Multicarrier Signals

EE161: Digital Communication Systems

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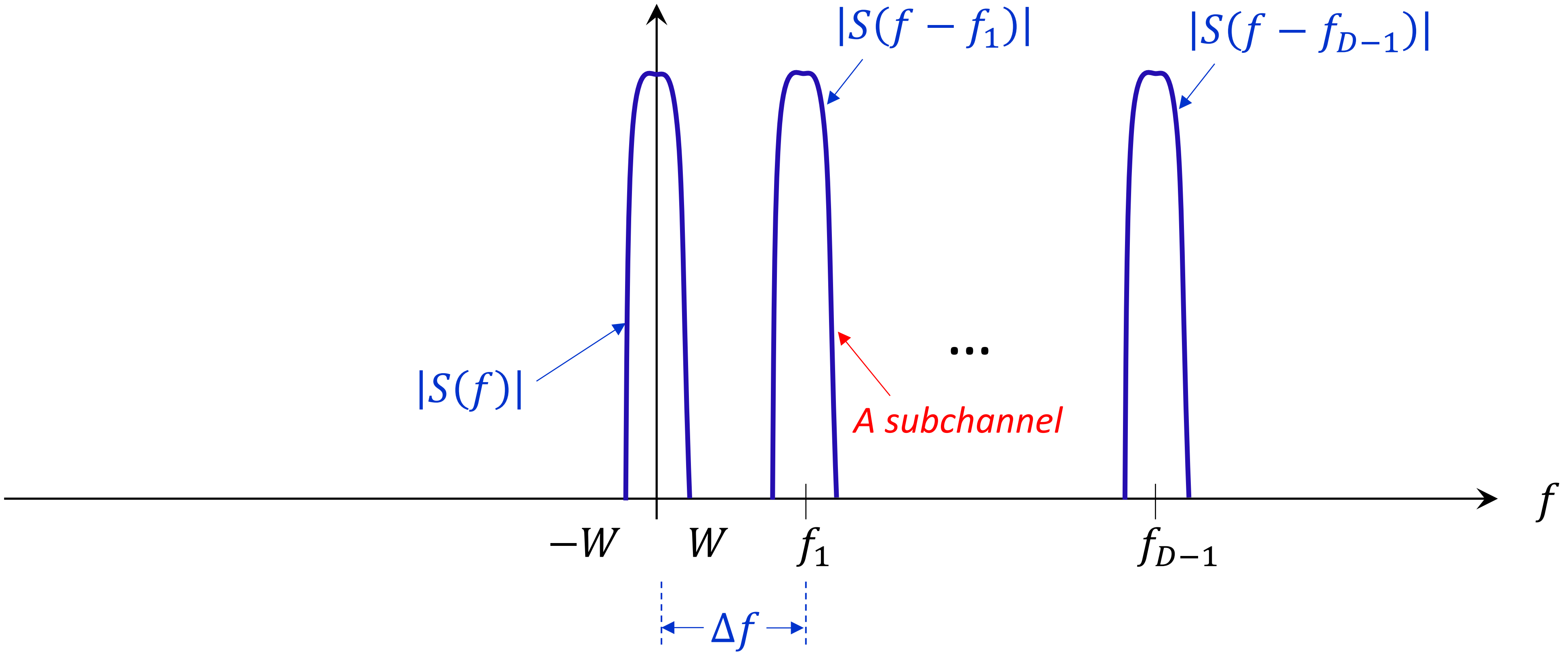
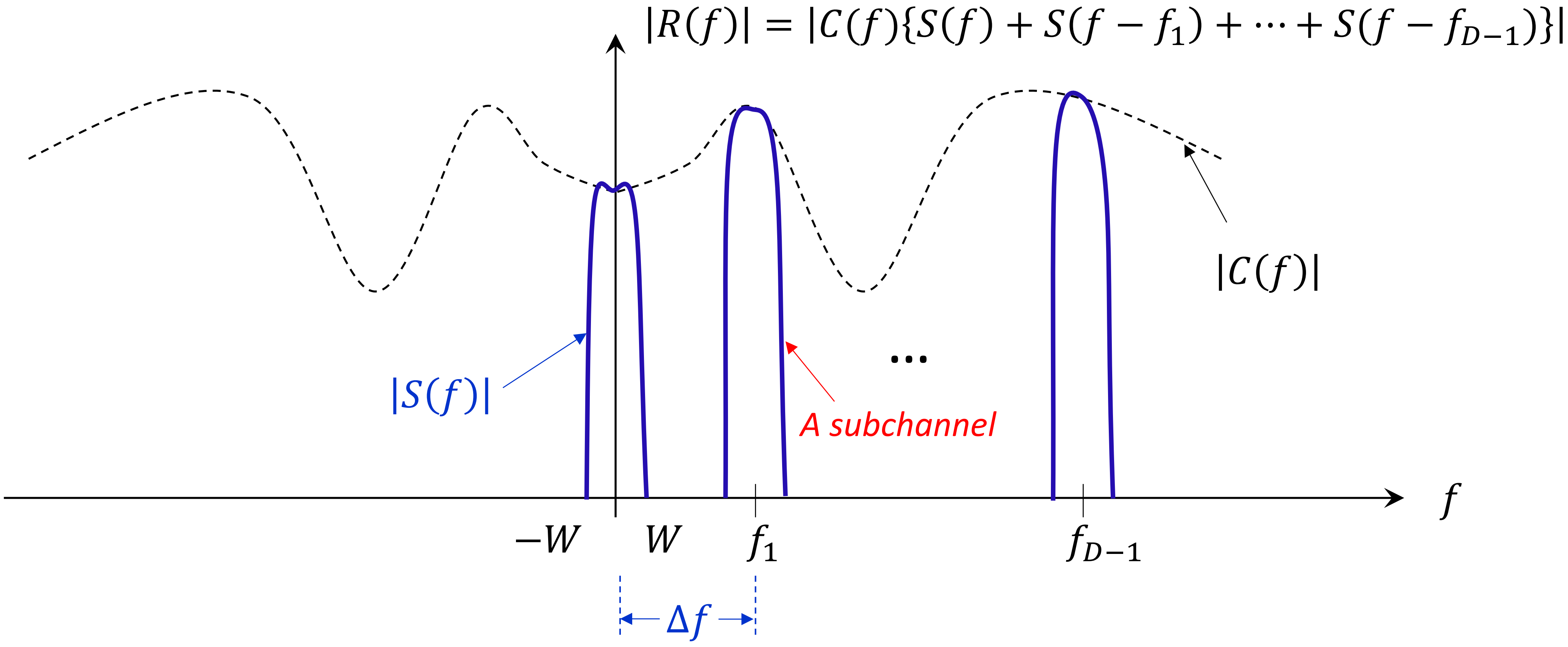
Frequency domain diversity

