Sharif University of Technology
Department of Chemical and Petroleum Engineering

Assignment (2) Report

- Advanced Well-testing -

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1 Methodology

In this report two of the most commonly used type-curves in the petroleum engineering literature have been re-derived using a computer code; Gringarten's storage and skin type-curve, and Bourdet's pressure derivative type-curve.

1.1 Analytical Inversion of Laplace Transform

The curves were computed from an analytical solution to the diffusivity equation representing the constant rate draw-down in a finite radius well. [1] The solution is first obtained in the Laplace domain and then inverted as follows:

$$P_D = \frac{4}{\pi^2}\int_0^{\infty}\frac{(1-e^{-u^2 t_D})du}{u^3\{[uC_D J_0(u)-(1-C_D s u^2)J_1(u)]^2+[uC_D Y_0(u)-(1-C_D s u^2)Y_1(u)]^2\}} \quad (1)$$

Where J, and Y are Bessel functions of the first and second kind, respectively.

The algorithm for derivation of the type-curves is in the following manner:

1. Define suitable t_D/C_D span covering the ranges of the original plot.
2. Define ranges of skin and storage that is needed to generate curves for ascending C_De^{2s}.
3. Calculate t_D of from the predefined range of t_D/C_D.
4. Numerically integrate the integral term of the above equation with respect to u.
5. Calculate P_D and plot against t_D/C_D on log-log coordinates.
6. For the derivative plot the data generated for P_D is numerically differentiated, multiplied by t_D/C_D and finally plotted against t_D/C_D on log-log scale.

The time steps defined in step 1 are finely meshed in earlier times and are coarser as time goes by. This is due to saving CPU time. The ranges for skin and storage in step 2 are between 0.01 and 10 for C_D and 0 to 60 for s, and are chosen such that the value of C_De^{2s} ranges from 1 up to 10^{50}. Simpson's 1/3 rule have been utilized for the numerical integration in step 4. The data has also been integrated by Simpson's 3/8, and the results were quite similar, however, CPU time was longer. It should be noted that for the Simpson's 1/3 rule the number of increments should be an even number.

1.2 Numerical Inversion of Laplace Transform

In this case, instead of Eq. (1), the original equation in Laplace space has been solved numerically using the Stehfest algorithm. [2] This equation is as follows:

$$L\{p_D\} = \frac{K_0(\sqrt{p})+s\sqrt{p}K_1(\sqrt{p})}{p\left[\sqrt{p}K_1(\sqrt{p})+C_D p[K_0(\sqrt{p})+s\sqrt{p}K_1(\sqrt{p})]\right]} \quad (2)$$

This approximation have been carried out with N = 10 for the numerical algorithm and had a good accuracy and much faster convergence than the analytical method. This method does not have the CPU time problem which was mentioned in the previous sub-section and converges in around 3 seconds. It is worth mentioning that the analytical solution had an integral in itself which was evaluated numerically, and therefore we can say that it was merely a semi-analytical solution to the problem and therefore had its own errors.

2 Results

The results are fairly similar to the original Gringarten et al. and Bourdet type-curves, however, with some minor modifications. Note that these results have not been investigated for negative skin and skin values are either zero or positive.

There may appear some sharp edges on curves which are as a result of shifting the time-steps at the end of each log-cycle to save more CPU time. As mentioned earlier, the time meshes become coarser (bigger step sizes) as time passes.

2.1 Analytical Inversion of Laplace Transform

In this case, if a relatively small number (say for instance 100) is chosen as the number of increments of the numerical integration, the curves will be somewhat misplaced, and moreover, the derivative curves will tend to zero at later times. Conversely, if the number of increments is too large (about 5000), the results, however accurate, take too much CPU time to be generated owing to the complexity of the above-mentioned equation.

The results for the integral step numbers of about 5000 are shown in Figures 1, 2, and 3 after 1750 seconds of CPU time. The type-curve shown in Figure 1 was not that sensitive to the chosen increment numbers after about 800 increments, however, the derivative plot shown in Figure 2 must be generated using at least 2000 increments or else it would present errors at late times.

