# EE 160: Principles of Communication Systems
# Experiment 1: Harmonics and intermodulation

## I. INTRODUCTION

a. Objectives

i. Gain familiarity with the operation of the spectrum analyzer
ii. Study harmonic generation with a nonlinear device (diode)
iii. Evaluate experimentally the input third order intercept point (IIP3) of an amplifier using a two-tone test procedure
iv. Measure the spectra of periodic signals

b. Required reading (follow the links in the web page)

i. Spectrum Analyzer Basics (Agilent)
ii. HP 8590 Spectrum Analyzer user's manual: All of section 2 and section 3, pages 3-1 to 3-3
iii. Digital Waveform Generator user's manual
iv. Digital Oscilloscope user's manual

c. List of parts

i. Spectrum analyzer coupling and attenuation circuit (**ATT-SA**)
   1. 1 kΩ resistor
   2. 56 Ω resistor
   3. 0.1 uF monolithic ceramic capacitor

ii. Two-sinusoidal adder
   1. Two 1 kΩ resistors

iii. Amplifier two-tone test
   1. LF347 (or equivalent) quad op amp
   2. Three 1 kΩ resistors
   3. Two 10 kΩ resistors

iv. Cosine waveform clipper
   1. 1 kΩ resistor
   2. Switching diode: 1N914 or 1N4148

## II. THEORY

This experiment serves three fundamental purposes: First, familiarize the student with the spectrum analyzer (SA) by identifying and resolving sinusoidal signals of relatively close center frequencies. Second, use the SA in measuring harmonics generated by a non-linearity. Finally, use a two-tone test in determining the nonlinear (third-order) characteristic of an amplifier.

A periodic waveform f(t) with period T can be expressed as an exponential Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

where

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

and $f_0$ the fundamental frequency. The Fourier coefficients $c_n$ can be computed as

$$c_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_0 t}\,dt$$

Note that the integral may be taken over any interval of length T. The power in the n-th harmonic (for n>0) is proportional to

$$P_n = 2\,|\,c_n\,|^2$$

The total harmonic distortion (THD) is defined as

$$THD = \frac{\sum_{n=2}^{\infty} P_n}{P_1} = \frac{\sum_{n=2}^{\infty} |\,c_n\,|^2}{|\,c_1\,|^2}$$

and typically expressed as a percentage.

Sequence $P_n$ can be plotted in a graph as a one-sided power line spectrum. The spectrum analyzer displays this spectrum with the lines broadened by the finite bandwidth resolution of the analyzer's variable frequency bandpass filter. When the spectrum analyzer is properly adjusted, the amplitude of each displayed spectral peak indicates the corresponding $P_n$.

### II. 1 The coupling-attenuation (ATT-SA) circuit

Of fundamental importance in the EE 160 lab is the ATT-SA circuit. Its function is to limit the input signal levels, block any DC components and match the input 50 Ω impedance of the spectrum analyzer. A diagram of the ATT-SA circuit is shown in Fig. 1.

