Time Diversity Techniques for Frequency-Selective Multipath Channels

EE161: Digital Communication Systems

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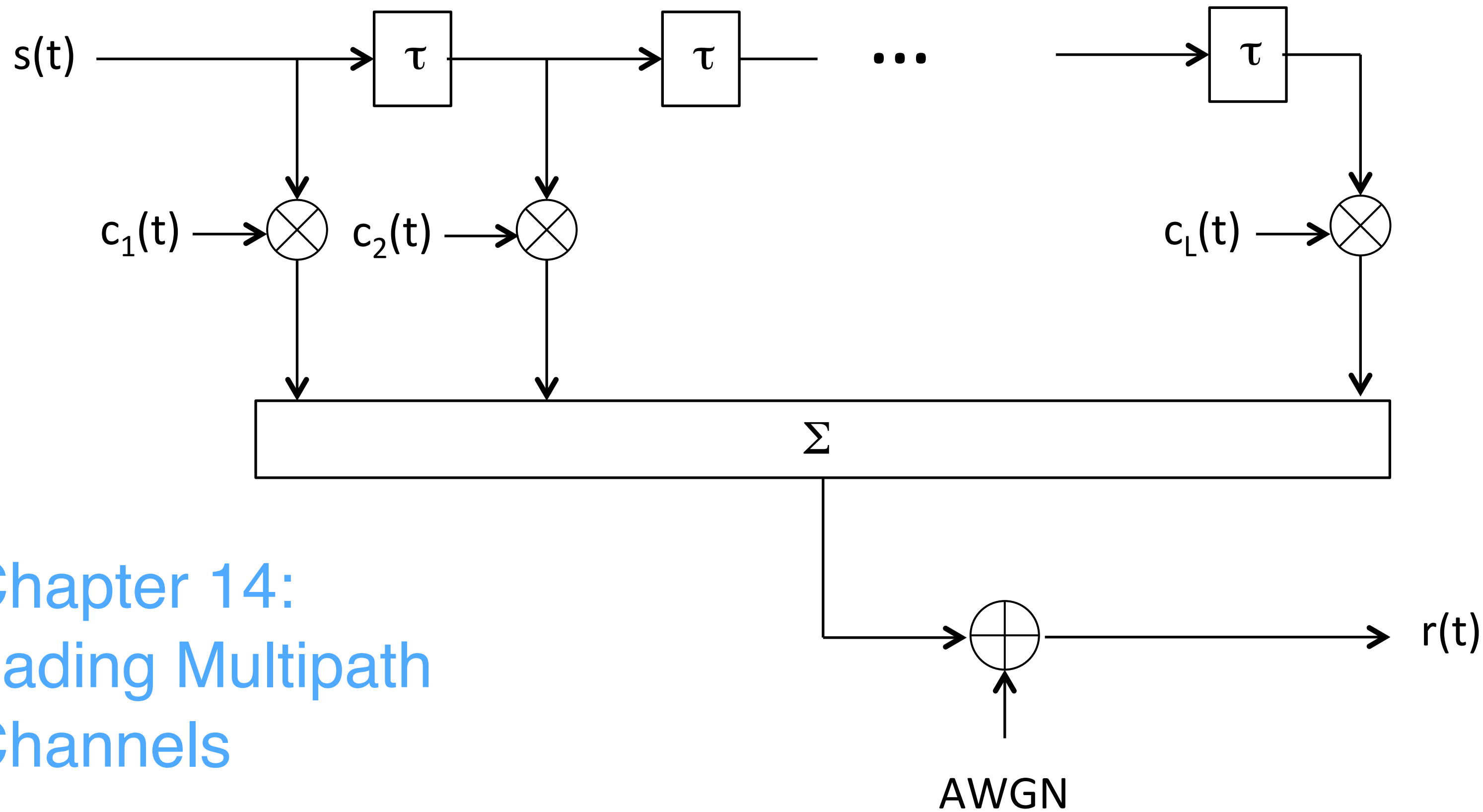
Time-diversity techniques

- Time-diversity techniques can be classified according to the frequency selectivity of the multipath channel
- Flat fading channels
  - Error correcting coding & interleaving (lecture notes)
  - Diversity order is equal to the minimum Hamming distance of the code
- Frequency-selective channels
  - RAKE demodulation
  - Linear adaptive equalization

