

Pulse Shaping Techniques for Baseband Binary Communication

EE 161: Digital Communication Systems

San Jose State University

Baseband binary communication

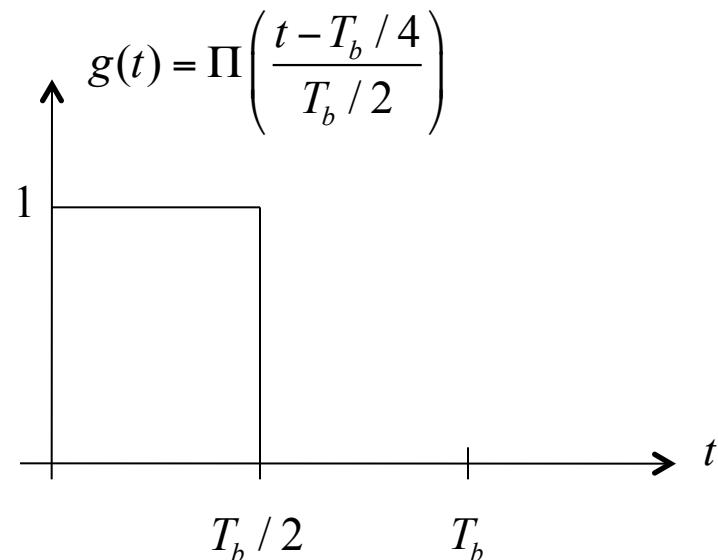
- In a *baseband* binary communication system, strings of bits need to be converted into *sequences of pulses* (*a waveform*) that are suitable for transmission over the lowpass channel
- The process of converting bits into pulses is known as ***pulse shaping*** (historically referred to as “line coding”). There are two components of this process:
 - 1. Pulse shape***
 - 2. Mapping of bits to amplitudes***
- In this lecture, only rectangular pulses are considered. Other pulses used in practice include square-root raised-cosine (SRRC) and Gaussian pulses

Basic pulse shapes/mappings

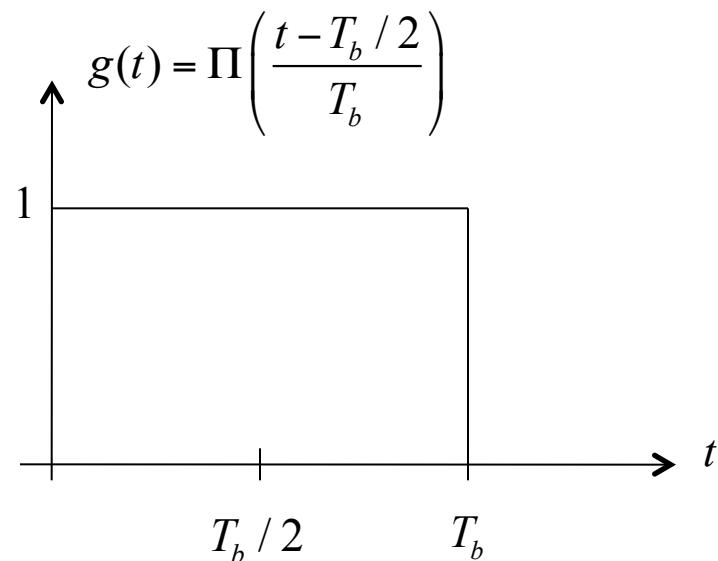
- **Pulse shapes**
 - Return-to-zero (**RZ**)
 - Non return-to-zero (**NRZ**)
 - **Manchester** (or split phase)
- **Mappings of bits to amplitudes**
 - **Unipolar**
 - **Polar**
 - Alternate-mark-inversion (**AMI**)

The above shapes/mappings induce a classification of schemes: *Unipolar NRZ*, *AMI RZ*, etc...

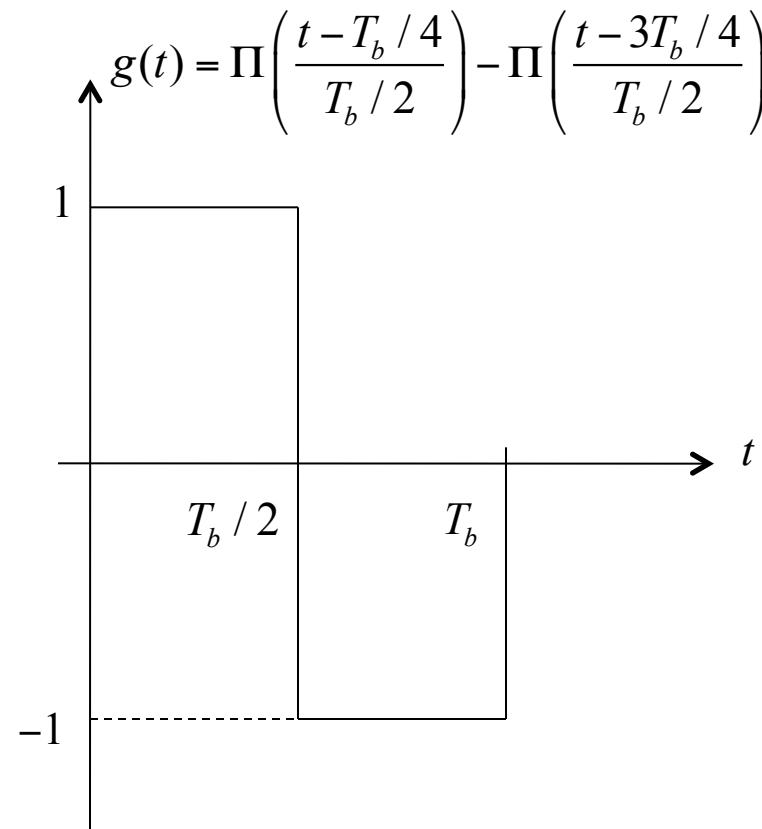
Return-to-zero (RZ) pulse



Non-return-to-zero (NRZ) pulse



Manchester pulse



Unipolar mapping

Bit	Amplitude
B_n	A_n
0	0
1	a

a: Amplitude

Example: $\underline{B} = \{0,0,1,1,0,1,0,\dots\}$ $\underline{A} = \{0,0,a,a,0,a,0,\dots\}$

