Coding and Modulation for Wireless Communications

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Outline

- Introduction: A simple two-path wireless channel
  - Flat and frequency selective fading
  - Fast and slow fading
- Diversity techniques for wireless channels
- IEEE 802 wireless network (PHY) standards
  - Bit-interleaved coded modulation
  - Modulations and codes used today

A wireless two-path channel

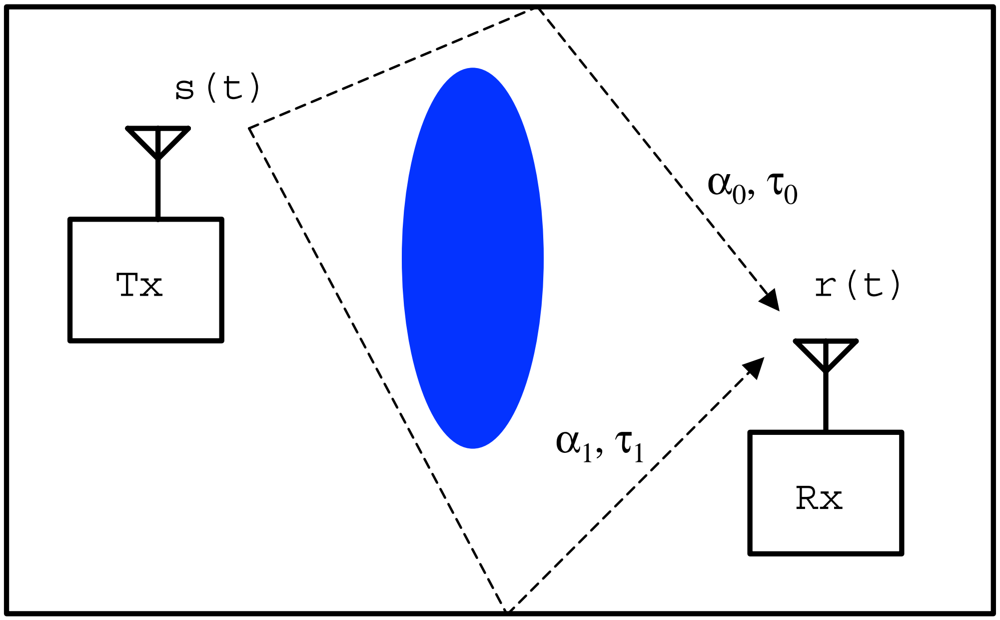
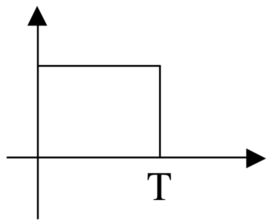


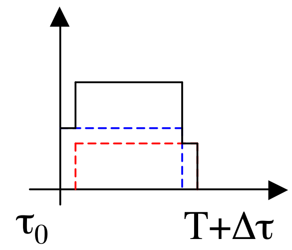
Figure 1: A wireless two-path channel.

Wireless two-path channel response to rectangular pulses

Narrowband pulses, $T > \Delta\tau \quad (\Delta\tau = \tau_1 - \tau_0)$



Input

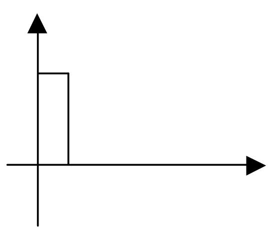


Output

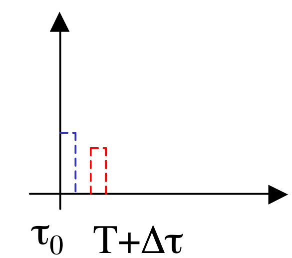
- Amplitude variation:
  Fading
- Distortion

FLAT FADING

Wideband pulses, $T < \Delta\tau$



Input

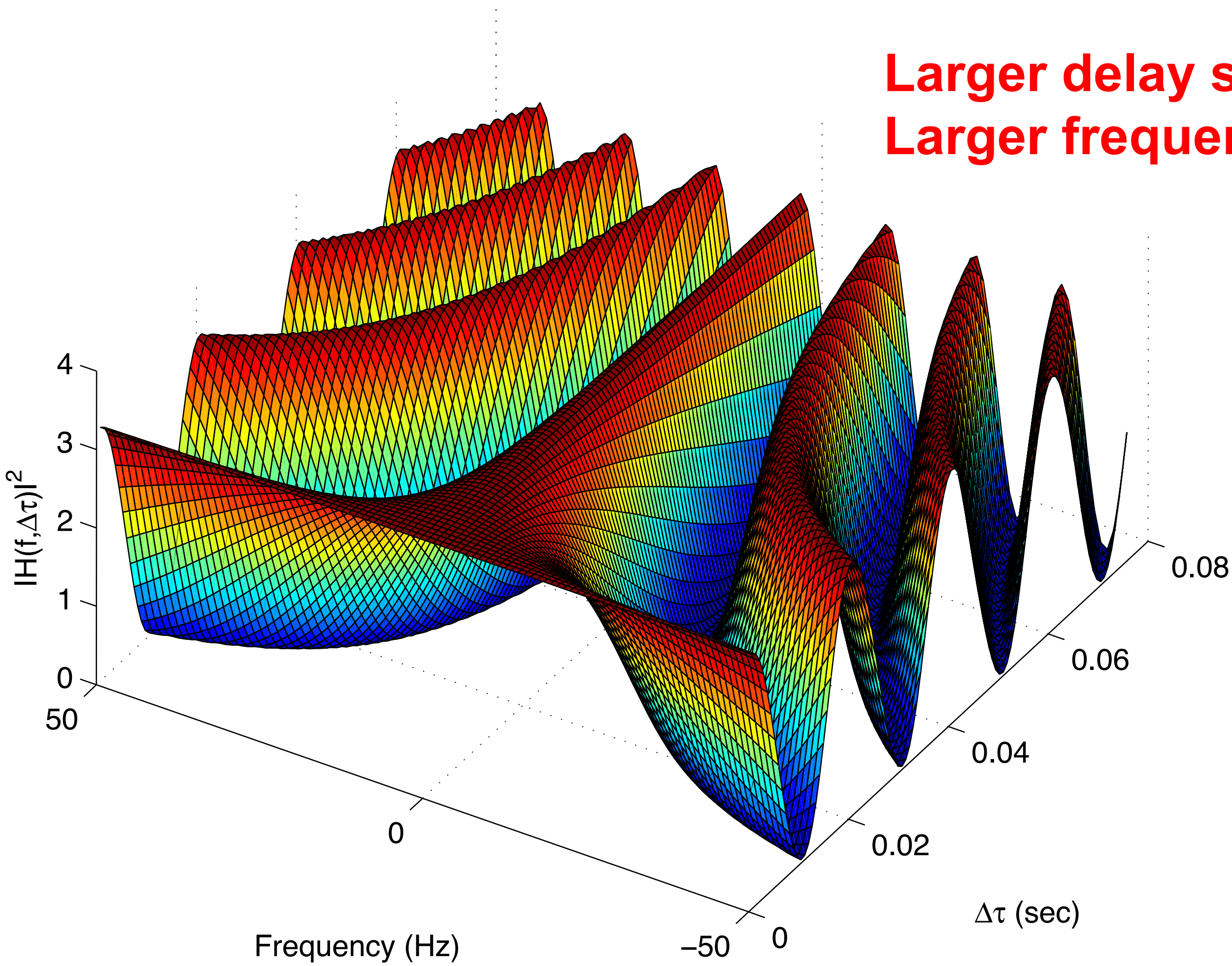


Output

- Amplitude variation:
  Fading (less)
- No distortion
- Overlap to next pulse:
  Interference

FREQUENCY-SELECTIVE FADING

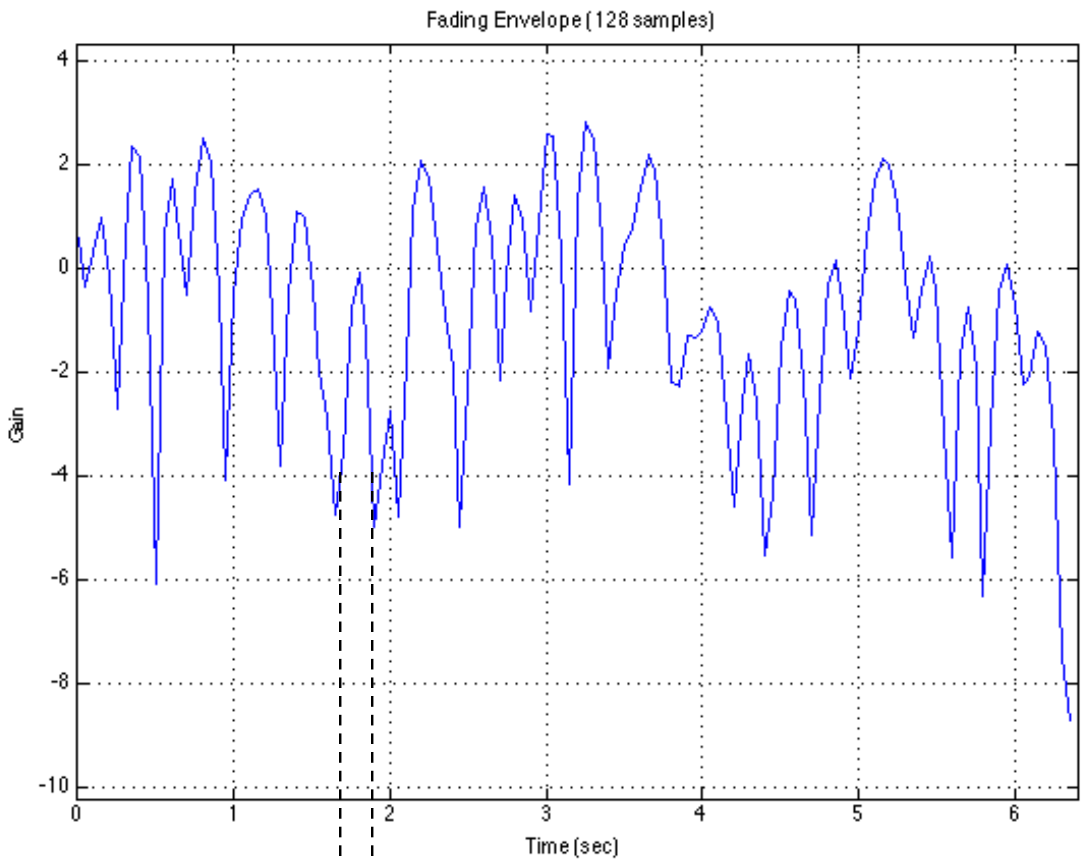
PSD of a wireless two-path channel



Larger delay spread
Larger frequency selectivity

Fading and time variations

- Variations in received power due to movement (Doppler):



T_c **Coherence time**

$$T_c = \frac{1}{B_D}$$

B_D : Doppler bandwidth

$$B_D = 2 v f_c / c$$

Slow fading: $T < T_c$

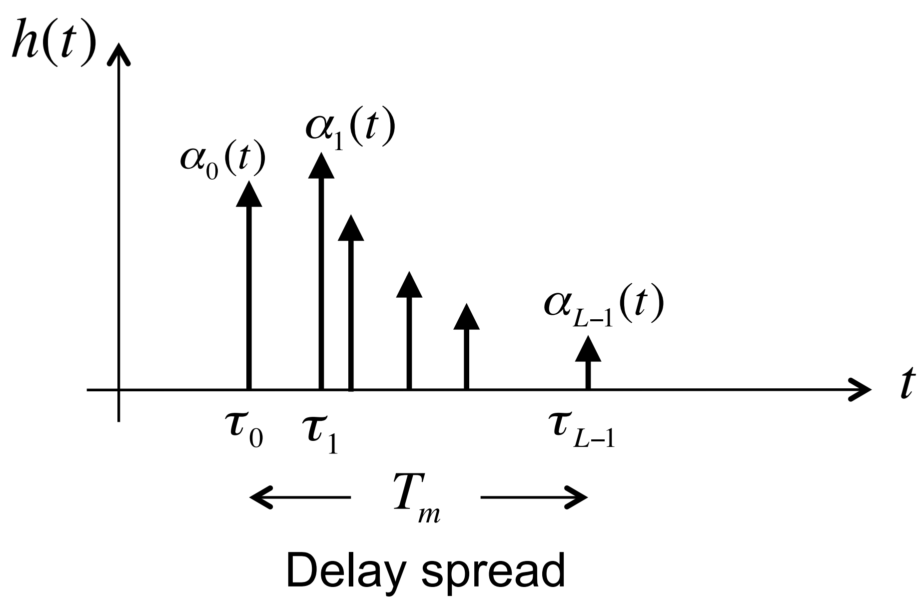
Fast fading: $T > T_c$

T is the symbol duration

Multipath effects

- Reflections (paths) of the transmitted electromagnetic signal on objects

- *L*-path channel impulse response:



Coherence bandwidth:

$$\boxed{B_c = \frac{1}{T_m}}$$

Phase rotation:

$$\boxed{\phi_i(t) = 2\pi f_c \tau_i(t)}$$

Basic types of fading

- Flat fading:

$$\boxed{B << B_c \quad \text{or} \quad 2\text{W} << \frac{1}{T_m}}$$

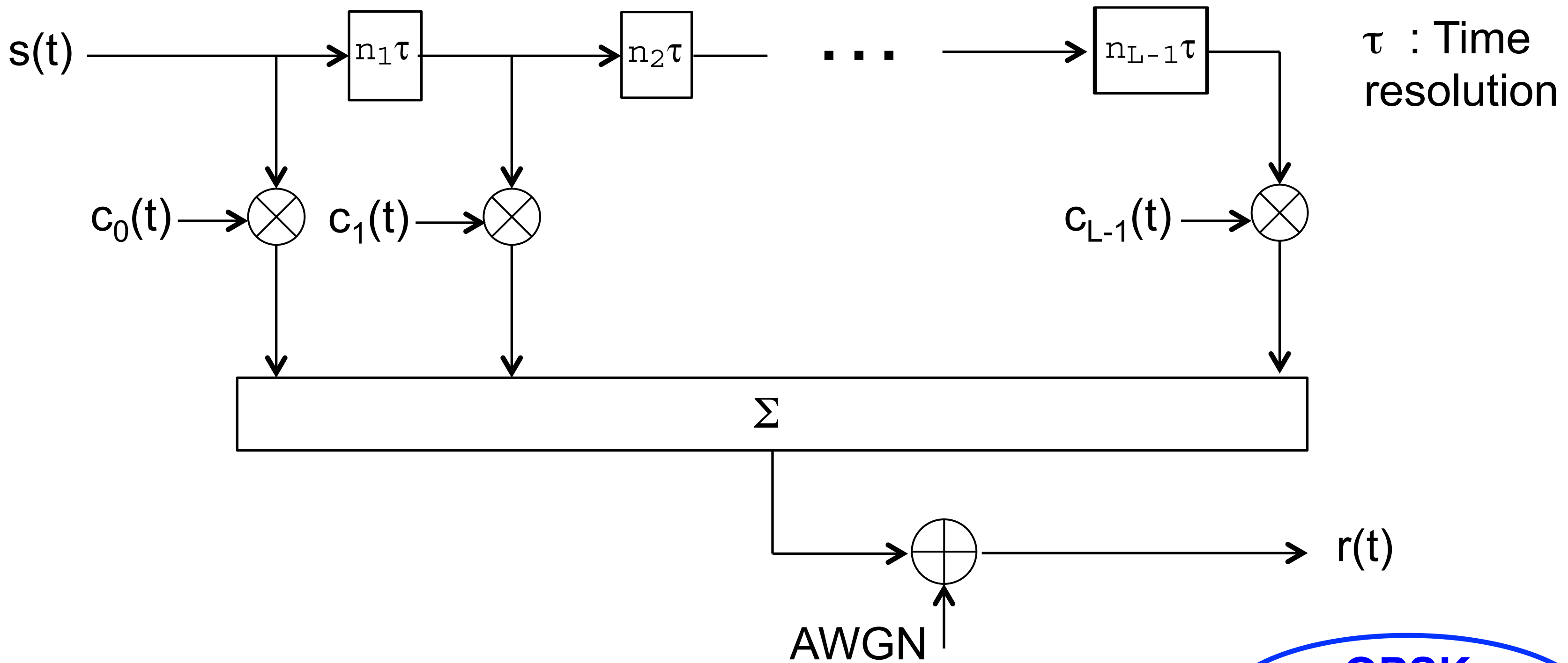
  – **Narrowband** signaling ***B=2W* is the signal bandwidth**

- Frequency-selective fading:

$$\boxed{B >> B_c \quad \text{or} \quad 2\text{W} >> \frac{1}{T_m}}$$

  – **Wideband** signaling

Complex baseband frequency-selective multipath channel model



QPSK Matlab demo

<u>Note</u>: L=1 and $c_0(t)=c_0$ gives flat fading

$c_i(t) = \alpha_i(t)\, e^{j\phi i(t)}$: i-th path (complex valued) gain, i=0,2,…,L-1

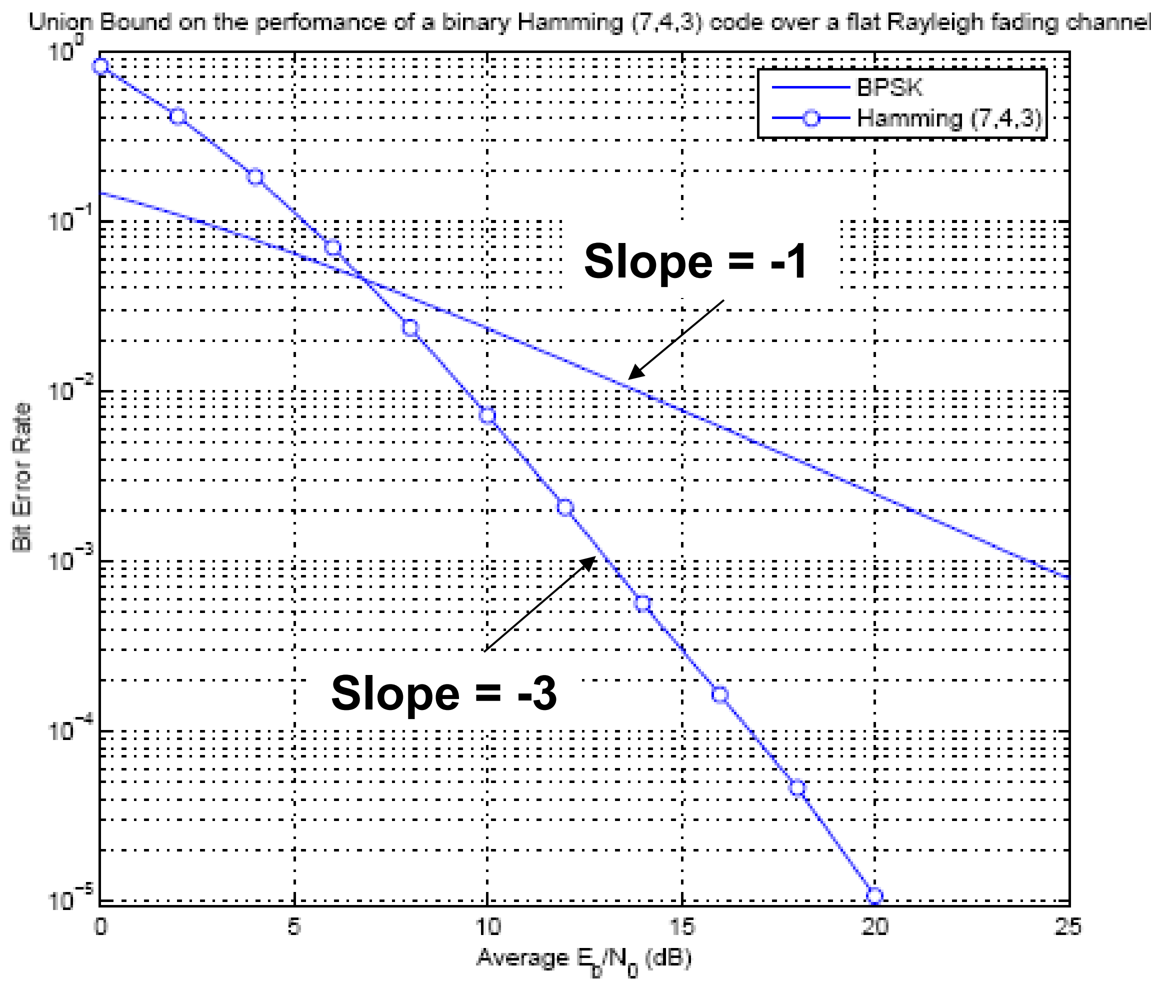
Time diversity for multipath channels

Time-diversity techniques

Time-diversity techniques can be classified according to the frequency selectivity of the multipath channel:

- Flat fading channels
  - ***Error correcting coding*** & ***interleaving*** **(More later)**
    - Diversity order equal to the ***minimum Hamming distance*** of the code
- Frequency-selective channels
  - ***ML sequence estimation: Viterbi equalization***
  - ***RAKE*** demodulation
  - Linear adaptive ***equalization***

Flat Rayleigh fading: ECC diversity with a Hamming (7,4,3) code



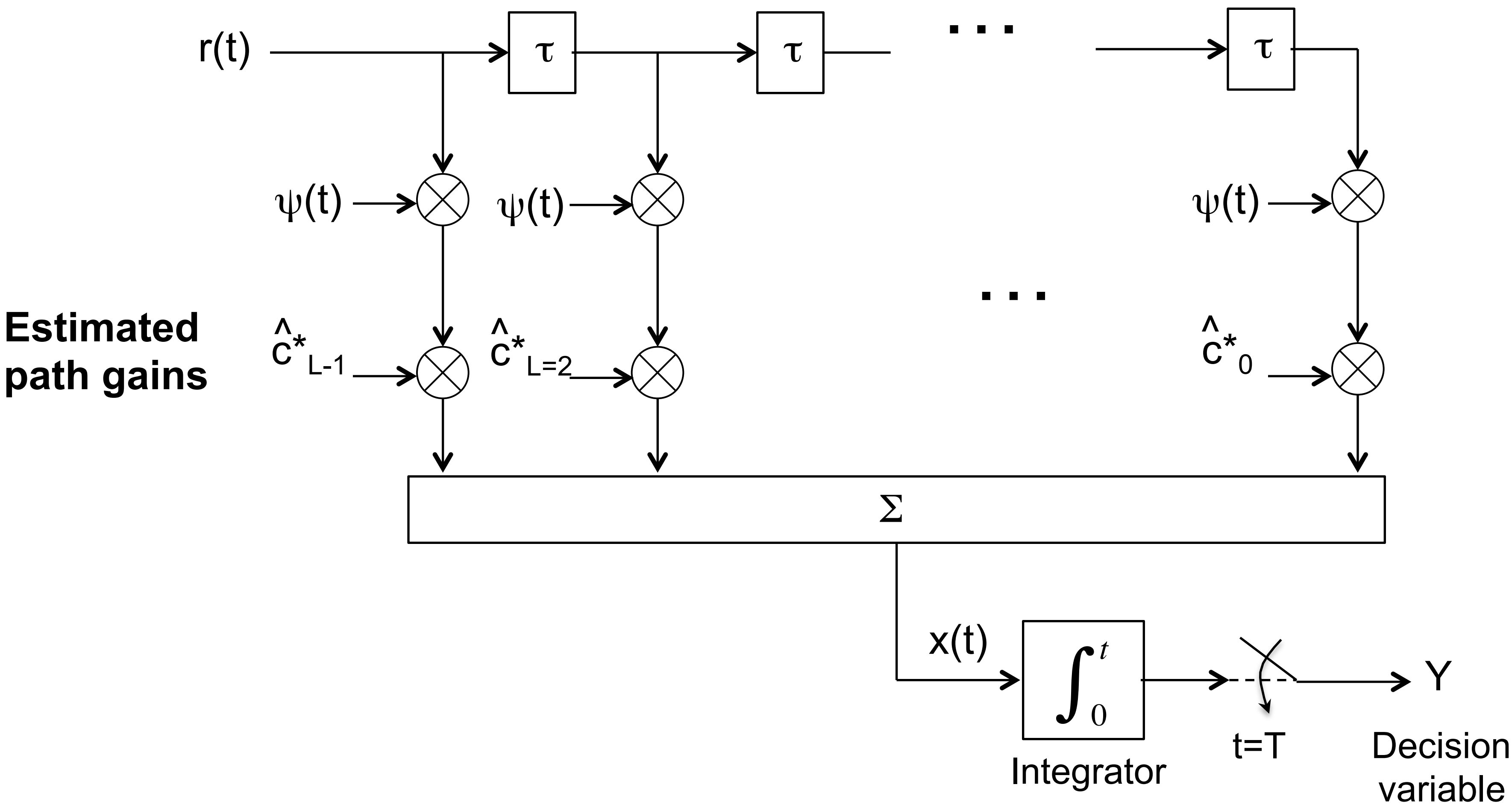
RAKE demodulator: Assumptions

- Slow fading: $T << T_c \quad \Rightarrow \quad c_i(t) = c_i, i = 0, \cdots, L-1$
- Frequency-selective fading: $W >> B_c$ (1)
- No intersymbol interference (ISI): $T >> T_m$ (2)

