Error performance of binary modulation methods

EE 161: Digital Communication Systems

San José State University

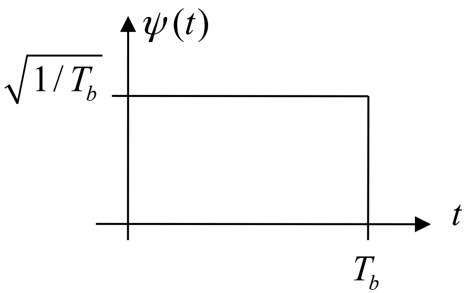
Average energy per bit

- Consider ***polar mapping*** with a pulse shape $\psi(t)$ such that

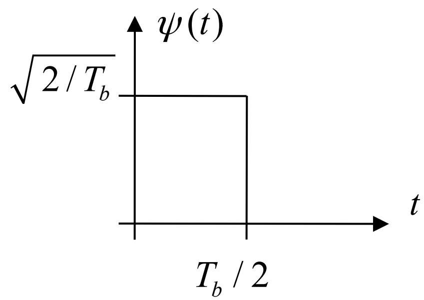
$$\int_0^{T_b} \psi^2(t)\,dt = 1$$

This type of pulse is known as a ***unit-energy pulse***.

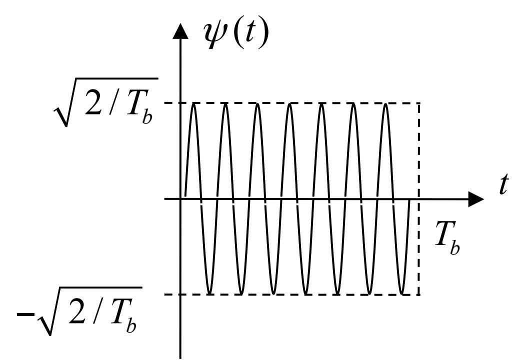
Examples of unit-energy pulses:



NRZ



RZ



sinusoidal

Polar mapping

- With ***polar mapping***, the transmitted signal waveforms are

$$s_1(t) = +a\psi(t), \quad \text{if } B = 0$$
$$s_2(t) = -a\psi(t), \quad \text{if } B = 1$$

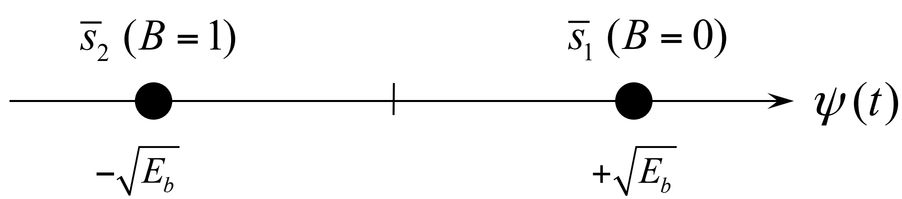
- Assuming equally likely bits, the ***average energy per bit*** E_b is

$$E_b = \frac{1}{2}E_{s1} + \frac{1}{2}E_{s2}$$

where

$$E_{s1} = E_{s2} = \int_0^{T_b} s_1^2(t)\,dt = \left(\pm a\right)^2 \int_0^{T_b} \psi^2(t)\,dt = a^2$$

Therefore, $E_b = a^2$ and $\boxed{a = \sqrt{E_b}}$ Geometric representation:



NOTE: ASK is also known as 2-PAM (or binary pulse amplitude modulation) and BPSK (or binary phase shift keying)

(Fig. 8.7) Signal constellation for polar mapping (ASK modulation for sinusoidal pulses)

Unipolar mapping

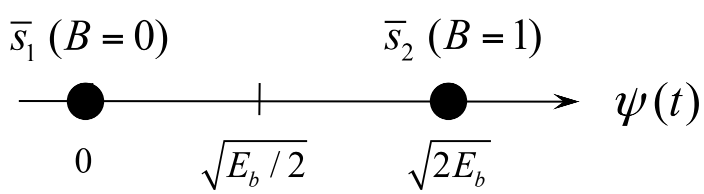
- With ***unipolar mapping***, the transmitted signal waveforms are

$$
\begin{aligned}
s_1(t) &= 0, && \text{if } B = 0 \\
s_2(t) &= +a\psi(t), && \text{if } B = 1
\end{aligned}
$$

