Diversity achieved by channel coding under slow flat fading

EE161: Digital Communication Systems
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Error correcting coding (ECC)

Board: Use 161 notation

- Communication system under flat fading using ECC:

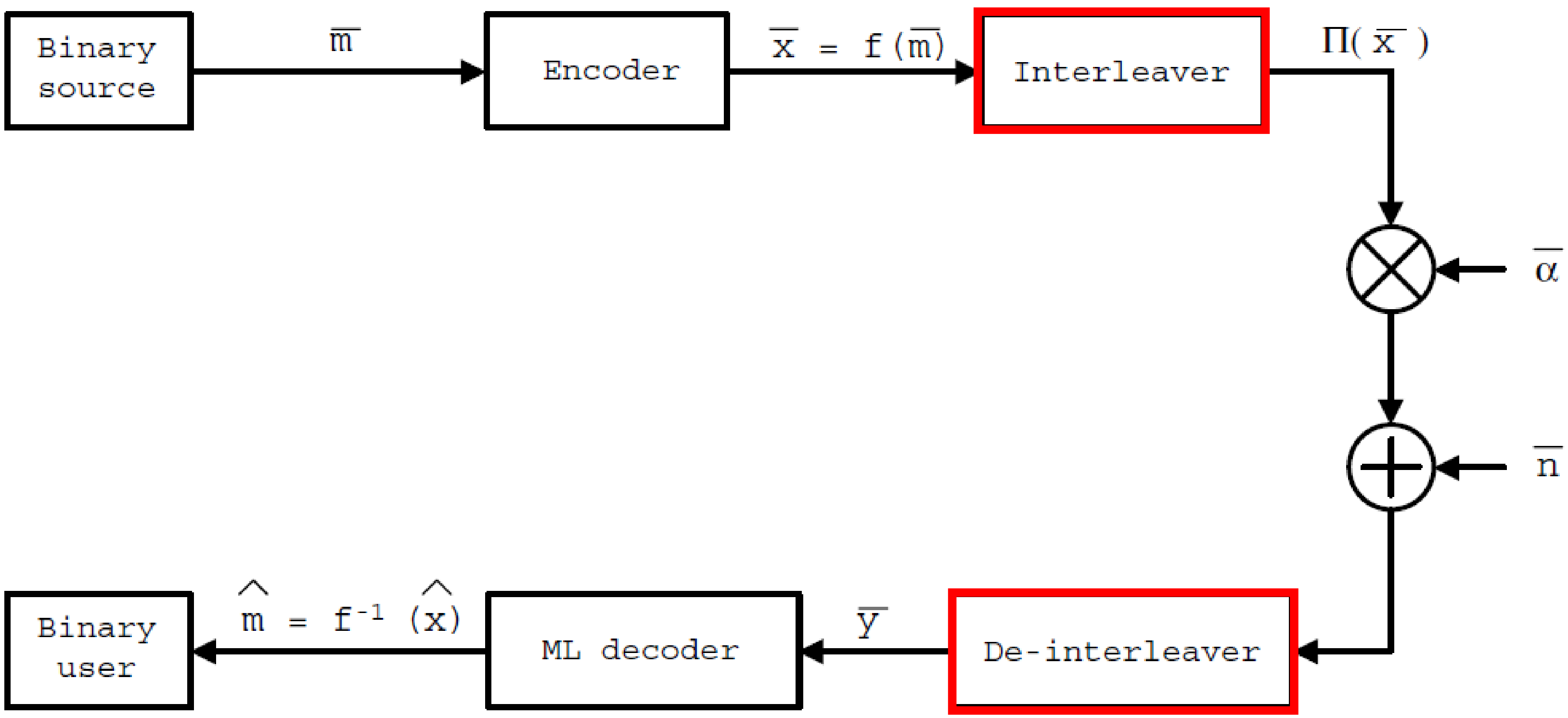


Figure 2: Communication system over a flat fading channel with error correcting coding.

Interleaving/De-interleaving ensures that ***amplitudes are independent***

Error probability

For BPSK modulation, the conditional PEP is [2]

$$\Pr\{\bar{x} \to \hat{x}|\bar{\alpha}\} = Q\left(\sqrt{\frac{2E}{N_0}\sum_{i=1}^{d}\alpha_i^2}\right), \tag{6}$$

where $d = |j : \hat{x}_j \neq x_j|$, is the Hamming distance between the vectors $\bar{x}, \hat{x} \in C$. Therefore,

$$P_2(d) \triangleq E_{\bar{\alpha}}\left\{\Pr\{\bar{x} \to \hat{x}|\bar{\alpha}\}\right\} = \int Q\left(\sqrt{\frac{2E}{N_0}\sum_{i=1}^{d}\alpha_i^2}\right) p_{\bar{\alpha}}(\bar{\alpha})\ d\bar{\alpha}. \tag{7}$$

Using the bound on the Q-function,

$$Q(x) < \frac{1}{2}e^{-x^2/2},$$

and assuming $\bar{\alpha}$ to be a vector of i.i.d. Rayleigh fading amplitudes, the PEP can be bounded as

$$P_2(d) < \frac{1}{2}\left[\frac{1}{1+\frac{E}{N_0}}\right]^d. \tag{8}$$

Error probability (cont.)