- (1) and (2) are satisfied by ***wideband pulses***, such as PPM or spread-spectrum

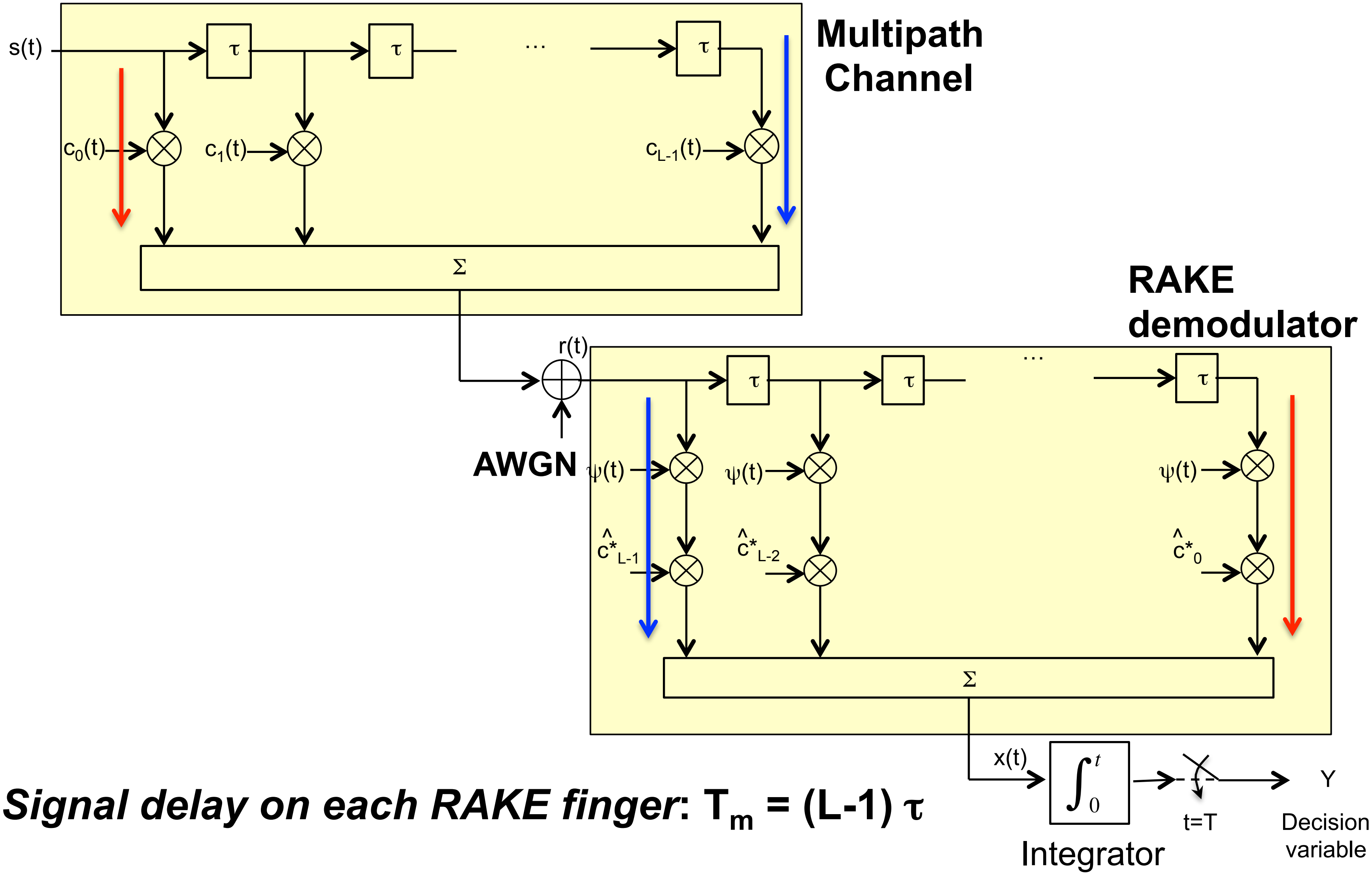
- Path gains and delays need to be known
  - Need ***channel estimation*** techniques ("*finger search*")

RAKE demodulator: Structure (BPSK)



→ **L fingers (diversity branches)**

Maximal-ratio combining property



***Signal delay on each RAKE finger*: T_m = (L-1) τ**

Maximal-ratio combining property (2)

- RAKE output (assuming no AWGN):

$$x(t) = r(t)\psi(t)c_{L-1}^{*} + r(t-\tau)\psi(t)c_{L-2}^{*} + \cdots + r\big(t-(L-1)\tau\big)\psi(t)c_{0}^{*}$$

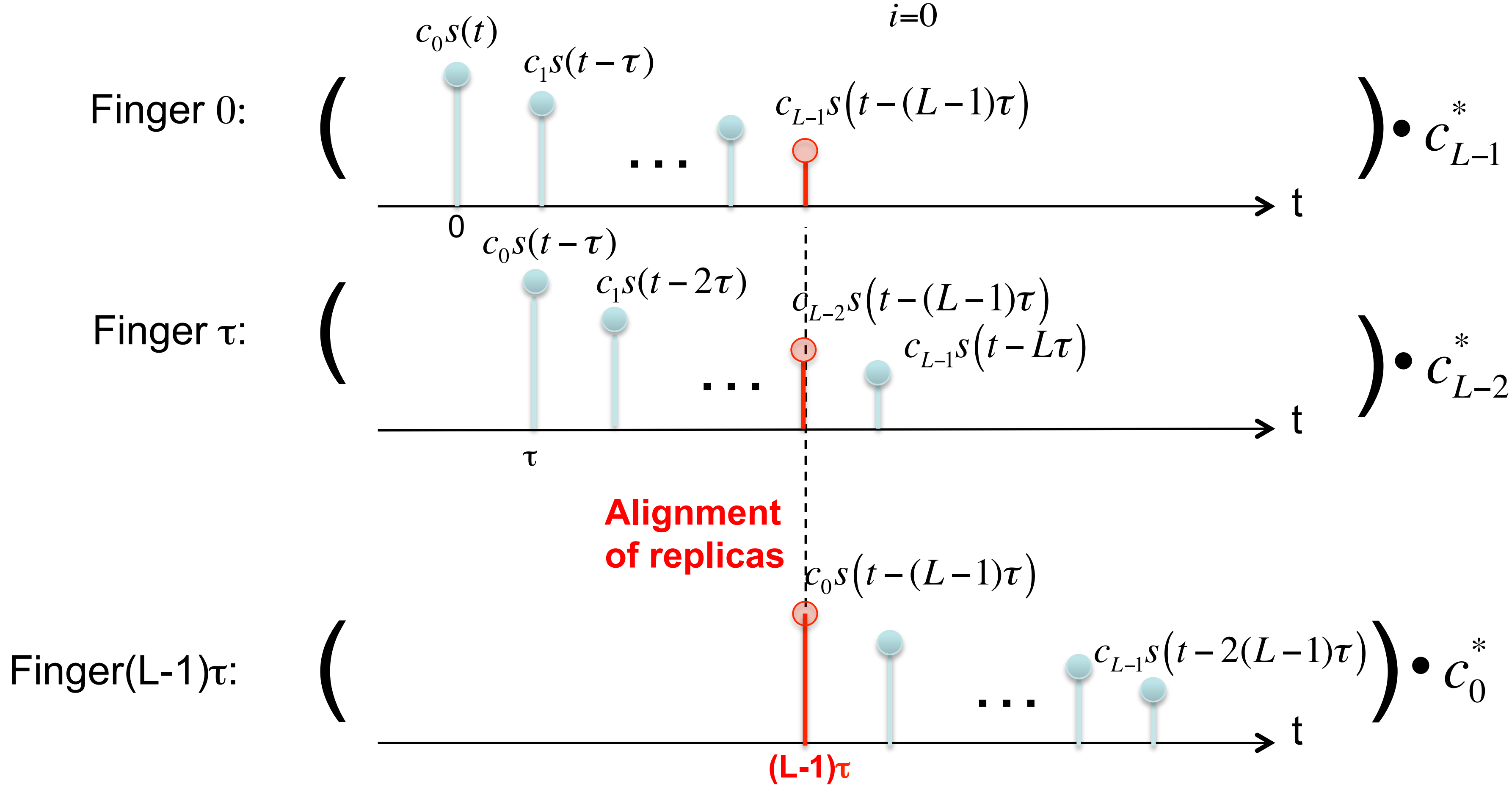
$$= \left[\sum_{i=0}^{L-1} c_i s\big(t-i\tau\big)\right]\psi(t)c_{L-1}^{*} + \left[\sum_{i=0}^{L-1} c_i s\big(t-(i+1)\tau\big)\right]\psi(t)c_{L-2}^{*}$$

$$+ \quad \cdots \quad + \left[\sum_{i=0}^{L-1} c_i s\Big(t-\big(i+L-1\big)\tau\Big)\right]\psi(t)c_{0}^{*}$$

$$\Rightarrow \boxed{x\big(t-(L-1)\tau\big) = \sum_{i=0}^{L-1} \left|c_i\right|^2 s\Big(t-\big(L-1\big)\tau\Big)\ \psi(t)}$$

Maximal-ratio combining property (3)

- Matched-filter output: $Y = S\sum_{i=0}^{L-1}|c_i|^2 + N$



RAKE performance under Rayleigh fading

- The signal goes through ***L*** **diversity branches**
- Probability of error for BPSK (polar) and BFSK (binary orthogonal or 2-PPM):

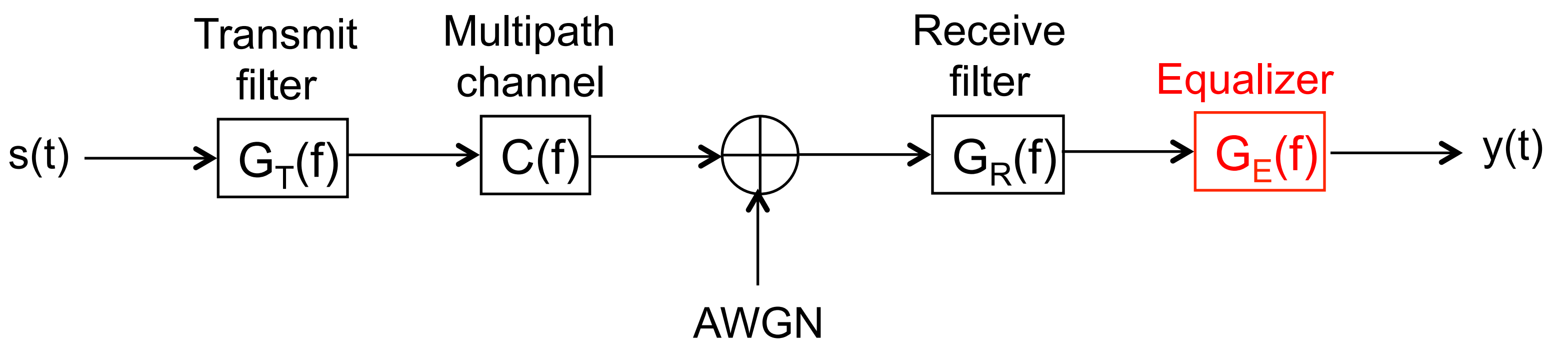
$$\boxed{P_b = \frac{(2L-1)!}{L!(L-1)!}\prod_{i=1}^{L}\left[\frac{1}{2(1-c)\bar{E}_{s,i}/N_0}\right]^{i+1}}$$

where $\bar{E}_{s,i}/N_0$ is the a verage signal energy-to-noise ratio of the i-th path, i=0,2, …, L-1, $c=-1$ for BPSK and $c=0$ for BFSK.

Linear adaptive equalizer: Assumptions

- Slow fading: $T << T_c \quad \Rightarrow \quad c_i(t) \approx c_i, i = 0, \cdots, L-1$
- Frequency-selective fading: $W >> B_c$
- Presence of ISI: $T < T_m$
  - Signaling using *narrowband pulses*
  - ***Example: Square-root raised-cosine (SRRC) pulses***
- Path gains unknown
  - Need pilot symbols (known at the receiver) and an adaptive algorithm to estimate gains
- Binary modulation (N =1 pulse)

Linear adaptive equalizer: System model



- Matched filter, assuming SRRC pulses:

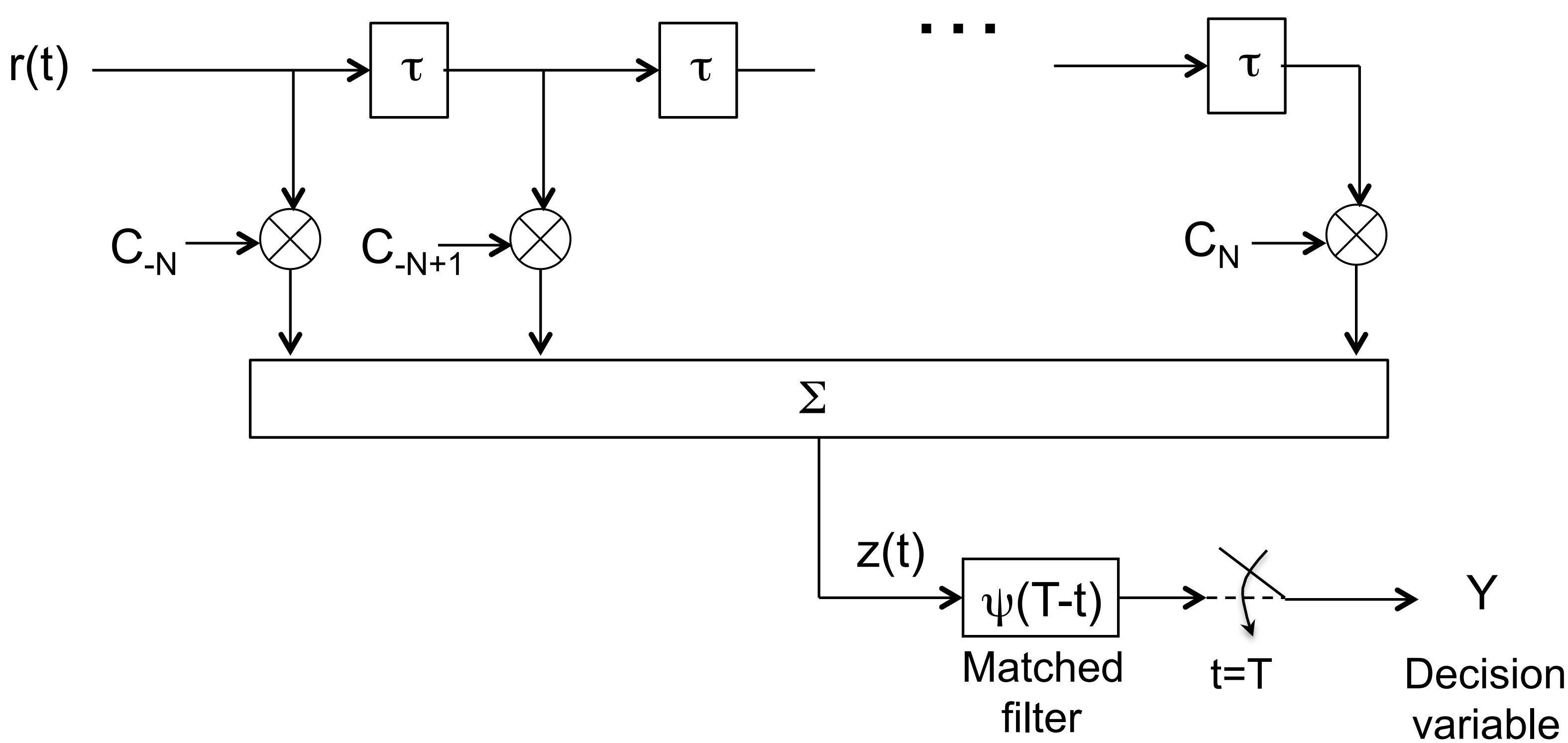
$$G_R(f) = G_T^*(f), \quad G_R(f)G_T(f) = X_{RC}(f) \quad \text{(Raised cosine)}$$

- Equalizer can be interpreted as a filter "matched" to the channel:

$$G_E(f) = \frac{1}{C(f)}$$

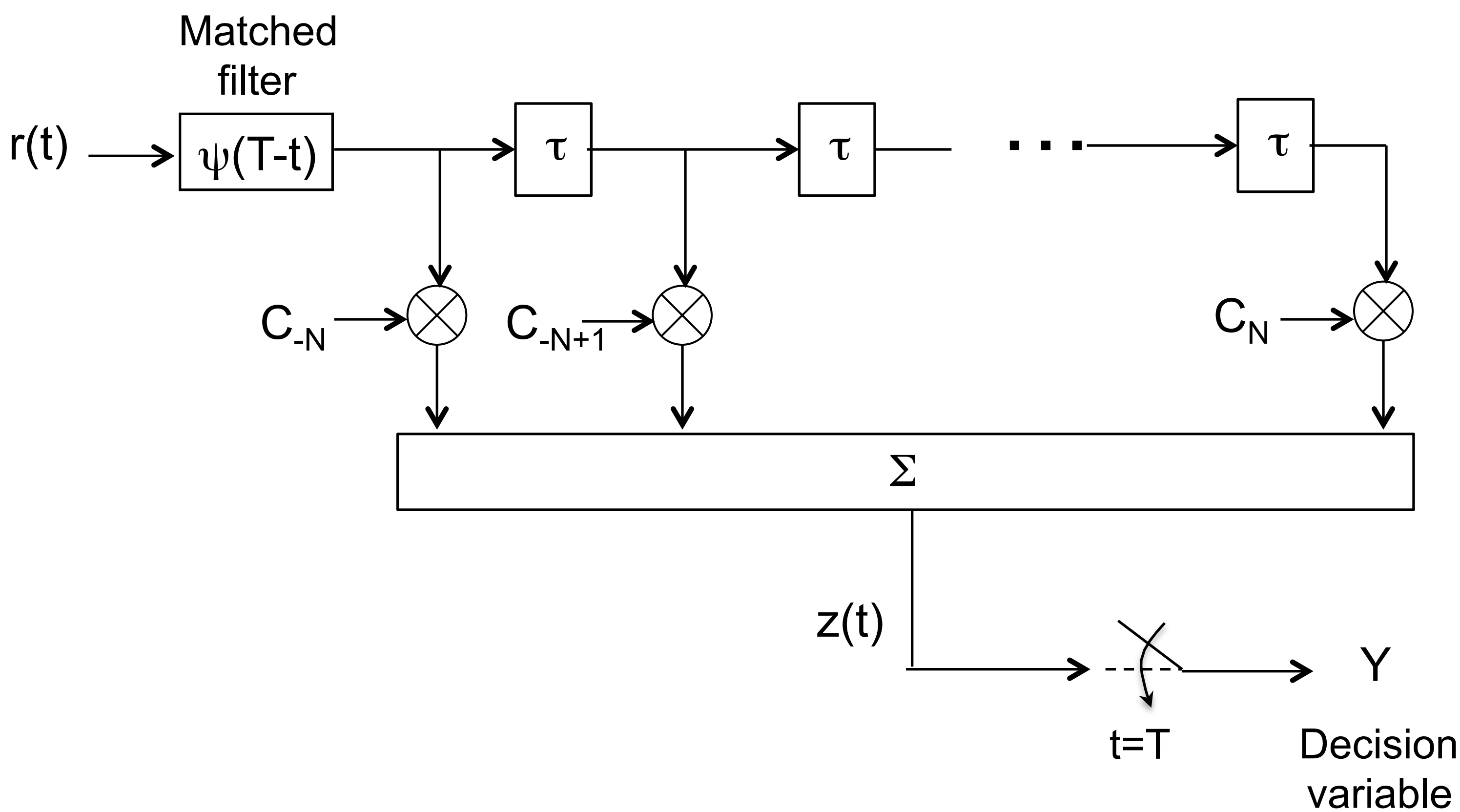
Linear adaptive equalizer: Pre-detector structure

- Similar to the RAKE demodulator:



- Equalizer (complex) coefficients updated based on error between Y and pilot symbols known to the receiver: ***MMSE criterion***.

Linear adaptive equalizer: Post-detector structure



Typically, τ = T (symbol spaced) or T/2 or T/4 (fractionally spaced)

The LMS algorithm

- Coefficients update at time t=(k+1)τ, for n=-2N,…,2N:

$$C_n[k+1] = \underbrace{C_n[k]}_{\text{Previous coefficient}} + \underbrace{\Delta}_{\text{Step size}}\, \underbrace{e[k]}_{\text{Error}} \cdot \underbrace{r_n[k]}_{\text{Received (stored) values}}$$

- Error signal (decision directed):

$$e[k] = \underbrace{s_P[k]}_{\text{Pilot symbol}} - \underbrace{Y[k]}_{\text{MF output}}$$

- The adaptive equalizer ***can track slow variations*** of multipath fading. Moreover, if nulls in the frequency response are too pronounced then the BER is high

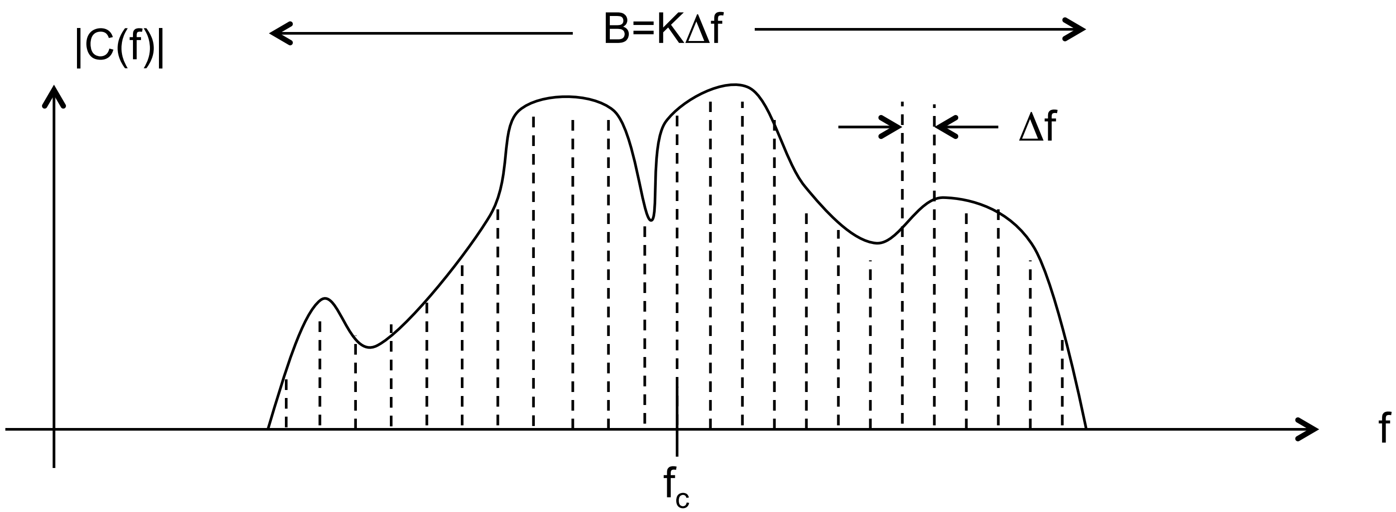
Matlab demo

Frequency diversity for frequency-selective multipath channels: OFDM

San José State University

Frequency-domain approach

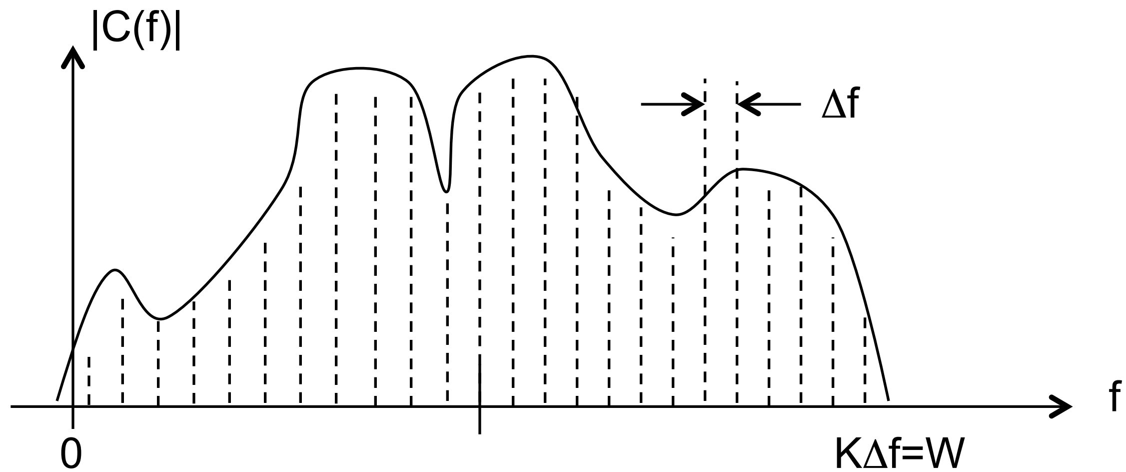
- Divide and conquer: Create K *subchannels* with frequency responses that are relatively constant (flat):



- Subcarrier frequencies:

$$f_k = f_c - \frac{K-1}{2T} + \frac{k}{T}, \quad k = 0,1,\cdots,K-1.$$

Complex baseband spectrum



- Each baseband channel has an associated basis signal

$$\psi_k(t) = e^{j\left(2\pi \frac{k}{T} t\right)}, \quad k = 0, 1, \cdots, K-1.$$

- Frequency separation and symbol duration (sinc pulses):

$$\Delta f = \frac{W}{K}, \quad T = \frac{1}{\Delta f} = \frac{K}{W} \qquad \rightarrow \quad \text{Symbol duration is proportional to K}$$

OFDM signal

- A large value of K results in T >> T_m and fading becomes ***flat***
  - Constant subchannel gains:

$$C\left(2\pi\frac{k}{T}\right) = C_k = A_k e^{j\phi_k}, \quad k = 0,1,\cdots, K-1.$$

- Each subcarrier is typically M-QAM mapped so that the signal transmitted over each subchannel is:

$$u_k(t) = \sqrt{\frac{2}{T}} S_{Ik} \cos\left(2\pi\frac{k}{T}t\right) + j\sqrt{\frac{2}{T}} S_{Qk} \sin\left(2\pi\frac{k}{T}t\right), \ 0 \le t \le T,$$

where $S_k = S_{Ik} + jS_{Qk}$ represent the modulation symbols.

