1 Methodology

This problem is solved according to the procedure presented in the second chapter of Ove Bratland's *Pipe Flow 2: Multi-phase Flow Assurance*. The mass and momentum conservation equations are to be derived for the condition of this problem which are as follows [2]:

- Only two fluids are presented in the pipe. No evaporation or condensation are to occur. No gas can be solved in the liquid.
- The pipe is not perforated no flow occurs through pipe wall. (RHS of mass conservation equation is zero)
- Flow is stratified.
- Flow is isothermal.
- Steady-state flow regime is considered.
- Gas and liquid properties are independent of pressure. (Incompressible)
- Pipe's inclination angle is constant.

Solving mass balance and momentum conservation for each phase considering the above conditions yields a matrix of residuals, F, in equation 3.6.9 of the book which has to be minimized in order to find the answer to this assignment.

1.1 Computer Program Development

The solution method presented in the book is to guess the value of the four unknowns, namely, void fraction, liquid hold-up, gas and liquid velocities in equation 3.6.10, (which are going to be called Y_0) and then obtain the vector of residuals by inputting Y_0 and well data into suitable formulas to derive closure relationships (frictional pressure gradients). The unknowns vector, Y is subsequently calculated by means of iteration such as Newton iteration in which case the Jacobian matrix has to be evaluated by changing the Y parameters slightly to re-derive F. The new Y is calculated as follows:

$$Y = Y - J^{-1}F$$

Where J is the Jacobian matrix. The iteration should continue until after a specific criteria (e.g. sum of squares of residuals vector less than tolerance) is met.

In this problem the value of k_{Gin} is not given, however, the term $\alpha_G v_G$ is given as an input (from 10 to 50) and can be used to define inlet mass rate as follows:

$$k_{Gin} = [\alpha_G v_G]\rho_G$$

. Moreover, the Jacobian matrix in some cases of $\alpha_G v_G$ turns out to be singular and this complicates the solution. Consequently, some modifications are carried out to ensure convergence. Instead of Newton iteration method, *Trust-region Dogleg* algorithm is employed to minimize F using initial guess of Y_0.

1.1.1 Functions and Algorithm

Two functions are defined: (1) *calcTPM*: based on well data and Y evaluates closure relationships for frictional pressure gradient on each interface. At first geometric parameters (β, height of liquid and gas in pipe, hydraulic radius for each phase) are calculated, and then Reynolds number which are used to obtain friction factors for each interface.

(2) *calcResiduals*: is a function that outputs F, which needs to be minimized using *Levenberg-Marquardt* method in built-in MATLAB function *fsolve*. This function basically uses the input Y and $\alpha_G v_G$ and the closures calculated via *calcTPM* and calculates and returns the vector of residuals, F, according to equation 3.6.9.

The algorithm to solve assignment #6 is in the following manner:

1. Set $\alpha_G v_G = 10$.
2. Guess $Y = Y_0$ (k_{Gin} is calculated knowing $\alpha_G v_G$); note that sum of the first two elements must add up to one (e.g. $Y0 = [0.6, 0.4, 10, 2]^T$).
3. Calculate geometric parameters using pipe data and Y.
4. Obtain Reynolds no. of each phase and friction factors for the interfaces. Volume-specific friction forces are then calculated.
5. Derive the residual terms and vector F using frictions, pipe data and Y.
6. Minimize F yielding optimum values for Y.
7. Calculate step 2 to 6 for $\alpha_G v_G = \alpha_G v_G + \Delta\alpha_G v_G$ until $\alpha_G v_G = 50$.
8. α_G, α_L, v_G, and v_L are extracted from Y, and pressure gradient is obtained via equation 3.6.7.
9. Variables in previous step are plotted against $\alpha_G v_G$.

It should be noted that while friction factor for gas-liquid interface is calculated via equations 3.5.11-12, friction factors for gas-wall and liquid-wall are calculated based on *Colebrook and White* equation. (Table 2.13.2 in [1])

2 Results

As demonstrated in Figures 1 and 2, five plots in total are produced by the computer code which evaluate five parameters with respect to superficial gas velocity.

