



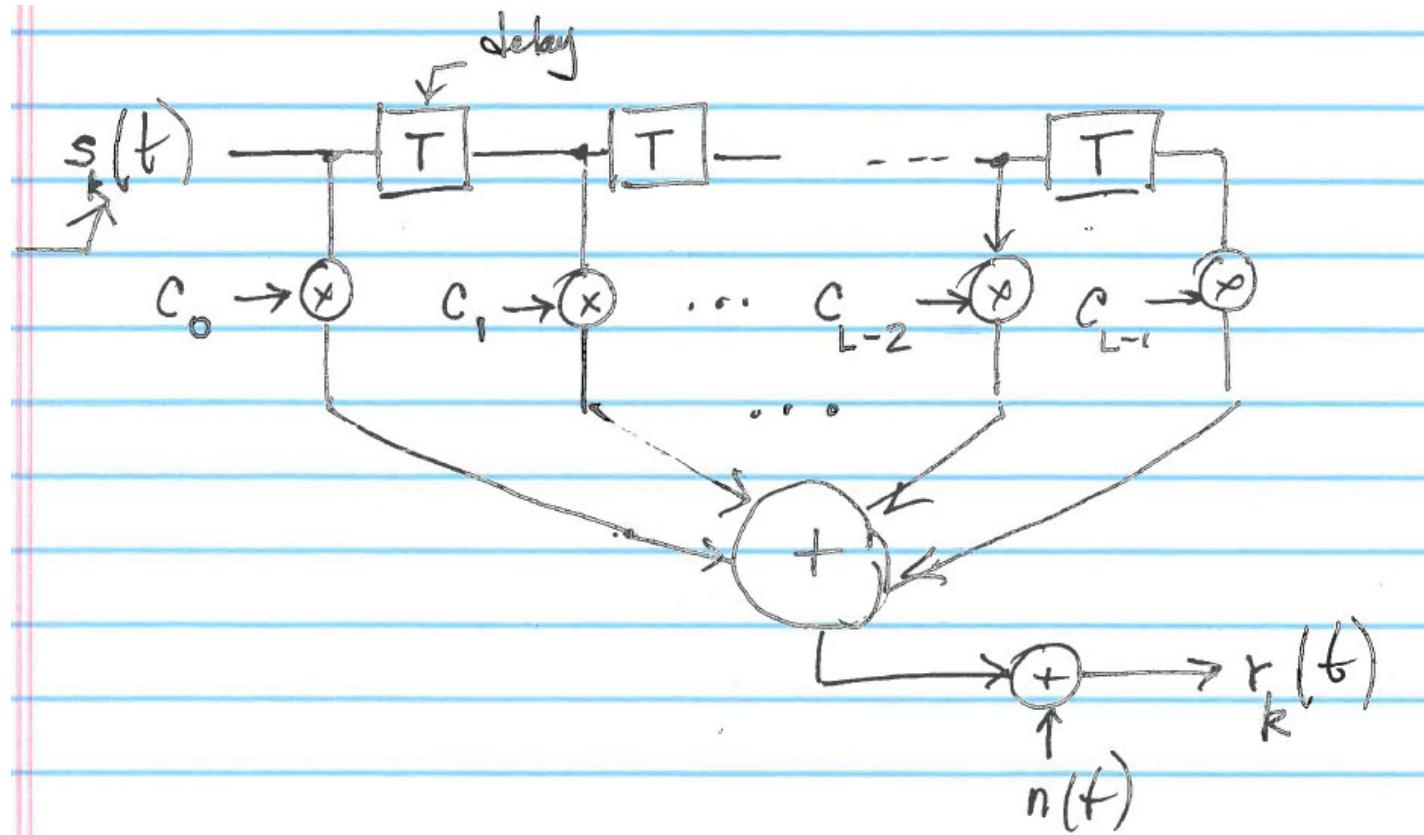
# OFDM: Cyclic Prefix and Channel Estimation

EE161: Digital Communication Systems  
San José State University

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# Symbol-spaced multipath channel model



# Frequency response



Impulse response

$$c(t) = c_0 \delta(t) + c_1 \delta(t-T) + \dots + c_{L-1} \delta(t-(L-1)T)$$

Frequency response

$$C(f) = \mathcal{F}\{c(t)\} = c_0 + c_1 e^{-j2\pi f T} + \dots + c_{L-1} e^{-j2\pi f(L-1)T}$$

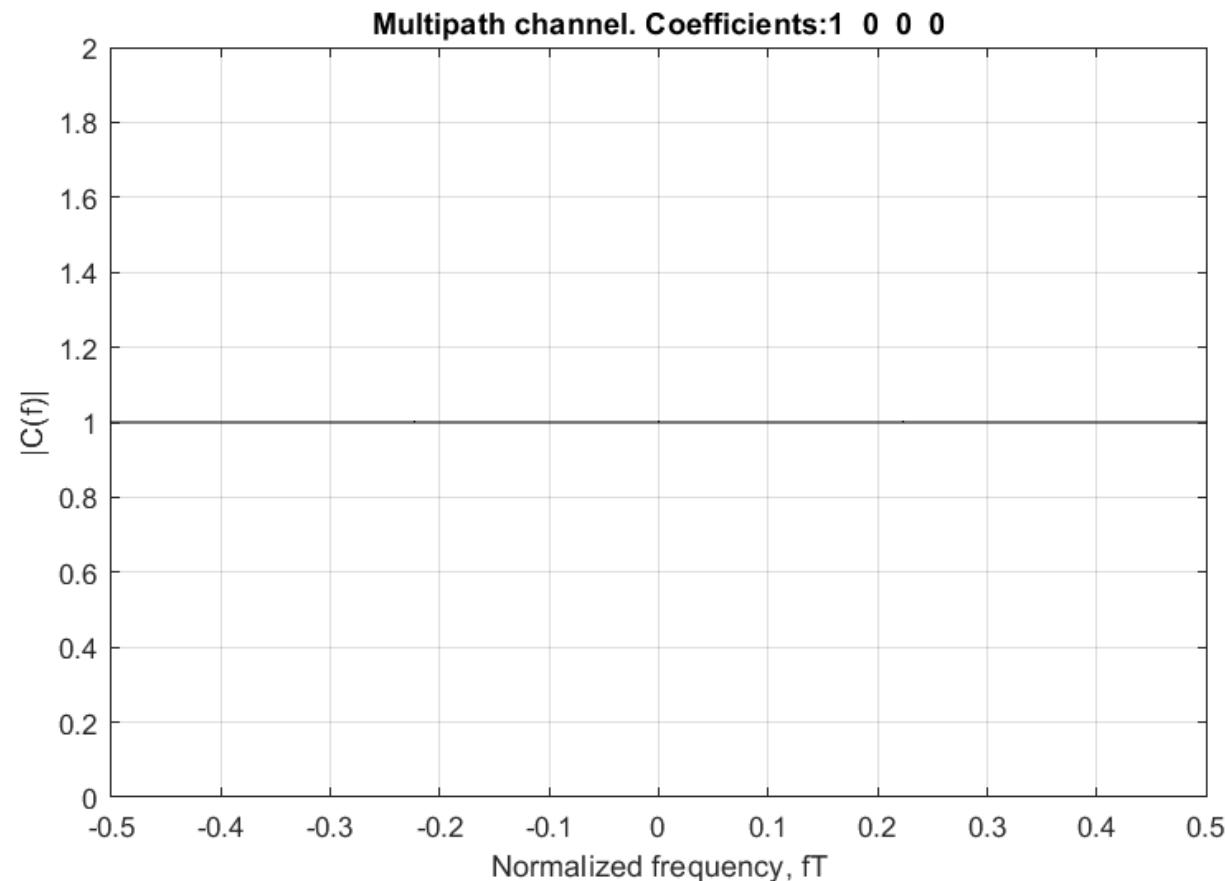
or

$$C(f) = \sum_{l=0}^{L-1} c_l e^{-j2\pi f l T}$$

(Matlab module `impul [c0 c1 .. cL-1]`)



