Harmonics and Intermodulation Distortion

EE 160 lecture

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Effects of a non-linearity

$$\underset{\text{Input}}{v_i} \longrightarrow \boxed{\text{Nonlinear system}} \longrightarrow \underset{\text{Output}}{v_o}$$

- In general, the output signal v_o of a non-linearity can be represented by a <u>Taylor series</u>:

$$\boxed{v_o = a_0 + a_1 v_i + a_2 v_i^{\,2} + a_3 v_i^{\,3} + \cdots} \qquad (1)$$

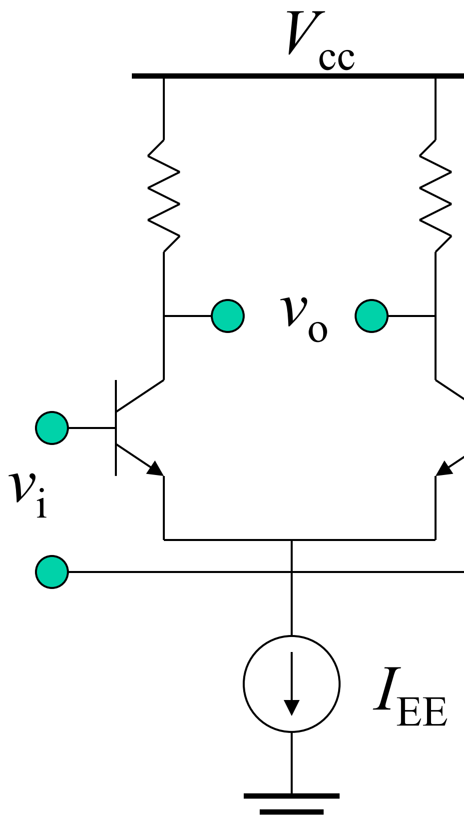
where v_i represents the input signal

- For simplicity, we will assume that a_0=0 and

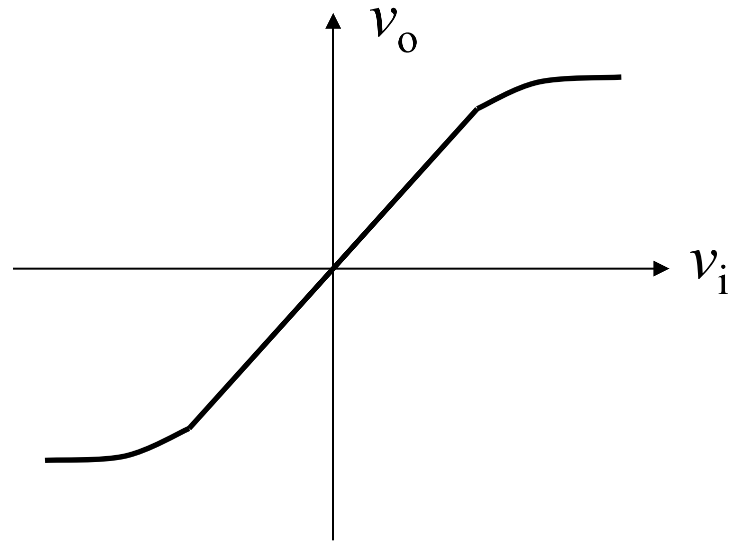
$$\boxed{v_o \approx a_1 v_i + a_2 v_i^{\,2} + a_3 v_i^{\,3}} \qquad (2)$$

Write on board

Example: Bipolar Differential Pair



$$v_o = RI_{EE}\tanh\left(\frac{v_i}{2V_T}\right)$$



Input-output characteristic

Single sinusoidal input: Harmonics - I

- If the input is $v_i = A\cos(2\pi f_0 t)$ then the *quadratic term* of (2) is

$$a_2 v_i^2 = a_2 A^2 \cos^2(2\pi f_0 t) = \frac{a_2 A^2}{2}\left[1 + \cos(4\pi f_0 t)\right]$$

giving the <u>second harmonic</u> term $2\omega_0 = 4\pi f_0$ and a DC component

- *Cubic term* of (2):

$$a_3 v_i^3 = a_3 A^3 \cos^3(2\pi f_0 t)$$

$$= \frac{3a_3 A^3}{4}\cos(2\pi f_0 t) + \frac{a_3 A^3}{4}\cos(6\pi f_0 t)$$

Single sinusoidal input: Harmonics - II

- Finally, the total output voltage from expression (2) is

FUNDAMENTAL

$(\omega_0 = 2\pi f_0)$

$$v_0(t) = \frac{a_2 A^2}{2} + \left(a_1 A + \frac{3 a_3 A^3}{4} \right) \cos(2\pi f_0 t)$$

$$+ \frac{a_2 A^2}{2} \cos(4\pi f_0 t) + \frac{a_3 A^3}{4} \cos(6\pi f_0 t) \qquad (3)$$

