Pulse Shaping Techniques for Baseband Binary Communication

EE 161: Digital Communication Systems

San Jose State University

Baseband binary communication

- In a *baseband* binary communication system, strings of bits need to be converted into *sequences of pulses (a waveform)* that are suitable for transmission over the lowpass channel
- The process of converting bits into pulses is known as pulse shaping (historically referred to as "line coding"). There are two components of this process:
    1. ***Pulse shaping***
    2. ***Mapping of bits to amplitudes***
- In this lecture, only <u>rectangular pulses</u> are considered

Basic pulse shapes/mappings

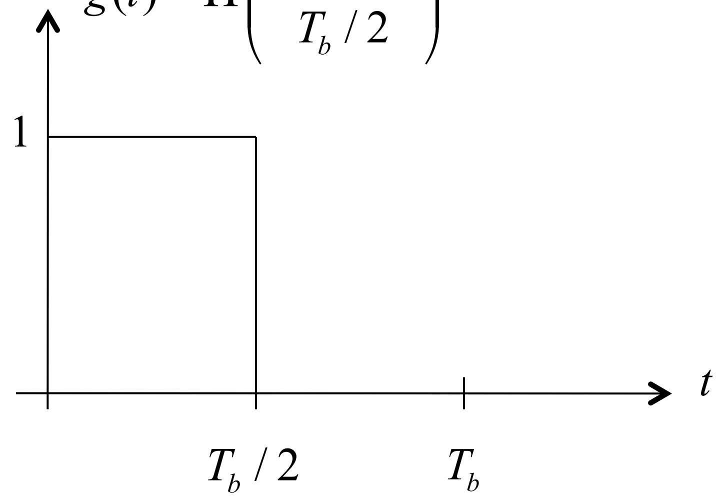
- **Pulse shapes**
  - Return-to-zero (***RZ***)
  - Non return-to-zero (***NRZ***)
  - ***Manchester*** (or split phase)

- **Mappings of bits to amplitudes**
  - ***Unipolar***
  - ***Polar***
  - Alternate-mark-inversion (***AMI***)

The above shapes/mapppings induce a classification of schemes: *Unipolar NRZ*, *AMI RZ*, etc…

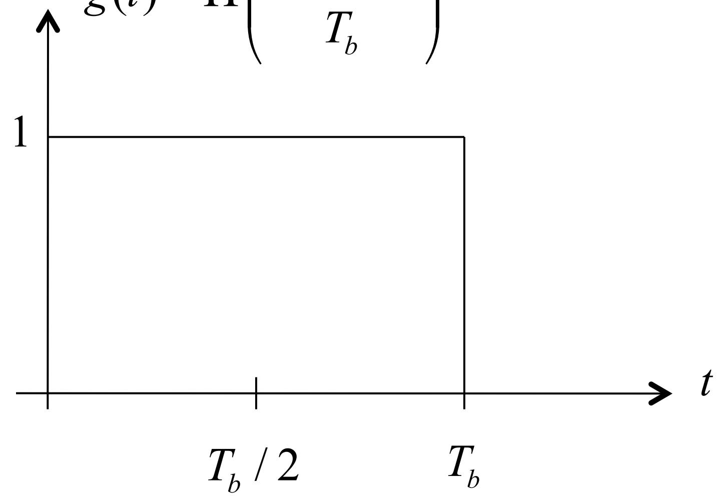
Return-to-zero (RZ) pulse

$$g(t) = \Pi\left(\frac{t - T_b/4}{T_b/2}\right)$$



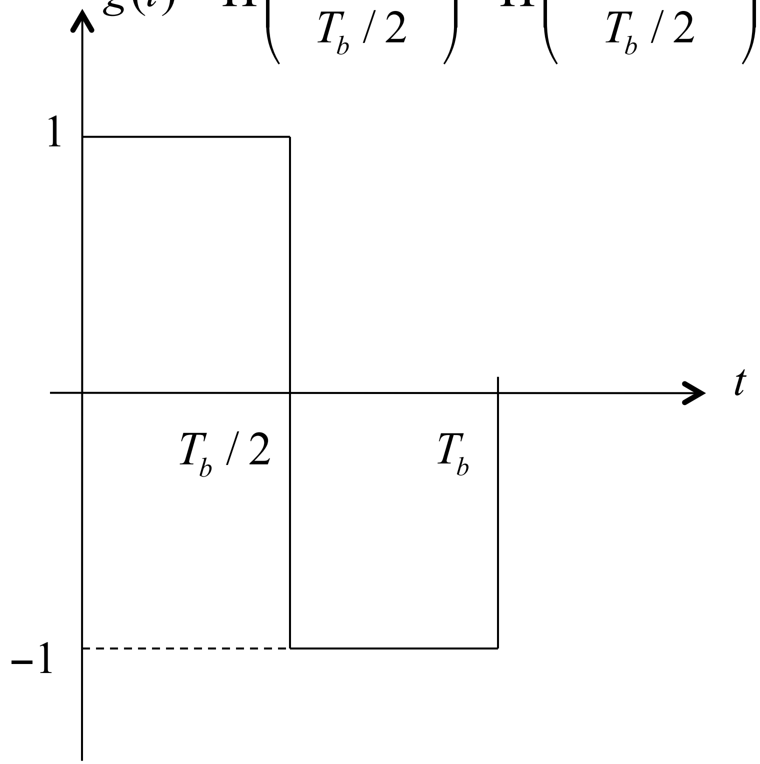
Non-return-to-zero (NRZ) pulse

$$g(t) = \Pi\left(\frac{t - T_b/2}{T_b}\right)$$



Manchester pulse

$$g(t) = \Pi\left(\frac{t - T_b/4}{T_b/2}\right) - \Pi\left(\frac{t - 3T_b/4}{T_b/2}\right)$$



Unipolar mapping

Bit	Amplitude
B_n	A_n
0	0
1	a

a: Amplitude

Example: $\underline{B} = \{0,0,1,1,0,1,0,\ldots\}$ $\underline{A} = \{0,0,a,a,0,a,0,\ldots\}$

Polar mapping

Bit	Amplitude
B_n	A_n
0	-a
1	a

a: Amplitude

Example: $\underline{B} = \{0,0,1,1,0,1,0,\ldots\}$ $\underline{A} = \{-a,-a,a,a,-a,a,-a,\ldots\}$

Note: The mapping that assigns a to 0 and -a to 1 is also valid

AMI (or bipolar) mapping

Bit	Amplitude
$\boldsymbol{B_n}$	$\boldsymbol{A_n}$
0	0
1	$A_n = -A_m, m$ is the largest index such that $m < n$ and $A_m \neq 0$.

