실습 5주차

문제 1

The probability of the difference of two events

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The probability of the difference of two events. Give a mathematical derivation of the formula

$$\mathbf{P}\Big((A \cap B^c) \cup (A^c \cap B)\Big) = \mathbf{P}(A) + \mathbf{P}(B) - 2 \cdot \mathbf{P}(A \cap B),$$

for the probability that exactly one of the events A and B will occur. Your derivation should be a sequence of steps, with each step justified by appealing to one of the probability axioms.

문제 2

Conditional probability example

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Conditional probability example. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

(a) Find the probability that doubles are rolled (i.e., both dice have the same number).

(b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled.

(c) Find the probability that at least one die roll is a 6.

(d) Given that the two dice land on different numbers, find the conditional probability that at least one die roll is a 6.

문제 3

The Monty Hall problem

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The Monty Hall problem. This is a much discussed puzzle, based on an old American game show. You are told that a prize is equally likely to be found behind any one of three closed doors in front of you. You point to one of the doors. A friend opens for you one of the remaining two doors, after making sure that the prize is not behind it. At this point, you can stick to your initial choice, or switch to the other unopened door. You win the prize if it lies behind your final choice of a door. Consider the following strategies:

- Stick to your initial choice.
- Switch to the other unopened door.
- You first point to door 1. If door 2 is opened, you do not switch. If door 3 is opened, you switch.

Which is the best strategy?

문제 4

The birthday problem

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The birthday problem. Consider n people who are attending a party. We assume that every person has an equal probability of being born on any day during the year, independently of everyone else, and ignore the additional complication presented by leap years (i.e., nobody is born on February 29). What is the probability that each person has a distinct birthday?

문제 5

PMF of a function of a random variable

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PMF of a function of a random variable. Consider a random variable X such that

$$p_X(x) = \begin{cases} \frac{x^2}{a}, & \text{for } x \in \{-3, -2, -1, 1, 2, 3\}, \\ 0, & \text{otherwise,} \end{cases}$$

where $a > 0$ is a real parameter.

1. Find a.
2. What is the PMF of the random variable $Z = X^2$?

문제 6

Joint PMF drill 3

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Joint PMF drill #3. The joint PMF of the random variables X and Y is given by the following table:

$y=3$	c	c	$2c$
$y=2$	$2c$	0	$4c$
$y=1$	$3c$	c	$6c$
	$x=1$	$x=2$	$x=3$

1. Find the value of the constant c.
2. Find $p_Y(2)$.
3. Consider the random variable $Z = YX^2$. Find $\mathbf{E}[Z \mid Y = 2]$.
4. Conditioned on the event that $X \neq 2$, are X and Y independent? Give a one-line justification.
5. Find the conditional variance of Y given that $X = 2$.