

# Digital Communications over Time-varying Multipath (mobile wireless) Channels

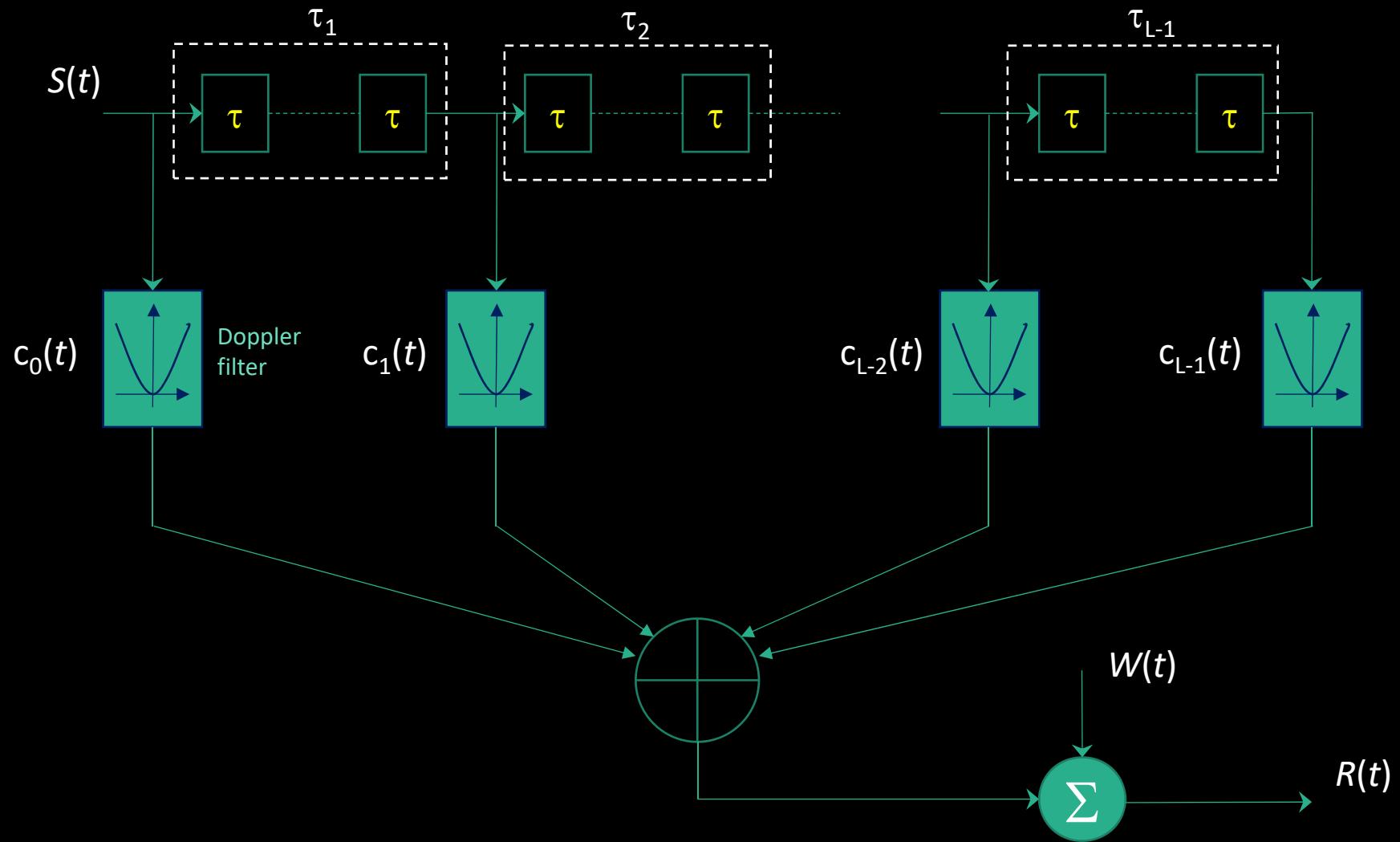
EE161: Digital Communication Systems

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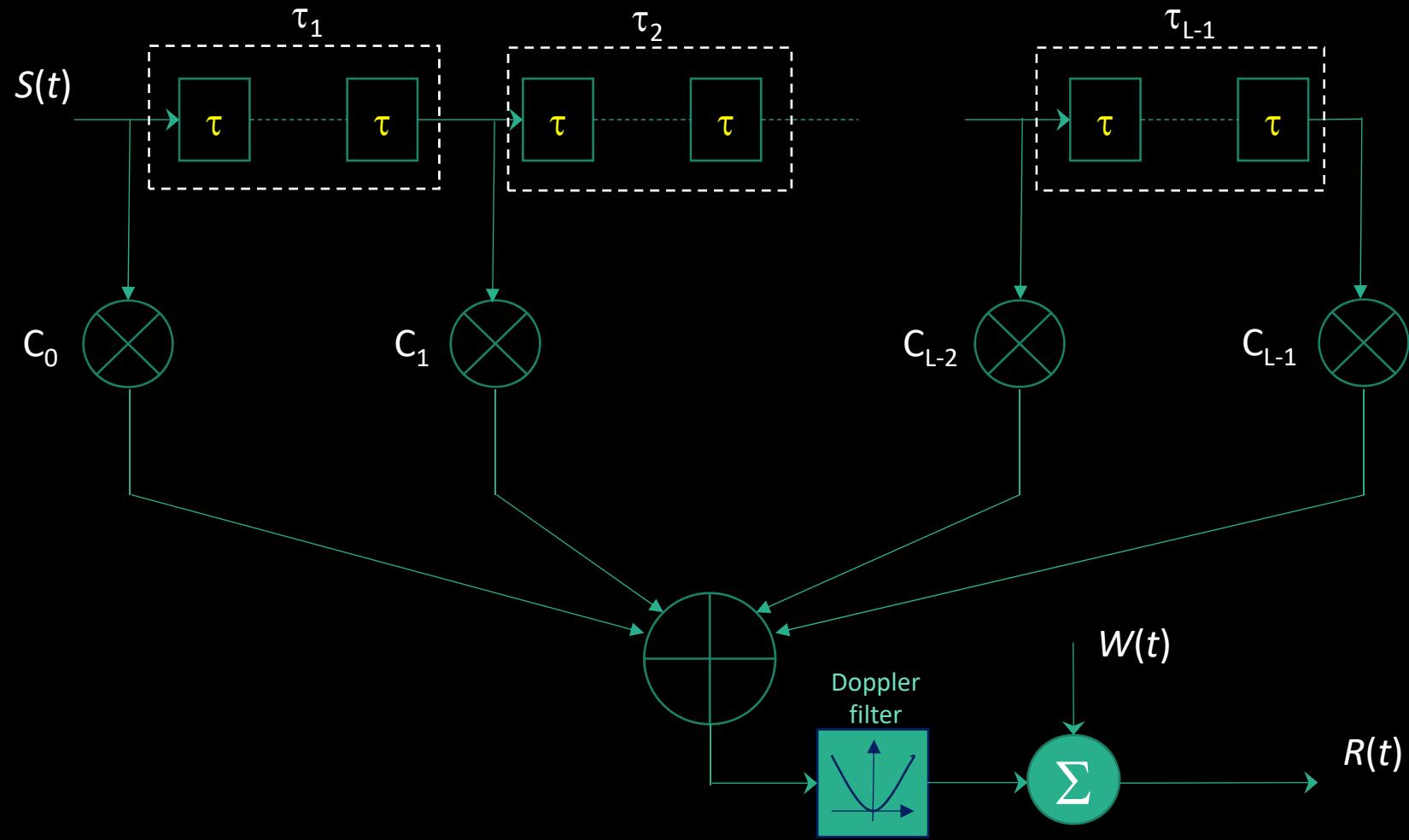
San Jose State University

Spring 2020

# General Multipath Channel Model



# Equivalent simplified model



# Symbol-spaced channel models

- In the case  $\tau = T$ , the channel is referred to as a ***symbol-spaced multipath channel***
- In some cases,  $\tau = T/N$ , where  $N$  is an integer (number of samples per symbol). Here the channel is known as a ***fractionally-spaced multipath channel***

