EE161 — Spring 2025 **San José State University**
Homework # 6 **Due: Thu. 3/20/25**

1. A systematic binary linear (5,2,3) code has generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

   (a) What is the parity-check matrix H of this code?

   (b) Build an encoding look-up table with two columns where all the 4 combinations of 2 information bits $\bar{B} = (B_1\ B_2)$ are listed in the first column (address) and the associated codewords $\mathbf{c} = \bar{B}G$ in the second column.

   (c) Build a hard-decision decoding lookup table with two columns: In the second column list all the six possible error vectors $\bar{e}$ of Hamming weight[1] up to 1 (no errors plus 5 single errors). In the first column (address) list the associated syndrome vectors $\bar{s} = \bar{e}H^T$.

   (d) The received vector is $\bar{r} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \end{pmatrix}$.

      i. Determine the syndrome vector $\bar{s}$

      ii. Determine the most likely error vector $\bar{e}$

      iii. Determine the most likely information bits $\hat{B} = (\hat{B}_1\ \hat{B}_2)$

2. Maximum-likelihood (soft-decision) decoding of a binary EPC (3,2,2) code

   Download MATLAB script `MLdecoder_322code.m` from Canvas. The script simulates the performance of soft-decision (SD) decoding of a binary EPC (3,2,2) code with polar mapping (or BPSK, 2-PAM) under AWGN.

   (a) Run the script with your student ID number an attach the resulting figure.

   (b) With reference to the figure from part (a), and at an average bit-error rate (BER) value equal to 10^{-4}, compare the simulated coding gain of SD decoding, with respect to uncoded BPSK, to the *real coding gain* (RCG) introduced in class

[1]The Hamming weight is equal to the number of ones in a vector.