- Complex baseband OFDM signal: $s(t) = \sum_{k=0}^{K-1} u_k(t)$

OFDM receiver processing

- For each subchannel, k=0,1, …, K-1, the ***received signal*** is

$$r_k(t) = \sqrt{\frac{2}{T}} A_k S_{k1} \cos\left(2\pi \frac{k}{T} t + \phi_k\right) + j\sqrt{\frac{2}{T}} A_k S_{k2} \sin\left(2\pi \frac{k}{T} t + \phi_k\right) + \underbrace{N_k(t)}_{\text{AWGN}},\ \ 0 \le t \le T,$$

with A_k the amplitude response and ϕ_k the phase response.

- ***Basis functions***:

$$\psi_{k1}(t) = \sqrt{\frac{2}{T}} \cos\left(2\pi \frac{k}{T} t\right), \quad \psi_{k2}(t) = \sqrt{\frac{2}{T}} \sin\left(2\pi \frac{k}{T} t\right),\ \ 0 \le t \le T.$$

- Corresponding ***matched filter outputs***:

$$Y_{k1} = A_k \cos(\phi_k) \cdot S_{k1} + W_{k1}, \quad Y_{k2} = A_k \sin(\phi_k) \cdot S_{k2} + W_{k2},\ \ \text{or}$$

$$\boxed{Y_k = C_k S_k + W_k,} \quad \text{as a complex number.}$$

Implementation with the FFT algorithm

- Let K = 2^m. The complex baseband OFDM signal is

$$x(t) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} S_k e^{j2\pi \frac{k}{T} t}, \quad 0 \le t \le T.$$

- Sampling at intervals $T_s = T/K$ results in K samples

$$x_n = x(nT_s) = x\left(n\frac{T}{K}\right) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} S_k e^{j2\pi k \frac{n}{K}}, \quad n = 0, 1, \cdots, K-1.$$

which is the ***K*-point inverse DFT** of the symbol sequence

- Implementation using the FFT (Fast-Fourier transform):

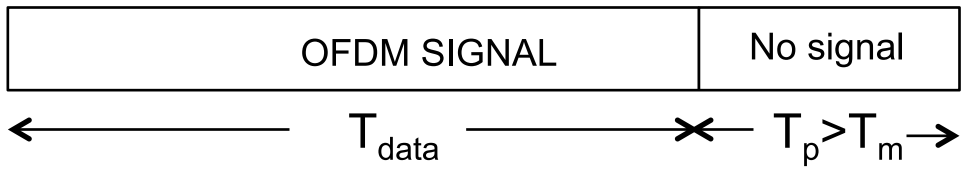
$$x_n = IFFT(S_k), \quad S_k = FFT(x_n)$$

Signal samples

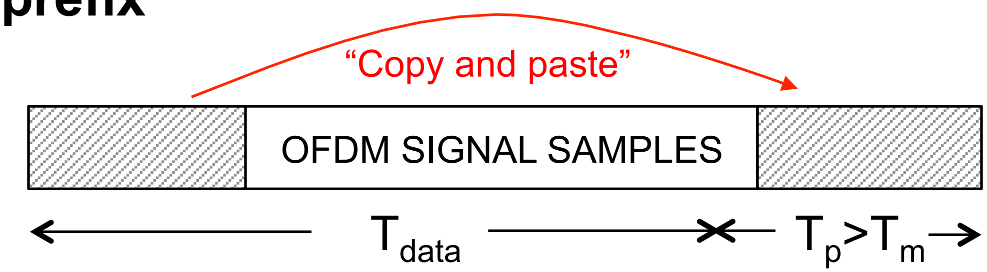
Modulated data symbols

ISI removal

- Effects of the delay spread T_m can be removed by using a ***prefix***
- Two choices:
  - Zero prefix (or time guardband)



  - **Cyclic prefix**



- The choice of a cyclic prefix offers the additional advantage that the ***discrete Fourier transform*** (implemented via the FFT algorithm) can be used

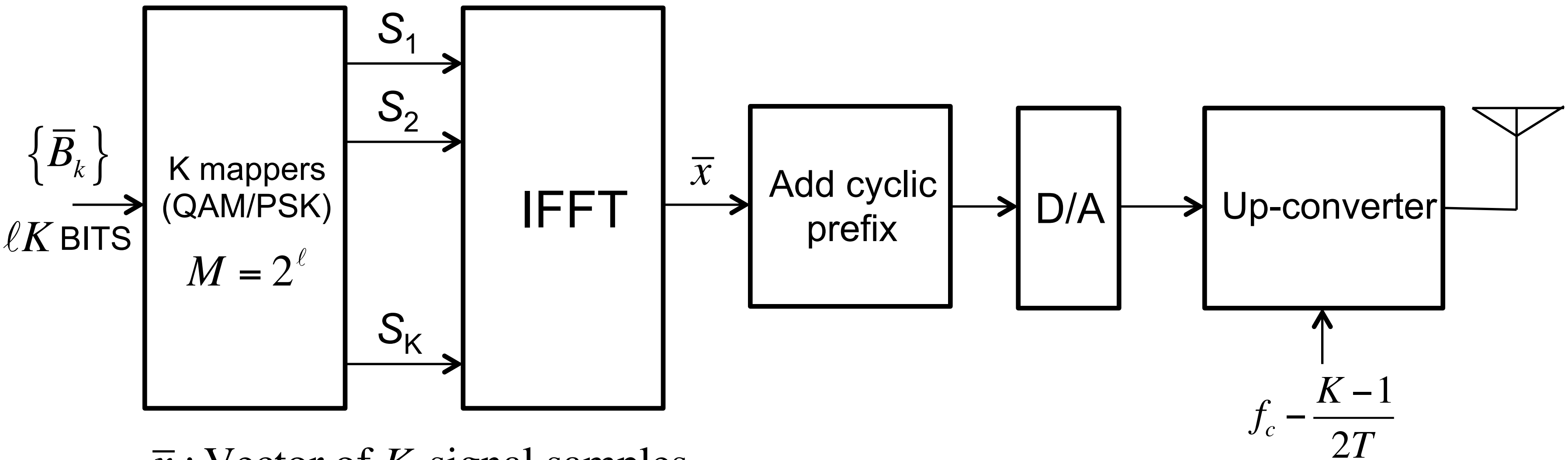
One-tap equalization

- The receiver estimates the subchannel gains using ***pilot symbols*** known to both transmitter and receiver

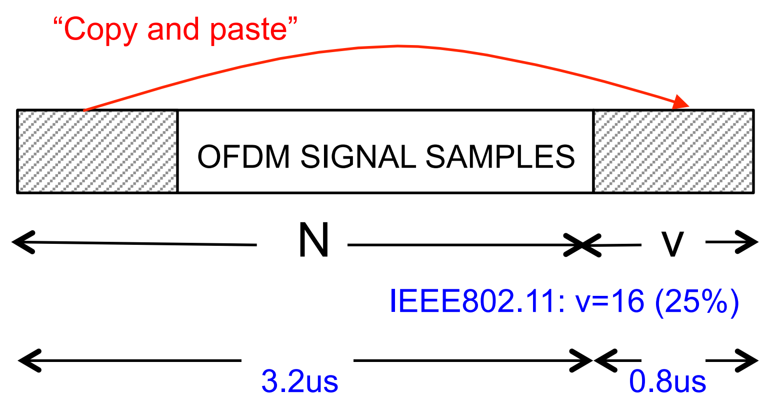
- Based on these estimates $\hat{C}_k$, the scaling of the transmitted symbols is removed by a process known in the literature as "***one-tap equalization***":

$$\boxed{Y_k' = \frac{\hat{C}_k^*}{\left|\hat{C}_k\right|^2} Y_k = \frac{\hat{C}_k^*}{\left|\hat{C}_k\right|^2}\left(C_k S_k + W_k\right) \approx S_k + W_k'}, \qquad k = 0,1,\cdots,K-1$$

OFDM transmitter

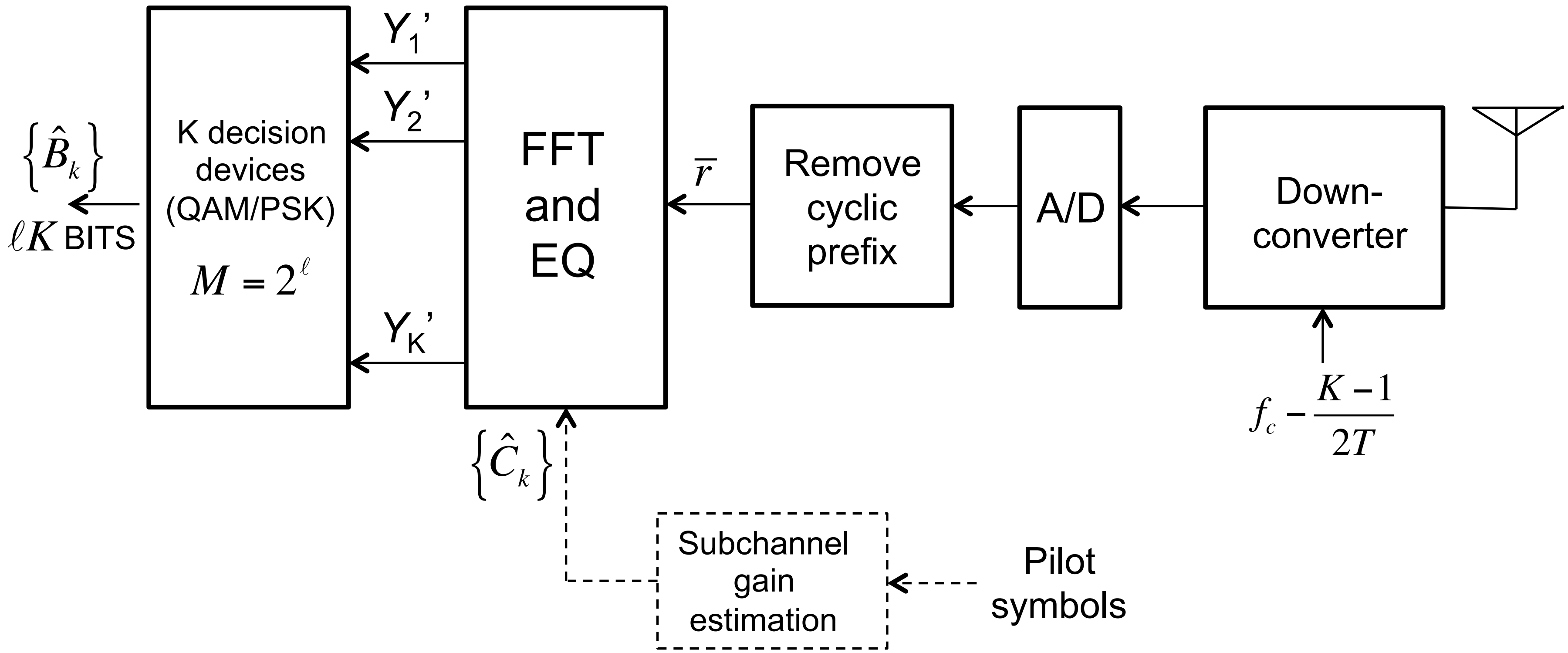


$\overline{x}$: Vector of K signal samples



IEEE 802.11 standard:
- 64-point FFT
- 48 data subcarriers
- 4 pilot subcarriers
- 12 turned-off (guardband)

OFDM receiver

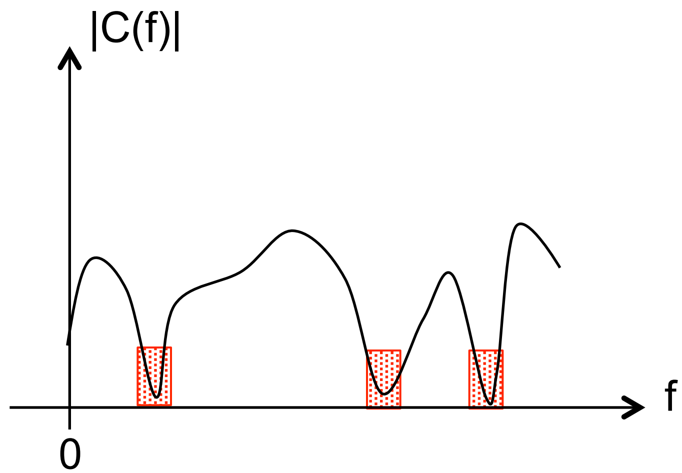


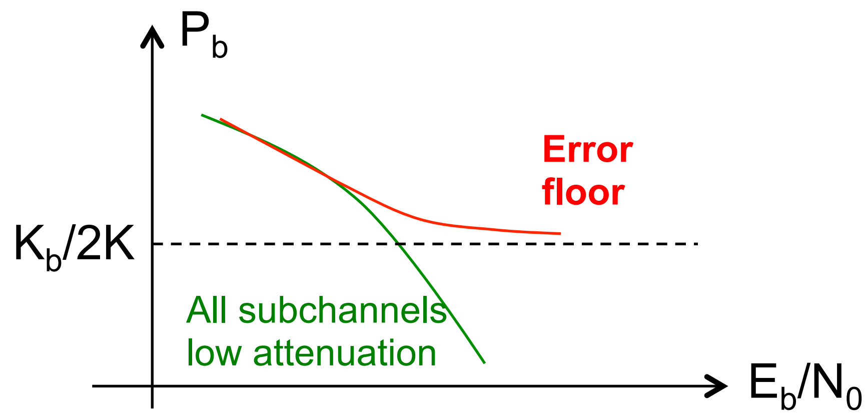
$\bar{r}$: Vector of K received signal samples

EQ : Array of K one-tap equalizers

Error floors in OFDM

- Subchannels (say K_b out of K) with high attenuation (low amplitude A_k) will experience large number of errors $(\approx 1/2)$



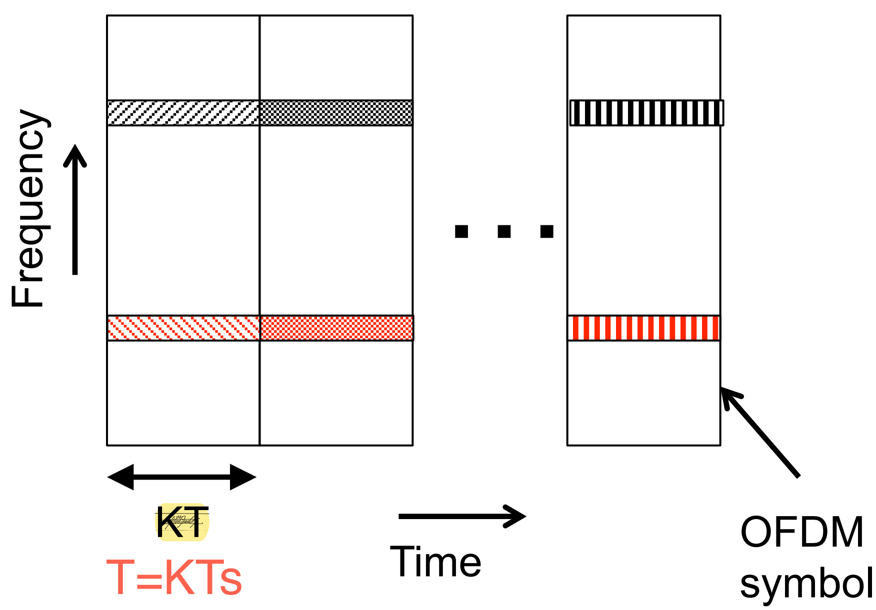


- Solution (e.g., all IEEE 802.11 physical layer specifications):
  - Scramble the symbols: ***Interleaving***
  - Use ***error correcting coding***

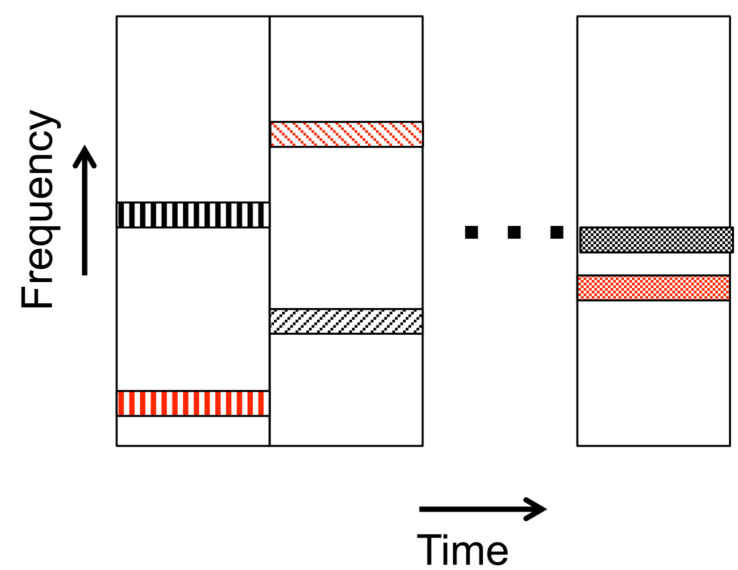
Time-Frequency Interleaving

Goal: Spread those subchannel symbols affected by frequency nulls (low energy) in channel response

Without interleaving:

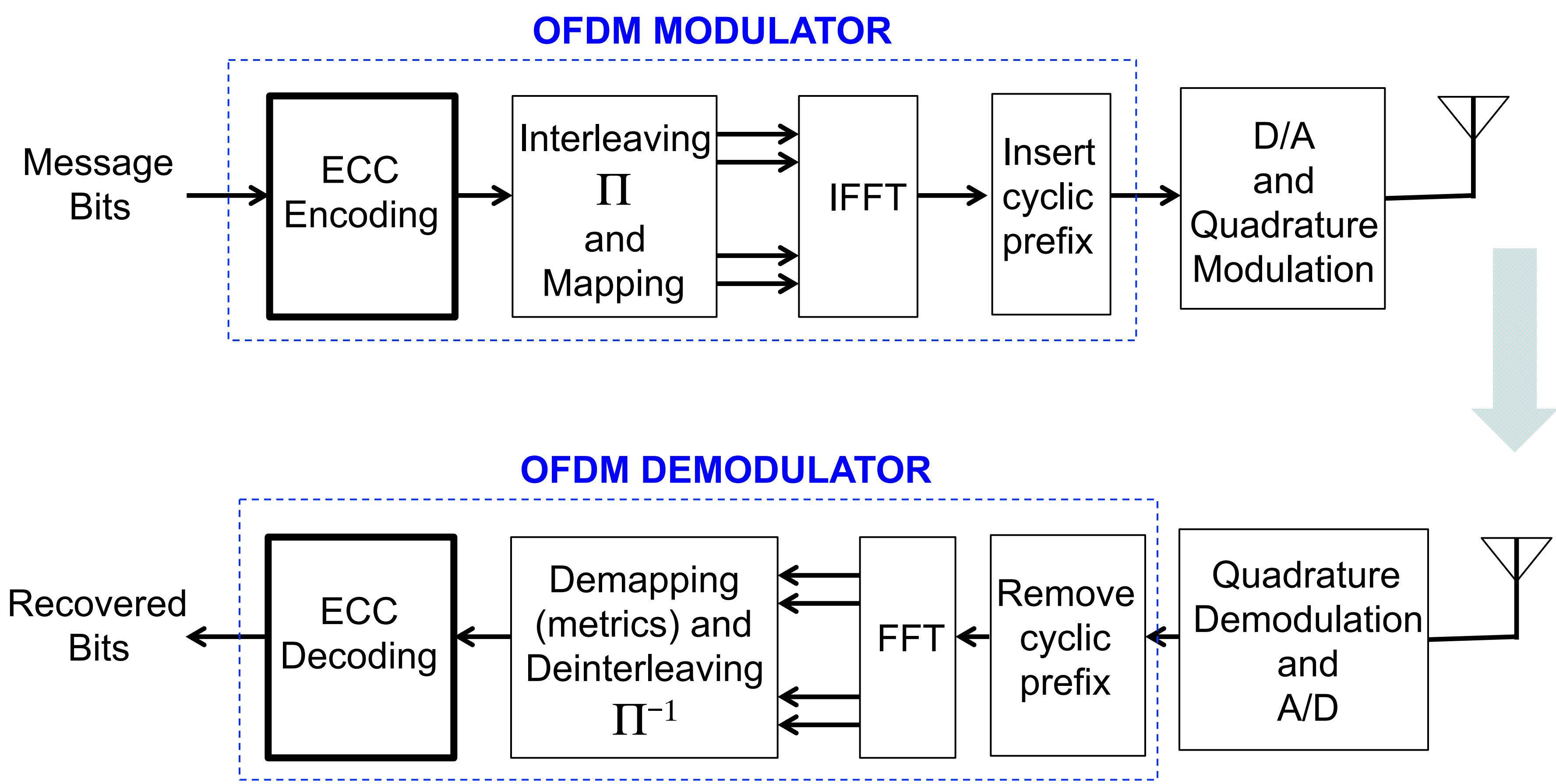


With interleaving:



Error Control Coding (ECC)

- Correct errors in symbols with low energy



Space diversity for multipath channels

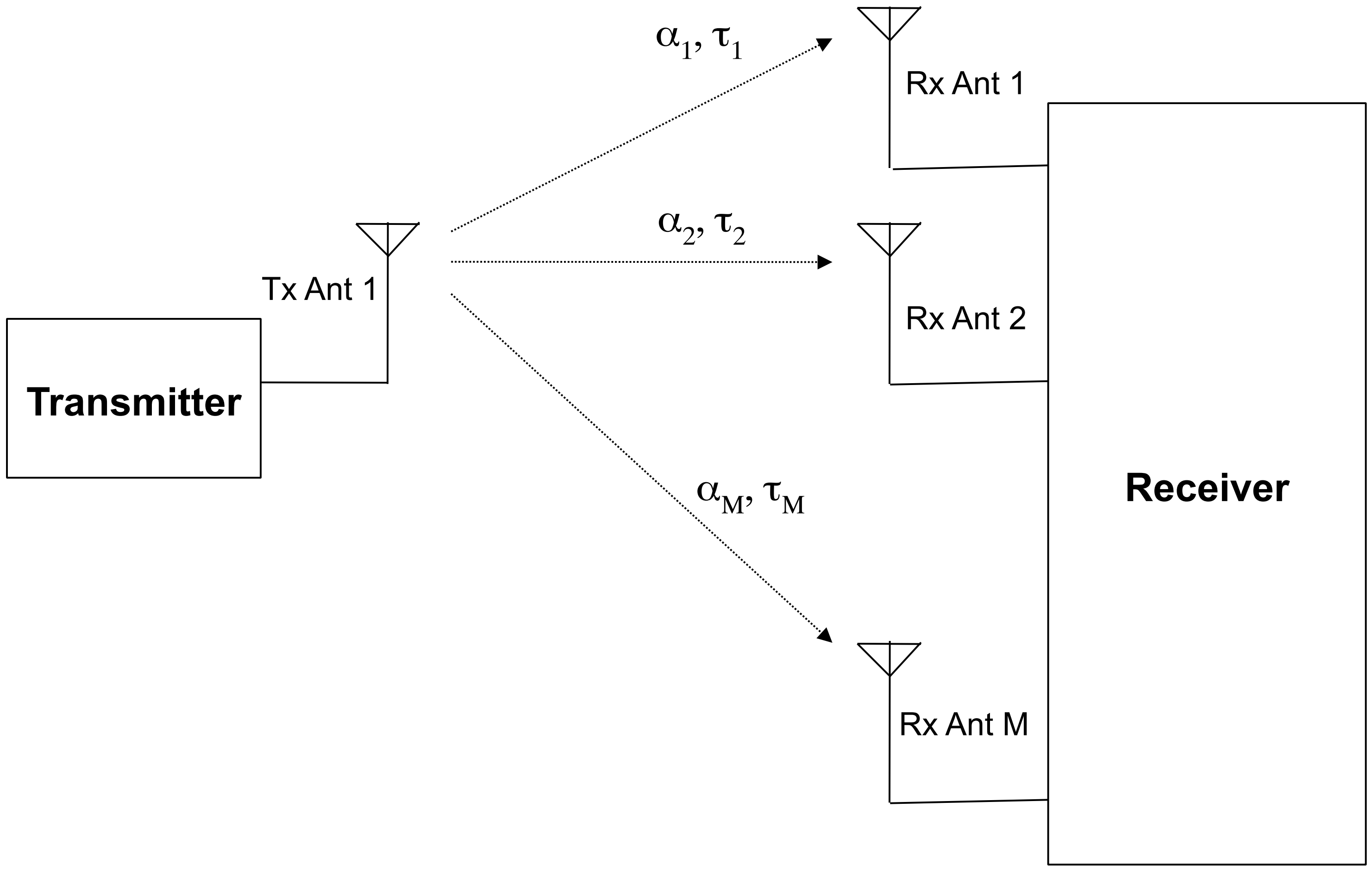
Multiple Antenna Systems

- Multiple antenna systems are designed to either achieve ***spatial diversity*** or increase data rate of wireless links