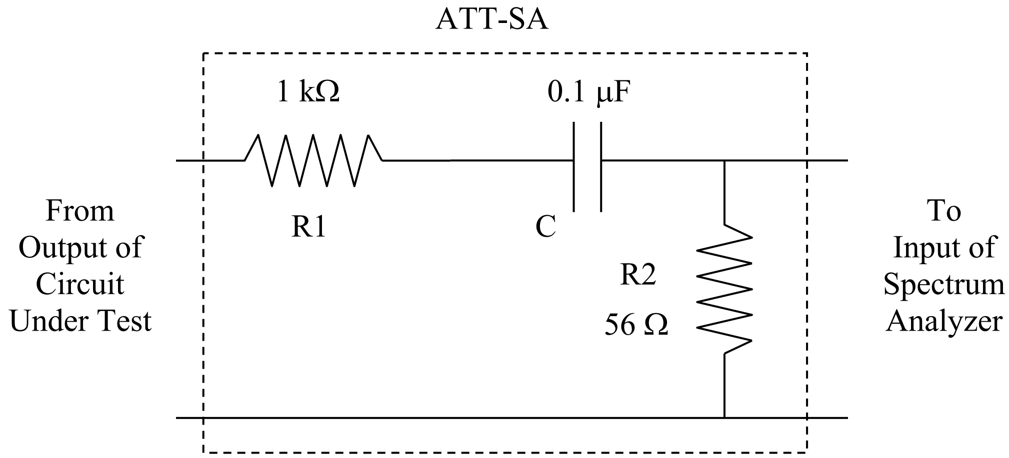


**Figure 1: Coupling-attenuator circuit (ATT-SA)**

As the ATT-SA will be used in future experiments, you must build it on an end of the breadboard and leave it permanently until the end of the semester. Please notice that **all input signals to the spectrum analyzer shall be applied through the ATT-SA**.

## III. INSTRUMENTS AND MATERIAL

Follow the procedure on pages 2-13 to 2-15 of the spectrum analyzer user's manual to learn how to make basic measurements. Make sure that you follow through the entire procedure. In particular, from section 2 you shall understand how to set the start and stop frequencies, adjusting the amplitude reference level and using markers to determine the dB ratio of two spectral components. Also read the user's manual of the digital oscilloscope from the web page.

## IV. PRE-LAB WORK

Read the class notes on nonlinear devices and harmonics. In particular, the section that deals with measuring the input third-order intercept point (IIP3) of an amplifier based on a two-sinusoidal input. This procedure is known as the two-tone test.

### IV.1 Triangle Wave

Either look up in your textbook or derive an exponential Fourier series coefficient formula for the symmetric triangular wave given by:

$$f(t) = 1 - \frac{4|t|}{t}, \qquad if\ |t| < \frac{T}{2}$$

as shown in Figure 2. Then evaluate the harmonic power ratio Pn/P1 (n = 2, 3, 4, and 5) in dB. Also, compute the THD due to the second through fifth harmonics (only).

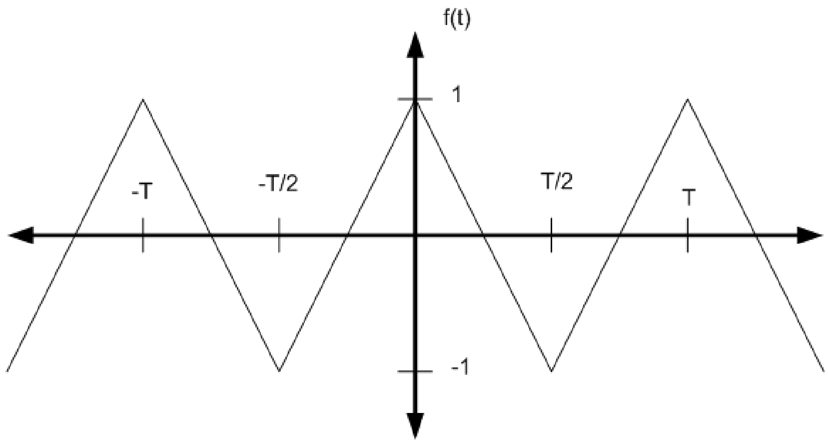


**Figure 2. Symmetric Triangular Wave.**

## IV.2 Variable Duty Cycle Rectangular Wave

Similarly, look up or derive an exponential Fourier coefficient formula for the square wave with duty cycle α (0 < α < 1):

$$f(t) = \begin{cases} 1, & \text{if } |t| < \dfrac{\alpha T}{2} \\ 0, & \text{if } \dfrac{\alpha T}{2} \le |t| < \dfrac{T}{2} \end{cases}$$

shown in Figure 3.