Complex baseband frequency-selective multipath channel model



Chapter 14: Fading Multipath Channels

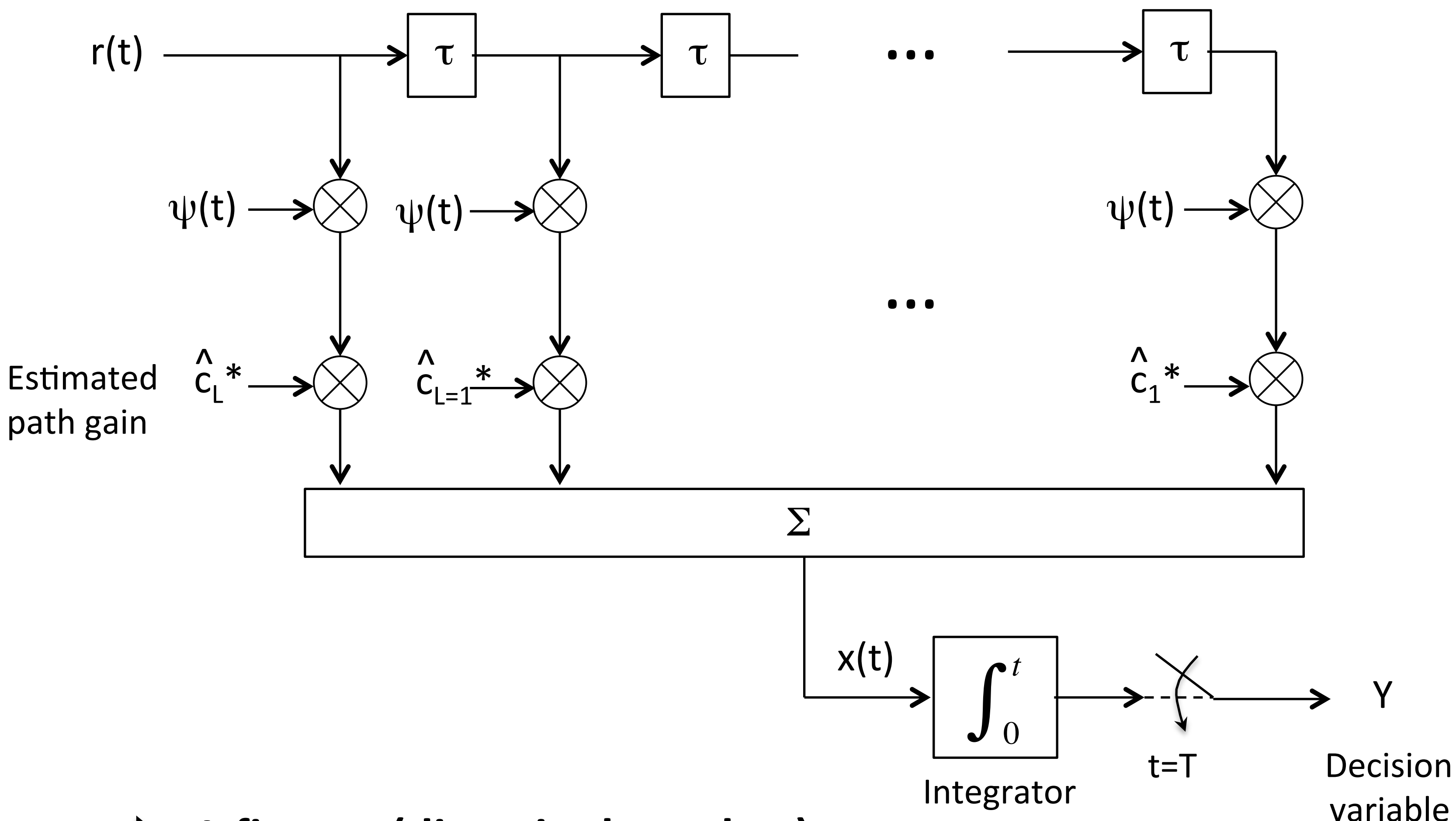
$c_i(t) = a_i(t)\, e^{j\phi i(t)}$: i-th path gain, i=1,2,…,L (see lecture notes)

RAKE demodulator: Assumptions

- Slow fading: $T << T_c \quad \Rightarrow \quad c_i(t) = c_i, i = 1, \cdots, L$
- Frequency-selective fading: $W >> B_c$ (1)
- No ISI: $T >> T_m$ (2)
  - Assumptions (1) and (2) are satisfied by *wideband pulses*, such as PPM or spread-spectrum
  - ***Examples (in classroom board)***
- Path gains known
  - Need channel estimation techniques (*finger search*)
- Binary modulation (N=1 pulse)

