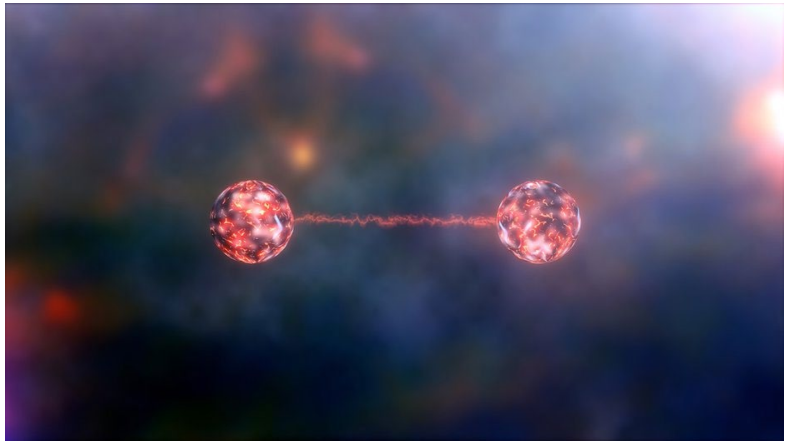


Quantum Communications

Robert Morelos-Zaragoza

EE160: Principles of Communication Systems

Department of Electrical Engineering

San José State University

SJSU SAN JOSÉ STATE UNIVERSITY

Light as both a waveform and photons [1]

- Classically, light belongs to the family of electromagnetic waves having a special property characterized by *polarization*
- Polarization is the orientation of oscillations in the plane perpendicular to a transverse wave's direction of travel
- Although quantum mechanically light behaves as a bunch of elementary particles (called photons), it preserves the polarization property

Light is both particles (photons) and an electromagnetic wave

Light as both a waveform and photons [1]

- Classically, light belongs to the family of electromagnetic waves having a special property characterized by *polarization*
- Polarization is the orientation of oscillations in the plane perpendicular to a transverse wave's direction of travel
- Although quantum mechanically light behaves as a bunch of elementary particles (called photons), it preserves the polarization property

Light is both particles (photons) and an electromagnetic wave

Quanta

Quantum states

Quantum states [2]

- Photons (light particles) have *quantum states*
- Light waves behave like particles and particles behave like waves (wave particle duality)
- Matter can go from one spot to another without moving through the intermediate space (quantum tunneling)
- Information can be moved across a vast distance without transmitting it through the intervening space (quantum teleportation)

Polarization of light photons [1]

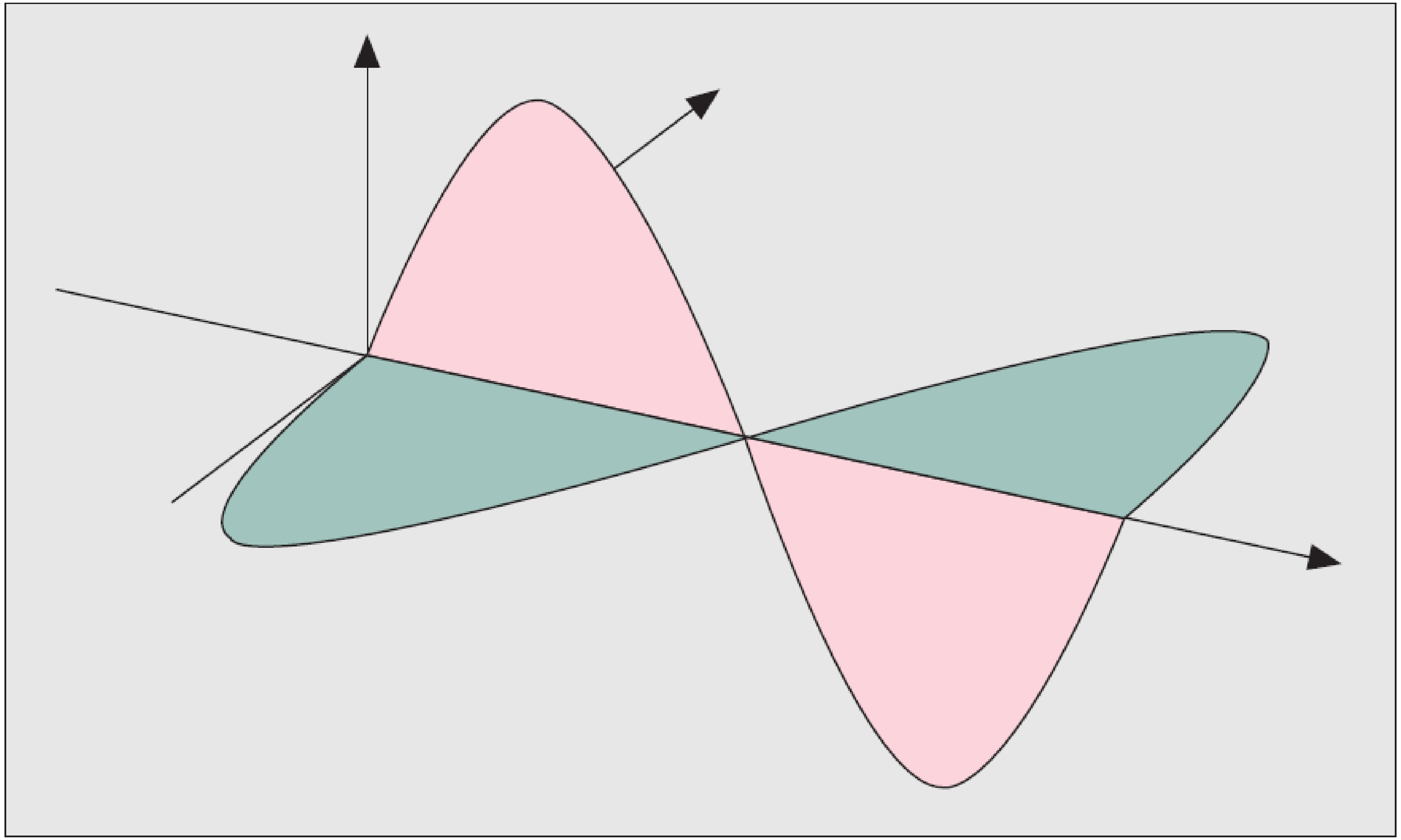


Figure 1. *Horizontal and vertical polarization of light.*

Vector representation [1]

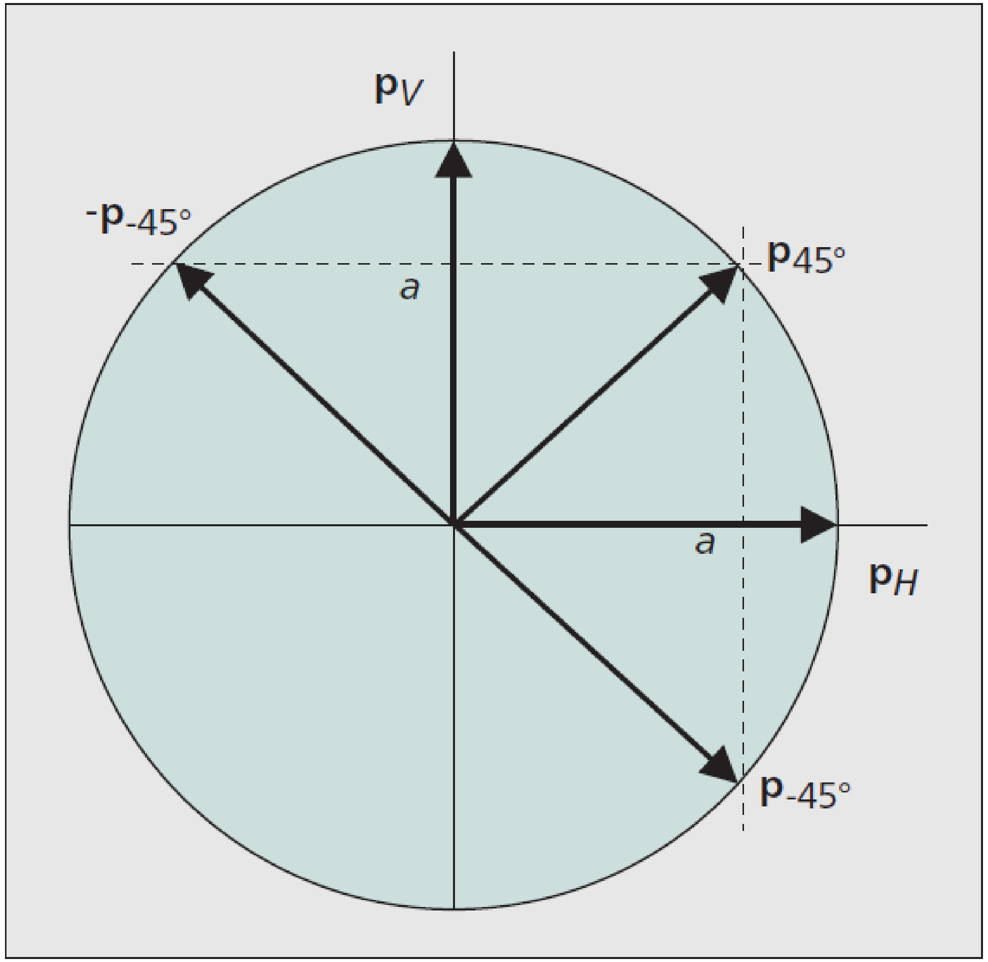


Figure 2. *Vector representation of different photon polarizations.*

Bloch sphere [3]

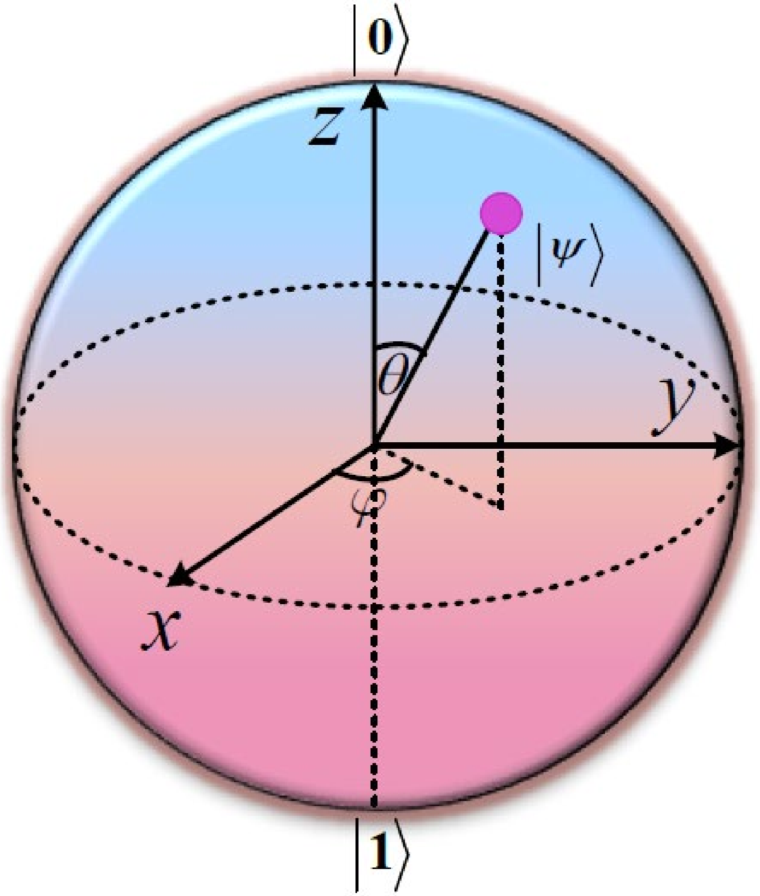


Fig. 4. The Bloch sphere representation of a single qubit.