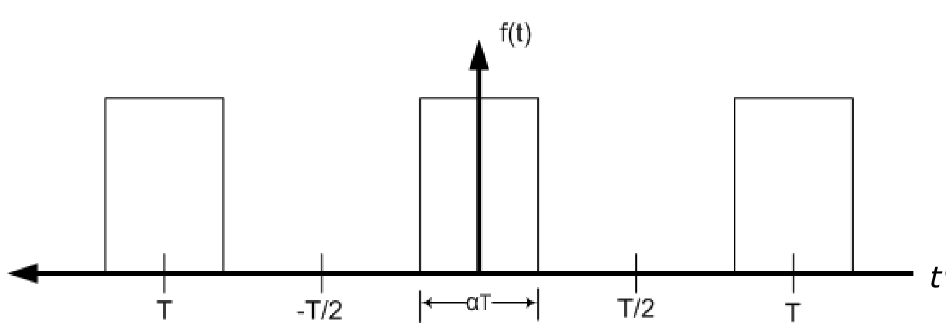


**Figure 3. Variable Duty Cycle Rectangular Wave**

From this formula, determine the duty cycle values α at which the second, third, and fourth harmonics vanish. For each of these three duty cycles, compute the harmonic power ratios $P_n/P_1$ (dB) and the THD (only for n = 2 to 5). Note that the harmonic power ratios for a unipolar (0 to +1) rectangular wave are identical to those for a bipolar (—1/2 to +1/2) rectangular wave with the same duty cycle. Their spectra differ only in the DC component, which cannot be measured with a spectrum analyzer.

## IV.3 Clipped Cosine Wave

Figure 4 shows a clipped cosine wave defined by:

$$f(t) = \max[0, (\cos(w_0 t) - \cos\theta)],$$

where $0 < \theta < \pi$. This waveform corresponds to the portion of a cosine wave with phase between $-\theta$ and $+\theta$. Note that the peak amplitude of this clipped wave increases monotonically with θ.

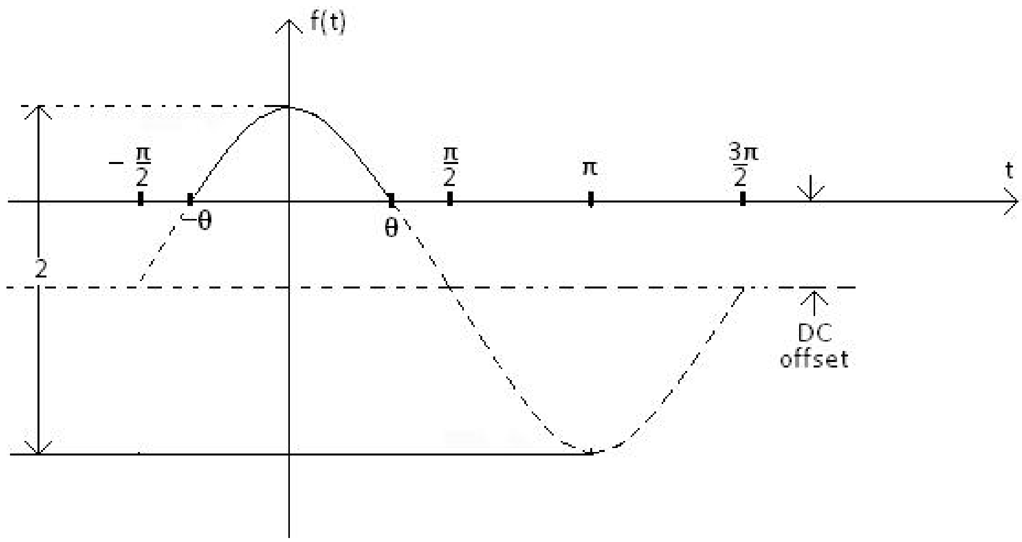


**Figure 4. Clipped Cosine Wave.**

Look up or derive an exponential Fourier coefficient formula for this periodic waveform. These coefficients will depend on θ as well as the period T. For n = 1 through 5, plot Pn(θ) vs. θ over the range $0 < \theta < \pi$. (you may use MATLAB to generate these plots.) From your plots, find the value(s) of θ at which each Pn reaches local maximum and minimum values. For some values of n you will see more than one sharp dip or "null" in the plots.