- This gives

$$E_b = \frac{1}{2} E_{s2} = \frac{1}{2} a^2$$

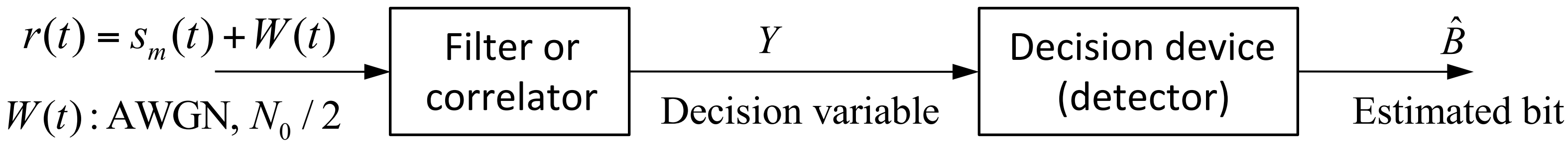
Therefore, $\boxed{a = \sqrt{2E_b}}$



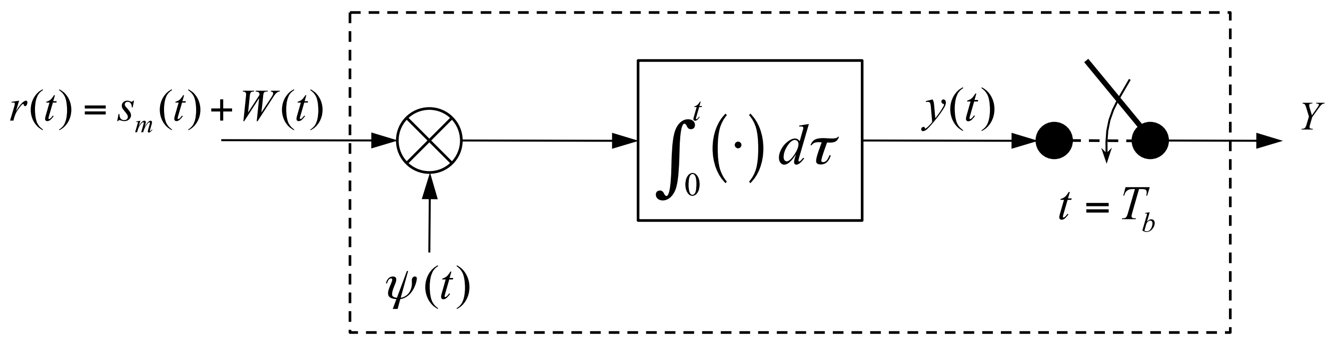
Signal constellation for unipolar mapping (OOK modulation)

Optimum receiver

- Receiver architecture for polar/unipolar mapping:



- Correlator structure:



(Fig. 8.17 of textbook)

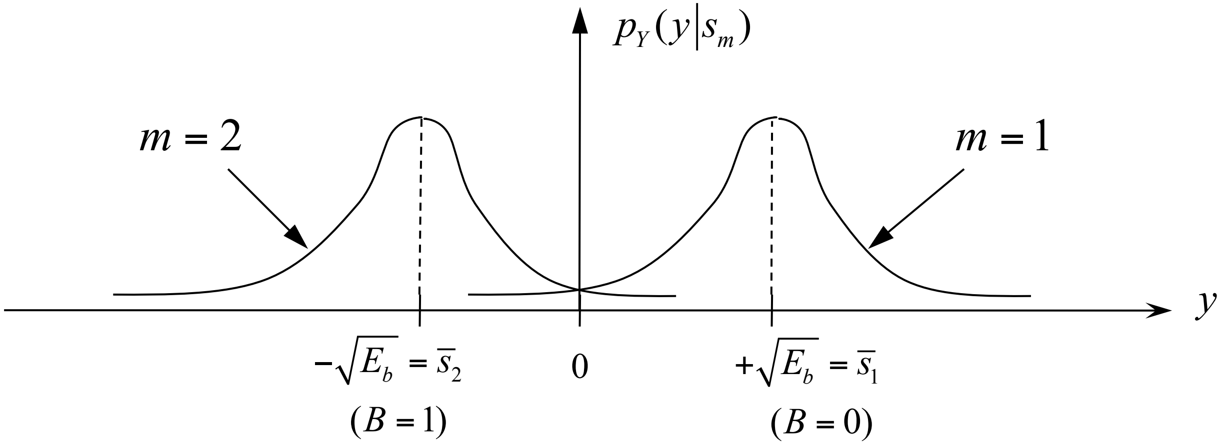
- An alternative structure, known as ***matched filter*** will be studied later

