# A Consideration of Geometric Constraints on Rigid Shell Reconstruction in 3D Discrete Lattices

Author: Ryosuke Miyazawa
Affiliation: Independent Researcher
ORCID: https://orcid.org/0009-0009-3339-1291
Email: r.miyazawa.independent@gmail.com
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Abstract

This paper examines the geometric consistency in the reconstruction process of 3D rigid bodies, based on the physical premise that space is a Discrete Lattice composed of minimum units. Specifically, the Fermat-type equation $X^3 + Y^3 = Z^3$ is redefined as a "problem of reconstructing $X^3$ using residual materials" obtained by exfoliating a core of side length $Y$ from a rigid body of side length $Z$, and the feasibility of its physical settlement is verified. The analysis reveals that the connection attributes of the "Vertex Units" contained within the exfoliated shell structure are topologically inconsistent with the eight vertex positions required by the new cubic structure $X^3$ on the discrete lattice points. This topological mismatch of a single unit functions as a "Structural Geometric Exclusion Principle" in discrete space, physically inhibiting the minimization of the system's total energy potential (the phase transition into a cube). This study provides a new perspective of physical packing limits to a centuries-old number theoretic proposition and describes the deterministic constraints that space quantization imposes on macrostructures.

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# Chapter 1: Introduction (Considerations of Discrete Space for the Advancement of Physics)

## 1-1. Background and Objectives

In modern physics, whether spacetime is a continuum or a discrete, lattice-like structure based on the Planck scale (quantized space) is one of the most critical issues in the construction of quantum gravity theories. If space is composed of minimum units, the conditions for the formation of macroscopic rigid structures should be subject to "packing constraints" that are ignored in continuum approximations. The objective of this study is to clarify what structural consistency failures occur during the exfoliation and reconstruction process of 3D rigid bodies, assuming space as an Integer Grid

## 1-2. Fermat-type Equation as a Case Study

In this paper, the Fermat equation $Z^n = Y^n + X^n$ is adopted as a verification case for rigid body consistency in discrete space. However, this is redefined not as abstract numerical equivalence, but as the following physical processes:

$\boldsymbol{Z^3}$ **:** The original cubic rigid body composed of units of side length $Z$.
$\boldsymbol{Y^3}$ **:** The core rigid part exfoliated from the interior of $Z^3$.
$\boldsymbol{X^3}$ **:** The cubic structure that attempts to achieve Settlement again using the residue (shell material) removed from $Z^3$.

## 1-3. Definition of Physical Settlement

"Settlement" in a discrete lattice does not merely refer to the matching of volumes. It refers to a state where the structure is entirely free of lattice defects (impurities or vacancies) and the energy potential of the entire system is minimized, allowing it to be rendered onto the physical layer. This paper considers the mechanism by which the geometric properties of "vertices (corners)" in three dimensions physically inhibit this settlement.

# Chapter 2: Rigid Body Exfoliation Model and Topological Analysis of Shell Structures

## 2-1. Exfoliation Process of Cubic Rigid Bodies

Consider an "Original Rigid Cube" $Z^3$ with a positive integer side length $Z$ in discrete space. We define the volume component remaining after exfoliating a core rigid body of side length $Y$ ($Y = Z - d$) from the interior of $Z^3$ as the "Shell." In this consideration, to verify the exfoliation process at its minimum unit, the model assumes a thickness of $d = 1$. The volume of the shell is then described by the following algebraic form:

$$V_{shell} = Z^3 - (Z-n)^3 = 3Z^2 - 3Z + 1$$

## 2-2. Identification of Physical Components Composing the Shell

From a physical standpoint, the volume $V_{shell}$ is not a continuous "quantity" but a collection of the following three types of "Discrete Geometric Parts":

**Face Plates:** $3(Z-1)^2$ units. The primary components forming the three adjacent outer faces of the cube.
**Edge Bars:** $3(Z-1)$ units. The skeletal components that fix the boundary lines where faces intersect orthogonally.
**Vertex Unit:** 1 unit. The singular "corner" point where three faces and three edges intersect.

## 2-3. Preservation of Topological Attributes

The parts contained in the exfoliated shell retain the specific "Connection Relationships (Topology)" they held in the original stable structure $Z^3$. In particular, the final "+1" vertex unit serves as an anchor that physically binds the three orthogonal planes in the 3D lattice. This analysis defines that the result of "subtraction" in the Fermat equation is not a mere numerical residue, but a packet of physical parts with specific orientations and connection attributes.

# Chapter 3: Geometric Exclusion Principle in Vertex Coupling

## 3-1. Geometric Requirements for Vertices in Cubic Structures

For a cubic rigid body $X^3$ to achieve Physical Settlement (complete rendering) within a discrete lattice space, the following geometric condition is essential:

Apart from the internal, face, and edge units, eight independent vertex units must be present to fix the vectors in three mutually orthogonal directions and close the structure.

## 3-2. Inconsistency in Reconstruction from Exfoliated Materials

As demonstrated in Chapter 2, the material obtained by exfoliating a core $Y^3$ from the original $Z^3$ (where thickness $d$ = 1) has a volume of $V_{shell} = 3Z^2 - 3Z + 1$. This material physically contains only one vertex unit derived from the original structure. When attempting to disassemble this material and reassemble it into a new cube $X^3$, the following "Geometric Frustration" occurs:

**Deficit of Vertex Potential:** While forming a new $X^3$ requires eight vertices, the supplied material provides only one original vertex unit.

