Quantitative Management Assignment#6

Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

Unit Shipping Cost				Unit Production Cost	Monthly Production Capacity
	Warehouse 1	Warehouse 2	Warehouse 3		
Plant A	\$22	\$14	\$30	\$600	100
Plant B	\$16	\$20	\$24	\$625	120
Monthly Demand	80	60	70		

Solution 1:

Assume that the plant A produces X11, X12 & X13 units of AEDs for Warehouse 1, Warehouse 2 and Warehouse 3 respectively. And plant B produces X21, X22 & X23 units of AEDs for Warehouse 1, Warehouse 2 and Warehouse 3 respectively. So, the total cost will be calculated as sum of Unit Shipping cost to each Warehouse and the Unit production cost:

$$Z = 600X11 + 600X12 + 600X13 + 625X21 + 625X22 + 625X23 + 22X11 + 14X12 + 30X13 + 16X21 + 20X22 + 24X23$$

Where, Z denotes the total cost.

Now, plant A has monthly production capacity of 100 AEDS. And plant B has monthly production capacity of 120 AEDs. Therefore,

$$X11 + X11 + X13 \leq 100$$

$$X21 + X22 + X23 \leq 120$$

Also, monthly demand for each warehouse is as follows

$$X11 + X21 = 80$$

$$X12 + X22 = 60$$

$$X13 + X23 = 70$$

Hence, the Linear programming model should be defined as:

The decision variables are:

X11 = units of AEDs shipped from Plant A to Warehouse 1
X12 = units of AEDs shipped from Plant A to Warehouse 2
X13 = units of AEDs shipped from Plant A to Warehouse 3
X21 = units of AEDs shipped from Plant B to Warehouse 1
X22 = units of AEDs shipped from Plant B to Warehouse 2
X23 = units of AEDs shipped from Plant B to Warehouse 3

Minimize $Z = 622X11 + 614X12 + 630X13 + 640X21 + 645X22 + 649X23$

subject to

$$X11 + X12 + X13 \leq 100$$
$$X21 + X22 + X23 \leq 120$$
$$X11 + X21 = 80$$
$$X12 + X22 = 60$$
$$X13 + X33 = 70$$

And $X11, X12, X13, X21, X22, X23 >= 0$

Solution 2 (DUMMY VARIABLES):

Since the monthly demand is less than the monthly production capacity. Let's create dummy Warehouse (Warehouse 4) which can accommodate 10 units. The unit shipping cost to the warehouse 4 is $0 since it is dummy warehouse.

Unit Shipping Cost					Unit	Monthly
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4 (Dummy)	Production Cost	Production Capacity
Plant A	$22	$14	$30	$0	$600	100
Plant B	$16	$20	$24	$0	$625	120

Monthly Demand	80	60	70	10		

So now Assume that the plant A produces X11, X12, X13 and X14 units of AEDs for Warehouse 1, Warehouse 2, Warehouse 3 and Warehouse 4 respectively. And plant B produces X21, X22, X23 & X24 units of AEDs for Warehouse 1, Warehouse 2, Warehouse 3 & Warehouse 4 respectively. So, the total cost will be calculated as sum of Unit Shipping cost to each Warehouse and the Unit production cost:

$Z = 600X11 + 600X12 + 600X13 + 625X21 + 625X22 + 625X23 + 22X11 + 14X12 + 30X13 + 0X14 + 16X21 + 20X22 + 24X23 + 0X24$

Where, Z denotes the total cost.

Now, plant A has monthly production capacity of 100 AEDS. And plant B has monthly production capacity of 120 AEDs. Therefore,

$$X11 + X11 + X13 + X14 = 100$$

$$X21 + X22 + X23 + X24 = 120$$

Also, monthly demand for each warehouse is as follows

$$X11 + X21 = 80$$

$$X12 + X22 = 60$$

$$X13 + X23 = 70$$

$$X14 + X24 = 10$$

Hence, the Linear programming model should be defined as:

The decision variables are:

X11 = units of AEDs shipped from Plant A to Warehouse 1
X12 = units of AEDs shipped from Plant A to Warehouse 2
X13 = units of AEDs shipped from Plant A to Warehouse 3
X14 = units of AEDs shipped from Plant A to Warehouse 4
X21 = units of AEDs shipped from Plant B to Warehouse 1
X22 = units of AEDs shipped from Plant B to Warehouse 2
X23 = units of AEDs shipped from Plant B to Warehouse 3

X24 = units of AEDs shipped from Plant B to Warehouse 4

Minimize $Z = 622X11 + 614X12 + 630X13 + 0X14 + 640X21 + 645X22 + 649X23 + 0X24$

subject to

$$X11 + X12 + X13 + X14 = 100$$
$$X21 + X22 + X23 + X24 = 120$$
$$X11 + X21 = 80$$
$$X12 + X22 = 60$$
$$X13 + X33 = 70$$
$$X14 + X24 = 10$$

And $X11, X12, X13, X14, X21, X22, X23, X24 >= 0$