Visualization of a quantum state [4]

- A practical way to visualize the quantum state of a single qubit is using the *Bloch sphere*. In general, the state of a qubit can be written as

$$|\psi\rangle \;=\; \cos\left(\frac{\vartheta}{2}\right)|0\rangle \;+ e^{-i\phi}\,\sin\left(\frac{\vartheta}{2}\right)|1\rangle$$

- The Bloch sphere represents the parameters ϑ and ϕ in spherical coordinates, respectively, as the *colatitude* with respect to the *z*-axis and the *longitude* with respect to the *x*-axis of the unit sphere.

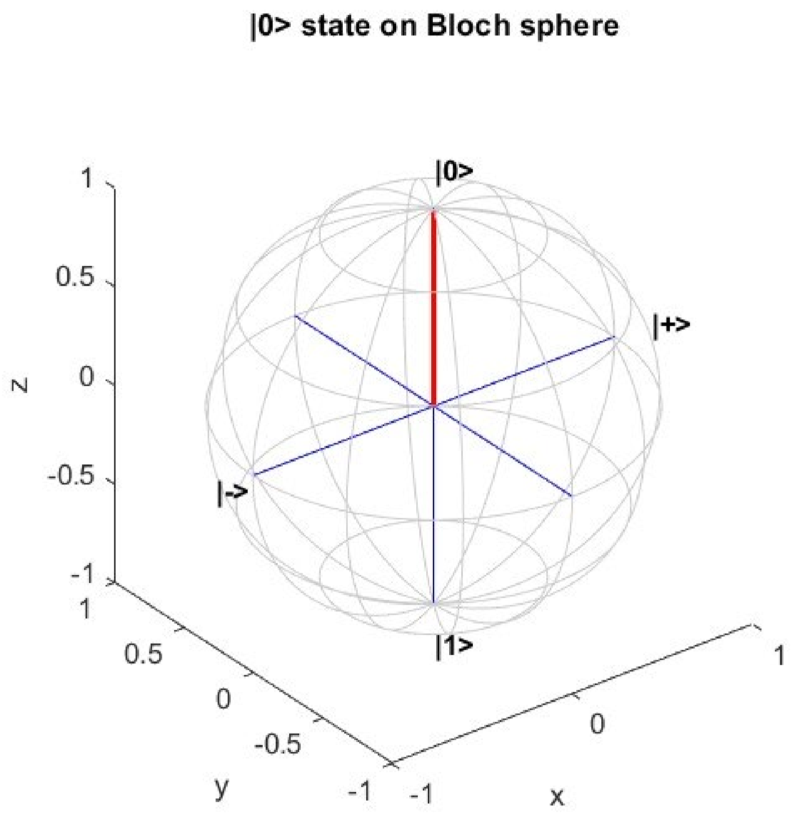
Visualization of a quantum state (cont.)

- The state is plotted as *a point **u** on the unit sphere* with coordinates

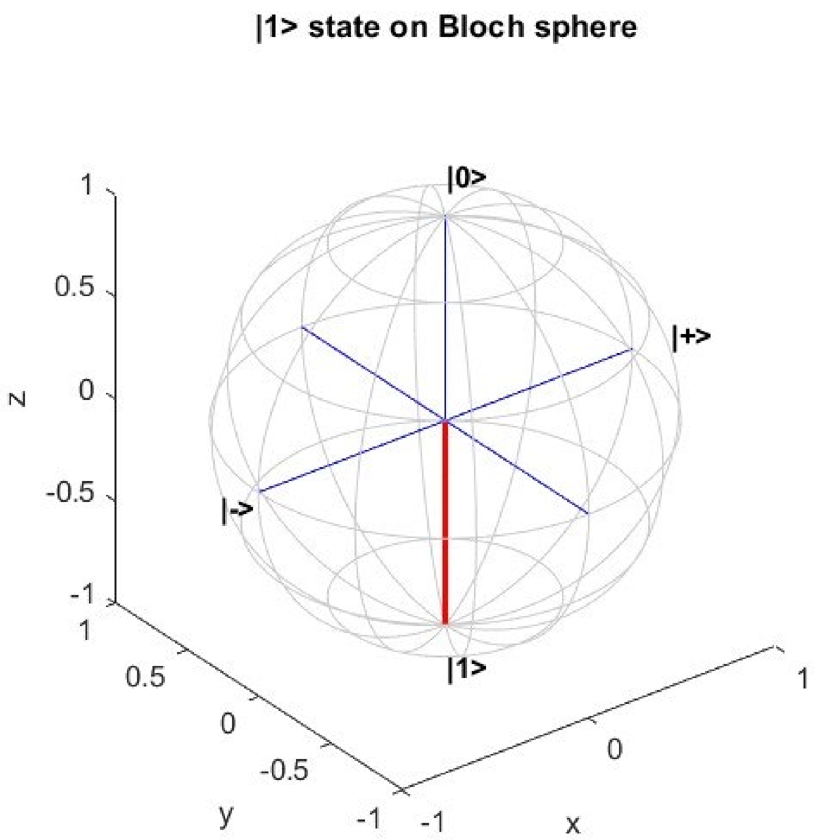
$$\mathbf{u} = (\sin\vartheta \cos\phi, \sin\vartheta \sin\phi, \cos\vartheta)$$

- By convention, the north pole is the $|0\rangle$ state, the south pole is the $|1\rangle$ state, and the equator is a linear combination of these two states with equal probability of measuring $|0\rangle$ or $|1\rangle$.

Bloch sphere quantum state representations [4]

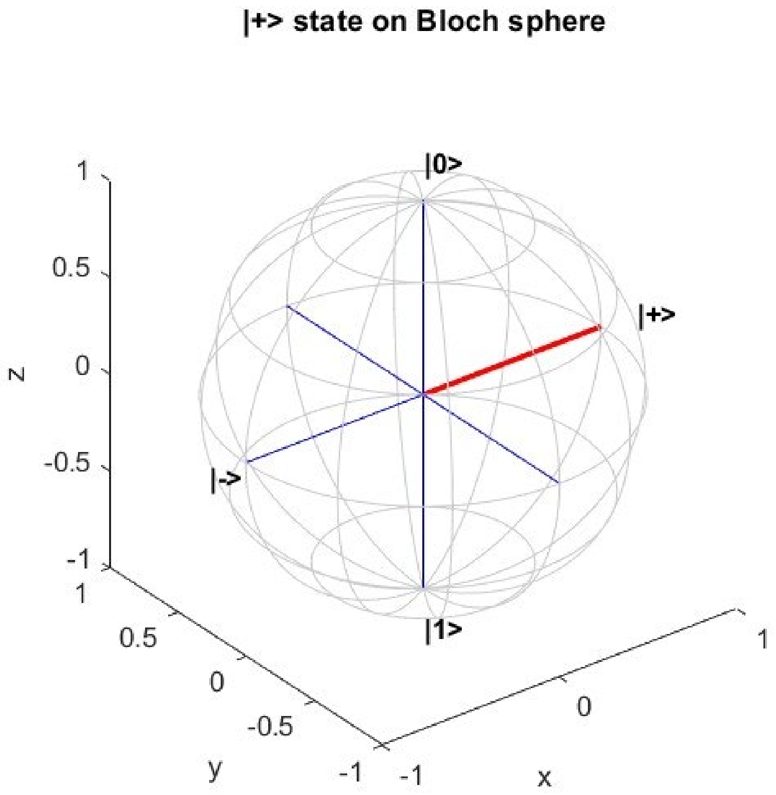


|0⟩ state on the Bloch sphere

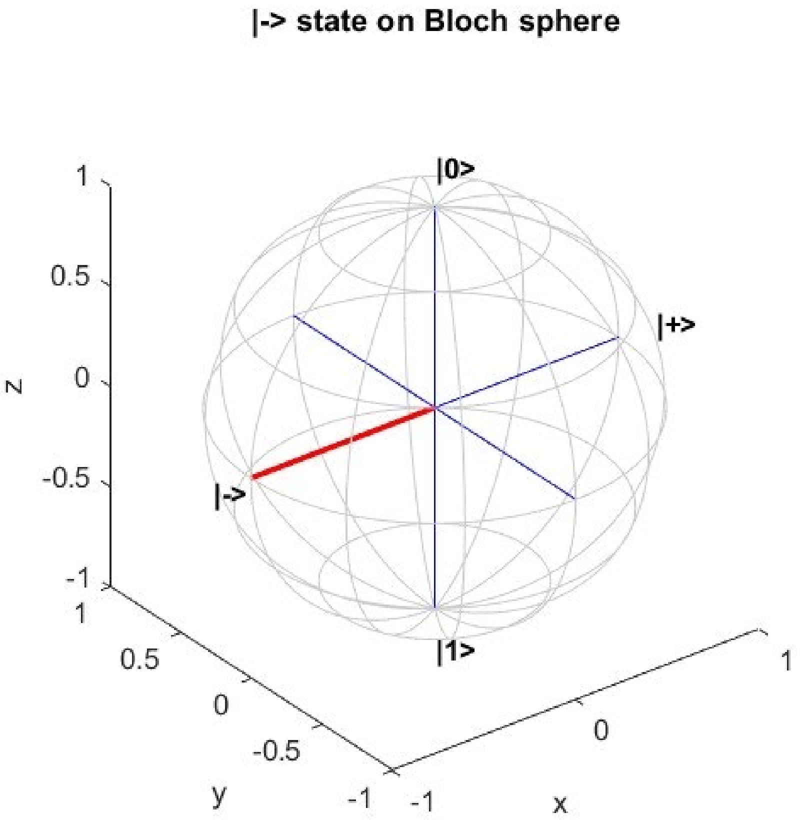


|1⟩ state on the Bloch sphere

Bloch sphere quantum state representations [4]