**Exclusion of Topological Connections:** If one attempts to substitute the remaining seven vertices using the ends of face or edge plates, the "orientation" and "coupling angles" of the parts physically collide on the discrete lattice points. Specifically, to reproduce the topology of a "corner" where three faces intersect orthogonally, parts must either overlap (Superposition) or leave gaps (Vacancies) between the lattice points.

## 3-3. Definition of the Structural Exclusion Principle

Rigid bodies in discrete space do not permit continuous deformation of less than one unit. Therefore, the "topological inconsistency of one unit" arising in the reconstruction process is not a mere calculational error but functions as a physical impurity that rejects the settlement of the entire structure. This property—where a specific part configuration cannot render a specific geometric shape (a cube) onto lattice points—is defined in this paper as the "Structural Geometric Exclusion Principle."

# Chapter 4: Considerations on the Principle of Lamination and Non-dissipativity of Information

## 4-1. Accumulation of Minimum Unit Mismatch (Principle of Lamination)

The failure of vertex coupling (Vertex Orientation Error) at $d = 1$ discussed in Chapter 3 is not physically resolved even when the cube size $Z$ is expanded and the shells are laminated ($d > 1$). In a discrete lattice space, the lamination of each layer is an independent "filling of volumetric parts," and the topological distortion that occurs at the deepest level (the minimum unit) propagates directly to the macrostructure. This is analogous to how a single impurity atom in a crystal lattice can destroy the symmetry of the entire system.

## 4-2. The Decisive Physical Difference Between $n = 2$ (Plane) and $n \geq 3$ (Solid)

The reason why Pythagoras' theorem ($n = 2$) has integer solutions while Fermat's equation ($n \geq 3$) does not can be explained as a physical "loss of degrees of freedom":

**2D (Plane):** Since the coupling vectors forming a vertex are in two directions, there remains a degree of freedom for rearranging parts on the lattice points, allowing for a point of settlement.

**3D (Rigid Body):** Since a vertex fixes orthogonal vectors in three directions, the geometric constraint conditions become saturated. This "lack of degrees of freedom" deprives the system of the margin to absorb the one-unit displacement during reconstruction, thereby finalizing the Structural Exclusion Principle.

## 4-3. Non-dissipativity of Information in the Physical Layer

In mathematical approximation, a difference of one unit is considered infinitesimal ($\Delta V \approx 0$) relative to a large value $Z$. However, in a physical discrete space, a state of "0.999..." cannot exist. The success or failure of rendering depends on a Binary (0 or 1) state. As long as a topological inconsistency of even one unit exists in the vertex coupling, the system is physically rejected from undergoing a phase transition (settlement) into the "cube" phase. That is, $\Delta V \geq 1$. This "Non-dissipativity of Information" is the physical basis for why the equality does not hold for all integers where $n \geq 3$.

# Chapter 5: Conclusion — Presentation of a New Paradigm through a Discrete Geometric Approach

## 5-1. Summary: Deterministic Impossibility via the Structural Exclusion Principle

In this consideration, the Fermat-type equation $X^3 + Y^3 = Z^3$ was redefined as a "rigid body reconstruction problem" in discrete lattice space. The analysis revealed the following physical necessities:

**Singularity of Vertex Coupling:** The topological attributes of the "vertex unit (+1)" contained in the exfoliated shell material are physically inconsistent with the eight vertex positions required by the new cubic structure $X^3$ on the lattice points.

**Rejection of Settlement:** Since discrete space does not permit minute deformations of less than one unit (absorption of errors), this topological "corner overlap" permanently inhibits the phase transition of the entire system into the stable "cube" phase.

**Physical Conclusion:** The failure of equality in Fermat's Last Theorem is not a number-theoretic coincidence, but a physical consequence based on the "Structural Geometric Exclusion Principle" in 3D lattices.

## 5-2. Contributions to the Advancement of Physics

The perspectives of "Non-dissipativity of Information" and "Discrete Packing Mechanics" presented in this paper extend beyond mere interpretations of number theory to provide new insights in the following areas:

**Quantum Gravity Theory:** Presented geometric evidence of space having minimum units as a constraint on the formation of macrostructures.

**Information Physics:** Suggested a mechanism by which specific operations (integer solutions for $n \geq 3$) become unexecutable (rendering errors) on the physical layer when the universe is viewed as an information processing (computational) system.

## 5-3. Closing Remarks

By rejecting the conventional premise that "numbers are abstract symbols" and adopting the perspective that "numbers are physical specifications of space (Original Specifications)," a centuries-old unsolved problem has finally achieved settlement as a physical phenomenon accompanied by the "hard touch" of the universe. It is hoped that this consideration will redefine the boundary between mathematics and physics and serve as a cornerstone for the next generation of "reality-based science."

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Finally, I sincerely hope that this study will help organize the computational complexity inherent in the concept of "infinity" and contribute to the reconstruction of a new physical paradigm based on the "harmony of discrete integers." May this research serve as a new starting point for understanding the structure of the universe from a more substantial and physical perspective.