Example: <u>B</u> = {0,0,1,1,0,1,0,…} <u>A</u> = {0,0,a,-a,0,a,0,…} (initial state=a)

Note: The sequence <u>A</u> = {0,0,-a,a,0,-a,0,…} is also valid (initial state=-a)

- This mapping has ***memory***. That is, the most recent *nonzero amplitude* level needs to be "remembered"
- An ***initial state*** (sign) is needed

Other mappings with memory

- Dicode (ternary mapping)
  - If there is a bit transition, then amplitude transition (polar)
  - Else, amplitude equal to zero

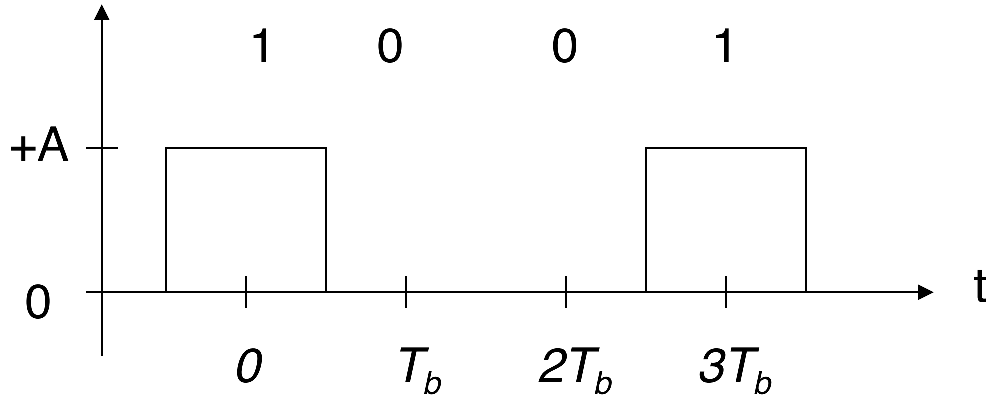
    Example: $\underline{B} = \{0,0,1,1,1,0,0,\ldots\}$ $\underline{A} = \{a,0,-a,0,0,a,0,\ldots\}$

- Mark code
  - "0" = No amplitude transition
  - "1" = Amplitude transition

    Example: $\underline{B} = \{0,0,1,1,1,0,0,\ldots\}$ $\underline{A} = \{a,a,-a,a,-a,-a,-a,\ldots\}$

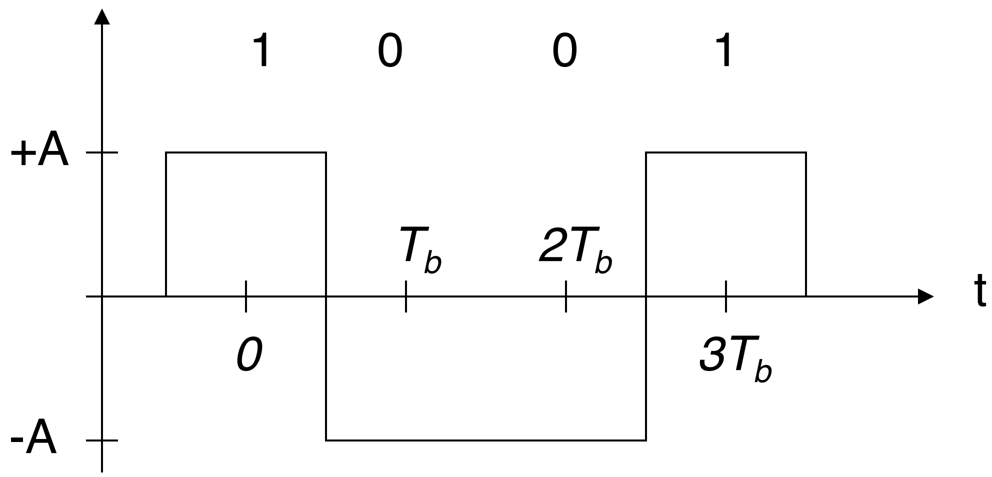
- Miller code **(Near-Field Communication or NFC)**
  - "1" = Transition in the middle of the bit duration ($T_b/2$)
  - "0" = Constant level
  - "0 to 0" = Transition at the end of the bit duration (T_b)

Unipolar NRZ signaling



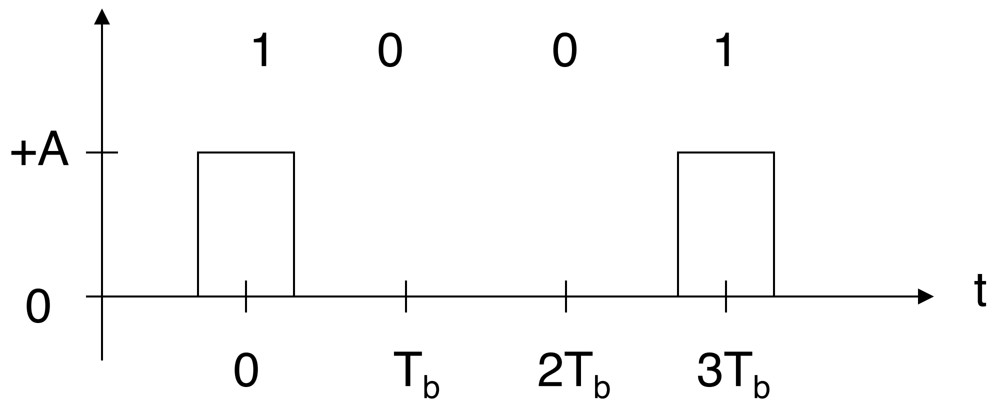
- No transitions if there is a long string of identical "0" or "1"
- This means it is difficult to recover the clock
- Strong DC component means power is wasted