A union bound on the codeword error probability of a linear error correcting code is given by

$$P_w \leq \sum_{d=d_H}^{n_c} A(d)\ P_2(d), \tag{9}$$

where $A(d)$ denotes the number of codewords at Hamming distance d from the transmitted codeword. Therefore, combining (8) and (9),

$$P_w \leq \sum_{d=d_H}^{n_c} \frac{A(d)}{2} \left[\frac{1}{1+\frac{E}{N_0}}\right]^d . \tag{10}$$

At high values of E/N_0,

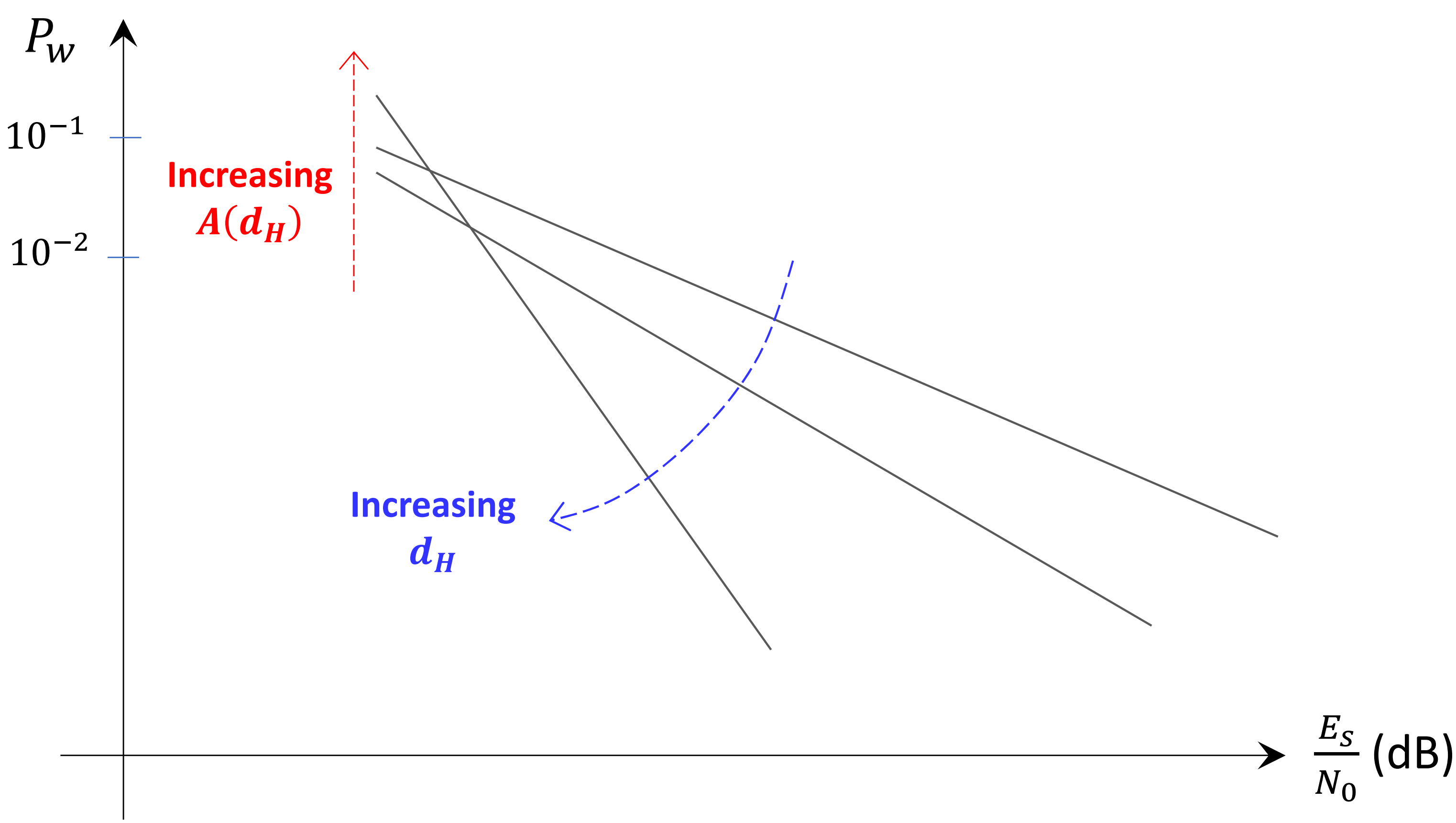
$$\boxed{P_w \approx \frac{A(d_H)}{2} \left[\frac{E}{N_0}\right]^{-d_H}}.$$

Therefore, with error correcting coding and transmission over a slow flat Rayleigh fading channel, the diversity order equals the minimum Hamming distance of the code.

