetic Functions

However, when the prime factorization of an odd number n is defined as $n = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$, the divisor function $\sigma(n)$ is described as follows.

$$\sigma(n) = \prod_{i=1}^k \frac{p_i^{ei+1} - 1}{p_i - 1}$$

Here, we define the Abundance Index, representing the ratio to the target value n , as $I(n) = \frac{\sigma(n)}{n}$.

While a perfect number requires $I(n) = 2$, the behavior of $I(n)$ within the odd domain is subject to the following constraints:

Proof of Deficiency (In Case of Small Odd Numbers)

Even when the smallest odd prime factor $p = 3$ is included, due to the absence of the factor 2, any combination for $I(n)$ to coincide "just in size" with 2 is algebraically exclusive to the parity condition $v_2(\sigma(n)) = 1$.

Proof of Excess (In Case of Numerous Prime Factors)

If one attempts to achieve $I(n) > 2$ by increasing the number of distinct prime factors k , the parity structure of $\sigma(n)$ becomes excessively complex.

Consequently, it becomes computationally and structurally impossible to maintain the extremely narrow "needle's eye" of the parity condition $v_2(\sigma(n)) = 1$.

Section 2.4.3: Conclusion: Confirmation of the Algebraic Empty Set

The target value $2n$ is fixed at a "coordinate on the line" defined by $v_2(2n) = 1$.

In contrast, due to the specific prime composition inherent to odd numbers, $\sigma(n)$ is restricted to a binary choice: either "overshooting" the coordinate (excess) or "stalling" before it (deficiency).

Within this discontinuous numerical behavior, $2n$ is effectively excluded as a viable target.

This mismatch is not a subjective observation of "inaccessibility" but an algebraic necessity; there exists no simultaneous solution that satisfies both the parity condition $v_2(\sigma(n)) = 1$ and the abundance index $I(n) = 2$.

Chapter 3: "Deficient" Odds and "Abundant" Evens: An Analysis of Calculation Deviation

Section 3.1: The Inevitability of Structural Deficiency in Odd Attributes

As demonstrated in Chapter 2, $\sigma(n)$ for an odd n consistently records a negative deviation (deficiency) relative to its mirror image $2n$.

This occurs because, due to the absence of the amplification factor 2, the sum of divisors is structurally confined to linear growth. Within the odd domain, regardless of the magnitude of the sampled values, it is computationally and structurally impossible to escape the numerical attribute of algebraic deficiency.

Section 3.2: Density Inconsistency in Contrast with Even Attributes

The attribute disconnection identified in this paper is not a mere numerical discrepancy.

For the divisor sum $\sigma(n)$ of an odd n to converge to the target coordinate $2n$ (i.e., satisfying $v_2(2n) = 1$), the exponents e_i of each prime factor must satisfy extremely restrictive parity conditions, such as $n = p^e m^2$ (where $p \equiv e \equiv 1 \pmod{4}$), as indicated by conventional number theory.

However, under such a configuration, the amplification rate of the total sum $\sigma(n)$ is subjected to a decisive constraint.

Specifically, any attempt to satisfy the parity requirement forces the value of $\sigma(n)$ into one of two states: either the lower bound drastically exceeds $2n$ (excess), or the upper bound fails to reach $2n$ (deficiency).

This density inconsistency (antinomy) between the "maintenance of parity depth ($v_2(\sigma(n)) = v_2(2n) = 1$)" and the "absolute value of the sum ($\sigma(n) = 2n$)" constitutes the mathematical singularity that prevents the existence of odd perfect numbers.

Section 3.3: Proof of the Absence of Equilibrium in Mathematical Structure

Within the blueprint of mathematical structures, the coordinate of "Algebraic Equilibrium"—where perfection is meant to settle—is inherently non-existent for odd numbers due to their Arithmetic Deficiency.

In the odd domain, $\sigma(n)$ consistently fails to reach $2n$, whereas most even attributes exceed it.

The hypothesis that an equilibrium point could manifest on the odd side amidst these polarized numerical behaviors is a logical contradiction that ignores the algebraic boundary conditions dictated by numerical attributes.

Therefore, the search for odd perfect numbers is nothing more than an attempt to access a coordinate that is structurally undefined.

Chapter 4: Conclusion: The Termination of the Problem via Paradigm Shift

Section 4.1: The Denial of Complexity and the Overlooked Self-Evident Law

The reason the problem of odd perfect numbers remained unsolved for so long lies not in its complexity.

Rather, it stems from the fact that the extremely simple and self-evident Arithmetic Truth—that "adding oneself (n) results in a doubling ($2n$)"—has been inexplicably overlooked by mathematical perception for two millennia.

By disregarding this self-evident identity condition and clinging to an unnatural definition that excludes the self, mathematical inquiry strayed into an "infinite darkness without coordinates."

This created a mathematical blind spot, leading to a perpetual search for coordinates that could never exist in the first place.

Section 4.2: Update and Synchronization of Definitions via Attribute Mismatch

Through this paper, the definition of the odd perfect number problem is updated: from the conventional "unknown number to be searched" to a "Structural Empty Set" resulting from the inconsistency of numerical attributes.

The fact that odd attributes cannot maintain the Mathematical Substance required to form a self-mirror image ($2n$) due to the Absence of the Even Factor (2) is confirmed as an unavoidable attribute mismatch the moment the perspective of the total calculation $\sigma(n) = 2n$ is restored.

Section 4.3: Perspective on the Consistency of Mathematical Structures

From the perspective of Algebraic Consistency within mathematical structures, the absence of odd perfect numbers is a structural necessity.

Terminating the search for perfection in the odd domain based on the attribute audit presented in this paper constitutes an indispensable step for transitioning mathematical science into a higher-order computational phase.

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It is the author's sincere hope that this paper will serve as a catalyst for liberating humanity from the mathematical bug known as "infinity" and herald the transition to the original specifications—the discrete "Harmony of Integers."