Polar NRZ signaling



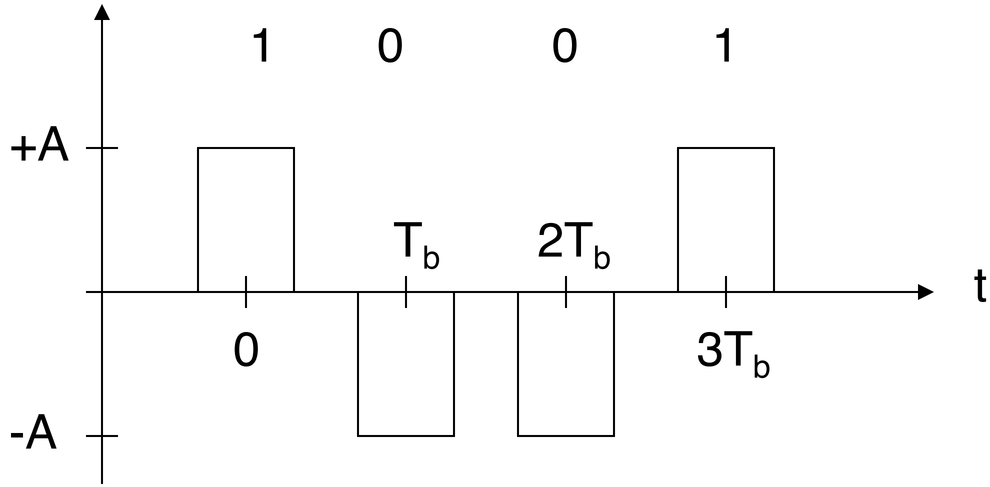
- No DC component for long strings of equally likely bits
- No transitions if there is a long string of identical “0” or “1”
- This means it is difficult to recover the clock

Unipolar RZ signaling



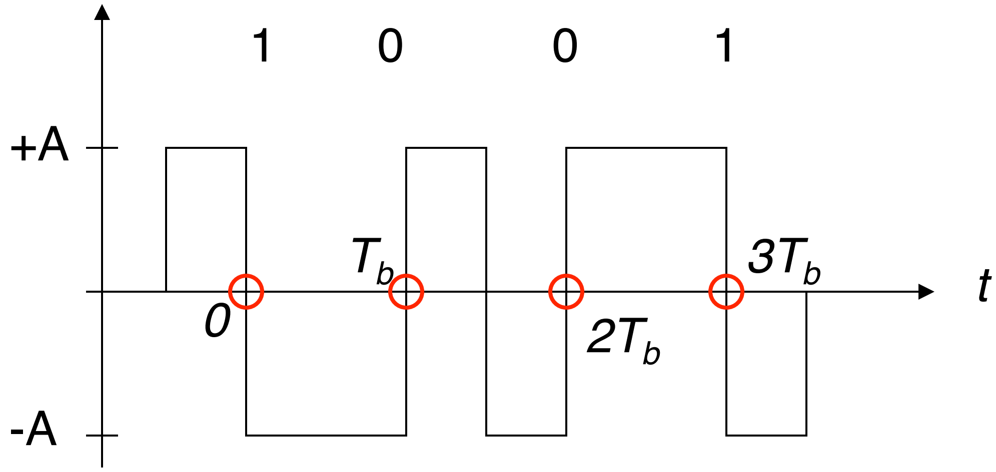
- Same as NRZ with pulses of half width
- Fixed problems with long string of "1"
- No transitions if there is a long string of "0"
- This means it may be difficult to recover the clock
- Strong DC component means power is wasted

Polar RZ signaling



- Same as polar NRZ with half-width pulses
- Fixes problems with long strings of “0” and “1”
- No DC component if “0” and “1” are balanced
- Power spectral density same as Unipolar RZ without impulses

Manchester signaling

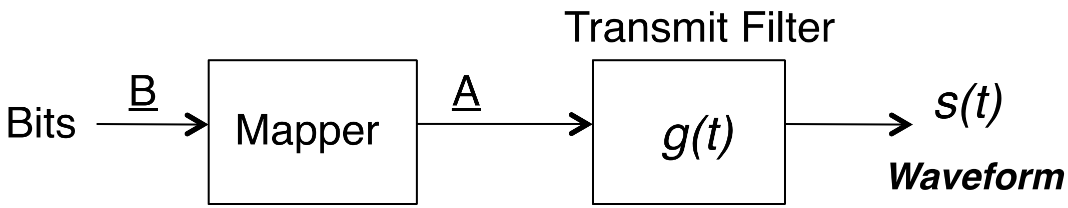


- Always have a transition every T_b seconds
- Easy to recover clock, independent of string of “0” and “1”
- No DC component, regardless of the bit string

Design objectives

- Pulse shaping and mapping are jointly designed to meet several objectives:
  - Self-synchronization
    - An ability to recover timing from the signal itself
    - Long series of ones and zeros could cause a problem
  - Low probability of bit error
    - The receiver needs to be able to distinguish the waveform associated with a zero from the waveform associated with a one, even if there is a considerable amount of noise and distortion in the channel
  - Spectrum shape suitable for the channel.
    - In some cases DC components should be avoided.
      - e.g. if the channel has a DC blocking capacitance or a transformer.
    - The transmission bandwidth should be minimized.

Pulse shaping



- The input to the transmit filter is a sequence of real values A_k from a ***mapper***

- The output of the transmit filter is a ***waveform***:

$$s(t) = \sum_{n=-\infty}^{\infty} A_n g(t - nT_b),$$

  where *g(t)* is the ***pulse shape*** and T_b is the ***bit period***

- The operational details of this process are set by the particular combination of mapper and transmit filter (pulse shape) used.

Power spectral density (PSD)

$$S_s(f) = \frac{|G(f)|^2}{T_b} \sum_{n=-\infty}^{\infty} R[n] e^{-j2\pi f n T_b} \qquad (1)$$

where

$G(f) \leftrightarrow g(t)$, and $R[n] = E\{A_k A_{k+n}\}$: Autocorrelation of $\{A_n\}$.