In the lab you will work with a simple circuit that generates a good approximation to this clipped sine wave. A practical application of this circuit is the generation of specific harmonics. For example, to design a frequency doubler, you might choose the value of θ that yields the maximum $P_2$, or you might choose a θ that produces a minimum $P_3$ while maintaining a relatively large $P_2$. You then follow the circuit with a lowpass or bandpass filter to remove the unwanted higher harmonics. Reducing the amplitude of the third harmonic, while maintaining a "good" second harmonic amplitude, would allow you to use a broader (less expensive) filter.

### IV.4 ATT-SA circuit

Download the LTspice model EE160_ATT_SA_circuit.asc of the ATT-SA circuit in Fig. 1 from the website of the lab. Open the model in one of the lab PCs using the application LTspice and run it, after modifying it with resistors and capacitor values measured with the digital multimeter. Upon completion of the simulation, move the mouse and click on the label Vout in the schematic diagram to display the amplitude and phase Bode plots. Print (and save as PDF file) the resulting figure. You will need these Bode plots in order to complete your report.

## V. MEASUREMENTS

### V.1 Coupling-attenuation circuit (ATT-SA)

To verify the proper operation of the ATT-SA perform the following measurements:

1. With the spectrum analyzer disconnected from the attenuator, measure the resistance of R1 and R2 and the capacitance of C1. From these values, compute the theoretical cutoff frequency.
2. Reconnect the spectrum analyzer to the ATT-SA. Use the function generator and the oscilloscope to determine the cutoff frequency of the attenuator, assuming a nominal input 50 Ω impedance of the spectrum analyzer. Make sure that the sinusoidal output from the function generator has sufficiently high amplitude (at least 5 Vpp).

3. Run the LTspice model of the ATT-SA circuit this time with the values of resistors and capacitor that you measured in part 1. Sketch or print carefully the magnitude plot and attach it to your report.

### V.2 Making measurements using the spectrum analyzer

Construct the circuit shown in Fig. 5 on your breadboard. Apply two sinusoidal signals of equal amplitudes (5Vpp) and center frequencies set to approximately 1 MHz and 1.1 MHz. Use the procedure outlined in the spectrum analyzer user's manual, pages 3-1 to 3-8, to identify the signals and resolve them as you change their amplitudes and the difference between their center frequencies.

**NOTE: The circuit in Fig. 5 replaces the setup shown in Fig. 3-1 of the user's manual.**

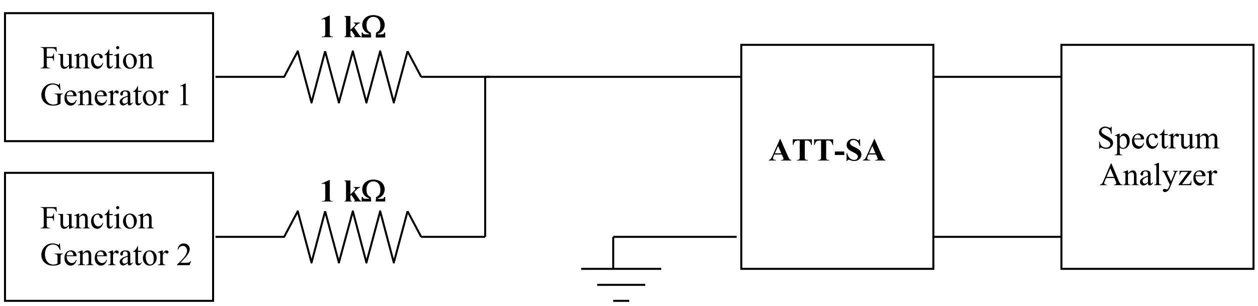


**Figure 5: Dual-input circuit**

Repeat this process with the two frequencies changed to approximately 10 MHz, while at the same time maintaining the same frequency difference of 100 kHz.

### V.3 Harmonic generation with a nonlinear device

Modify the circuit of Fig. 5 by inserting a diode and a 56Ω resistor as shown in Fig. 6.

