Examples of Autocorrelation and Power Spectral Density

EE160: Principles of Communication Systems

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Sinusoidal

- Autocorrelation:

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A\cos(2\pi f_0 t)\, A\cos(2\pi f_0 (t+\tau))\, dt = \frac{A^2}{2} \cos(2\pi f_0 \tau)$$

- Power Spectral Density:

$$S_x(f) = F\{R_x(\tau)\} = \frac{A^2}{4} [\delta(f+f_0) + \delta(f-f_0)]$$

Periodic signal

- Autocorrelation:

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t+\tau)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{n=-\infty}^{\infty} x_n e^{j2\pi n f_0 t} \sum_{m=-\infty}^{\infty} x^*{}_m e^{-j2\pi m f_0 (t+\tau)}\; dt = \sum_{m=-\infty}^{\infty} |x_m|^2 e^{-j2\pi m f_0 \tau}$$

- Power Spectral Density:

$$S_x(f) = F\{R_x(\tau)\} = \sum_{m=-\infty}^{\infty} |x_m|^2 \delta(f + m f_0)$$