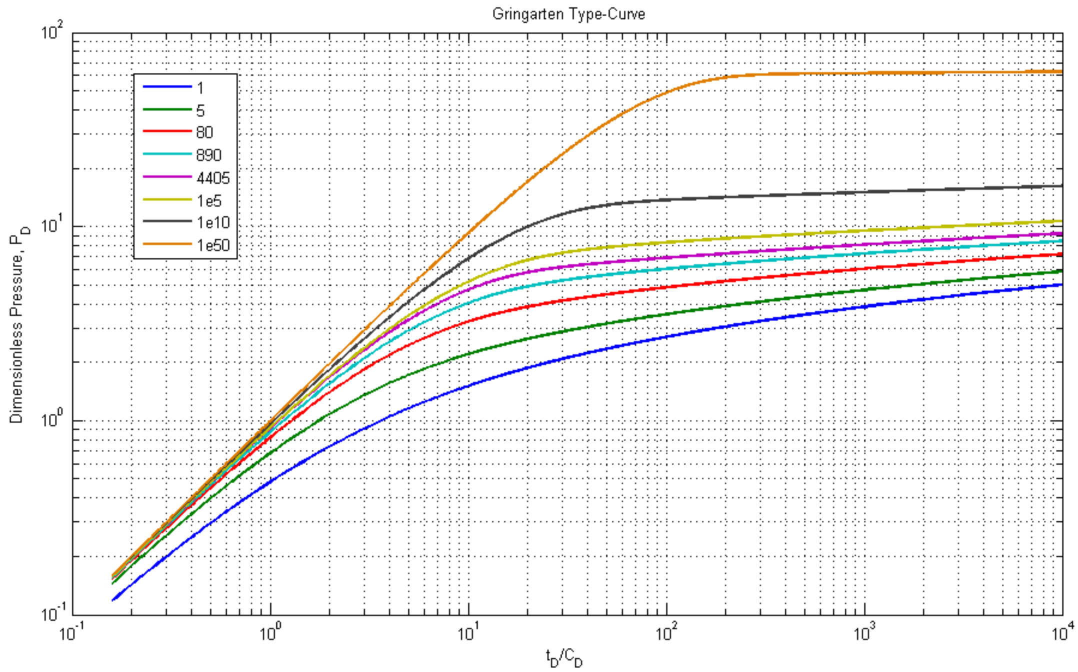

Figure 1: Regeneration of Gringarten's type-curve.