- If $\{A_n\}$ are uncorrelated, then $R[n] = \begin{cases} \sigma_A^2 + m_A^2, & n = 0 \\ m_A^2, & n \neq 0 \end{cases}$

$$S_s(f) = \frac{|G(f)|^2}{T_b} \left[\sigma_A^2 + \frac{m_A^2}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b} \right) \right] \qquad (2)$$

Example: PSD of Polar NRZ

- Let $p_0 = \Pr\{B_n = 0\}$, $p_1 = \Pr\{B_n = 1\}$. Then

$$R[n] = \begin{cases} p_0(-a)^2 + p_1(a)^2 = a^2, & n = 0 \\ p_0^2(-a)^2 + p_1^2(a)^2 & \\ \quad + 2p_0p_1(a)(-a) = 0, & n \neq 0 \end{cases} \quad \Rightarrow \quad R[n] = a^2\,\delta[n]$$

- From **equation (1)**: $S_s(f) = \dfrac{a^2 \left|G(f)\right|^2}{T_b}$

- For an NRZ rectangular pulse: $\left|G(f)\right| = T_b \operatorname{sinc}(fT_b)$. Thus

$$\boxed{S_s(f) = a^2 T_b \operatorname{sinc}^2\left(fT_b\right)}$$

Example: PSD of Unipolar NRZ

- Here $m_A = \frac{a}{2}, \sigma_A^2 = \frac{a^2}{4}$
- From **equation (2)**:

$$\boxed{S_s(f) = \frac{a^2 T_b}{4} \mathrm{sinc}^2\left(fT_b\right) + \frac{a^2}{4}\delta(f)}$$

Example: PSD of AMI NRZ/RZ

- Here

$$R[n] = \begin{cases} a^2/2, & n = 0 \\ -a^2/4, & n = \pm 1 \\ 0, & |n| > 1 \end{cases}$$

- From **equation (1)**:

$$S_s(f) = \frac{1}{T_b}|G(f)|^2 \left(\frac{a^2}{2} - \frac{a^2}{4} e^{+j2\pi f T_b} - \frac{a^2}{4} e^{-j2\pi f T_b} \right)$$

$$= \frac{1}{T_b}|G(f)|^2 \left[\frac{a^2}{2} - \frac{a^2}{2} \cos(2\pi f T_b) \right] = \frac{a^2}{T_b}|G(f)|^2 \sin^2(\pi f T_b)$$

$$\boxed{\begin{aligned} S_s(f) &= a^2 T_b \operatorname{sinc}^2(f T_b) \sin^2(\pi f T_b), \quad \text{for NRZ pulses} \\ S_s(f) &= \frac{a^2 T_b}{4} \operatorname{sinc}^2\left(\frac{f T_b}{2}\right) \sin^2(\pi f T_b), \quad \text{for RZ pulses} \end{aligned}}$$

Example: PSD of Manchester code

- Manchester code refers to the combination of a Manchester pulse with polar mapping

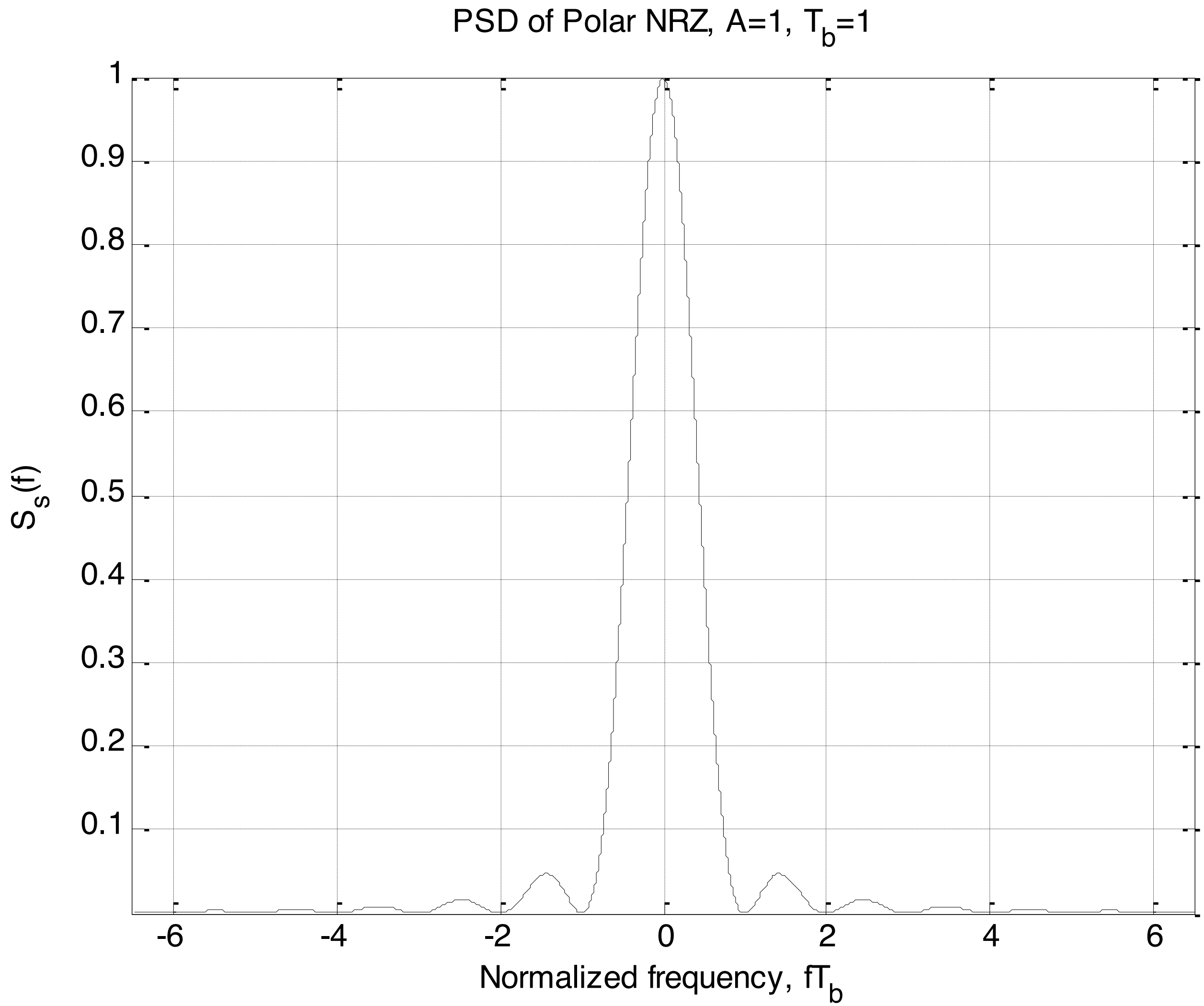
- Pulse spectrum:

$$g(t) = \Pi\left(\frac{t - T/4}{T/2}\right) - \Pi\left(\frac{t - 3T/4}{T/2}\right) \quad \leftrightarrow \quad G(f) = T_b \operatorname{sinc}\left(\frac{fT_b}{2}\right) \sin\left(\frac{\pi}{2} fT_b\right) e^{j\frac{\pi}{2}}$$

