

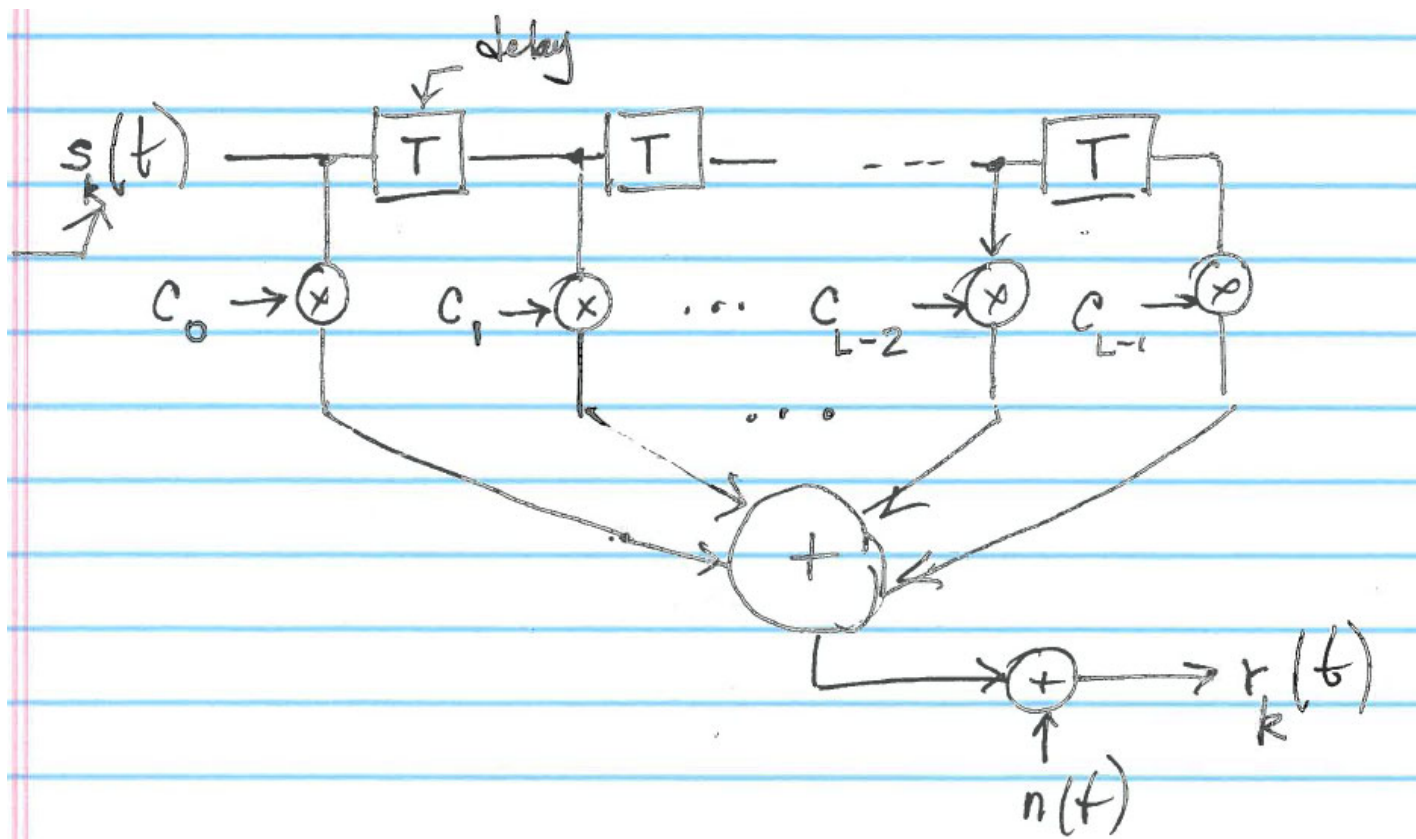


OFDM: Cyclic Prefix and Channel Estimation

EE161: Digital Communication Systems
San José State University



Symbol-spaced multipath channel model



Frequency response



Impulse response

$$c(t) = c_0 \delta(t) + c_1 \delta(t-T) + \dots + c_{L-1} \delta(t-(L-1)T)$$

Frequency response

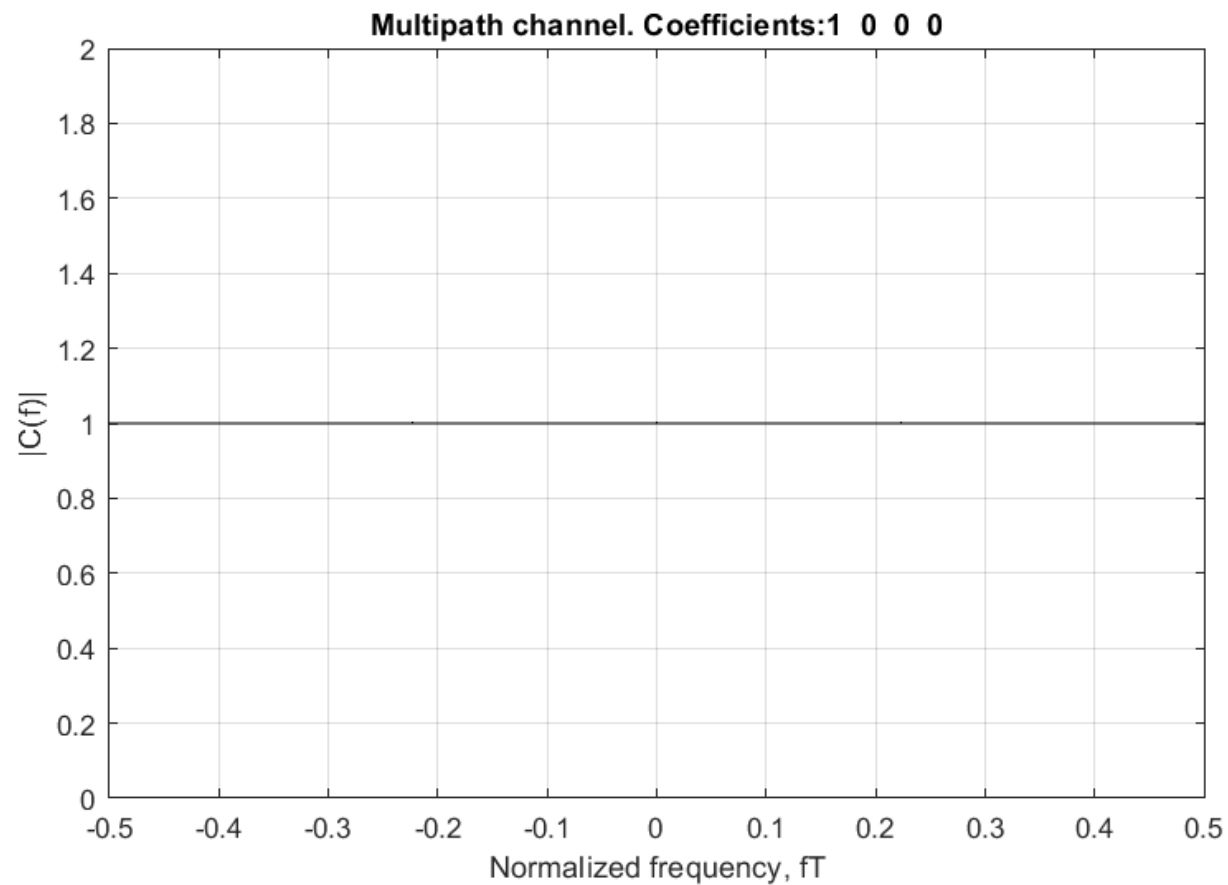
$$c(f) = \mathcal{F}\{c(t)\} = c_0 + c_1 e^{-j2\pi f T} + \dots + c_{L-1} e^{-j2\pi f (L-1)T}$$

or

$$c(f) = \sum_{l=0}^{L-1} c_l e^{-j2\pi f l T}$$

(matlab models input $[c_0 \ c_1 \ \dots \ c_{L-1}]$)