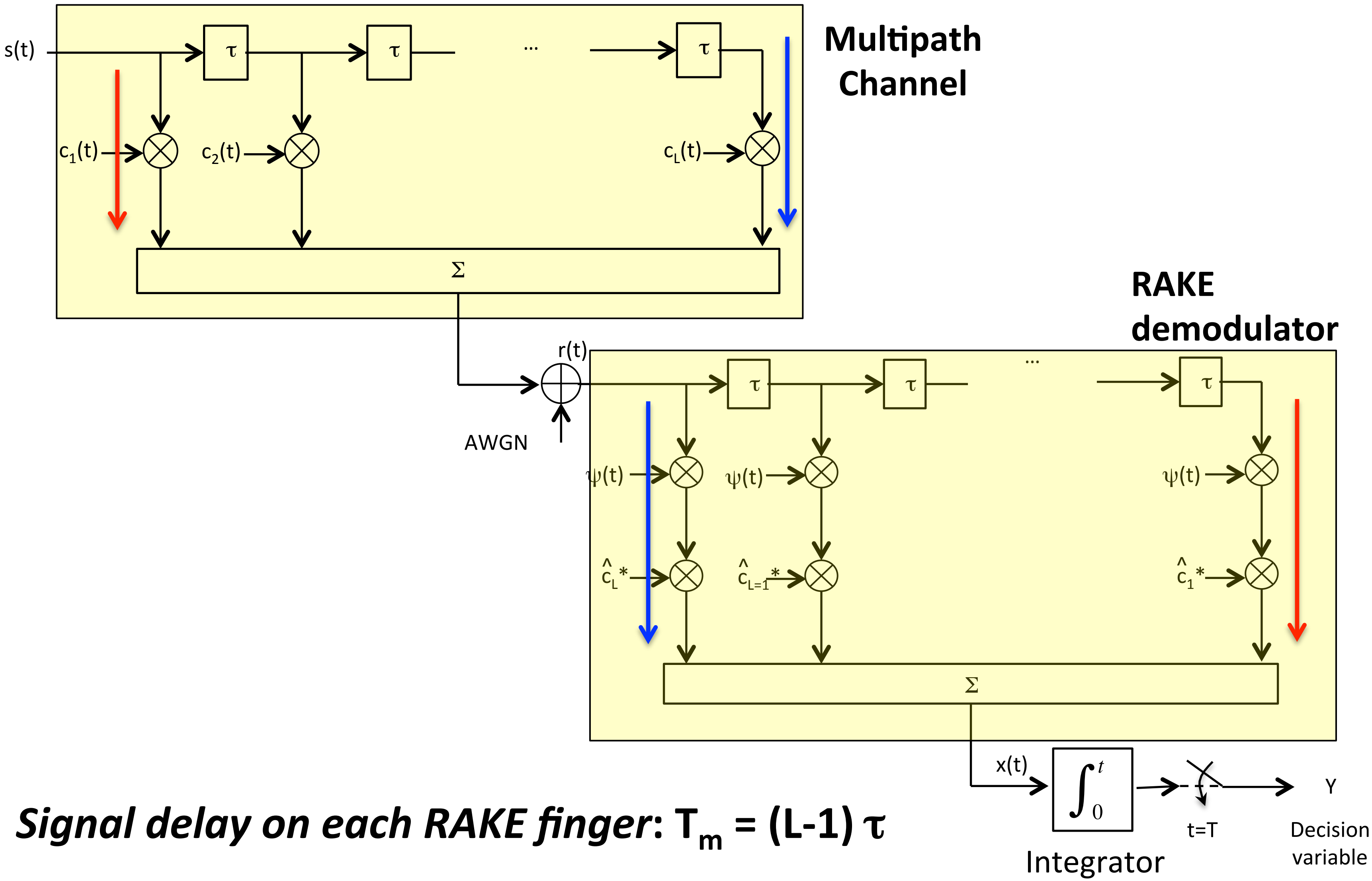
Assuming same delay between replicas in CIR

RAKE demodulator: Structure



➞ **L fingers (diversity branches)**

Maximal-ratio combining property



Signal delay on each RAKE finger: $T_m = (L-1)\,\tau$

Maximal-ratio combining property (cont.)

