Sharif University of Technology
Department of Chemical and Petroleum Engineering

Assignment (1) Report

- Advanced Well-testing -

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1 Introduction

The Matthews-Brons-Hazebroek (MBH) method is a well-known method for determination of the average pressure in a bounded reservoir. They have created various plots corresponding to different reservoir dimensions and well positioning within them. The method has a straight forward algorithm for obtaining the average reservoir pressure based on a false pressure read from the extrapolation of the semi-logarithmic Horner plot generated during pressure transient analysis. The derivation of average reservoir pressure using MBH algorithm is not however the subject of this discussion. This report investigates the ways and means of generation of the MBH plots for various shapes of bounded reservoirs and well positions.

The formulae for obtaining MBH plots are put into two categories based on their drainage area: (1) circular reservoirs, and (2) non-circular reservoirs. [2] These relationships are first derived in terms of dimensionless variables for each case, and then, a computer code is written and run re-generating some of MBH plots based on the aforementioned formulae.

2 Methodology

2.1 Formulation

The formulae for the case of circular reservoir were described in class and are available in lecture notes and therefore are not brought up here to avoid repetition. However, the formulation used for the computer code will be explained in detail in the next sub-section.

Derivation of formulae for non-circular reservoirs is hand-written to save time and will be attached to this report. The final formula however will be explained in the next sub-section.

2.2 Computer Code Development

2.2.1 Circular Reservoir

According to the previous derivations, $P_{D,MBH}$ for a circular drainage area is defined as:

$$P_{D,MBH} = ln(4\pi t_{pDA}) + 3/2 - 0.5772 + 4\sum exp(\frac{-x^2\pi t_{pDA})}{x^2 J_0^2(x)})$$

Where J_0 is the 0 order Bessel function of the first kind and x is a vector containing roots of the 1st order Bessel function of the first kind, J_1. Therefore, it is necessary to solve these unknowns prior to substituting for dimensionless time. 1st step is then to calculate roots of the 1st order Bessel function and then substituting them in 0 order Bessel function to obtain J_0.

This leaves no other unknowns so now we are able to feed different values of dimensionless time to the above equation and get their corresponding $P_{D,MBH}$ and plot them against each other. The generated plots are discussed in the third section.

2.2.2 Non-circular Reservoir

The hand-written derivations lead to the final formula for non-circular drainage area as follows:

$$P_{D,MBH} = 4\pi t_{pDA} + \sum_{-\infty}^{\infty} Ei(\frac{a_{m,n}^2}{-4At_{pDA}})$$

Where the second term is the sum of the Ei function of an infinite number of image wells. Therefore, to obtain the dimensionless pressure for this case two steps are necessary: (1) calculating the distances of image wells to the source well, and (2) approximating the Exponential Integral function for each distance.

There has been created a MATLAB function to carry out each of the above-mentioned steps. The first one called "imageDistance" divides the x-y coordinate system into 4 types of points. According to MBH's 1954 work these points are as follows: $(2ma, 2nb)$, $(2(m+\alpha)a, 2nb)$, $(2ma, 2(n+\beta)b)$, $(2(m+\alpha)a, 2(n+\beta)b)$. Where m and n are indices, a and b are length and width of rectangular reservoir respectively, and α and β define well distance from the sides and are in terms of fractions of length and width. Figure 1 is a demonstration of this system.