Example: $L=4$. $c = [1 \ 0 \ 0 \ 0]$

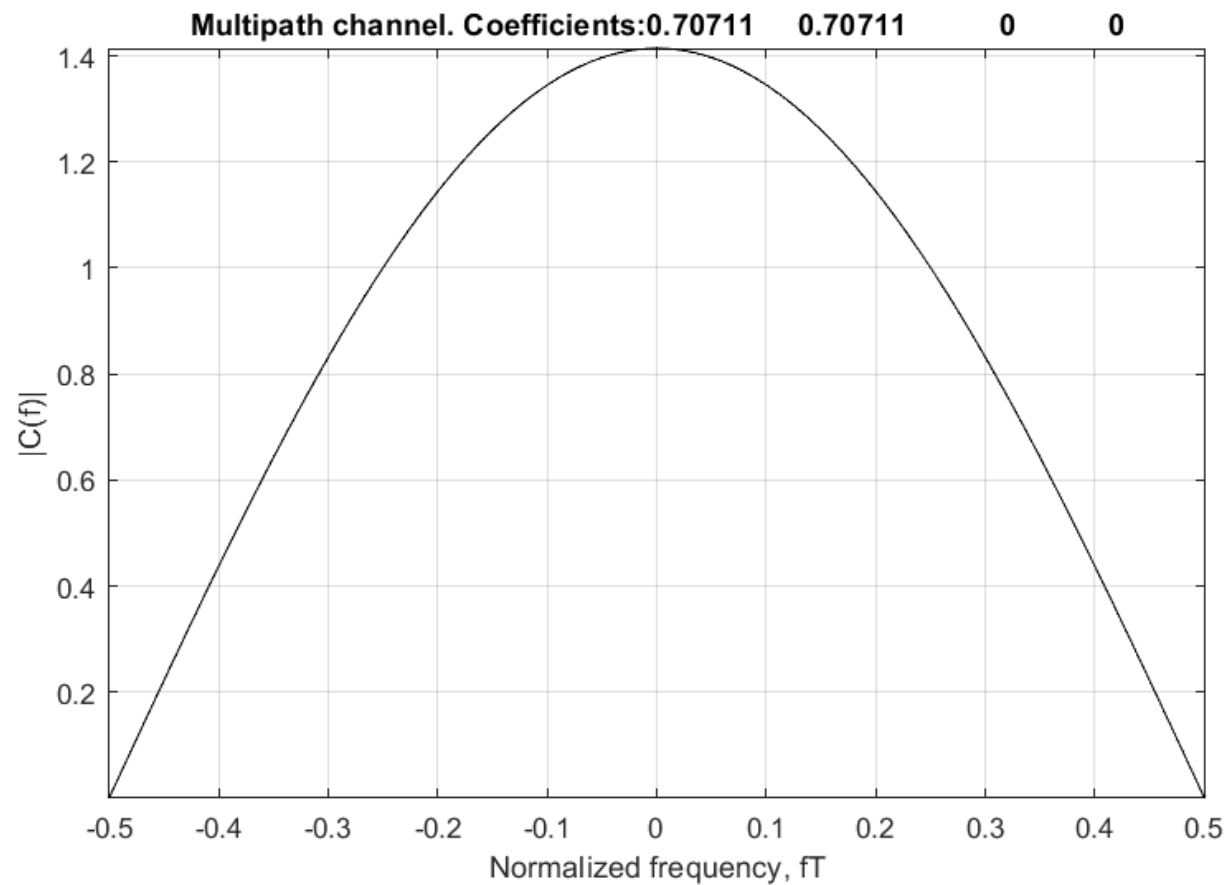


[frequency_response_multipath.m](#)

EE161



Example: $L=4$. $c = [1 \ 1 \ 0 \ 0] / \sqrt{2}$

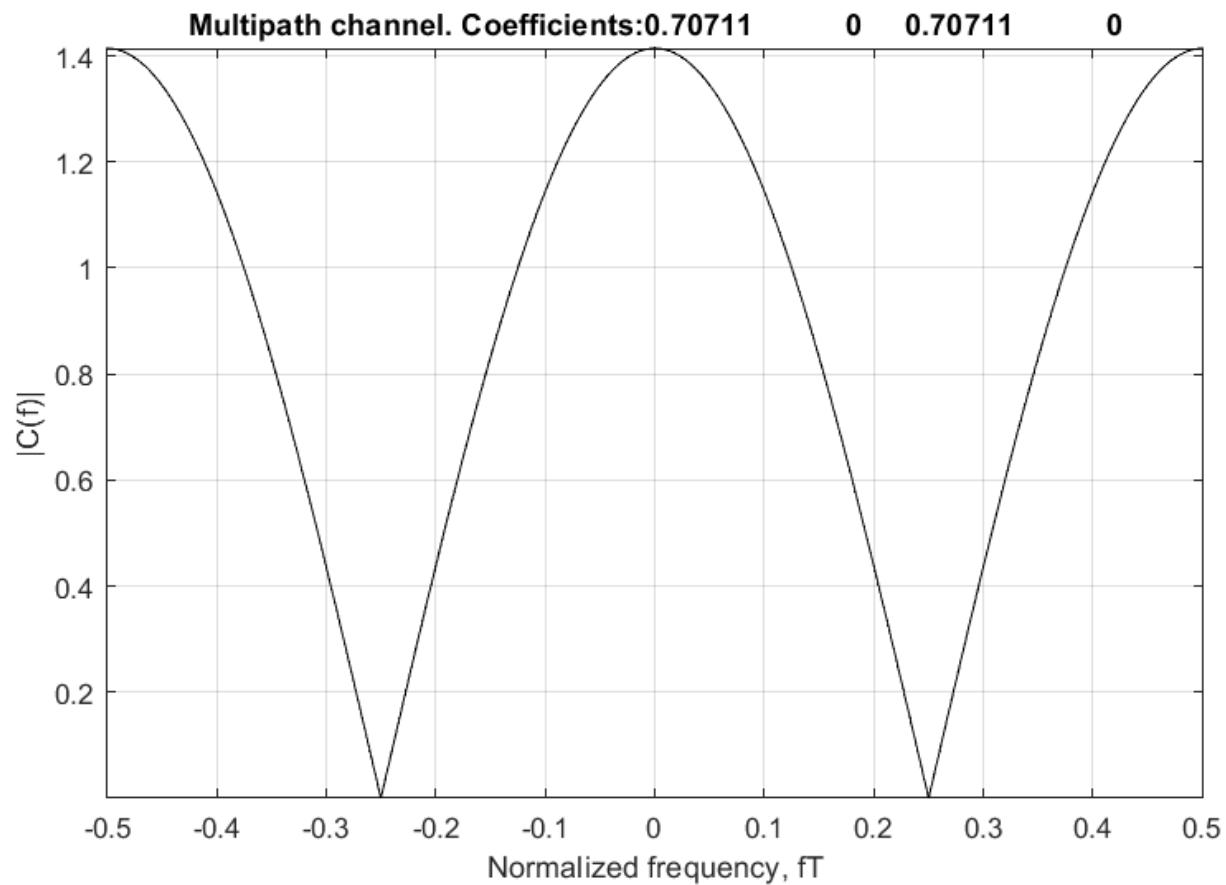


[frequency_response_multipath.m](#)