- *Pure diversity* is achieved by sending the same signal over multiple frequencies. Example of a diversity order D scheme:



Frequency domain diversity (cont.)

- Received signal:



Some observations

- The time signal of this “pure-diversity” frequency domain scheme is *complex valued*:

$$S(f) + S(f - f_1) + \cdots + S(f - f_{D-1}) \overset{F}{\leftrightarrow} s(t) + s(t)e^{j2\pi f_1 t} + \cdots + s(t)e^{j2\pi f_{D-1} t}$$

$$\boxed{= s(t)\left[1 + e^{j2\pi f_1 t} + \cdots + e^{j2\pi f_{D-1} t}\right]}$$

- This is similar to a (*D*,1,*D*) repetition code
- *Spectral efficiency* decreases by a factor of at least *D* (Coding rate: *R* = 1/*D*)
- To achieve full diversity, it is needed to have a spectral separation equal to $\Delta f = f_{i+1} - f_i$, for $i = 0, 1, D-2$, (assumed constant) greater than the coherence bandwidth:

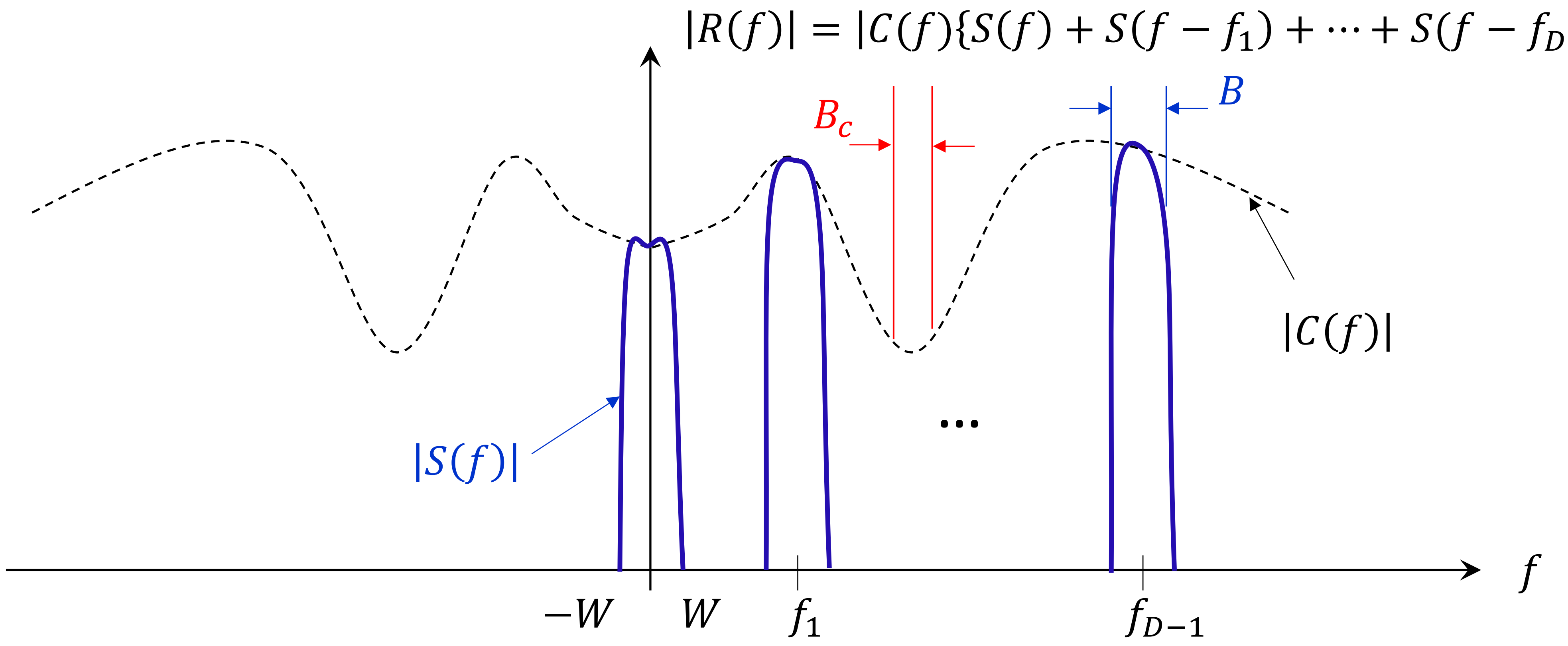
$$\boxed{\Delta f > B_c}$$

Some observations (cont.)

- *Diversity* is achieved only if the signal bandwidth $B = 2W$ satisfies

$$\boxed{B \ll B_c}$$

where B_c is the coherence bandwidth: *Narrowband (low rate) signal under flat fading*



Some observations (cont.)

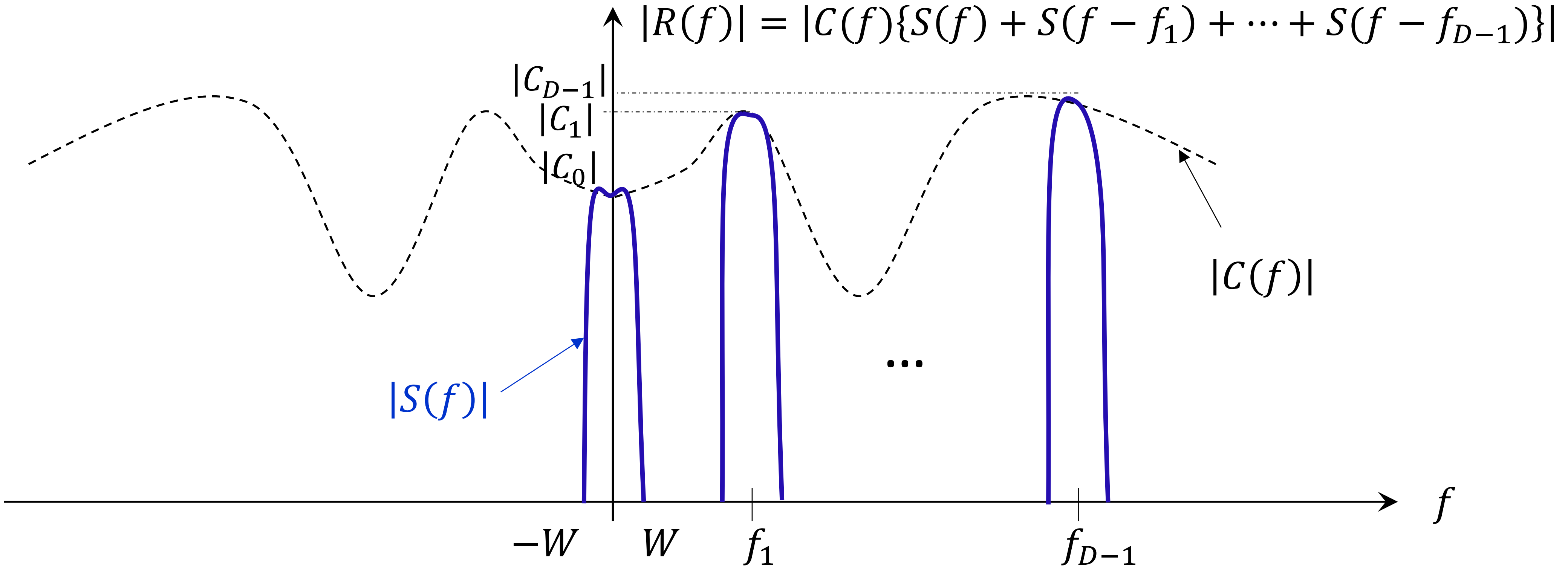
- If the signal bandwidth B is not sufficiently small, then subchannels become frequency-selective

