1 Methodology

The objective of this assignment is to reproduce the plots presented in Figure 6.2.1 of the reference book. [3] The algorithm carried out in this simulation is fairly similar to the one implemented in assignment #6, and so are the assumptions. Except the previous assignment was essentially a two fluid model, where in this one we are dealing with the three-fluid model, i.e. the core gas phase, liquid film on pipe's surface, and liquid droplets floating around in the core gas phase. Therefore, the closure relationships for calculating the friction forces are more complex and two parameters involving droplets deposition and liquid entrainment shall be derived.

Also the vector of unknowns, Y, and the residuals, F, are now comprised of 6 elements instead of 4 to account for liquid droplets' fraction and velocity. The assumptions and simplifications for the simulation are as follows:

- Two types of fluid are present in the pipe. No evaporation or condensation are to occur. No gas can be solved in the liquid.
- Liquid film and liquid droplets floating in gas can interact via entrainment and deposition.
- The pipe is not perforated no flow occurs through pipe wall. (RHS of mass conservation equation is zero)
- Annular flow regime is present.
- Flow is isothermal.
- Steady-state flow regime is considered.
- Gas and liquid properties are independent of pressure. (Incompressible)
- Pipe's inclination angle is constant.

Again, two equations of mass balance and momentum conservation for each phase have to be solved considering the above conditions which yield a vector of residuals, F, which has to be minimized.

1.1 Computer Program Development

The procedure is similar to assignment #6 and the modifications are exactly as equations presented in Chapter 4 through 6 of the book, except for the ones which are mentioned bellow.

1.1.1 Modifications

Equation 6.1.11 has a misprint in the second line of the residual vector which is supposed to be ρ_L instead of ρ_G.

Inlet liquid mass flow is given, however, inlet gas mass flow needs to be calculated as

$$k_{Gin} = \alpha_G v_G$$

Where $\alpha_G v_G$ is the model input which is supposed to change from 10 to 50, and is different from the α_G, and v_G which are calculated separately using the estimated and then iterated values and then multiplied.

The general friction factor correlation used to obtain equations 5.1.9 and 5.1.15 is *Haaland's* explicit approximation of *Colebrook and White*'s original equation. [2] This equation is as follows:

$$1/\sqrt{f} = -1.8log[6.9/Re + (\frac{\epsilon}{3.7d})^{1.11}]$$

Surface roughness is assumed to be zero since no data has been given, however, it can be altered to analyze the effect of pipe roughness on different plots.

Any relationship describing the value of gas core diameter was not found in the reference book, and therefore the equation bellow has been used for estimating gas-dispersed core diameter after *Alipchenkov et al.* (2004) [1]:

$$d_i = (1 - \alpha_L)^{1/2} d$$

Where d is the hydraulic (pipe) diameter.

Instead of the Newton iteration method, *Trust-region dogleg* algorithm was used to minimize the residual terms, due to difficulties occurring while calculating and implementing the *Jacobian* matrix.

1.1.2 Functions

Two functions are defined: (1) *calcTFM*: based on well data and Y evaluates closure relationships for frictional forces and also entrainment and deposition coefficients, Γ_{DL}, and Γ_{LD}. Equations presented in Chapter 5 are all coded in this function.

Two mini-functions, so to speak, are defined at the end which carry out iterative methods to solve for droplet size. The average droplet diameter is calculated by running two algorithms to define maximum stable droplet diameter due to average velocity difference and due to turbulence. Then the minimum of the two values are put into an equation which gives the final average droplet diameter, d_D.

(2) *calcResiduals*: is a function that outputs F, which needs to be minimized in order to solve the problem i.e. calculate the optimum Y. This function basically uses the input $Y0$ and $\alpha_G v_G$ and the relevant relationships are then calculated via *calcTFM*. This function, subsequently, returns the vector of residuals, F, according to equation 6.1.11.