# Example: $L=4$ . $c = [1 \ 0 \ 0 \ 0]$



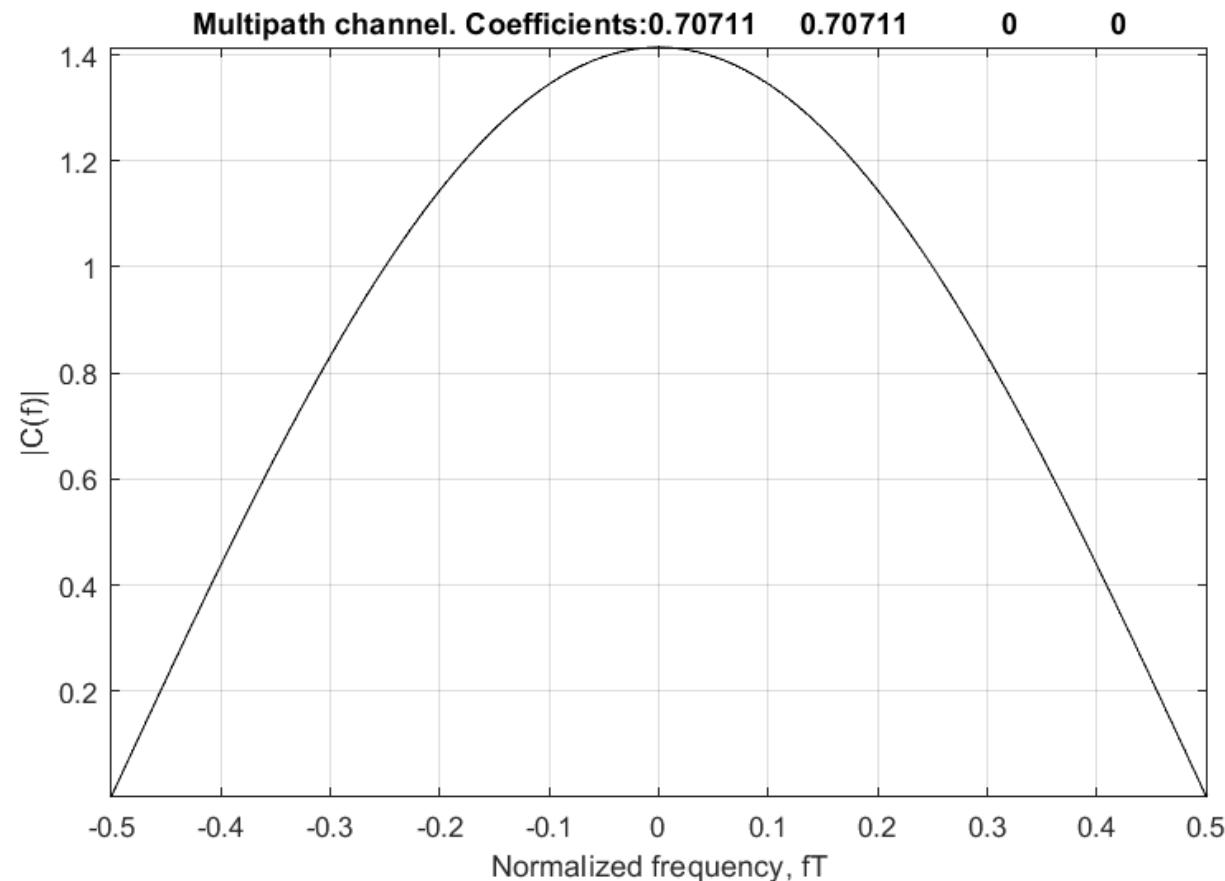
[\*frequency\\_response\\_multipath.m\*](#)

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Example:  $L=4$ .  $c = [1 \ 1 \ 0 \ 0] / \sqrt{2}$



[frequency\\_response\\_multipath.m](#)

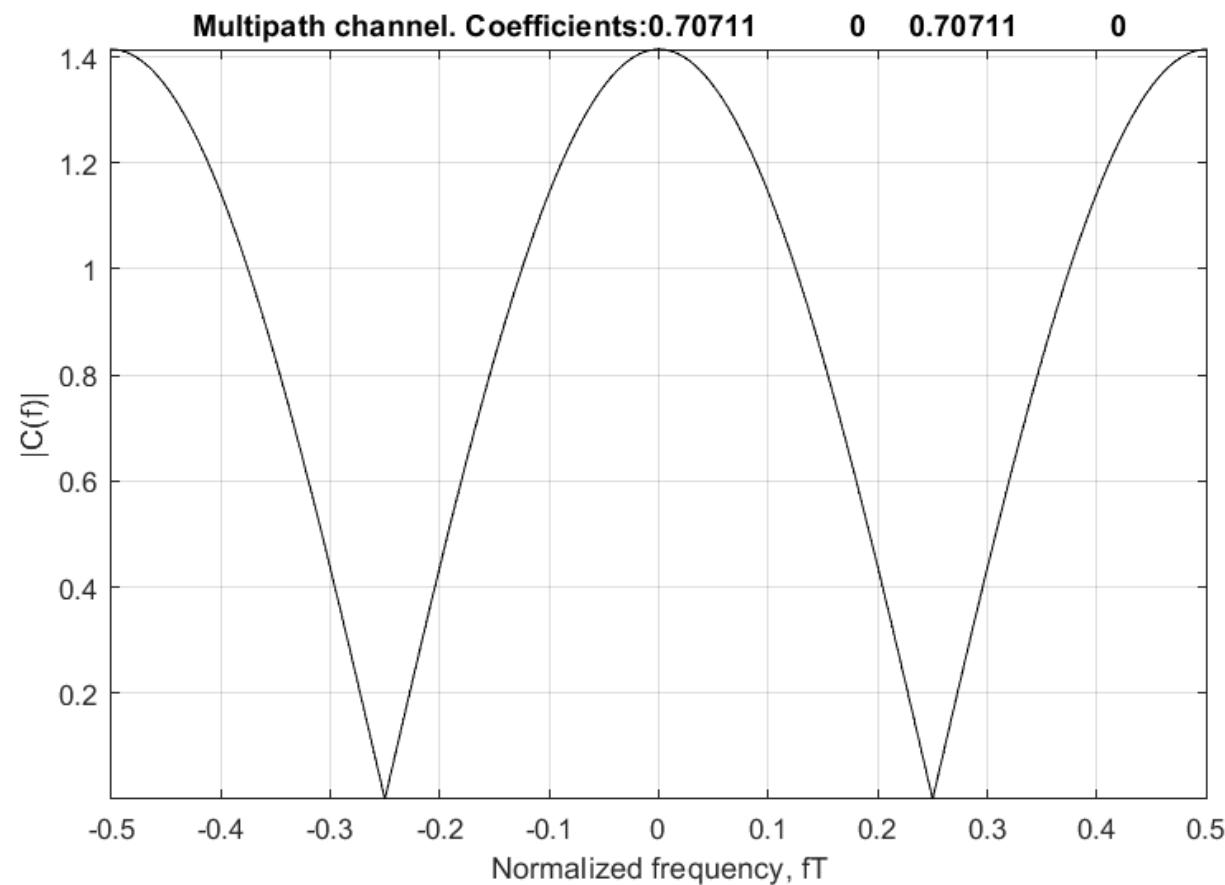
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Example:  $L=4$ .  $c = [1 \ 0 \ 1 \ 0] / \sqrt{2}$