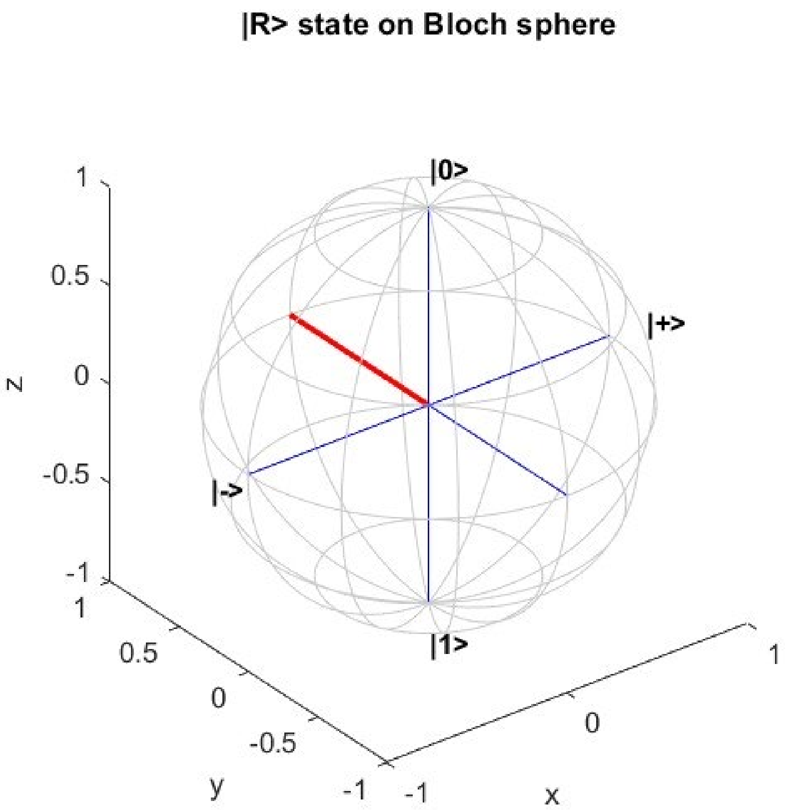


|+⟩=(|0⟩+|1⟩)/√2 state on the Bloch sphere

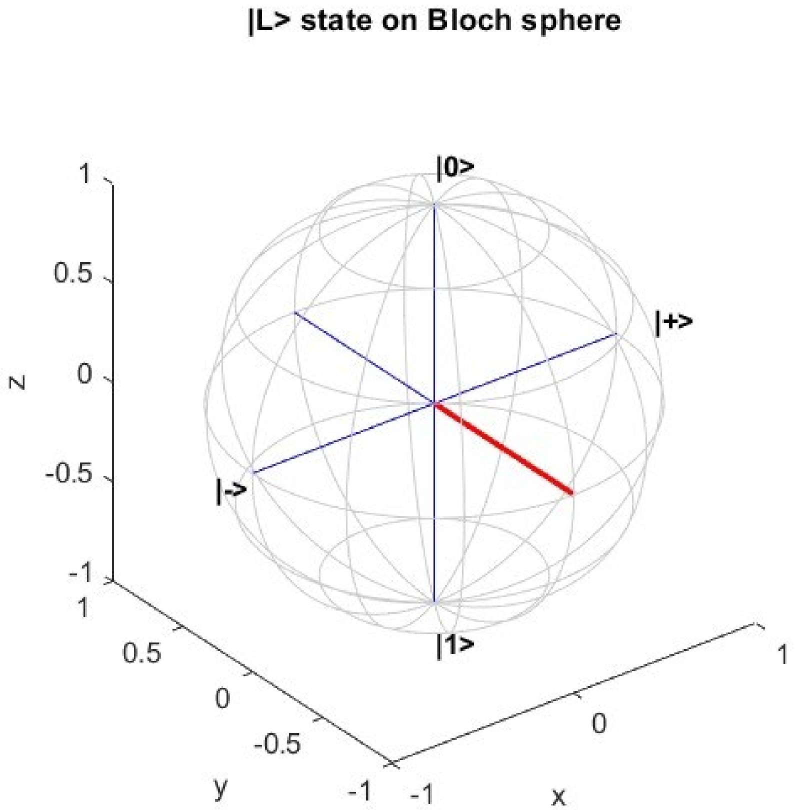


|−⟩=(|0⟩−|1⟩)/√2 state on the Bloch sphere

Bloch sphere quantum state representations [4]

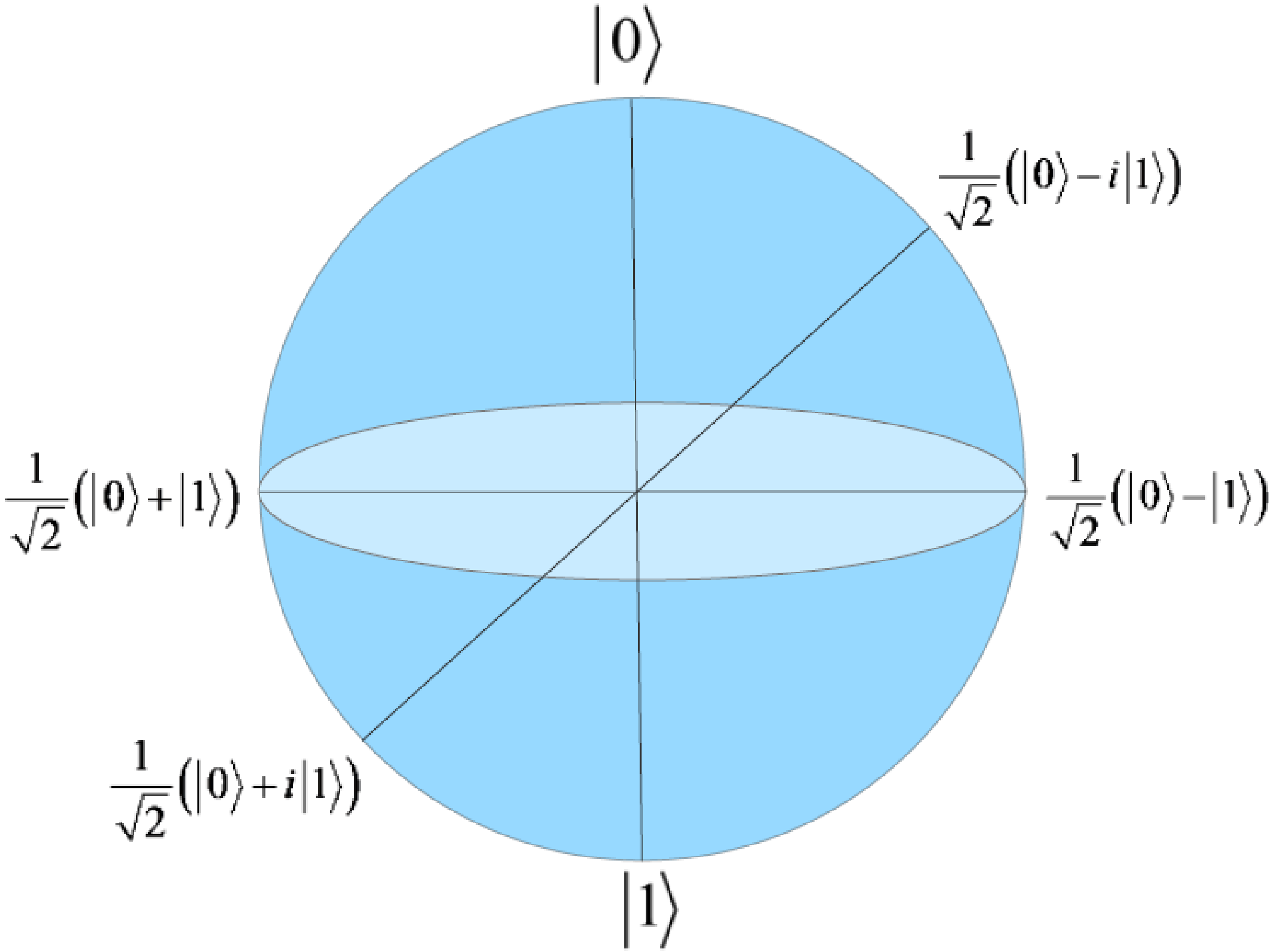


|R⟩=(|0⟩+i|1⟩)/√2 state on the Bloch sphere



|L⟩=(|0⟩-i|1⟩)/√2 state on the Bloch sphere

Three orthogonal bases for quantum states [6]



Quantum bits: Qubits [2]

- Just like a classical bit with state either 0 or 1, a *qubit* also has states $|0\rangle$ and $|1\rangle$

- Besides states $|0\rangle$ and $|1\rangle$, a qubit may take the superposition states,

$$|\psi\rangle = \alpha_o\,|0\rangle + \ \alpha_1\,|1\rangle,$$

  where α_o and α_1 are complex numbers called amplitudes satisfying

$$|\alpha_o|^2 + |\alpha_1|^2 = 1.$$

- The state of a qubit can be interpreted as a ***two-dimensional complex valued vector***

Classical bits and qubits [2]

- For a classical bit we can examine it to determine whether it is in the state 0 or 1
- However, for a qubit we cannot determine its state and find the values of α_o and α_1 by examining it.

Qubit measurements [2]

- For qubit state $|\psi\rangle$ the only measurable quantities are the *probabilities* $|\alpha_o|^2$ and $|\alpha_1|^2$, since $\left|e^{i\theta}\alpha_x\right|^2 = |\alpha_x|^2$, where θ is a real number

- From the viewpoint of the qubit measurements, states $e^{i\theta}|\psi\rangle$ and $|\psi\rangle$ are identical. That is, multiplying a qubit state by a global phase factor $e^{i\theta}$ bears no observational consequence

Quantum computation

Quantum computers [2]

- A superposition state is a state of matter which we may think of as both one and zero at the same time
- Quantum computers use the *strange superposition* states and *quantum entanglements* to do the trick of performing simultaneous calculations and extracting the calculated results
- The ***spooky phenomena*** of quantum entanglement and superposition are the key that enables quantum computers to be superfast and vastly outperform classical computers.

