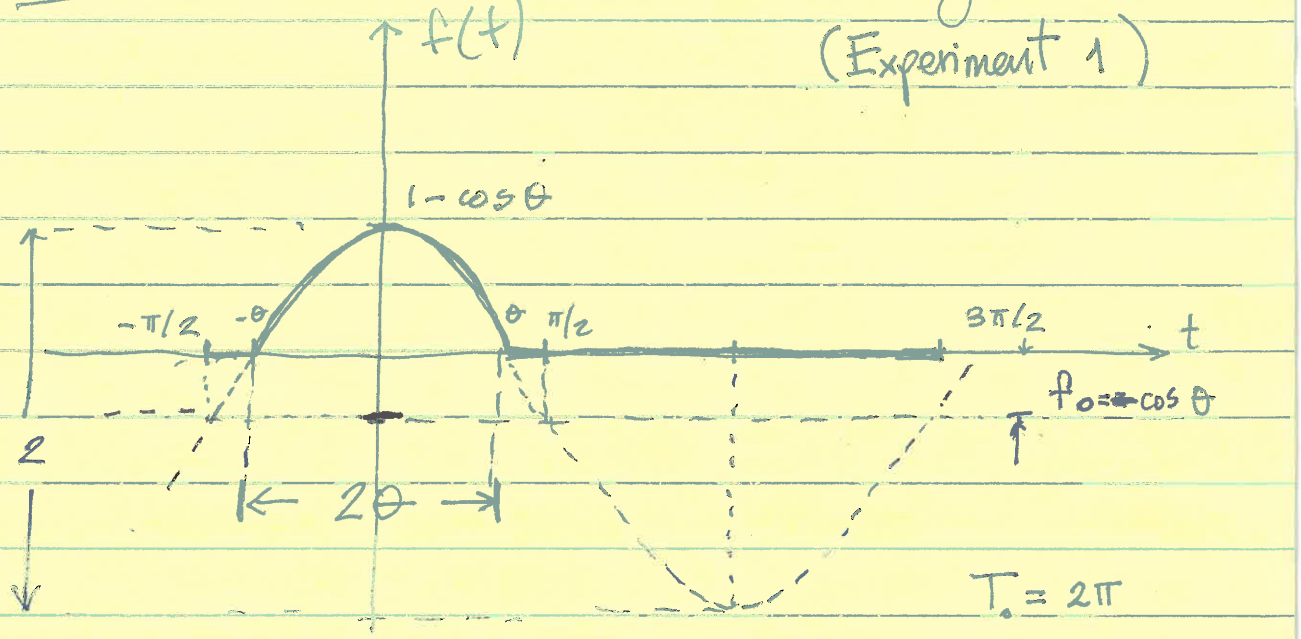
Clipped sinusoidal signal

(Experiment 1)

CLIPPED SINUSOIDAL



$T_0 = 2\pi$

$\omega_0 = 1$

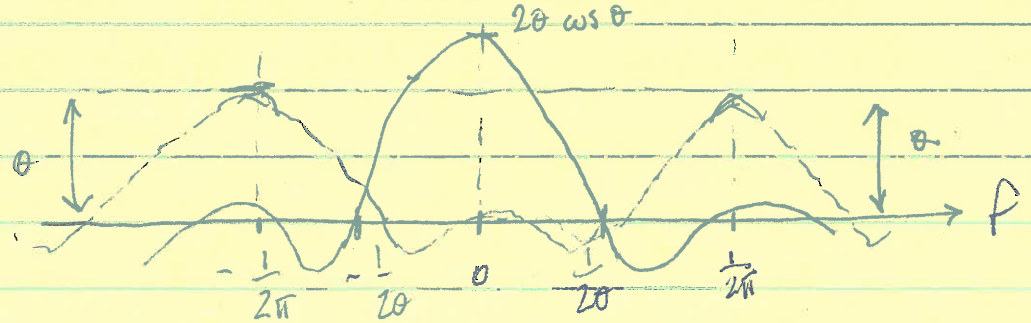
$f_0 = \frac{1}{2\pi}$

$x(t)$ $f(t) = \max\{0, \cos(t) - \cos\theta\}$

$x_T(t)$ $f_T(t) = \begin{cases} \cos(t) - \cos\theta, & |t| \le \theta; \\ 0, & |t| > \theta. \end{cases} = [\cos(t) - \cos\theta]\,\Pi\left(\frac{t}{2\theta}\right)$