# Normalizing the channel energy

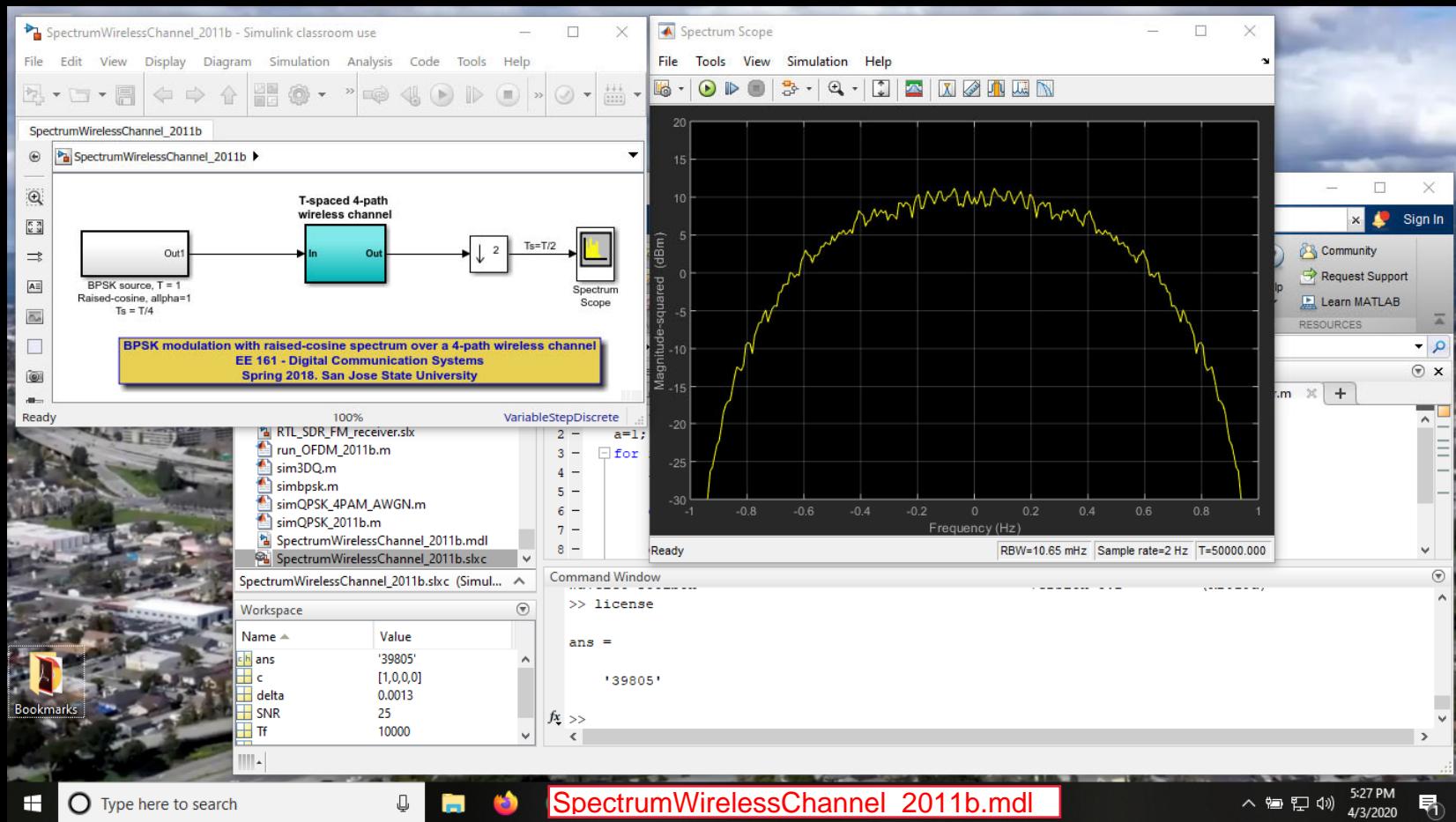
- In simulating error performance over an  $L$ -path channel, the channel gains are normalized so that the ***channel energy*** is equal to one:

$$\sum_{i=0}^{L-1} |c_i|^2 = 1$$

- This allows comparison of performance of different mappings under different time-varying multipath fading scenarios (including no fading)