- There are three basic system types:
  - Single-input multiple-output (**SIMO**)
  - Multiple-input single-output (**MISO**)
  - Multiple-input multiple-output (**MIMO**)

For the purpose of explanation, flat fading is assumed

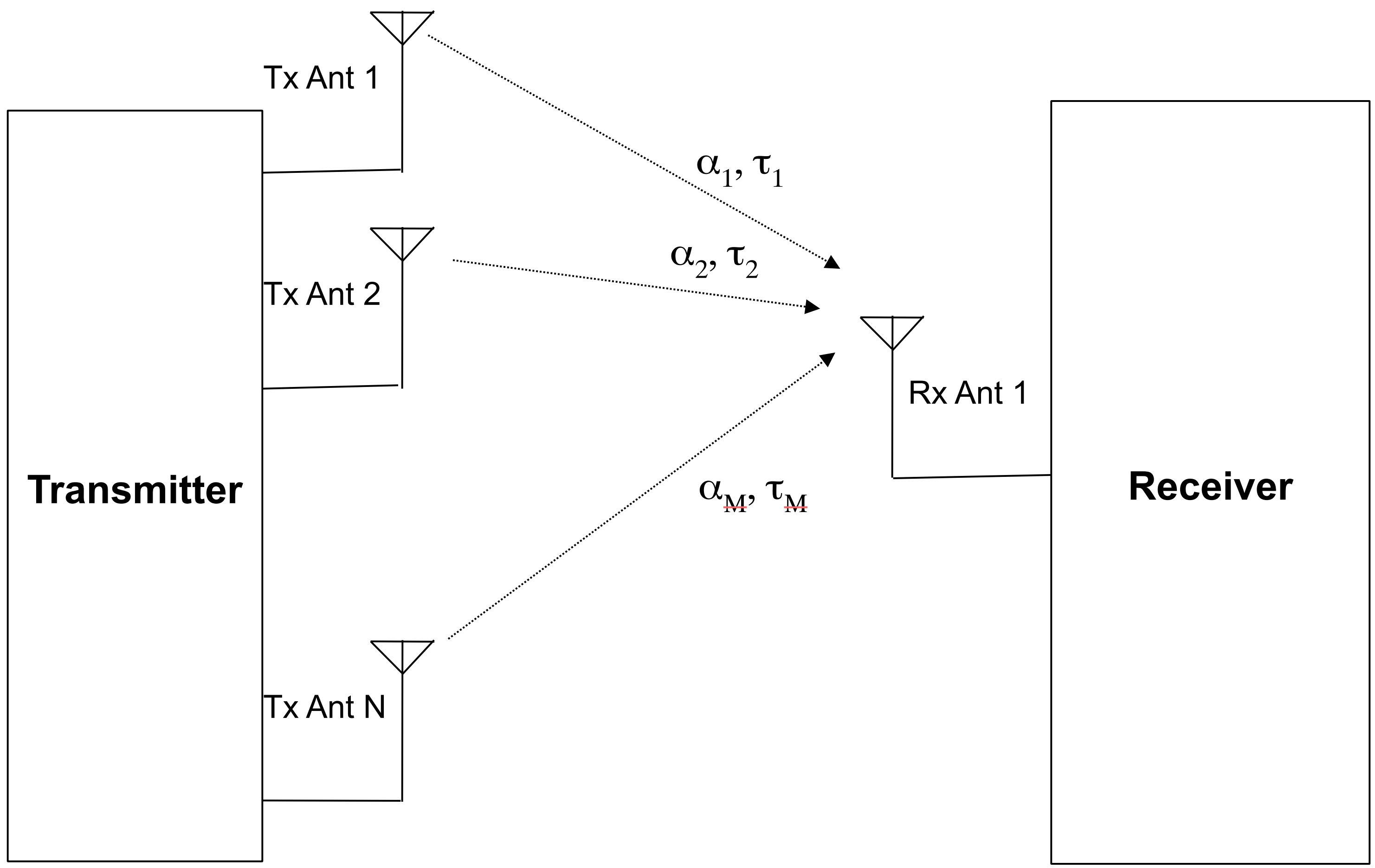
SIMO Systems



SIMO Systems (2)

- Historically first to achieve M-order diversity
  - Antenna separation large enough (> 5λ)
- Receive beamforming possible
  - Small antenna separation (< λ/2)
  - Used in multiple user systems

MISO Systems

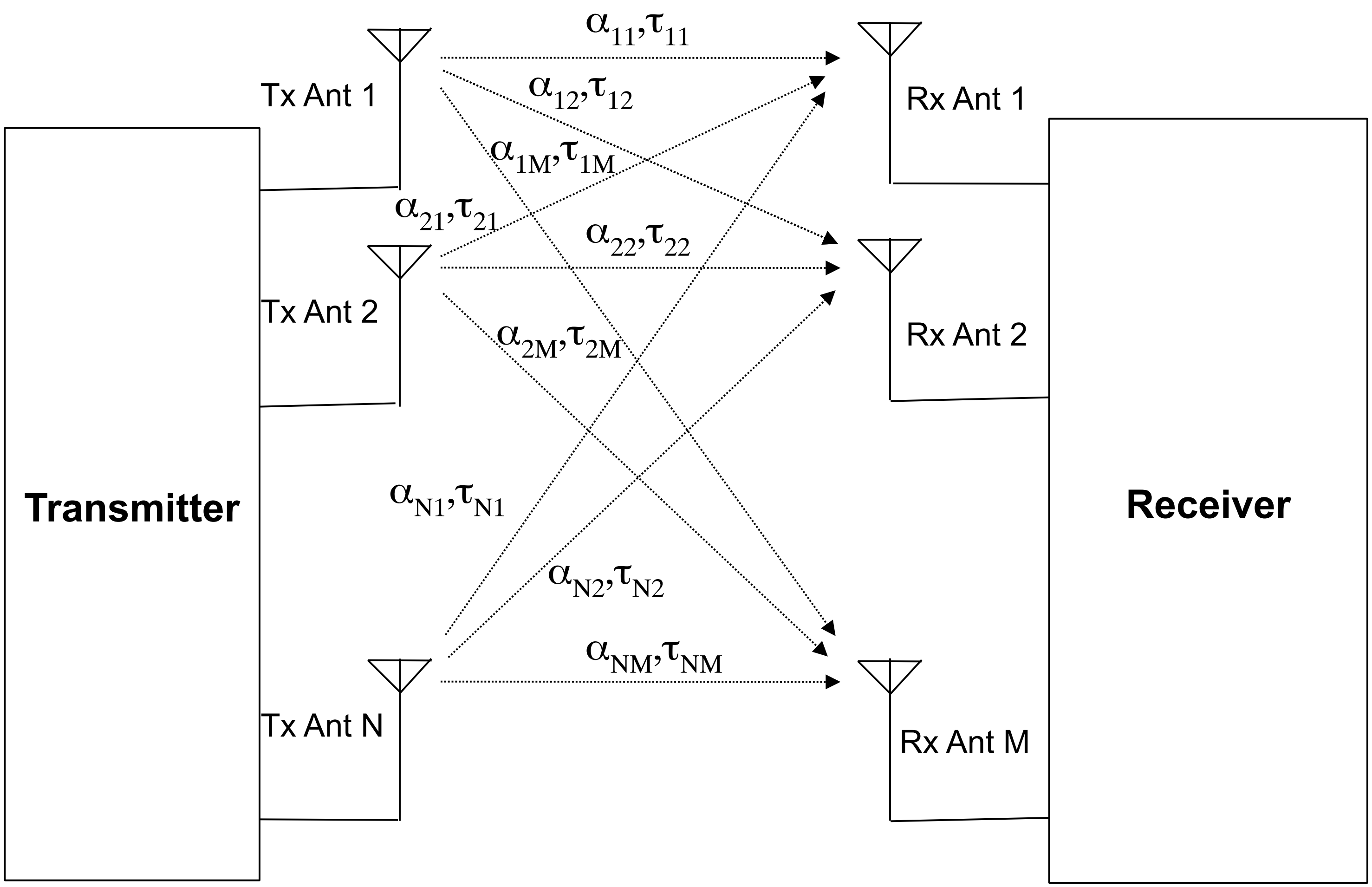


MISO Systems (2)

- Traditionally have used for transmit beamforming
  - Small antenna separation (< $\lambda/2$)
  - Used in multiple user systems

- N-order diversity with transmission in time/frequency and space (***Block Coding***)
  - Large antenna separation (> 5λ)
  - Example: Alamouti's 2x1 transmit diversity

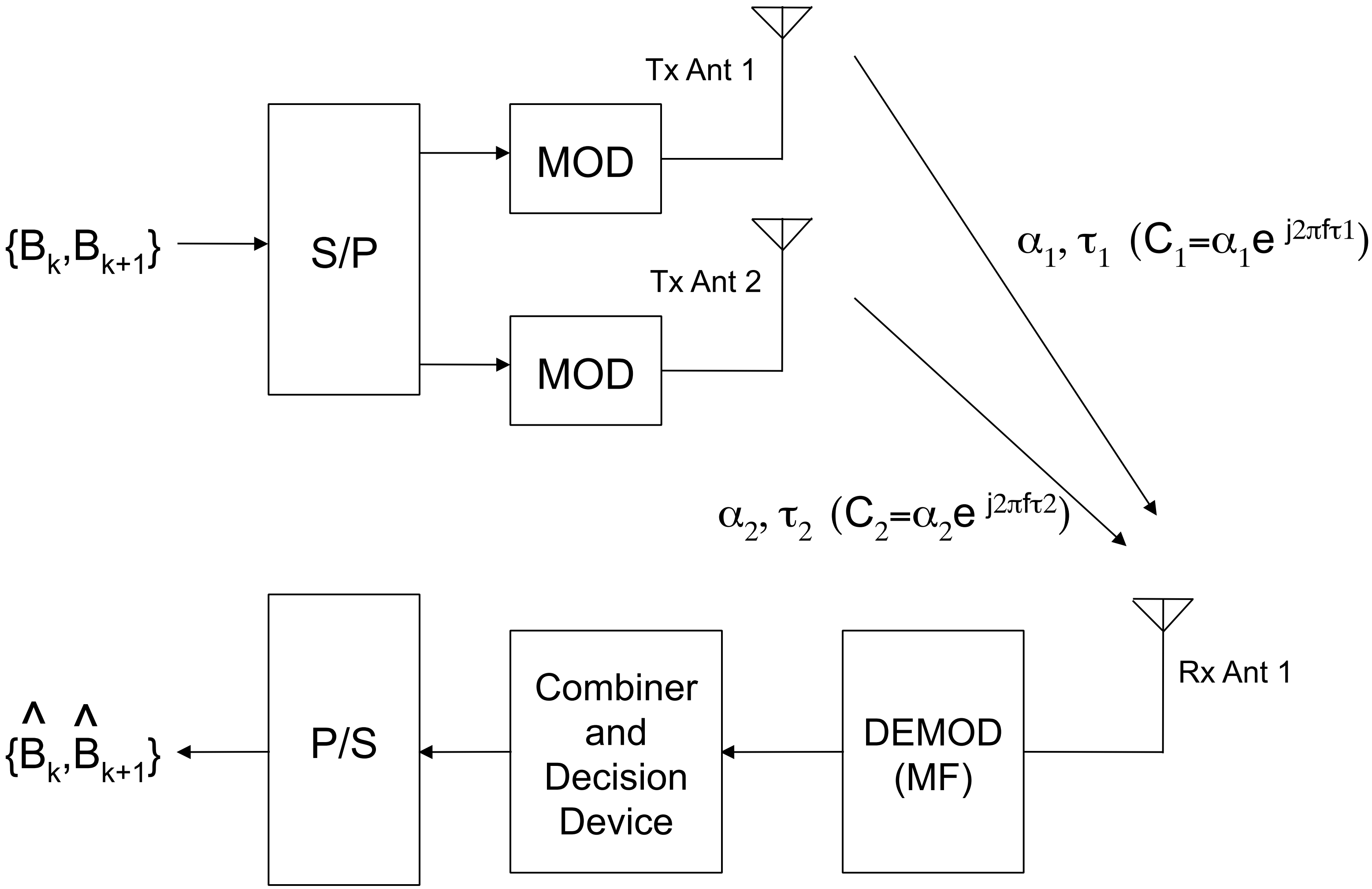
MIMO Systems



MIMO Systems (2)

- Several choices
  - Transmit beamforming/Receive diversity
  - Receive ***beamforming***
  - D=~~min{N,M}~~-order ***spatial diversity*** **D=MN**
  - Rate increase up to min{N,M} with ***interference cancellation*** to create parallel channels
  - Example: A 2x2 MIMO system

Example 1: Alamouti Code



Example 1: Alamoui Code (2)

- First adopted in 3GPP specification (2000)
  - Known as Space-Time Block-Coding (STBC)
- Transmitted symbols:

Antenna	Interval kT	Interval (k+1)T
Tx Ant 1	$S_{1,k}$	$S^*_{2,k}$
Tx Ant 2	$S_{2,k}$	$-S^*_{1,k}$

- Received symbols:

$$Y_k = C_1 S_{1,k} + C_2 S_{2,k} + N_k \quad (N_k : \text{Gaussian})$$

$$Y_{k+1} = C_1 S^*_{2,k} - C_2 S^*_{1,k} + N_{k+1} \quad (N_{k+1} : \text{Gaussian})$$

Example 1: Alamouti Code (3)

- NOTE: Complex channel gains C_1, C_2 (slow and flat fading assumed) need to be ***known*** at the receiver. (Channel estimation)

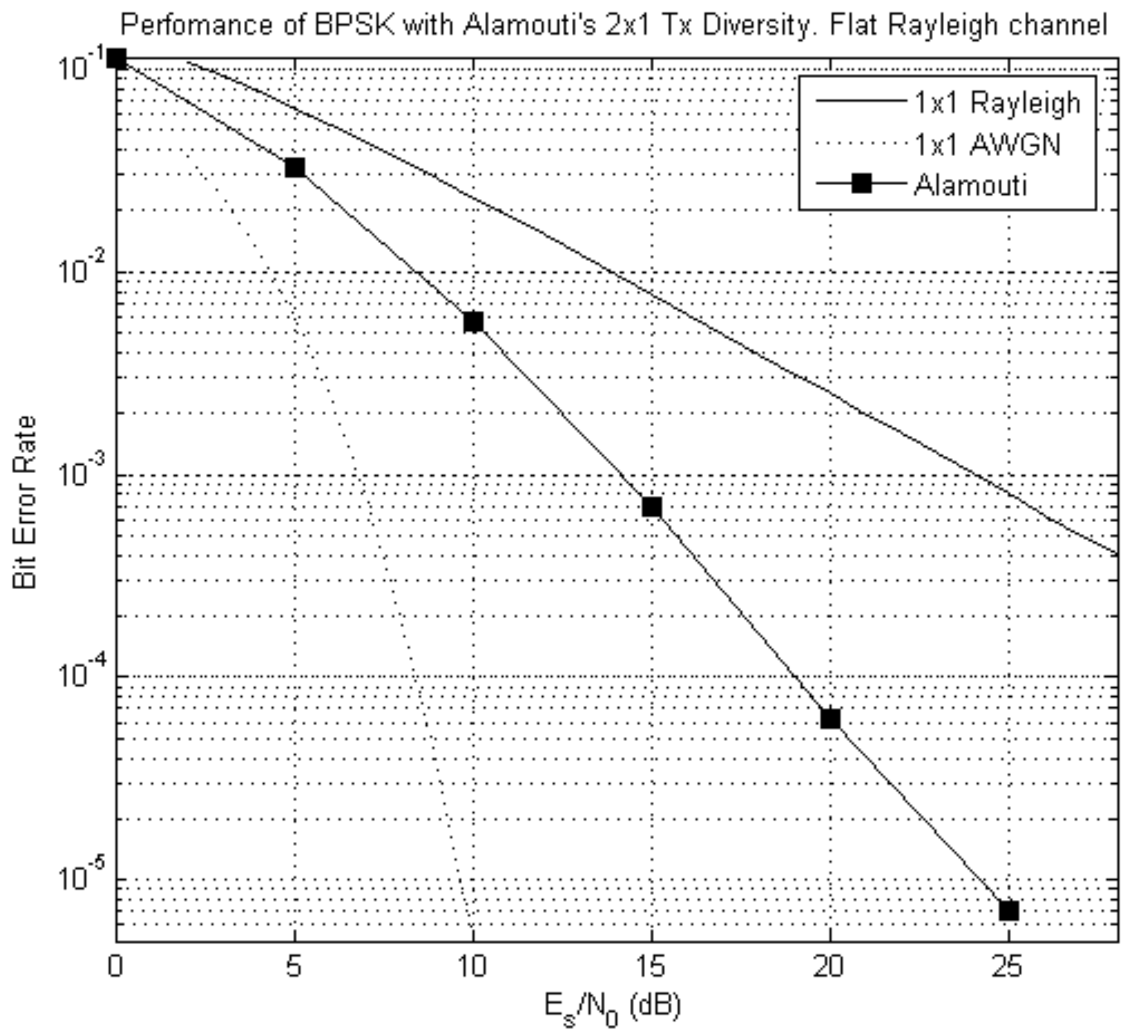
Combiner:

$$
\begin{aligned}
Y_k &= C^*_1 Y_k - C_2 Y^*_{k+1} \\
&= (|C_1|^2 + |C_2|^2)\; S_{1,k} + (C^*_1 N_k - C_2 N^*_{k+1}) \\
Y_{k+1} &= C^*_2 Y_k + C_1 Y^*_{k+1} \\
&= (|C_1|^2 + |C_2|^2)\; S_{2,k} + (C^*_2 N_k + C_1 N^*_{k+1})
\end{aligned}
$$

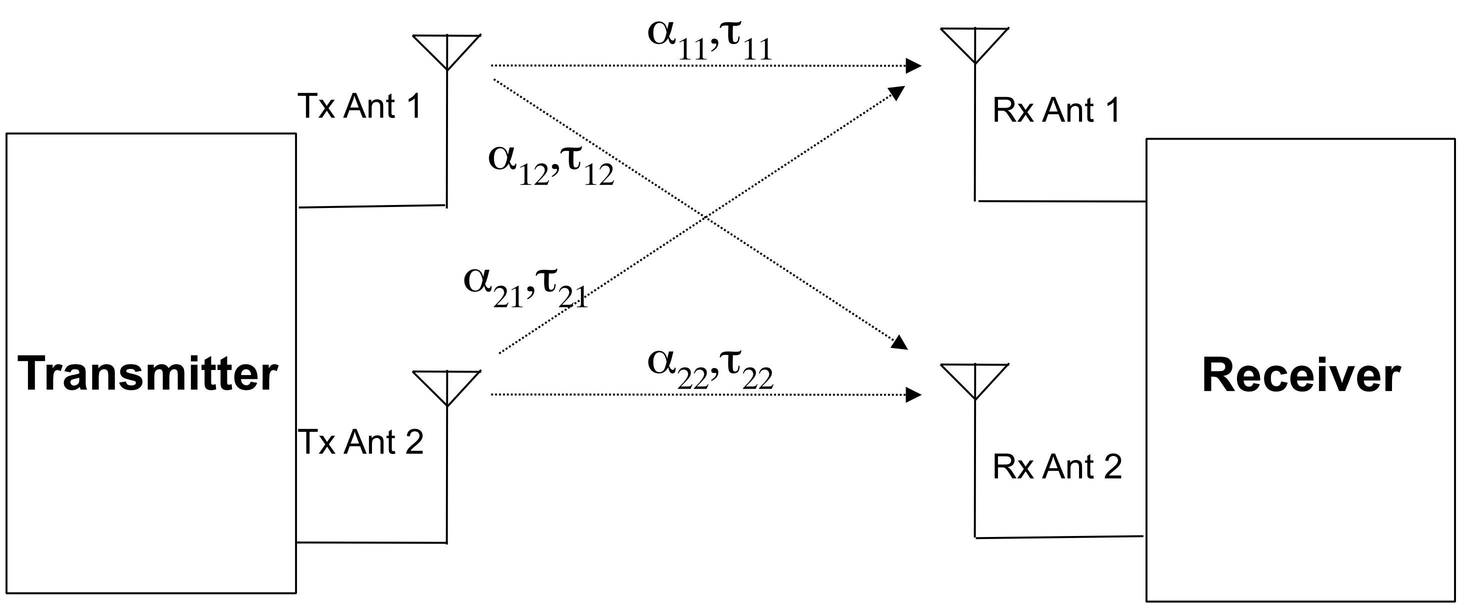
Gaussian random variables $(0; N_0/2)$, if $|C_1|^2+|C_2|^2=1$

- Same performance as maximal ratio combining D=2 !!!