Quantum states: Representation

As another example, create a quantum state where the two elements of the vector have different amplitudes and different absolute values. If you call `quantum.gate.QuantumState` with a general complex vector, then the constructor creates a quantum state that is normalized. For example, create a quantum state and find its measurement probabilities.

```
state = quantum.gate.QuantumState([3 -4i]);
state.Amplitudes
```

```
ans =

   0.6000 + 0.0000i
   0.0000 - 0.8000i
```

$$3\,|0\succ \;-\; 4i\,|1> \;\longrightarrow\; 3\,|0\succ \;-\; 4i\,|1>$$

$$\longrightarrow\; \frac{3}{\sqrt{3^2+4^2}}\,|0\succ \;-\; \frac{4i}{\sqrt{3^2+4^2}}\,|1> \;=\; 0.6\,|0\succ -0.8i\,|1>$$

```
[states,probabilities] = querystates(state)
```

```
states =

  2×1 string array

    "0"
    "1"


probabilities =

    0.3600
    0.6400
```

$= (0.6)^2$

$= (0.8)^2$

Quantum gates

Pauli-X gate [11]

- Equivalent to a binary NOT gate or a "bit-flip"
- Matrix representation:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- Operation: Express a qubit $\boxed{Q = a_0 \; |0> \; + a_1 \; |1>}$ in vector form

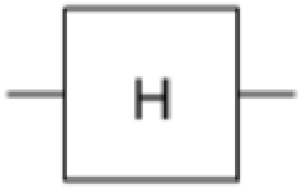
$$Q = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

- Then

$$\bar{Q} = QX = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}, \text{ or } \boxed{\bar{Q} = a_1 \; |0> \; + a_0 \; |1>}$$

Hadamard gate [2,4]

$$|0\rangle \to \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |1\rangle \to \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

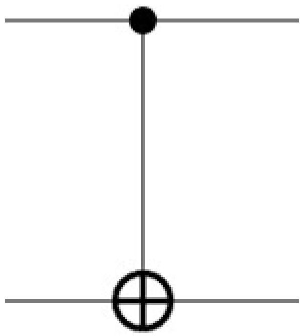
Creation Function	Gate Name	No. of Qubits	Matrix Representation
 `hGate`	Hadamard gate	1	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

CNOT gate [11]

The CNOT gate is two-qubit operation, where the first qubit is usually referred to as the control qubit and the second qubit as the target qubit. Expressed in basis states, the CNOT gate:

- leaves the control qubit unchanged and performs a Pauli-X gate on the target qubit when the control qubit is in state $|1\rangle$;
- leaves the target qubit unchanged when the control qubit is in state $|0\rangle$.

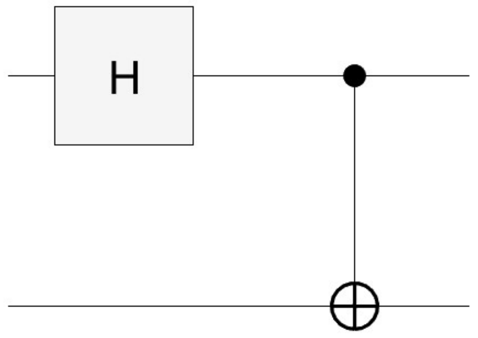
$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



Quantum states: MATLAB Simulation

MATLAB Quantum State Simulation: Hadamard and controlled X gates to entangle two qubits [8]

MATLAB script	**Result**

gates = [hGate(1); cxGate(1,2)];
c = quantumCircuit(gates);
 	
figure(100), plot(c)
	

% Simulate the circuit using the default initial state % where **each qubit is in the |0⟩ state**. s = simulate(c)	s = QuantumState with properties: BasisStates: [4×1 string] Amplitudes: [4×1 double] NumQubits: 2

MATLAB Quantum State Simulation: Hadamard and controlled X gates to entangle two qubits [8] – Cont.

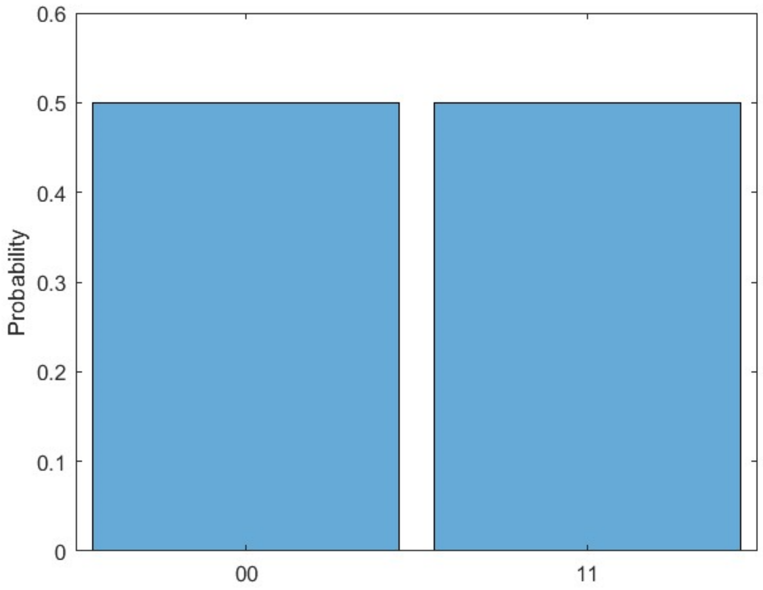
MATLAB script	Result
% Display the basis states by inspecting the % properties of the resulting quantum gate s.BasisStates	ans = 4×1 string array **"00"** "01" "10" **"11"** "Given that the initial values are 0"
% Display the amplitudes, by inspecting the % properties of the resulting quantum gate s.Amplitudes	ans = **0.7071** 0 0 **0.7071**

SJSU SAN JOSÉ STATE UNIVERSITY

MATLAB Quantum State Simulation: Hadamard and controlled X gates to entangle two qubits [8] – Cont.

MATLAB script	Result

% Show the final state of the circuit.
 	
formula(s)
	`ans =`
	
	`"0.70711 * |00> +`
	`0.70711 * |11>"`

% Plot the histogram of probabilities to measure % possible states from the final state of the circuit. histogram(s)	

Hadamard and controlled X gates: *All states*

MATLAB script	Result
% Create a quantum circuit that consists of a % controlled X gate. c = quantumCircuit(cxGate(1,2));	

% Simulate the circuit using initial states of
% "00", "01", "10", and "11". Show the final
% state of the circuit for each initial state
% after running the circuit.
 	
for ket=["00", "01", "10", "11"]
s = simulate(c,ket);
disp("|" + ket + "> -> " + formula(s));
end
	>> test2
	
	|00> -> 1 * |00>
	|01> -> 1 * |01>
	|10> -> 1 * |11>
	|11> -> 1 * |10>

MATLAB Application: Quantum Neural Network

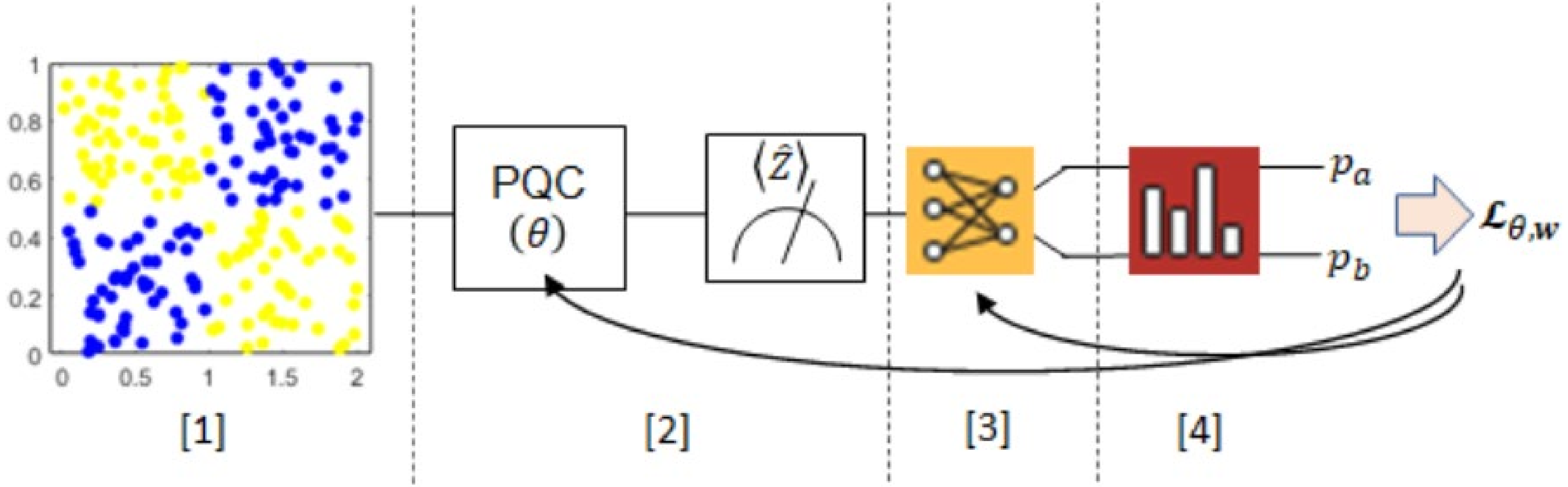
Quantum Neural Network (QNN) [9]

- A QNN is a machine learning model that ***combines quantum computing layers and classical layers***. This example shows how to train such a hybrid network for a classification problem that is nonlinearly separable, such as the **exclusive-OR (XOR) problem**
- In the XOR problem, two-dimensional (2-D) data ***points are classified based on the region of their x- and y-coordinates*** using a mapping function that resembles the XOR function. ***If the x- and y-coordinates are both in region* 0 *or* 1, *then the data are classified into class* "0"**. Otherwise, the data are classified into class "1".