1.1.3 Algorithm

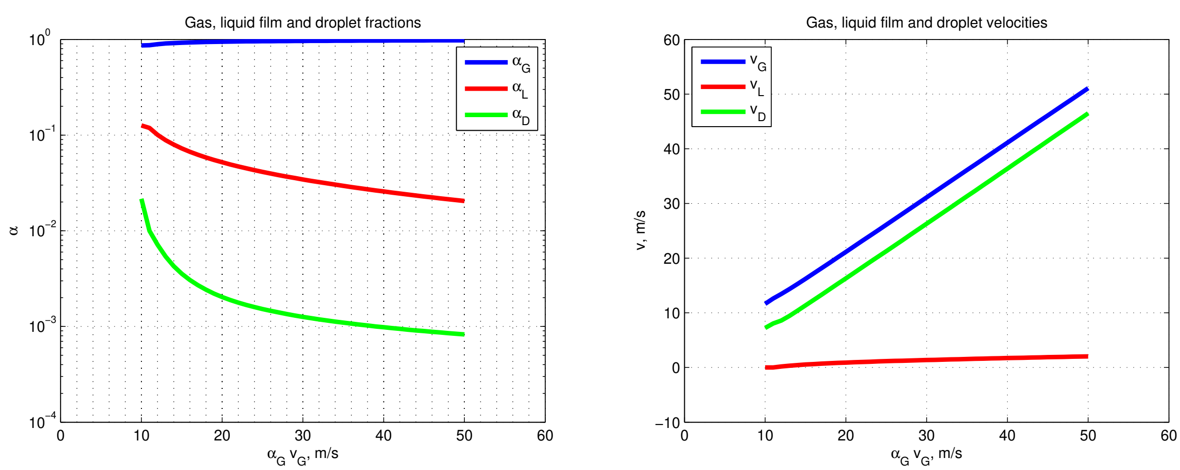
The algorithm to solve assignment #9 is in the following manner:

1. Set $\alpha_G v_G = 10$.

2. Guess $Y = Y_0$ ($k_G in$ is calculated knowing $\alpha_G v_G$); note that sum of the first three elements must add up to one (e.g. $Y0 = [0.9, 0.099, 0.001, 30, 2, 25]^T$).
3. Set $\alpha_G = Y(1)$, $\alpha_L = Y(2)$, $\alpha_D = Y(3)$, $v_G = Y(4)$, $v_L = Y(5)$, and $v_D = Y(6)$.
4. Calculate $R_{GD}, R_{LW}, R_{GL}, R_{DL}, \Gamma_{DL}, \Gamma_{LD}, d_D$ through *calcTFM* and using the input pipe data and info gathered in step 3.
5. Derive F_G, F_L, F_D according to equations 6.1.5 through 6.1.7.
6. Construct the residuals vector, F, as in 6.1.11.
7. Iterate 2 through 6 to minimize F yielding optimum values for Y.
8. Add the new Y column vector to the i'th column of the matrix with the same name.
9. Calculate step 2 to 8 for $\alpha_G v_G = \alpha_G v_G + \Delta\alpha_G v_G$ until $\alpha_G v_G = 50$.
10. α_G, α_L, α_D, v_G, v_L, and v_D are extracted from the matrix Y and plotted on vertical axes against $\alpha_G v_G$.
11. $\Gamma_{DL}, \Gamma_{LD}, d_D$ are also calculated knowing the optimum value of Y and are plotted in the same manner as previous step.