Example 1: Alamouti Code (4)



Example 2: A 2x2 MIMO system



Example 2: A 2x2 MIMO system (2)

- Received symbols

$$Y_1 = S_1 H_{11} + S_2 H_{21} + N_1$$

$$Y_2 = S_1 H_{12} + S_2 H_{22} + N_2$$

- Vector channel:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

H_{ij} : Channel gain from Tx Ant. i to Rx Ant. j

N_i : AWGN sample from receiver j

Example 2: A 2x2 MIMO system (3)

- Using channel estimates $\underline{H}_{ij}$, apply inverse of channel matrix

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} \underline{H}_{11} & \underline{H}_{21} \\ \underline{H}_{12} & \underline{H}_{22} \end{bmatrix}^{-1} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

$$\approx \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \underline{H}_{11} & \underline{H}_{21} \\ \underline{H}_{12} & \underline{H}_{22} \end{bmatrix}^{-1} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}}_{\text{Gaussian samples}}$$

Example 2: A 2x2 MIMO system (4)

- Computations:

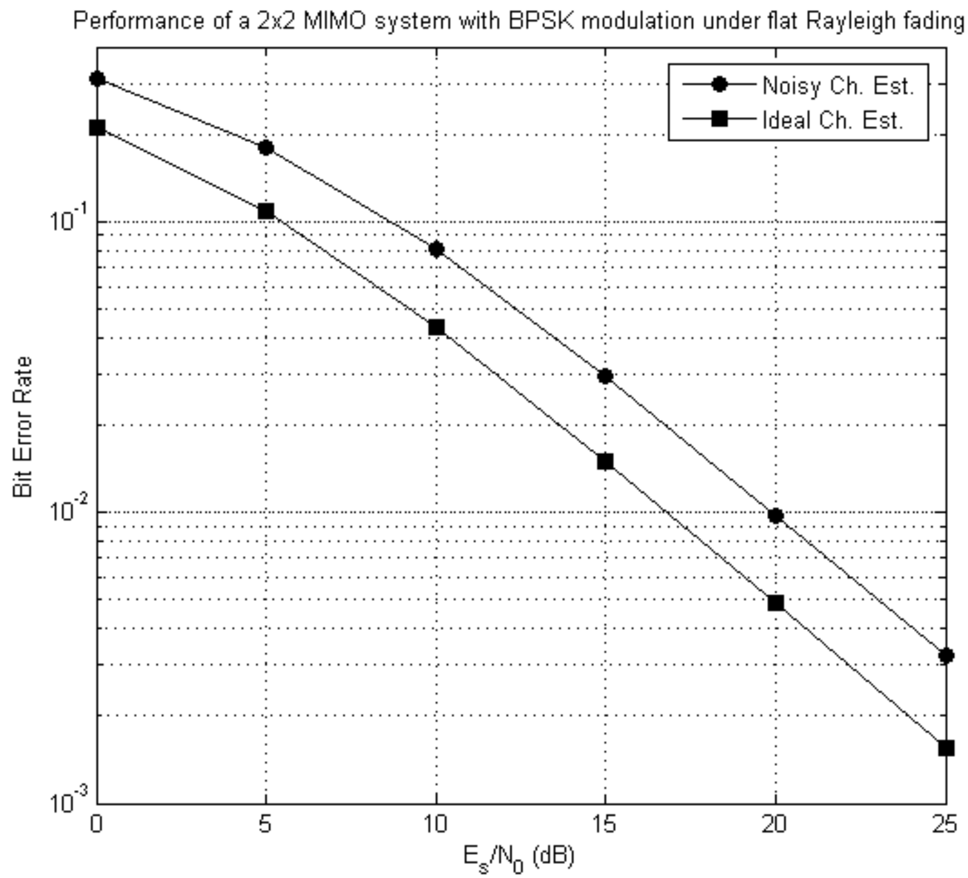
$$S_1 = \frac{\underline{H}_{22} Y_1 - \underline{H}_{21} Y_2}{\Delta}, \quad S_2 = \frac{-\underline{H}_{12} Y_1 + \underline{H}_{11} Y_2}{\Delta}$$

$$\Delta = \underline{H}_{11}\underline{H}_{22} - \underline{H}_{12}\underline{H}_{21}$$

- Noisy estimates modeled as Gaussians:

$$\underline{H}_{ij} = H_{ij} + N_{ij}, \;\; N_{ij}: \;\; \text{AWGN sample}$$

Example 2: A 2x2 MIMO system (5)



A 2x2 MIMO scheme based on SVD (beamforming)

- Similar to the idea of OFDM with a cyclic prefix, use singular-value decomposition of the channel matrix

$$H = \mathbf{P}\Lambda\mathbf{R}^{T}$$

- Transmitter multiplies by $\mathbf{P}^{-1}$ and sends

$$X = S\mathbf{P}^{-1}$$

- Receiver multiplies by $(\mathbf{R}^{T})^{-1}$ to obtain

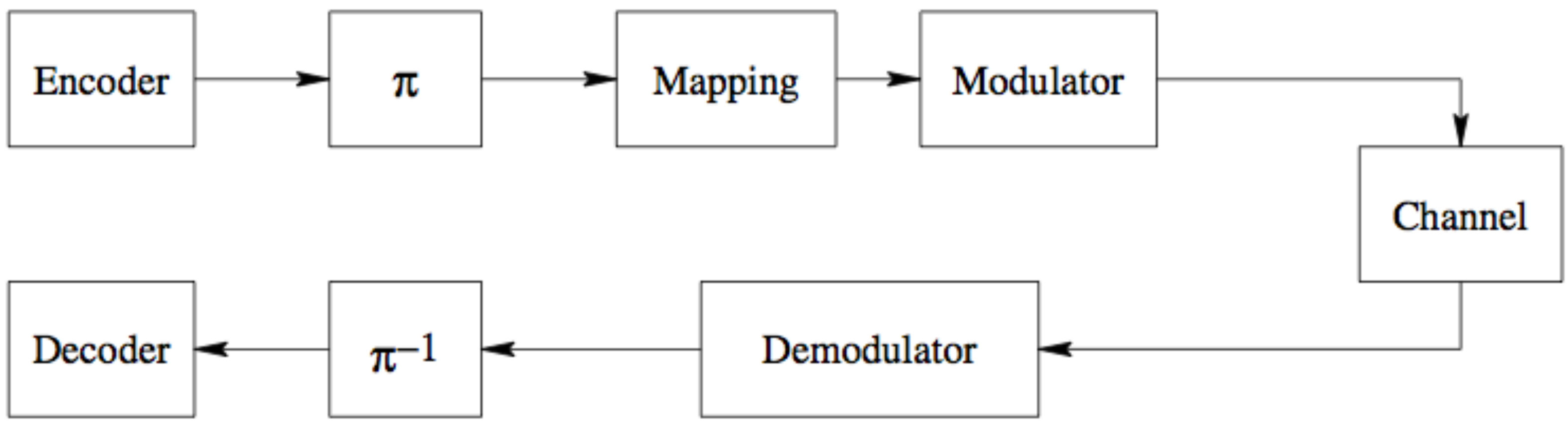
$$Y = R\mathbf{(R}^{T}\mathbf{)}^{-1} = (XH + N)\mathbf{(R}^{T}\mathbf{)}^{-1} = \Lambda S + N(\mathbf{R}^{T})^{-1}$$

- N parallel channels are obtained !!

Coding and Modulation in IEEE 802 Wireless Network Standards

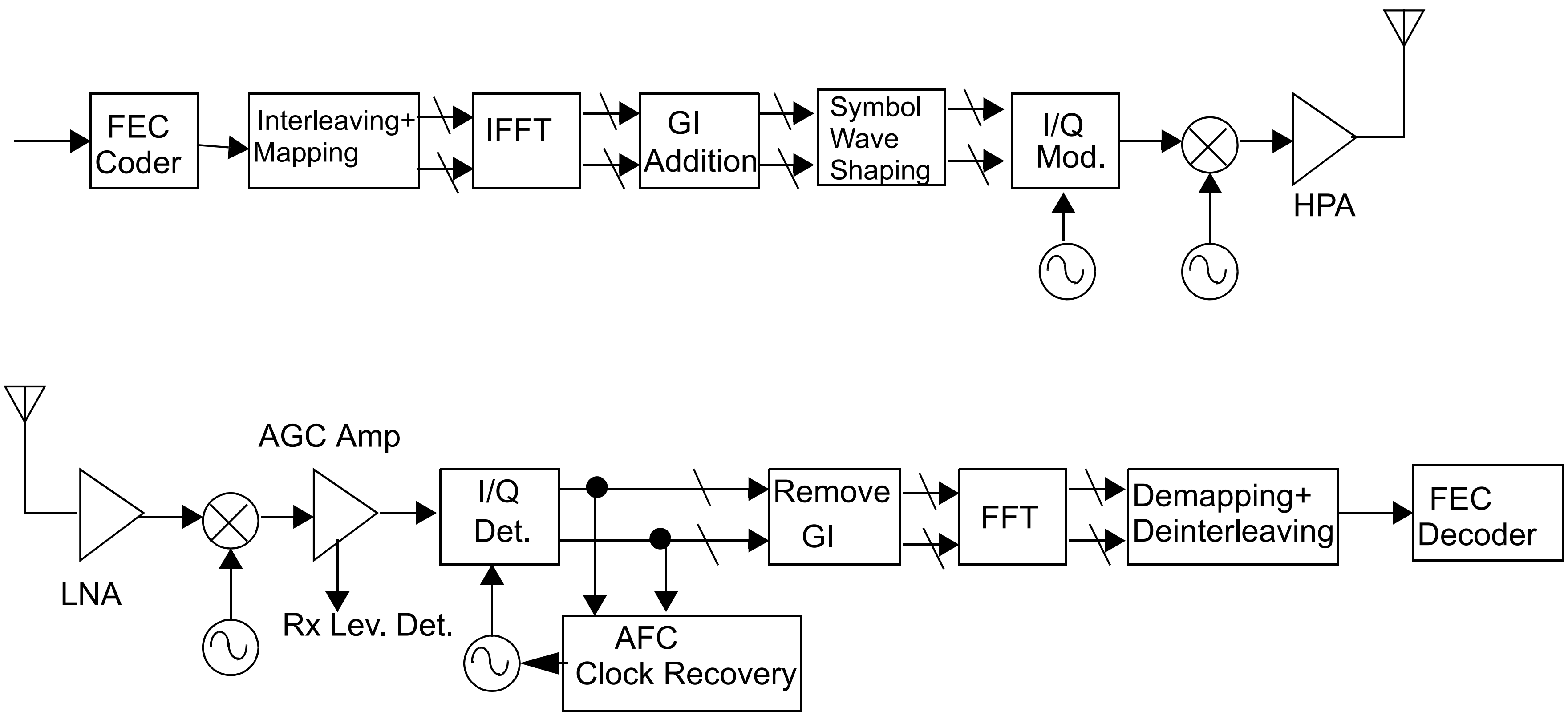
San José State University

Bit-interleaved coded modulation

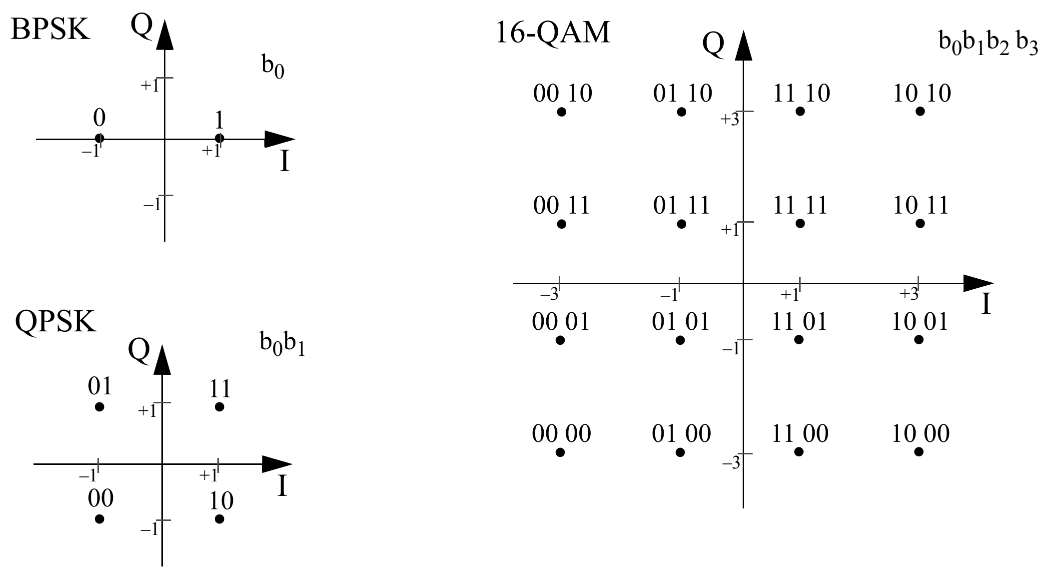


- ***Gray mapping*** of bits to *modulation symbols*
- Demapping to produce *binary metrics (LLR values)*
- Practically all IEEE 802 standards use it

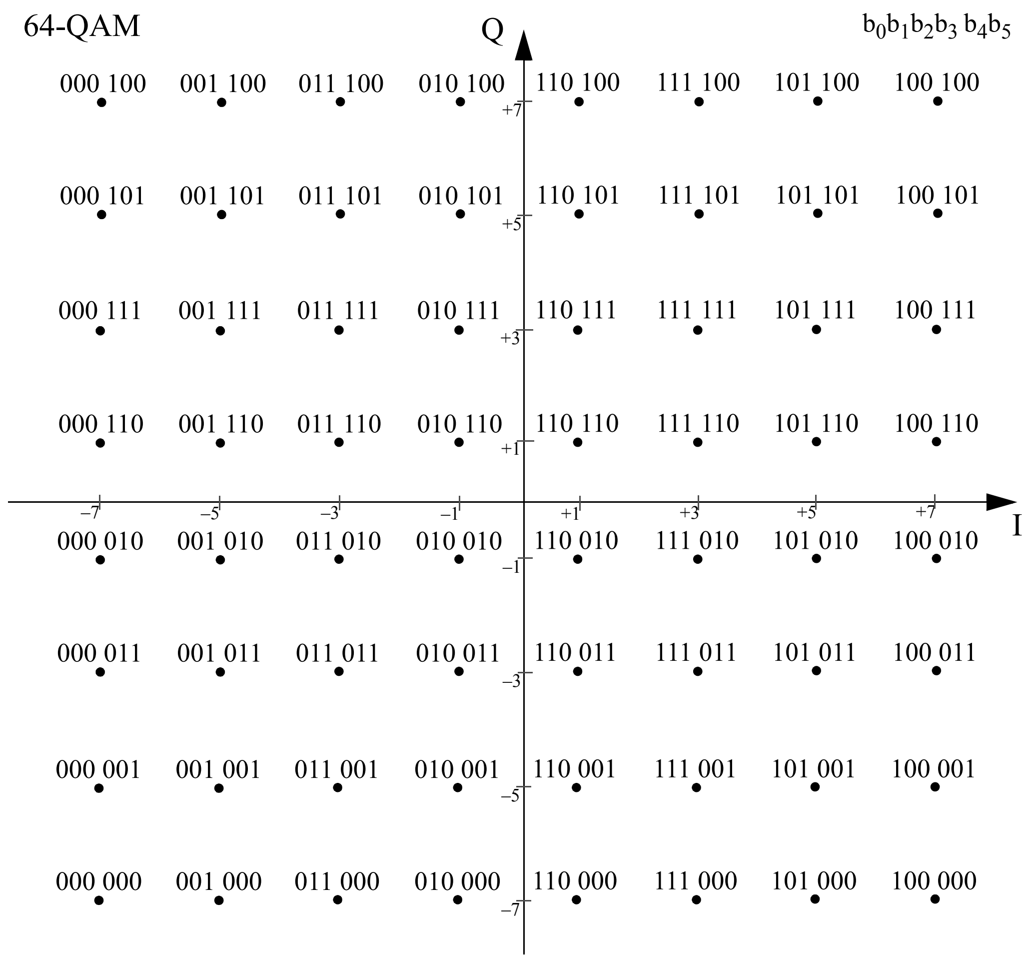
IEEE 802.11-2012



IEEE 802.11-2012: Constellations (1)



IEEE 802.11-2012: Constellations (2)



IEEE 802.11-2012: OFDM and Rates

Table 17-3—Modulation-dependent parameters

Modulation	Coding rate (R)	Coded bits per subcarrier (N_{BPSC})	Coded bits per OFDM symbol (N_{CBPS})	Data bits per OFDM symbol (N_{DBPS})	Data rate (Mb/s) (20 MHz channel spacing)	Data rate (Mb/s) (10 MHz channel spacing)	Data rate (Mb/s) (5 MHz channel spacing)
BPSK	1/2	1	48	24	6	3	1.5
BPSK	3/4	1	48	36	9	4.5	2.25
QPSK	1/2	2	96	48	12	6	3
QPSK	3/4	2	96	72	18	9	4.5
16-QAM	1/2	4	192	96	24	12	6
16-QAM	3/4	4	192	144	36	18	9
64-QAM	2/3	6	288	192	48	24	12
64-QAM	3/4	6	288	216	54	27	13.5

Table F-1—Matrix prototypes for codeword block length n=648 bits, subblock size is Z = 27 bits

Permutation matrix

Parity-check matrices

(a) Coding rate R = 1/2.

0	-	-	-	0	0	-	-	0	-	-	0	1	0	-	-	-	-	-	-	-	-	-	-
22	0	-	-	17	-	0	0	12	-	-	-	-	0	0	-	-	-	-	-	-	-	-	-
6	-	0	-	10	-	-	-	24	-	0	-	-	-	0	0	-	-	-	-	-	-	-	-
2	-	-	0	20	-	-	-	25	0	-	-	-	-	-	0	0	-	-	-	-	-	-	-
23	-	-	-	3	-	-	-	0	-	9	11	-	-	-	-	0	0	-	-	-	-	-	-
24	-	23	1	17	-	3	-	10	-	-	-	-	-	-	-	-	0	0	-	-	-	-	-
25	-	-	-	8	-	-	-	7	18	-	-	0	-	-	-	-	-	0	0	-	-	-	-
13	24	-	-	0	-	8	-	6	-	-	-	-	-	-	-	-	-	-	0	0	-	-	-
7	20	-	16	22	10	-	-	23	-	-	-	-	-	-	-	-	-	-	-	0	0	-	-
11	-	-	-	19	-	-	-	13	-	3	17	-	-	-	-	-	-	-	-	-	0	0	-
25	-	8	-	23	18	-	14	9	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
3	-	-	-	16	-	-	2	25	5	-	-	1	-	-	-	-	-	-	-	-	-	-	0

(b) Coding rate R = 2/3.

25	26	14	-	20	-	2	-	4	-	-	8	-	16	-	18	1	0	-	-	-	-	-	-
10	9	15	11	-	0	-	1	-	-	18	-	8	-	10	-	-	0	0	-	-	-	-	-
16	2	20	26	21	-	6	-	1	26	-	7	-	-	-	-	-	-	0	0	-	-	-	-
10	13	5	0	-	3	-	7	-	-	26	-	-	13	-	16	-	-	-	0	0	-	-	-
23	14	24	-	12	-	19	-	17	-	-	-	20	-	21	-	0	-	-	-	0	0	-	-
6	22	9	20	-	25	-	17	-	8	-	14	-	18	-	-	-	-	-	-	-	0	0	-
14	23	21	11	20	-	24	-	18	-	19	-	-	-	-	22	-	-	-	-	-	-	0	0
17	11	11	20	-	21	-	26	-	3	-	-	18	-	26	-	1	-	-	-	-	-	-	0

(c) Coding rate R = 3/4.

16	17	22	24	9	3	14	-	4	2	7	-	26	-	2	-	21	-	1	0	-	-	-	-
25	12	12	3	3	26	6	21	-	15	22	-	15	-	4	-	-	16	-	0	0	-	-	-
25	18	26	16	22	23	9	-	0	-	4	-	4	-	8	23	11	-	-	-	0	0	-	-
9	7	0	1	17	-	-	7	3	-	3	23	-	16	-	-	21	-	0	-	-	0	0	-
24	5	26	7	1	-	-	15	24	15	-	8	-	13	-	13	-	11	-	-	-	-	0	0
2	2	19	14	24	1	15	19	-	21	-	2	-	24	-	3	-	2	1	-	-	-	-	0

(d) Coding rate R = 5/6.

17	13	8	21	9	3	18	12	10	0	4	15	19	2	5	10	26	19	13	13	1	0	-	-
3	12	11	14	11	25	5	18	0	9	2	26	26	10	24	7	14	20	4	2	-	0	0	-
22	16	4	3	10	21	12	5	21	14	19	5	-	8	5	18	11	5	5	15	0	-	0	0
7	7	14	14	4	16	16	24	24	10	1	7	15	6	10	26	8	18	21	14	1	-	-	0

Permutation matrix example

- Z=27, p=22

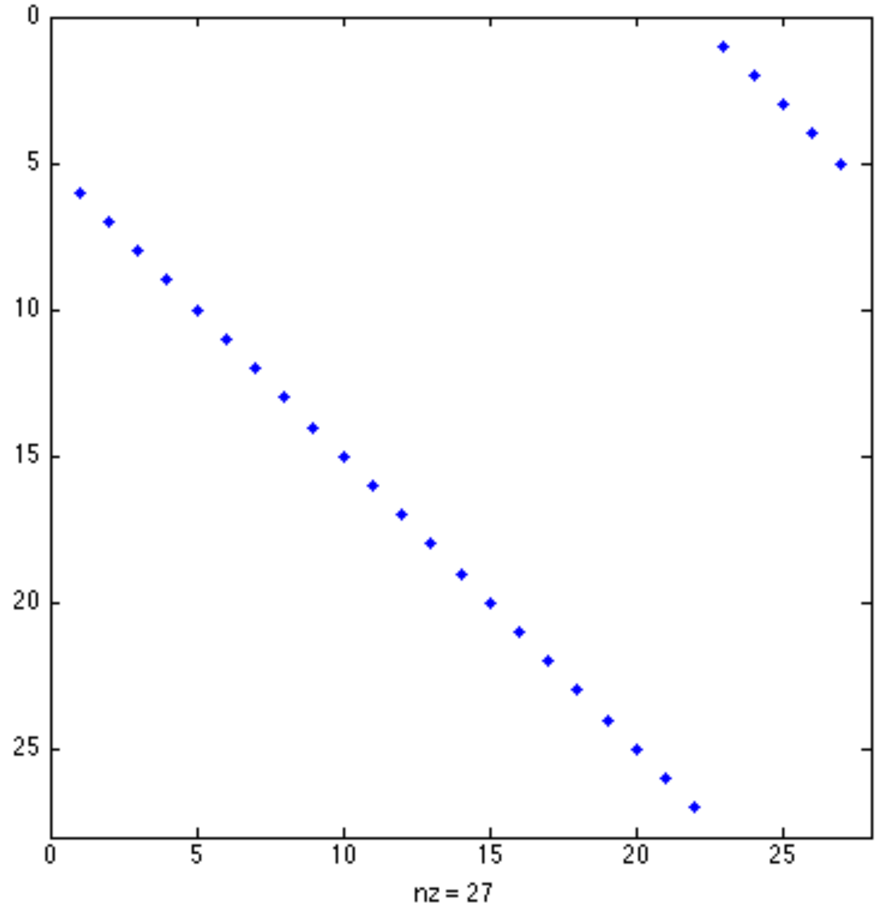


Table F-2—Matrix prototypes for codeword block length *n*=1296 bits, subblock size is *Z*= 54 bits

(a) Coding rate R = 1/2.

40	-	-	-	22	-	49	23	43	-	-	-	1	0	-	-	-	-	-	-	-	-	-	-
50	1	-	-	48	35	-	-	13	-	30	-	-	0	0	-	-	-	-	-	-	-	-	-
39	50	-	-	4	-	2	-	-	-	-	49	-	-	0	0	-	-	-	-	-	-	-	-
33	-	-	38	37	-	-	4	1	-	-	-	-	-	-	0	0	-	-	-	-	-	-	-
45	-	-	-	0	22	-	-	20	42	-	-	-	-	-	-	0	0	-	-	-	-	-	-
51	-	-	48	35	-	-	-	44	-	18	-	-	-	-	-	-	0	0	-	-	-	-	-
47	11	-	-	-	17	-	-	51	-	-	-	0	-	-	-	-	-	0	0	-	-	-	-
5	-	25	-	6	-	45	-	13	40	-	-	-	-	-	-	-	-	-	0	0	-	-	-
33	-	-	34	24	-	-	-	23	-	-	46	-	-	-	-	-	-	-	-	0	0	-	-
1	-	27	-	1	-	-	-	38	-	44	-	-	-	-	-	-	-	-	-	-	0	0	-
-	18	-	-	23	-	-	8	0	35	-	-	-	-	-	-	-	-	-	-	-	-	0	0
49	-	17	-	30	-	-	-	34	-	-	19	1	-	-	-	-	-	-	-	-	-	-	0

(b) Coding rate R = 2/3.

39	31	22	43	-	40	4	-	11	-	-	50	-	-	-	6	1	0	-	-	-	-	-	-
25	52	41	2	6	-	14	-	34	-	-	-	24	-	37	-	-	0	0	-	-	-	-	-
43	31	29	0	21	-	28	-	-	2	-	-	7	-	17	-	-	-	0	0	-	-	-	-
20	33	48	-	4	13	-	26	-	-	22	-	-	46	42	-	-	-	-	0	0	-	-	-
45	7	18	51	12	25	-	-	-	50	-	-	5	-	-	-	0	-	-	-	0	0	-	-
35	40	32	16	5	-	-	18	-	-	43	51	-	32	-	-	-	-	-	-	-	0	0	-
9	24	13	22	28	-	-	37	-	-	25	-	-	52	-	13	-	-	-	-	-	-	0	0
32	22	4	21	16	-	-	-	27	28	-	38	-	-	-	8	1	-	-	-	-	-	-	0

(c) Coding rate R = 3/4.

39	40	51	41	3	29	8	36	-	14	-	6	-	33	-	11	-	4	1	0	-	-	-	-
48	21	47	9	48	35	51	-	38	-	28	-	34	-	50	-	50	-	-	0	0	-	-	-
30	39	28	42	50	39	5	17	-	6	-	18	-	20	-	15	-	40	-	-	0	0	-	-
29	0	1	43	36	30	47	-	49	-	47	-	3	-	35	-	34	-	0	-	-	0	0	-
1	32	11	23	10	44	12	7	-	48	-	4	-	9	-	17	-	16	-	-	-	-	0	0
13	7	15	47	23	16	47	-	43	-	29	-	52	-	2	-	53	-	1	-	-	-	-	0

(d) Coding rate R = 5/6.

