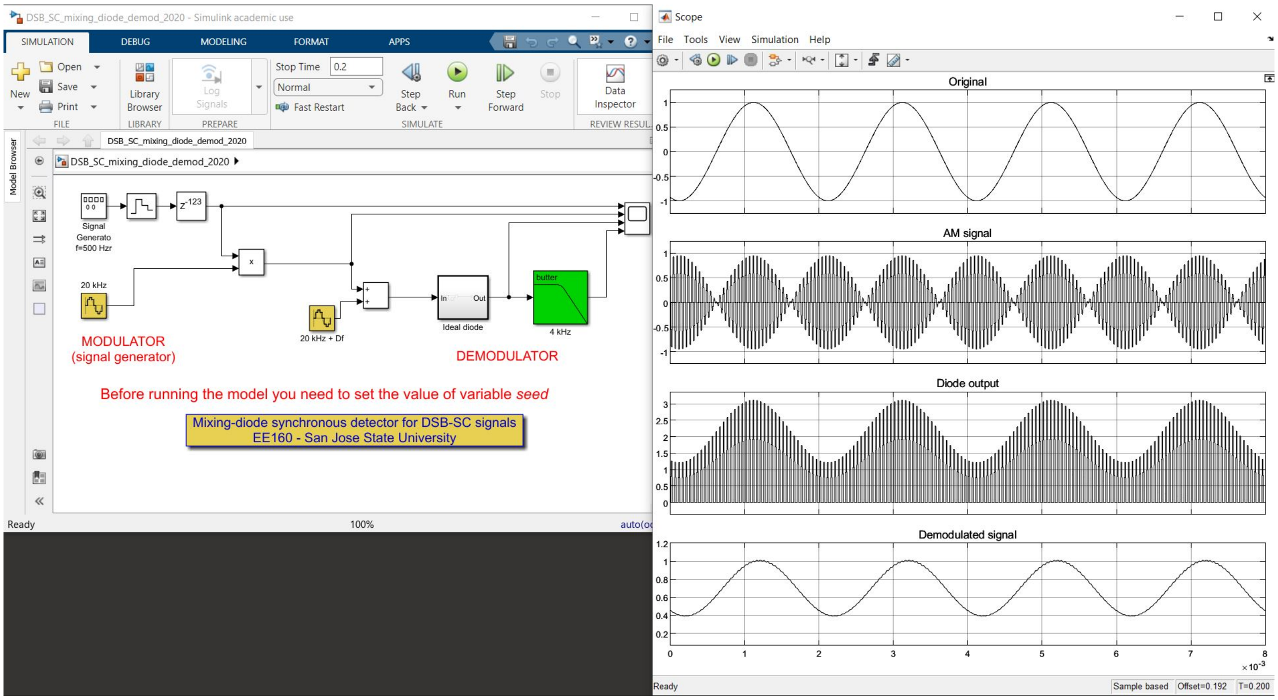
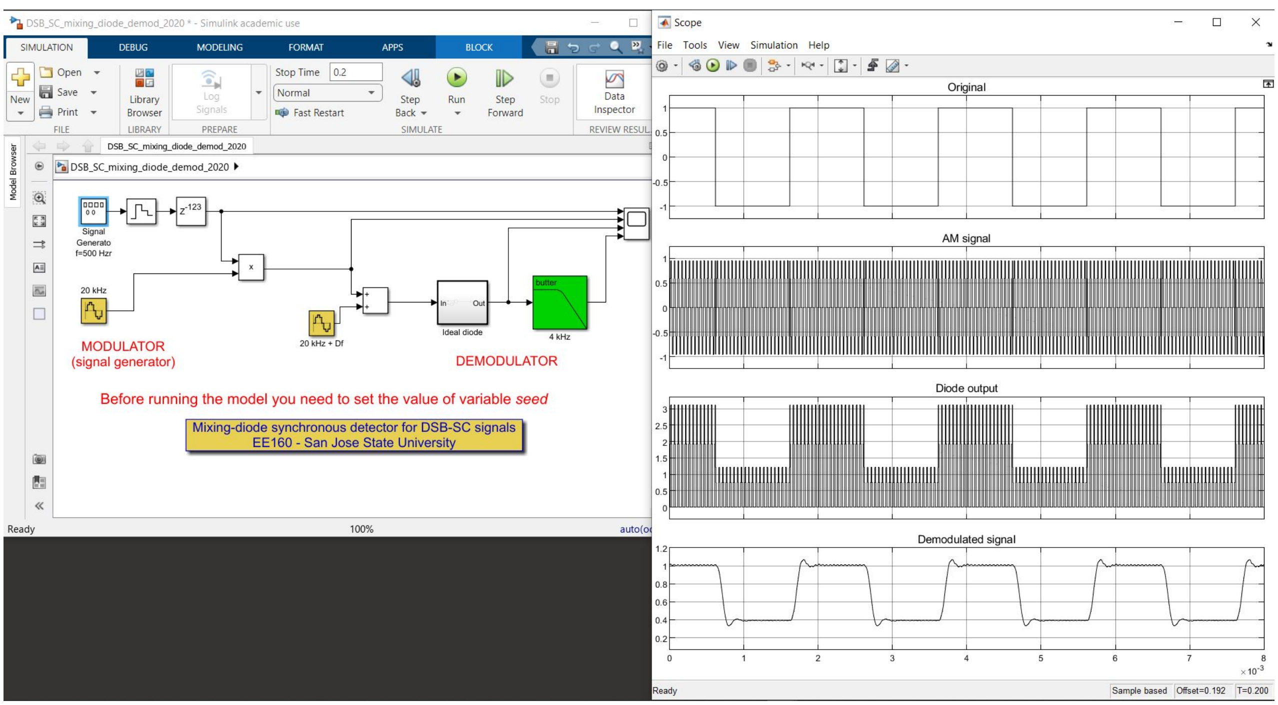
Solution of Homework # 5