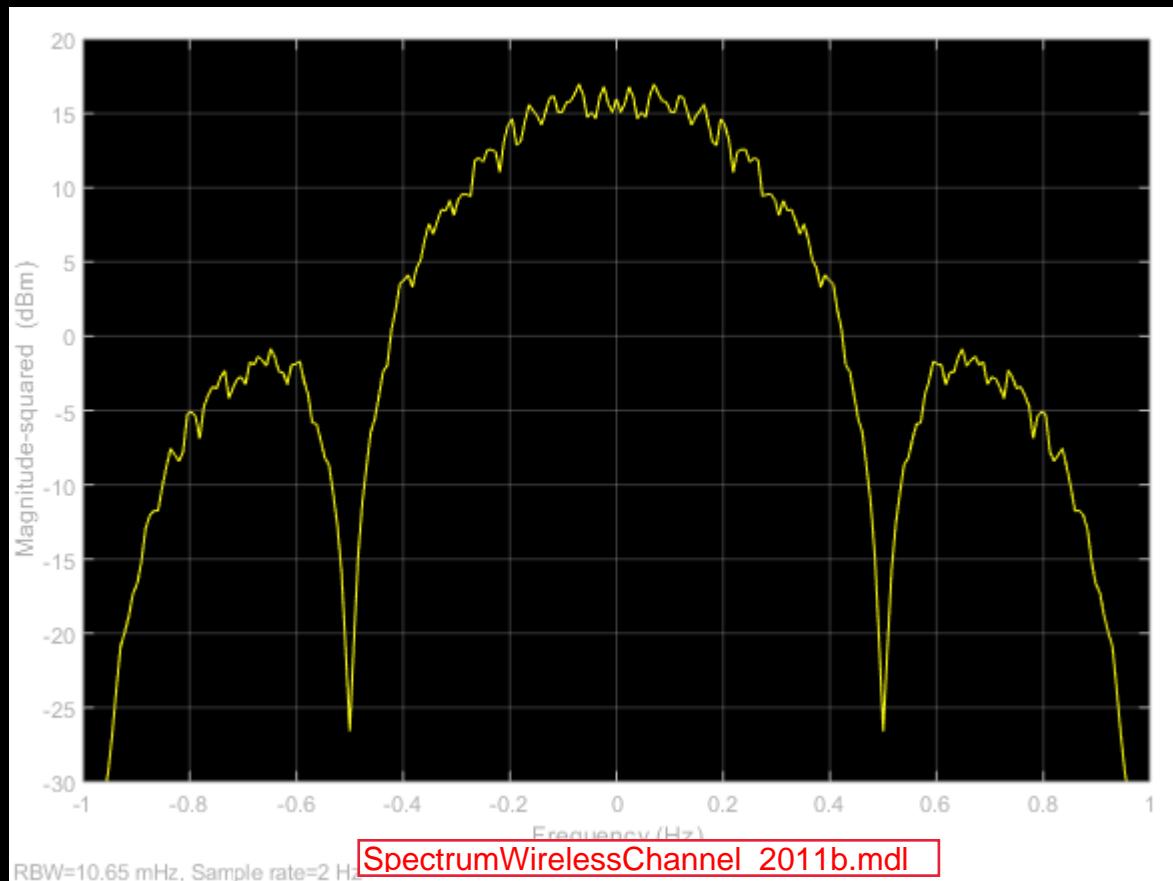
# Spectral effects of multiple paths

Polar mapping (BPSK). Raised-cosine spectrum, rolloff factor  $\alpha = 1$ . One path ( $c_0 = 1$ ),  $\tau = T$