- RAKE output (assuming no AWGN):

$$x(t) = r(t)\psi(t)c_L^* + r(t-\tau)\psi(t)c_{L-1}^* + \cdots + r\big(t-(L-1)\tau\big)\psi(t)c_1^*$$

$$= \left[\sum_{i=1}^{L} c_i s\big(t-(i-1)\tau\big)\right]\psi(t)c_L^* + \left[\sum_{i=1}^{L} c_i s(t-i\tau)\right]\psi(t)c_{L-1}^*$$

$$+ \quad \cdots \quad + \left[\sum_{i=1}^{L} c_i s\big(t-(i+L-2)\tau\big)\right]\psi(t)c_1^*$$

$$\Rightarrow \quad x\big(t-(L-1)\tau\big) = \sum_{i=1}^{L} |c_i|^2 s\big(t-(L-1)\tau\big)\ \psi(t)$$

MRC performance

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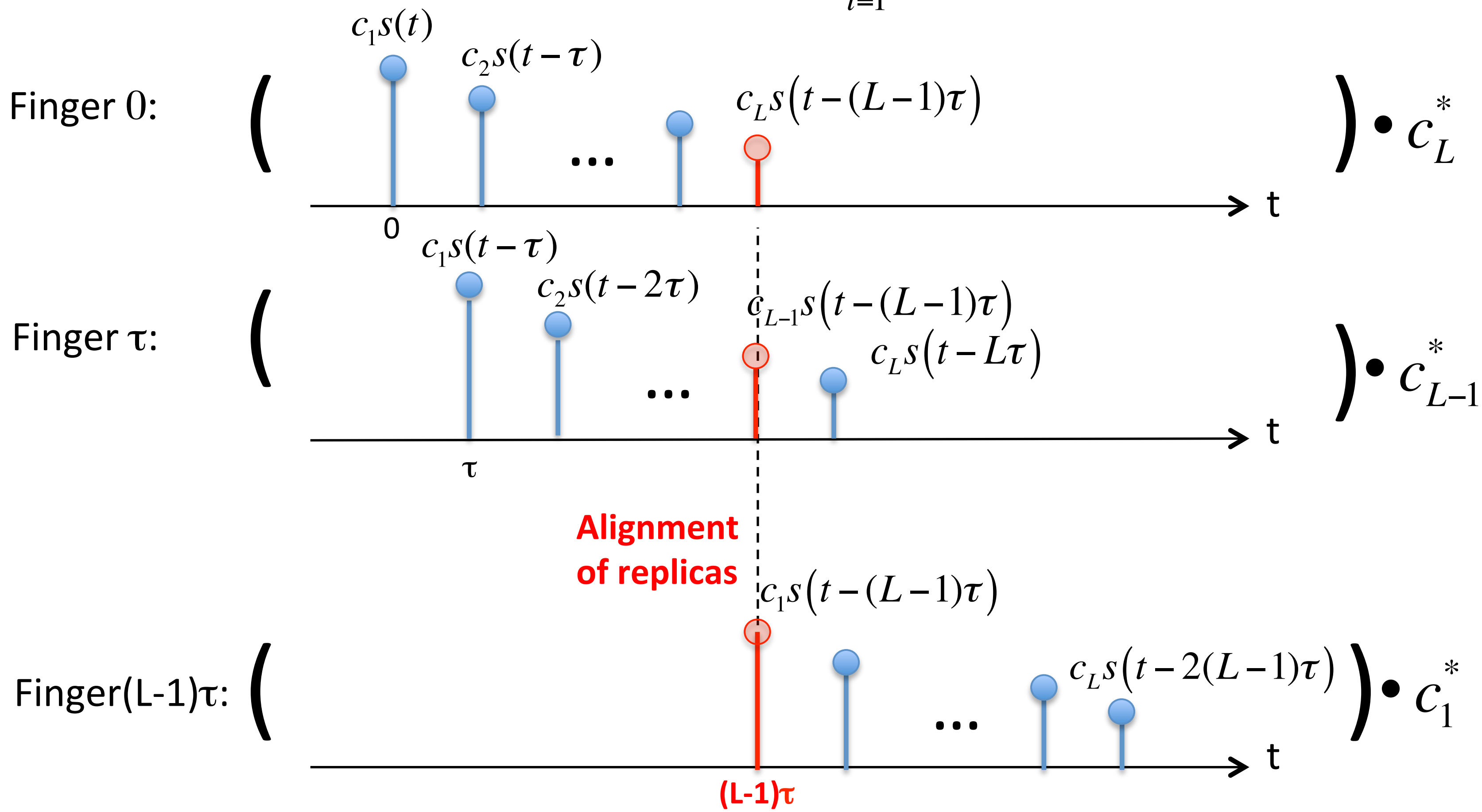
Maximal-ratio combining property (cont.)