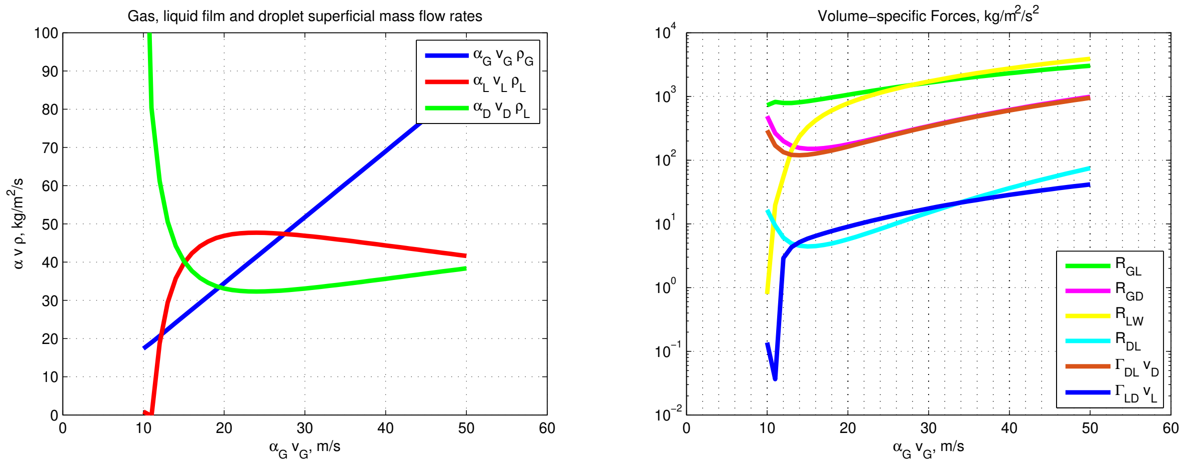
2 Results

The plots presented in the Figures 6.2.1 and 6.2.2 of the reference have been reproduced with a reasonable accuracy and are demonstrated in Figures 1 through 3.



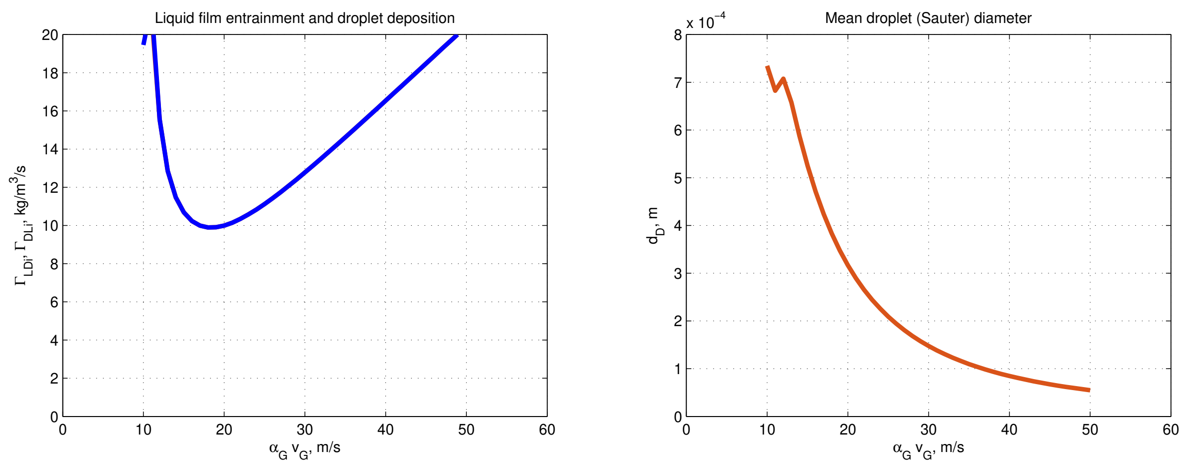
(a) (b)

Figure 1: Gas, liquid, and droplet phase fractions and velocities.



(a) (b)

Figure 2: Gas, liquid, and droplet phase superficial mass flow-rates and different volumetric forces of the momentum equation.



(a) (b)

Figure 3: Droplet deposition and liquid entrainment and mean droplet diameter with respect to superficial gas velocity.

Figure 3a actually consists of two curves that have overlapped. These curves represent Γ_{DL} and Γ_{LD} which should be equal as one of the boundary conditions of the problem.

References

[1] V.M. Alipchenkov, R.I. Nigmatulin, S.L. Soloviev, O.G. Stonik, L.I. Zaichik, and Y.A. Zeigarnik. A three-fluid model of two-phase dispersed-annular flow. *International Journal of Heat and Mass Transfer*, 47(24):5323 – 5338, 2004.

[2] Ove Bratland. Pipe flow 1: single-phase flow assurance. *Chapter*, 2:21–92, 2009.

[3] Ove Bratland. Pipe flow 2: multi-phase flow assurance, 2010.