Quantum Neural Network (QNN) (cont.) [9]

- In this problem, a single linear decision boundary cannot solve the classification problem. Instead, nonlinear decision boundaries are required to classify the data
- Training a QNN using a local simulation. The QNN in this example consists of four layers:

Matlab example [9]

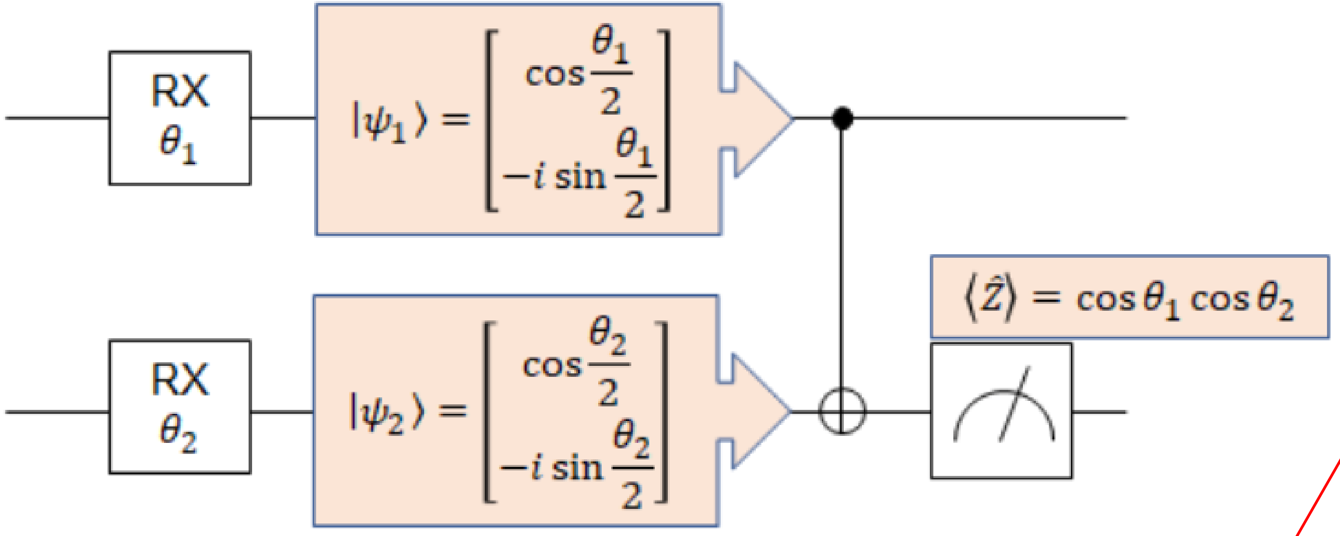


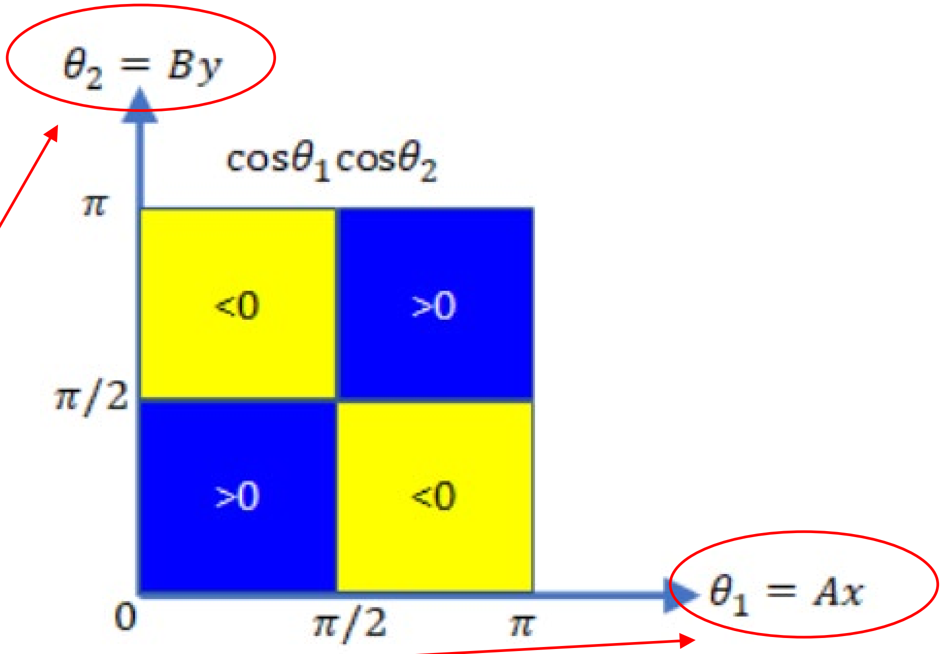
A feature input layer for the XOR problem

A parameterized quantum circuit (PQC) to prepare states of qubits according to the coordinates of the input data and function to compute probabilities

Two-output softmax layer outputs probabilities for data classification.

Parameterized quantum circuit





These gates prepare the states of qubits according to the coordinates of the input data by introducing two adjustable learnable parameters A and B in the rotation angles.

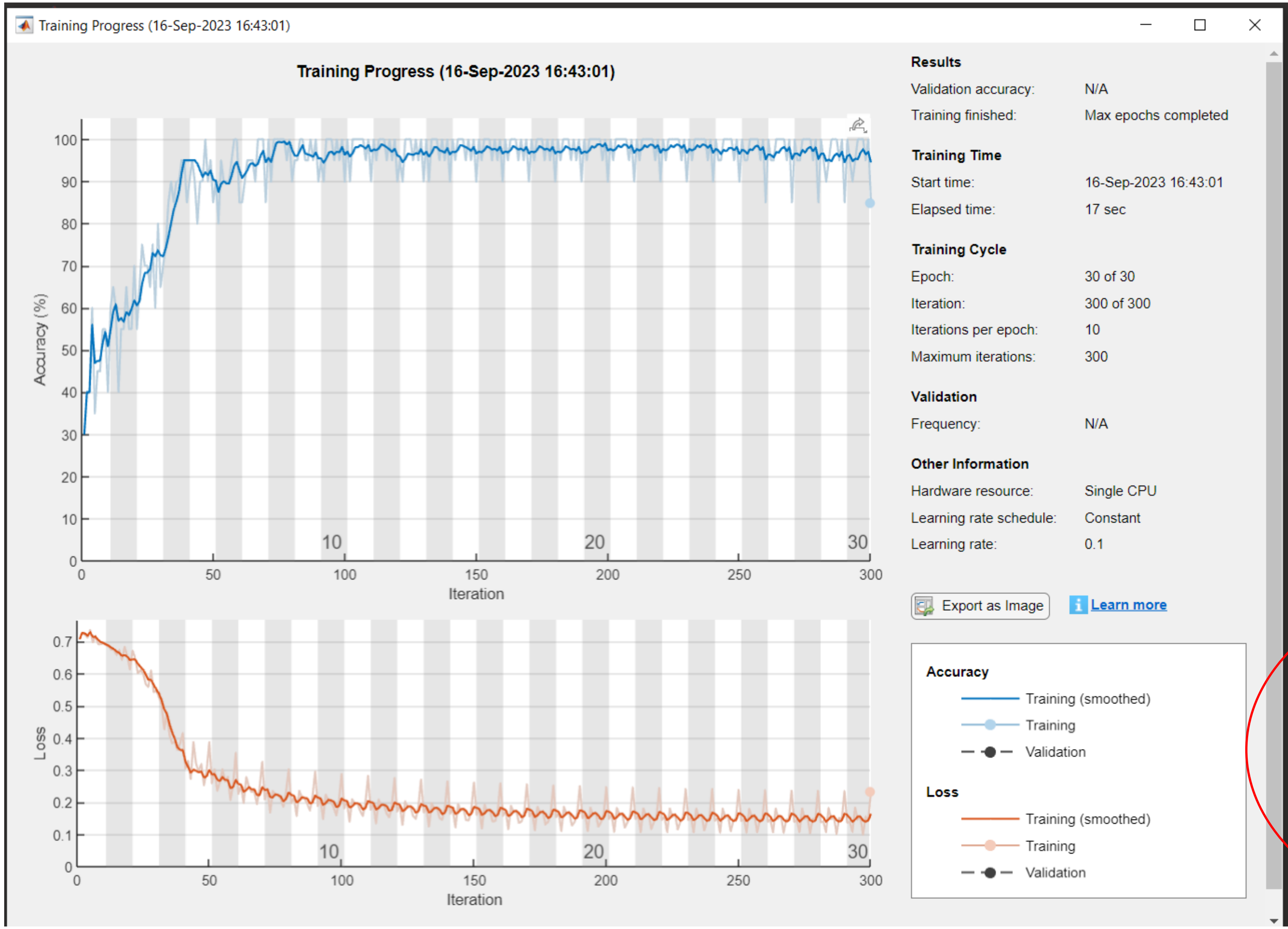
Parameterized quantum circuit (cont.)

- The quantity of interest is the *magnetization of the second qubit*, which is the difference in counts of this qubit being in the $|0\rangle$ state and the $|1\rangle$ state

- For this quantum circuit, the measured quantity $\langle Z \rangle$ has a predicted form of $\cos\theta_1 \cos\theta_2$ based on the states of the qubits.

- The condition $\langle Z \rangle = 0$ determines the classification boundaries of the XOR problem.

Results [9]



Training Progress (16-Sep-2023 16:43:01)
Training Progress (16-Sep-2023 16:43:01)
Accuracy (%)
Iteration
Loss
Iteration
Results
Validation accuracy: N/A
Training finished: Max epochs completed
Training Time
Start time: 16-Sep-2023 16:43:01
Elapsed time: 17 sec
Training Cycle
Epoch: 30 of 30
Iteration: 300 of 300
Iterations per epoch: 10
Maximum iterations: 300
Validation
Frequency: N/A
Other Information
Hardware resource: Single CPU
Learning rate schedule: Constant
Learning rate: 0.1
Export as Image
Learn more
Accuracy
Training (smoothed)
Training
Validation
Loss
Training (smoothed)
Training
Validation

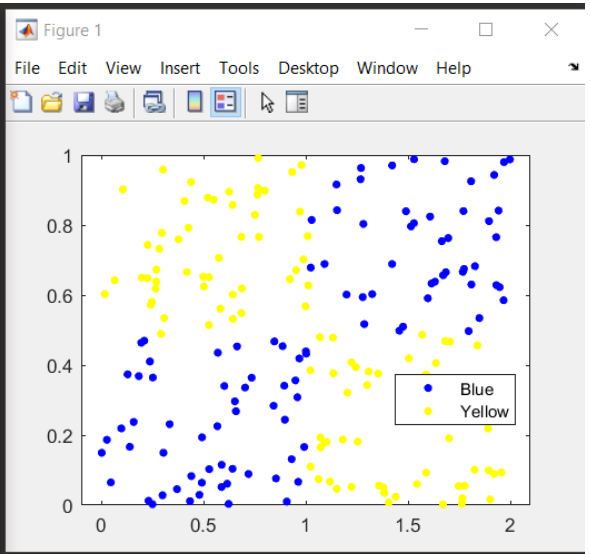


Figure 1
File Edit View Insert Tools Desktop Window Help
Blue
Yellow

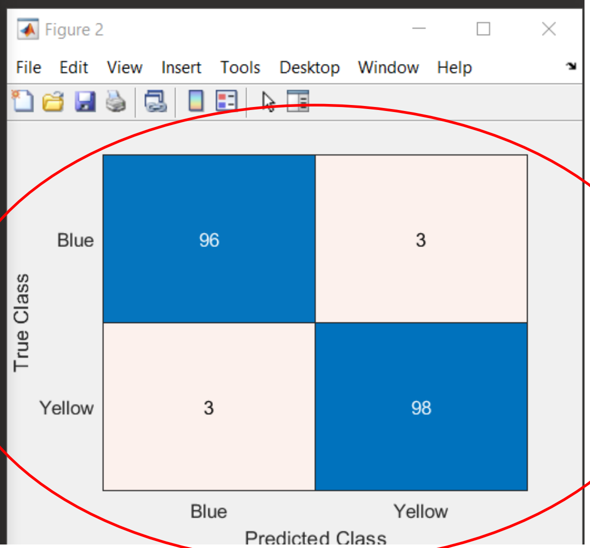


Figure 2
File Edit View Insert Tools Desktop Window Help
True Class
Blue
Yellow
96
3
3
98
Blue
Yellow
Predicted Class