SECOND ORDER HARMONIC

$(2\omega_0 = 4\pi f_0)$

THIRD ORDER HARMONIC

$(3\omega_0 = 6\pi f_0)$

- Matlab spectrum
- Show spectrum in board

Two observations on harmonics

- Even-order harmonics vanish (a_j=0 for even integers j) if the input-output characteristic has *odd symmetry*. A system with such nonlinearity is said to be *differential* or *balanced*.

Write equation for amplifier

- For small values of amplitude A, the n-th order harmonic grows approximately in proportion to A^n.

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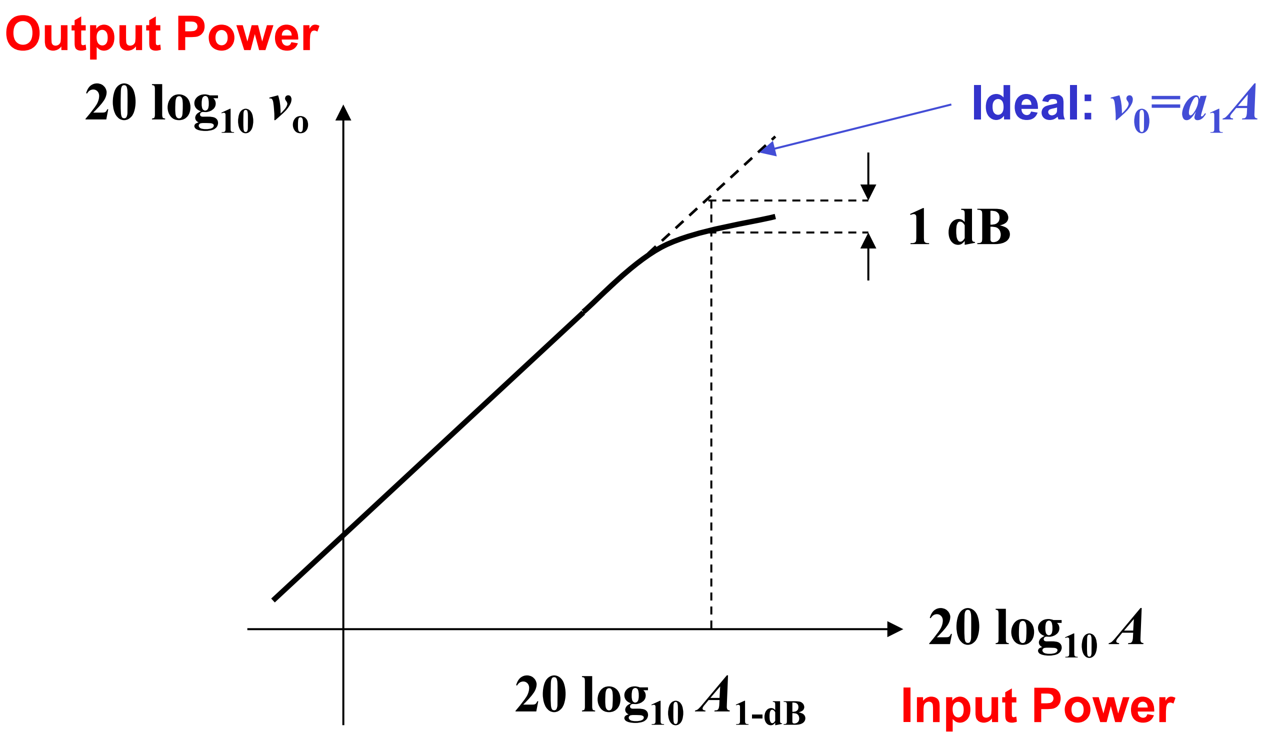
Gain compression

- *Small-signal* voltage gain is obtained under the assumption that harmonics are negligible.
  - For the differential pair:

$$v_o = \frac{RI_{EE}}{2V_T} v_i$$

- As the input signal amplitude increases, the gain begins to change.
- The gain becomes a function of the input level and approaches zero for large input levels