[frequency\\_response\\_multipath.m](#)

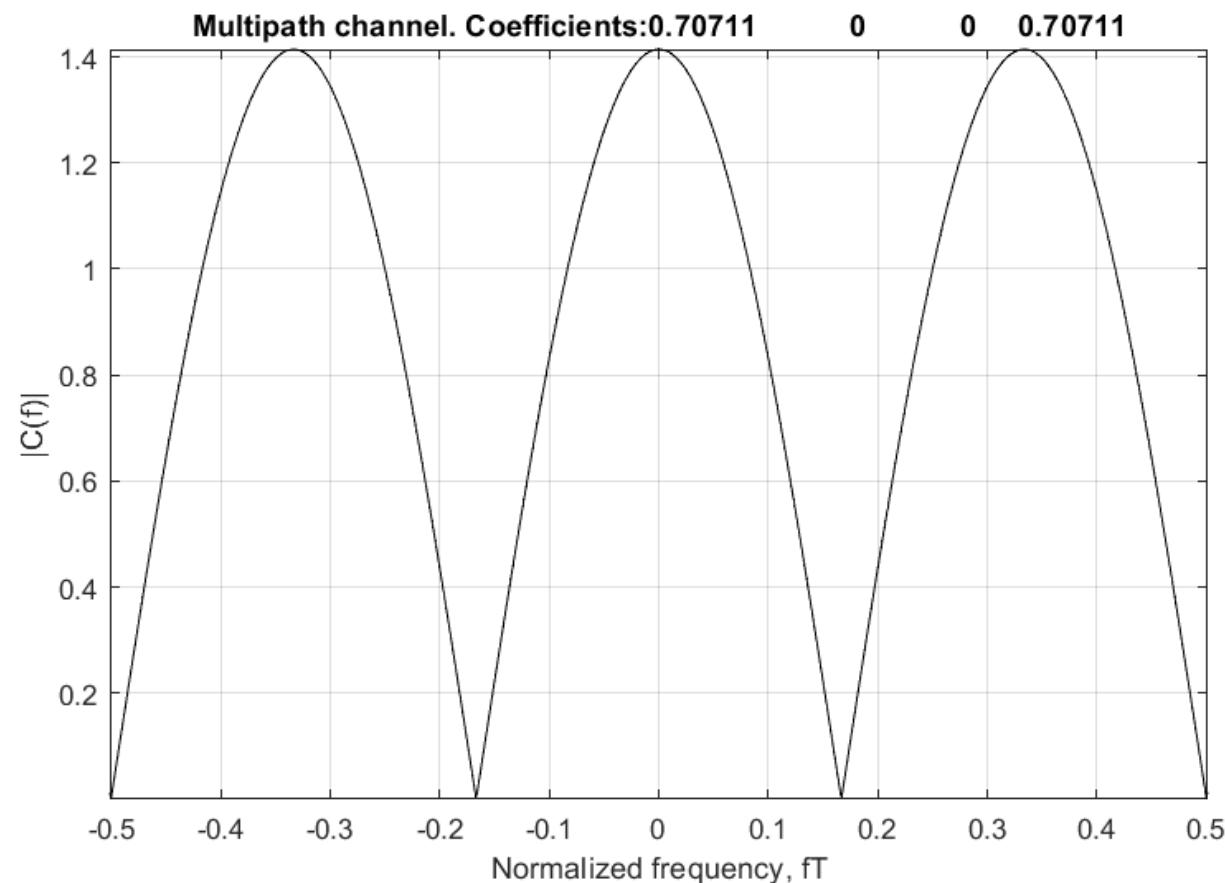
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Example:  $L=4$ .  $c = [1 \ 0 \ 0 \ 1] / \sqrt{2}$



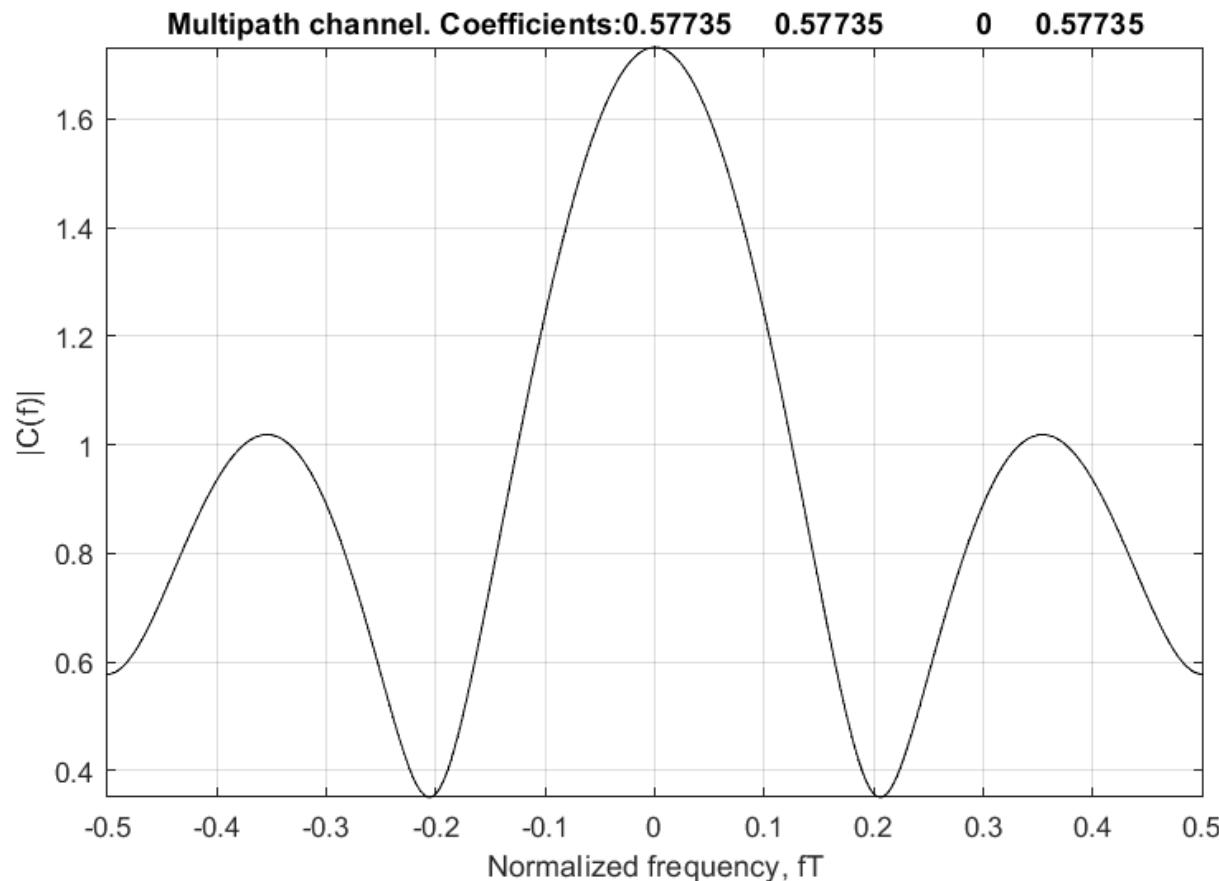
[frequency\\_response\\_multipath.m](#)