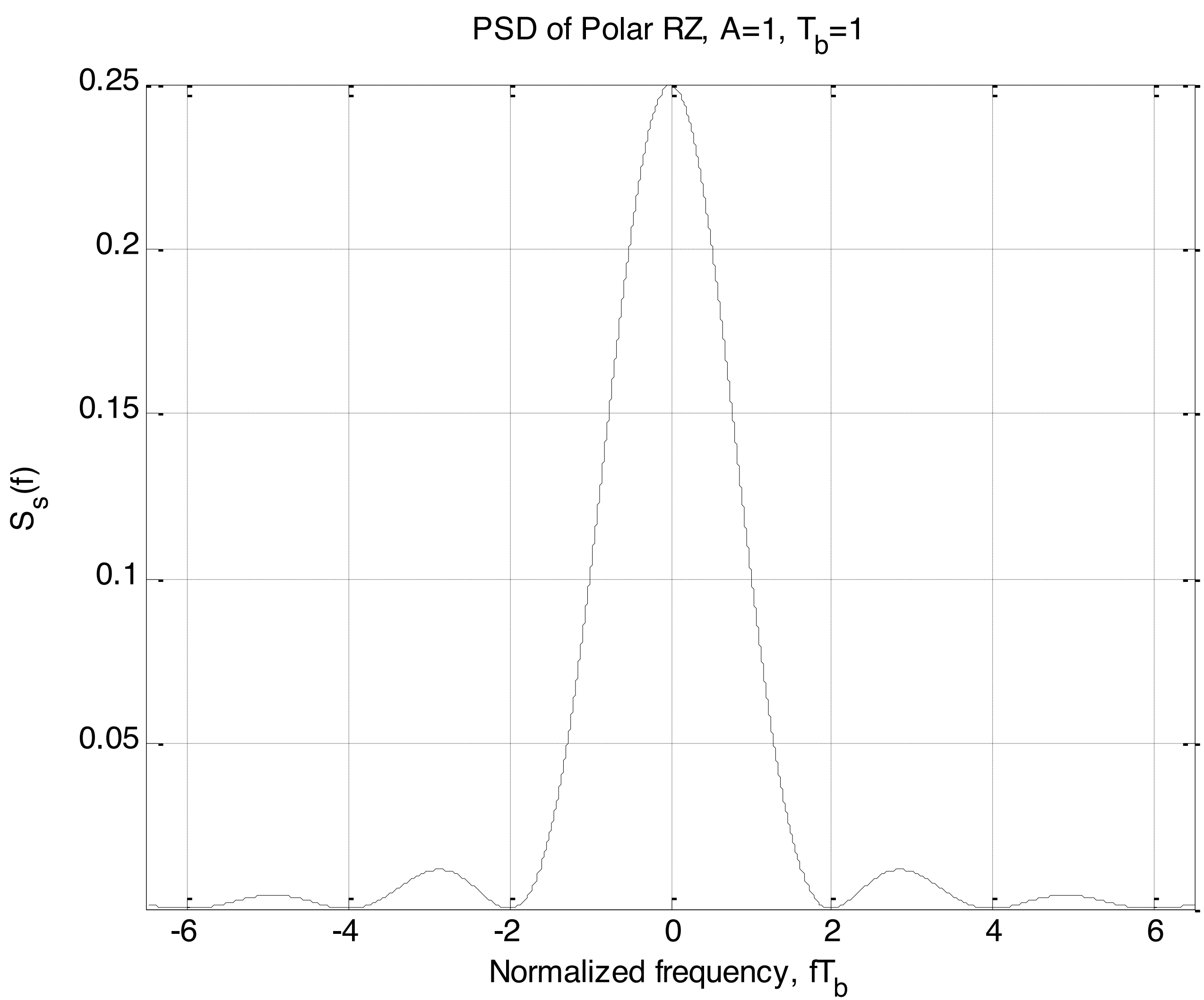
- Polar mapping: $R[n] = a^2\,\delta[n]$. Therefore,

$$\boxed{S_s(f) = a^2 T_b \operatorname{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2\left(\frac{\pi}{2} fT_b\right)}$$

