Name: Rameen **Roll No:** 2023-EE-03

EE-322L Analog and Digital Communication Marks Obtained: ________

Lab Report

Experiment No. 3

Auto-correlation and Energy Spectral Density of a deterministic Signal

Note:

- **Don't forget to include the rubrics table (available at the end in this document), otherwise reports will not be graded.**
- **Copy-pasted and plagiarized reports will get zero marks**

Ensure proper comments are there in the source code of each task

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1) Objective

To verify, using MATLAB, that the Energy Spectral Density (ESD) of a deterministic energy signal is equal to the Fourier transform of its autocorrelation. This is done by computing the DTFT of a given finite-length pulse, calculating its ESD, computing the autocorrelation of the same pulse, and then taking its DTFT for comparison.

2) Technical Background

An energy signal is a signal with finite total energy. Its frequency content can be analyzed using the Discrete-Time Fourier Transform (DTFT). According to the Wiener–Khinchin theorem, the Energy Spectral Density (ESD) of an energy signal can be obtained in two equivalent ways: either by taking the magnitude-squared of its DTFT, or by taking the Fourier transform of its autocorrelation. If both approaches produce the same spectral result, it confirms the theoretical relationship between ESD and autocorrelation for deterministic energy signals.

3) Task-1

3.1. ***Description of Task-1***

For the discrete signal:

$$u[n] = [1, 1, 1, 1, 1]$$

where n = [-2, -1, 0, 1, 2]

Find **Discrete-Time Fourier Transform** (DTFT) is given by:

$$U\left(e^{-j\omega}\right) = \sum_{n=-2}^{n=2} u[n]\, e^{-j\omega n}$$

Find **Energy Spectral Density** (ESD) is computed as:

$$S_{uu}(e^{-j\omega}) = \left|U\left(e^{-j\omega}\right)\right|^2$$

where ω ranges from -2π to 2π.

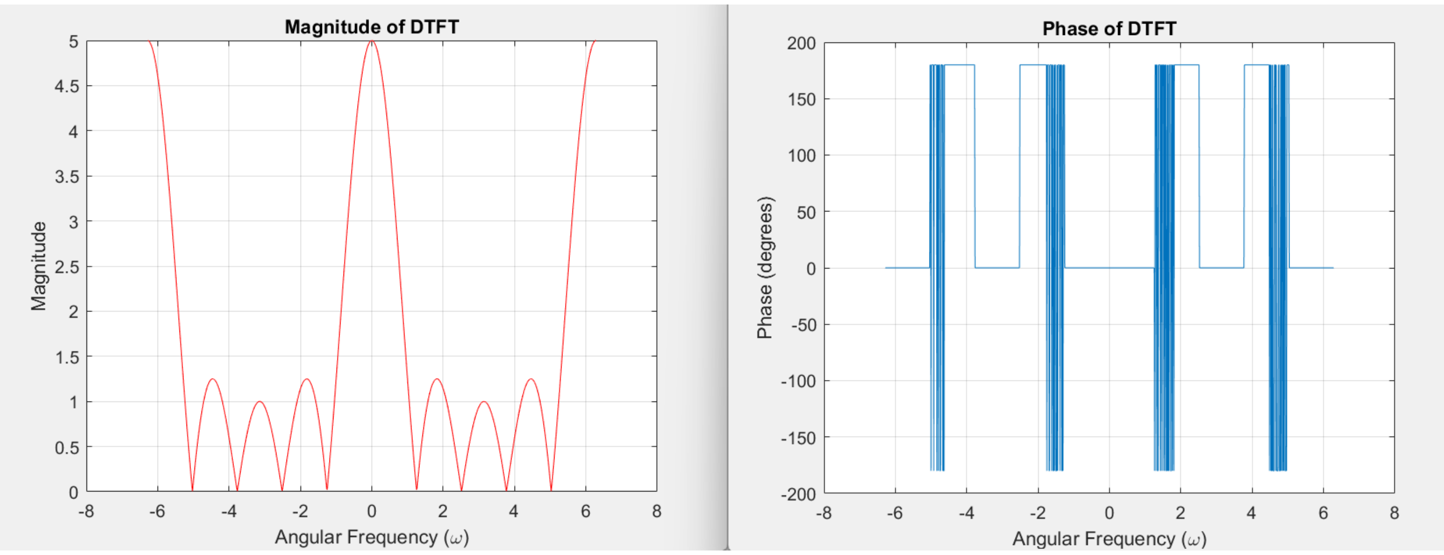
Plot magnitude and phase of $U(e^{-jw})$ and *ESD*