EE161



Example: $L=4$. $c = [1 \ 0 \ 1 \ 0] / \sqrt{2}$

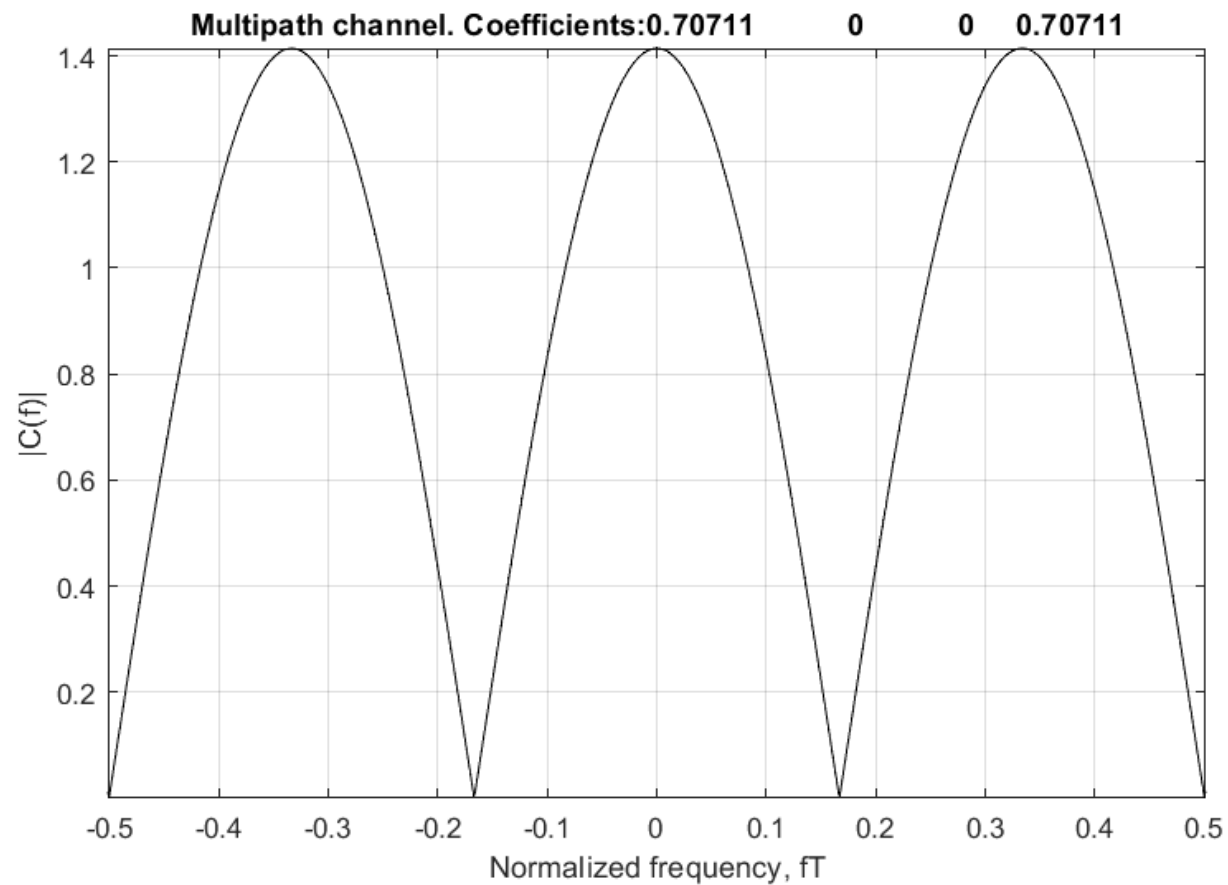


[frequency_response_multipath.m](#)

EE161



Example: $L=4$. $c = [1 \ 0 \ 0 \ 1] / \sqrt{2}$

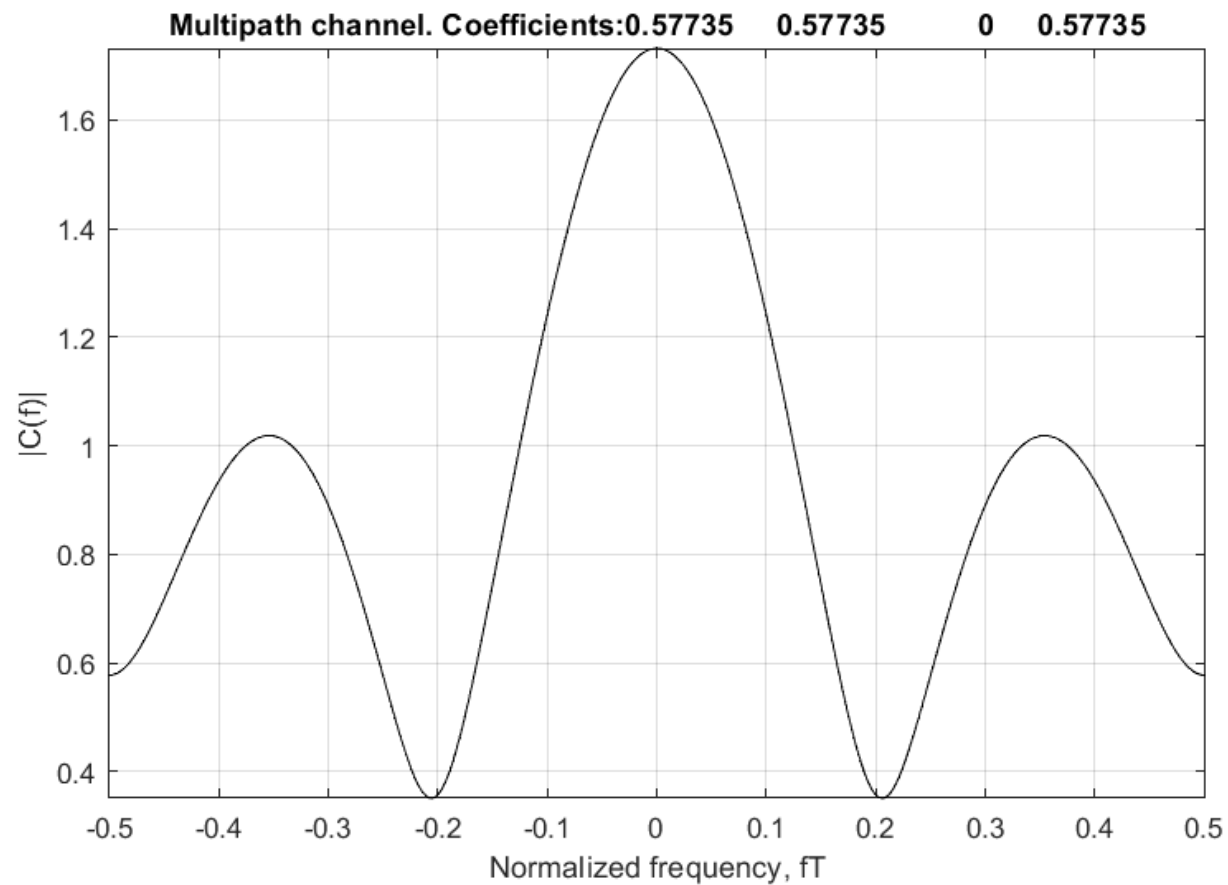


[frequency_response_multipath.m](#)