1. A mixing-diode synchronous detector of DSB-SC signals

   (a) Results with `seed = 1234567`:



   (b) Square message signal:

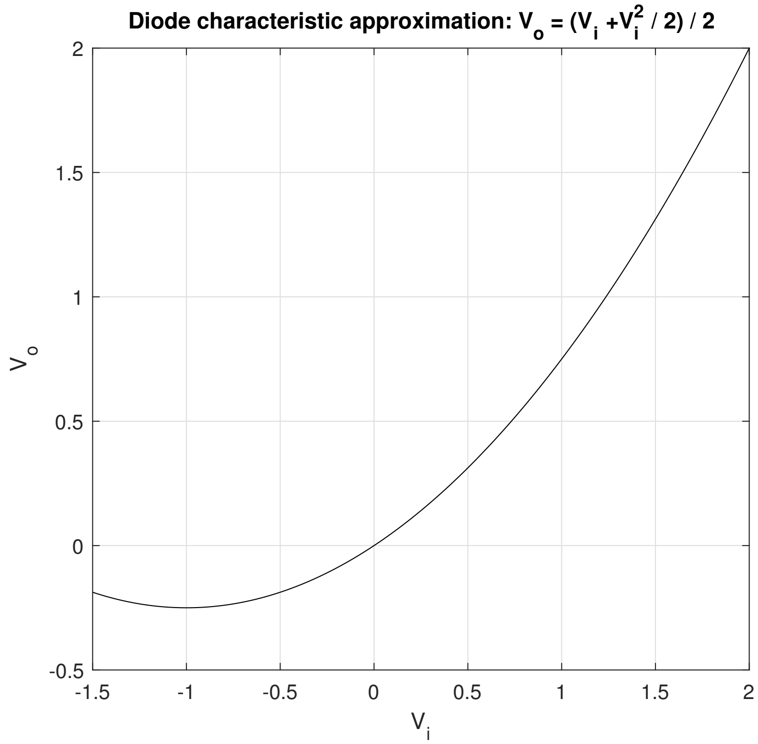


(c) The diode is a nonlinear device and thus produces harmonics when the input is a periodic signal. The output of lowpass filter consists of a DC component and the sinusoidal signal.

To see why. let $m(t)$ be an arbitrary message signal. A relatively crude approximation of the diode input-output characteristic is

$$V_o = \frac{1}{2}\left[V_i + \frac{V_i^2}{2}\right].$$

A plot of this characteristic is shown below:



Now let $V_i = c(t) + u(t) = A_c \cos(2\pi f_c t) + A_c\, m(t) \cos(2\pi f_c t)$, with the local oscillator signal equal to the carrier signal $c(t) = A_c \cos(2\pi f_c t)$. Then, using the trigonometric identity $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$, we obtain

$$V_o = \frac{1}{4}\left[A_c^2\left(1 + 2\,m(t) + m^2(t)\right) + A_c^2\left(1 + 2\,m(t) + m^2(t)\right)\cos(4\pi f_c t)\right].$$