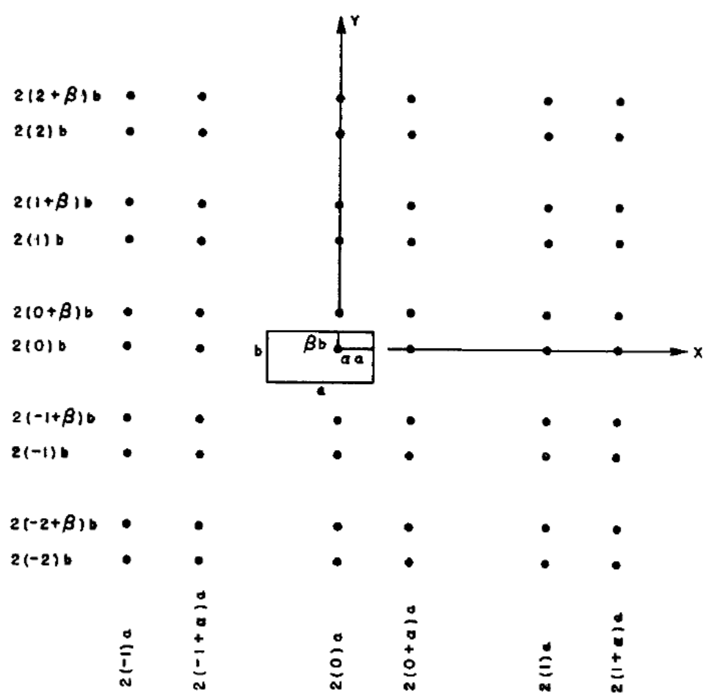


Figure 1: Image wells linked to corresponding coordinates. [2]

This function takes 'U' which is an auxiliary parameter to determine the shape of reservoir and well-positioning by changing a, b, α, and β as an input. Also takes a parameter deciding values of m and n which are supposed to be infinite values in theory, however, are given limits, because in practice, after a certain number of images are passed the effect of image wells on the source well becomes negligible due to large distance. This parameter is considered to work well for values above 5. The square of distances from source well is returned as a convenience.

The second function is basically an algorithm to approximate the Exponential Integral function. MATLAB's built-in Ei function calculates the return values so slowly that a basic run of the above formula (if reachable at all) would take about an hour on a regular laptop memory. Therefore, the value of Ei function should be approximated. I have come across several approximations and at first used the ones presented in Tarek Ahmed's handbook but they returned inaccurate and odd results. The algorithm to approximate the Ei function in this computer code is based on work done by DA Barry (2000). [1]

By implementing the two mentioned MATLAB functions the code will be ready to calculate the dimensionless pressure based on the time which is fed. The results are presented in a semi-logarithmic scale and are the topic of discussion in the subsequent section.

3 Results

Figure 2 shows the MBH plot for a circular drainage area with the well located in the middle.

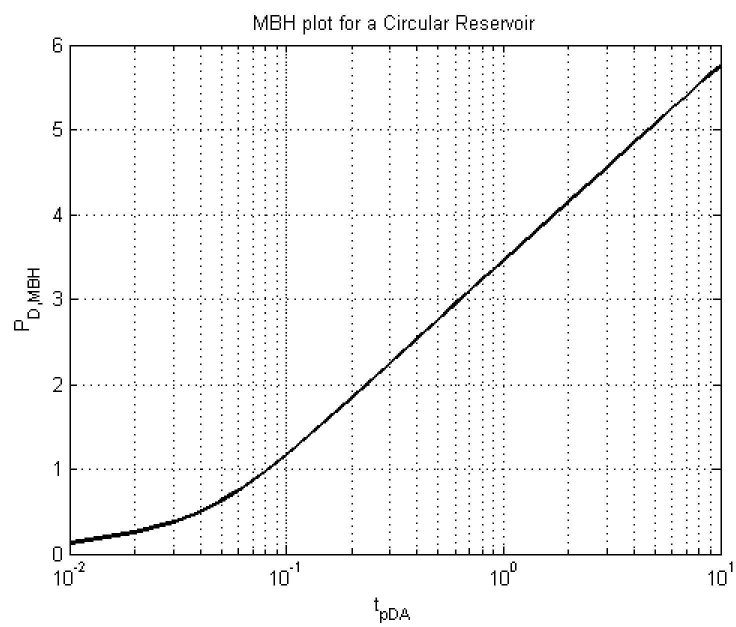


Figure 2: Obtained MBH plot for circular drainage area.

Comparing to the original MBH plots shown in Figure 3, the generated plots demonstrates a good match.