EE161



Example: $L=4$. $c = [1 \ 1 \ 0 \ 1] / \sqrt{3}$

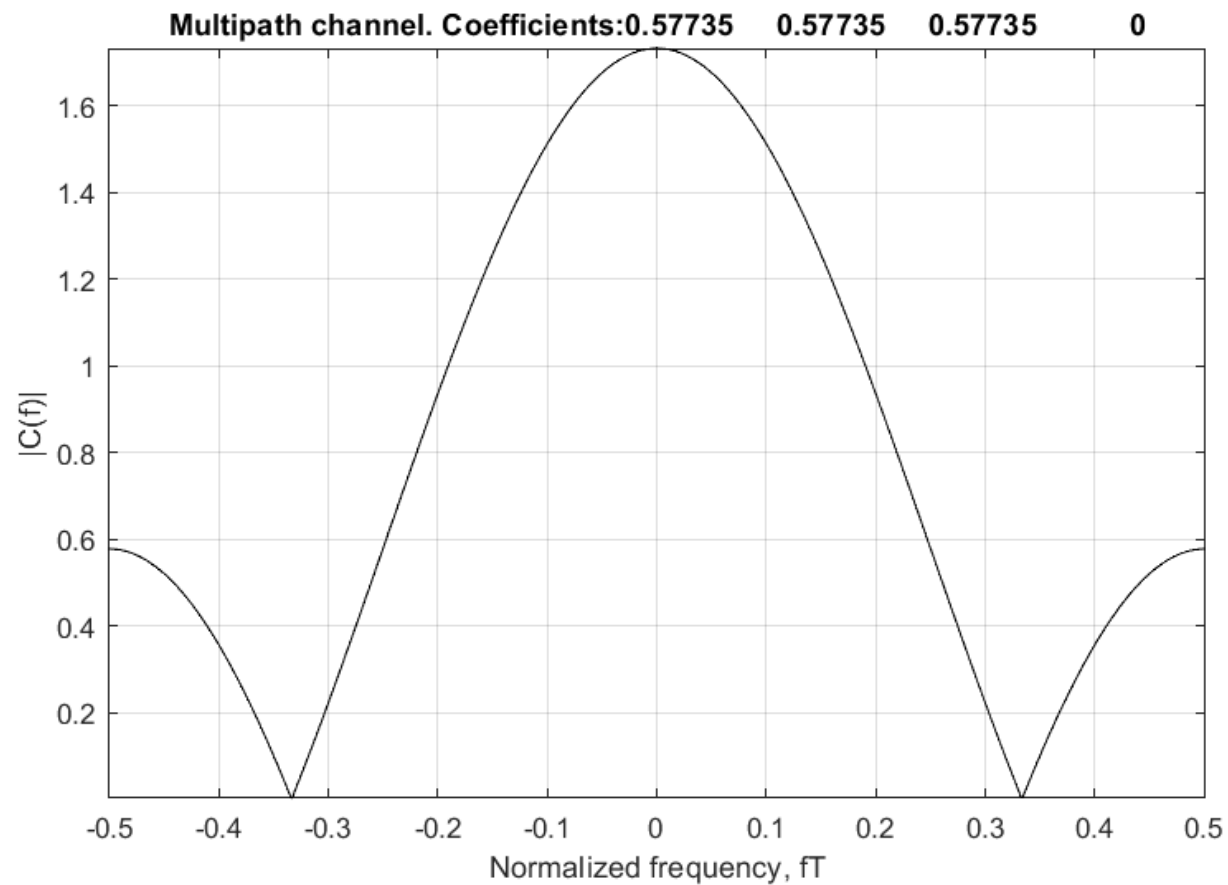


[frequency_response_multipath.m](#)

EE161



Example: $L=4$. $c = [1 \ 1 \ 1 \ 0] / \sqrt{3}$

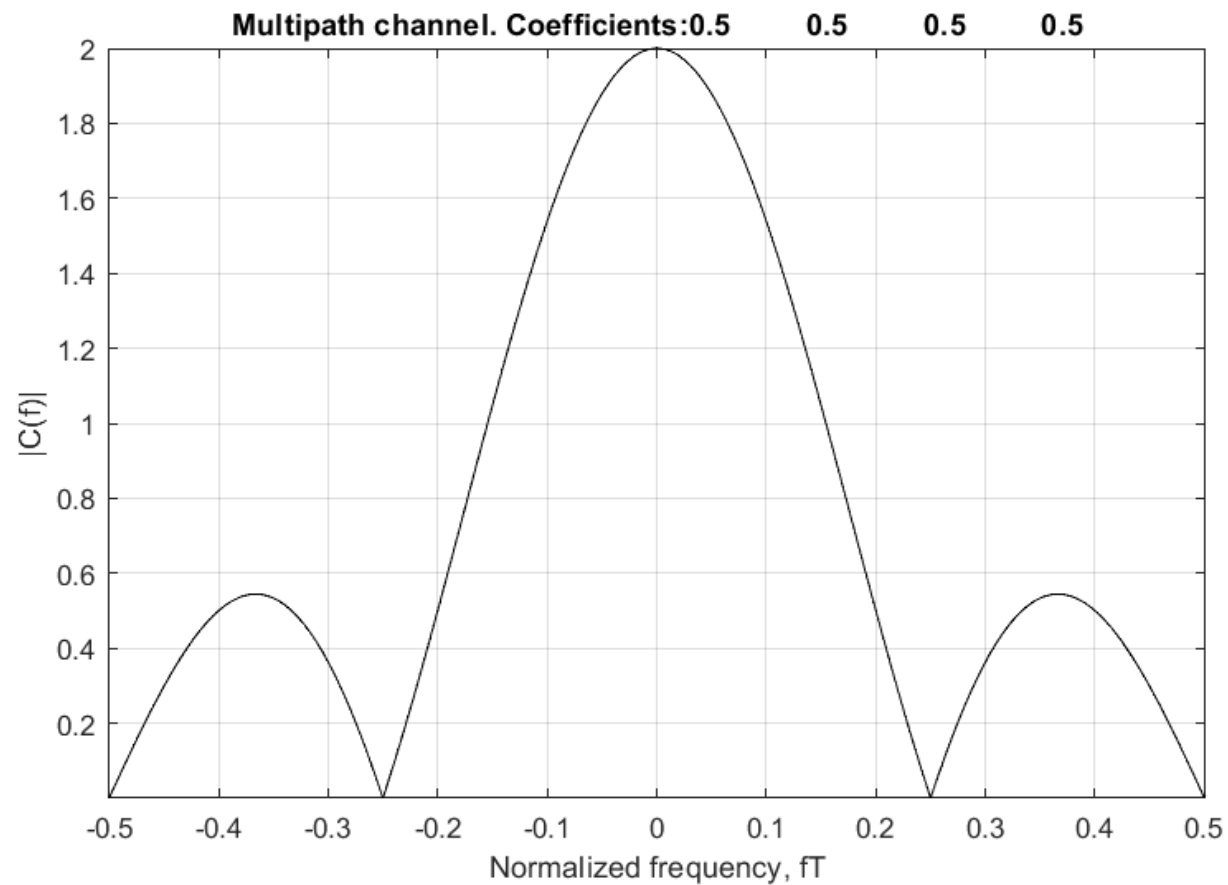


[frequency_response_multipath.m](#)