Polar mapping

Bit	Amplitude
B_n	A_n
0	-a
1	a

a: Amplitude

Example: $\underline{B} = \{0,0,1,1,0,1,0,\dots\}$ $\underline{A} = \{-a,-a,a,a,-a,a,-a,\dots\}$

Note: The mapping that assigns a to 0 and -a to 1 is also valid

AMI (or bipolar) mapping

Bit	Amplitude
B_n	A_n
0	0
1	$A_n = -A_m, m \text{ is the largest index}$ $\text{such that } m < n \text{ and } A_m \neq 0.$

Example: $\underline{B} = \{0,0,1,1,0,1,0,\dots\}$ $\underline{A} = \{0,0,a,-a,0,a,0,\dots\}$ (initial state=a)

Note: The sequence $\underline{A} = \{0,0,-a,a,0,-a,0,\dots\}$ is also valid (initial state=-a)

- This mapping has ***memory***. That is, the most recent *nonzero amplitude* level needs to be “remembered”
- An ***initial state*** (sign) is needed

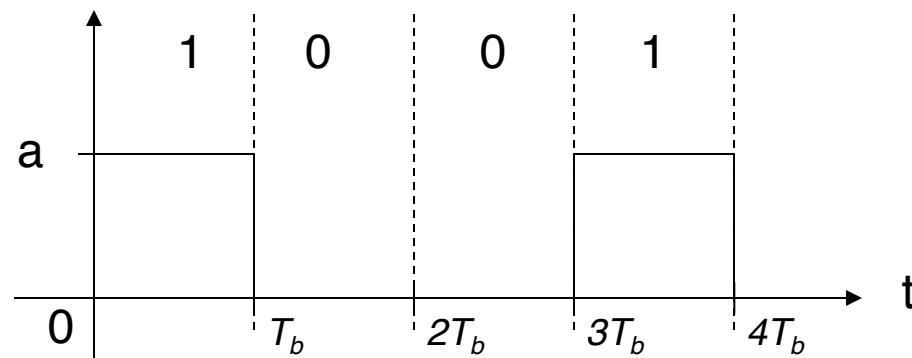
Other mappings with memory

- Dicode (ternary mapping)
 - If there is a bit transition, then amplitude transition (polar)
 - Else, amplitude equal to zero

Example: $\underline{B} = \{0,0,1,1,1,0,0,\dots\}$ $\underline{A} = \{a,0,-a,0,0,a,0,\dots\}$
- Mark code
 - “0” = No amplitude transition
 - “1” = Amplitude transition

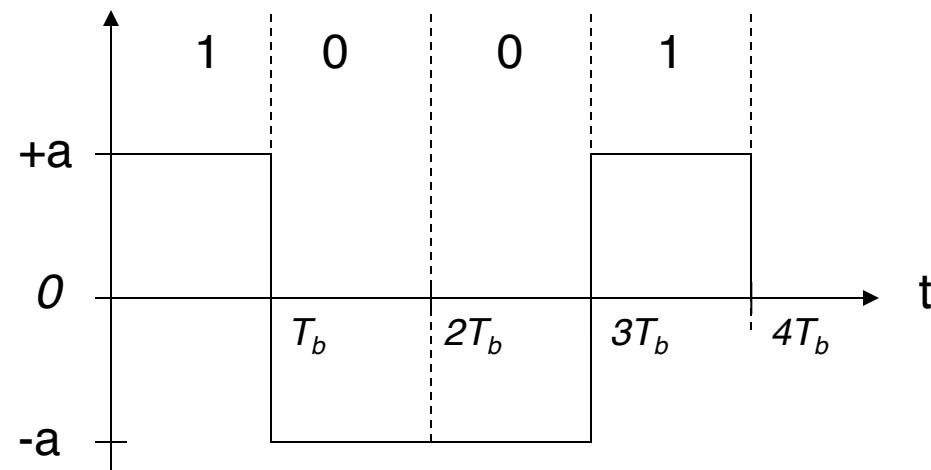
Example: $\underline{B} = \{0,0,1,1,1,0,0,\dots\}$ $\underline{A} = \{a,a,-a,a,-a,-a,-a,\dots\}$
- Miller code
 - “1” = Transition in the middle of the bit duration ($T_b/2$)
 - “0” = Constant level
 - “0 to 0” = Transition at the end of the bit duration (T_b)

Unipolar NRZ signaling



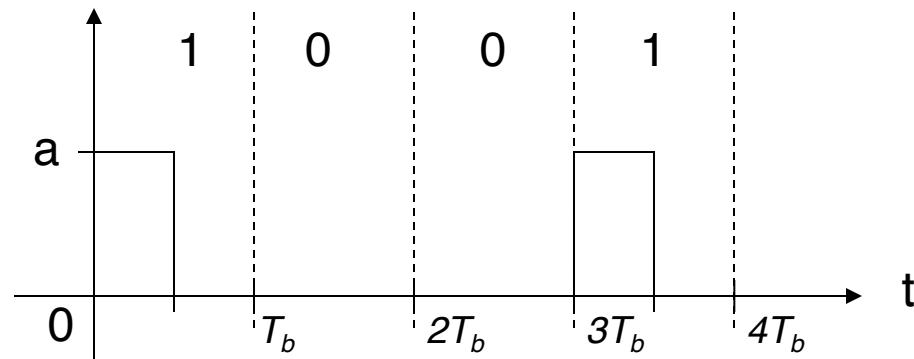
- No transitions if there is a long string of identical “0” or “1”
- This means it is difficult to recover the clock
- Strong DC component means power is wasted