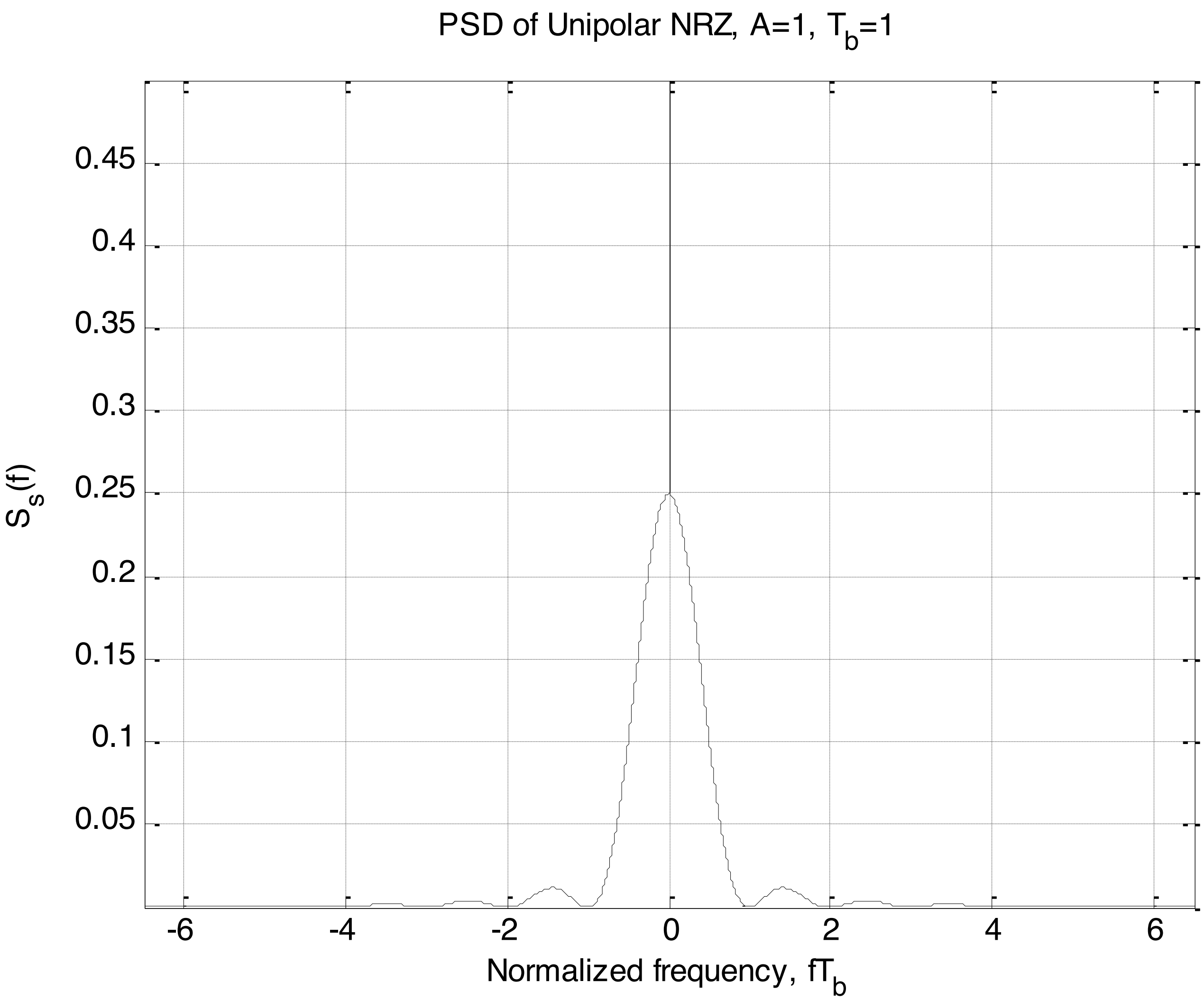
PSD of polar NRZ



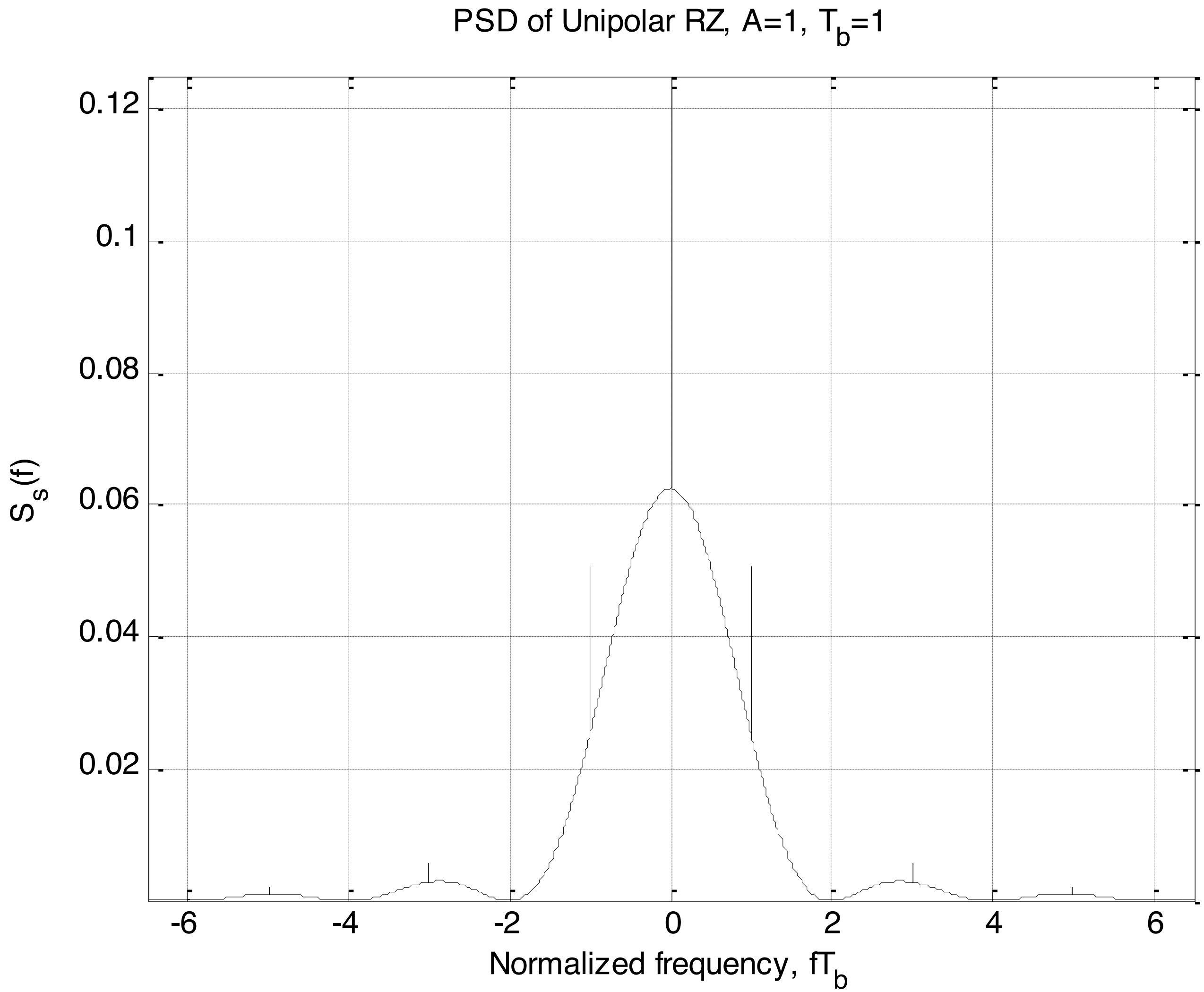
PSD of polar RZ



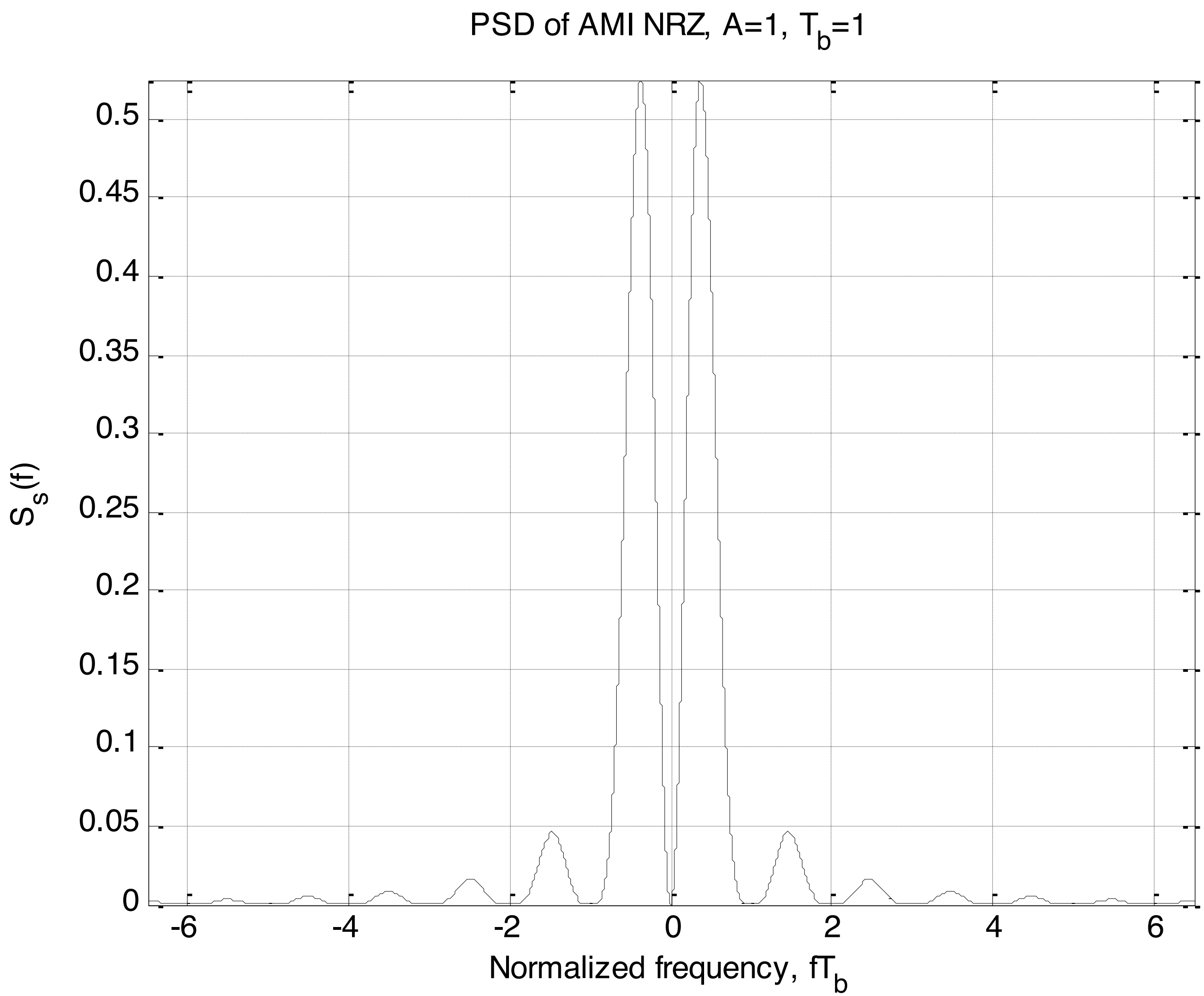
PSD of unipolar NRZ



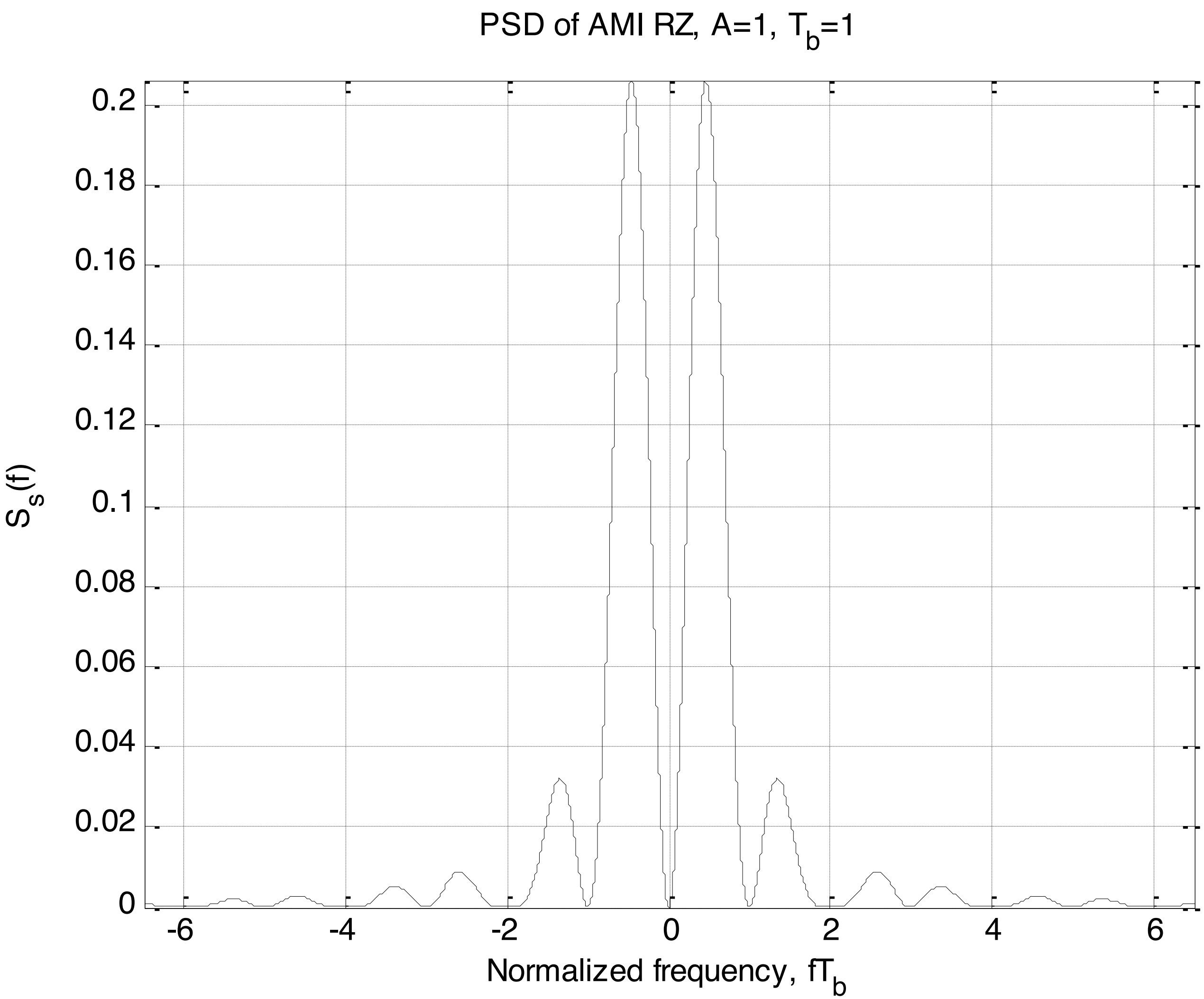
PSD of unipolar RZ



PSD of AMI NRZ

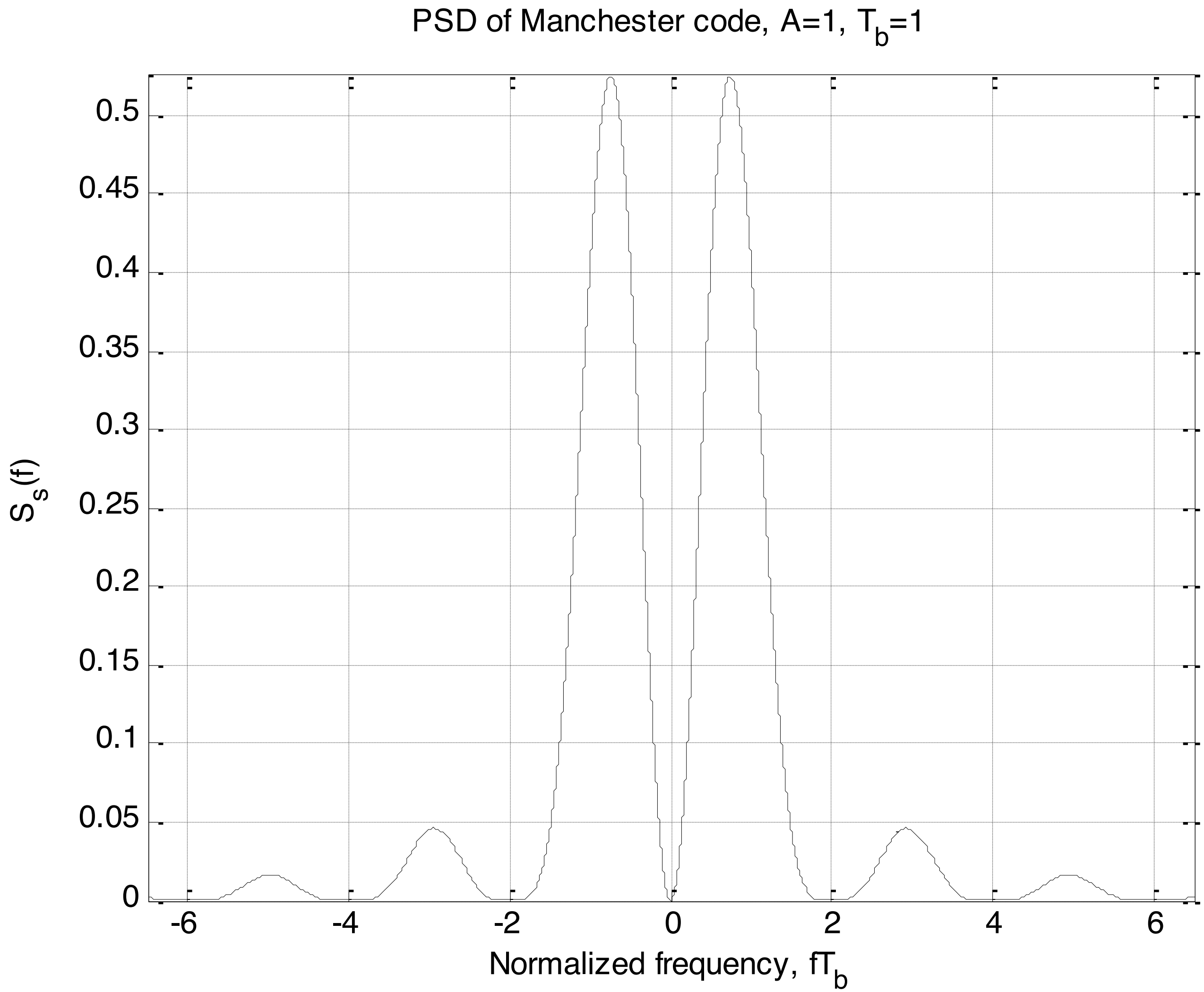


PSD of AMI RZ



PSD of AMI RZ, A=1, T_b=1
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-6
-4
-2
0
2
4
6
$S_s(f)$
Normalized frequency, fT_b

PSD of Manchester code



NOTE: *The Manchester code was used in the first generation of Ethernet (IEEE 802.3 standard) and is being used (as of 2010) in second generation RFID systems*

Summary of power spectral density of line codes

- DC Components