EE161



Example: $L=4$. $c = [1 \ 1 \ 1 \ 1] / \sqrt{4}$



[frequency_response_multipath.m](#)

EE161





Normalized channel energy

- The channel energy is normalized to unity:

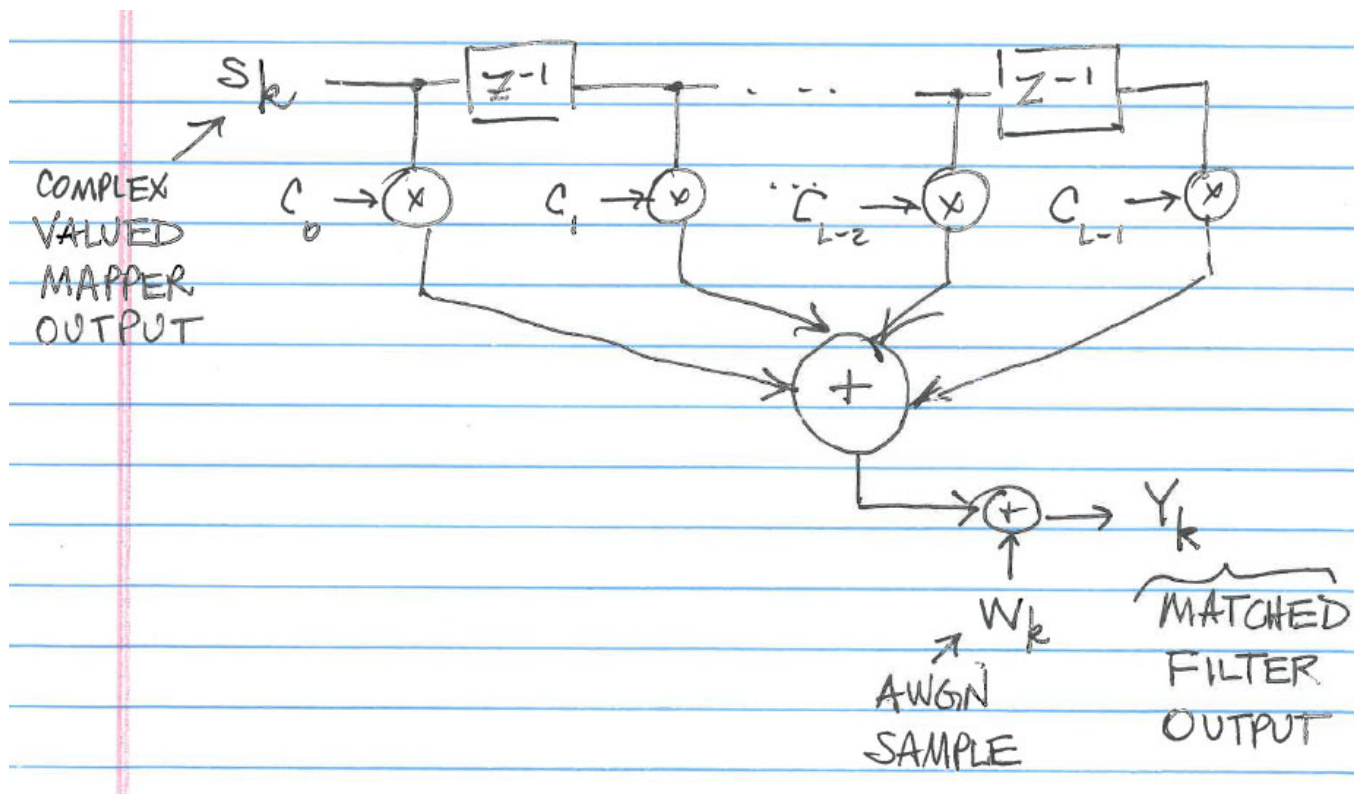
$$E_c = \sum_{\ell=0}^{L-1} |c_\ell|^2 = 1$$

- This allows us to compare different channel scenarios for the same average signal energy



Equivalent discrete-time model

- This model is obtained when looking at the matched-filter output:





OFDM subchannel average signal energies

- Over each subchannel, $k = 0, 1, \dots, K - 1$, we have

$$\text{SNR}_k = \frac{E_{s,k}}{N_0} |C_k|^2$$

where $E_{s,k}$ is the energy of mapper output in the k -th subchannel

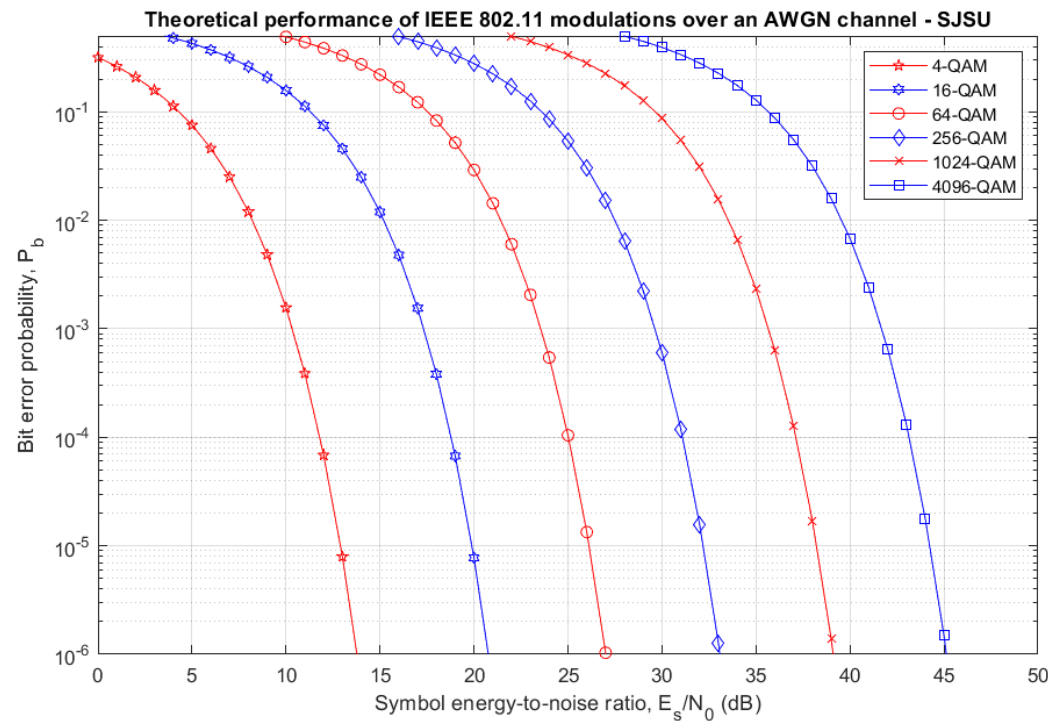
- Due to the frequency selectivity nature of a multipath channel, these SNR (signal energy-to-noise ratio) values can be very low, resulting in an unacceptably high bit error rate (BER)



Approaches to low subchannel energy

1. Adaptive modulation (e.g., DSL modems)

For each subchannel, select a modulation format (mapper output symbols) according to average SNR value to achieve a given BER



Approaches to low subchannel energy (cont.)



