### The wireless two-path channel

Consider a two-path channel, as depicted in Fig. 1. The signal $s(t)$ sent by the transmitter (Tx) goes through two different propagation paths, and experiences attenuations ($\alpha_i$, $i = 0, 1$) and delays ($\tau_i$, $i = 0, 1$.) At the receiver, the signal $r(t)$ is obtained as the superposition of the two signal components, denoted $s_0(t)$ and $s_1(t)$, one per propagation path.

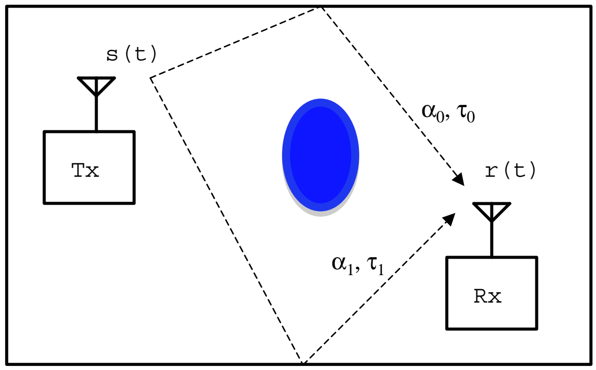


**Figure 1:** A wireless two-path channel.

The signal at the receiver is given by

$$\begin{aligned} r(t) &= s_0(t) + s_1(t) = \alpha_0 s(t - \tau_0) + \alpha_1 s(t - \tau_1) \\ &= s(t) \star [\alpha_0\, \delta(t - \tau_0) + \alpha_1\, \delta(t - \tau_1)]\,. \end{aligned} \quad (1)$$

Consequently, the impulse response of the channel is given by

$$h(t) = \alpha_0\, \delta(t - \tau_0) + \alpha_1\, \delta(t - \tau_1),$$

where, for $i = 0, 1$, the attenuation $\alpha_i$ is inversely proportional to the $n$-th power (for some value of an integer $n$) of the path distance $d_i$ between transmitter and receiver, and the delay of an electromagnetic wave is $\tau_i = d_i/c$, where $c = 3 \times 10^8$ (m/s) is the speed of light.

The transfer function of the channel is derived next. Without loss of generality, assume that $\tau_1 > \tau_0$. Let $\beta \triangleq \alpha_1/\alpha_0$ and $\Delta\tau \triangleq \tau_1 - \tau_0$. Consider the following scaled and delayed impulse response

$$h_d(t) = \frac{1}{\alpha_0}\, h(t + \tau_0). \quad (2)$$

Then

$$h_d(t) = \delta(t) + \beta\delta(t - \Delta\tau),$$

with Fourier transform

$$\begin{aligned} H_d(f) &= 1 + \beta e^{-j2\pi f \Delta\tau} \\ &= e^{-j\pi f \Delta\tau} \left[ e^{j\pi f \Delta\tau} + \beta e^{-j\pi f \Delta\tau} \right] \\ &= e^{-j\pi f \Delta\tau} \left[ (1 + \beta) \cos(\pi f \Delta\tau) + j(1 - \beta) \sin(\pi f \Delta\tau) \right], \end{aligned} \quad (3)$$

where in the third equation above Euler's identity ($e^{j\theta} = \cos\theta + j\sin\theta$) has been used. From (2), it follows that $h(t) = \alpha_0\, h_d(t-\tau_0)$. This combined with (3), and observing that $2\tau_0 + \Delta\tau = \tau_0 + \tau_1$, results in the *transfer function of the channel*,

$$\begin{aligned} H(f) &= \alpha_o H_d(f) e^{-j\pi f(\tau_0+\tau_1)} \\ &= \alpha_0 \left[(1+\beta)\cos(\pi f\Delta\tau) + j(1-\beta)\sin(\pi f \Delta\tau)\right] e^{-j\pi f(\tau_0+\tau_1)}. \end{aligned} \tag{4}$$

The magnitude square of the transfer function of the channel is

$$\begin{aligned} |H(f)|^2 &= \alpha_0^2 \left[(1+\beta)^2\cos^2(\pi f\Delta\tau) + (1-\beta)^2\sin^2(\pi f\Delta\tau)\right] \\ &= \alpha_0^2 \left[1+\beta^2+2\beta\left\{\cos^2(\pi f\Delta\tau) - \sin^2(\pi f\Delta\tau)\right\}\right] \\ &= \alpha_0^2 \left[1+\beta^2+2\beta\cos(2\pi f\Delta\tau)\right]. \end{aligned} \tag{5}$$

Note that $|H(f)|^2$ is a periodic function with period $1/\Delta\tau$. It has a maximum value given by $\alpha_0^2(1+\beta)^2 = (\alpha_0+\alpha_1)^2$ when the cosine term equals 1, and a minimum value $\alpha_0^2(1-\beta)^2 = (\alpha_0-\alpha_1)^2$ when the cosine term equals -1. Fig. 2 shows a sketch of this function.