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Example:  $L=4$ .  $c = [1 \ 1 \ 0 \ 1] / \sqrt{3}$



[\*frequency\\_response\\_multipath.m\*](#)

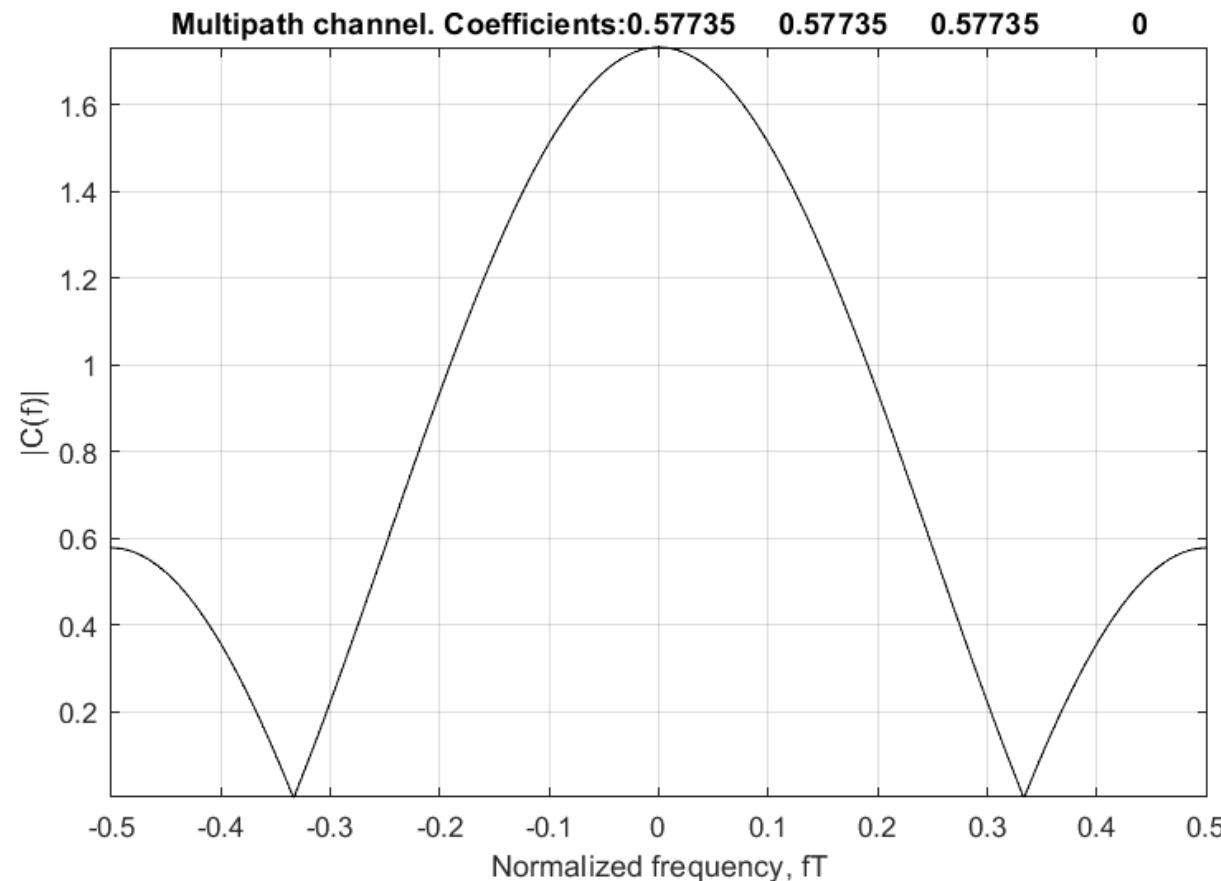
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Example:  $L=4$ .  $c = [1 \ 1 \ 1 \ 0] / \sqrt{3}$



[frequency\\_response\\_multipath.m](#)

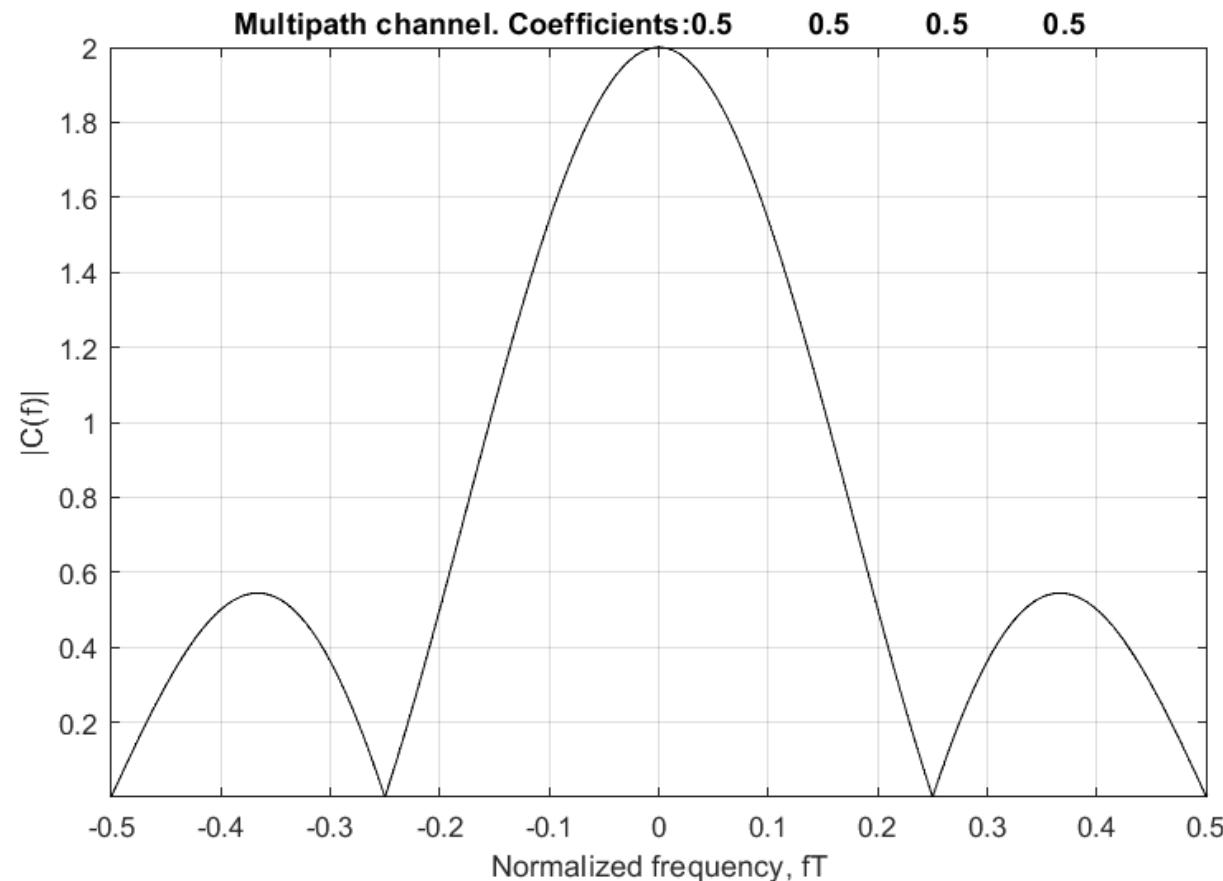
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Example:  $L=4$ .  $c = [1 \ 1 \ 1 \ 1] / \sqrt{4}$



