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MAE 207
Final Examination
Fall 2019

This exam is open book and open notes, and you need to show all work and state all assumptions. Your solutions are due by 9:00 p.m. on 12/10/2019. Return your answers digitally via Canvas as pdf files as well as executable .R files, with comments, corresponding to your answers.

While you need to solve all three problems on this exam, only two problems will be graded.

You can discuss these problems with other students, but the answers you turn in need to be written by you.

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1. The ACME Pipe Insulation Company manufactures cylindrical insulation pieces that are to be used on the exterior surfaces of round pipes. Each insulation piece comes in a standard 0.1 m length with a 2 cm inside diameter and a 3 cm outside diameter. Test engineer A evaluated the thermal resistance θ of randomly selected pieces of insulation by measuring the radial rate of heat transfer Q through each piece of insulation, where the temperatures at the inner and outer surfaces, i.e., T_1 and T_2, were also measured for each piece. The equation relating thermal resistance to the measured values is as follows.

$$\theta = \frac{T_1 - T_2}{Q}$$

The temperatures on the inside and outside surfaces of a piece of insulation, i.e., T_1 and T_2, were each measured with different 12-bit digital instruments that had four readout digits and full-scale operating ranges of 250 - 350 K. The instrument used to measure T_1 had a systematic error β_{T_1} with b_{T_1} = 0.5 K. The instrument used to measure T_2 had a systematic error β_{T_2} with b_{T_2} = 1 K.

The measurements of Q were made with a 12-bit digital instrument that had four readout digits and a full-scale operating range of 0 - 200 W. This instrument had a systematic error β_Q with b_Q = 0.5 K.

Measured values for insulation pieces are available in the following file on Canvas (download this file).

mae_207_final_exam_f_2019_problem_1_data.csv

(a) (10 points) - After downloading the data and loading it into R, use Chauvenet's criterion to identify outliers in the T_1 data. Perform this same operation for the T_2 and Q data. If any measurand has outliers, delete the data in the row that corresponds to these outliers. For example, if data points 3 and 7 in the T_2 column are outliers, then delete all data corresponding to pieces 3 and 7 in the T_1, T_2, and Q columns. List all outliers as well as the rows of data that have been deleted.

(b) (30 points) - Using the outlier-free data from (a), evaluate the mean and standard deviation of μ_θ with a Monte Carlo approach that uses resampling for T_1, T_2, and Q. Assume that all systematic errors follow a normal distribution.

(c) (10 points) - Suppose test engineer B had concluded from independent experiments that μ_θ follows a normal distribution with a mean of 0.551 K/W and a standard deviation of 0.074 K/W. Assuming that test engineer's A results for μ_θ are normal with the mean and standard deviation from part (b) above, combine the pdfs of engineers A and B and determine the mean and standard deviation of the combined pdfs. Also plot the pdfs for A and B as well as the combined pdf. Assume that A and B both used uninformative priors.

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2. (50 points) – Water flowing through a pipe enters a flowmeter (flowmeter 1). The water exiting this flowmeter is directed into two other pipes, with each pipe having its own flowmeter (flowmeters 2 and 3). Measured data values in units of kg/s are in the following file for each flowmeter. This file is available on Canvas (download this file).

mae_207_final_exam_f_2019_problem_2_data.csv

Each flowmeter has a systematic error β_{ins} from installation, with b_{ins} = 0.02 kg/s (these errors are uncorrelated). In addition, all flowmeters were calibrated using the same standard, yielding correlated β_{cal} values, with b_{cal} = 0.04 kg/s. All other systematic errors are negligible.

Perform a balance check on mass conservation at a 95% confidence level. Use a Taylor series approach with the coverage factor k = 2 (you do not need to use the WS equation). Note that if mass is conserved, the mass flow rate through flowmeter 1 equals the sum of the mass flow rates through flowmeters 2 and 3.

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3. Consider the pdf below.

$$p(x) = p(x|\lambda) = \lambda e^{-\lambda x}, x \geq 0, \lambda > 0$$

(a) (5 points) - By integrating over all possible values of x, show that $\lambda = 1/\mu_x$.

(b) (5 points) - Create a set of random numbers corresponding to this pdf by executing the R commands below.

```
set.seed(10)
x <- rexp(n=20, rate=0.2)
```

Note that "rate" in the rexp() command corresponds to λ.

(c) (20 points) - Calculate μ_λ using a Monte Carlo approach with resampling from the random numbers created in part (b). Assume that systematic errors are negligible.

(d) (20 points) - Use the conditional pdf $p(\lambda|\vec{x})$ and the random numbers from part (b) to calculate μ_λ (this is a Bayesian approach). Use an uninformative prior. Note that $\int_0^\infty x^m e^{-nx} dx = m!\, n^{-m-1}$ if m is a positive integer and $n > 0$.