48	29	37	52	2	16	6	14	53	31	34	5	18	42	53	31	45	-	46	52	1	0	-	-
17	4	30	7	43	11	24	6	14	21	6	39	17	40	47	7	15	41	19	-	-	0	0	-
7	2	51	31	46	23	16	11	53	40	10	7	46	53	33	35	-	25	35	38	0	-	0	0
19	48	41	1	10	7	36	47	5	29	52	52	31	10	26	6	3	2	-	51	1	-	-	0

Table F-3—Matrix prototypes for codeword block length *n*=1944 bits, subblock size is *Z* = 81 bits

(a) Coding rate R = 1/2.

57	-	-	-	50	-	11	-	50	-	79	-	1	0	-	-	-	-	-	-	-	-	-	-
3	-	28	-	0	-	-	-	55	7	-	-	-	0	0	-	-	-	-	-	-	-	-	-
30	-	-	-	24	37	-	-	56	14	-	-	-	-	0	0	-	-	-	-	-	-	-	-
62	53	-	-	53	-	-	3	35	-	-	-	-	-	-	0	0	-	-	-	-	-	-	-
40	-	-	20	66	-	-	22	28	-	-	-	-	-	-	-	0	0	-	-	-	-	-	-
0	-	-	-	8	-	42	-	50	-	-	8	-	-	-	-	-	0	0	-	-	-	-	-
69	79	79	-	-	-	56	-	52	-	-	-	0	-	-	-	-	-	0	0	-	-	-	-
65	-	-	-	38	57	-	-	72	-	27	-	-	-	-	-	-	-	-	0	0	-	-	-
64	-	-	-	14	52	-	-	30	-	-	32	-	-	-	-	-	-	-	-	0	0	-	-
-	45	-	70	0	-	-	-	77	9	-	-	-	-	-	-	-	-	-	-	-	0	0	-
2	56	-	57	35	-	-	-	-	-	12	-	-	-	-	-	-	-	-	-	-	-	0	0
24	-	61	-	60	-	-	27	51	-	-	16	1	-	-	-	-	-	-	-	-	-	-	0

(b) Coding rate R = 2/3.

61	75	4	63	56	-	-	-	-	-	-	8	-	2	17	25	1	0	-	-	-	-	-	-
56	74	77	20	-	-	-	64	24	4	67	-	7	-	-	-	-	0	0	-	-	-	-	-
28	21	68	10	7	14	65	-	-	-	23	-	-	-	75	-	-	-	0	0	-	-	-	-
48	38	43	78	76	-	-	-	-	5	36	-	15	72	-	-	-	-	-	0	0	-	-	-
40	2	53	25	-	52	62	-	20	-	-	44	-	-	-	-	0	-	-	-	0	0	-	-
69	23	64	10	22	-	21	-	-	-	-	-	68	23	29	-	-	-	-	-	-	0	0	-
12	0	68	20	55	61	-	40	-	-	-	52	-	-	-	44	-	-	-	-	-	-	0	0
58	8	34	64	78	-	-	11	78	24	-	-	-	-	-	58	1	-	-	-	-	-	-	0

(c) Coding rate R = 3/4.

48	29	28	39	9	61	-	-	-	63	45	80	-	-	-	37	32	22	1	0	-	-	-	-
4	49	42	48	11	30	-	-	-	49	17	41	37	15	-	54	-	-	-	0	0	-	-	-
35	76	78	51	37	35	21	-	17	64	-	-	-	59	7	-	-	32	-	-	0	0	-	-
9	65	44	9	54	56	73	34	42	-	-	-	35	-	-	-	46	39	0	-	-	0	0	-
3	62	7	80	68	26	-	80	55	-	36	-	26	-	9	-	72	-	-	-	-	-	0	0
26	75	33	21	69	59	3	38	-	-	-	35	-	62	36	26	-	-	1	-	-	-	-	0

(d) Coding rate R = 5/6.

13	48	80	66	4	74	7	30	76	52	37	60	-	49	73	31	74	73	23	-	1	0	-	-
69	63	74	56	64	77	57	65	6	16	51	-	64	-	68	9	48	62	54	27	-	0	0	-
51	15	0	80	24	25	42	54	44	71	71	9	67	35	-	58	-	29	-	53	0	-	0	0
16	29	36	41	44	56	59	37	50	24	-	65	4	65	52	-	4	-	73	52	1	-	-	0

The parity-check matrix interpreted as the incidence matrix of a graph

<u>Example</u>: Hamming (7,4,3) code

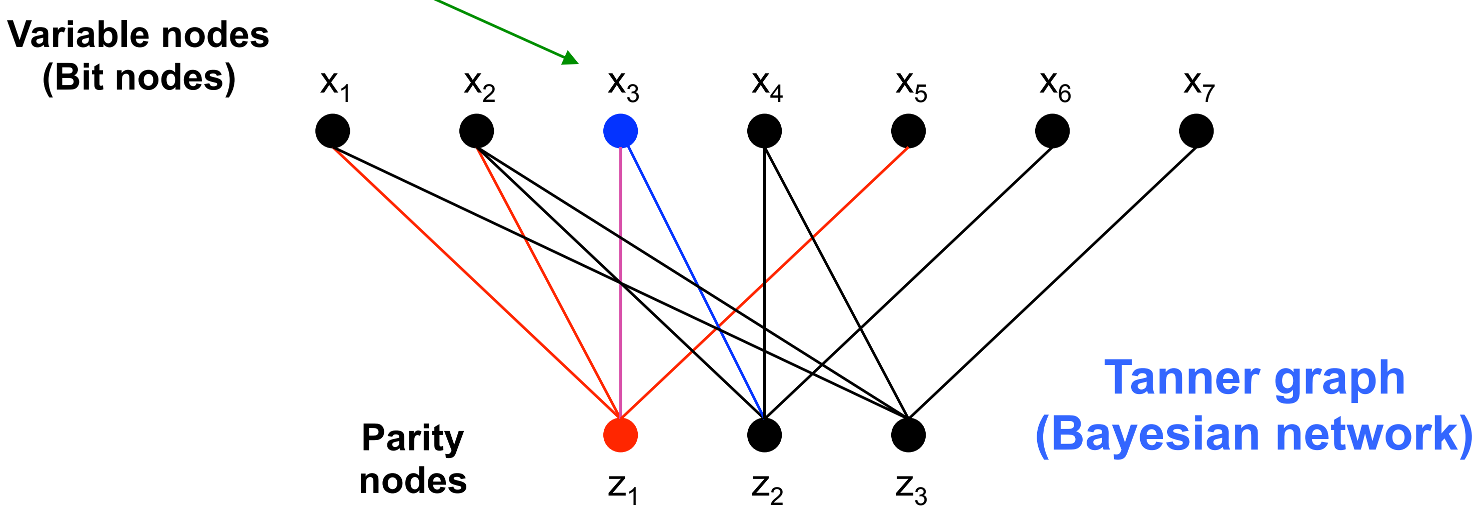
Parity-check matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$

Parity-check equations (syndromes)

$$z_1 = x_1 + x_2 + x_3 + x_5$$
$$z_2 = x_2 + x_3 + x_4 + x_6$$
$$z_3 = x_1 + x_2 + x_4 + x_7$$

Incidence matrix



Iterative decoding of LDPC codes using Tanner graph

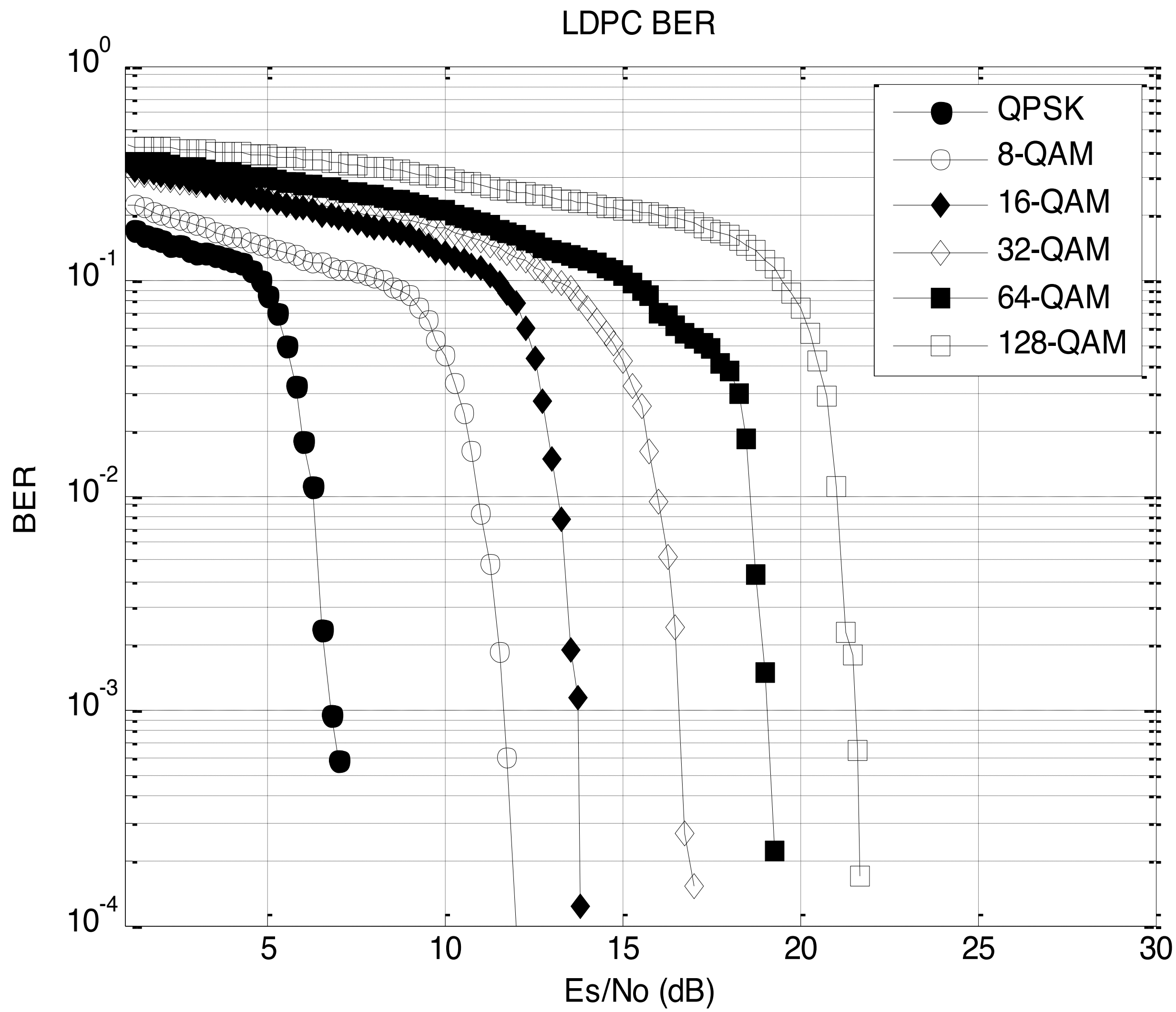
- <u>Hard-decision</u>

  Preliminary ("hard") decisions: ***bit-flip***

- <u>Soft-decision</u>

  Channel outputs (matched filter): ***belief propagation***

More constellations: Illustration using binary (273,191,17) EG code



Add 8-, 32- and 128-QAM to the mix !!

IEEE 802.11n-2009: Code rates

Table 20-21—Allowed relative constellation error versus constellation size and coding rate

Modulation	Coding rate	Relative constellation error (dB)
BPSK	1/2	–5
QPSK	1/2	–10
QPSK	3/4	–13
16-QAM	1/2	–16
16-QAM	3/4	–19
64-QAM	2/3	–22
64-QAM	3/4	–25
64-QAM	5/6	–28

IEEE 802.11n: Space-Time Block Coding

Table 20-17—Constellation mapper output to spatial mapper input for STBC

N_{STS}	HT-SIG MCS field (bits 0–6 in HT-SIG$_1$)	N_{SS}	HT-SIG STBC field (bits 4–5 in HT-SIG$_2$)	i_{STS}	$\tilde{d}_{k,i,2m}$	$\tilde{d}_{k,i,2m+1}$
2	0–7	1	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
3	8–15, 33–38	2	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
4	8–15	2	2	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
				4	$-d^*_{k,2,2m+1}$	$d^*_{k,2,2m}$
4	16–23, 39, 41, 43, 46, 48, 50	3	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
				4	$d_{k,3,2m}$	$d_{k,3,2m+1}$

802.11ad-2012: OFDM

Table 21-14—Modulation and coding scheme for OFDM

MCS index	Modulation	Code rate	N_{BPSC}	N_{CBPS}	N_{DBPS}	Data rate (Mbps)
13	SQPSK	1/2	1	336	168	693.00
14	SQPSK	5/8	1	336	210	866.25
15	QPSK	1/2	2	672	336	1386.00
16	QPSK	5/8	2	672	420	1732.50
17	QPSK	3/4	2	672	504	2079.00
18	16-QAM	1/2	4	1344	672	2772.00
19	16-QAM	5/8	4	1344	840	3465.00
20	16-QAM	3/4	4	1344	1008	4158.00
21	16-QAM	13/16	4	1344	1092	4504.50
22	64-QAM	5/8	6	2016	1260	5197.50
23	64-QAM	3/4	6	2016	1512	6237.00
24	64-QAM	13/16	6	2016	1638	6756.75

802.11ad-2012: LDPC codes (1)

- **Rate 1/2:**

N = 16x42 = 672
N-K = 8x42 = 336

K = 336

21.3.8.2 Rate-1/2 LDPC code matrix H = 336 rows x 672 columns, Z = 42

Table 21-6—Rate 1/2 LDPC code matrix
(Each nonblank element i in the table is the cyclic permutation matrix P_i of size $Z \times Z$; blank entries represent the zero matrix of size $Z \times Z$)

40		38		13		5		18							
34		35		27			30	2	1						
	36		31		7		34		10	41					
	27		18		12	20				15	6				
35		41		40		39		28			3	28			
29		0			22		4		28		27		23		
	31		23		21		20			12			0	13	
	22		34	31		14		4				13		22	24

802.11ad-2012: LDPC codes (2)

- **Rate 5/8:**

N = 16x42 = 672
N-K = 6x42 = 252

K = 420

21.3.8.3 Rate-5/8 LDPC code matrix H = 252 rows x 672 columns, Z = 42

Table 21-7—Rate 5/8 LDPC code matrix
(Each nonblank element i in the table is the cyclic permutation matrix P_i of size $Z \times Z$; blank entries represent the zero matrix of size $Z \times Z$)

20	36	34	31	20	7	41	34		10	41					
30	27		18		12	20	14	2	25	15	6				
35		41		40		39		28			3	28			
29		0			22		4		28		27	24	23		
	31		23		21		20		9	12			0	13	
	22		34	31		14		4						22	24

802.11ad-2012: LDPC codes (3)

- **Rate 3/4:**

N = 16x42 = 672
N-K = 4x42 = 168

K = 504

21.3.8.4 Rate-3/4 LDPC code matrix H = 168 rows x 672 columns, Z = 42

Table 21-8—Rate 3/4 LPDC code matrix
(Each nonblank element i in the table is the cyclic permutation matrix P_i of size $Z \times Z$; blank entries represent the zero matrix of size $Z \times Z$)

35	19	41	22	40	41	39	6	28	18	17	3	28			
29	30	0	8	33	22	17	4	27	28	20	27	24	23		
37	31	18	23	11	21	6	20	32	9	12	29		0	13	
25	22	4	34	31	3	14	15	4		14	18	13	13	22	24

802.11ad-2012: LDPC codes (4)

- **Rate 13/16:**

N = 16x42 = 672
N-K = 3x42 = 126

K = 546

21.3.8.5 Rate-13/16 LDPC code matrix H = 126 rows x 672 columns, Z = 42

Table 21-9—Rate 13/16 LDPC code matrix
(Each nonblank element i in the table is the cyclic permutation matrix P_i of size $Z \times Z$; blank entries represent the zero matrix of size $Z \times Z$)

29	30	0	8	33	22	17	4	27	28	20	27	24	23		
37	31	18	23	11	21	6	20	32	9	12	29	10	0	13	
25	22	4	34	31	3	14	15	4	2	14	18	13	13	22	24

802.11ad-2012: Single-Carrier (1)

Table 21-18—Modulation and coding scheme for SC

MCS index	Modulation	N_{CBPS}	Repetition	Code rate	Data rate (Mbps)
1	π/2-BPSK	1	2	1/2	385
2	π/2-BPSK	1	1	1/2	770
3	π/2-BPSK	1	1	5/8	962.5
4	π/2-BPSK	1	1	3/4	1155
5	π/2-BPSK	1	1	13/16	1251.25
6	π/2-QPSK	2	1	1/2	1540
7	π/2-QPSK	2	1	5/8	1925
8	π/2-QPSK	2	1	3/4	2310
9	π/2-QPSK	2	1	13/16	2502.5
10	π/2-16QAM	4	1	1/2	3080
11	π/2-16QAM	4	1	5/8	3850
12	π/2-16QAM	4	1	3/4	4620

802.11ad-2012: Single-Carrier (2)

Table 21-19—LDPC code rates

Code rate	Codeword size	Number of data bits
1/2	672	336
5/8	672	420
3/4	672	504
13/16	672	546

Table 21-22—Low-power SC modulation and coding schemes

MCS	Modulation	Effective code rate	Coding scheme	N_{CPB}	Rate (Mbps)
25	π/2-BPSK	13/28	RS(224,208)+Block-Code(16,8)	392	626
26	π/2-BPSK	13/21	RS(224,208)+Block-Code(12,8)	392	834
27	π/2-BPSK	52/63	RS(224,208)+SPC(9,8)	392	1112
28	π/2-QPSK	13/28	RS(224,208)+Block-Code(16,8)	392	1251
29	π/2-QPSK	13/21	RS(224,208)+Block-Code(12,8)	392	1668
30	π/2-QPSK	52/63	RS(224,208)+SPC(9,8)	392	2224
31	π/2-QPSK	13/14	RS(224,208)+Block-Code(8,8)	392	2503

IEEE 802.15.3c-2009: Code Rates

Table 103—MCS dependent parameters

MCS class	MCS identifier	Data rate (Mb/s) with pilot word length = 0	Data rate (Mb/s) with pilot word length = 64	Modulation	Spreading factor, L_{SF}	FEC type
Class1	0	25.8 (CMS)	—	π/2 BPSK/(G)MSK[a]	64	RS(255,239)
	1	412	361		4	
	2	825	722		2	
	3	1650 (MPR)	1440		1	
	4	1320	1160	π/2 BPSK/(G)MSK	1	LDPC(672,504)
	5	440	385	π/2 BPSK/(G)MSK	2	LDPC(672,336)
	6	880	770		1	
Class2	7	1760	1540	π/2 QPSK	1	LDPC(672,336)
	8	2640	2310	π/2 QPSK	1	LDPC(672,504)
	9	3080	2700	π/2 QPSK	1	LDPC(672,588)
	10	3290	2870	π/2 QPSK	1	LDPC(1440,1344)
	11	3300	2890	π/2 QPSK	1	RS(255,239)
Class3	12	3960	3470	π/2 8-PSK	1	LDPC(672,504)
	13	5280	4620	π/2 16-QAM	1	LDPC(672,504)

IEEE 802.15.3c-2009: Unequal Error Protection !!

12.4 Audio/Visual mode of mmWave PHY

The Audio/Visual (AV) PHY is implemented with two PHY modes, the high-rate PHY (HRP) and low-rate PHY (LRP), both of which use orthogonal frequency domain multiplexing (OFDM). The data rates supported by the HRP are defined in Table 134.

Table 134—HRP data rates and coding

HRP mode index	Coding mode	Modulation	Inner code rate		Data rate (Gb/s)
			MSB [7] [6] [5] [4]	LSB [3] [2] [1] [0]	
0	EEP	QPSK	1/3		0.952
1	EEP	QPSK	2/3		1.904
2	EEP	16-QAM	2/3		3.807
3	UEP	QPSK	4/7	4/5	1.904
4	UEP	16-QAM	4/7	4/5	3.807
5	MSB-only retransmission	QPSK	1/3	N/A	0.952
6	MSB-only retransmission	QPSK	2/3	N/A	1.904

IEEE 802.15.3c-2009: Constellations

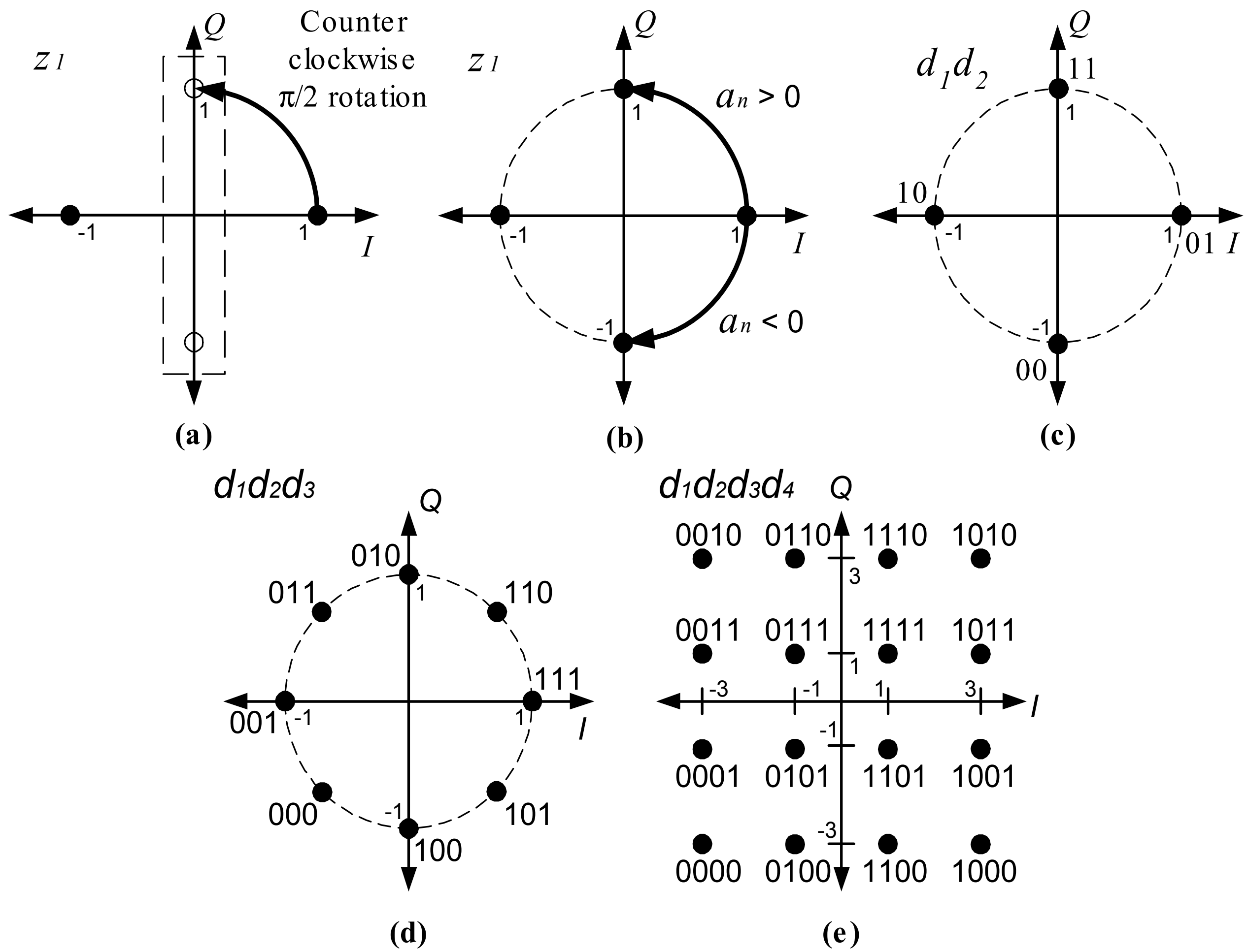


Figure 164—Constellation maps for modulations: (a) π/2 BPSK, (b) pre-coded (G)MSK, (c) π/2 QPSK, (d) π/2 8-PSK, (e) π/2 16-QAM

IEEE 802.15.7-2011: OOK and PPM (1)

Table 73—PHY I operating modes

Modulation	RLL code	Optical clock rate	FEC: Outer code (RS)	FEC: Inner code (CC)	Data rate
OOK	Manchester	200 kHz	(15,7)	1/4	11.67 kb/s
			(15,11)	1/3	24.44 kb/s
			(15,11)	2/3	48.89 kb/s
			(15,11)	none	73.3 kb/s
			none	none	100 kb/s
VPPM	4B6B	400 kHz	(15,2)	none	35.56 kb/s
			(15,4)	none	71.11 kb/s
			(15,7)	none	124.4 kb/s
			none	none	266.6 kb/s

IEEE 802.15.7-2011: OOK and PPM (2)