- ~~The larger the number~~ $\cancel{K}$ ~~of subchannel frequencies~~ $f_i, i = 1,2, \ldots, K$, the less frequency selective they become

- In the general case with a relatively large number of subchannels, the diversity order may be smaller: $\boldsymbol{D \leq K}$

Channel estimation

- For diversity combining (e.g., MRC), the subchannel (complex-valued) gains estimates are needed: $\hat{C}_i$, where $C_i = C(f_i)$ for $i = 0, 1, \ldots, D-1$



- This means that ***pilot symbols*** known at the receiver need to be transmitted

Signal spectrum in the pure diversity scheme

- The spectrum $S(f)$ of the signal can be either

  1. *Bandwidth-limited* (like with square-root raised-cosine transmit filters), or

  2. *Raised-cosine pulses in the time domain*, provided that the complex exponentials $e^{j2\pi f_i t}, i = 1,2, \dots, K$, are orthogonal

- The second option is common in a scheme known as **orthogonal frequency-division multiplexing** (OFDM)

- Most of modern wireless networks today use OFDM

OFDM: Divide and conquer approach

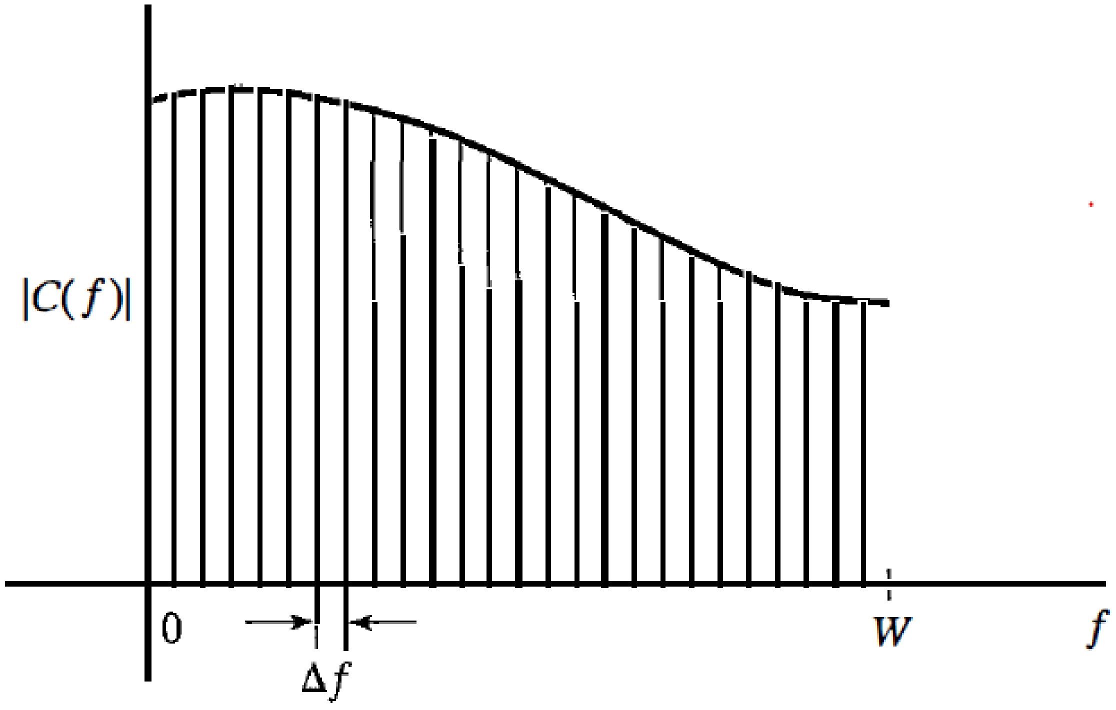


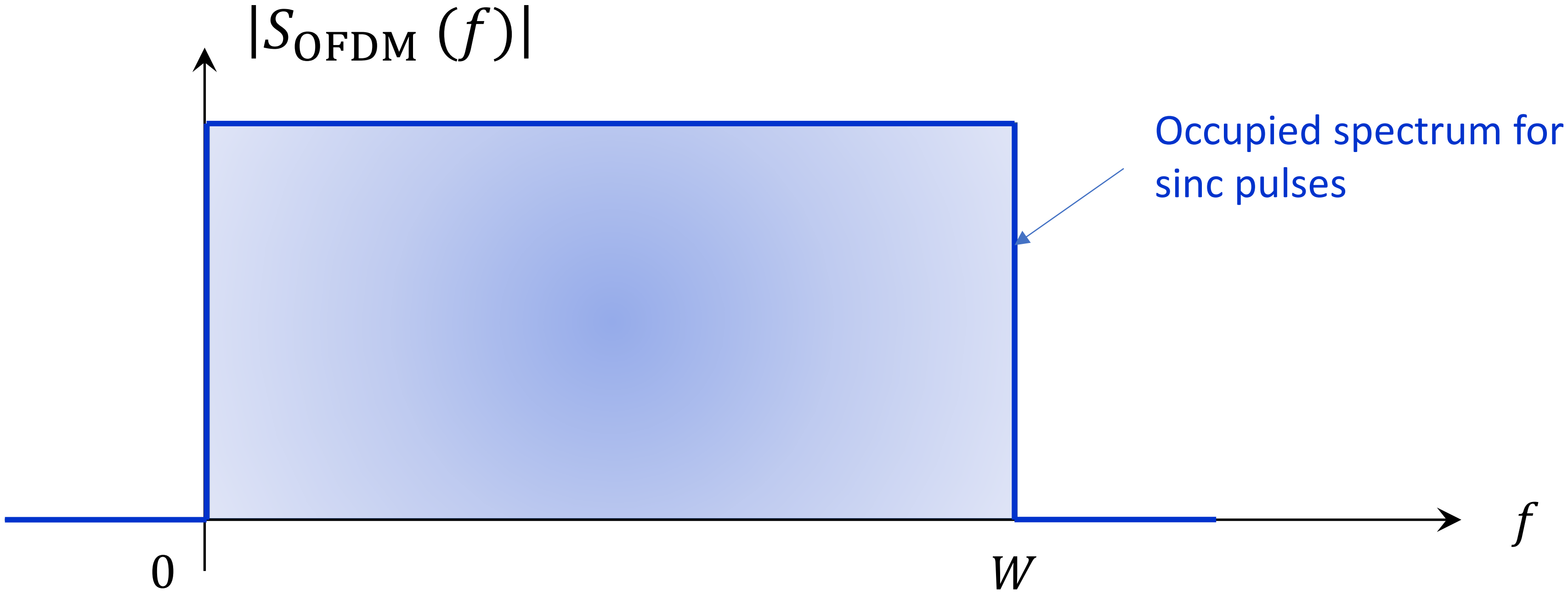
Figure 11.1 Subdivision of the channel bandwidth W into narrowband subchannels of equal width Δf.

- ***No spacing*** between subchanels

Figure from textbook

OFDM: Complex-signal spectrum

- The OFDM signal occupied spectrum is that of a complex signal, i.e., the amplitude spectrum is not even-symmetric !



- In other words, the RF bandwidth is the same as the lowpass bandwidth:

$$\boxed{B = W}$$

OFDM: Signal multiplexing

- Unlike the pure-diversity scheme, in OFDM each subchannel carries an independent message signal:

$$\boxed{s_{\mathrm{OFDM}}(t) = s_0(t) + s_1(t)e^{j2\pi f_1 t} + \cdots + s_{K-1}(t)e^{j2\pi f_{K-1} t}}$$

- The number of subcarriers *K* is such that *each subchannel is relatively flat*
  - ✓ In this sense, OFDM is equivalent to quantizing the frequency domain