The second term on the right-hand side of the above equation is removed by the lowpass filter (LPF) and the demodulated signal is

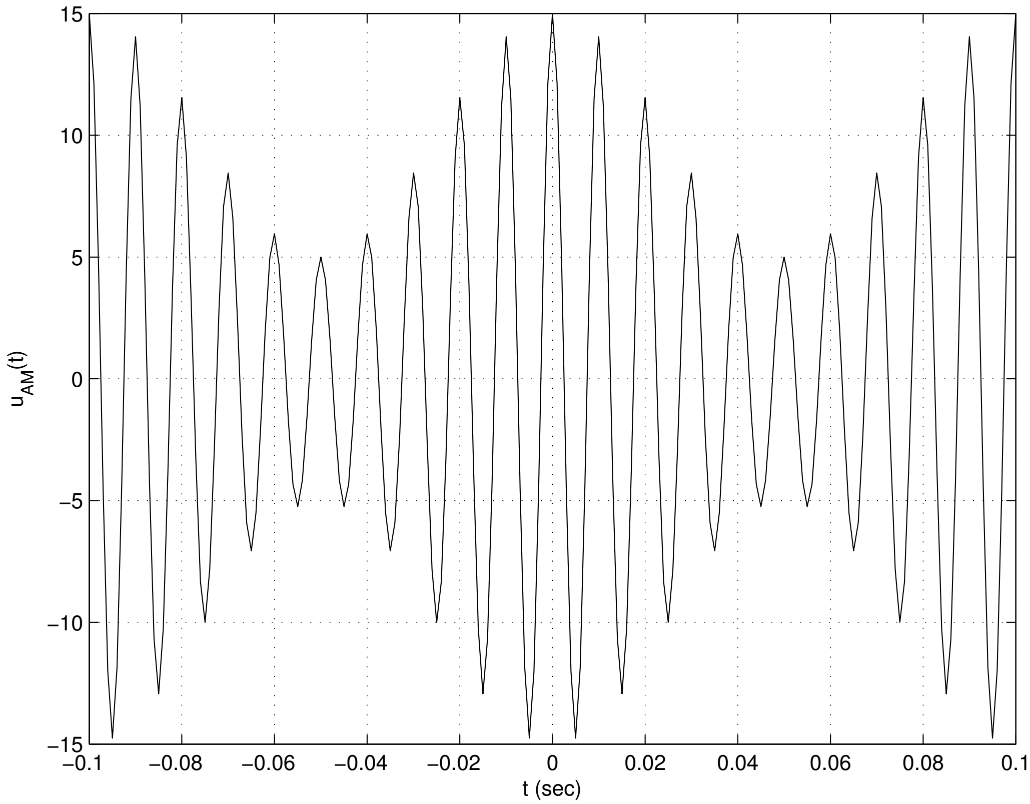
$$y_\ell(t) = \frac{A_\ell\, A_c^2}{4}\left(1 + 2\,m(t) + m^2(t)\right),$$

where A_ℓ depends on the gain of the filter. In the case of a sinusoidal modulating (or message) signal $m(t) = \cos(2\pi f_m t)$, we have $m^2(t) = \frac{1}{2}\left[1 + \cos(4\pi f_m t)\right]$ and the LPF output is of the form

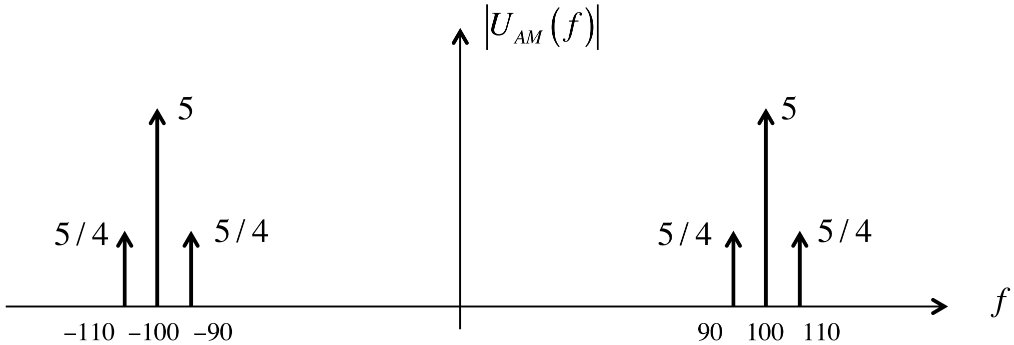
$$y_\ell(t) = \mathrm{DC} + k \cdot \cos(2\pi f_m t),$$

where DC and k are constants, if the filter is designed such that the double-frequency terms $\pm 2 f_m$ are removed.

