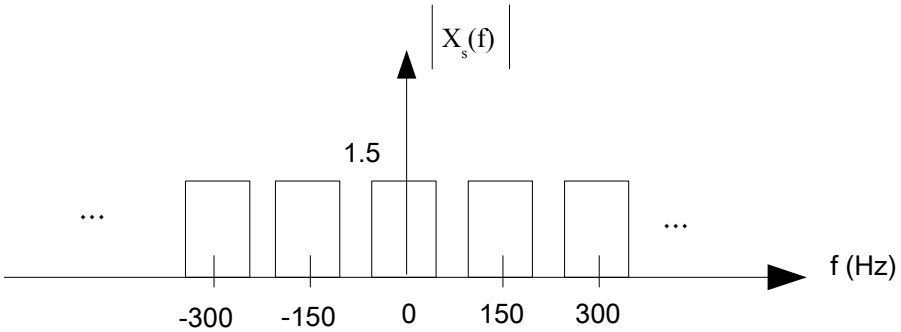
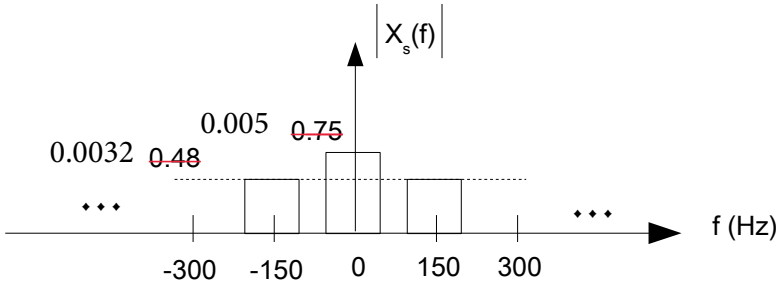
Solution of Homework # 4

1. (a) $f_{s,\min} = 100$ Hz.

   (b) A guardband of 50 Hz gives $f_s = 150$ Hz. Sketch:



   (c) Practical sampling with 50% duty cycle. Sketch:



2. (a) Transfer function:

$$H(f) = \begin{cases} \frac{1}{150}, & |f| \le 50, \\ g(f), & 50 < |f| \le 100, \\ 0, & |f| > 100, \end{cases}$$

   where $g(f)$ is an arbitrary function.

   (b) Aliasing occurs. Assuming an ideal lowpass filter of bandwidth $W' = 50$ Hz, i.e., $g(f) = 0$ (no rolloff) in part (a), the spectral density of the output is given by

$$Y(f) = \frac{1}{150}\left[\frac{3}{2}\Pi\left(\frac{f}{50}\right) + \frac{6}{2}\left(\Pi\left(\frac{f+37.5}{25}\right) + \Pi\left(\frac{f-37.5}{25}\right)\right)\right].$$