MATLAB Quantum Computation Homework

- **Download MATLAB 2023a or later version**
- **Canvas under *files/Matlab/Quantum***

Quantum communication

Quantum communication [7]

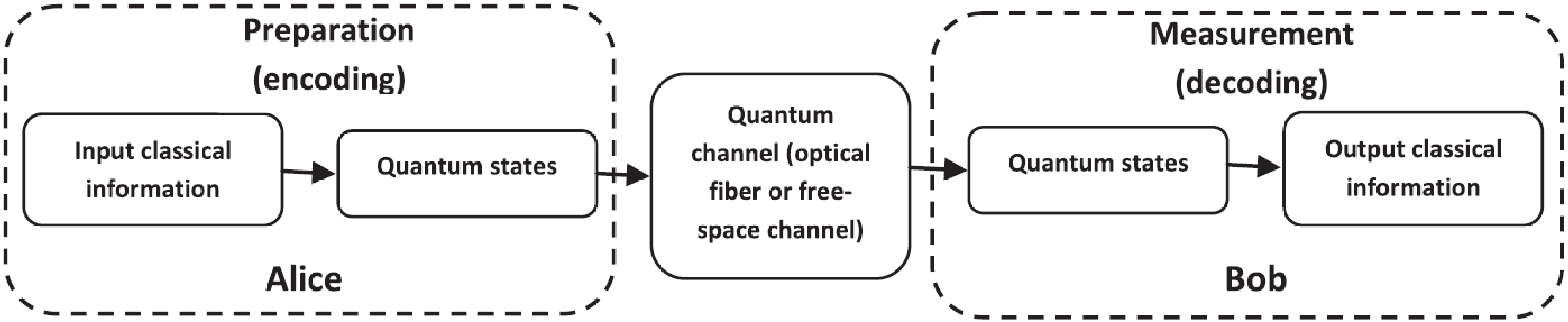
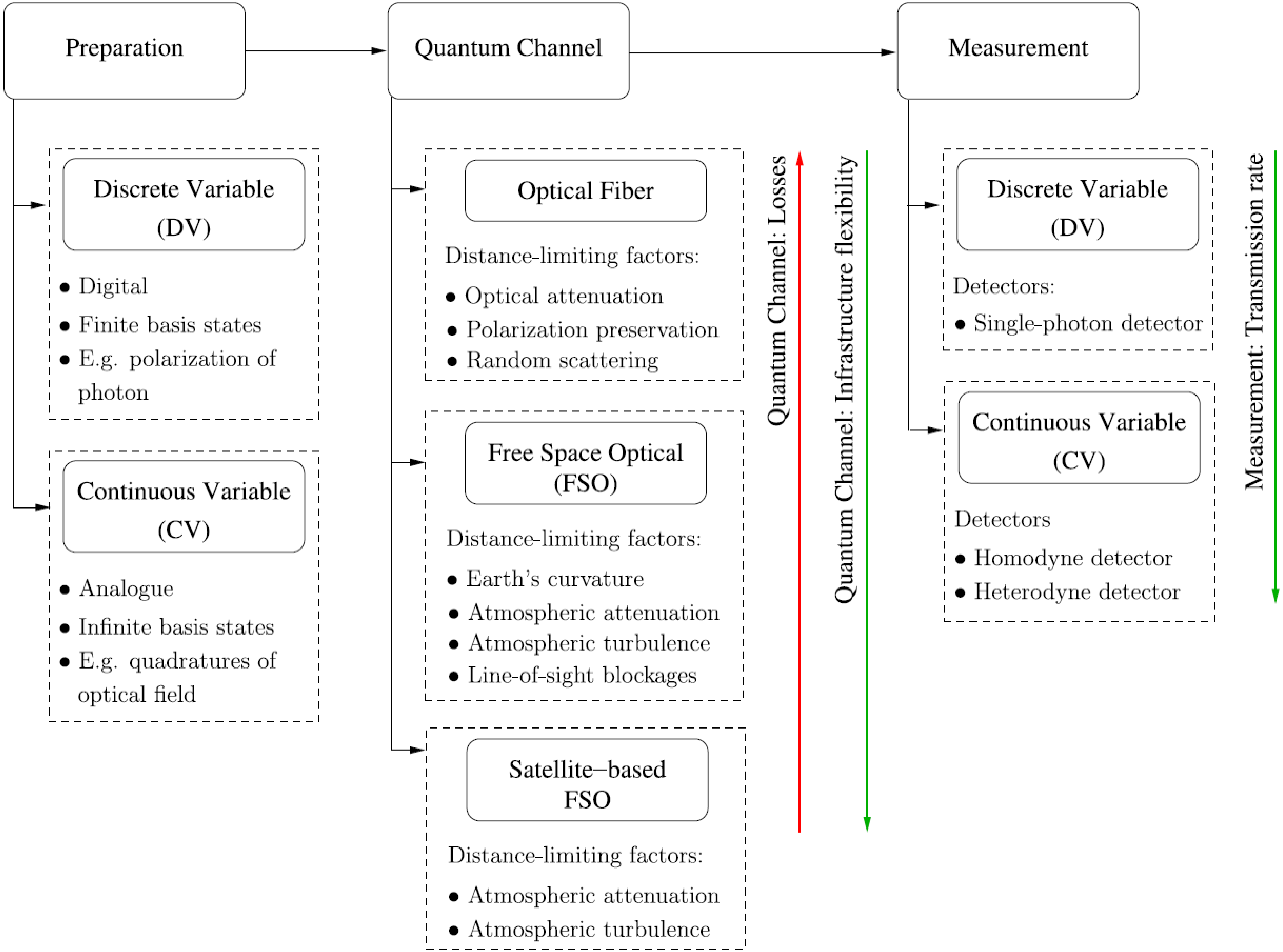
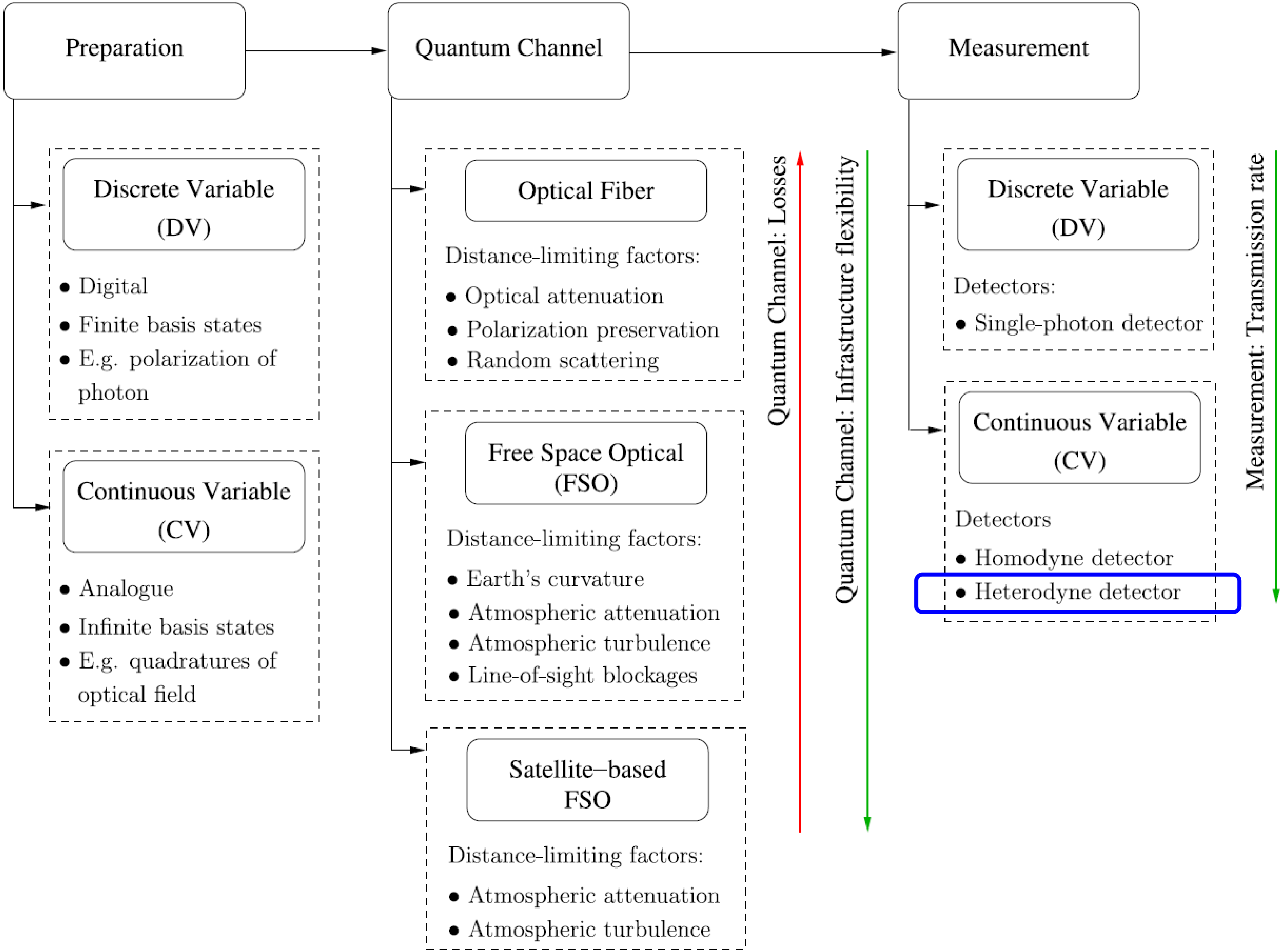