Decision rule

- The output of the correlator is a bi-modal ***Gaussian random variable***

$$Y = \int_0^{T_b} r(\tau)\psi(\tau)\,d\tau = \bar{s}_m + N$$

where N is a sample of AWGN with $\sigma_N^2 = N_0 / 2$ and conditional PDF's:



(Fig. 8.18 of textbook: Decision variable PDF's for polar mapping)

- Decision rule:

$$\boxed{\hat{B} = \begin{cases} 0, & Y \geq 0 \\ 1, & Y < 0 \end{cases}}$$

Probability of a bit error

$$P_b = P\left[\hat{B} \neq B\right] = \frac{1}{2}P\left[\hat{B} \neq 0 \middle| B = 0\right] + \frac{1}{2}P\left[\hat{B} \neq 1 \middle| B = 1\right]$$

- By symmetry:

$$P_b = P\left[\hat{B} \neq 1 \middle| B = 1\right] = P\left[\hat{B} = 0 \middle| B = 1\right] = P\left[Y \geq 0 \middle| B = 1\right]$$

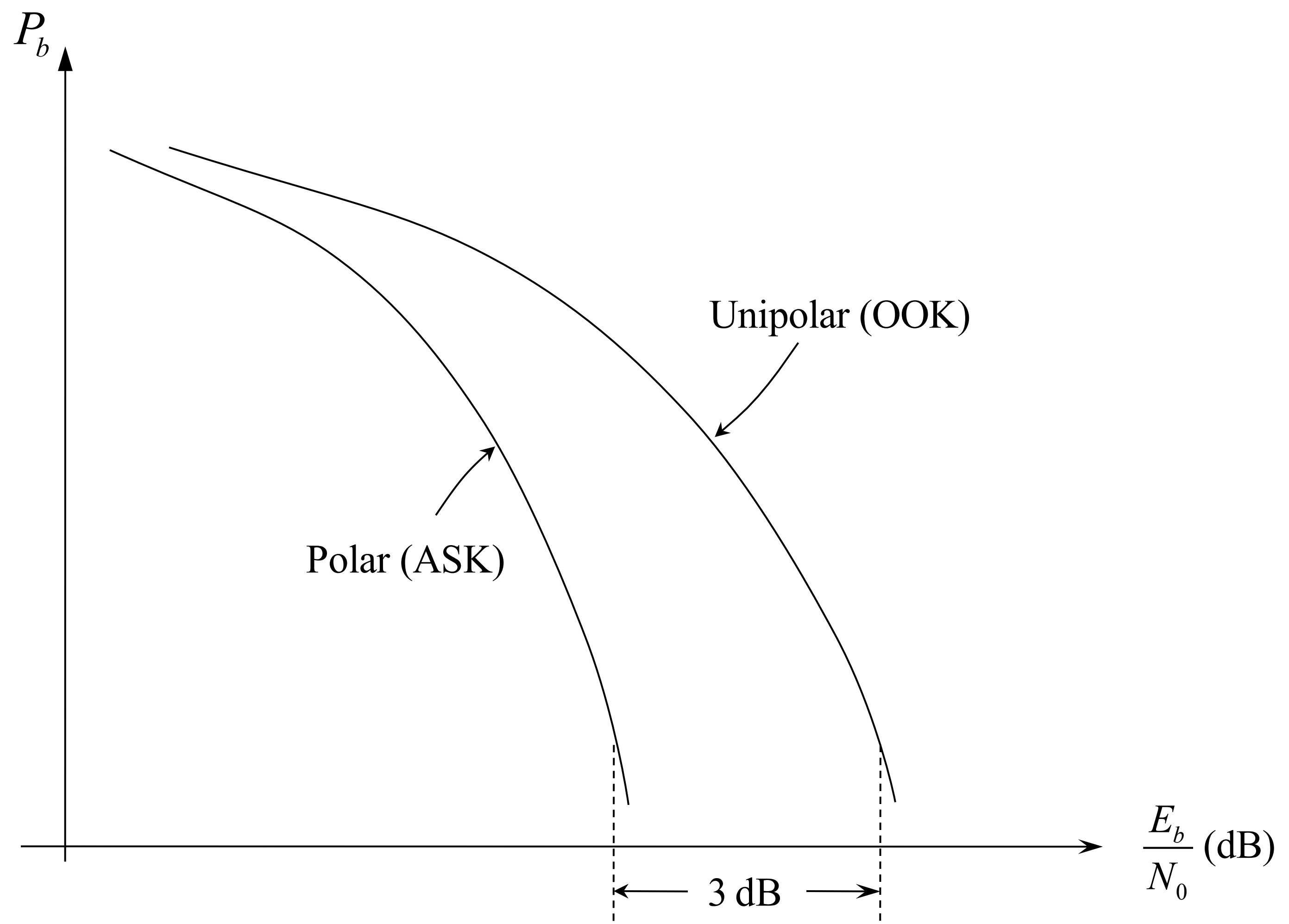
$$= Q\left(\frac{0-\left(-\sqrt{E_b}\right)}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- For unipolar mapping:

$$P_b = P\left[\hat{B} \neq 0 \middle| B = 0\right] = P\left[\hat{B} = 1 \middle| B = 0\right] = P\left[Y \geq \sqrt{E_b/2} \middle| B = 0\right]$$

$$= Q\left(\frac{\sqrt{E_b/2}}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Error performance of ASK and OOK



Binary orthogonal modulation

- In this method the transmitted pulses are

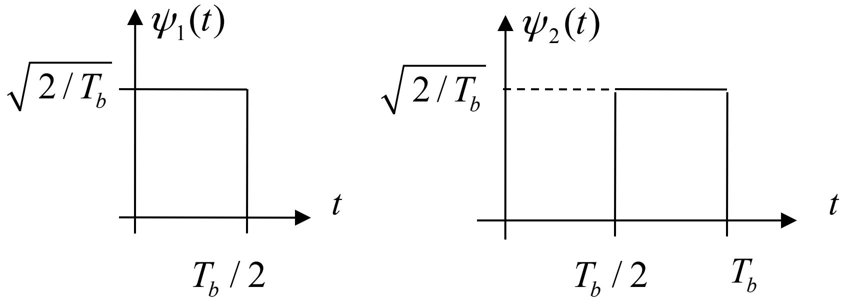
$$s_1(t) = \sqrt{E_b}\,\psi_1(t), \quad \text{if } B = 0$$
$$s_2(t) = \sqrt{E_b}\,\psi_2(t), \quad \text{if } B = 1$$

with

$$\int_0^{T_b} \psi_1(t)\psi_2(t)\,dt = 0, \quad \text{(orthogonal)}$$

<u>Examples</u>:

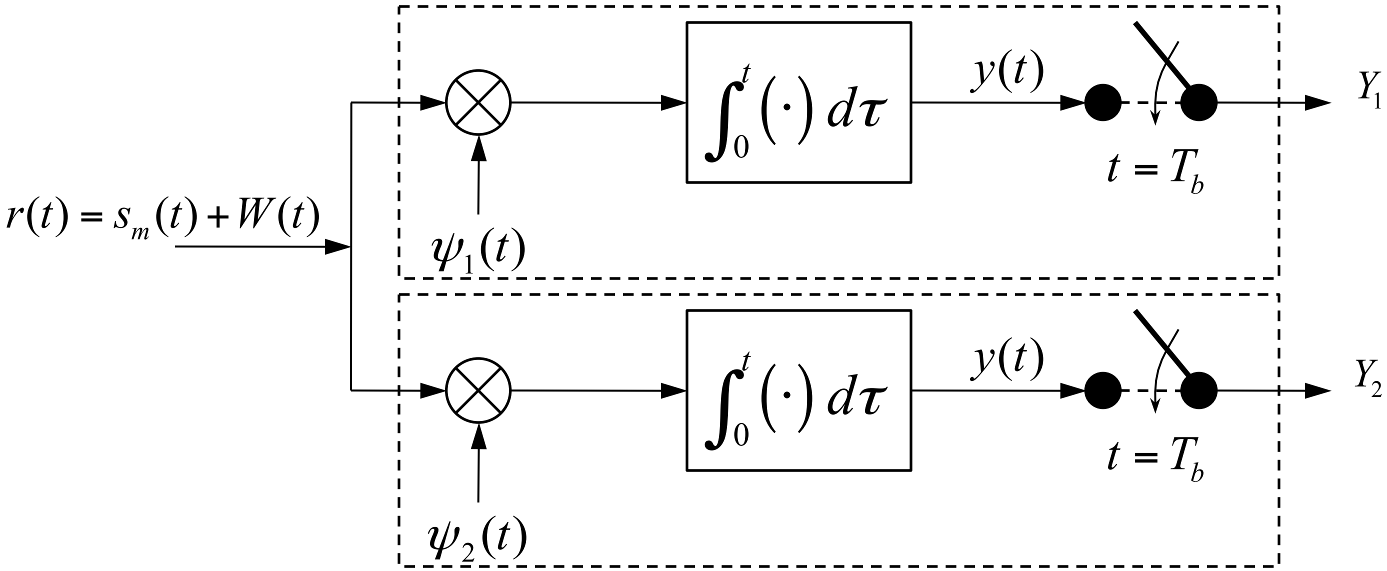
1. Pulse position modulation (2-PPM):



2. Binary frequency-shift keying (BFSK):

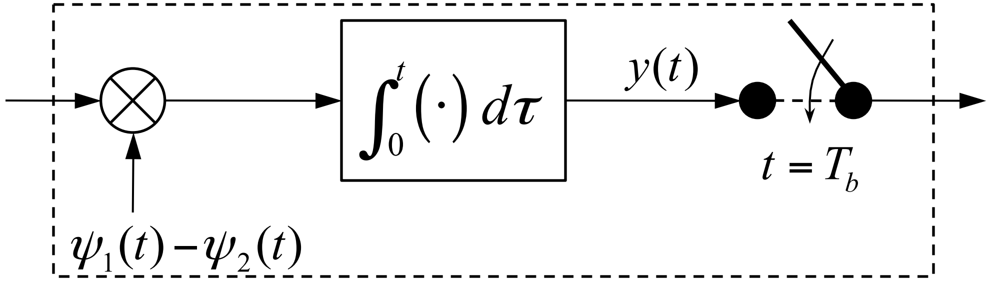
$$\psi_1(t) = \sqrt{\frac{2}{T_b}}\cos\left(2\pi f_1 t\right), \psi_2(t) = \sqrt{\frac{2}{T_b}}\cos\left(2\pi f_2 t\right), \qquad f_1 - f_2 = \frac{m}{2T_b}, \quad m:\text{integer}$$

Optimum receiver for BFSK



(Fig. 8.20 of textbook)

- Equivalent:



(Fig. 8.22 of textbook)