Polar NRZ signaling



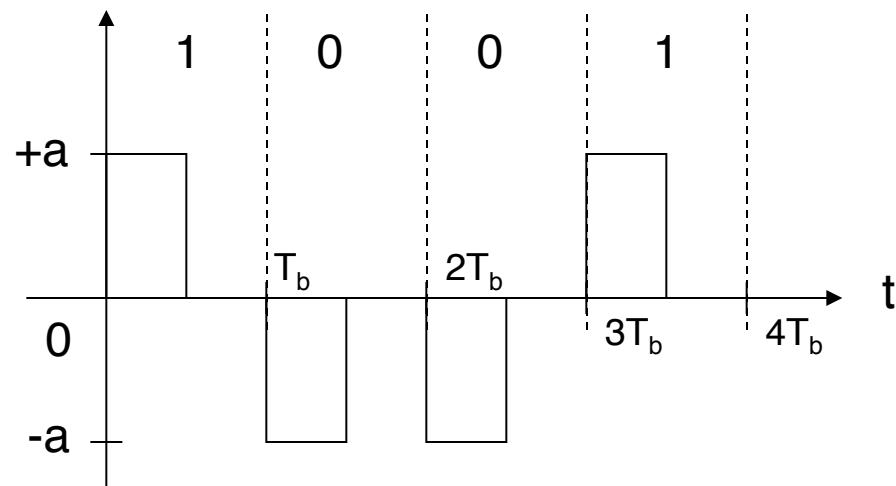
- No DC component for long strings of equally likely bits
- No transitions if there is a long string of identical “0” or “1”
- This means it is difficult to recover the clock

Unipolar RZ signaling



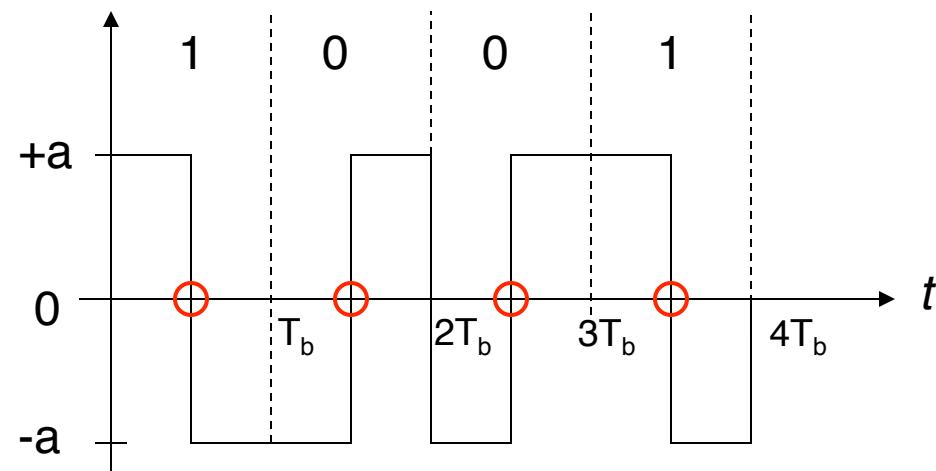
- Same as NRZ with pulses of half width
- Fixed problems with long string of “1”
- No transitions if there is a long string of “0”
- This means it may be difficult to recover the clock
- Strong DC component means power is wasted

Polar RZ signaling



- Same as polar NRZ with half-width pulses
- Fixes problems with long strings of “0” and “1”
- No DC component if “0” and “1” are balanced
- Power spectral density same as Unipolar RZ without impulses

Manchester signaling

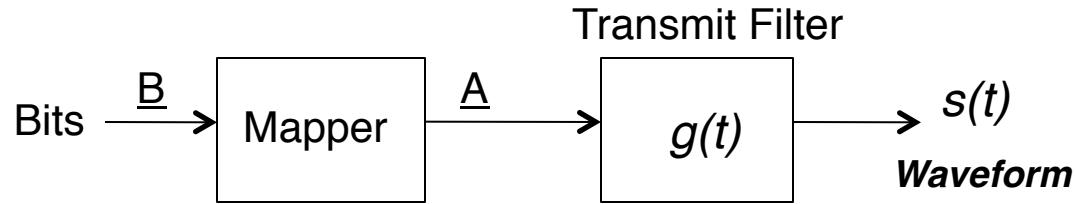


- Always have a transition every T_b seconds
- Easy to recover clock, independent of string of “0” and “1”
- No DC component, regardless of the bit string

Design objectives

- Pulse shaping and mapping are jointly designed to meet several objectives:
 - **Self-synchronization**
 - An ability to recover timing from the signal itself
 - Long series of ones and zeros could cause a problem
 - **Low probability of bit error**
 - The receiver needs to be able to distinguish the waveform associated with a zero from the waveform associated with a one, even if there is a considerable amount of noise and distortion in the channel
 - **Spectrum shape** suitable for the channel.
 - In some cases DC components should be avoided.
 - e.g. if the channel has a DC blocking capacitance or a transformer.
 - The transmission bandwidth should be minimized.

Pulse shaping



- The input to the transmit filter is a sequence of real values A_k from a ***mapper***
- The output of the transmit filter is a ***waveform***:

$$s(t) = \sum_{n=-\infty}^{\infty} A_n g(t - nT_b),$$

where $g(t)$ is the ***pulse shape*** and T_b is the ***bit period***

- The operational details of this process are set by the particular combination of mapper and transmit filter (pulse shape) used.

Power spectral density (PSD)

$$S_s(f) = \frac{|G(f)|^2}{T_b} \sum_{n=-\infty}^{\infty} R[n] e^{-j2\pi f n T_b} \quad (1)$$

where

$G(f) \Leftrightarrow g(t)$, and $R[n] = E\{A_k A_{k+n}\}$: Autocorrelation of $\{A_n\}$.