Table 74—PHY II operating modes

Modulation	RLL code	Optical clock rate	FEC	Data rate
VPPM	4B6B	3.75 MHz	RS(64,32)	1.25 Mb/s
			RS(160,128)	2 Mb/s
		7.5 MHz	RS(64,32)	2.5 Mb/s
			RS(160,128)	4 Mb/s
			none	5 Mb/s
OOK	8B10B	15 MHz	RS(64,32)	6 Mb/s
			RS(160,128)	9.6 Mb/s
		30 MHz	RS(64,32)	12 Mb/s
			RS(160,128)	19.2 Mb/s
		60 MHz	RS(64,32)	24 Mb/s
			RS(160,128)	38.4 Mb/s
		120 MHz	RS(64,32)	48 Mb/s
			RS(160,128)	76.8 Mb/s
			none	96 Mb/s

IEEE 802.15.7-2011: CSK (Color-Shift Keying)

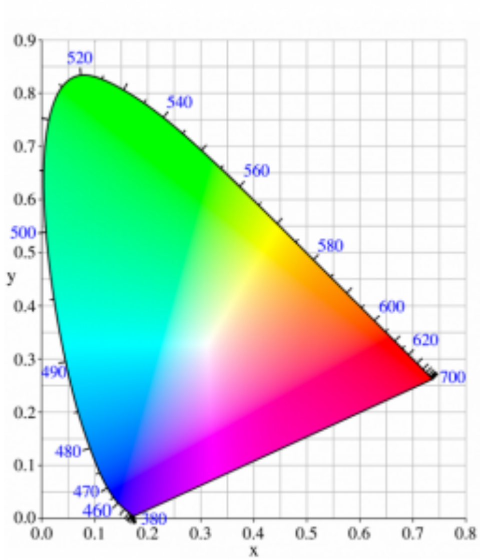
Table 75—PHY III operating modes

Modulation	Optical clock rate	FEC	Data rate
4-CSK	12 MHz	RS(64,32)	12 Mb/s
8-CSK		RS(64,32)	18 Mb/s
4-CSK	24 MHz	RS(64,32)	24 Mb/s
8-CSK		RS(64,32)	36 Mb/s
16-CSK		RS(64,32)	48 Mb/s
8-CSK		none	72 Mb/s
16-CSK		none	96 Mb/s

IEEE 802.15.7-2011: Colors

Table 106—*xy* color coordinates

Band (nm)	Code	Center (nm)	(*x*, *y*)
380–478	000	429	(0.169, 0.007)
478–540	001	509	(0.011, 0.733)
540–588	010	564	(0.402, 0.597)
588–633	011	611	(0.669, 0.331)
633–679	100	656	(0.729, 0.271)
679–726	101	703	(0.734, 0.265)
726–780	110	753	(0.734, 0.265)



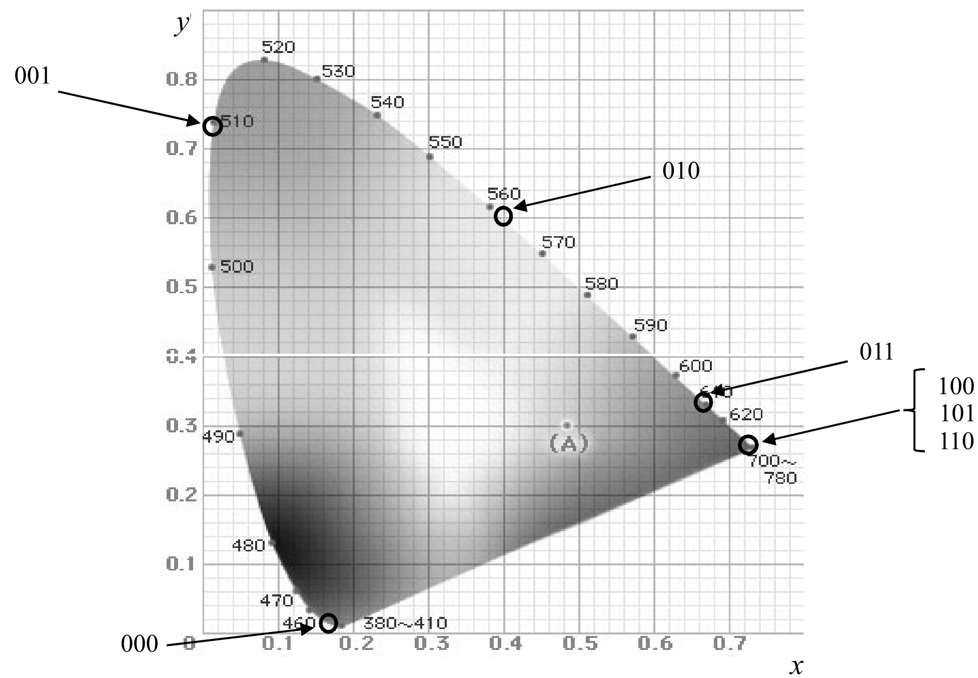


Figure 137—Center of color bands on *xy* color coordinates

IEEE 802.16-2009: Alamouti (STBC)

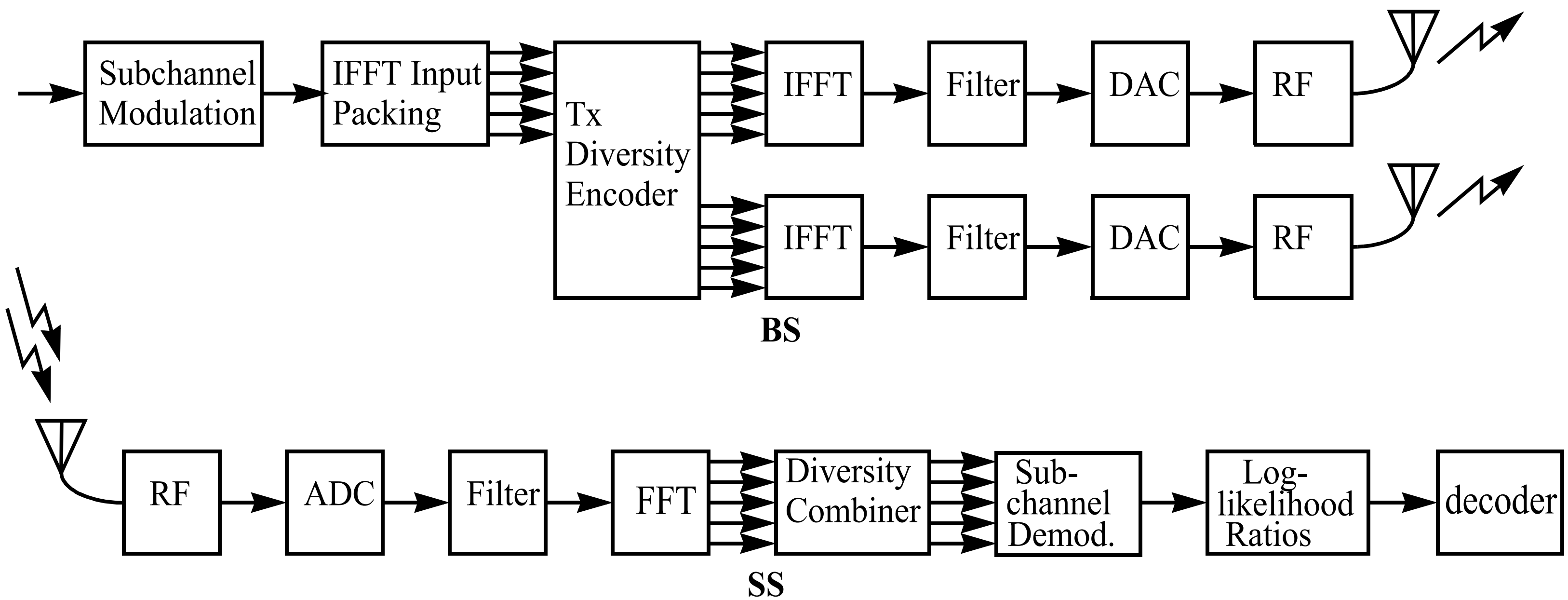


Figure 259—Illustration of STC

IEEE 802.20-2008: Modulations

Table 434—Modulation and coding rates

ModClass	Bits/Sym	Signal Set	Puncture	Shaper	Block Code
0	0.5	π/2 BPSK	Repeat	—	—
1	0.67	π/2 BPSK	1 of 4	—	—
2	1	QPSK	—	—	—
3	1.5	QPSK	2 of 6	—	—
4	2	8-PSK	—	—	(64,57)
5	2.5	8-PSK	—	—	(64,57)
6	3	12-QAM	2 of 6	3/4	(48,47)
7	3.5	16-QAM	2 of 6	4/4	(64,63)
8	4	24-QAM	2 of 6	5/4	(80,79)
9	4.5	32-QAM	2 of 6	5/5	(80,79)
10	5.5	64-QAM	2 of 5	6/6	(80,79)
11–15	RESERVED				

References

[1] IEEE 802 Part 3: Carrier sense multiple access with Collision Detection (CSMA/CD) Access Method and Physical Layer Specifications, IEEE Computer Society, 2008.

[2] IEEE 802 Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, IEEE Computer Society, 2007.

[3] IEEE 802 Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications— Amendment 5: Enhancements for Higher Throughput, IEEE Computer Society, 2009.

[4] IEEE 802 Part 15.3: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for High Rate Wireless Personal Area Networks (WPANs) – Amendment 2: Millimeter-wave-based Alternative Physical Layer Extension, 2009

[5] IEEE 802 Part 15.7: Short-Range Wireless Optical Communication Using Visible Light, IEEE Computer Society, 2011.

[6] IEEE 802 Part 16: Air Interface for Broadband Wireless Access Systems, IEEE Computer Society, 2009.

[7] IEEE 802 Part 20: Air Interface for Mobile Broadband Wireless Access Systems Supporting Vehicular Mobility — Physical and Media Access Control Layer Specification, IEEE Computer Society, 2008.

[8] IEEE 802 Part 22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Policies and Procedures for Operation in the TV Bands, IEEE Computer Society, 2011.

Support slides

Low-density parity-check (LDPC) codes

Recall the following facts about linear block codes:

- ✓ **C**: A linear block (n,k,d_{min}) code
- ✓ **G**: Generator matrix of code **C**, using k basis vectors $\mathbf{g_1}, \mathbf{g_2}, \ldots, \mathbf{g_k}$ as rows such that

$$\bar{\mathbf{v}} = \bar{\mathbf{u}}\,\mathbf{G}$$

  where $\bar{\mathbf{u}}$ is an information bit vector of length k, and $\bar{\mathbf{v}}$ is a codeword of length n

- ✓ **H:** Parity-check matrix of the code, using n-k basis vectors in the null (dual) space of **C**, such that

$$\mathbf{G}\mathbf{H}^{\mathbf{T}} = \mathbf{0}_{k\mathrm{x}(n\text{-}k)}$$

  In particular,

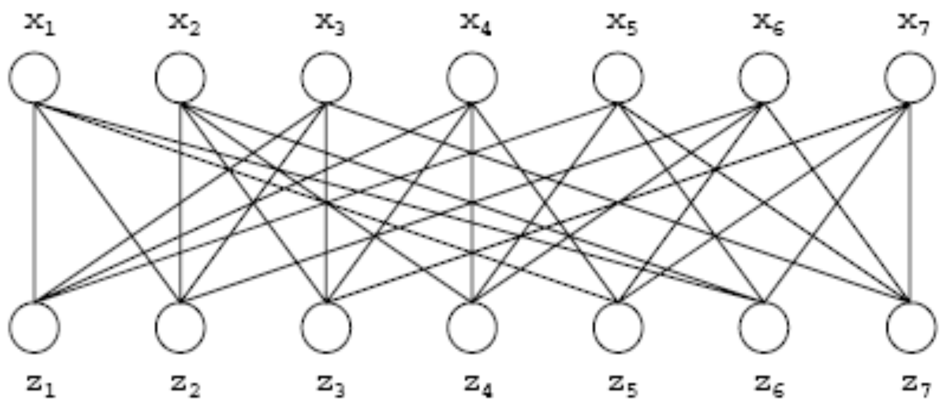
$$\bar{\mathbf{v}}\,\mathbf{H}^{\mathbf{T}} = \mathbf{0}_{1\mathrm{x}(n\text{-}k)}$$

The extended parity-check matrix (Lin, 1999)

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Rows 4 to 7 are the linear combinations of the first three rows

This matrix generates the same null (dual) space: rank(H)=3

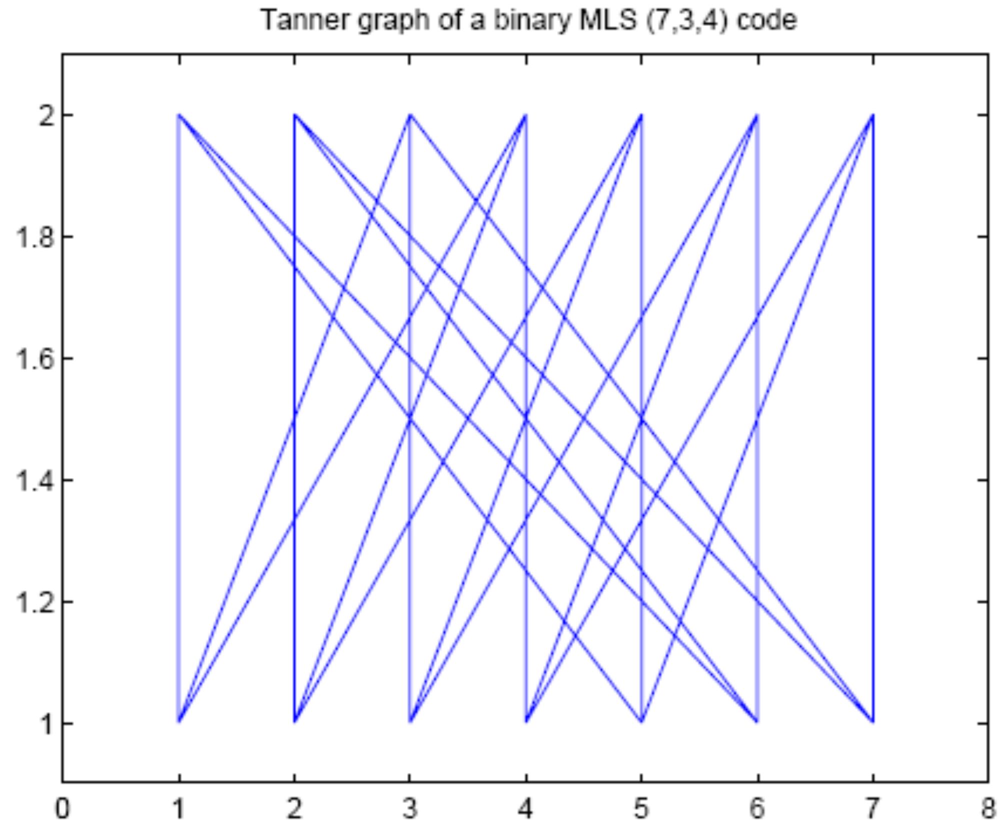


The new Tanner graph has a <u>regular structure</u>:

- ✓ **All <u>variable nodes</u> have degree (edges connected to them) equal to FOUR**
- ✓ **All <u>parity nodes</u> have degree equal to four**

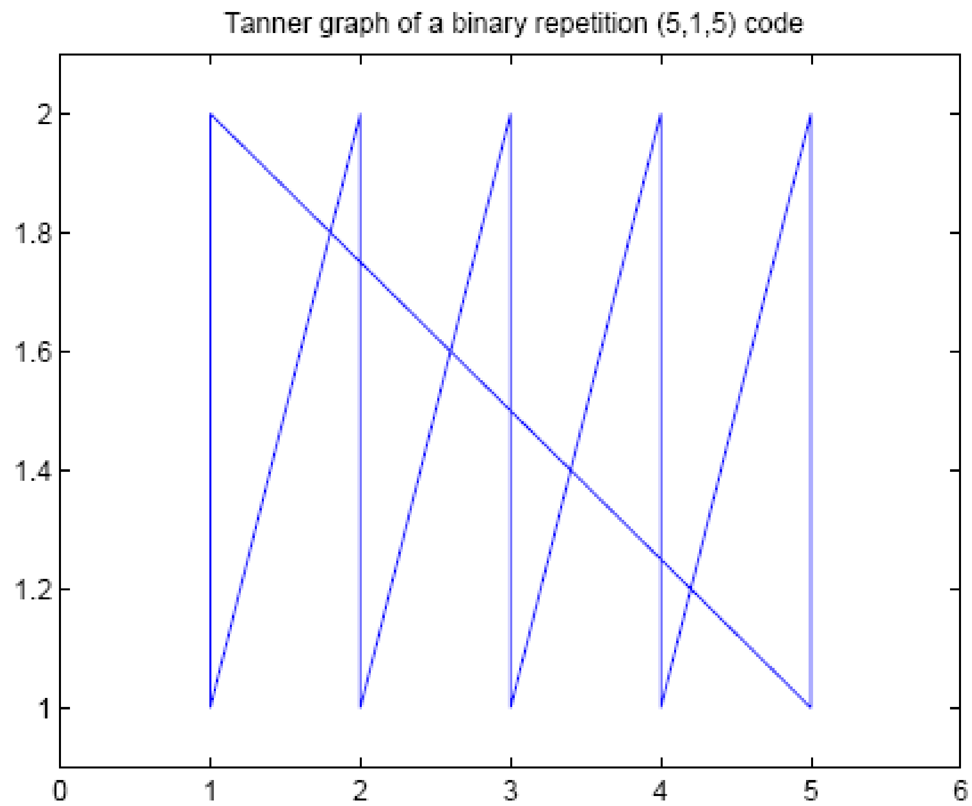
Example: Tanner graph of the dual (7,3,4) code of the Hamming (7,4,3) code

$$H = \begin{pmatrix} 1011000 \\ 0101100 \\ 0010110 \\ 0001011 \\ 1000101 \\ 1100010 \\ 0110001 \end{pmatrix}$$

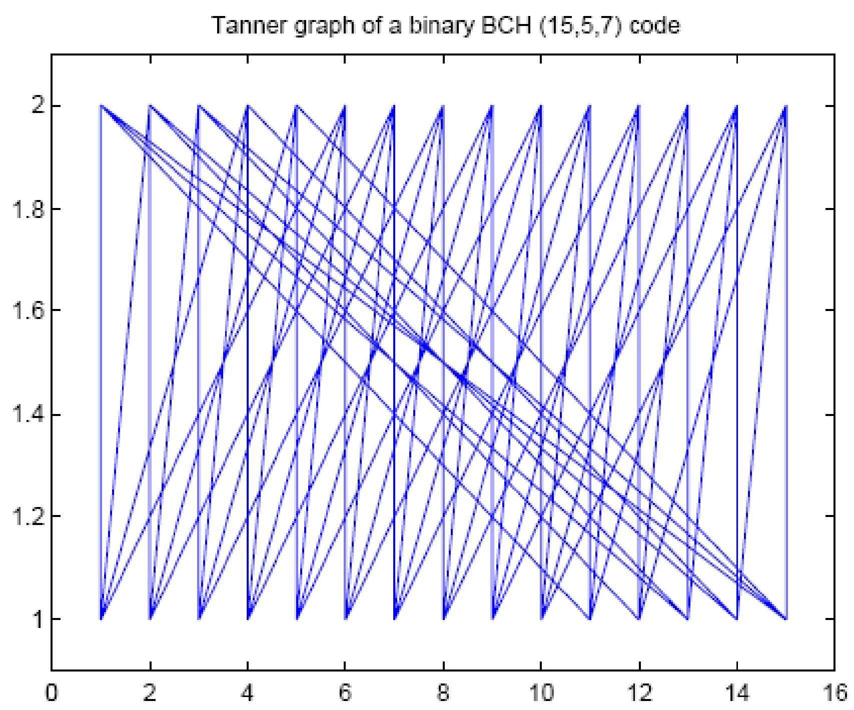


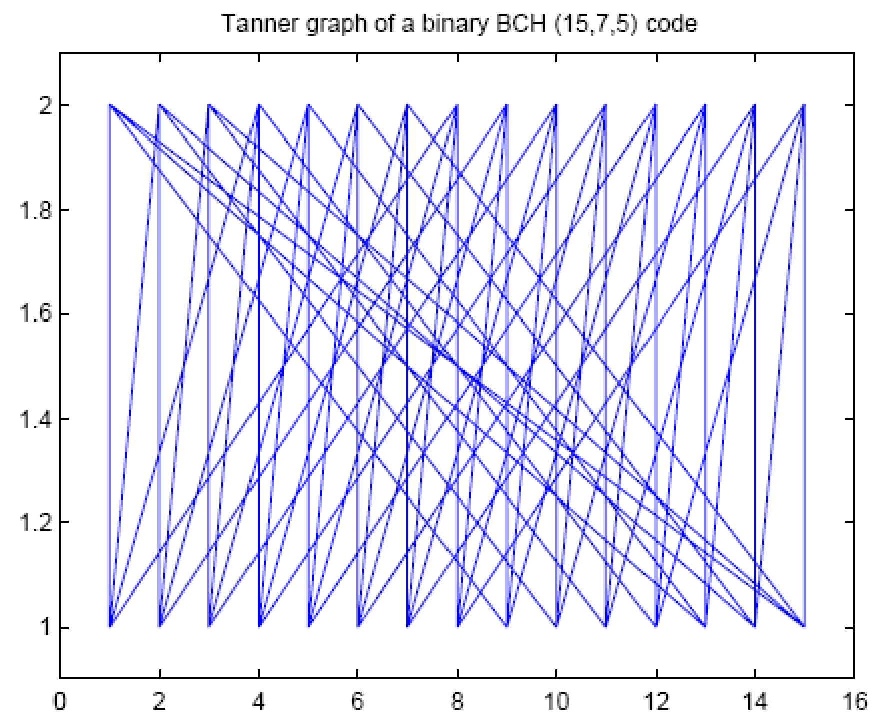
Tanner graph of the repetition (5,1,5) code

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$



Tanner graphs of binary BCH (15,5,7) code and BCH (15,7,5) code





Bit-flip decoding (Gallager 1962)

- **Step 1**: <u>Calculate</u> the parity-check equations from preliminary (hard) decisions from the demodulator (matched filter or correlator)
- **Step 2**: <u>Compare</u> each result from step 1 with each received parity bit. If they are different then the equation is not satisfied
- **Step 3**: <u>Complement</u> information bits involved in more than T parity equations that are not satisfied.