The 1-dB compression point – I



From (3):

$$20\log_{10}\left|a_1+\frac{3a_3A_{1-dB}^2}{4}\right|=20\log_{10}|a_1|-1 \quad\Rightarrow\quad \boxed{A_{1-dB}=\sqrt{0.145\left|\frac{a_1}{a_3}\right|}}$$

The 1-dB compression point – II

- The 1-dB compression point is a measure of the *maximum input range* of a circuit

- In front-end RF amplifiers, typical values range from **–25 to –20 dBm** (or 35.6 to 63.2 $\mathrm{mV_{pp}}$ in 50 Ω systems)

Note: A dBm is a logaritmic measure of power with respect to 1 mW

$$1\,\mathrm{dBm} = 10\log_{10}\left(\frac{P}{1\,\mathrm{mW}}\right)$$

Two sinusoidal inputs: Intermodulation - I

- Two-tone test: the input is a *two-tone* signal:

$$v_i = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t) \tag{4}$$

- Quadratic term of (2):

$$\begin{aligned} a_2 v_i^2 &= a_2\left[A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)\right]^2 \\ &= \frac{a_2(A^2+B^2)}{2} + \frac{a_2 A^2}{2}\cos(4\pi f_1 t) + \frac{a_2 B^2}{2}\cos(4\pi f_2 t) \\ &\quad + a_2 AB\left\{\cos\left[2\pi(f_1+f_2)t\right] + \cos\left[2\pi(f_1-f_2)t\right]\right\} \end{aligned} \tag{5}$$

Two sinusoidal inputs: Intermodulation - II

- Cubic term of (2) is much more elaborated
  - Term due to *DC term* of (5) (with coefficient a_3):

$$\frac{a_3(A^2+B^2)}{2}+\left[A\cos\left(2\pi f_1 t\right)+B\cos\left(2\pi f_2 t\right)\right] \tag{6a}$$

  - *Double frequency* terms of (5) (with coefficient a_3):

$$\begin{aligned} &a_3\left[\frac{A^2}{2}\cos\left(4\pi f_1 t\right)+\frac{B^2}{2}\cos\left(4\pi f_2 t\right)\right]\left[A\cos(2\pi f_1 t)+B\cos(2\pi f_2 t)\right] \\ &=a_3\left[\frac{A^3}{2}\cos\left(4\pi f_1 t\right)\cos\left(2\pi f_1 t\right)\right]+a_3\left[\frac{B^3}{2}\cos\left(4\pi f_2 t\right)\cos\left(2\pi f_2 t\right)\right] \\ &+a_3\left[\frac{A^2B}{2}\cos\left(4\pi f_1 t\right)\cos\left(2\pi f_2 t\right)\right]+a_3\left[\frac{AB^2}{2}\cos\left(4\pi f_2 t\right)\cos\left(2\pi f_1 t\right)\right] \end{aligned} \tag{6b}$$

Two sinusoidal inputs: Intermodulation - III

– *Sum and difference terms* of (5) (with coefficient a_3):

$$a_3AB\left[\cos[2\pi(f_1+f_2)t]+\cos[2\pi(f_1-f_2)t]\right]\left[A\cos(2\pi f_1t)+B\cos(2\pi f_2t)\right]$$