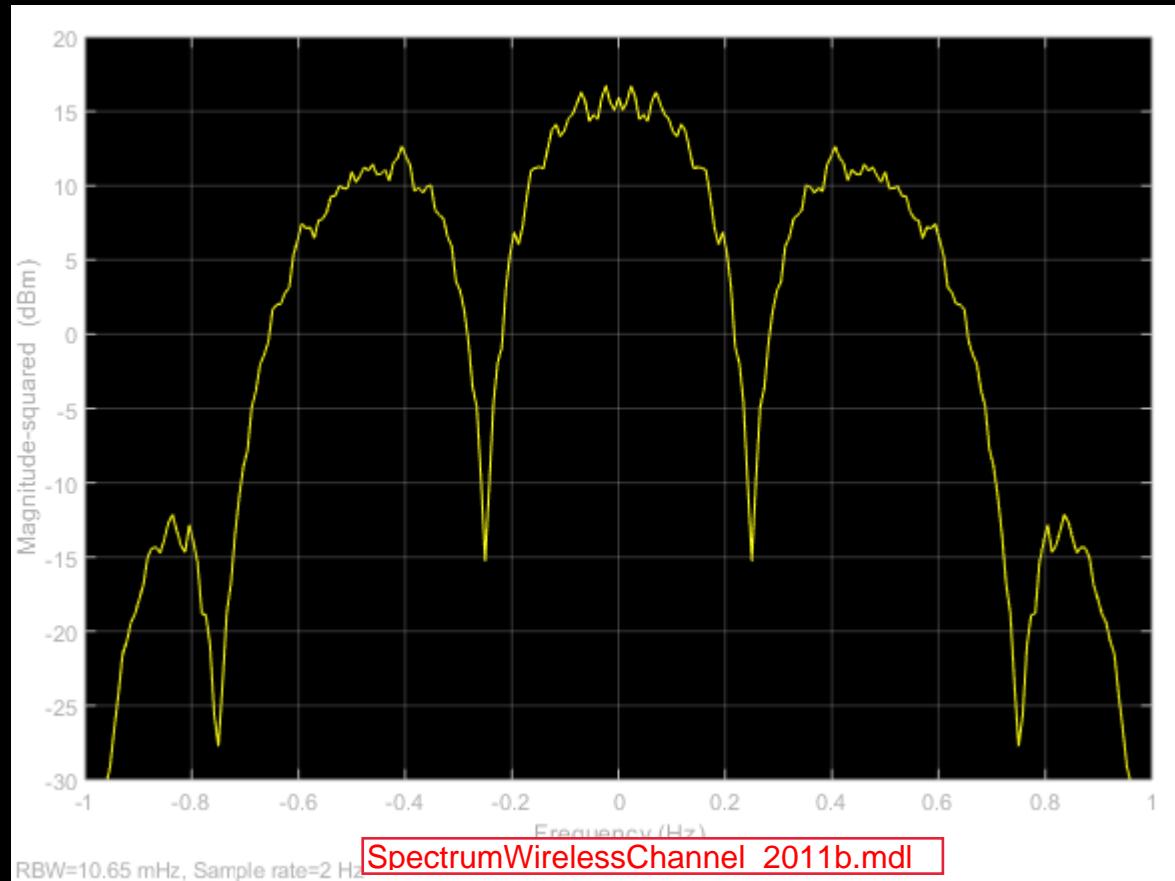
# Two-paths, symbol-spaced, $T_m = T$

Two paths,  $c_0 = c_1 = 1/\sqrt{2}$ . Delay spread:  $T_m = T$ . MATLAB:  $c = [1 \ 1 \ 0 \ 0]$