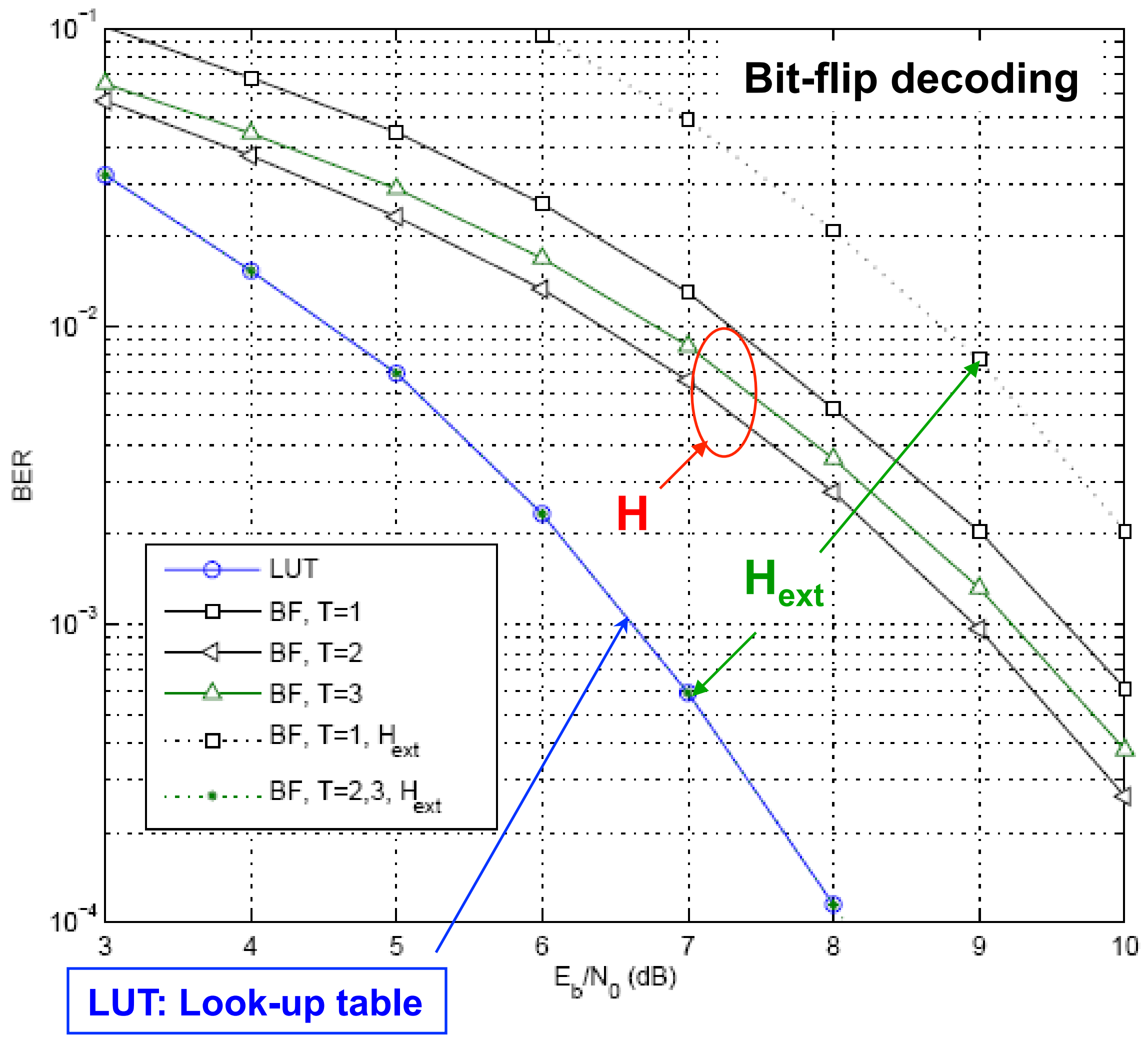
T is a design parameter selected in advance

Repeat steps 1 to 3 until either

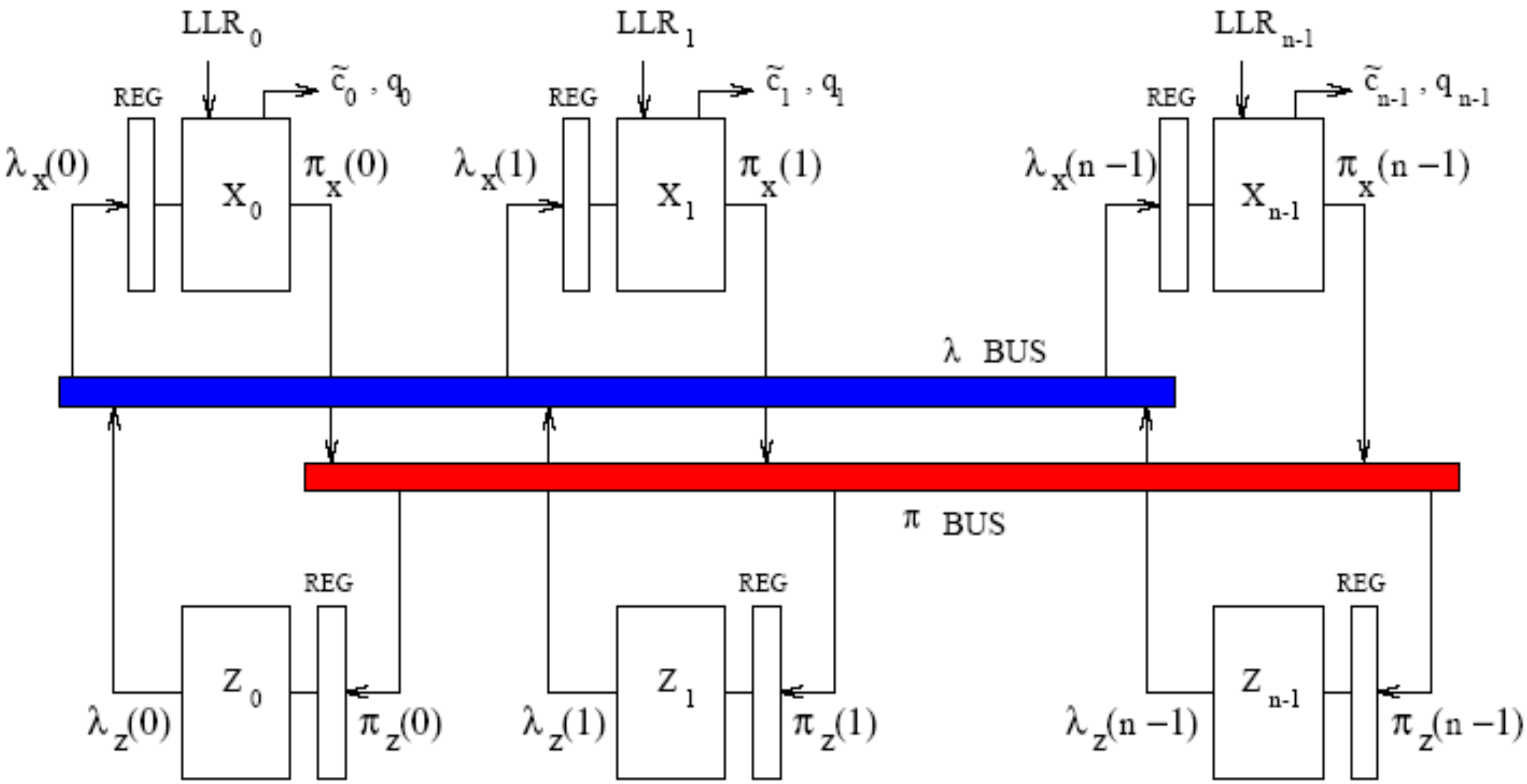
(1) all parity equations are satisfied, or

(2) the number of iterations reaches a limit value

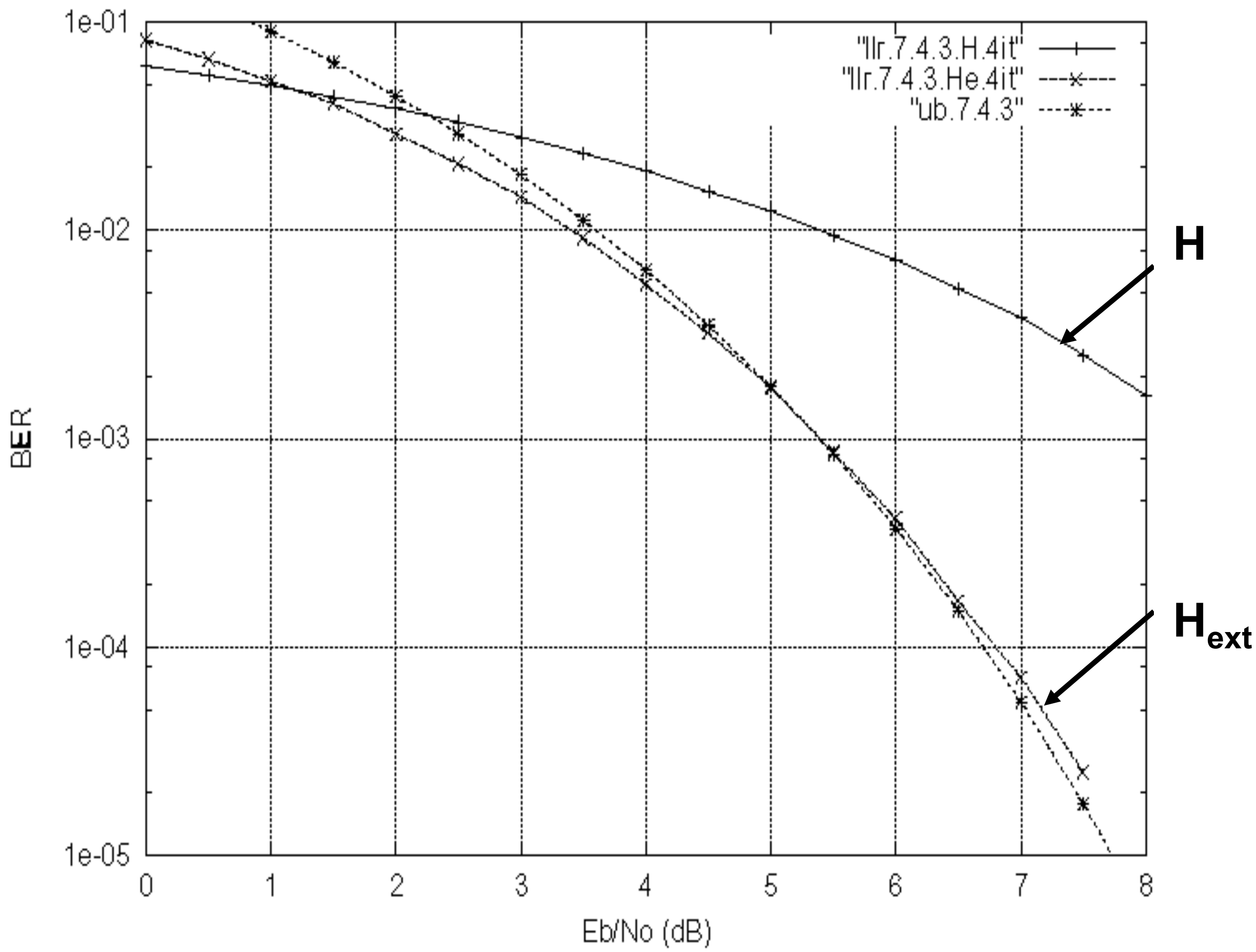
Hamming (7,4,3) code with original (H) and extended (H_{ext}) parity-check matrices



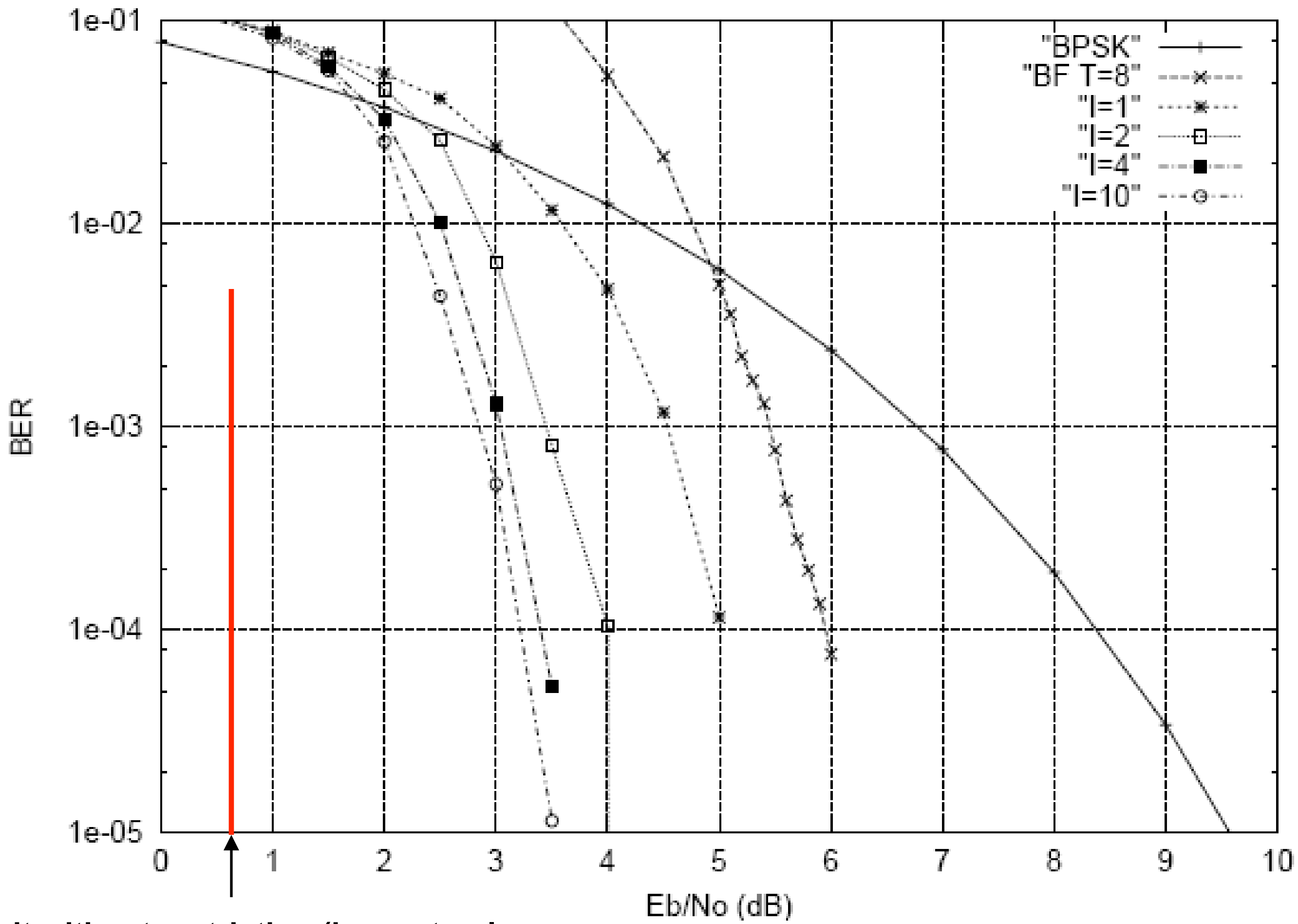
Parallel architecture of decoder



Hamming (7,4,3) code with BP decoding (4 iterations)

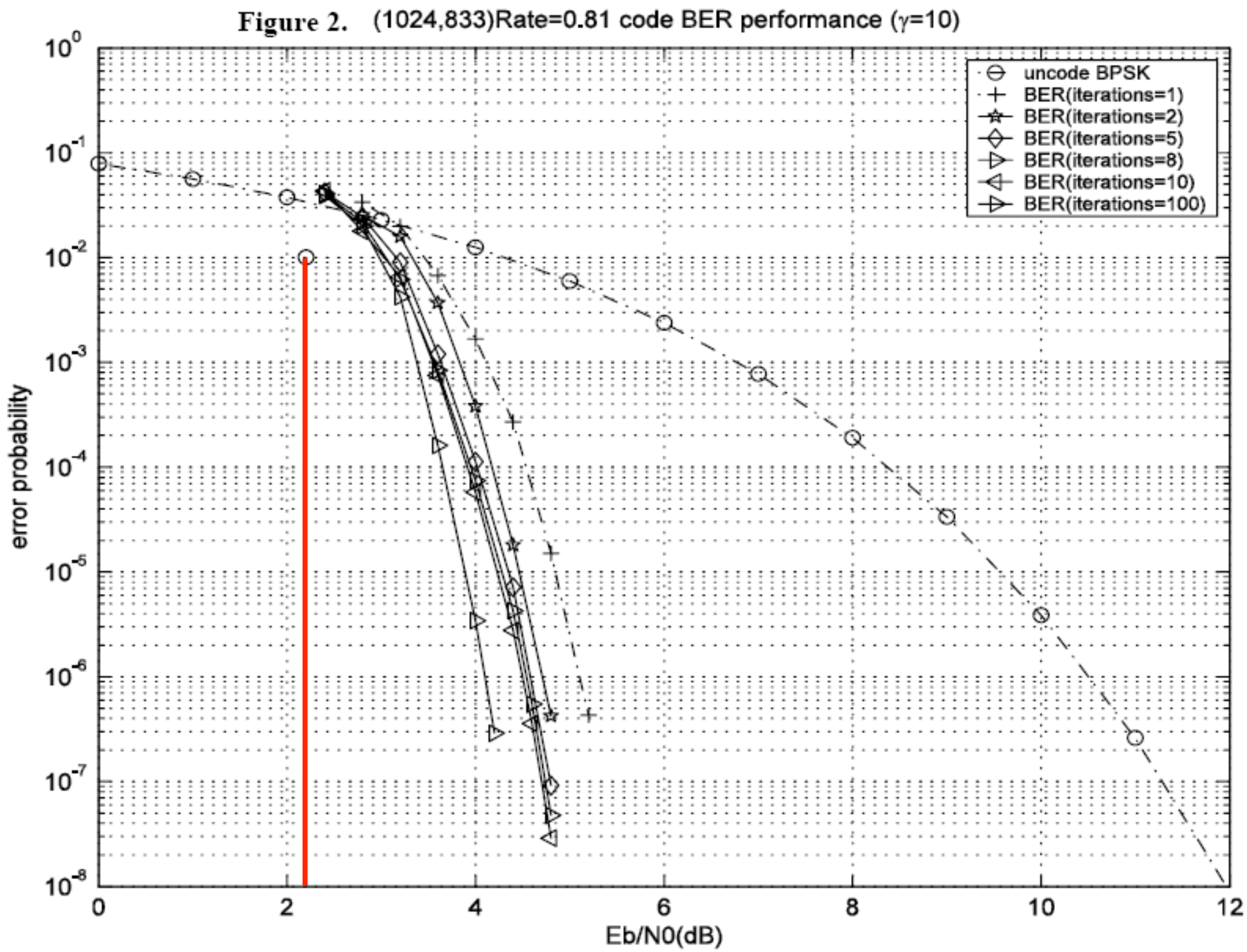


Cyclic (273,191,17) code from finite Euclidean geometry (Lin, 1999)



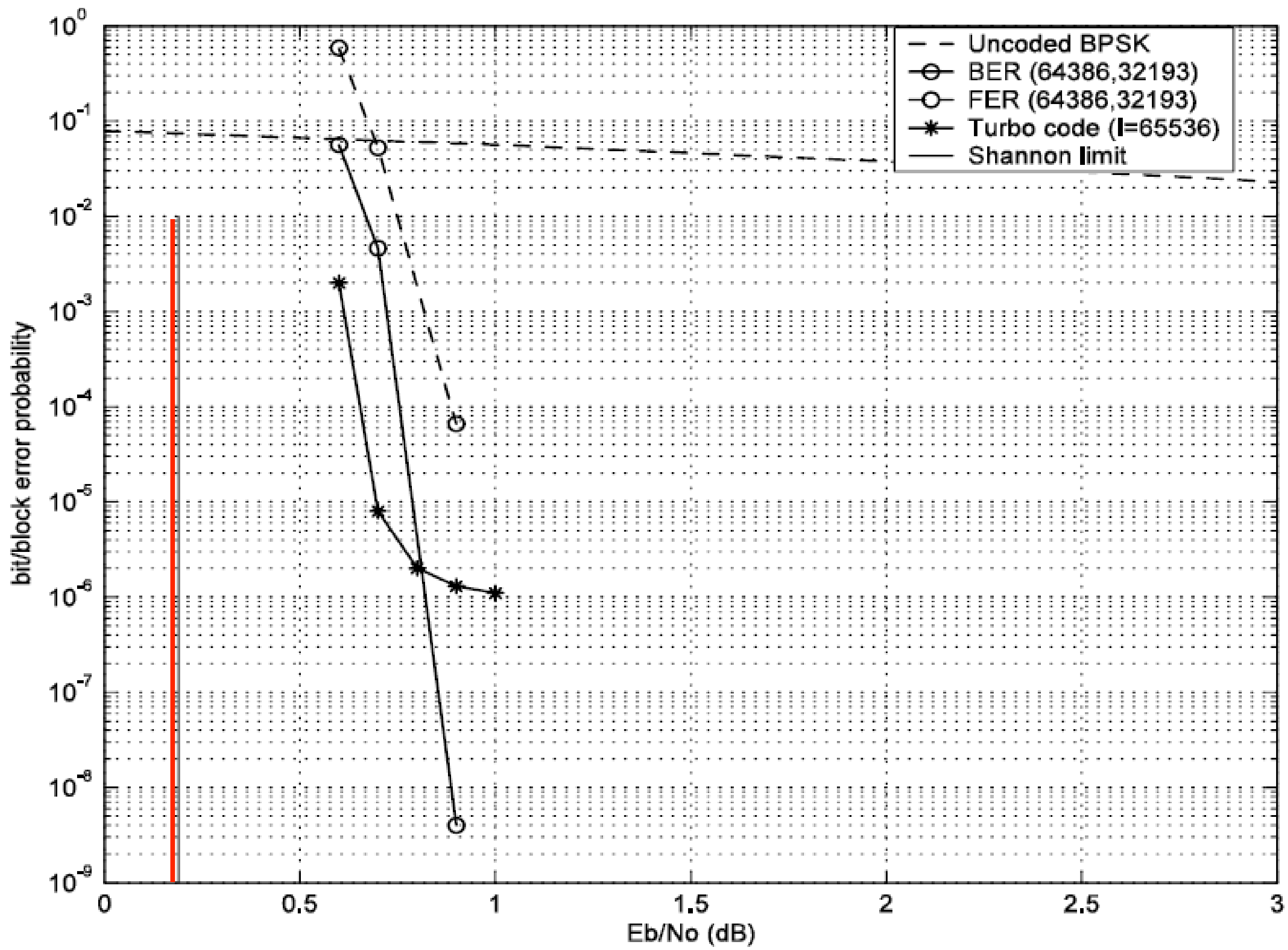
Shannon limit without restriction (i.e., not only BPSK) for R=0.7 id Eb/No=0.68 dB

Effect of the number of iterations in BP decoding (Lin, 2004)

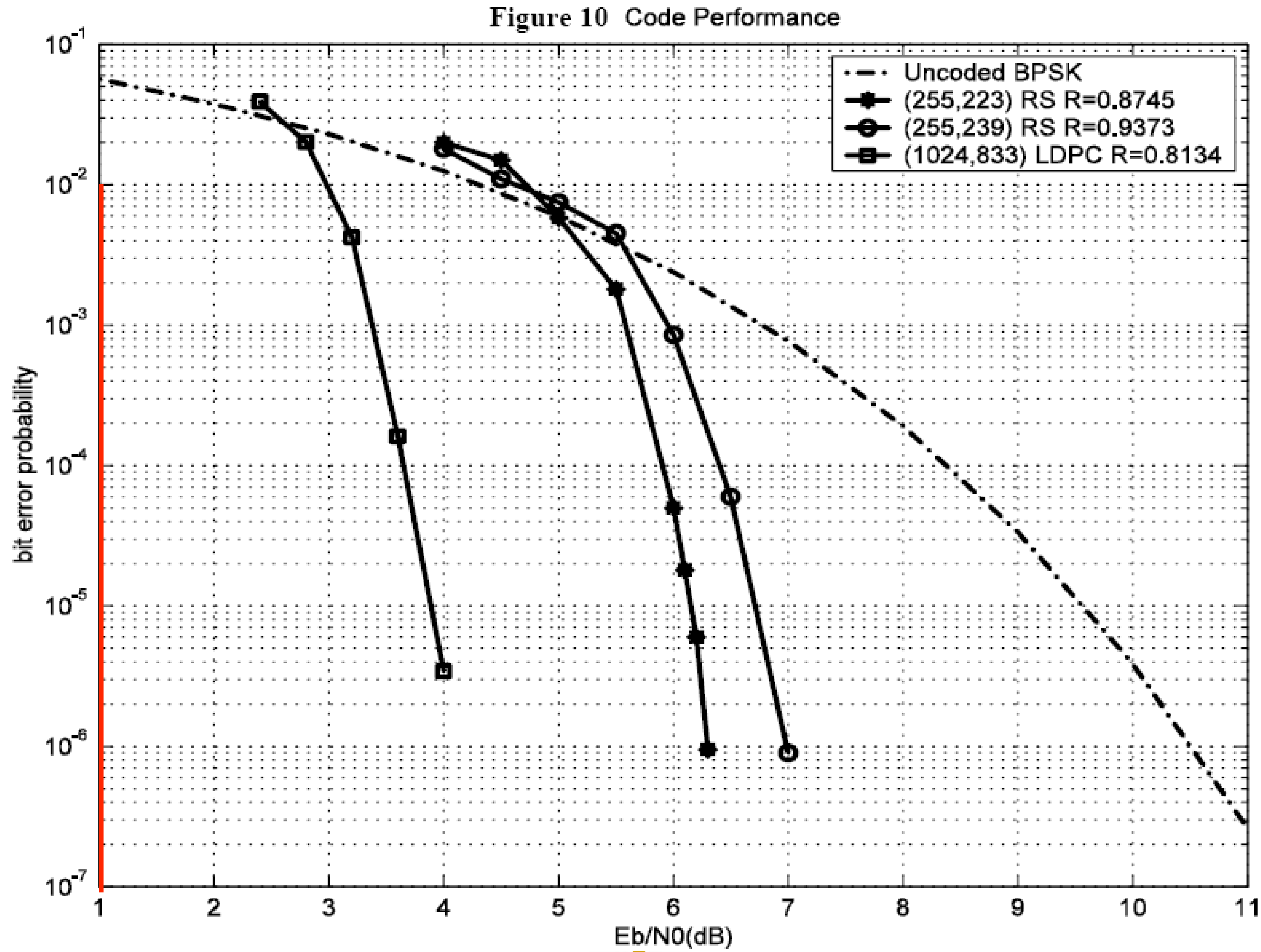


LDPC and turbo codes (Lin, 2004)

Figure 8. LDPC(64386, 32193) Code Performance



LDPC y Reed-Solomon codes (Lin, 2004)



Observations on LDPC codes

- LDPC codes with irregular Tanner graphs have best performance
- Short cycles in the Tanner graph **limit the efficiency** of iterative decoding algorithms working on it
- On the other hand, LDPC codes of large minimum distance have many short cycles in their Tanner graphs
- The number of required computations is reduced with LLR values
- Complexity (number of required computations and memory size) increases linearly with the code length !!!