- If $\{A_n\}$ are uncorrelated, then $R[n] = \begin{cases} \sigma_A^2 + m_A^2, & n = 0 \\ m_A^2, & n \neq 0 \end{cases}$

$$S_s(f) = \frac{|G(f)|^2}{T_b} \left[\sigma_A^2 + \frac{m_A^2}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \right] \quad (2)$$

Example: PSD of Polar NRZ

- Let $p_0 = \Pr\{B_n = 0\}$, $p_1 = \Pr\{B_n = 1\}$. Then

$$R[n] = \begin{cases} p_0(-a)^2 + p_1(a)^2 = a^2, & n = 0 \\ p_0^2(-a)^2 + p_1^2(a)^2 \\ \quad + 2p_0p_1(a)(-a) = 0, & n \neq 0 \end{cases} \Rightarrow R[n] = a^2 \delta[n]$$

- From **equation (1)**: $S_s(f) = \frac{a^2 |G(f)|^2}{T_b}$
- For an NRZ rectangular pulse: $G(f) = T_b \text{sinc}(fT_b)$. Thus

$$S_s(f) = a^2 T_b \text{sinc}^2(fT_b)$$

Example: PSD of Unipolar NRZ

- Here $m_A = \frac{a}{2}$, $\sigma_A^2 = \frac{a^2}{4}$
- From **equation (2)**:

$$S_s(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2(f T_b) + \frac{a^2}{4} \delta(f)$$

Example: PSD of AMI NRZ/RZ

- Here

$$R[n] = \begin{cases} a^2 / 2, & n = 0 \\ -a^2 / 4, & n = \pm 1 \\ 0, & |n| > 1 \end{cases}$$

- From **equation (1)**:

$$\begin{aligned} S_s(f) &= \frac{1}{T_b} |G(f)|^2 \left(\frac{a^2}{2} - \frac{a^2}{4} e^{+j2\pi f T_b} - \frac{a^2}{4} e^{-j2\pi f T_b} \right) \\ &= \frac{1}{T_b} |G(f)|^2 \left[\frac{a^2}{2} - \frac{a^2}{2} \cos(2\pi f T_b) \right] = \frac{a^2}{T_b} |G(f)|^2 \sin^2(\pi f T_b) \end{aligned}$$

$$S_s(f) = a^2 T_b \operatorname{sinc}^2(f T_b) \sin^2(\pi f T_b), \quad \text{for NRZ pulses}$$

$$S_s(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2\left(\frac{f T_b}{2}\right) \sin^2(\pi f T_b), \quad \text{for RZ pulses}$$

Example: PSD of Manchester code

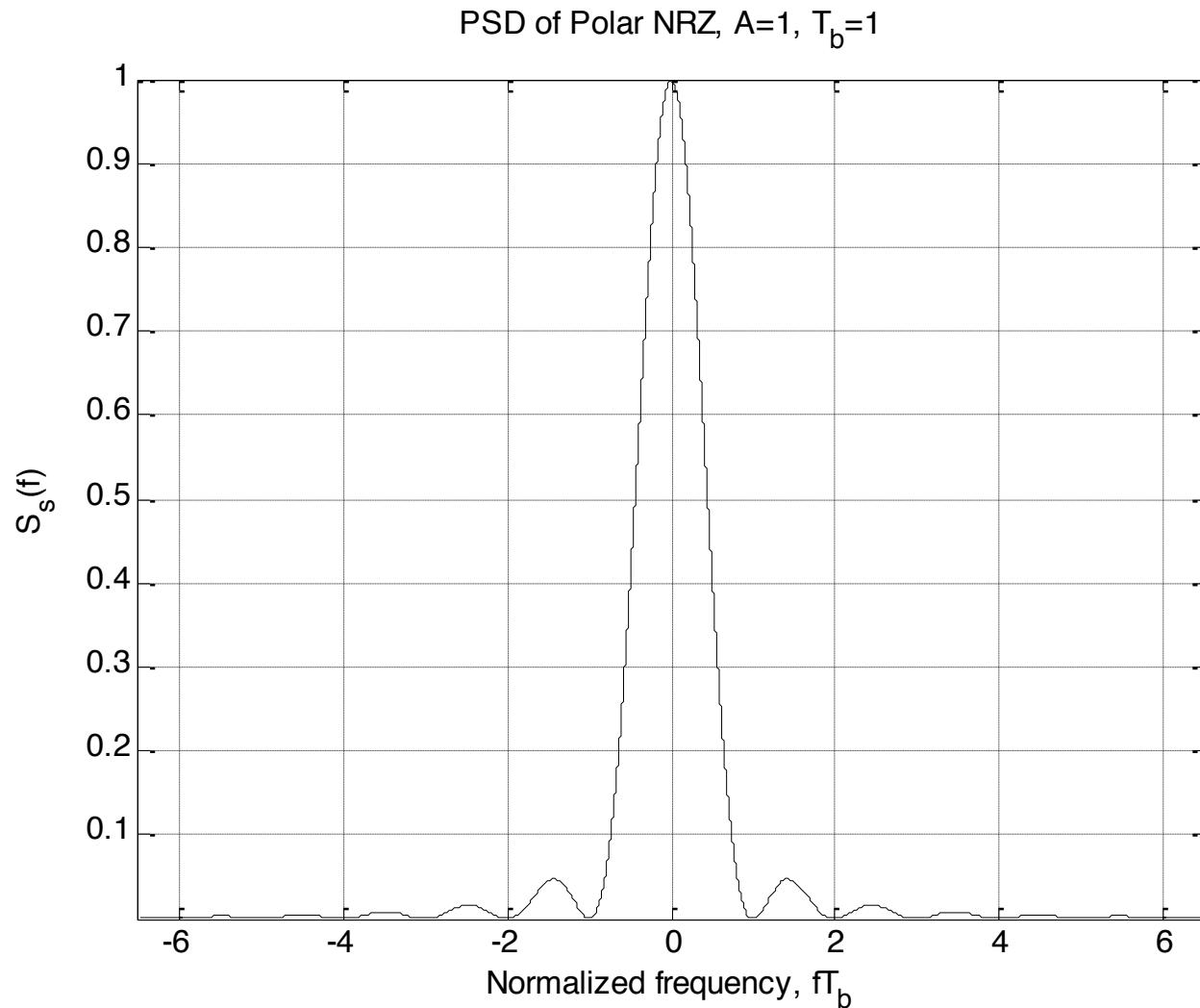
- Manchester code refers to the combination of a Manchester pulse with polar mapping
- Pulse spectrum:

$$g(t) = \Pi\left(\frac{t - T/4}{T/2}\right) - \Pi\left(\frac{t - 3T/4}{T/2}\right) \Leftrightarrow G(f) = T_b \operatorname{sinc}\left(\frac{fT_b}{2}\right) \sin\left(\frac{\pi}{2} f T_b\right) e^{j\frac{\pi}{2}}$$