- Matched-filter output: $Y = S\sum_{i=1}^{L}|c_i|^2 + N$



RAKE performance under Rayleigh fading

- The signal goes through ***L* diversity branches**
- Probability of error for BPSK (polar) and BFSK (binary orthogonal or 2-PPM):

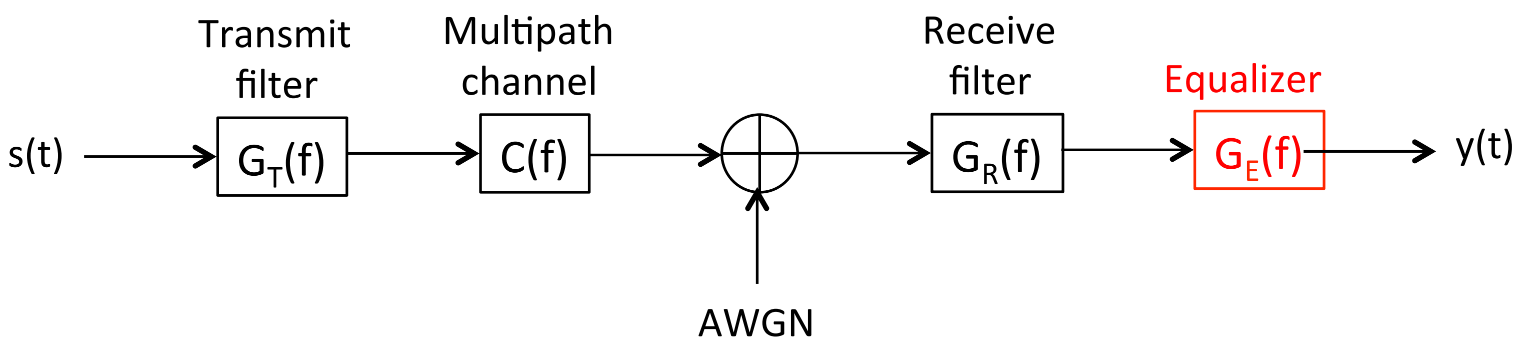
$$\boxed{P_b = \frac{(2L-1)!}{L!(L-1)!}\prod_{i=1}^{L}\left[\frac{1}{2(1-c)\bar{E}_{s,i}/N_0}\right]^i}$$

where $\bar{E}_{s,i}/N_0$ is the average signal energy-to-noise ratio of the i-th path, i=1,2, ..., L, $c=-1$ for BPSK and $c=0$ for BFSK.

Linear adaptive equalizer: Assumptions

- Slow fading: $T << T_c \quad \Rightarrow \quad c_i(t) \approx c_i, i = 1, \cdots, L$
- Frequency-selective fading: $W >> B_c$
- Presence of ISI: $T < T_m$
  - Signaling using *narrowband pulses*
  - ***Example: Square-root raised-cosine (SRRC) pulses***
- Path gains unknown
  - Need pilot symbols (known at the receiver) and an adaptive algorithm to estimate gains
- Binary modulation (N=1 pulse)

Linear adaptive equalizer: System model



- Matched filter, assuming SRRC pulses:

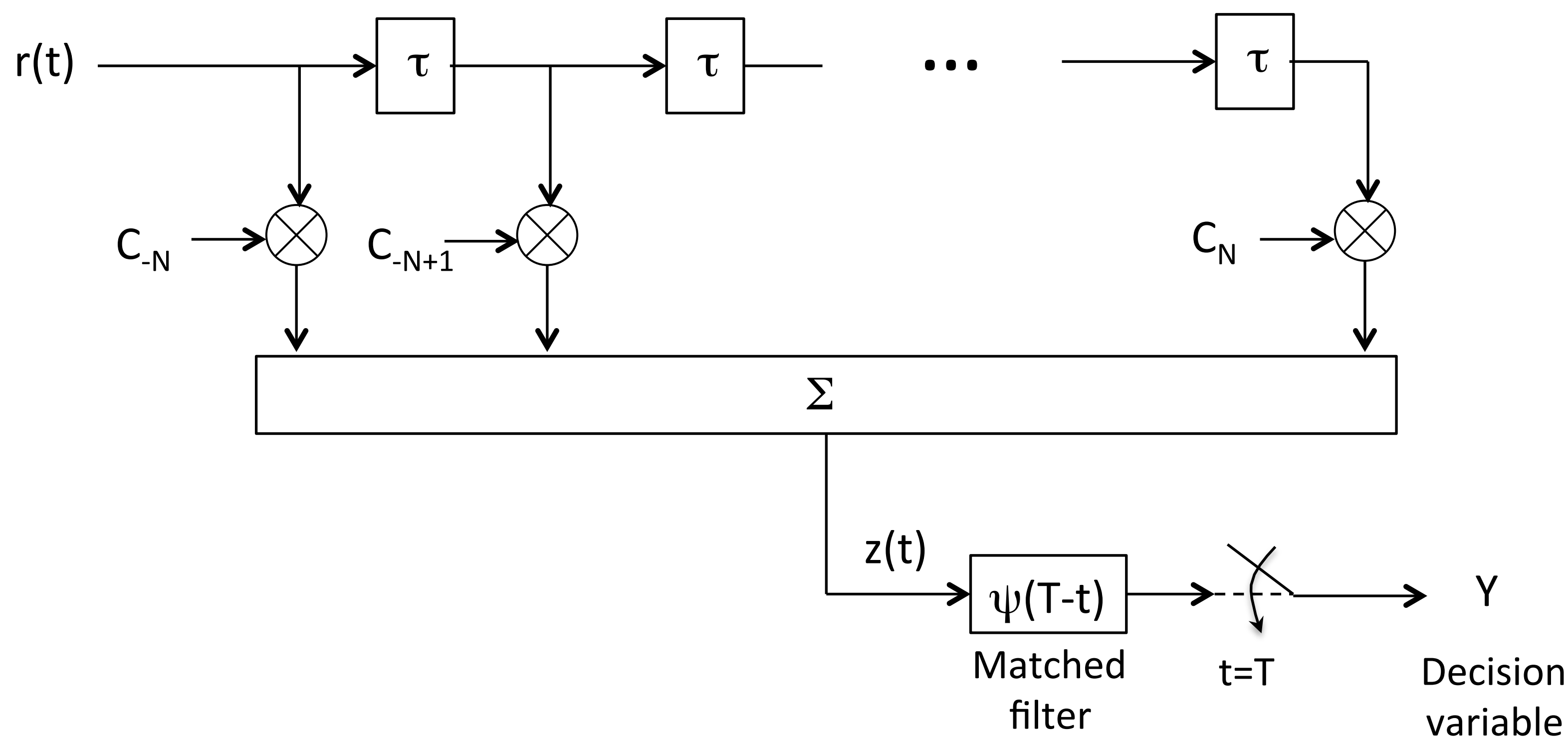
$$G_R(f) = G_T^*(f), \quad G_R(f)G_T(f) = X_{RC}(f) \quad \text{(Raised cosine)}$$

- Equalizer can be interpreted as a filter "matched" to the channel:

$$G_E(f) = \frac{1}{C(f)}$$

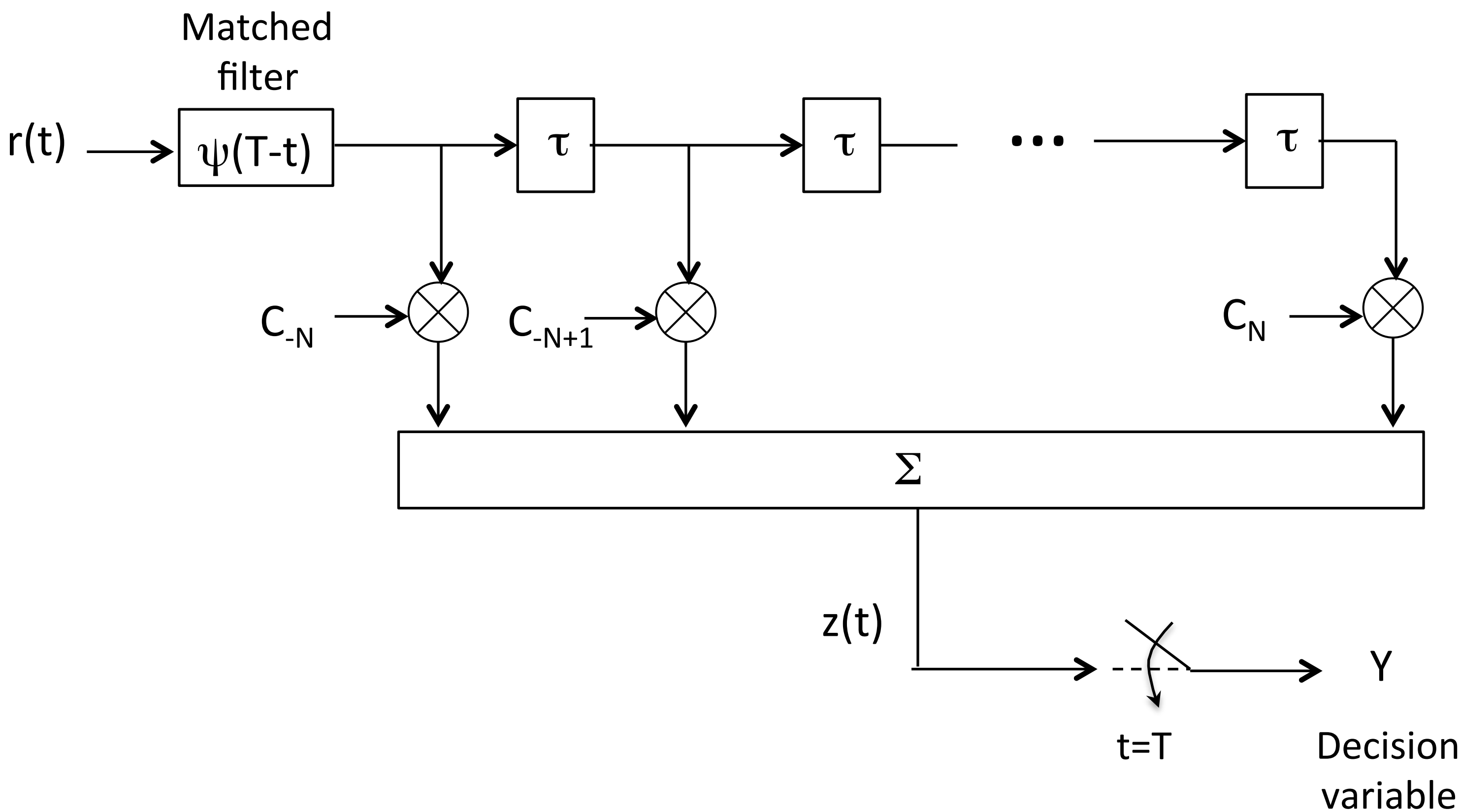
Linear adaptive equalizer: Pre-detector structure

- Similar to the RAKE demodulator:



- Equalizer (complex) coefficients may be updated based on an error signal between Y and pilot symbols known to the receiver: ***MMSE criterion***. See Figure 9.22 of textbook

Linear adaptive equalizer: Post-detector structure



Typically, τ = T (symbol spaced) or T/2 or T/4 (fractionally spaced)

The LMS algorithm

- <u>Coefficients update</u> at time t=(k+1)τ, for n=-2N,...,2N:

$$C_n[k+1] = C_n[k] + \Delta e[k] \cdot r_n[k]$$

Previous coefficient

Step size

Error

Received (stored) values

- <u>Error signal</u> (decision directed):

$$e[k] = s_P[k] - Y[k]$$

Pilot symbol
(a[k] in textboook)

MF output

- The adaptive equalizer can track ***slow variations*** of multipath fading. Moreover, if nulls in the frequency response are too pronounced then the BER is high

MATLAB demo

- Linear adaptive (LMS) equalizers work in two regimes
  - ***Training***: Use *pilot symbols* to set coefficients values
  - ***Equalization***: Combine replicas and estimate *data symbols*
- Matlab model ***Equalizer_Multipath.mdl*** from webpage of class
  - Demonstration in class
  - Post-detector structure
  - Frames of 3000 BPSK symbols (could be any number)
    - First 700 symbols (23% overhead) are pilot symbols (derived from a linear-feedback shift register or LFSR)
    - Remaining 2300 symbols are randomly generated data symbols