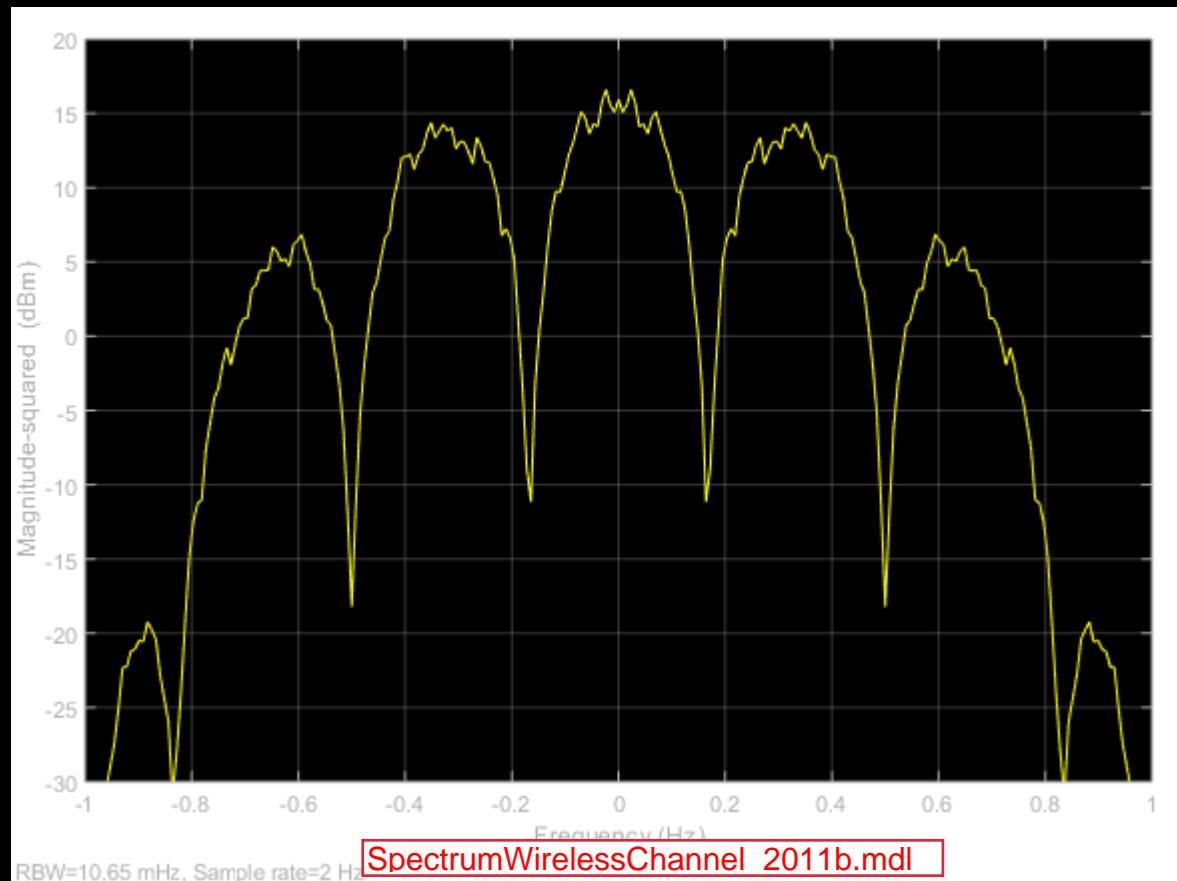
# Two-paths, symbol-spaced, $T_m=2T$

Two paths,  $c_0 = c_2 = 1/\sqrt{2}$ ,  $c_1 = 0$ . Delay spread:  $T_m = 2T$ . MATLAB:  $c = [1 \ 0 \ 1 \ 0]$



# Two-paths, symbol-spaced, $T_m=3T$

Two paths,  $c_0 = c_3 = 1/\sqrt{2}$ ,  $c_1 = c_2 = 0$ . Delay spread:  $T_m = 3T$ . MATLAB:  $c = [1 \ 0 \ 0 \ 1]$



# Slow and flat Rayleigh fading

- $L=1$  path (*flat*)
- $C_0(t) = C_0$ , independent of  $t$  (*slow*)
- Moreover, the *channel gain*  $C_0$  is a complex Gaussian random variable, of Rayleigh amplitude  $A$  and uniform phase  $\Theta$ :

$$p_A(a) = 2ae^{-2a^2}$$

$$p_\theta(\theta) = \frac{1}{2\pi}$$

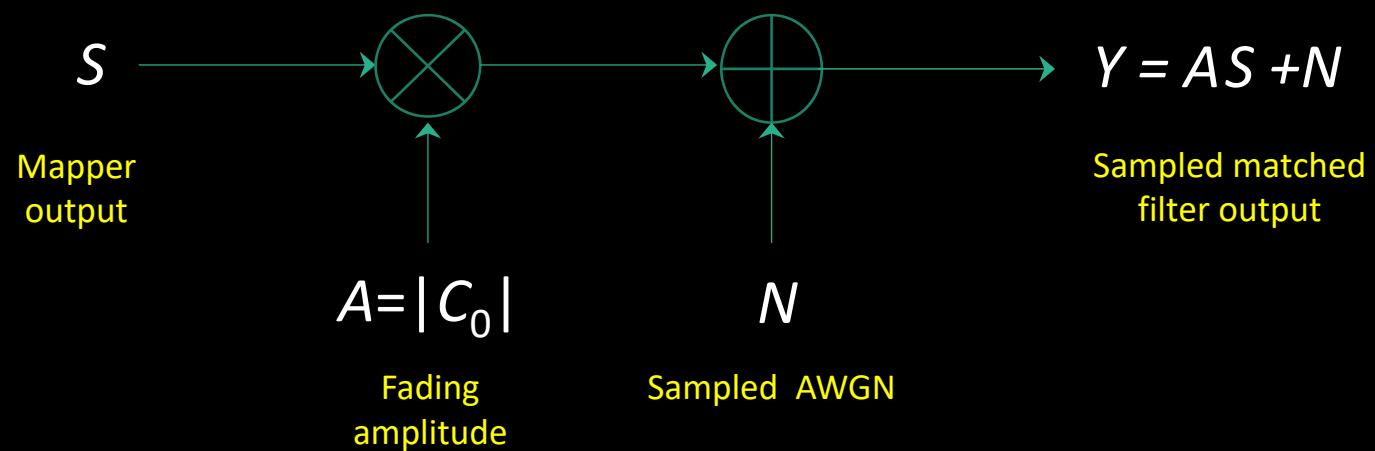
- Central Limit Theorem:  $C_i(t) = C_{Ri}(t) + j C_{li}(t)$ , for  $i = 0, 1, \dots, L-1$ , assuming that  $C_{Ri}(t)$  and  $C_{li}(t)$  are *zero-mean* Gaussian random variables of variance  $\frac{1}{2}$  (so that the channel has *unit energy*)