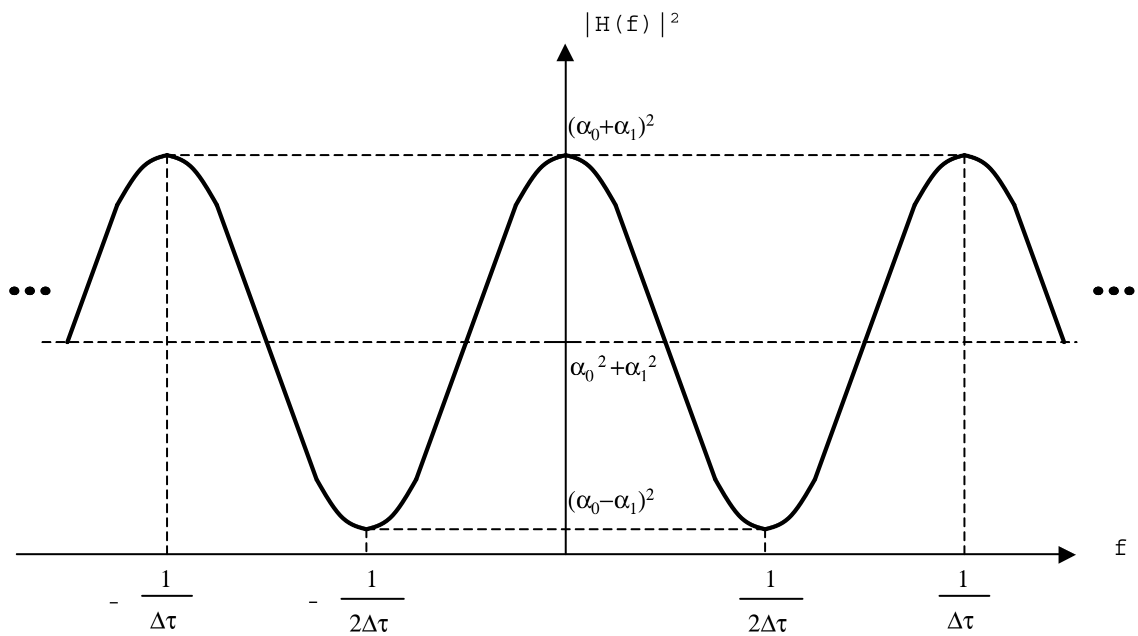


**Figure 2:** Sketch of $|H(f)|^2$.

We say that this channel is *frequency selective* in that the transfer function is no longer flat (constant). This means that some frequency components are severely attenuated while others are accentuated. In addition, the frequency values where the low values of the transfer function occur (known in the literature as *nulls*) are a function of the delay $\Delta\tau$ between the two paths. The PSD $|H(f,\Delta\tau)|^2$ of the channel as a function of both frequency $f$ and delay spread $\Delta\tau$ is depicted in Fig. 2 below.

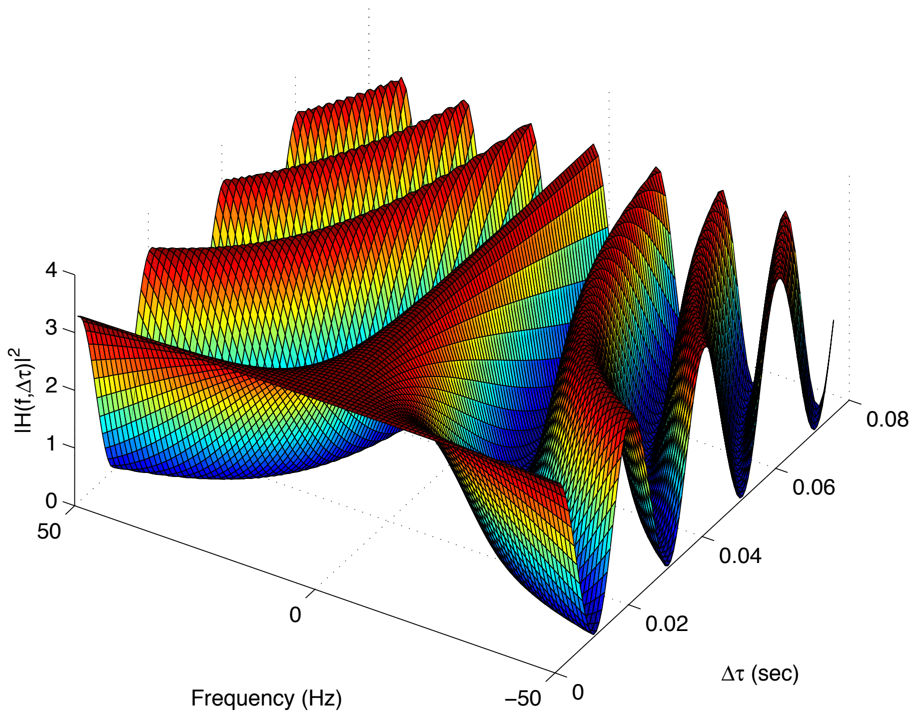


**Figure 2:** PSD of a wireless two-path channel.