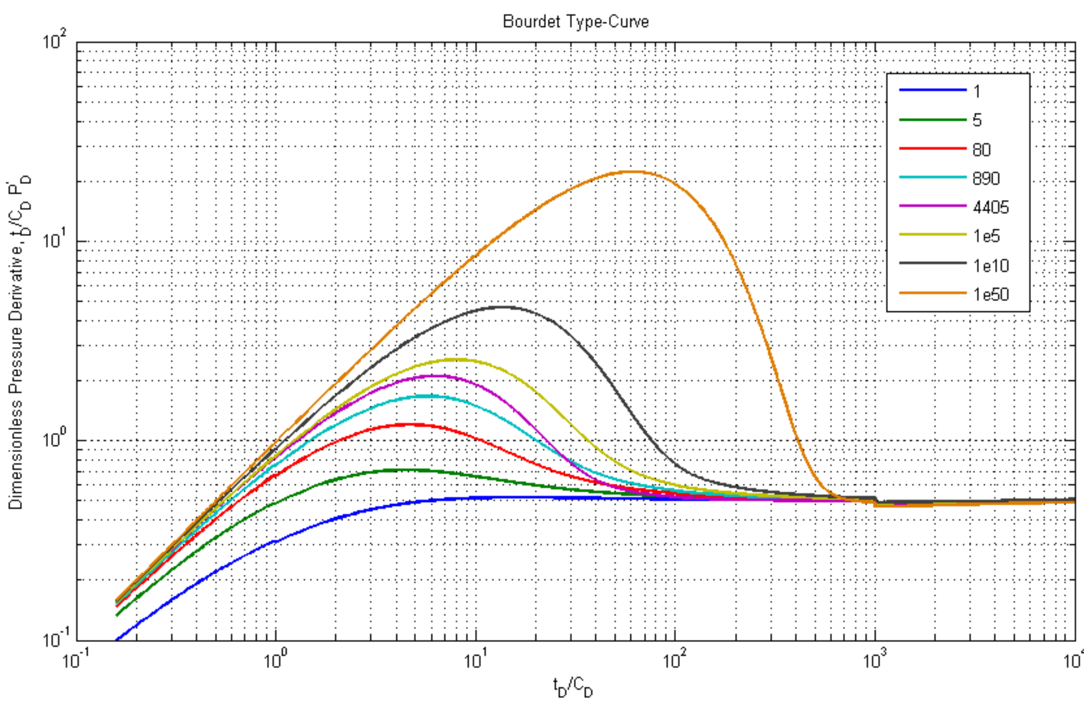

Figure 2: Regeneration of Bourdet's type-curve

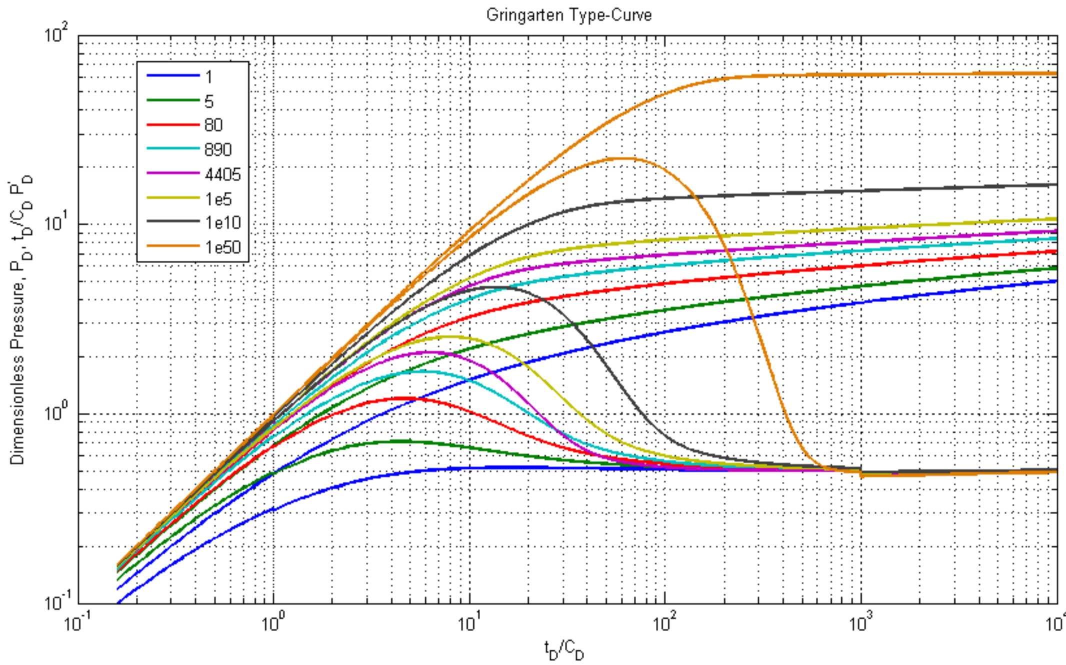

Figure 3: Final dimensionless pressure and derivative type-curve.

2.2 Numerical Inversion of Laplace Transform

The results obtained for the numerical inversion using the Stehfest method is slightly different and may even be more accurate. Figure 4 is the type-curve generated in this case.

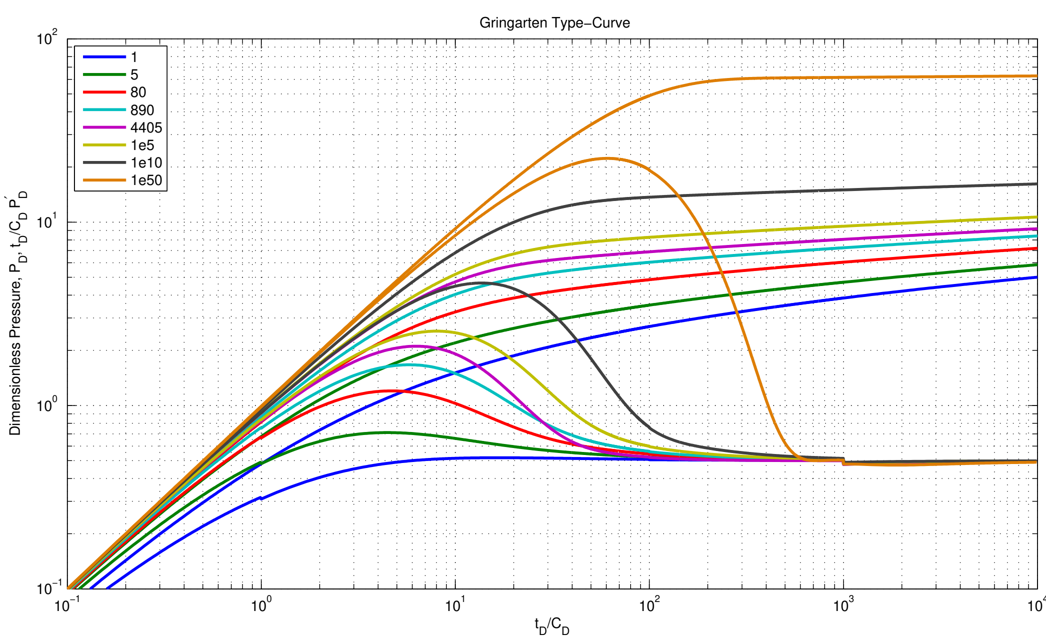

Figure 4: Final dimensionless pressure and derivative type-curve, using Stehfest algorithmm.

References

[1] Alain C Gringarten, Dominique P Bourdet, Pierre A Landel, Vladimir J Kniazeff, et al. A comparison between different skin and wellbore storage type-curves for early-time transient analysis. In *SPE Annual Technical Conference and Exhibition*. Society of Petroleum Engineers, 1979.

[2] Harald Stehfest. Algorithm 368: Numerical inversion of laplace transforms [d5]. *Communications of the ACM*, 13(1):47–49, 1970.