$X_T(f)$ $F_T(f) = \left[\frac{1}{2}\delta\left(f - \frac{1}{2\pi}\right) + \frac{1}{2}\delta\left(f + \frac{1}{2\pi}\right) - \cos\theta\,\delta(f)\right] * 2\theta\,\mathrm{sinc}(2\theta f)$

$= \theta\,\mathrm{sinc}\left(2\theta\left(f - \frac{1}{2\pi}\right)\right) + \theta\,\mathrm{sinc}\left(2\theta\left(f + \frac{1}{2\pi}\right)\right) - 2\theta\cos\theta\,\mathrm{sinc}(2\theta f)$



$x_n \quad C_n = \frac{1}{T}\left[F_T(f)\right]_{f=\frac{n}{T}}, \qquad T = \frac{1}{2\pi}$ **2π**

$x_n \;\therefore\; C_n = \frac{1}{2\pi}\left[\theta\,\mathrm{sinc}\left(2\theta\left(\frac{n-1}{2\pi}\right)\right) + \theta\,\mathrm{sinc}\left(2\theta\left(\frac{n+1}{2\pi}\right)\right) - 2\theta\cdot\cos\theta\cdot\mathrm{sinc}\left(\frac{2\theta n}{2\pi}\right)\right]$

$x_n \quad \boxed{C_n = \frac{\theta}{2\pi}\left[\mathrm{sinc}\left(\frac{\theta}{\pi}(n-1)\right) + \mathrm{sinc}\left(\frac{\theta}{\pi}(n+1)\right) - 2\cos\theta\,\mathrm{sinc}\left(\frac{\theta}{\pi}n\right)\right]}$

Let $\boxed{d \triangleq \frac{\theta}{\pi}}, \quad 0 < \theta \leq \pi \quad \left(\text{or } d = \frac{2\theta}{2\pi} = \frac{\theta}{\pi}\right)$

$x_n \rightarrow \boxed{C_n = \frac{d}{2}\left[\mathrm{sinc}\left(d(n-1)\right) + \mathrm{sinc}\left(d(n+1)\right) - 2\cos\theta\,\mathrm{sinc}(dn)\right]}$

NOTE: The average (DC component) is $x_0 = -\cos(\theta)$

```
% Evaluation of the coefficients of the complex exponential Fourier
% series of a clipped sinusoidal signal. (Lab. experiment 1, EE 161)
% Copyright (c) 2008. Robert Morelos-Zaragoza. SJSU.
theta = 0:pi/100:pi;              % Conduction angle:
d = theta/pi;                     % Normalized conduction angle;
n=0;   % DC component
c0 = (d/2).*(sinc(d*(n-1))+sinc(d*(n+1))-2*cos(theta).*sinc(d*n));
n=1;   % Fundamental component
c1 = (d/2).*(sinc(d*(n-1))+sinc(d*(n+1))-2*cos(theta).*sinc(d*n));
n=2;   % Second harmonic component
c2 = (d/2).*(sinc(d*(n-1))+sinc(d*(n+1))-2*cos(theta).*sinc(d*n));
n=3;   % Third harmonic component
c3 = (d/2).*(sinc(d*(n-1))+sinc(d*(n+1))-2*cos(theta).*sinc(d*n));
n=4;   % Fourth harmonic component
c4 = (d/2).*(sinc(d*(n-1))+sinc(d*(n+1))-2*cos(theta).*sinc(d*n));
%
subplot(5,1,1), plot(theta,c0.^2), ylabel('|c_0|^2' ), axis tight, grid on
subplot(5,1,2), plot(theta,c1.^2), ylabel('|c_1|^2' ), axis tight, grid on
subplot(5,1,3), plot(theta,c2.^2), ylabel('|c_2|^2' ), axis tight, grid on
subplot(5,1,4), plot(theta,c3.^2), ylabel('|c_3|^2' ), axis tight, grid on
subplot(5,1,5), plot(theta,c4.^2), ylabel('|c_4|^2' ); axis tight, grid on
xlabel('\theta (rad)')
```

Matlab script "clipped_sinusoidal_FS.m"