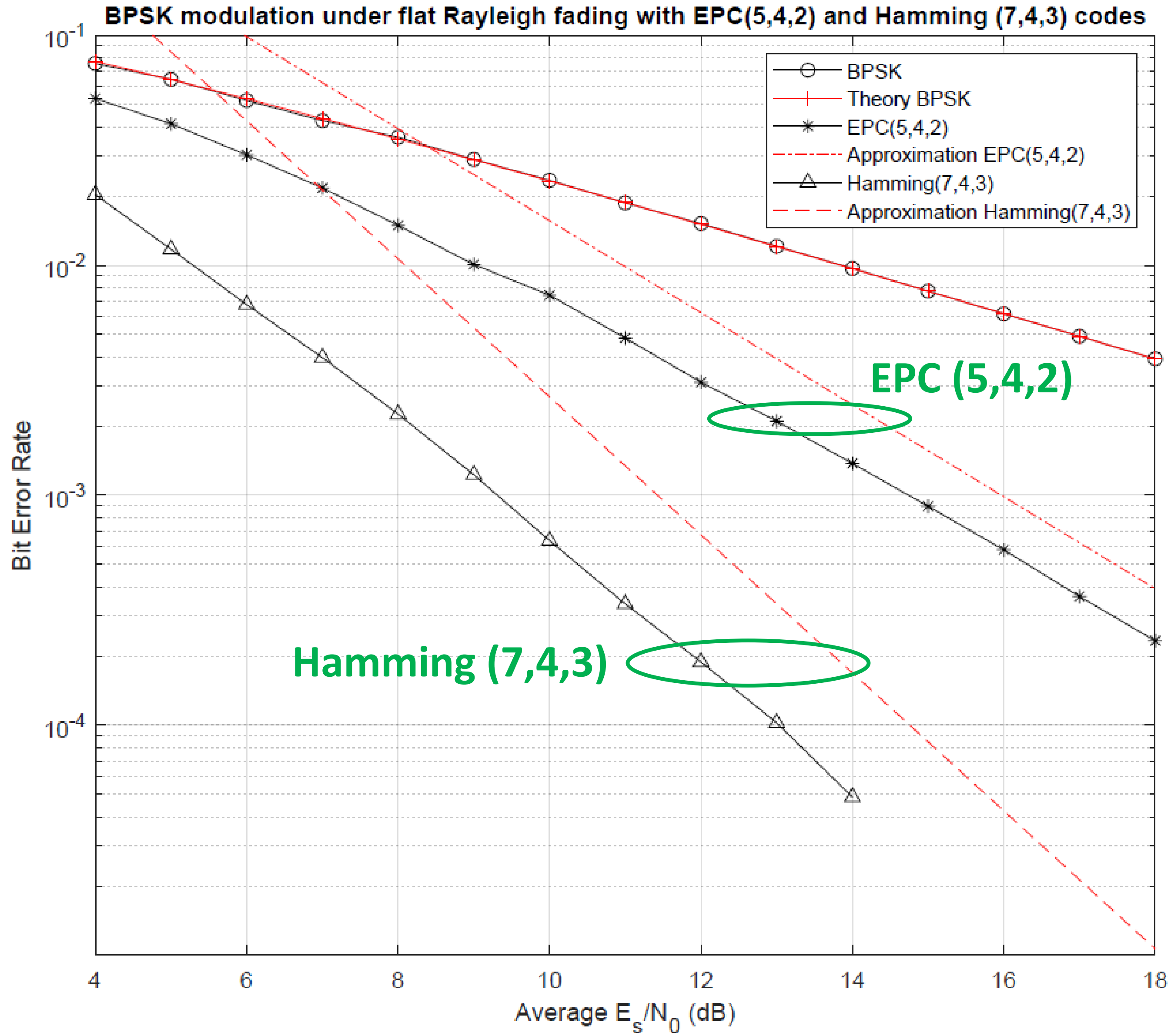
Diversity is the minimum Hamming distance

- Approximation on the (word) error probability:

$$\boxed{10\log_{10}(P_w) \approx 10\log_{10}\left(\frac{A(d_H)}{2}\right) - \boldsymbol{d_H} \cdot 10\log_{10}\left(\frac{E_s}{N_0}\right)}$$



Example: EPC (5,4,2) and Hamming (7,4,3) codes



References

[1] Stüber, *Principles of Mobile Communication*, Kluwer Academic, 1996.

[2] E.K. Hall and G. Wilson, "Design and Analysis of Turbo Codes on Rayleigh Fading Channels," *IEEE J. Sel. Areas in Comm.*, vol. 16, no. 2, pp. 160-174, Feb. 1998.

[3] S. Benedetto and E. Biglieri, *Principles of Digital Transmission*, Kluwer Academic, 1999.