[\*frequency\\_response\\_multipath.m\*](#)

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# Normalized channel energy

- The channel energy is normalized to unity:

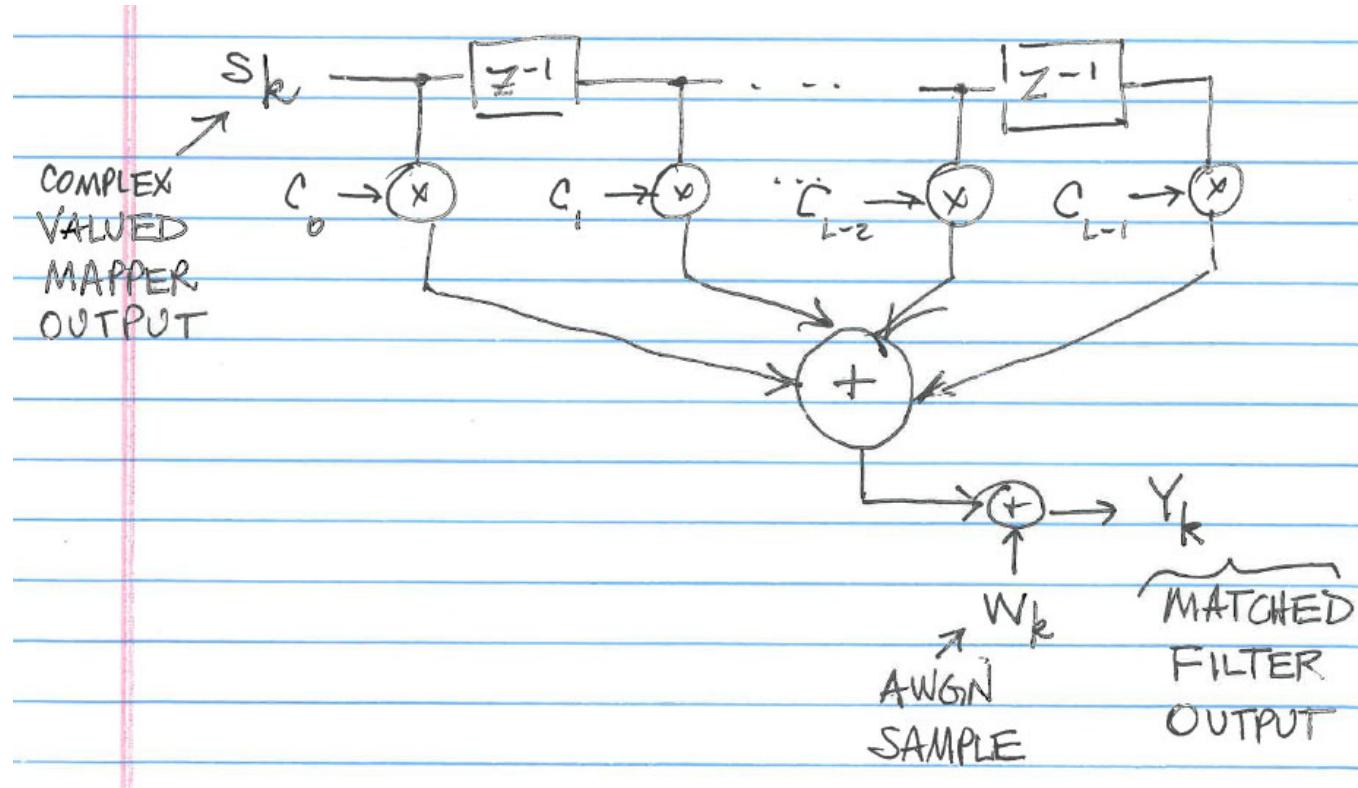
$$E_c = \sum_{\ell=0}^{L-1} |c_\ell|^2 = 1$$

- This allows us to compare different channel scenarios for the same average signal energy



# Equivalent discrete-time model

- This model is obtained when looking at the matched-filter output:





# OFDM subchannel average signal energies

- Over each subchannel,  $k = 0, 1, \dots, K - 1$ , we have

$$\text{SNR}_k = \frac{E_{s,k}}{N_0} |C_k|^2$$

where  $E_{s,k}$  is the energy of mapper output in the  $k$ -th subchannel

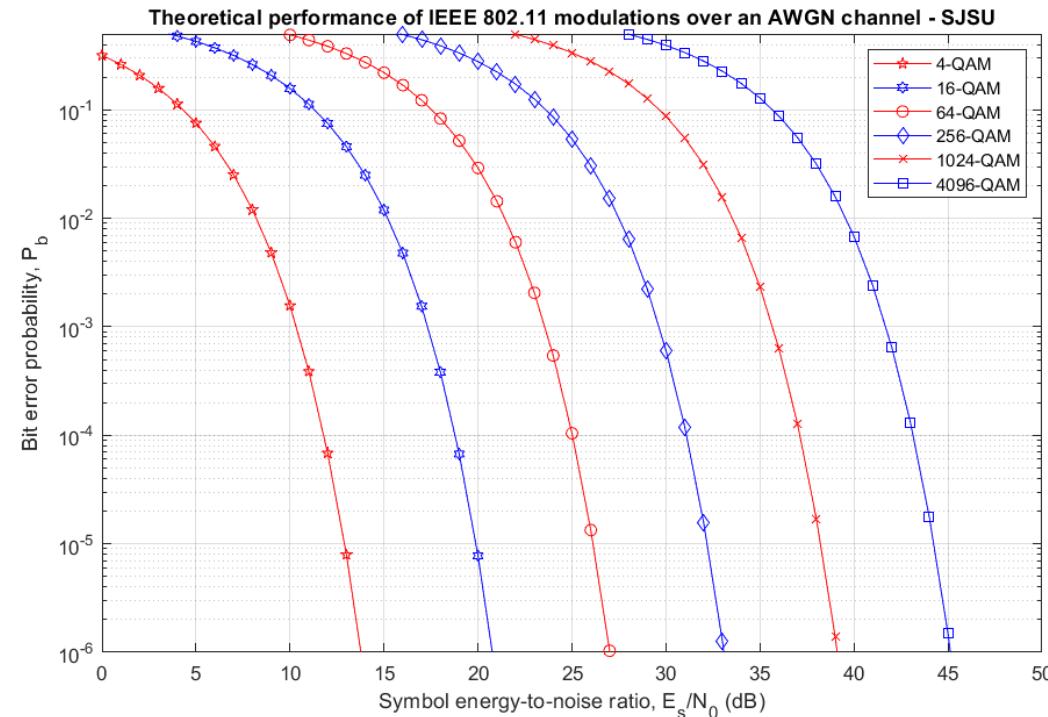
- Due to the frequency selectivity nature of a multipath channel, these SNR (signal energy-to-noise ratio) values can be very low, resulting in an unacceptably high bit error rate (BER)