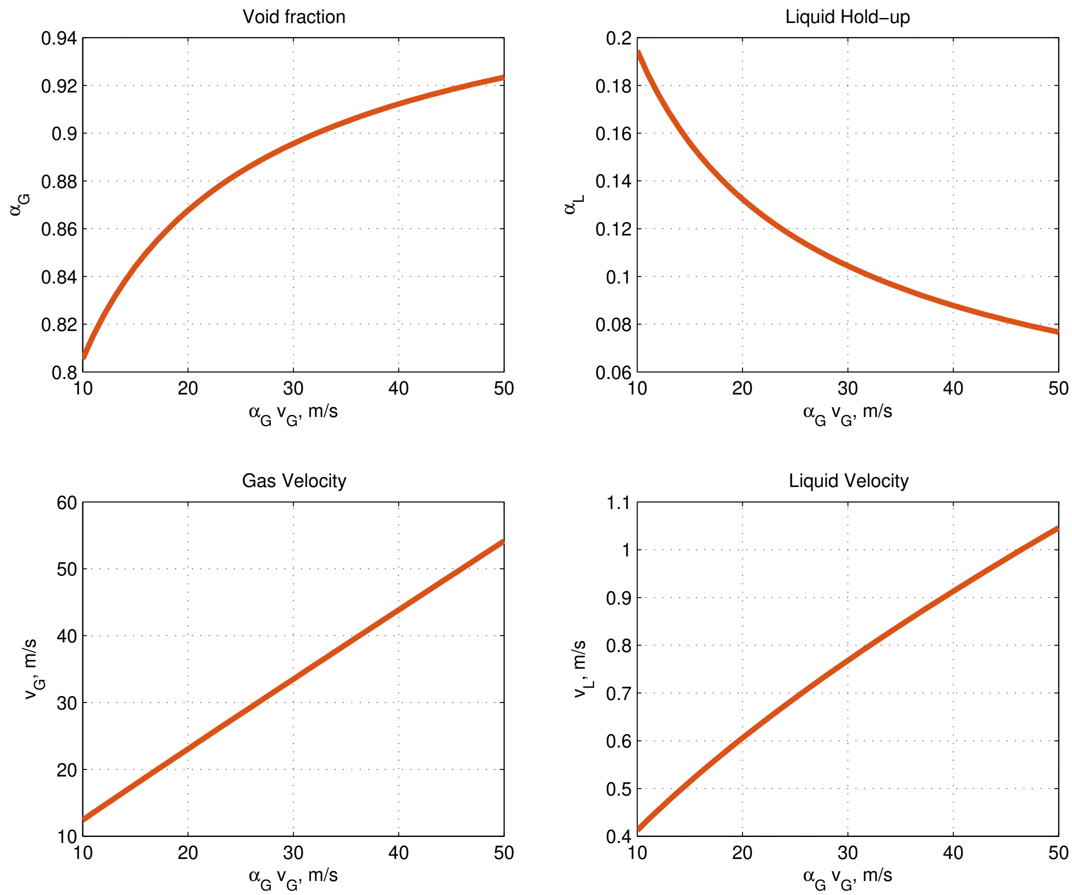

Figure 1: Plots generated by computer program.

The performance of the generated curves are reasonable within the simplifications of the problem. As evident in Figure 1, gas void fraction increases with increasing gas superficial velocity. Also gas velocity is increased which is also accurate. Liquid hold-up reduces as superficial gas velocity increases due to rising void fraction, and finally, liquid velocity is increased by a small amount due to diminishing liquid cross-section.

Figure 2 shows the pressure gradient increasing with rising gas superficial velocity. Considering the flow is of horizontal stratified nature, gravity forces do not play a huge role in determining the pipe's pressure gradient. Also pipe is assumed to have a surface roughness, albeit small. So by increasing gas superficial velocity, and therefore gas velocity, frictional pressure loss increases, and consequently the total pressure gradient increases.

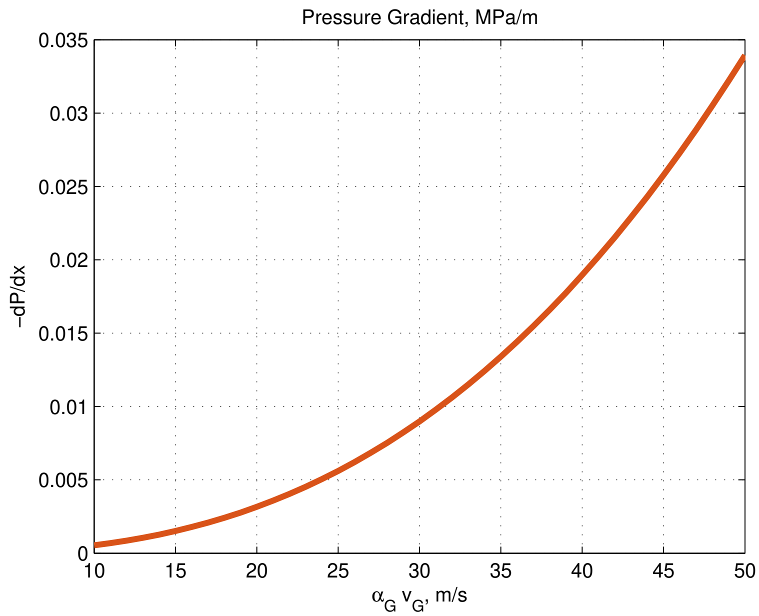

Figure 2: Pressure gradient changes with respect to increasing superficial gas velocity.

References

[1] Ove Bratland. Pipe flow 1: single-phase flow assurance. *Chapter*, 2:21–92, 2009.

[2] Ove Bratland. Pipe flow 2: multi-phase flow assurance, 2010.