2. OFDM one-tap equalization

Estimate the subchannel gains $C_k, k = 0, 1, \dots, K - 1$, and “equalize” them

More next lecture



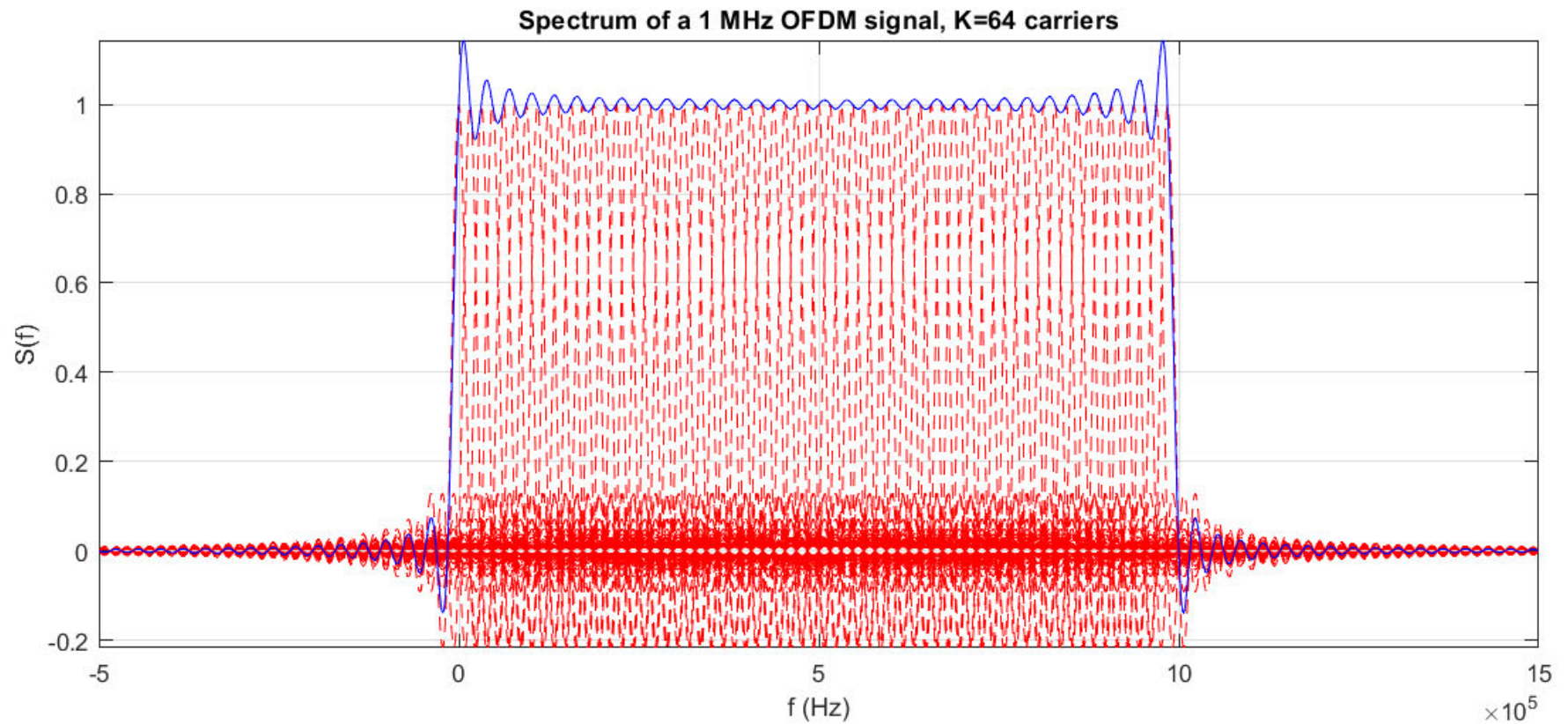
Subchannel (OFDM symbol) spectrum

- There are several options for the spectrum of each subchannel
 - Rectangular spectrum
 - Raised-cosine spectrum
 - “sinc”-like spectrum (pulse shapes as an RC spectrum)
- Regardless of the shape, the *subchannels become orthogonal* provided they are sampled in the frequency domain at intervals multiples of

$$f_s = \frac{1}{T} = \frac{W}{K}$$

See: Slide 29 of [13b_Coding_Modulation_Wireless.pdf](#)

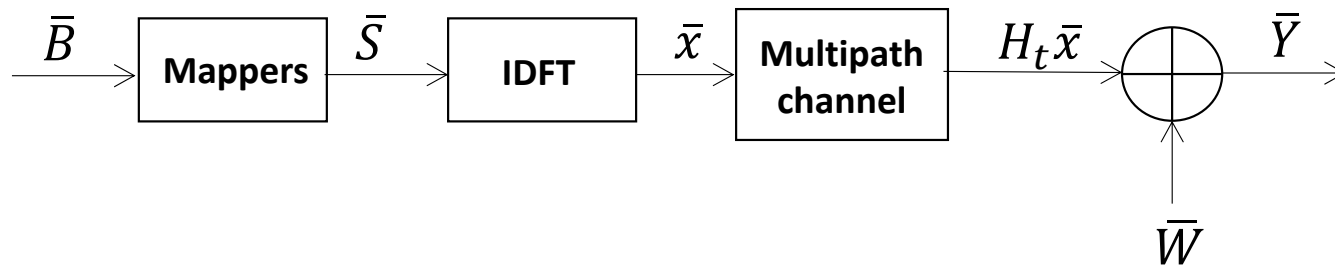
Example of *sinc*-shaped spectrum





Vector channel concept

- Remarkably, sampling in the frequency domain gives the *inverse DFT* (Discrete Fourier Transform) of the subchannel modulation symbols, $S_k, k = 0, 1, \dots, K - 1$
- This creates K orthogonal/parallel channels:





Channel matrix

- The channel output is a vector

$$\bar{Y} = H_t \bar{x} + \bar{W}$$

- The channel matrix H_t is diagonal:

$$H_t = \begin{bmatrix} c_0 & c_1 & \cdots & c_{L-1} & & & 0 \\ & c_0 & c_1 & \cdots & c_{L-1} & & \\ & & & \ddots & & & \\ 0 & & & & c_0 & c_1 & \cdots & c_{L-1} \end{bmatrix}$$

