A Note on Total Harmonic Distortion

In class we introduced the definition of total harmonic distortion (THD) based of a periodic waveform $x(t)$ of period T_0 and $x_0 = 0$ (zero average amplitude). For an arbitrary periodic waveform $y(t)$ of nonzero average amplitude, such as a periodic train of rectangular pulses with duty cycle $d = 1/2$ taking amplitude values 0 and 1, proceed as follows:

Define the signal $x(t) = y(t) - y_0$, where y_0 is the Fourier series coefficient of $y(t)$ for $n = 0$, or average amplitude, given by

$$y_0 = \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} y(t)\, dt.$$

Then $x_0 = 0$ and $x_n = y_n$ for $n \neq 0$. The total harmonic distortion is just as defined in class:

$$\mathrm{THD} = \frac{\frac{1}{2} P_x - |x_1|^2}{|x_1|^2},$$

with

$$P_x = \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} x^2(t)\, dt.$$

Stated in other words, the THD can be defined for any arbitrary periodic waveform $y(t)$ provided that its amplitude is modified to null the average. The THD is given by

$$\mathrm{THD} = \frac{\frac{1}{2} P_y - |y_1|^2}{|y_1|^2},$$

with

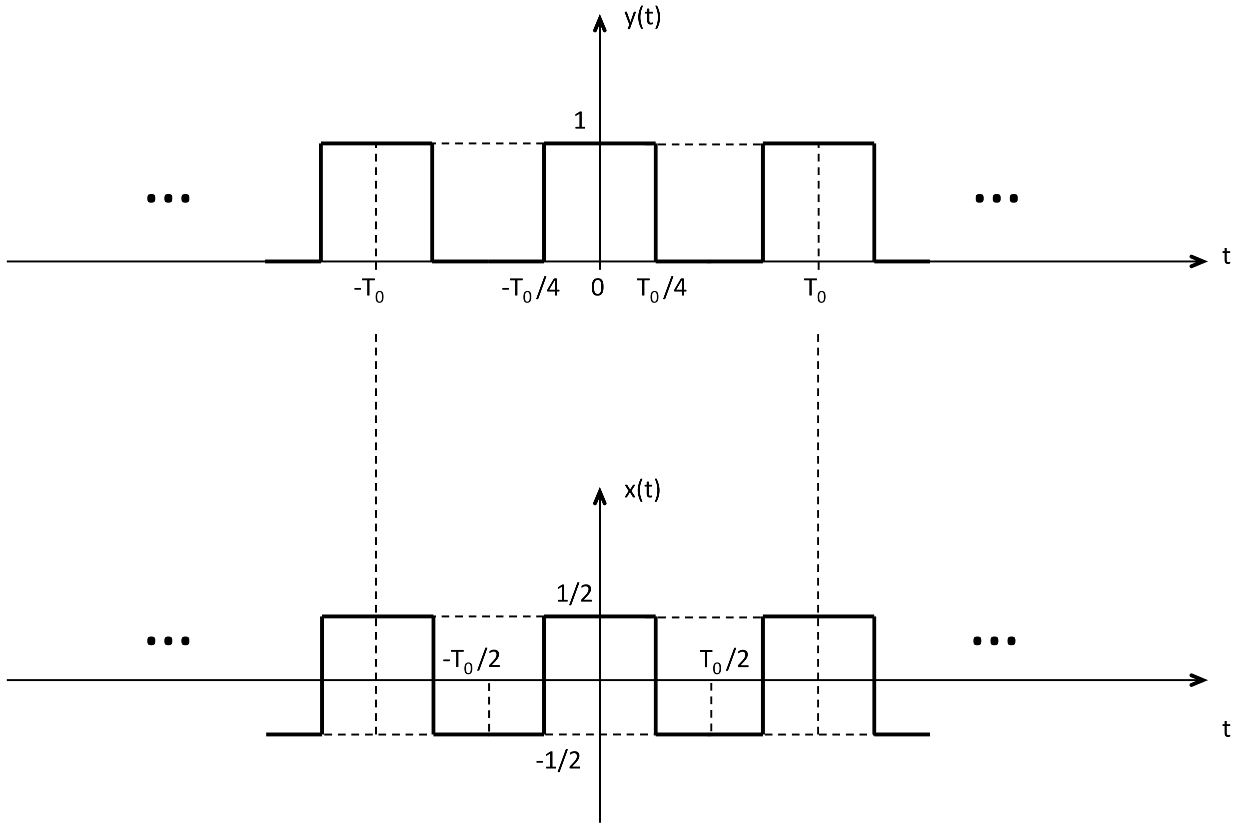
$$P_y = \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} \left[y(t) - y_0\right]^2\, dt.$$

Example: A periodic train of rectangular pulses with duty cycle $d = 1/2$ has average amplitude $y_0 = \frac{1}{2}\mathrm{sinc}(0) = \frac{1}{2}$ and (see the figure in the next page) we have

$$P_y = \frac{1}{T_0} \int_{-T_0/2}^{-T_0/4} \left(-\frac{1}{2}\right)^2 dt + \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} \left(\frac{1}{2}\right)^2 dt + \frac{1}{T_0} \int_{T_0/4}^{T_0/2} \left(-\frac{1}{2}\right)^2 dt = \frac{1}{16} + \frac{1}{8} + \frac{1}{16} = \frac{1}{4}.$$

It follows that

$$\mathrm{THD} = \frac{\frac{1}{2} P_y - |y_1|^2}{|y_1|^2} = \frac{\frac{1}{2}\left(\frac{1}{4}\right) - \left[\frac{1}{2}\,\mathrm{sinc}\left(\frac{1}{2}\right)\right]^2}{\left[\frac{1}{2}\ \mathrm{sinc}\left(\frac{1}{2}\right)\right]^2} = \frac{\frac{1}{8} - \left(\frac{1}{\pi}\right)^2}{\left(\frac{1}{\pi}\right)^2} = \frac{\pi^2}{8} - 1 = 0.234$$



y(t)
1
...
...
t
-T0
-T0/4
0
T0/4
T0
x(t)
1/2
...
...
-T0/2
T0/2
t
-1/2