3.2. ***Source Code for Task-1***

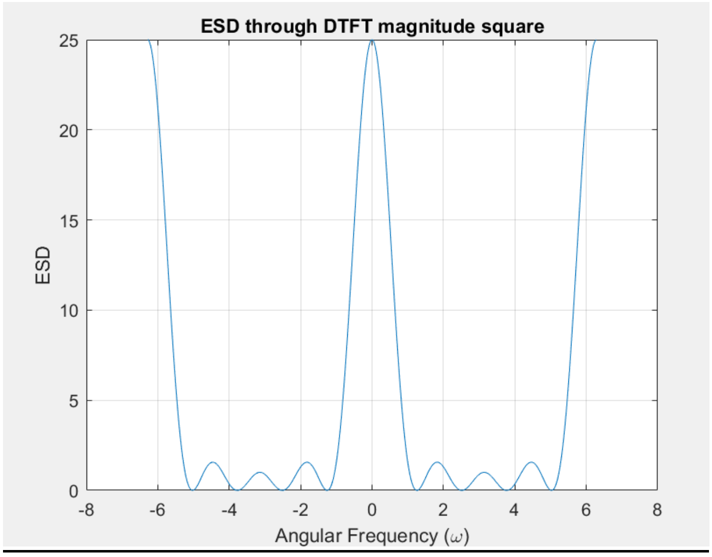
```
u_n = [1, 1, 1, 1, 1];
n = -2:1:2;

w = linspace(-2*pi, 2*pi, 1000);

U_ejw = zeros(size(w));

for w_i = 1:length(w)
    % vectorized sum over n
    U_ejw(w_i) = sum(u_n .* exp(-1j * w(w_i) * n));
end


magnitude_U_ejw = abs(U_ejw);
phase_U_ejw = angle(U_ejw) * (180/pi);


ESD = magnitude_U_ejw .^ 2;

%  Plotting
% Magnitude plot
figure;
plot(w, magnitude_U_ejw, 'r');
xlabel('Angular Frequency (\omega)');
ylabel('Magnitude');
title('Magnitude of DTFT');
grid on;

% Phase plot
figure;
plot(w, phase_U_ejw);
xlabel('Angular Frequency (\omega)');
ylabel('Phase (degrees)');
title('Phase of DTFT');
grid on;

% Energy Spectral Density plot
figure;
plot(w, ESD);
xlabel('Angular Frequency (\omega)');
ylabel('ESD');
title('ESD through DTFT magnitude square');
grid on;
```

3.3. ***Results and Discussions for Task-1***

1. *Magnitude and phase of $U(e^{-jw})$:*



2. *Plot of ESD:*



From –π to π, the strongest spectral component of u[n] is at ω = 0, which can be seen from the amplitude graph of its DTFT and that of its ESD.

4) Task-2

4.1. ***Description of Task-2***

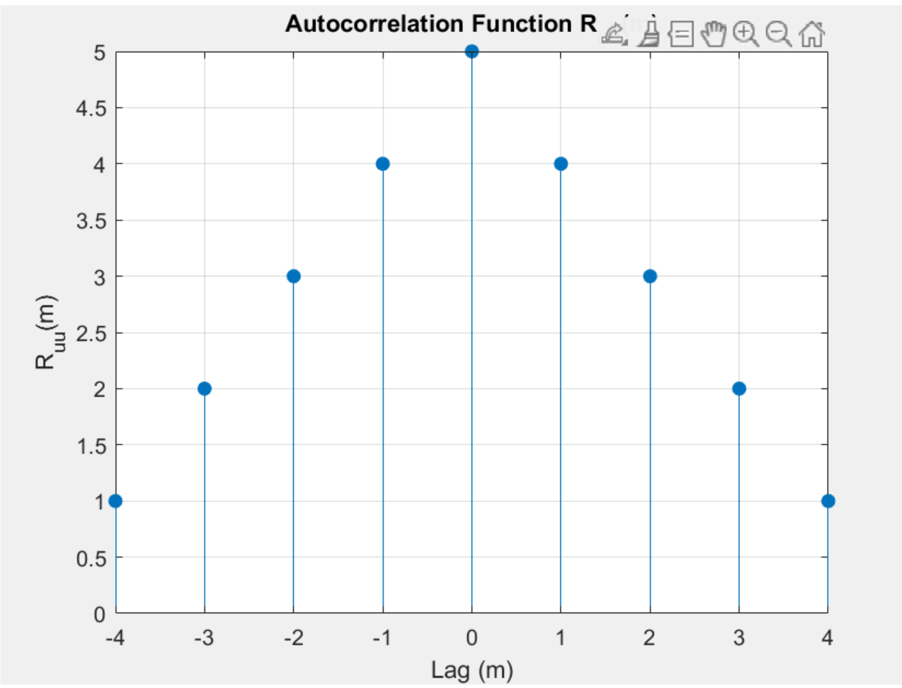
Find the autocorrelation Ruu of above $u[n]$ by using your own code. Find DTFT of Ruu and verify that it is equivalent to $Suu(e{-}j\omega)$.