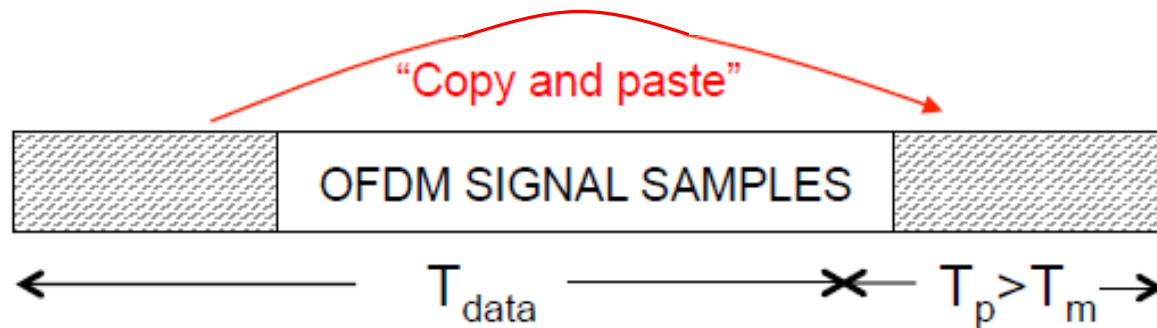
↑
K rows
↓

← $K+\nu$ columns →

where $\nu = L = \text{int}\left(\frac{T_m}{T}\right)$, and T_m is the delay spread



Adding a cyclic prefix



- The first ν symbols are appended to the end of an OFDM symbol
- This produces a *circulant* channel matrix:



Cyclic prefix = Circulant channel matrix

$$H = \begin{bmatrix} c_0 & c_1 & \cdots & c_{L-1} & & & 0 \\ & c_0 & c_1 & \cdots & c_{L-1} & & \\ & & \ddots & & & & \\ & & & c_0 & c_1 & \cdots & c_{L-1} \\ c_{L-1} & & & 0 & & & \\ c_{L-2} & c_{L-1} & & & c_0 & c_1 & \cdots & c_{L-2} \\ & c_0 & c_1 & \cdots & c_{L-3} & & & \\ c_1 & c_2 & \cdots & c_{L-1} & & & & c_0 \end{bmatrix}$$

K rows

K columns



Singular value decomposition (SVD)

- Singular value decomposition of circulant matrix H :

$$H = Q\Lambda Q^*$$

where Q^* : **inverse DFT** matrix, $Q^*Q = QQ^* = I_K$, and

$$\Lambda = \text{diag}(C_0, C_1, \dots, C_{K-1})$$

where $\bar{C} = Q\bar{c} = (C_0, C_1, \dots, C_{K-1})$ is the **DFT** of the channel impulse

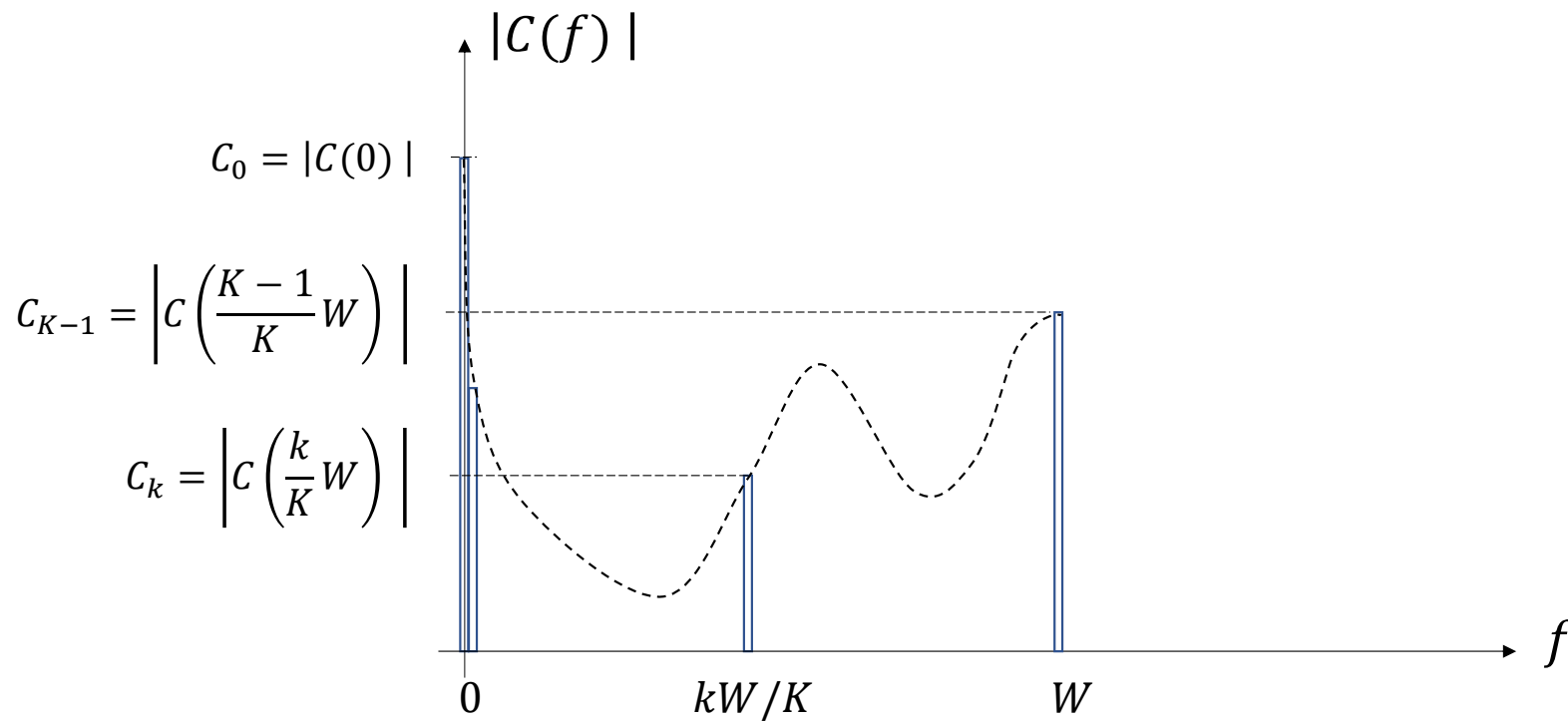
response $\bar{c} = (c_0, c_1, \dots, c_{\nu-1}, \boxed{0 \ 0 \ \dots \ 0})$