$Y = Y_1 - Y_2$

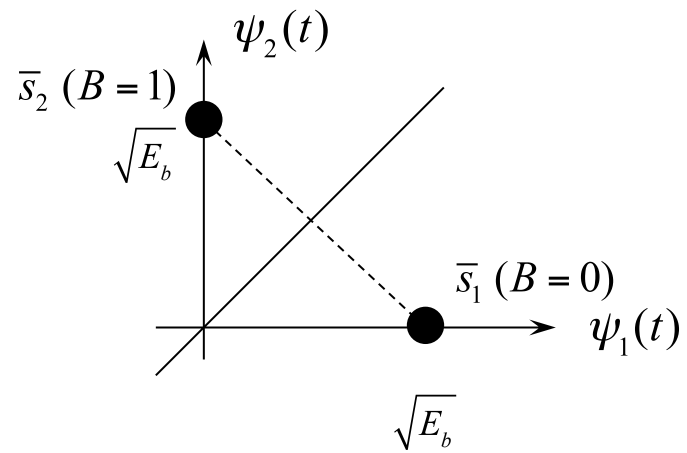
Gaussian with

$\mu_Y = \pm\sqrt{E_b}$

$\sigma_Y^2 = 2\left(\frac{N_0}{2}\right) = N_0$

Error performance of BFSK

- Signal constellation:



- Decision rule:

$$\boxed{\hat{B} = \begin{cases} 0, & Y \geq 0 \ \ (Y_1 \geq Y_2) \\ 1, & Y < 0 \ \ (Y_1 < Y_2) \end{cases}}$$

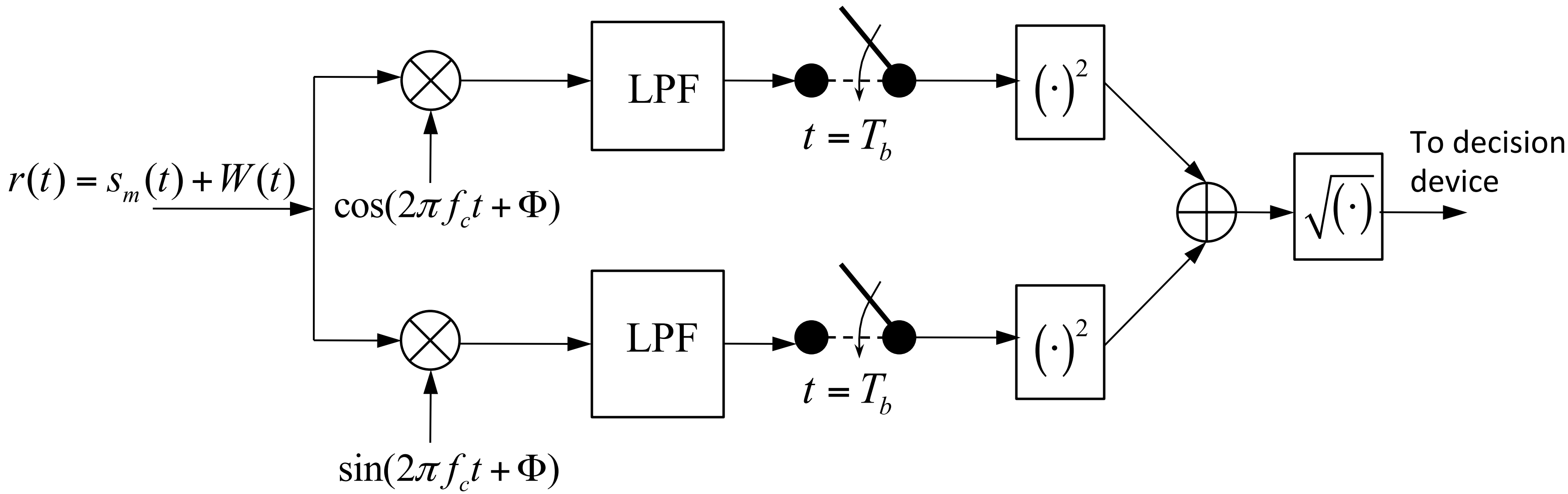
- Probability of a bit error:

$$P_b = P\left[\hat{B} \neq 1 \middle| B=1\right] = P\left[\hat{B} = 0 \middle| B=1\right] = P\left[Y \geq 0 \middle| B=1\right]$$

$$= Q\left(\frac{0 - \left(-\sqrt{E_b}\right)}{\sqrt{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- Same as OOK modulation!

Non-coherent detection of OOK



Energy detector

- Performance is only marginally inferior to coherent BFSK (less than 1 dB)[1]
  - This is one of the modulation methods used in the wireless experiments 5 and 6 of the EE160 lab

[1] See http://www.engr.sjsu.edu/rmorelos/ee252s08/solhomework3.pdf