$$=a_3A^2B\left(\cos[2\pi(f_1+f_2)t]\cos(2\pi f_1t)\right)$$

$$+a_3AB^3\left(\cos[2\pi(f_1+f_2)t]\cos(2\pi f_2t)\right)$$

$$+a_3A^2B\left(\cos[2\pi(f_1-f_2)t]\cos(2\pi f_1t)\right)$$

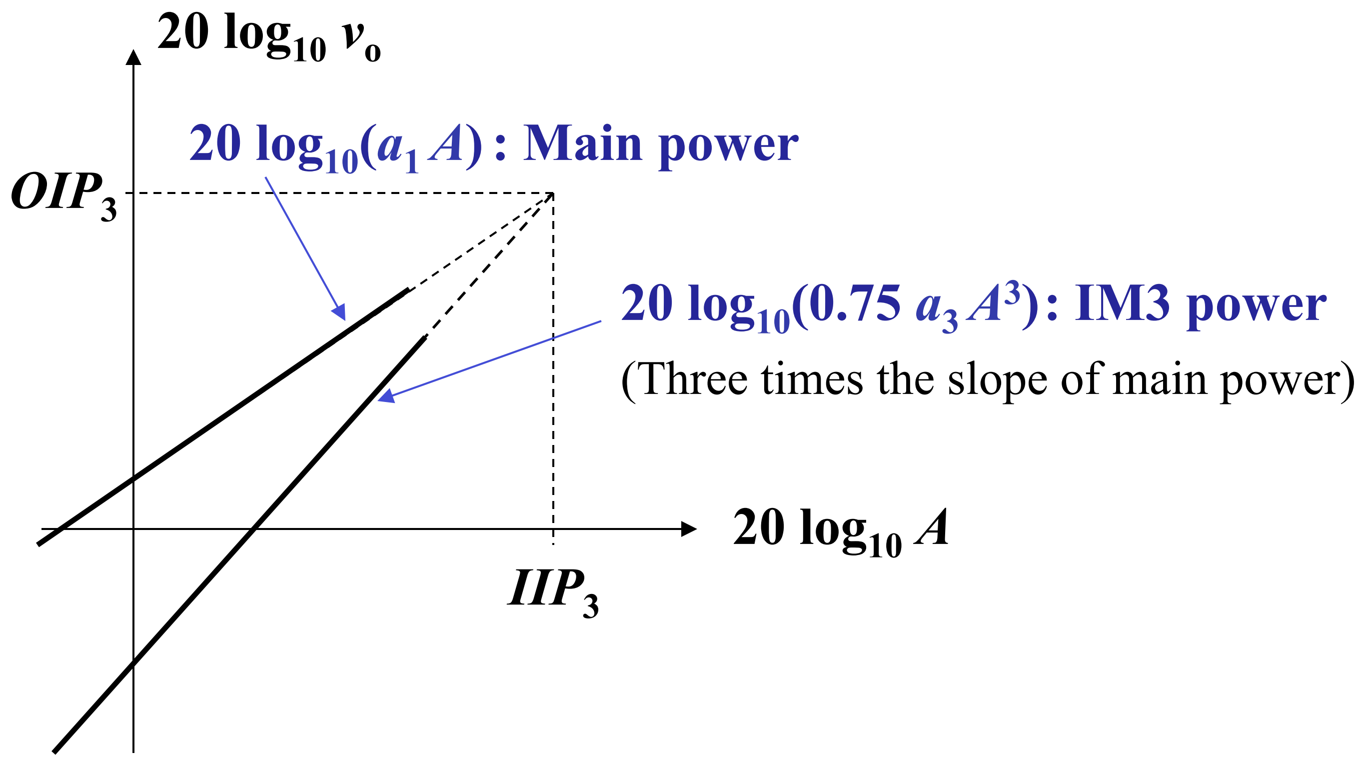
$$+a_3AB^3\left(\cos[2\pi(f_1-f_2)t]\cos(2\pi f_2t)\right) \qquad (6c)$$

- Equation (6b) includes frequencies (<u>intermodulation products</u>): $\omega_1,\ \omega_2, 3\omega_1, 3\omega_2, 3\omega_1\pm\omega_2, 3\omega_2\pm\omega_1$
- Equation (6c) includes frequencies: $\omega_1,\ \omega_2, 2\omega_1-\omega_2, 2\omega_2\pm\omega_1$

See Fig. 2.3 in note "02"RFsignals_nonlinear_amps.pdf"

The third intercept point (IP_3)

- As the amplitude of the input sinusoidals $A(=B)$ increases, the third order intermodulation (IM) products increase proportionally to A^3



Computation of IIP$_3$ and OIP$_3$ (part 1)

- Let the input be a sum of two <u>equal-amplitude</u> sinusoidals:

$$v_i(t) = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t)$$

- Then from (3), with $a_2 = 0$, we have

$$\boxed{\begin{aligned} v_0(t) &= A\left(a_1 + \frac{9a_3A^2}{4}\right)\left(\cos(2\pi f_1 t) + \cos(2\pi f_2 t)\right) \\ &\quad + \frac{3a_3A^3}{4}\cos\left[2\pi\left(2f_1 - f_2\right)t\right] + \frac{3a_3A^3}{4}\cos\left[2\pi\left(2f_2 - f_1\right)t\right] \end{aligned}} \qquad (7)$$

- <u>We assumed</u> that $a_1 >> \frac{9|a_3|A^2}{4}$ (small signal)