# Approaches to low subchannel energy

## 1. Adaptive modulation (e.g., DSL modems)

For each subchannel, select a modulation format (mapper output symbols) according to average SNR value to achieve a given BER



# Approaches to low subchannel energy (cont.)



## 2. OFDM one-tap equalization

Estimate the subchannel gains  $C_k, k = 0, 1, \dots, K - 1$ , and “equalize” them

**More next lecture**



# Subchannel (OFDM symbol) spectrum

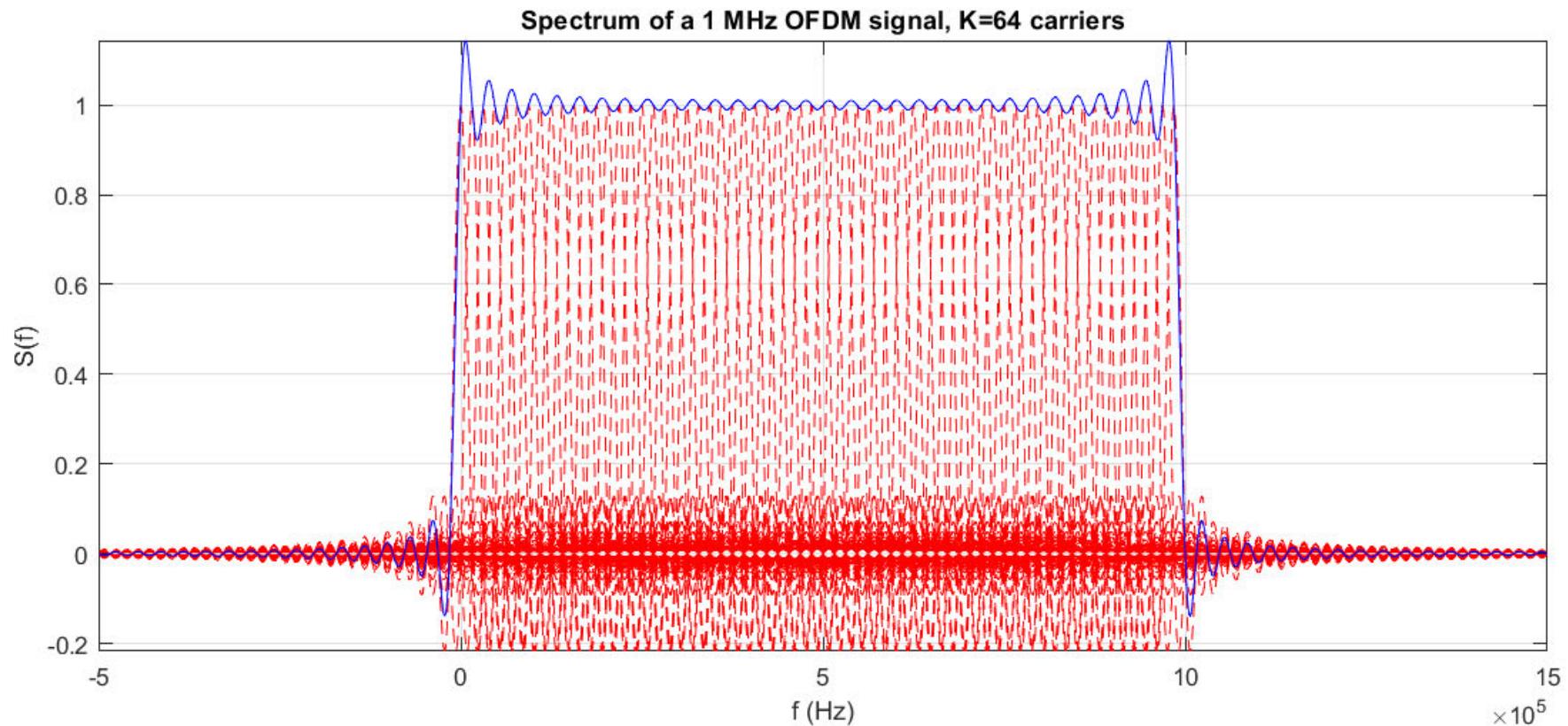
- There are several options for the spectrum of each subchannel
  - Rectangular spectrum
  - Raised-cosine spectrum
  - “sinc”-like spectrum (pulse shapes as an RC spectrum)
- Regardless of the shape, the ***subchannels become orthogonal*** provided they are sampled in the frequency domain at intervals multiples of

$$f_s = \frac{1}{T} = \frac{W}{K}$$

See: Slide 29 of [\*13b\\_Coding\\_Modulation\\_Wireless.pdf\*](#)



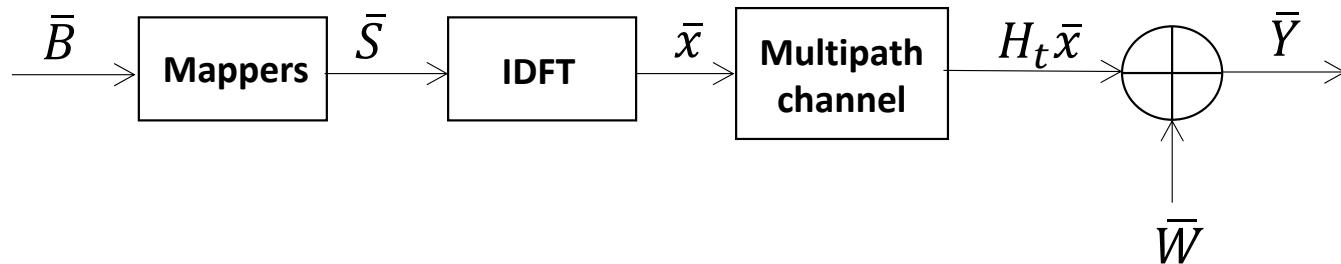
# Example of *sinc*-shaped spectrum





# Vector channel concept

- Remarkably, sampling in the frequency domain gives the **inverse DFT** (Discrete Fourier Transform) of the subchannel modulation symbols,  $S_k, k = 0, 1, \dots, K - 1$
- This creates  $K$  orthogonal/parallel channels:





# Channel matrix

- The channel output is a vector

$$\bar{Y} = H_t \bar{x} + \bar{W}$$

- The channel matrix  $H_t$  is diagonal:

