Quant Management Assignment#11

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Integer Programming Problem:

AP is a shipping service that guarantees overnight delivery of packages in the continental US. The company has various hubs at major cities and airports across the country. Packages are received at hubs, and then shipped to intermediate hubs or to their final destination. The manager of the AP hub in Cleveland is concerned about labor costs, and is interested in determining the most effective way to schedule workers. The hub operates seven days a week, and the number of packages it handles varies from one day to another. The table below provides an estimate of the number of workers needed each day of the week.

Day of the Week	Workers Required
Sunday	18
Monday	27
Tuesday	22
Wednesday	26
Thursday	25
Friday	21
Saturday	19

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is $750 per week. Workers working on Saturday or Sunday receive an additional $25 per day. The possible shifts and salaries for package handlers are:

Shift	Days off	Wage
1	Sunday and Monday	$755
2	Monday and Tuesday	$800
3	Tuesday and Wednesday	$800
4	Wednesday and Thursday	$800
5	Thursday and Friday	$800
6	Friday and Saturday	$775
7	Saturday and Sunday	$750

Questions:

The manager wants to keep the total wage expenses as low as possible while ensuring that there are sufficient number of workers available each day. Formulate and solve the problem. What was the total cost? How many workers are available each day?

Answers:

Lets Formulate the problem

Let us consider the decision variables as following:
x1= the number of workers assigned to shift 1
x2= the number of workers assigned to shift 2
x3= the number of workers assigned to shift 3
x4= the number of workers assigned to shift 4
x5= the number of workers assigned to shift 5
x6= the number of workers assigned to shift 6
x7= the number of workers assigned to shift 7

Minimize the total wage expense (Objective Function):
Min = 775x1 + 800x2 + 800x3 + 800x4 + 800x5 + 775x6 + 750x7

Workers Required each day (Constraints):

Day of Week	Workers
Sunday	0x1 + 1x2 + 1x3 + 1x4 + 1x5 + 1x6 + 0x7 >= 18
Monday	0x1 + 0x2 + 1x3 + 1x4 + 1x5 + 1x6 + 1x7 >= 27
Tuesday	1x1 + 0x2 + 0x3 + 1x4 + 1x5 + 1x6 + 0x7 >= 22
Wednesday	1x1 + 1x2 + 0x3 + 0x4 + 1x5 + 1x6 + 1x7 >= 26
Thursday	1x1 + 1x2 + 1x3 + 0x4 + 0x5 + 1x6 + 1x7 >= 25
Friday	1x1 + 1x2 + 1x3 + 1x4 + 0x5 + 0x6 + 1x7 >= 21
Saturday	1x1 + 1x2 + 1x3 + 1x4 + 1x5 + 0x6 + 0x7 >= 19

x1, x2, x3, x4, x5, x6, x7 >=0

Lets solve the IP problem,

```
# Load the library
library(lpSolveAPI)
# Note that we had 7 decision variables and 7 constraints.
lprec <- make.lp(7, 7)
# Set the minimization objective function
set.objfn(lprec, c(775, 800, 800, 800, 800, 775, 750))
lp.control(lprec,sense='min')
```

```
## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"       "dynamic"      "rcostfixing"
##
## $break.at.first
```

```
## [1] FALSE
##
## $break.at.value
## [1] -1e+30
##
## $epsilon
##       epsb       epsd      epsel     epsint epsperturb   epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##    1e-11    1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"    "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"   "equilibrate" "integers"
##
## $sense
## [1] "minimize"
##
## $simplextype
## [1] "dual"   "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

```
# Set values for the rows (set the Left hand side constraints)
set.row(lprec, 1, c(0, 1, 1, 1, 1, 1, 0), indices = c(1, 2, 3, 4, 5, 6, 7))
```