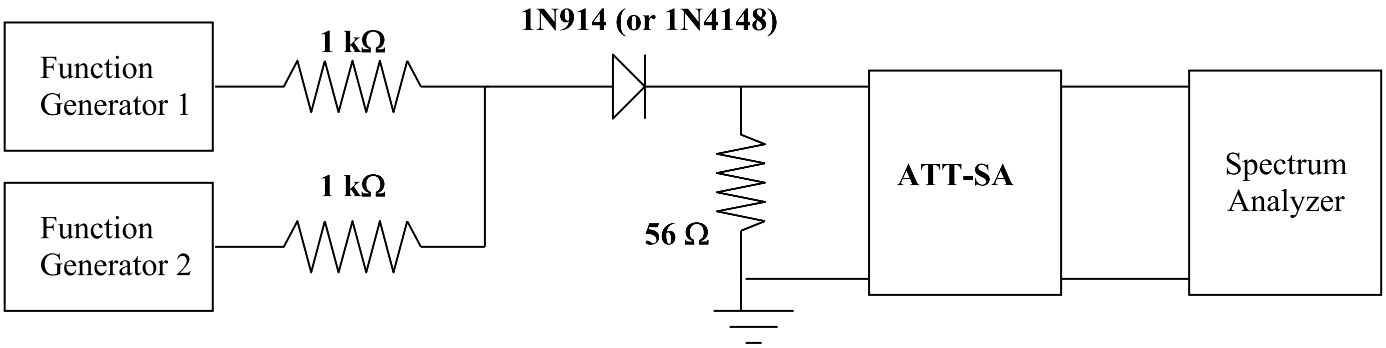


**Figure 6: Dual-input nonlinear circuit**

Using only one of the inputs to the dual-input nonlinear circuit of Fig. 6, and the other input to ground (or disconnect the second function generator and the second 1 kΩ resistor), apply a sinusoidal signal of sufficiently large amplitude (at least 8 Vpp) and center frequency equal to 1 MHz to the circuit. Find and record the amplitudes of the fundamental and all harmonics that you can measure. Note that at some point in frequency the values become so small they are no longer observable. Change the amplitude of the fundamental frequency by 3 dB and repeat the measurements. Repeat again by changing the amplitude of the fundamental an additional 3 dB.

Apply two sinusoidal signals, of equal amplitude and fundamental frequencies set to 1 MHz and 1.1 MHz, to the dual-input nonlinear circuit of Fig. 3. The amplitudes shall be set high enough (5 Vpp) so that all frequency terms in the range between 1 MHz and 3.5 MHz should be visible in the spectrum analyzer. Take data on the frequency values and corresponding amplitudes. In general, harmonics related to fourth and fifth order terms of the non-linearity should be visible. Try to decrease the amplitudes of the input signals so that these terms disappear. Next, decrease the amplitude of one of the inputs by 10 dB and take data. Now, with the amplitudes of both inputs equal, make them high enough so that they generate components from the fourth and fifth order terms. Take data. Finally, reduce one of the input amplitude levels by 10 dB and repeat your measurements.

### V.4 Two-tone test of an amplifier

Modify the circuit of Fig. 6 by replacing the two 1 kΩ resistors, diode and 56Ω resistor with the amplifier circuit shown in Fig. 7. Make sure to use the triple-output power supply at –8V and +8V.

