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The Big Short: Interest Rates Explained
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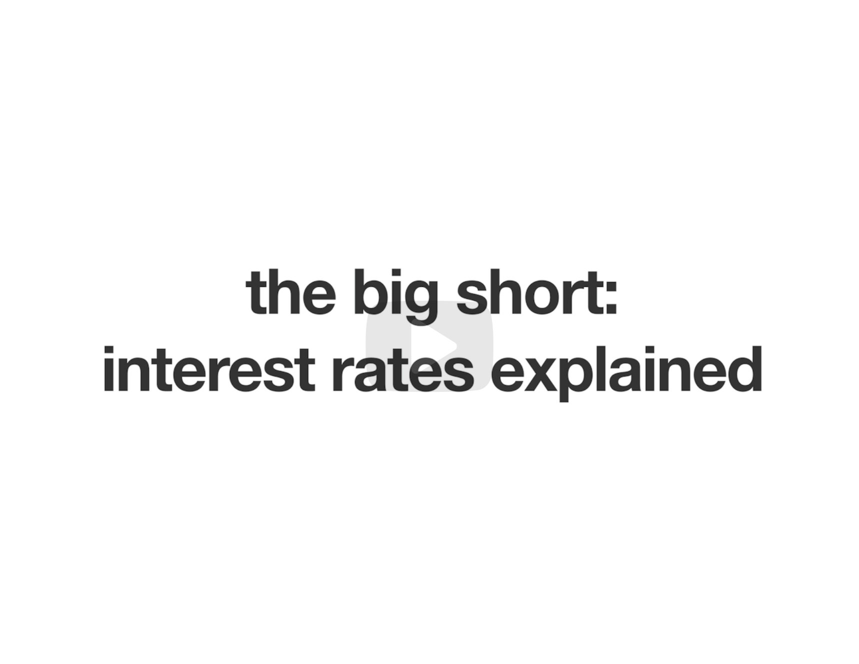

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Start of transcript. Skip to the end.

RAFAEL IRIZARRY: In a way, the sampling models we've been talking about

are also used by banks to decide interest rates.

Let's see how this could be.

Suppose you run a small bank that has a history of identifying

potential homeowners that can be trusted to make payments.

Video
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Textbook link

This video corresponds to the textbook section on interest rates.

Correction

At 2:35, the displayed results of the code are incorrect. Here are the correct values:

```
n*(p*loss_per_foreclosure + (1-p)*0)
[1] -4e+06
sqrt(n)*abs(loss_per_foreclosure)*sqrt(p*(1-p))
[1] 885438
```

Key points

- Interest rates for loans are set using the probability of loan defaults to calculate a rate that minimizes the probability of losing money.
- We can define the outcome of loans as a random variable. We can also define the sum of outcomes of many loans as a random variable.
- The Central Limit Theorem can be applied to fit a normal distribution to the sum of profits over many loans. We can use properties of the normal distribution to calculate the interest rate needed to ensure a certain probability of losing money for a given probability of default.

Code: Interest rate sampling model

```
n <- 1000
loss_per_foreclosure <- -200000
p <- 0.02
defaults <- sample( c(0,1), n, prob=c(1-p, p), replace = TRUE)
sum(defaults * loss_per_foreclosure)
```

Code: Interest rate Monte Carlo simulation

```
B <- 10000
losses <- replicate(B, {
    defaults <- sample( c(0,1), n, prob=c(1-p, p), replace = TRUE)
  sum(defaults * loss_per_foreclosure)
})
```

Code: Plotting expected losses

```
library(tidyverse)
data.frame(losses_in_millions = losses/10^6) %>%
    ggplot(aes(losses_in_millions)) +
    geom_histogram(binwidth = 0.6, col = "black")
```

Code: Expected value and standard error of the sum of 1,000 loans

```
n*(p*loss_per_foreclosure + (1-p)*0)    # expected value
sqrt(n)*abs(loss_per_foreclosure)*sqrt(p*(1-p))    # standard error
```

Code: Calculating interest rates for expected value of 0

We can calculate the amount x to add to each loan so that the expected value is 0 using the equation $lp + x(1-p) = 0$. Note that this equation is the definition of expected value given a loss per foreclosure l with foreclosure probability p and profit x if there is no foreclosure (probability $1-p$).

We solve for $x = -\frac{lp}{1-p}$ and calculate x:

```
x = - loss_per_foreclosure*p/(1-p)
x
```

On a $180,000 loan, this equals an interest rate of:

```
x/180000
```

Equations: Calculating interest rate for 1% probability of losing money

We want to calculate the value of x for which $\Pr(S < 0) = 0.01$. The expected value $\mathrm{E}[S]$ of the sum of $n = 1000$ loans given our definitions of x, l and p is:

$$\mu_S = (lp + x(1-p)) * n$$

And the standard error of the sum of n loans, $\mathrm{SE}[S]$, is:

$$\sigma_S = \mid x - l \mid \sqrt{np(1-p)}$$

Because we know the definition of a Z-score is $Z = \frac{x-\mu}{\sigma}$, we know that $\Pr(S < 0) = \Pr(Z < -\frac{\mu}{\sigma})$. Thus, $\Pr(S < 0) = 0.01$ equals:

$$\Pr\left(Z < \frac{-\{lp + x(1-p)\}n}{(x-l)\sqrt{np(1-p)}}\right) = 0.01$$

`z<-qnorm(0.01)` gives us the value of z for which $\Pr(Z \leq z) = 0.01$, meaning:

$$z = \frac{-\{lp + x(1-p)\}n}{(x-l)\sqrt{np(1-p)}}$$

Solving for x gives:

$$x = -l\frac{np - z\sqrt{np(1-p)}}{n(1-p) + z\sqrt{np(1-p)}}$$

Code: Calculating interest rate for 1% probability of losing money

```
l <- loss_per_foreclosure
z <- qnorm(0.01)
x <- -l*( n*p - z*sqrt(n*p*(1-p)))/ ( n*(1-p) + z*sqrt(n*p*(1-p)))\x
x/180000    # interest rate
loss_per_foreclosure*p + x*(1-p)    # expected value of the profit per
n*(loss_per_foreclosure*p + x*(1-p)) # expected value of the profit ove
```

Code: Monte Carlo simulation for 1% probability of losing money

Note that your results will vary from the video because the seed is not set.

```
B <- 100000
profit <- replicate(B, {
    draws <- sample( c(x, loss_per_foreclosure), n,
                        prob=c(1-p, p), replace = TRUE)
    sum(draws)
})
mean(profit)    # expected value of the profit over n loans
mean(profit<0)    # probability of losing money
```