K- ν zeros



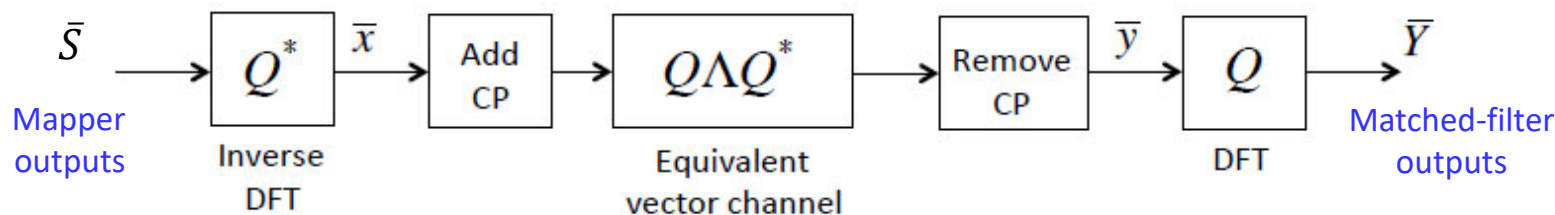
Subchannel gains

$$\bar{C} = Q\bar{c} = (C_0, C_1, \dots, C_{K-1})$$





CP-OFDM parallel channels with cyclic prefix



Since $QQ^* = Q^*Q = I_K$, **K parallel channels are created!**

$$\bar{Y} = \Lambda \bar{S}, \quad Y_k = C_k S_k, \quad 0 \leq k \leq K-1$$

where $C_k = [C(f)]_{f=k/T}$



CP-OFDM parallel channels

- To see why K parallel channels are created, write

$$\begin{aligned}\bar{Y} &= \bar{y}Q = (\bar{x}H)Q = (\bar{S}Q^*)HQ \\ &= (\bar{S}Q^*)(Q\Lambda Q^*)Q = \bar{S}I_K\Lambda I_K = \bar{S}\Lambda\end{aligned}$$

or

$$Y_k = S_k C_k, \quad k = 0, 1, \dots, K - 1$$



OFDM channel estimation

- Pilot symbols: P_k , $0 \leq k \leq K-1$, known at both Transmitter and Receiver

- $Y_k = C_k P_k + W_k$

- Receiver computes, after the FFT,

$$\hat{C}_k = \frac{P_k^*}{|P_k|^2} Y_k = \frac{P_k^*}{|P_k|^2} (C_k P_k + W_k), \quad 0 \leq k \leq K-1$$

$$\rightarrow \hat{C}_k = C_k + W_k' \approx C_k, \quad \text{for } \text{var}\{W_k'\} = \frac{N_0}{2} \text{ small.}$$



OFDM channel estimation (cont.)

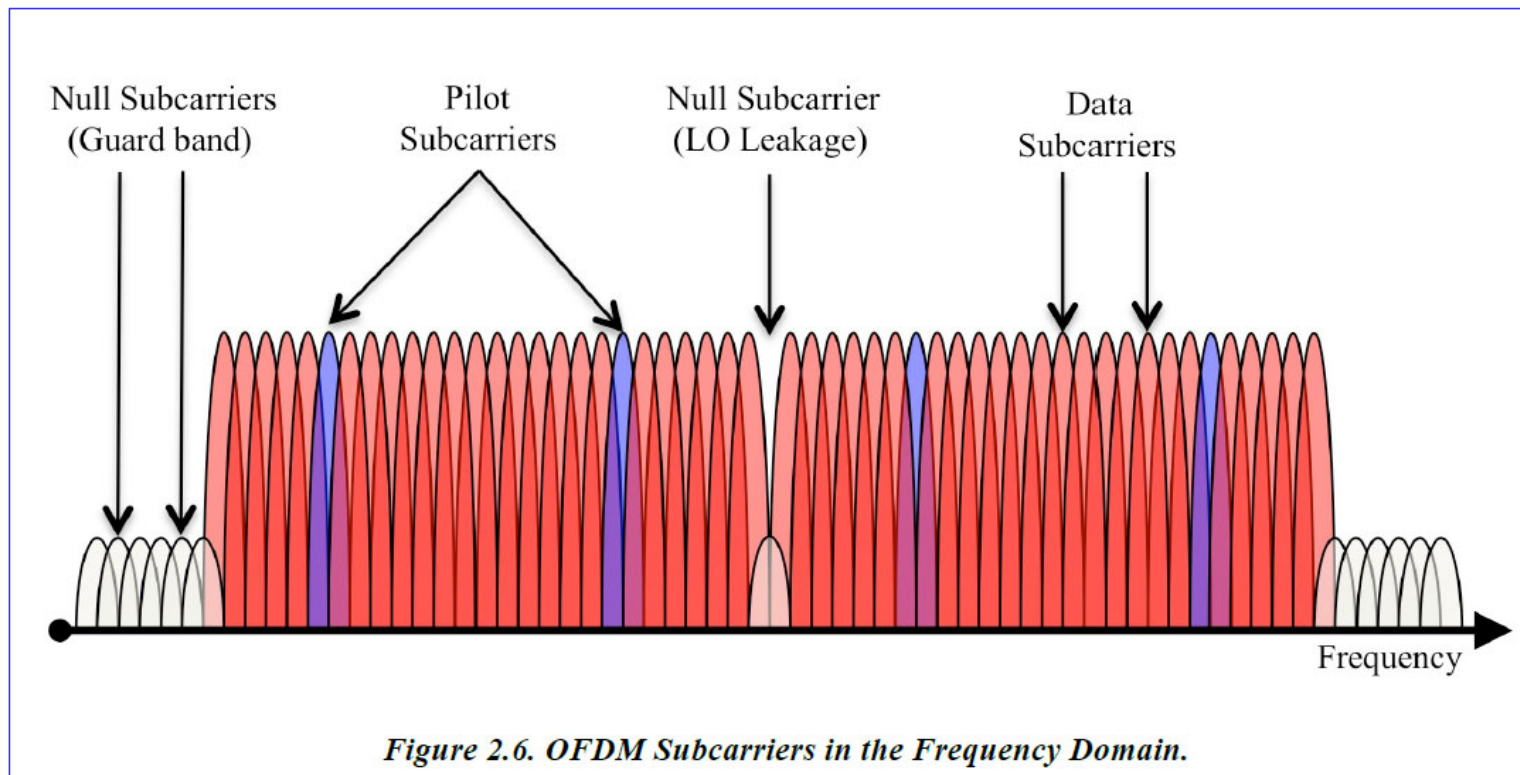
- In practice, several OFDM symbols carry pilot symbols and the receiver computes an average

$$\hat{G}_k = \frac{1}{N_p} \sum_{i=1}^{N_p} \hat{C}_{k,i}$$

where N_p is the number of times a pilot symbol is sent over the k -th subchannel.

- As an example, in the IEEE 802.11a (WiFi-2) specification, there are $K = 64$ subchannels of which 4 (four) are used for pilot symbols. (See next figure)

IEEE 802.11a pilot subchannels



Reference: https://download.ni.com/evaluation/rf/Introduction_to_WLAN_Testing.pdf



IEEE 802.11a pilot subchannels (cont.)

For the most common 802.11 implementations used in Wi-Fi, such as 802.11a/g, 802.11n, and 802.11ac, the OFDM transmission uses a constant symbol rate (and therefore constant subcarrier spacing) for all bandwidth configurations. **Table 2.4** shows how wider bandwidth options are implemented by using a larger number of subcarriers through a larger FFT size.

Bandwidth	FFT Size	Data Subcarriers	Pilot Subcarriers
20 MHz	64	52	4
40 MHz	128	108	6
80 MHz (VHT only)	256	234	8
160 MHz (VHT only)	512	468	16

Table 2.4. Bandwidth Configurations and FFT Sizes for HT and VHT PHY.

Reference: https://download.ni.com/evaluation/rf/Introduction_to_WLAN_Testing.pdf

Next lecture: Equalization and ECC