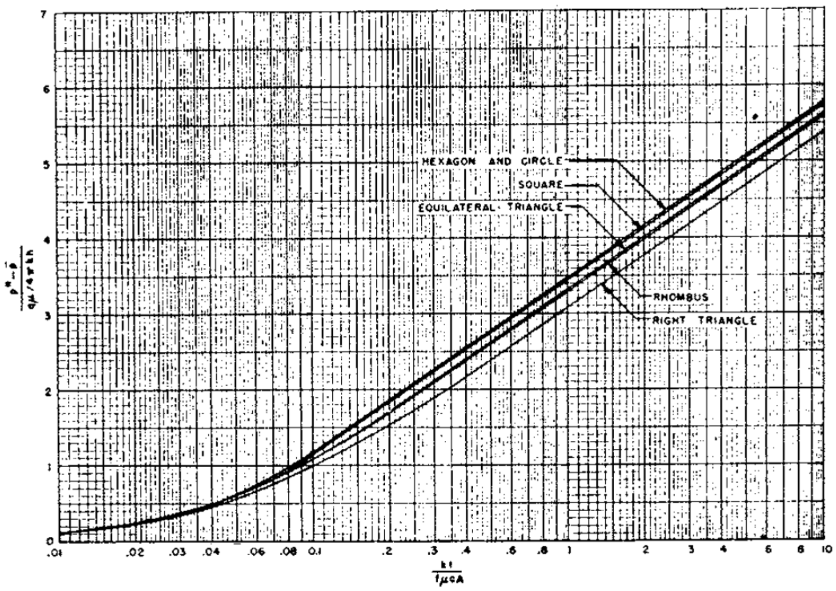


Figure 3: Original MBH plots for circular and non-circular drainage area.

The second plot generated by the application is for a square drainage area with the well located in the middle and is shown in Figure 4. Also in comparison to Figure 3, the application shows reliability in predicting dimensionless pressure for a square shaped reservoir.

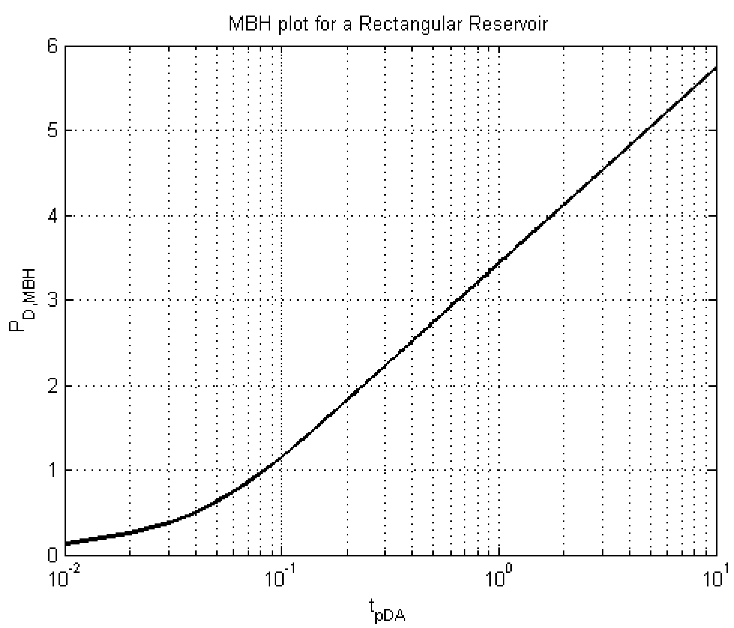


Figure 4: Generated MBH plot for a well in the middle of a square shaped reservoir.

Moreover, an MBH plot is generated for curve III in Figure 7a, which is an exact fit and is illustrated in Figure 5.Same is done for curve IV of Figure 7b and the generated MBH plot for this type of reservoir is shown in Figure 6.

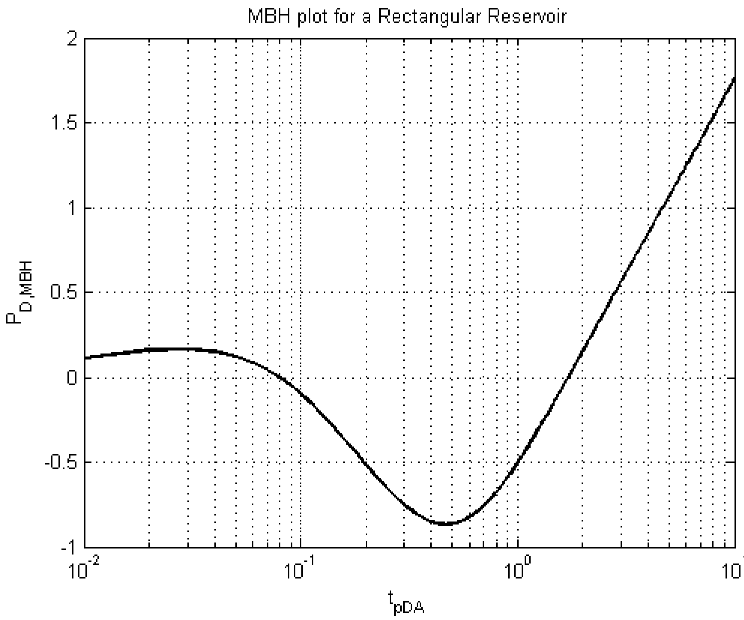


Figure 5: Generated MBH plot for a well in the side of a rectangular reservoir. (a)-III