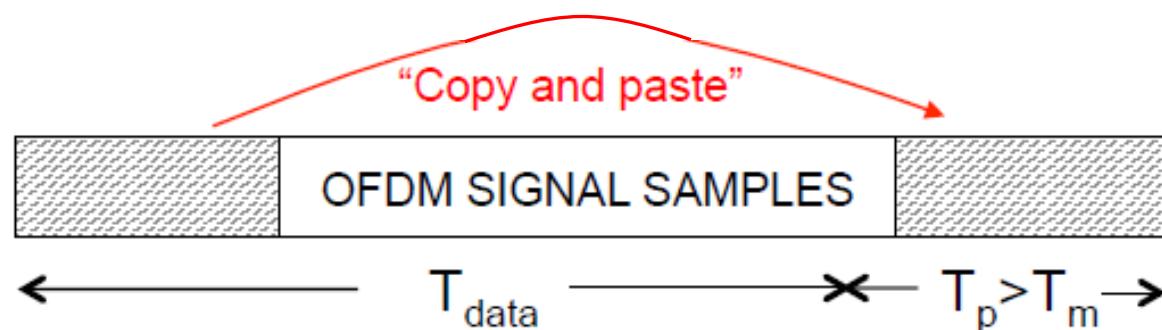
$$H_t = \begin{bmatrix} c_0 & c_1 & \cdots & c_{L-1} \\ & c_0 & c_1 & \cdots & c_{L-1} \\ & & \ddots & & \\ 0 & & & c_0 & c_1 & \cdots & c_{L-1} \end{bmatrix}$$

$\xleftarrow{\quad K+\nu \text{ columns} \quad}$

$\uparrow$   
 $K$  rows  
 $\downarrow$

where  $\nu = L = \text{int}\left(\frac{T_m}{T}\right)$ , and  $T_m$  is the delay spread

# Adding a cyclic prefix



- The first  $\nu$  symbols are appended to the end of an OFDM symbol
- This produces a *circulant* channel matrix:



# Cyclic prefix = Circulant channel matrix

$$H = \begin{bmatrix} c_0 & c_1 & \cdots & c_{L-1} \\ c_0 & c_1 & \cdots & c_{L-1} \\ \ddots & & & \\ & & c_0 & c_1 & \cdots & c_{L-1} \\ c_{L-1} & & & c_0 & c_1 & \cdots & c_{L-2} \\ c_{L-2} & c_{L-1} & & c_0 & c_1 & \cdots & c_{L-3} \\ c_1 & c_2 & \cdots & c_{L-1} & & & c_0 \end{bmatrix}$$

$\xleftarrow{\quad K \text{ columns} \quad}$   $\xrightarrow{\quad K \text{ rows} \quad}$



# Singular value decomposition (SVD)

- Singular value decomposition of circulant matrix  $H$ :

$$H = Q\Lambda Q^*$$

where  $Q^*$ : **inverse DFT** matrix,  $Q^*Q = QQ^* = I_K$ , and

$$\Lambda = \text{diag}(C_0, C_1, \dots, C_{K-1})$$

where  $\bar{C} = Q\bar{c} = (C_0, C_1, \dots, C_{K-1})$  is the **DFT** of the channel impulse

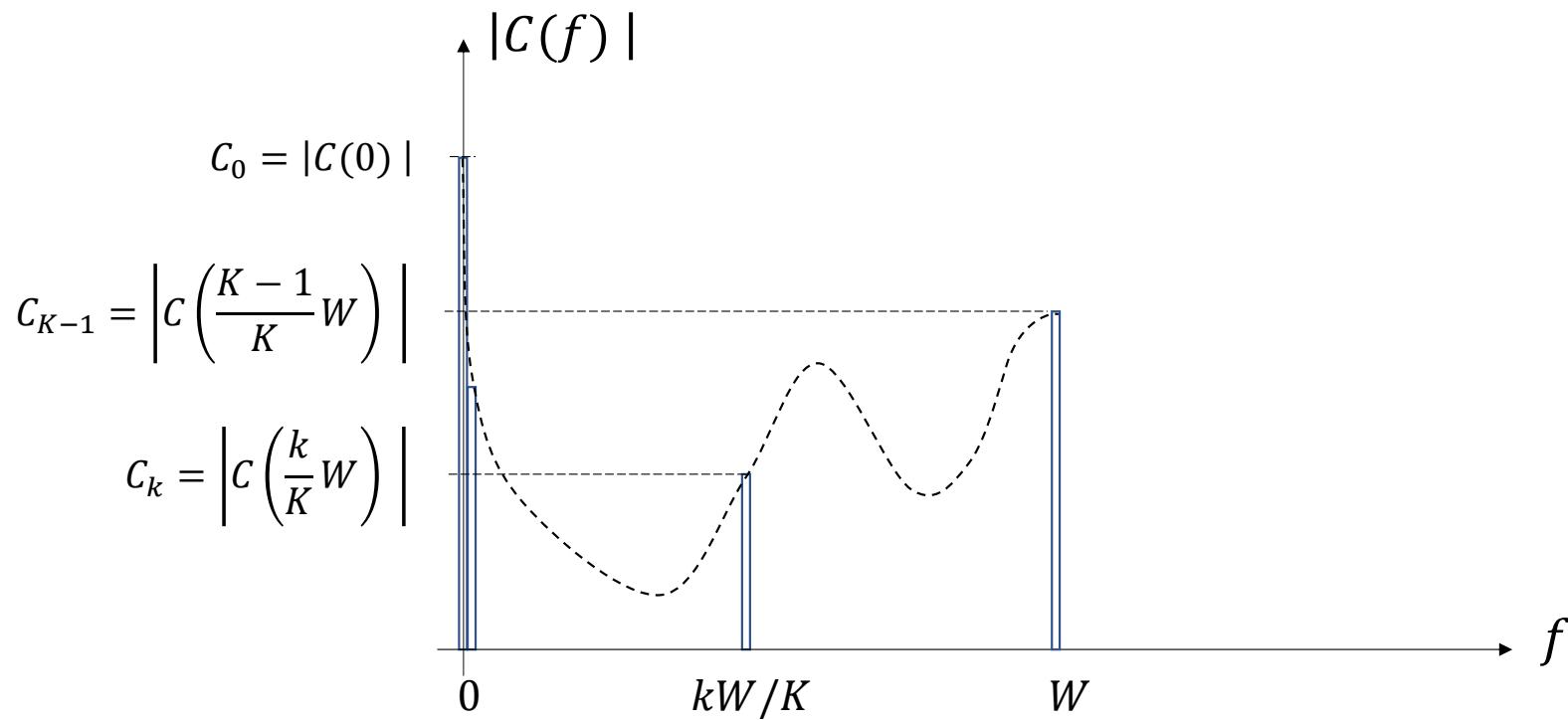
response  $\bar{c} = (c_0, c_1, \dots, c_{\nu-1}, \boxed{0 \ 0 \ \cdots \ 0})$