Computation of IIP_3 and OIP_3 (part 2)

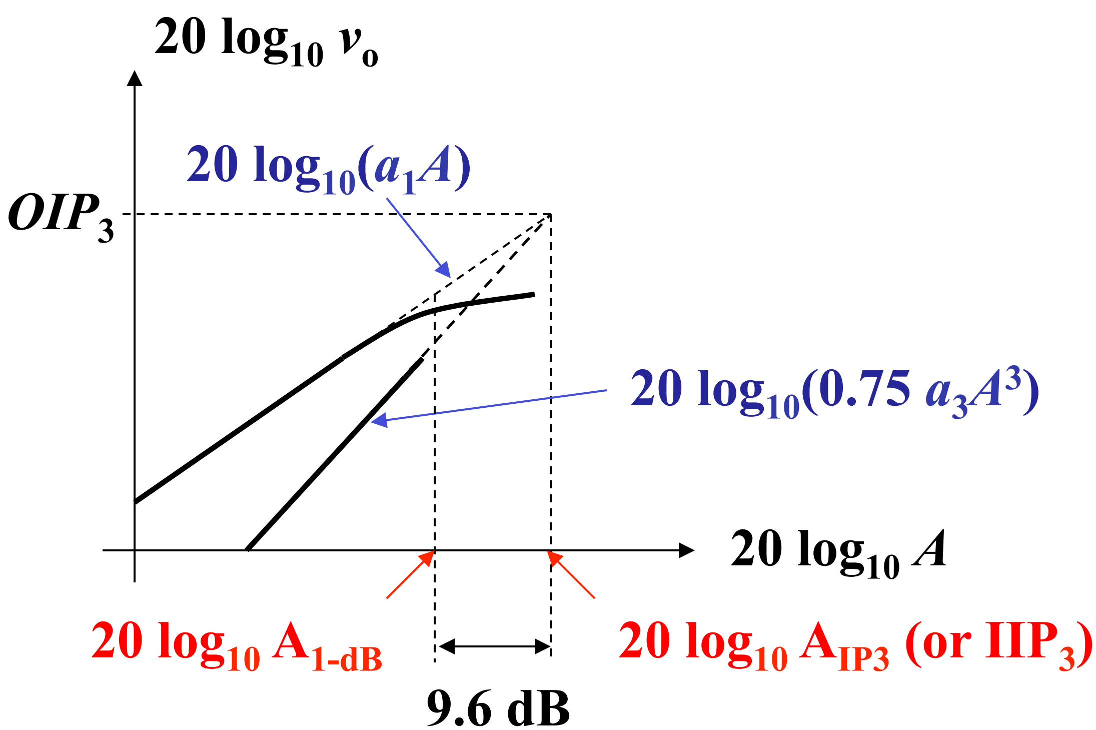
- The input level A_{IP3} for which the output (fundamental) components at f_1 and f_2 have the same amplitude as those (third order IM products) at $2f_1$-f_2 and $2f_2$-f_1 is

$$|a_1| A_{IP3} = \frac{3}{4} |a_3| A_{IP3}^3 \quad \Rightarrow \quad \boxed{A_{IP3} = \sqrt{\frac{4}{3} \left| \frac{a_1}{a_3} \right|}}$$

- Thus IIP3 = 20 $\log_{10}(A_{IP3})$ and OIP3 = 20 $\log_{10}(a_1 A_{IP3})$

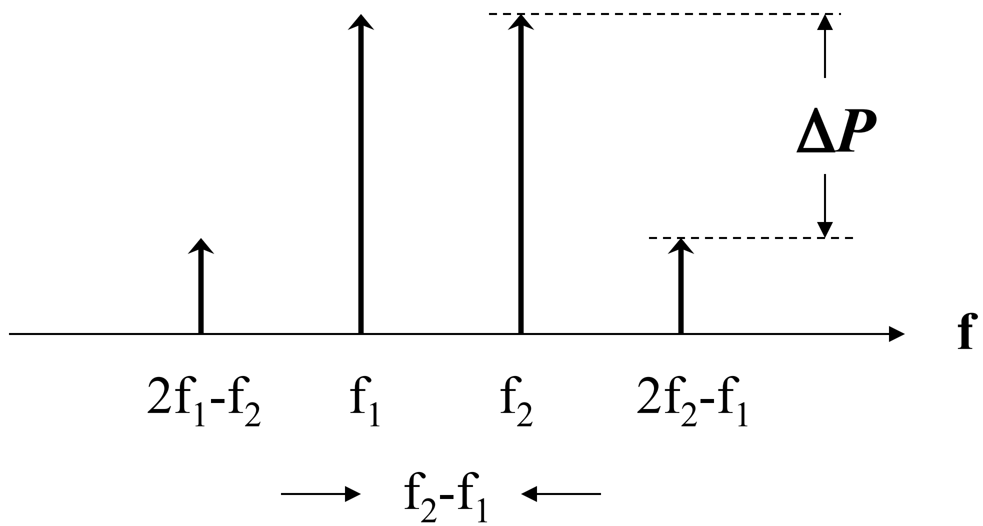
Relationship between $A_{1\text{-dB}}$ and A_{IP3}