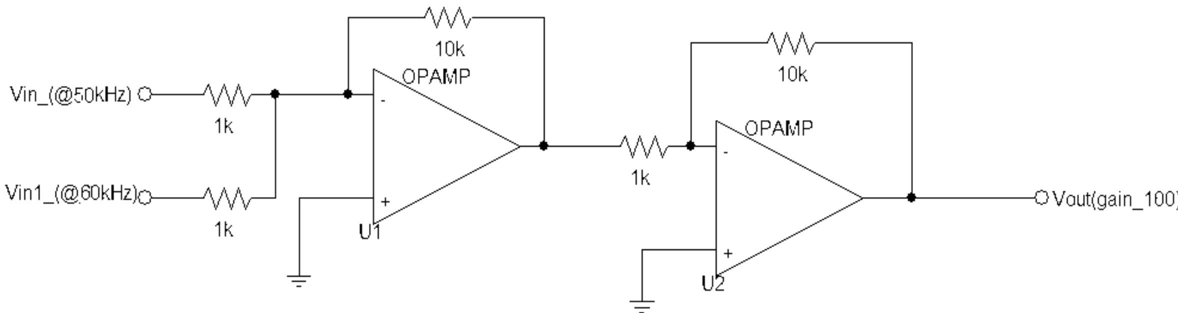


**Figure 7: An amplifier circuit with gain of 100**

Generate two sinusoidal signals of equal amplitude and fundamental frequency values set to 50 kHz and 60 kHz. **Important: You must make sure that the waveform generator's amplitude does not exceed 80 mVpp with a load impedance set to High. Set the function generator amplitude to the lowest value (20 mVpp) and its output impedance to "High Z". You can change this by pressing "Output Setup" and then selecting "Load".**

Increase gently the input amplitudes so that IM3 components at 40 kHz and 70 kHz are visible in the spectrum analyzer. Using the spectrum analyzer, measure the difference in amplitude (dB) between the 60 kHz and 70 kHz components. Repeat your measurements with both input amplitudes set to 0.5 dB higher and 0.5 dB lower.

### V.5 Periodic waveform spectra

#### V.5.1 Triangular Wave Spectrum

Use the function generator to produce a 5 Vp-p 200 kHz triangle wave at the ATT-SA input terminals. Make sure the amplitudes are sufficiently large. Set the spectrum analyzer to a 0 – 1.2 MHz span. Verify that the second harmonic is practically zero when the function generator's symmetry control is set to 50%. Measure and record the amplitudes of the fundamental and second through fifth harmonics.

### V.5.2 Rectangular wave Spectrum

Use the function generator to produce a 10 Vp-p 200 kHz square wave at the ATT-SA input terminals. Perform the following:

1. Null the second harmonic by changing the duty cycle of the wave. Refer to the user manual of the function generator for how to achieve this. Verify the rectangular wave's duty cycle with the oscilloscope. The duty cycle $\alpha$ is the ratio of the "on" time to the period T. Measure and record the amplitude of all harmonics up to the fifth in the spectrum analyzer.
2. Null the third harmonic. Verify again the duty cycle and perform harmonic amplitude measurements.
3. Null the fourth harmonic and repeat the measurements.
4. Keeping the duty cycle as in the previous step, increase the analyzer's span to 0 – 10 MHz. Sketch the resulting spectrum: measure the amplitude of some (but not all) of the higher order harmonics.

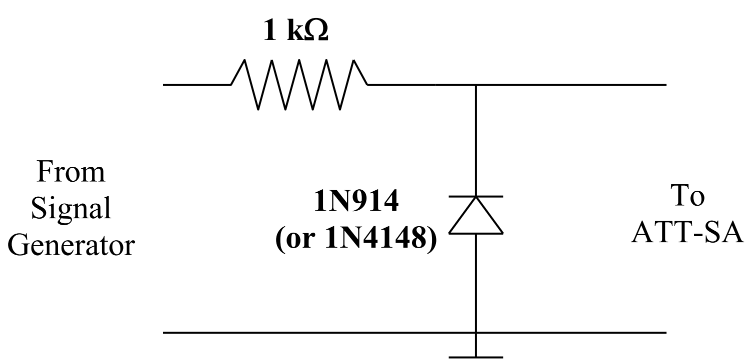


**Figure 8. Clipper Circuit**

### V.5.3 Clipped Cosine Wave

Build the clipper circuit shown in Figure 8, and connect it as shown in Figure 9. If D1 were an ideal diode, this circuit would pass to the spectrum analyzer only the positive portion of the input signal. A real diode introduces some distortion into the clipping process because the transition from the non-conducting to highly conducting state is not sharp.