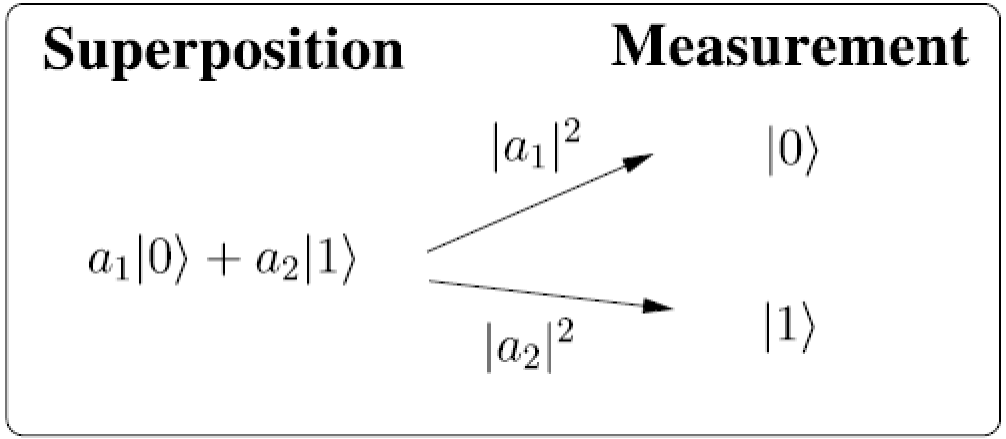
Fig. 2. Basic quantum communications schematic for transmitting classical information over a secure quantum channel. **Preparation**: Encoding classical information into quantum states. **Channel**: Secure quantum transmission using optical fiber or free space optical. **Measurement**: Decoding the received quantum states, yielding classical information.


Preparation
Quantum Channel
Measurement
Discrete Variable (DV)
• Digital
• Finite basis states
• E.g. polarization of photon
Continuous Variable (CV)
• Analogue
• Infinite basis states
• E.g. quadratures of optical field
Optical Fiber
Distance-limiting factors:
• Optical attenuation
• Polarization preservation
• Random scattering
Free Space Optical (FSO)
Distance-limiting factors:
• Earth's curvature
• Atmospheric attenuation
• Atmospheric turbulence
• Line-of-sight blockages
Satellite–based FSO
Distance-limiting factors:
• Atmospheric attenuation
• Atmospheric turbulence
Quantum Channel: Losses
Quantum Channel: Infrastructure flexibility
Discrete Variable (DV)
Detectors:
• Single-photon detector
Continuous Variable (CV)
Detectors
• Homodyne detector
• Heterodyne detector
Measurement: Transmission rate



Preparation
Quantum Channel
Measurement
Discrete Variable (DV)
• Digital
• Finite basis states
• E.g. polarization of photon
Continuous Variable (CV)
• Analogue
• Infinite basis states
• E.g. quadratures of optical field
Optical Fiber
Distance-limiting factors:
• Optical attenuation
• Polarization preservation
• Random scattering
Free Space Optical (FSO)
Distance-limiting factors:
• Earth's curvature
• Atmospheric attenuation
• Atmospheric turbulence
• Line-of-sight blockages
Satellite–based FSO
Distance-limiting factors:
• Atmospheric attenuation
• Atmospheric turbulence
Quantum Channel: Losses
Quantum Channel: Infrastructure flexibility
Discrete Variable (DV)
Detectors:
• Single-photon detector
Continuous Variable (CV)
Detectors
• Homodyne detector
• Heterodyne detector
Measurement: Transmission rate

Qubit superposition and measurement [7]



Superposition & Measurement: A qubit exists in superposition of the states $|0\rangle$ and $|1\rangle$. However, when measured, it collapses to the state $|0\rangle$ with a probability of $|a_1|^2$ and the state $|1\rangle$ with a probability of $|a_2|^2$. Hence, measurement of the qubit perturbs its coherent superposition.

Qubit no-cloning [7]

No–cloning Theorem

$$\mathcal{U}_c|0\rangle \rightarrow |0\rangle|0\rangle$$
$$\mathcal{U}_c|1\rangle \rightarrow |1\rangle|1\rangle$$
$$\mathcal{U}_c(a_1|0\rangle + a_2|1\rangle) \neq a_1\mathcal{U}_c|0\rangle + a_2\mathcal{U}_c|1\rangle$$

No-cloning Theorem: An arbitrary quantum state cannot be cloned. Assume a hypothetical cloning operator $\mathcal{U}_c$, it is straightforward to show that cloning of a state $|\psi\rangle$ is not equivalent to cloning the constituent basis states, hence a quantum cloning operator $\mathcal{U}_c$ does not exist.

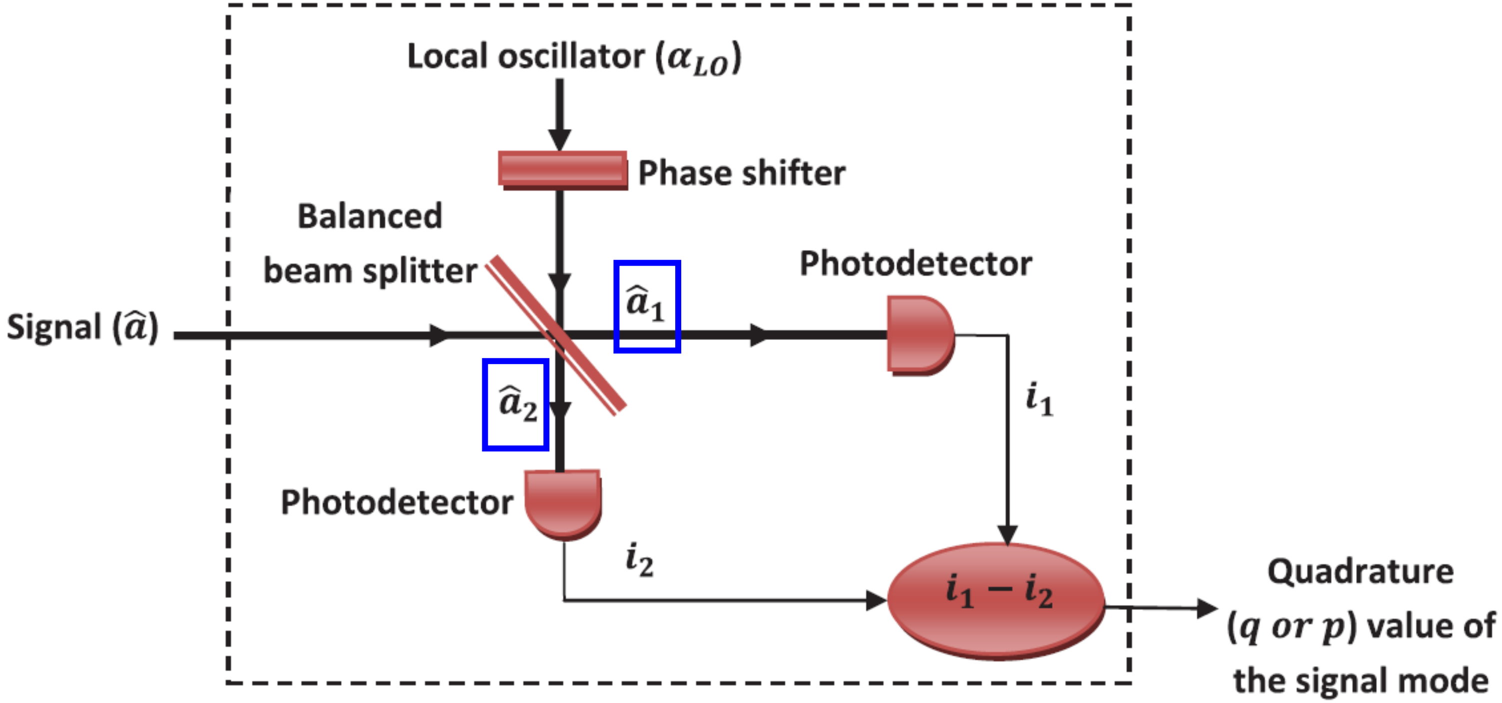
Qubit entanglement [7]

Entanglement

$$a_1|00\rangle + a_2|11\rangle \begin{array}{l} \xrightarrow{|a_1|^2} |00\rangle \\ \xrightarrow[|a_2|^2]{} |11\rangle \end{array}$$

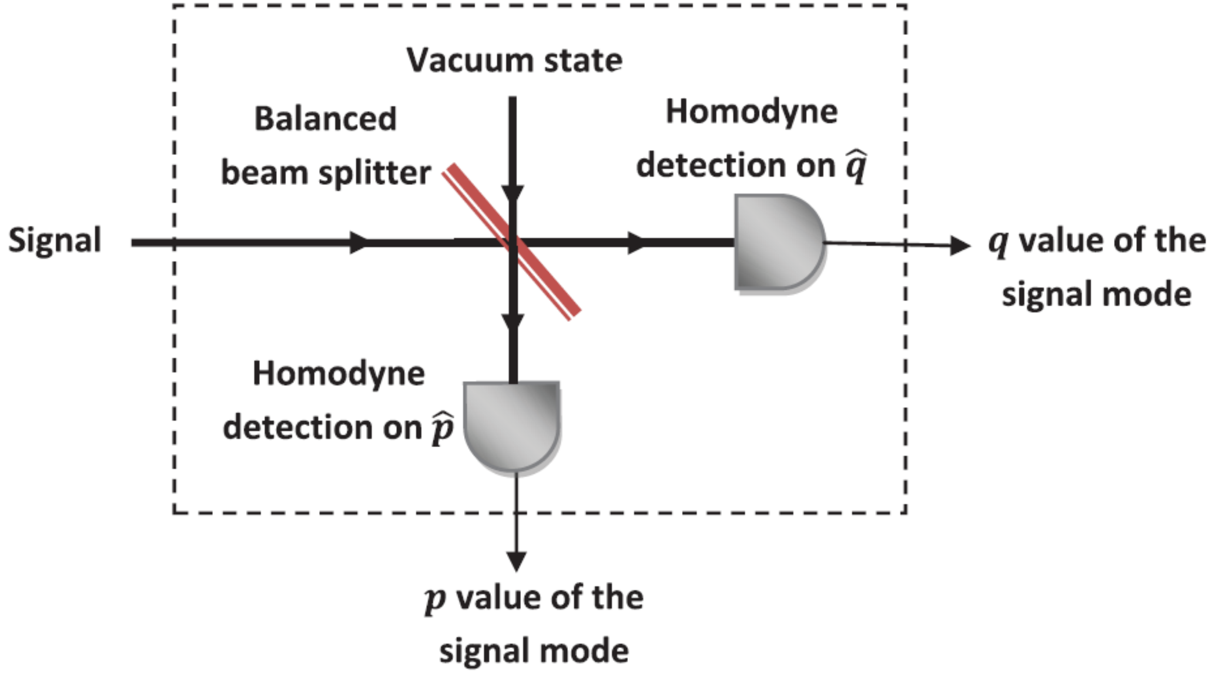
Entanglement: Qubits are said to be entangled, if measuring one qubit reveals information on the value of the other. In the example given, if the first qubit is found to be in the state $|0\rangle$ (or $|1\rangle$) upon measurement, then the second qubit also exists in the state $|0\rangle$ (or $|1\rangle$), hence a mysterious relation exists between the two entangled qubits.