# Random phase

- To handle the random phase introduced in wireless communications, there are two approaches:
  - Phase estimation* and correction (EE252)
  - Noncoherent modulation* (e.g., DPSK)
- In the following, we will assume that the *phase is known* and corrected
- In a subsequent lecture, we will consider *differential BPSK* (DPSK) and analyze its performance

# Slow flat Rayleigh fading

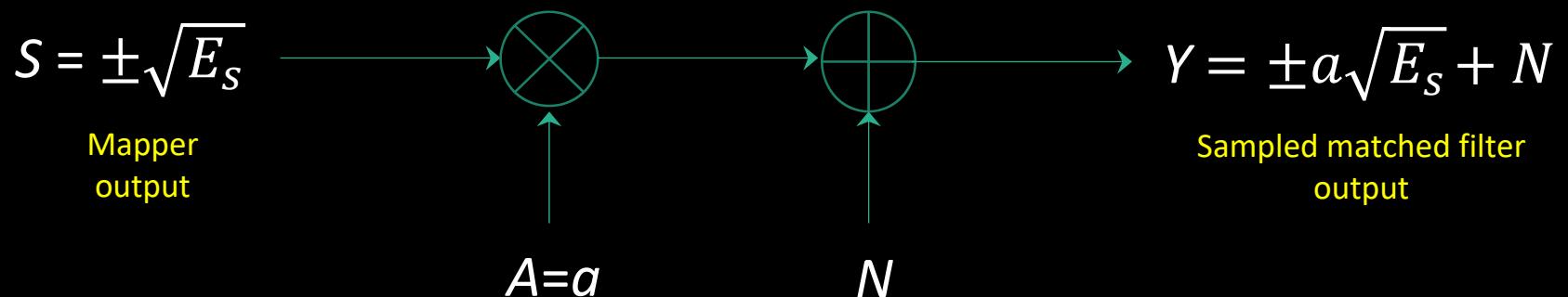
- Equivalent (complex baseband) model:



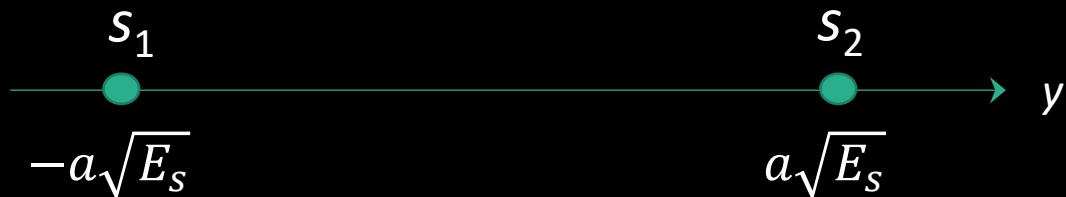
# BPSK performance under slow flat Rayleigh fading

- Approach

—**Condition** the fading amplitude  $A=a$  and then remove the condition by integrating over its PDF  $p_A(a)$



# Average bit error probability



$$P[\text{error} | A=a] = Q \left( \sqrt{\frac{d_{12}^2}{2N_0}} \right) = Q \left( a \sqrt{\frac{2E_s}{N_0}} \right)$$