$$\sqrt{\frac{4}{3}}A_{1\text{-dB}} = \sqrt{0.145}A_{IP3} \quad \Rightarrow \quad \boxed{\frac{A_{1\text{-dB}}}{A_{IP3}} = 0.33 \quad (-9.6\text{ dB})}$$



The two-tone test

- From equation (7), if the difference between f_1 and f_2 is small then the third-order terms $2f_1 - f_2$ and $2f_2 - f_1$ appear close together:

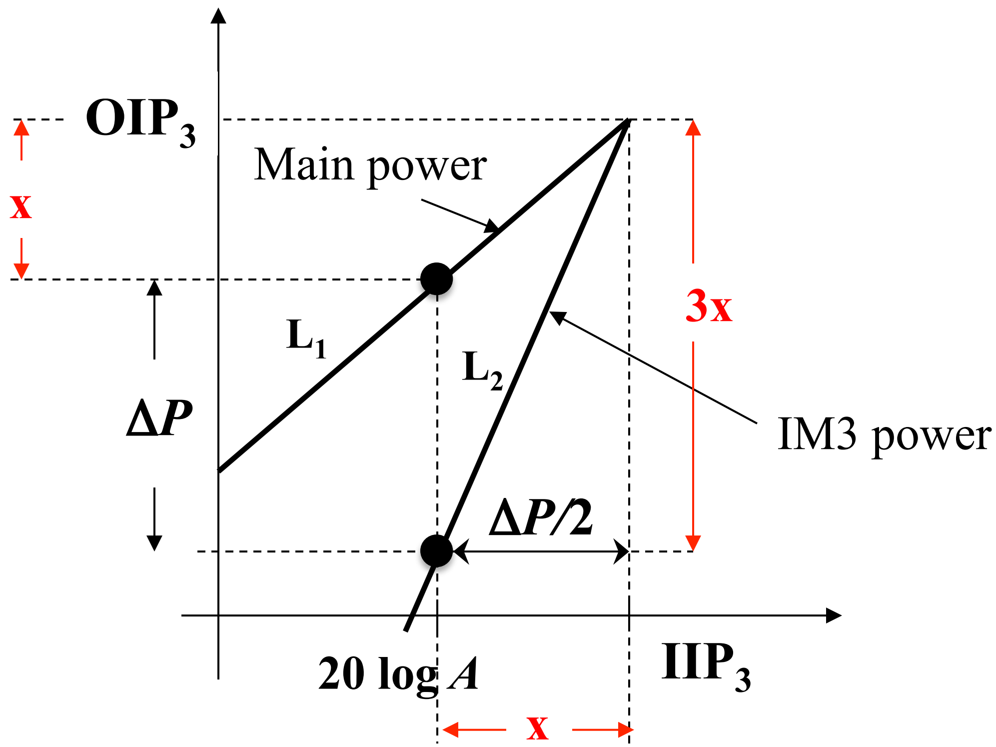


- This is useful in measuring the IIP3 in a laboratory

The two-tone test (cont.)

- The *IIP3* in dBm can then be approximated as (see figure)

$$\boxed{\mathrm{IIP3\ (dBm)} = \frac{\Delta P}{2}\ \mathrm{(dB)} + 20\log A\ \mathrm{(dBm)}}$$



$x + \Delta P = 3x$

$\rightarrow \boxed{x = \Delta P/2}$

Reference

- Section 2.1 of

  B. Razavi, *RF Microelectronics*, Prentice Hall, 1998.

"02_Book_RF_Razavi.pdf" in Canvas: Files/_Lectures