BIOL 274 Homework 1

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Question 1

> Assume that once a drug is administered to a patient, it leaves the body following exponential decay. After administration of a dose, the concentration of the drug in the body decreases by 50% in 30 hours. Approximately how long does it take for the drug to decrease to 1% of its initial value (round to the nearest hour)?

We can represent the concentration of drug in the body at time t as $x(t) = x_0 e^{kt}$. Since the drug leaves the body according to an exponential decay model, that means k should be negative.

We are given $\frac{x}{x_0} = 0.5$ at $t = 30$ hours. Plugging into our equation, we get

$$0.5 = e^{30k}$$

$$\ln(0.5) = \ln(e^{30k})$$

$$\ln(0.5) = 30k$$

$$k = \frac{\ln(0.5)}{30}$$

```r
k = log(0.5)/30
k
```

```
## [1] -0.02310491
```

Now, we want to know how long it will take for x to be 1% of x_0, or at what time t we will have $\frac{x(t)}{x(0)} = 0.01$.

$$0.01 = e^{kt}$$

$$t = \frac{\ln(0.01)}{k}$$

```r
t = log(0.01)/k
round(t, 0)
```

```
## [1] 199
```

It will take approximately 199 hours for the drug to decrease to 1% of its initial value.

Question 2

Consider the population model: $\frac{dP}{dt} = 0.3\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)P$.

a. For what values of P is the population increasing?
b. For what values of P is the population decreasing?

If the population is increasing, that means the rate of change $\frac{dP}{dt}$ is positive, while a decreasing population means that $\frac{dP}{dt}$ is negative. First we need to find the equilibrium points, or the values of P for which $\frac{dP}{dt} = 0.3\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)P = 0$.

We can see that this will clearly occur if $P = 0$; i.e., the population has no organisms in it. The other equilibrium points occur when

$$1 - \frac{P}{200} = 0$$
$$1 = \frac{P}{200}$$
$$P = 200$$

and similarly when $1 = \frac{P}{50}$ or $P = 50$. So we need to determine the sign of $\frac{dP}{dt}$ on the intervals $(0, 50)$, $(50, 200)$, and $(200, \infty)$. Let's evaluate some convenient values within these intervals: 25, 100, and 400.

```r
eqn <- function(P){
  0.3*(1-(P/200))*((P/50-1))*P
}
eqn(25)
```

```
## [1] -3.28125
```

```r
eqn(100)
```

```
## [1] 15
```

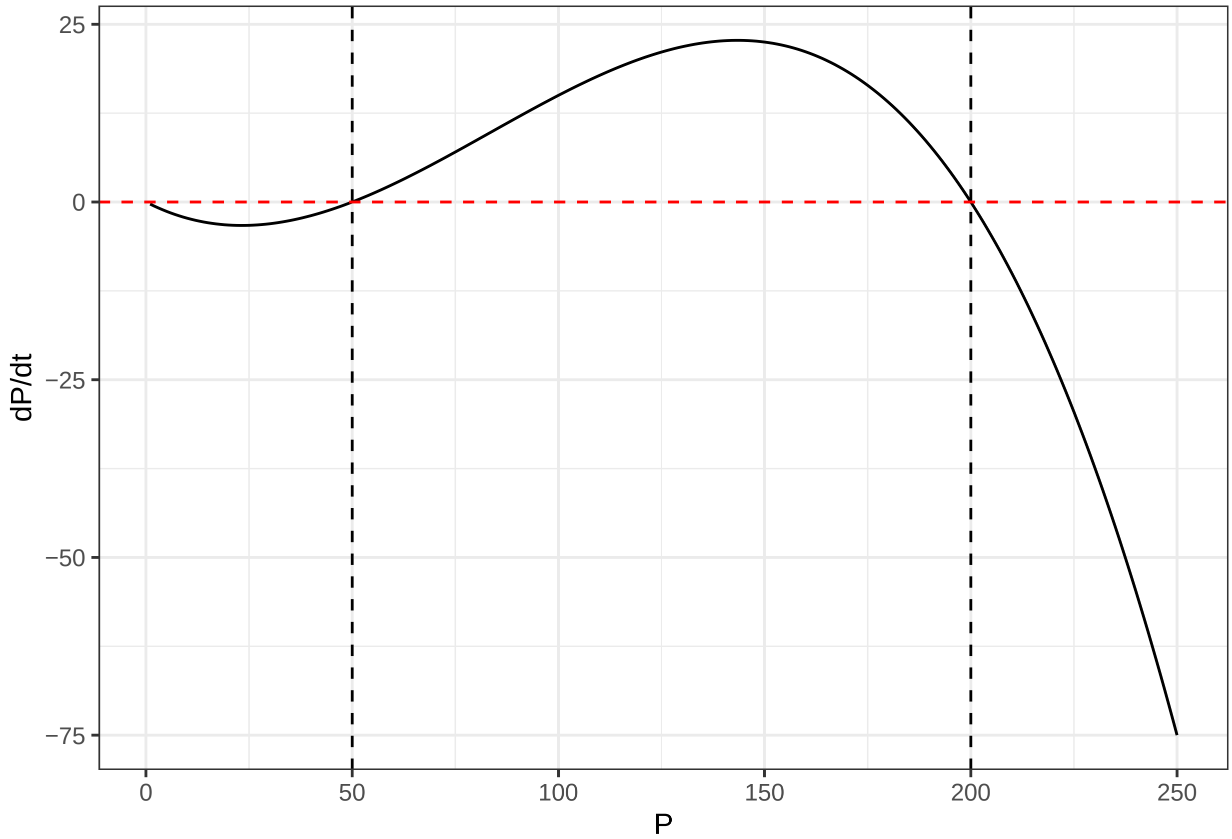
```r
eqn(400)
```

```
## [1] -840
```

So the population is decreasing when $0 < P < 50$ or $P > 200$ and the population is increasing when $50 < P < 200$. Let's confirm this with a quick plot:

```r
x=c(1:250)

ggplot()+
  geom_line(aes(x=x, y=0.3*(1-(x/200))*((x/50-1))*x))+
  geom_vline(xintercept = c(50, 200), linetype="dashed")+
  geom_hline(yintercept = 0, linetype="dashed", color="red")+
  theme_bw()+
  labs(x="P", y="dP/dt")
```



Question 3

Find the general solution to each of the following DEs:

a

$$\frac{dy}{dt} = \frac{3t+1}{2y}$$

$$2y\,dy = (3t+1)dt$$

$$\int 2y\,dy = \int (3t+1)dt$$

$$y^2 = \frac{3t^2}{2} + t + C$$

$$y = \sqrt{\frac{3t^2}{2} + t + C}$$

b

$$\frac{dy}{dx} = x\sqrt[3]{y}$$

$$y^{-\frac{1}{3}}\,dy = x\,dx$$

$$\int y^{-\frac{1}{3}}\,dy = \int x\,dx$$
$$\frac{3}{2}y^{\frac{2}{3}} = \frac{x^2}{2} + C_1$$
$$y^{\frac{2}{3}} = \frac{x^2 + C_2}{3}$$
$$y = \left(\frac{x^2 + C_2}{3}\right)^{\frac{3}{2}} = \frac{(x^2 + C_2)^{\frac{3}{2}}}{3\sqrt{3}}$$

c

$\frac{dp}{dt} = \frac{1}{3tp^2}$