  - Unipolar NRZ, polar NRZ, and unipolar RZ all have a DC component
  - AMI RZ and Manchester NRZ do not have DC component
- Null-to-null Bandwidth (NNB)
  - Unipolar NRZ, polar NRZ, and **bipolar** all have NNB equal to $R_b = 1/T_b$
  - Unipolar RZ has NNB equal to of $2R_b$
  - **Manchester** NRZ also has NNB equal to $2R_b$, although the spectrum becomes very low at approximately $1.6R_b$

Shortcut Method for Finding the PSD of a Line Code

- If a line code has data symbols that are equiprobable and independent, then the PSD of the line code is related to the F.T. of the pulse shape P(f) by:
- ***Polar*** line codes: $$G_x(f) = \frac{A^2}{T_b}\left|P(f)\right|^2$$
- ***Unipolar*** line codes: $$G_x(f) = \frac{A^2}{4T_b}\left|P(f)\right|^2\left(1 + \frac{1}{T_b}\sum_{k=-\infty}^{\infty}\delta\left(f - \frac{k}{T_b}\right)\right)$$
- ***Bipolar*** line codes: $$G_x(f) = \frac{A^2}{T_b}\left|P(f)\right|^2 \sin^2(\pi f T_b)$$

Reproduced from notes of Prof. M. Valenti, West Virginia U.

Example: PSD of Unipolar Line Code with NRZ Pulse Shapes

- The PSD of a unipolar line code is:

$$G_x(f) = \frac{A^2}{4T_b}\left|P(f)\right|^2\left(1 + \frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)\right)$$

- If the pulse shape is NRZ, then:

$$P(f) = 0 \quad \text{for } f = \frac{n}{T_b} \text{ when } n \neq 0$$

- Thus the PSD of unipolar NRZ is:

$$G_x(f) = \frac{A^2}{4T_b}\left|P(f)\right|^2\left(1 + \frac{1}{T_b}\delta(f)\right)$$

Reproduced from notes of Prof. M. Valenti, West Virginia U.