   Consequently,

$$y(t) = \frac{1}{2}\,\mathrm{sinc}(50t) + \mathrm{sinc}(25t)\,\cos(75\pi t).$$

   A sketch is shown below using Matlab script

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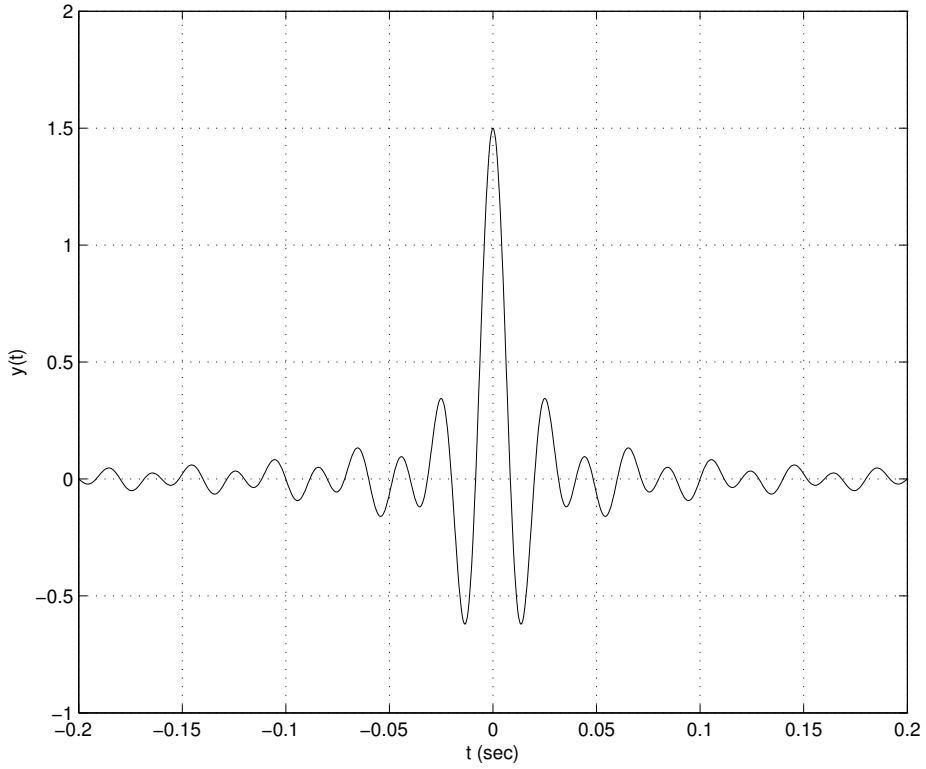
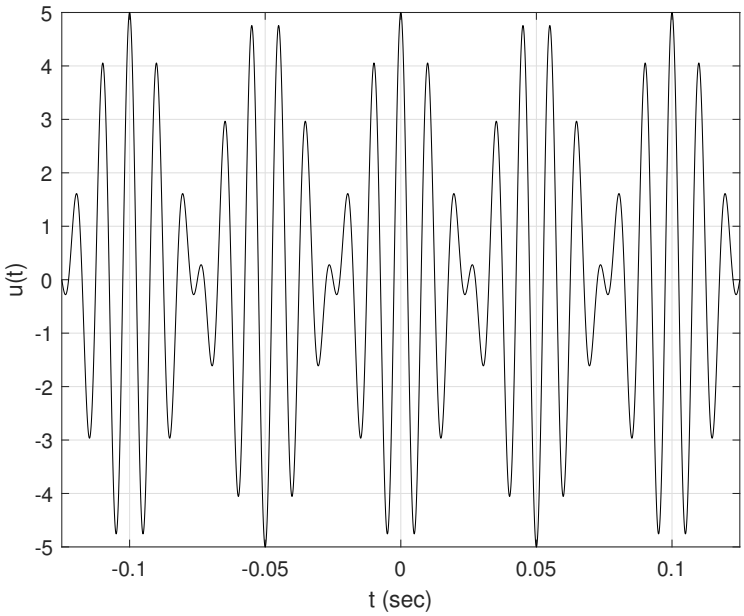
t=-1/5:1/2500:1/5; y = 0.5*sinc(50*t)+sinc(25*t).*cos(75*pi*t);