K- $\nu$  zeros

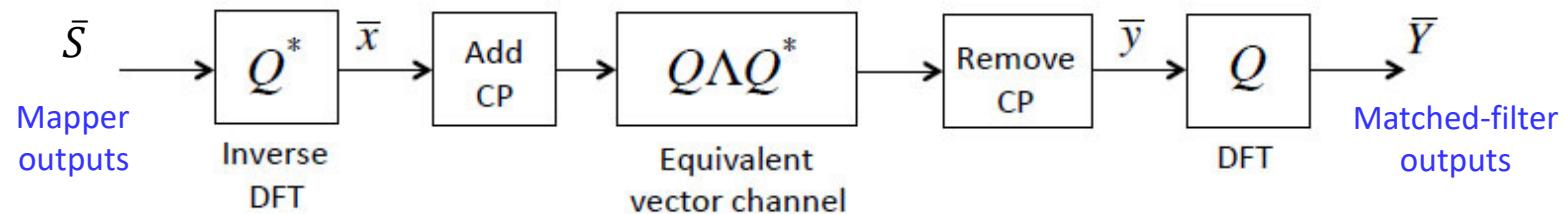


# Subchannel gains

$$\bar{C} = Q\bar{c} = (C_0, C_1, \dots, C_{K-1})$$



# CP-OFDM parallel channels with cyclic prefix



Since  $QQ^* = Q^*Q = I_K$ , **K parallel channels are created!**

$$\bar{Y} = \Lambda \bar{S}, \quad Y_k = C_k S_k, \quad 0 \leq k \leq K-1$$

where  $C_k = [C(f)]_{f=k/T}$



# CP-OFDM parallel channels

- To see why  $K$  parallel channels are created, write

$$\begin{aligned}\bar{Y} &= \bar{y}Q = (\bar{x}H)Q = (\bar{S}Q^*)HQ \\ &= (\bar{S}Q^*)(Q\Lambda Q^*)Q = \bar{S}I_K\Lambda I_K = \bar{S}\Lambda\end{aligned}$$

or

$$Y_k = S_k C_k, \quad k = 0, 1, \dots, K - 1$$



# OFDM channel estimation

- Pilot symbols :  $P_k$ ,  $0 \leq k \leq K-1$ , known at both Transmitter and Receiver
- $Y_k = C_k P_k + W_k$
- Receiver computes, after the FFT,

$$\hat{C}_k = \frac{P_k^*}{|P_k|^2} Y_k = \frac{P_k^*}{|P_k|^2} (C_k P_k + W_k), \quad 0 \leq k \leq K-1$$

$$\rightarrow \hat{C}_k = C_k + W_k' \approx C_k, \quad \text{for } \text{var}\{W_k\} = \frac{N_0}{2} \text{ small.}$$



# OFDM channel estimation (cont.)

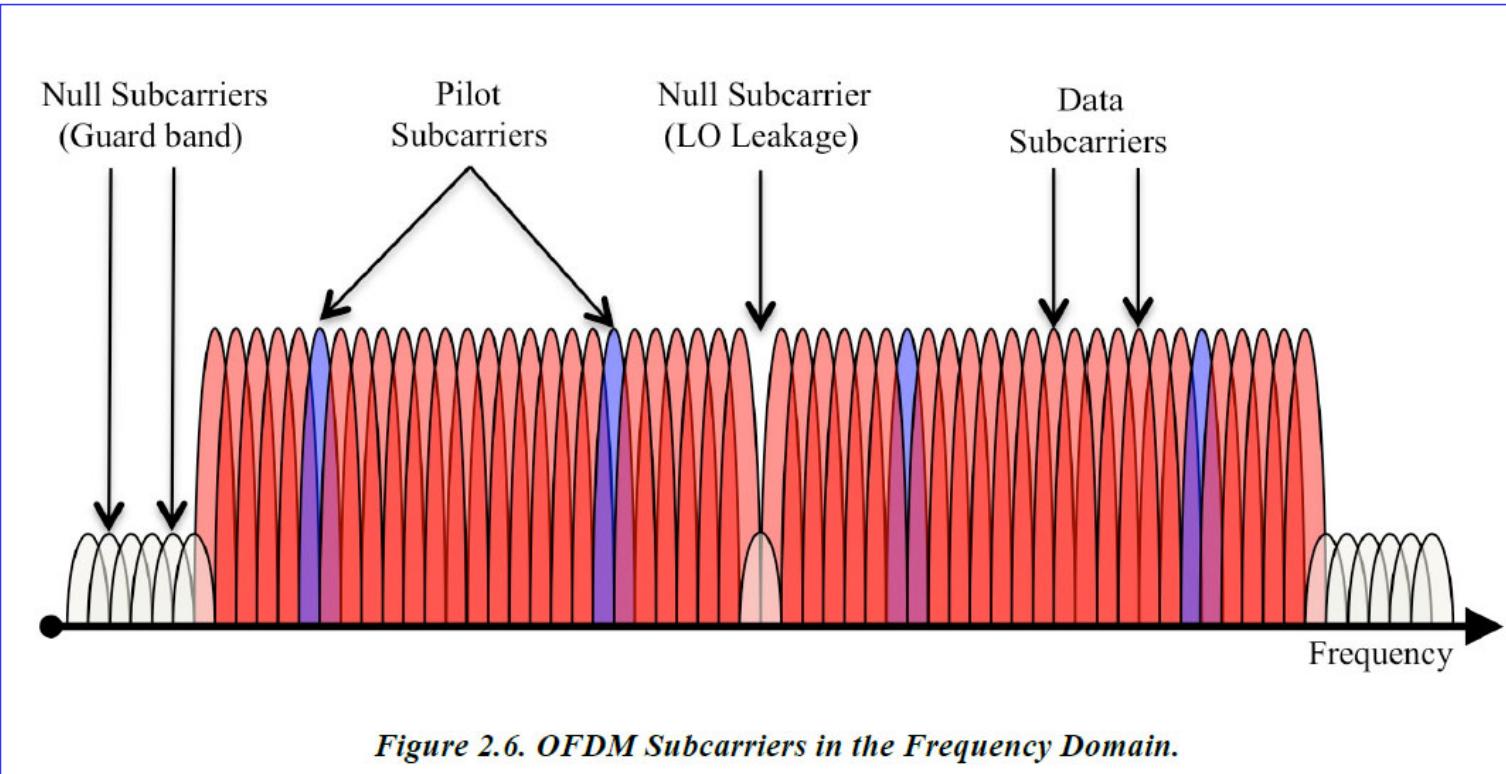
- In practice, several OFDM symbols carry pilot symbols and the receiver computes an average

$$\hat{G}_k = \frac{1}{N_p} \sum_{i=1}^{N_p} \hat{C}_{k,i}$$

where  $N_p$  is the number of times a pilot symbol is sent over the  $k$ -th subchannel.

- As an example, in the IEEE 802.11a (WiFi-2) specification, there are  $K = 64$  subchannels of which 4 (four) are used for pilot symbols. (See next figure)

# IEEE 802.11a pilot subchannels



Reference: [https://download.ni.com/evaluation/rf/Introduction\\_to\\_WLAN\\_Testing.pdf](https://download.ni.com/evaluation/rf/Introduction_to_WLAN_Testing.pdf)

# IEEE 802.11a pilot subchannels (cont.)



For the most common 802.11 implementations used in Wi-Fi, such as 802.11a/g, 802.11n, and 802.11ac, the OFDM transmission uses a constant symbol rate (and therefore constant subcarrier spacing) for all bandwidth configurations. **Table 2.4** shows how wider bandwidth options are implemented by using a larger number of subcarriers through a larger FFT size.

Bandwidth	FFT Size	Data Subcarriers	Pilot Subcarriers
20 MHz	64	52	4
40 MHz	128	108	6
80 MHz (VHT only)	256	234	8
160 MHz (VHT only)	512	468	16

*Table 2.4. Bandwidth Configurations and FFT Sizes for HT and VHT PHY.*

Reference: [https://download.ni.com/evaluation/rf/Introduction\\_to\\_WLAN\\_Testing.pdf](https://download.ni.com/evaluation/rf/Introduction_to_WLAN_Testing.pdf)



# Next lecture: Equalization and ECC