Homodyne detector [7]



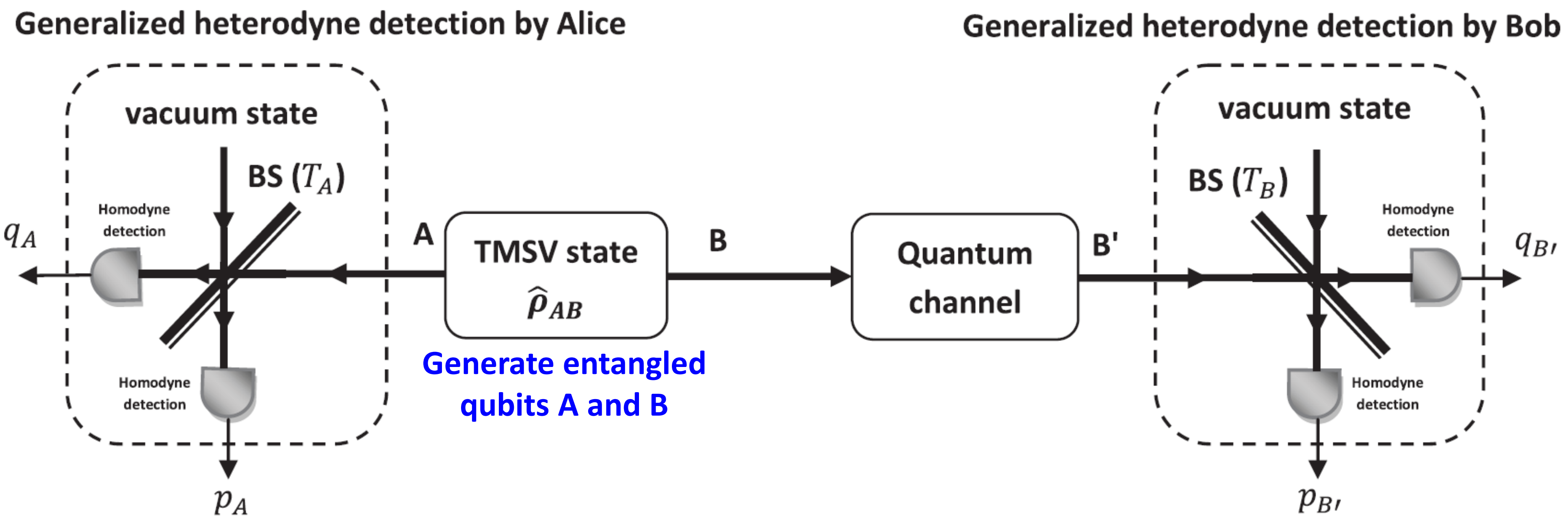
Homodyne detection: The signal mode is combined with the local oscillator in a balanced beam splitter. Each output mode of the beam splitter is then measured using a photodetector, which generates a photo-current proportional to the photon numbers of the output mode. By measuring the difference between the two photo-currents, the $\hat{q}$ (or $\hat{p}$) quadrature operator of the signal mode can be measured depending on the phase of the local oscillator.

Heterodyne detector [7]



Heterodyne detection: The signal mode interacts with a vacuum mode in a balanced beam splitter. By applying homodyne detection to the conjugate quadratures of the two output modes, both the quadrature operators of the signal mode can be measured simultaneously at the price of introducing an additional noise term into the measurements.

Entangled qubit transmission (Crypto app)



Satellite-based quantum communication – I

(a) (b) (c)

(a), (b) Quantum states transmitted to/from satellite and ground station (GS)

(c) Quantum states transmitted from GS to satellite and then reflected by satellite

Satellite-based quantum communication – II

(d)

(e)

(d) Quantum states generated by satellite and transmitted to two separate GS’s

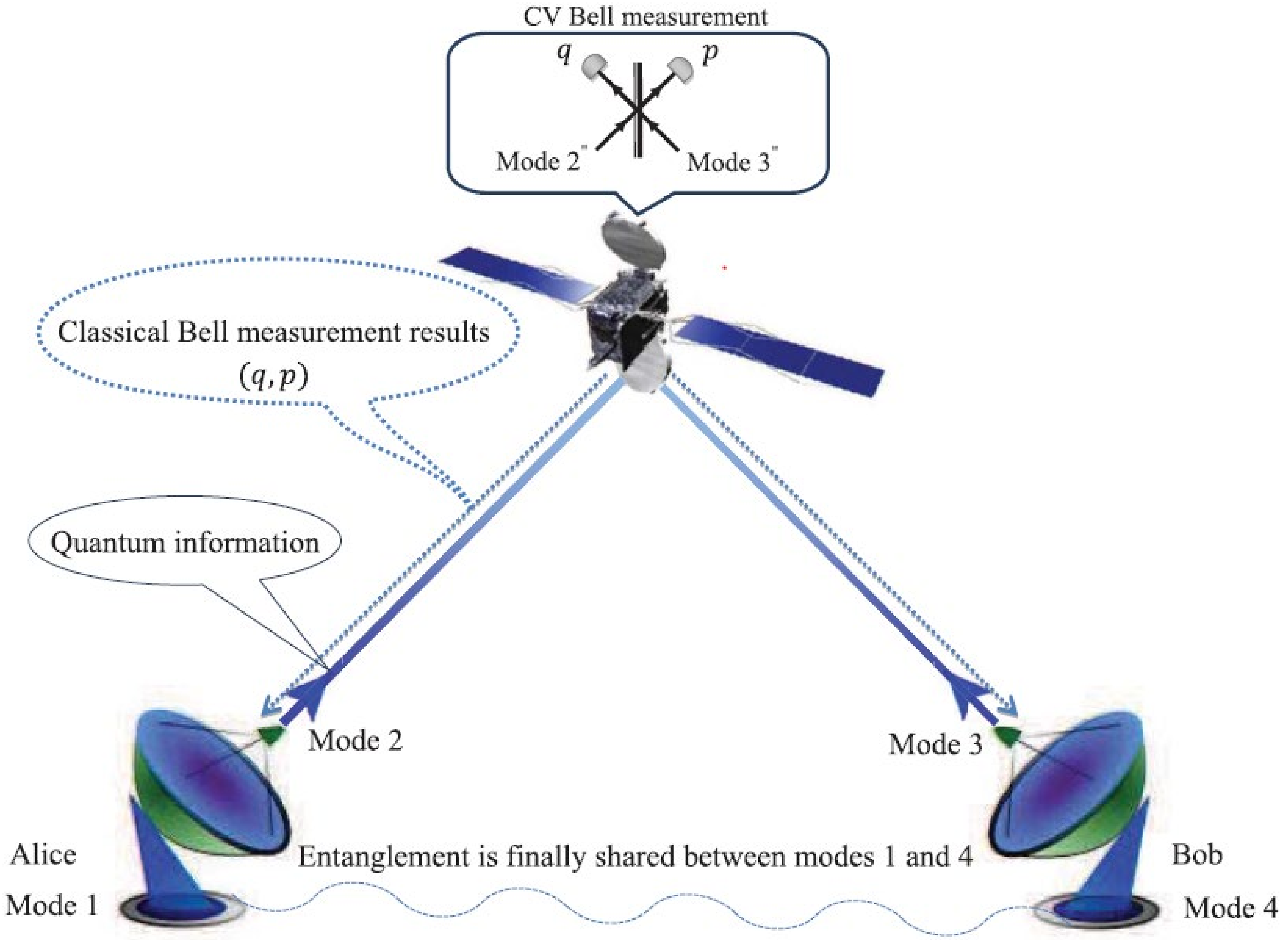
(e) Two separate GS’s transmit independent quantum states to a satellite which performs measurements and retransmits results

Entanglement between two ground stations [7]

Secure communication scheme based on scheme (e) above.

The two modes are mixed through a balanced beam splitter: The **q** quadrature of one of the output modes of the beam splitter and the **p** quadrature of the output mode are separately measured by two homodyne detectors

Alice and Bob displace their modes, according to the measurement outcome



References

1. S. Imre, “Quantum communications: explained for communication engineers,” *IEEE Communications Magazine*, vol. 51, no. 8, pp. 28-35, August 2013, doi: 10.1109/MCOM.2013.6576335
2. Y. Wang, “Quantum Computation and Quantum Information,” *Statistical Science*, vol. 27, no. 3, August 2012, doi: 10.1214/11-sts378
3. Z. Li et al., "Entanglement-Assisted Quantum Networks: Mechanics, Enabling Technologies, Challenges, and Research Directions," *IEEE Communications Surveys & Tutorials*, doi: 10.1109/COMST.2023.3294240.
4. *MATLAB Support Package for Quantum Computing*, MATLAB v. 2023b
5. *IEEE Transactions on Quantum Engineering*

References (cont.)

6. S. R. Hasan et al., “Quantum Communication Systems: Vision, Protocols, Applications, and Challenges,” *Quantum Physics*, arXiv:2212.13333 [quant-ph]

7. N. Hosseinidehaj, Z. Babar, R. Malaney, S. X. Ng and L. Hanzo, "Satellite-Based Continuous-Variable Quantum Communications: State-of-the-Art and a Predictive Outlook," *IEEE Communications Surveys & Tutorials*, vol. 21, no. 1, pp. 881-919, First quarter 2019, doi: 10.1109/COMST.2018.2864557.

8. *Local Quantum State Simulation,* MATLAB 2023b Documentation

9. *Solve XOR Problem Using Quantum Neural Network (QNN)*, MATLAB 2023b Documentation

10. *IEEE Quantum website*

11. *https://www.quantum-inspire.com/*