plot(t,y,'-k'), axis([-0.2 0.2 -1 2]), xlabel('t (sec)'), ylabel('y(t)'), grid on
```

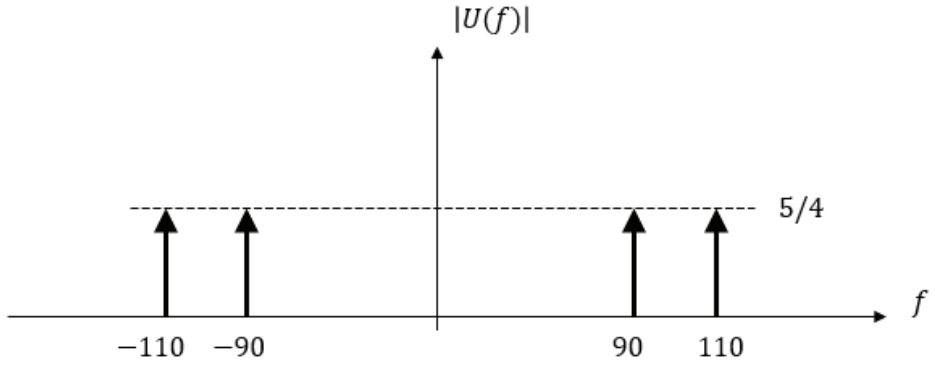


Alternative solution: Write $Y(f) = \dfrac{1}{150}\left[3\,\Pi\left(\dfrac{f}{100}\right) - \dfrac{3}{2}\,\Pi\left(\dfrac{f}{50}\right)\right]$. Then $y(t) = 2\text{sinc}(100t) - \dfrac{1}{2}\,\text{sinc}(50t)$. You are invited to verify that this gives the same waveform as above.