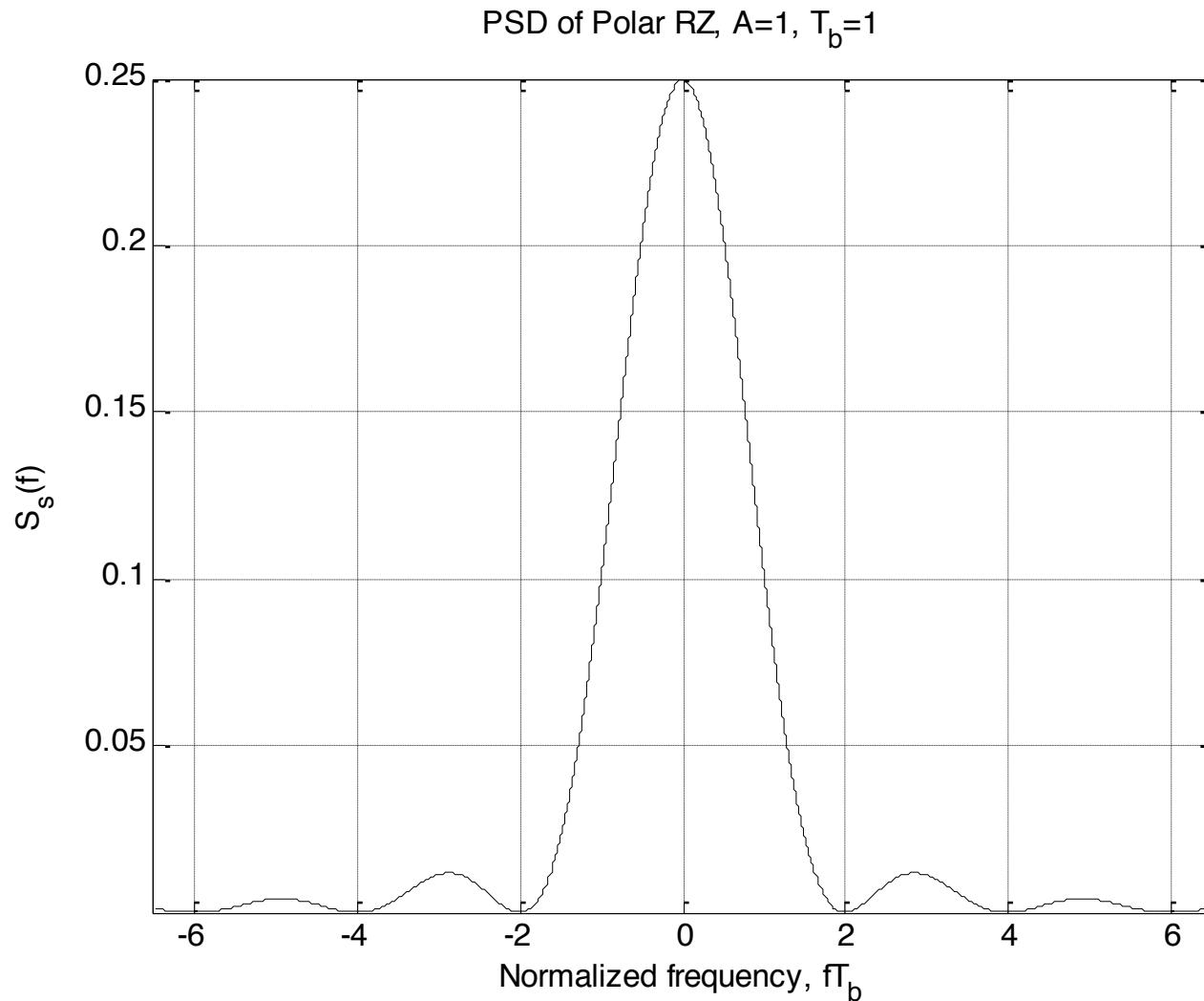
- Polar mapping: $R[n] = a^2 \delta[n]$. Therefore,

$$S_s(f) = a^2 T_b \operatorname{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2\left(\frac{\pi}{2} f T_b\right)$$