```
set.row(lprec, 2, c(0, 0, 1, 1, 1, 1, 1), indices = c(1, 2, 3, 4, 5, 6, 7))
set.row(lprec, 3, c(1, 0, 0, 1, 1, 1, 1), indices = c(1, 2, 3, 4, 5, 6, 7))
set.row(lprec, 4, c(1, 1, 0, 0, 1, 1, 1), indices = c(1, 2, 3, 4, 5, 6, 7))
set.row(lprec, 5, c(1, 1, 1, 0, 0, 1, 1), indices = c(1, 2, 3, 4, 5, 6, 7))
set.row(lprec, 6, c(1, 1, 1, 1, 0, 0, 1), indices = c(1, 2, 3, 4, 5, 6, 7))
set.row(lprec, 7, c(1, 1, 1, 1, 1, 0, 0), indices = c(1, 2, 3, 4, 5, 6, 7))
# Set the right hand side values
rhs <- c(18, 27, 22, 26, 25, 21, 19)
set.rhs(lprec, rhs)
# Set constraint type and set variable types and bound
set.constr.type(lprec, c(">=", ">=", ">=", ">=", ">=", ">=", ">="))
# Set lower bound as zero
set.bounds(lprec, lower = rep(0, 7))
# Set variable type as integer
set.type(lprec,1:7,"integer")
# Finally, name the decision variables (column) and constraints (rows)
lp.rownames <- c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")
lp.colnames <- c("Shift1", "Shift2", "Shift3","Shift4", "Shift5", "Shift6","Shift7")
dimnames(lprec) <- list(lp.rownames, lp.colnames)
# View the linear program object to make sure it's correct
lprec
```

```
## Model name:
##            Shift1  Shift2  Shift3  Shift4  Shift5  Shift6  Shift7
## Minimize      775     800     800     800     800     775     750
## Sunday          0       1       1       1       1       1       0  >=  18
## Monday          0       0       1       1       1       1       1  >=  27
## Tuesday         1       0       0       1       1       1       1  >=  22
## Wednesday       1       1       0       0       1       1       1  >=  26
## Thursday        1       1       1       0       0       1       1  >=  25
## Friday          1       1       1       1       0       0       1  >=  21
## Saturday        1       1       1       1       1       0       0  >=  19
## Kind          Std     Std     Std     Std     Std     Std     Std
## Type          Int     Int     Int     Int     Int     Int     Int
## Upper         Inf     Inf     Inf     Inf     Inf     Inf     Inf
## Lower           0       0       0       0       0       0       0
```

```
# Save this into a file
write.lp(lprec, filename = "QA11.lp", type = "lp")
# Now solve the model
solve(lprec)
```

```
## [1] 0
```

```
# Show the value of objective function, variables and constraints
get.objective(lprec)
```

```
## [1] 25675
```

```
get.variables(lprec)
```

```
## [1]  2  4  5  0  8  1 13
```

```
get.constraints(lprec)
```

```
## [1] 18 27 24 28 25 24 19
```

Also, We now read the lp formulation using an lp file saved above

```
# Read the IP formulation file
y <- read.lp("QA11.lp")
# Print the model
y
```

```
## Model name:
##            Shift1  Shift2  Shift3  Shift4  Shift5  Shift6  Shift7
## Minimize      775     800     800     800     800     775     750
## Sunday          0       1       1       1       1       1       0  >=  18
## Monday          0       0       1       1       1       1       1  >=  27
## Tuesday         1       0       0       1       1       1       1  >=  22
## Wednesday       1       1       0       0       1       1       1  >=  26
## Thursday        1       1       1       0       0       1       1  >=  25
## Friday          1       1       1       1       0       0       1  >=  21
## Saturday        1       1       1       1       1       0       0  >=  19
## Kind          Std     Std     Std     Std     Std     Std     Std
## Type          Int     Int     Int     Int     Int     Int     Int
## Upper         Inf     Inf     Inf     Inf     Inf     Inf     Inf
## Lower           0       0       0       0       0       0       0
```

```
# Solve the model
solve(y)
```

```
## [1] 0
```

```
# Get the objective function value
get.objective(y)
```

```
## [1] 25675
```

The total cost is $25675

```
# Get the variables value
get.variables(y)
```

```
## [1]  2  4  5  0  8  1 13
```

```
D <- data.table::data.table(Decision_Variables=lp.rownames<-c("x1","x2","x3","x4","x5","x6","x7"),value
D
```

```
##    Decision_Variables values
## 1:                 x1      2
## 2:                 x2      4
## 3:                 x3      5
## 4:                 x4      0
## 5:                 x5      8
## 6:                 x6      1
## 7:                 x7     13
```

```
# Get the constraints value
get.constraints(y)
```

```
## [1] 18 27 24 28 25 24 19
```

```
C <- data.table::data.table(Days_of_week=lp.rownames<-c("Sunday","Monday","Tuesday","Wednesday","Thursd
C
```

```
##    Days_of_week Workers_available
## 1:       Sunday                18
## 2:       Monday                27
## 3:      Tuesday                24
## 4:    Wednesday                28
## 5:     Thursday                25
## 6:       Friday                24
## 7:     Saturday                19
```

The above table shows the number of workers available each day

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