2. (a) Modulated signal:



(b) Amplitude spectrum:



(c) Modulation index:

i.

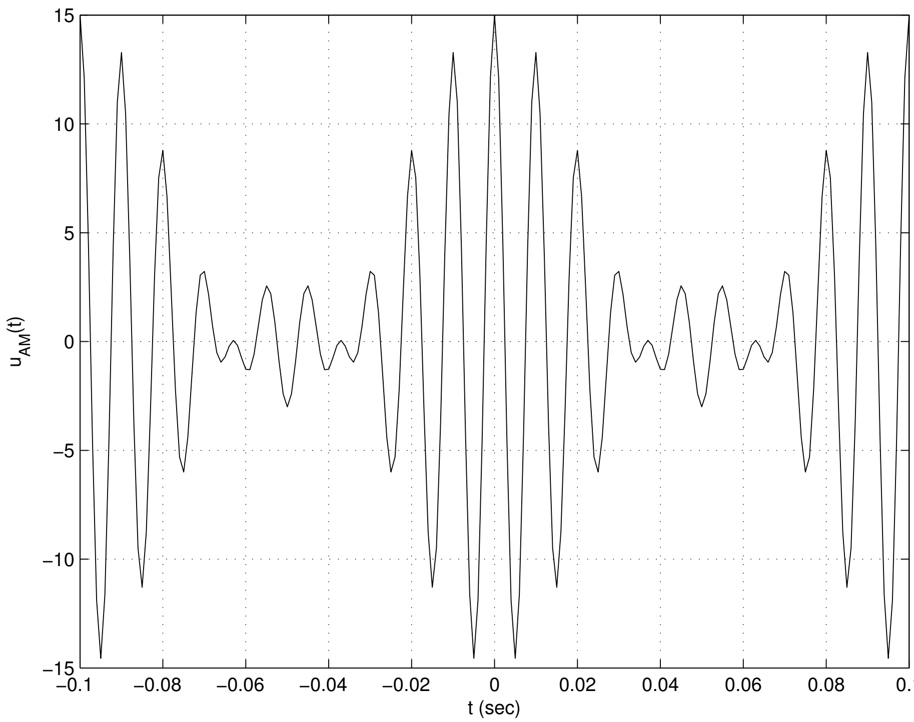
$$a = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} = \frac{10-5}{10+5} = \frac{1}{2}.$$

ii. Express the AM signal as

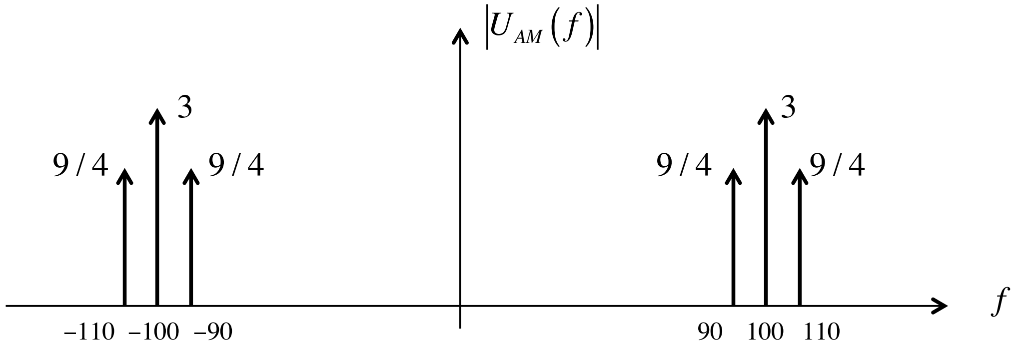
$$u_{AM}(t) = 10\left[1 + \frac{1}{2}\cos(20\pi t)\right]\cos(200\pi t) \quad \longrightarrow \quad a = \frac{1}{2}.$$

(d) The modulation index is $a < 1$ and thus the signal is <u>under-modulated</u>.

3. (a) Modulated signal:



(b) Amplitude spectrum:



(c) i. Since the envelope does not have a sinusoidal shape, $E_{\max}$ and $E_{\min}$ cannot be used in the formula.

ii. Express the AM signal as

$$u_{AM}(t) = 6\left[1 + \frac{3}{2}\cos(20\pi t)\right]\cos(200\pi t) \quad \longrightarrow \quad a = \frac{3}{2}.$$