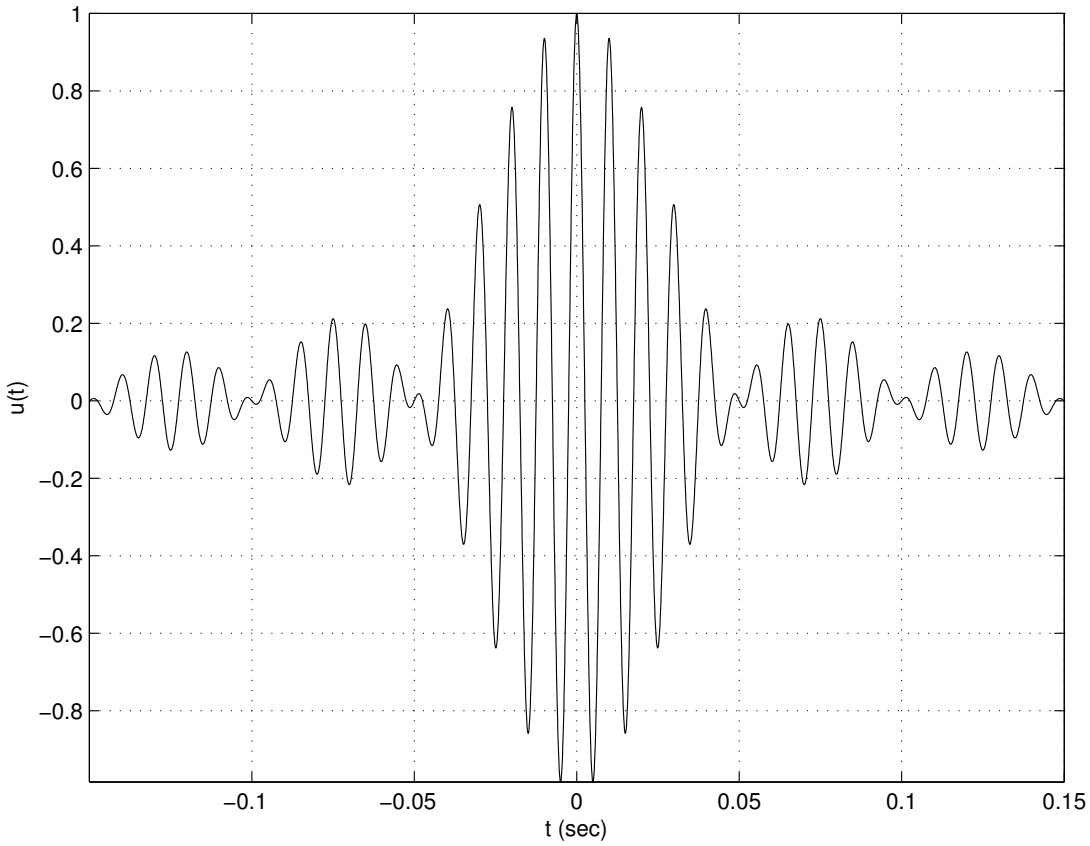
3. (a) Sketch:



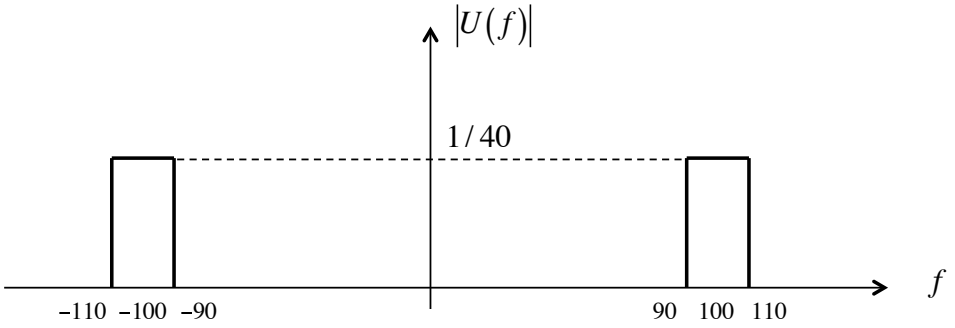
(b) Sketch:



4. (a) Sketch:



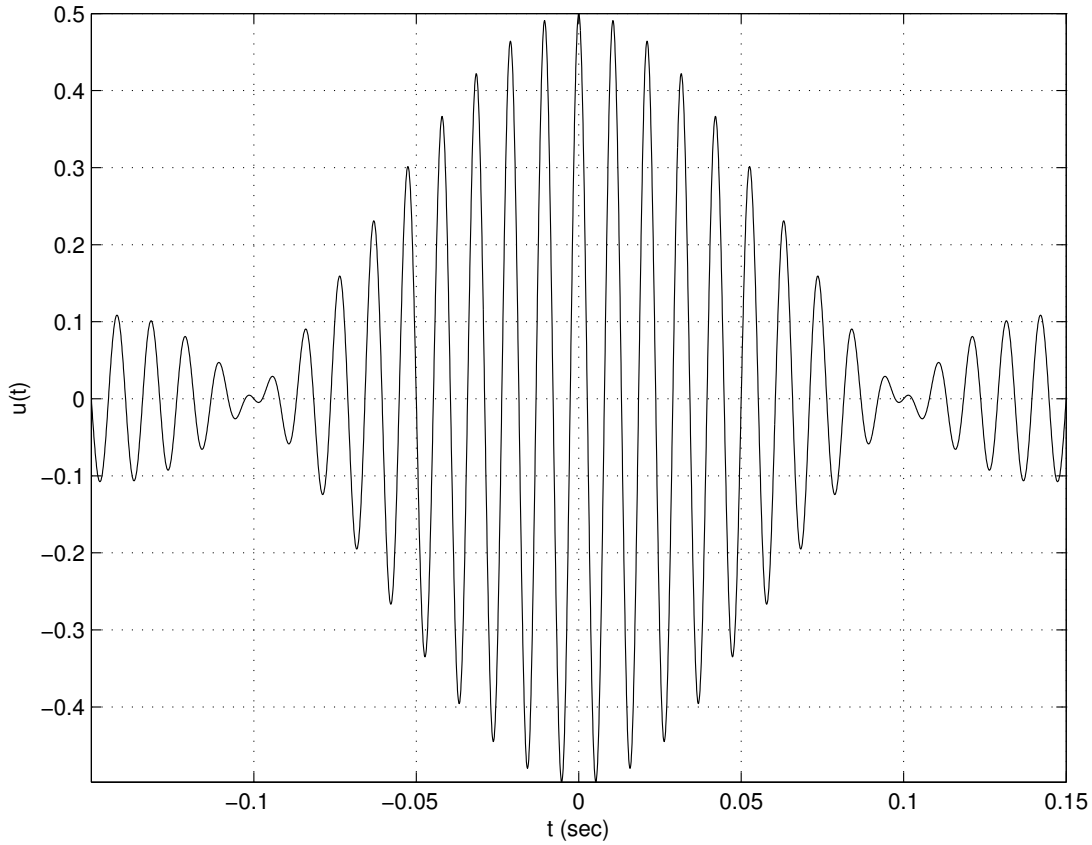
(b) Sketch:



(c) Lower sideband: The shape of the spectral density $U_\ell(f)$ is a rectangular pulse of width 10 Hz centered at 95 Hz. Therefore,