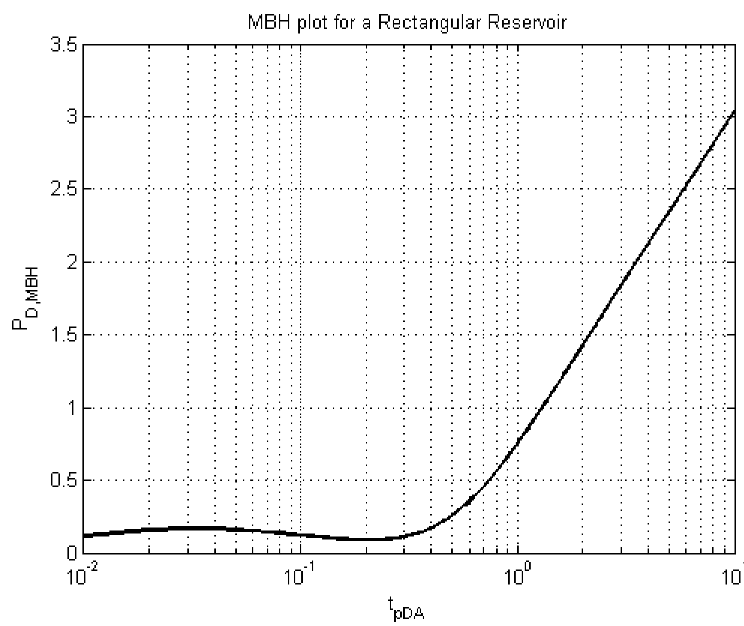
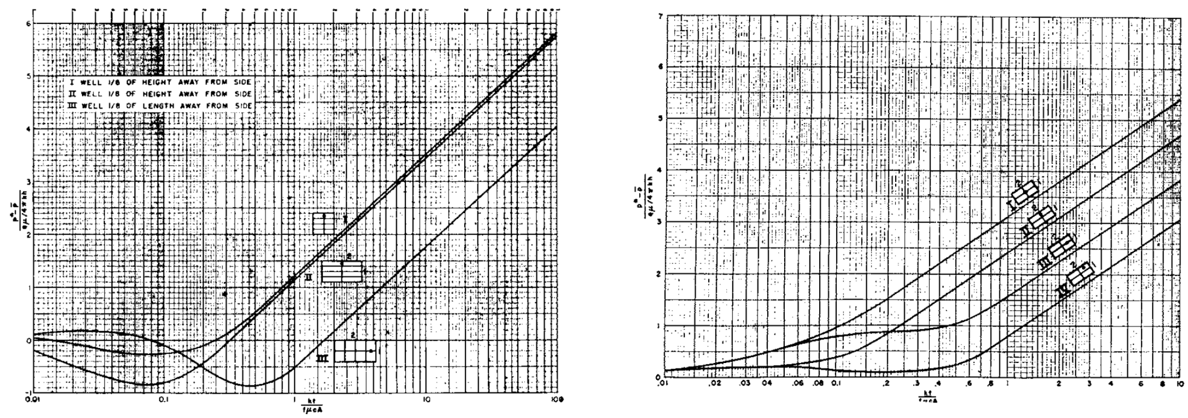


Figure 6: Generated MBH plot for a well in the side of a rectangular reservoir. (b)-IV



(a) (b)

Figure 7: Original MBH plot for rectangular reservoirs.

References

[1] DA Barry, J-Y Parlange, and L Li. Approximation for the exponential integral (theis well function). *Journal of Hydrology*, 227(1-4):287–291, 2000.

[2] CS Matthews, F Brons, P Hazebroek, et al. A method for determination of average pressure in a bounded reservoir. 1954.