4.2. ***Source Code for Task-2***

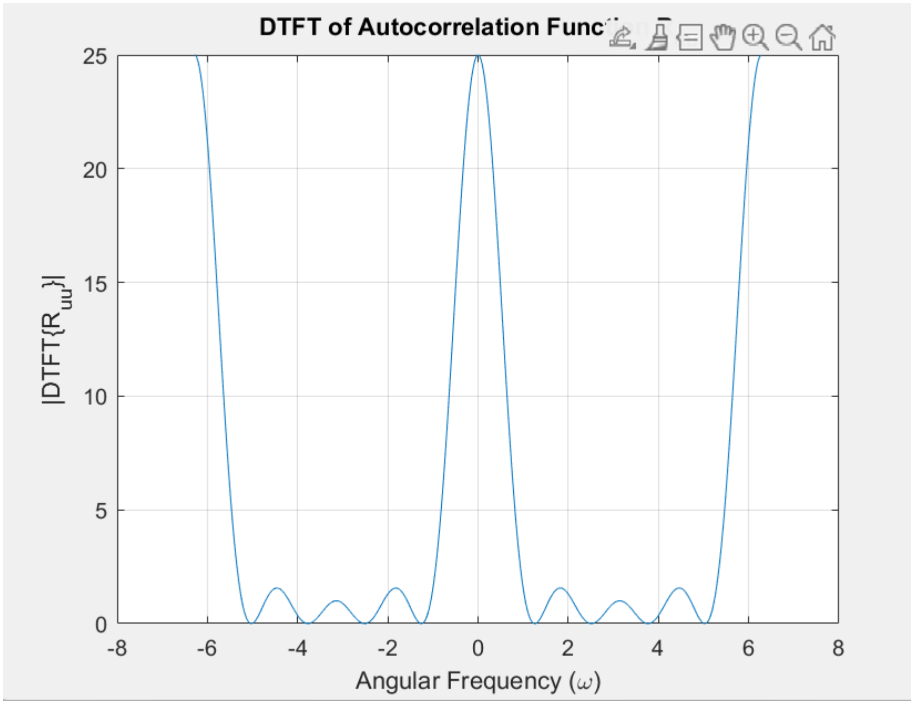
```matlab

u_n = [1, 1, 1, 1, 1];
M = 5;

% --- Autocorrelation ---
R_uu = xcorr(u_n, 'none');
m = -(M-1):(M-1);


figure;
stem(m, R_uu, 'filled');
xlabel('Lag (m)');
ylabel('R_{uu}(m)');
title('Autocorrelation Function R_{uu}(m)');
grid on;

% --- DTFT of R_uu ---
w = linspace(-2*pi, 2*pi, 1000);
U_ejw = zeros(size(w));

for w_i = 1:length(w)
    U_ejw(w_i) = sum(R_uu .* exp(-1j * w(w_i) * m));
end

% --- Plot DTFT of Autocorrelation ---
figure;
plot(w, abs(U_ejw));
xlabel('Angular Frequency (\omega)');
ylabel('|DTFT\{R_{uu}\}|');
title('DTFT of Autocorrelation Function R_{uu}');
grid on;

```

4.3. ***Results and Discussions for Task-2***

1. Autocorrelation of u[n]



2. DTFT of Ruu



The ESD obtained in Task-1 matches the DTFT of the autocorrelation computed in Task-2. This confirms the Wiener–Khinchin theorem, which states that the energy spectral density of an energy signal is equal to the Fourier transform of its autocorrelation.

5) Conclusion

In this experiment, the DTFT of a finite rectangular pulse was computed and its Energy Spectral Density (ESD) was obtained from the magnitude-squared of the DTFT. The autocorrelation of the same signal was then calculated and its DTFT was taken. The resulting spectrum matched the ESD obtained earlier. This verified the Wiener–Khinchin theorem, demonstrating that the ESD of an energy signal is equal to the Fourier transform of its autocorrelation. Through this, the theoretical relationship between time-domain correlation and frequency-domain energy distribution was successfully confirmed.

Rubrics for Experiment No.

Performance	**Exceeds expectation (2)**	**Meets expectation (1)**	**Does not meet expectation (0.5)**	**Marks**
R1: Knowledge of required functions for code design. **Marks: 0-2**	Has required knowledge for code	Has partial knowledge for code	Has no knowledge for code	
R2: Simulation of experiment **Marks: 0-2**	Simulates all the tasks correctly by himself	Needs guidance to simulate the tasks correctly	Incapable to simulate the tasks correctly by himself even with guidance	
R3: Demonstrate proper results with justification **Marks: 0-2**	Correct results are provided with required justification	Results are provided with minor errors and/or with little justification	Results are provided with major errors and/or with no justification	

Rubrics for Lab Manual No.

Performance	**Exceeds expectation (0.5)/(0.25)**	**Meets expectation (-)/(-)**	**Does not meet expectation (0)/(0)**	**Marks**
R1: Timely submission **Marks: 0-0.5**	The submission is on time	---	Late submission	
R2: Report completeness **Marks: 0-0.25**	All relevant calculations, specifications, code, graphs, and results are provided with proper explanation.	All the relevant calculations, specifications, code, graphs and results are provided but with little explanation and justification.	Most of the relevant graphs, results, calculations, specifications, and code are missing, as well as their proper explanation and justification is also missing.	
R3: Error-free writeup **Marks: 0-0.25**	The submitted assignment is without any plagiarism and formatting errors.	Some parts of the submitted assignment contain formatting errors and plagiarized material.	The submitted assignment is mostly plagiarized and contain formatting errors.	