$$u(t) = \frac{1}{2}\,\mathrm{sinc}(10t)\,\cos(190\pi t).$$

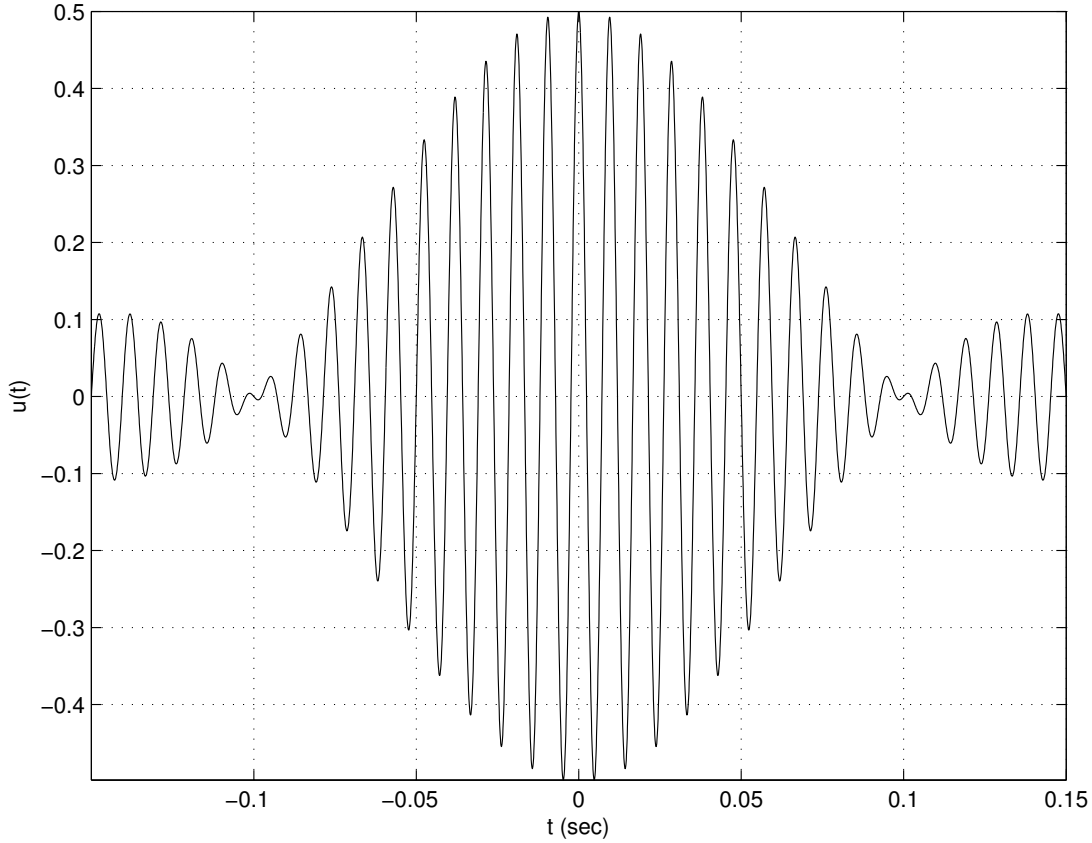
Sketch:



(d) Upper sideband: The shape of the spectral density $U_u(f)$ is a rectangular pulse of width 10 Hz but now centered at 105 Hz. Therefore,

$$u(t) = \frac{1}{2}\,\mathrm{sinc}(10t)\,\cos(210\pi t).$$

Sketch:



(e) The plots (not shown) are left as an exercise.