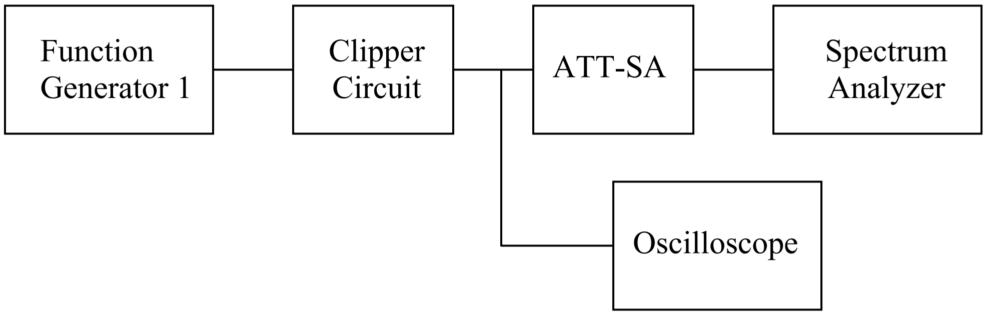


**Figure 9. Measurement setup for a truncated sinusoidal waveform**

Set the function generator to produce a 200 kHz sine wave with the amplitude control at its maximum setting. Observe the waveform at the input of the ATT-SA on your oscilloscope. Add just enough positive DC offset so that the negative peaks are not clipped by the diode. As the DC offset is increased, you may have to reduce the amplitude to avoid clipping the positive peaks (inside the function generator). Modify the amplitude and DC offset so as to obtain a sine wave with (1) the largest possible amplitude and (2) no clipping at either peak. The use of a relatively large amplitude helps to minimize the distortion caused by the nonlinear characteristic of the diode.

**NOTE: FOR THE REMAINDER OF THIS SECTION, DO NOT ADJST AGAIN THE AMPLITUDE OR FREQUENCY CONTROLS. USE ONLY THE DC OFFSET CONTROL.**

As you vary the DC offset control, you should see on the oscilloscope a clipped cosine wave very similar to that in Figure 4. You should be able to vary the conduction angle $\theta$ from 0 (DC only) to $\pi$ (undistorted cosine wave). Note that as you change the value of $\theta$, the amplitude of each harmonic passes through maximum and minimum values. After you have obtained a general sense of how the spectrum behaves, make and record the following measurements:

1. Adjust the DC offset to determine the conduction angles $\theta$ at which the first harmonic (fundamental) amplitude reaches its (local) maximum and minimum values(s). The maxima (peaks) will be very broad, but the minima (nulls) will be quite sharp. Measure the angle $\theta$ by adjusting the oscilloscope to display a single cycle across the full width, and estimating the fraction of a period in which the diode is conducting.

2. Repeat step 1 for the second through the fifth harmonics. The values of θ at which the maxima and minima occur will depend on the order of the harmonic that you are measuring.

## VI. ANALYSIS OF RESULTS

- Discuss how the two frequencies are resolved with the aid of the spectrum analyzer. Using the data collected in the lab, compare the measurements with the theoretical expressions presented in class, using a relationship between input waveform voltage and the output power measured by the spectrum analyzer.
- Discuss how the two-tone test helps in measuring the third order non-linearity of an amplifier. Using the measure data sets, determine the average IIP3 value of the circuit amplifier in the experiment.
- Verify the attenuation and cutoff frequency values measured as described in section V.0. In the same graph, compare the Bode amplitude plot obtained from the LTspice model with the laboratory measurements.
- In the lab you measured six periodic waveforms: 2 sine waves, 1 triangular wave, and 3 rectangular waves. For each of these waveforms you are asked to:
    - Compute the amplitude of each harmonic in dB relative to the fundamental (which will be the 0dB reference). You may already have done this in the lab as you made your measurements.
    - Compute the percent THD. Ignore the power in all harmonics above the fifth.
    - Compare the theoretical power spectrum with the measure spectrum. You may use tables(s) or graph(s) (or both) to present the results of your comparisons.
- Compare the spectrum you sketched and labeled in section V.5.2.4 with theoretical expectations.
- Prepare a table comparing the theoretical and measured values of the conduction angles at which each harmonic (n = 1 to 5) reaches its local maximum and minimum values.
- If the theoretical and measured results in Section V.5.3 differ by more than 10%, discuss possible reasons for these differences. In particular, what effect would you expect from the non-ideal diode characteristics?