PSD of polar NRZ

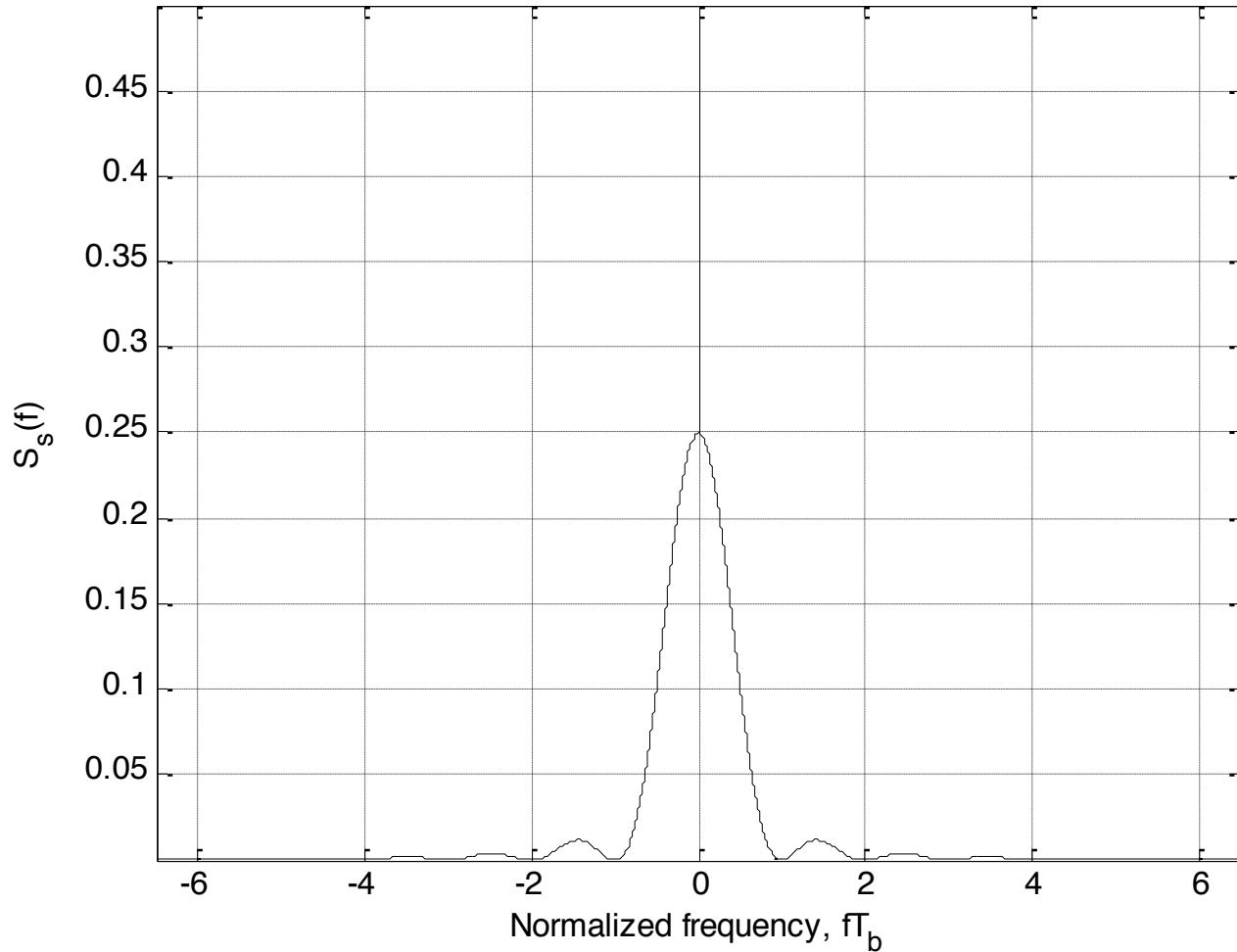


PSD of polar RZ



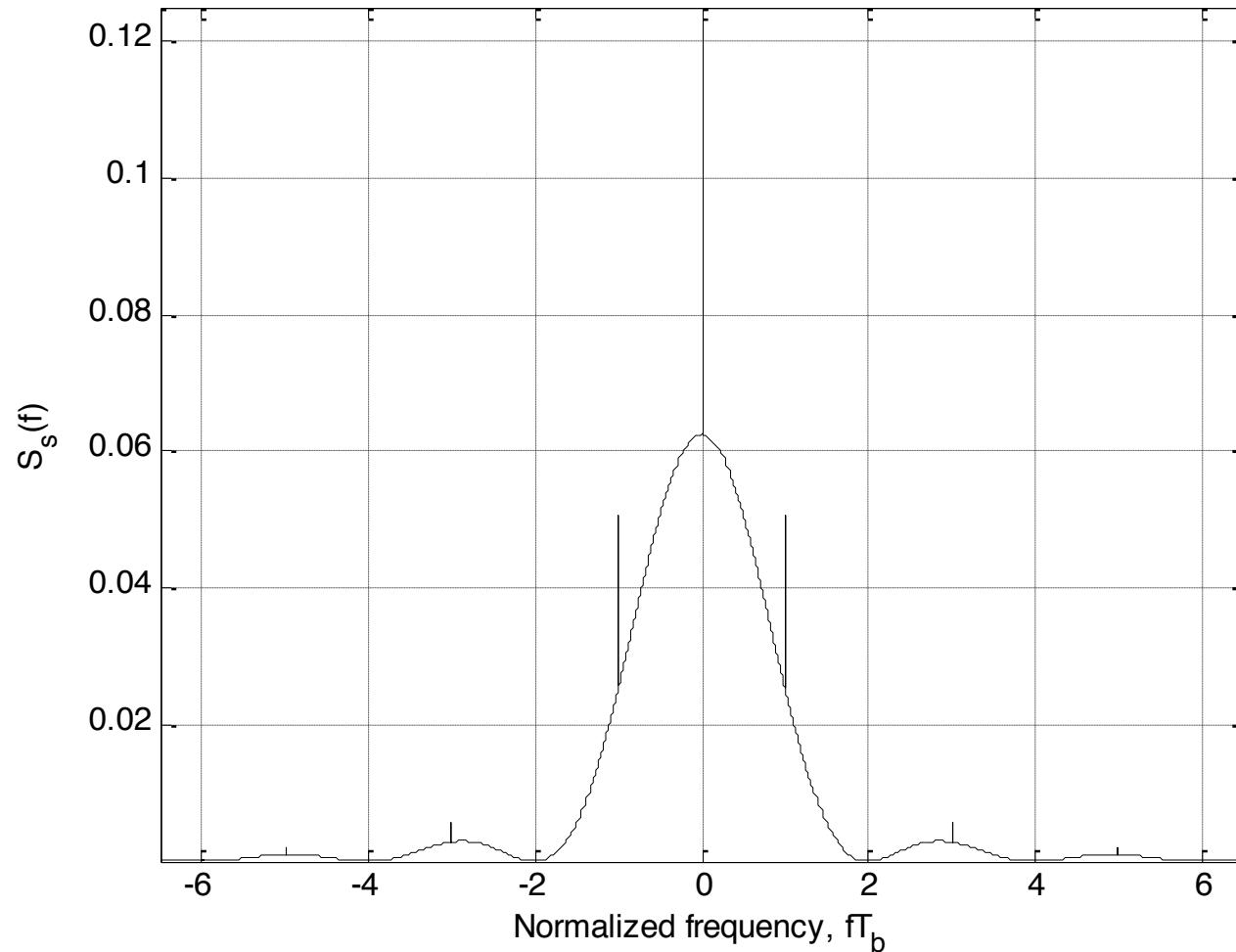
PSD of unipolar NRZ

PSD of Unipolar NRZ, $A=1$, $T_b=1$

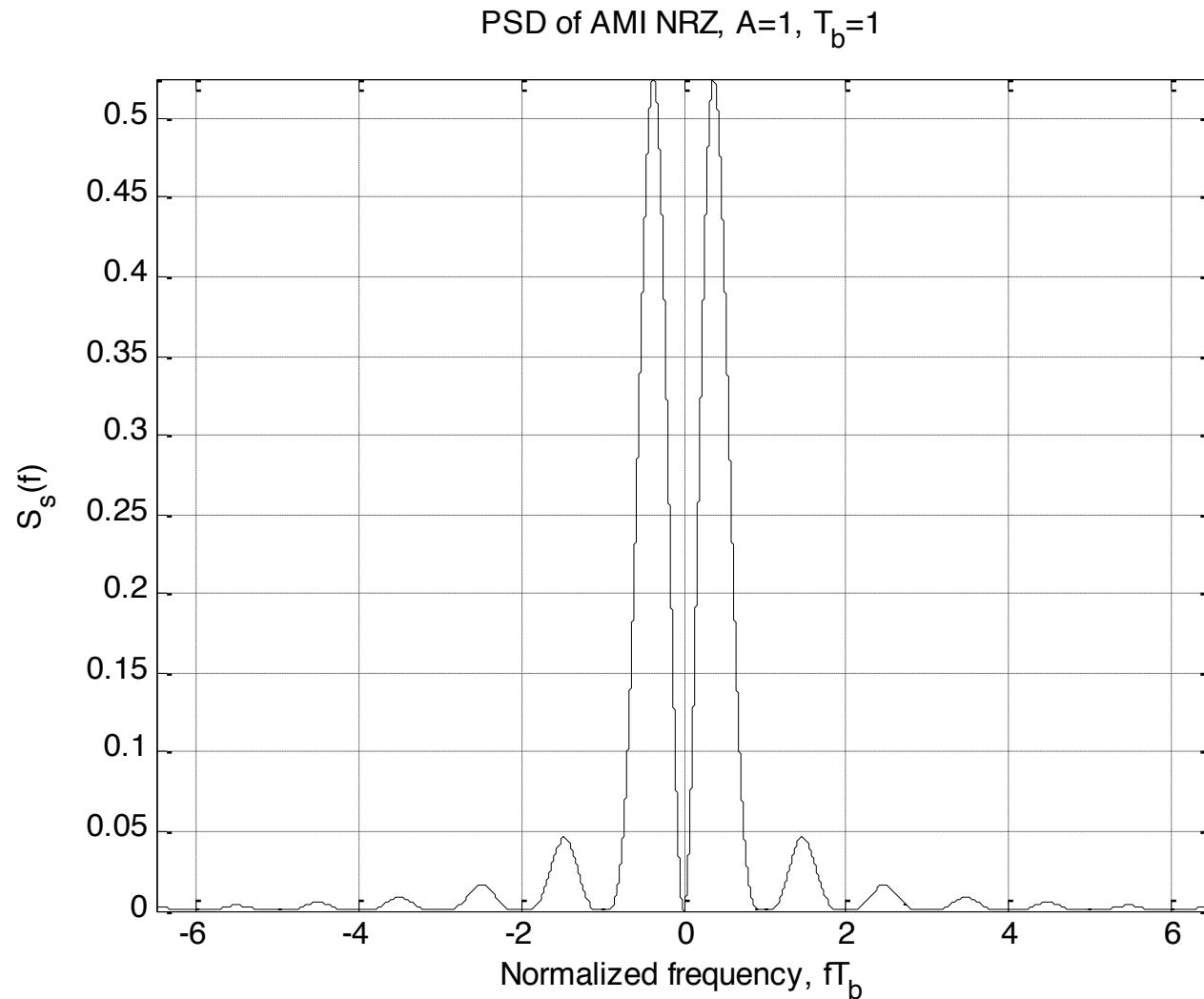


PSD of unipolar RZ

PSD of Unipolar RZ, A=1, $T_b=1$

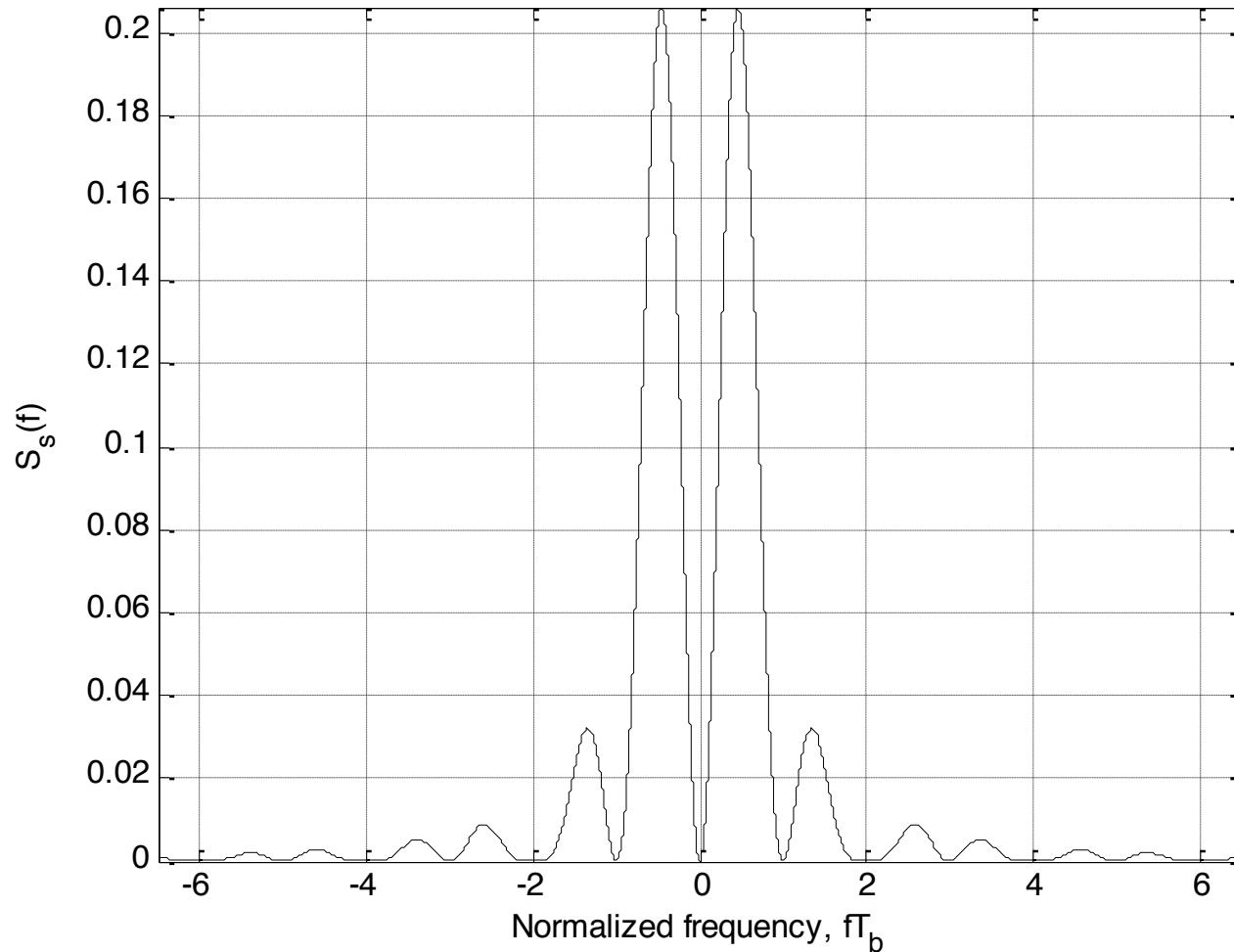


PSD of AMI NRZ



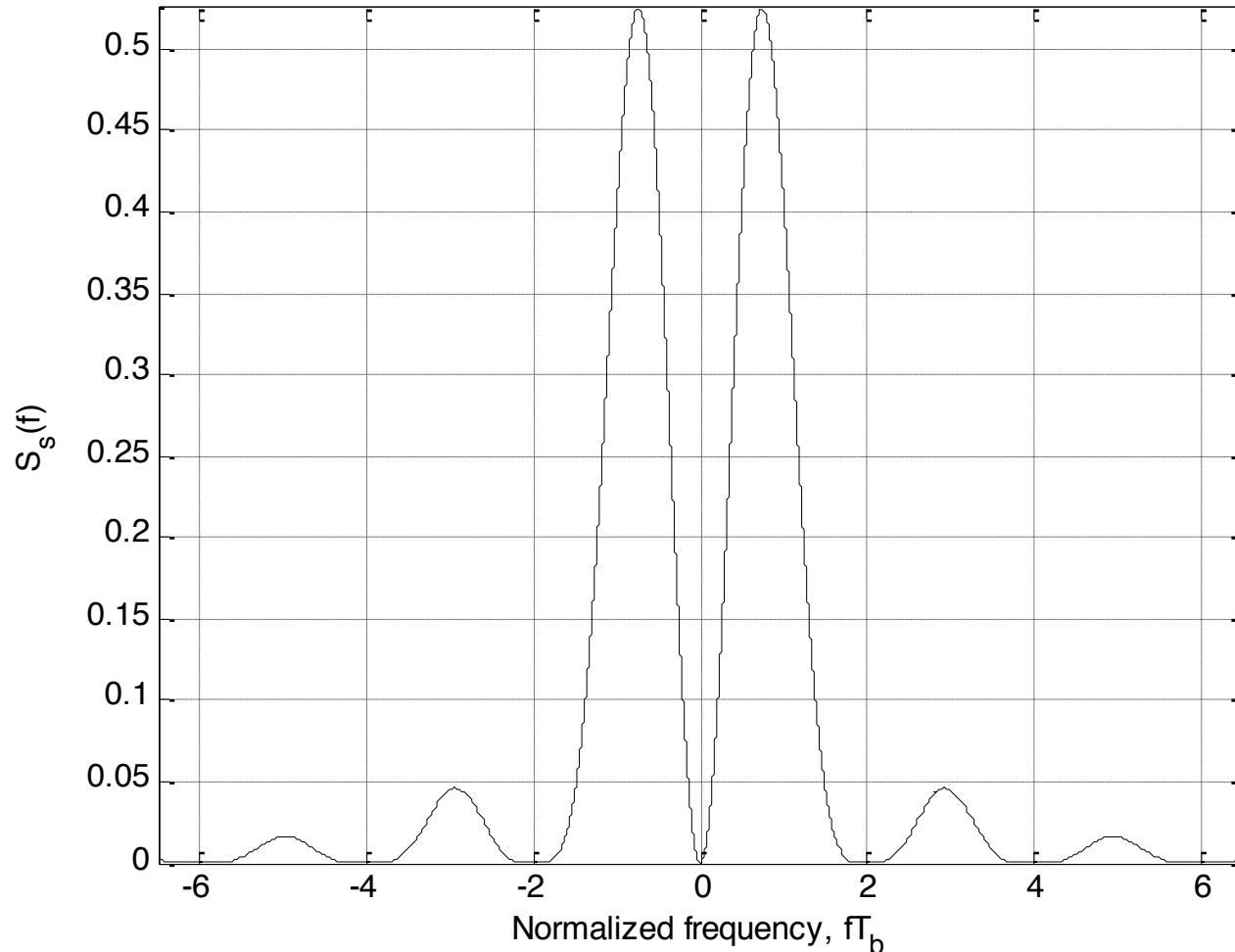
PSD of AMI RZ

PSD of AMI RZ, A=1, $T_b=1$



PSD of Manchester code

PSD of Manchester code, $A=1$, $T_b=1$



NOTE: *The Manchester code was used in the first generation of Ethernet (IEEE 802.3 standard) and is being used (as of 2010) in second generation RFID systems*

Summary of power shaping

- DC Components
 - Unipolar NRZ, polar NRZ, and unipolar RZ all have a DC component
 - AMI RZ and Manchester NRZ do not have DC component
- Null-to-null Bandwidth (NNB)
 - Unipolar NRZ, polar NRZ, and **bipolar** all have NNB equal to $R_b = 1/T_b$
 - Unipolar RZ has NNB equal to of $2R_b$
 - **Manchester** NRZ also has NNB equal to $2R_b$, although the spectrum becomes very low at approximately $1.6R_b$