# Average bit error probability (cont.)

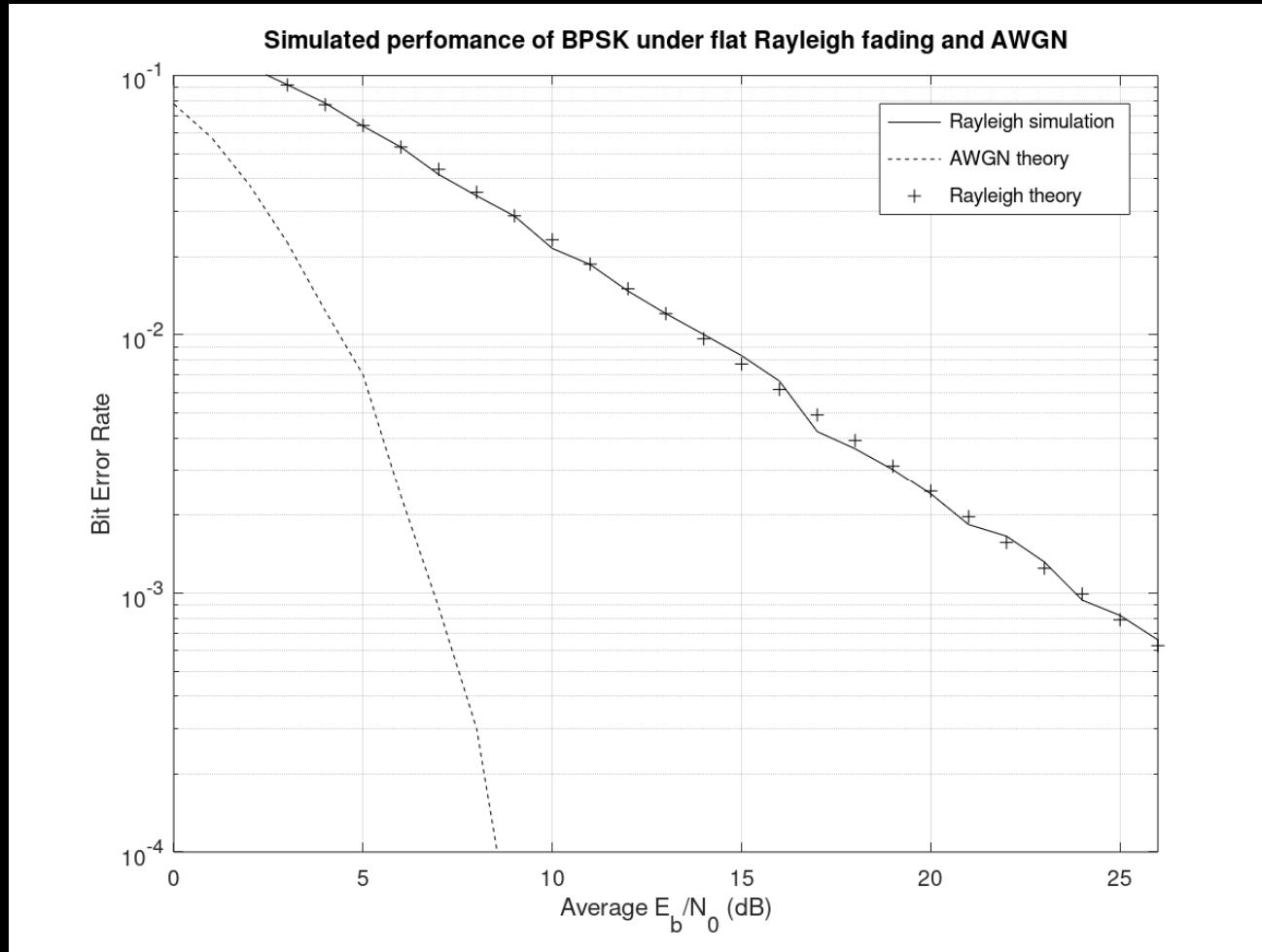
- Average over the PDF of the fading amplitude gives:

$$\begin{aligned} P_b &= \int_0^{\infty} P[\text{error}|A = a] p_A(a) da \\ &= E_A \left\{ Q \left( A \sqrt{\frac{2E_s}{N_0}} \right) \right\} = \boxed{\frac{1}{2} \left[ 1 - \sqrt{\frac{\rho_0}{1+\rho_0}} \right] \cong \frac{1}{4\rho_0}} \end{aligned}$$

where  $\rho_0 = \frac{E_s}{N_0} E\{A^2\}$  is the **average** signal energy-to-noise ratio

- NOTE: Unit channel energy condition gives  $E\{A^2\} = 1$

# BPSK: Simulation results



MATLAB script: "bpsk\_rayleigh.m"

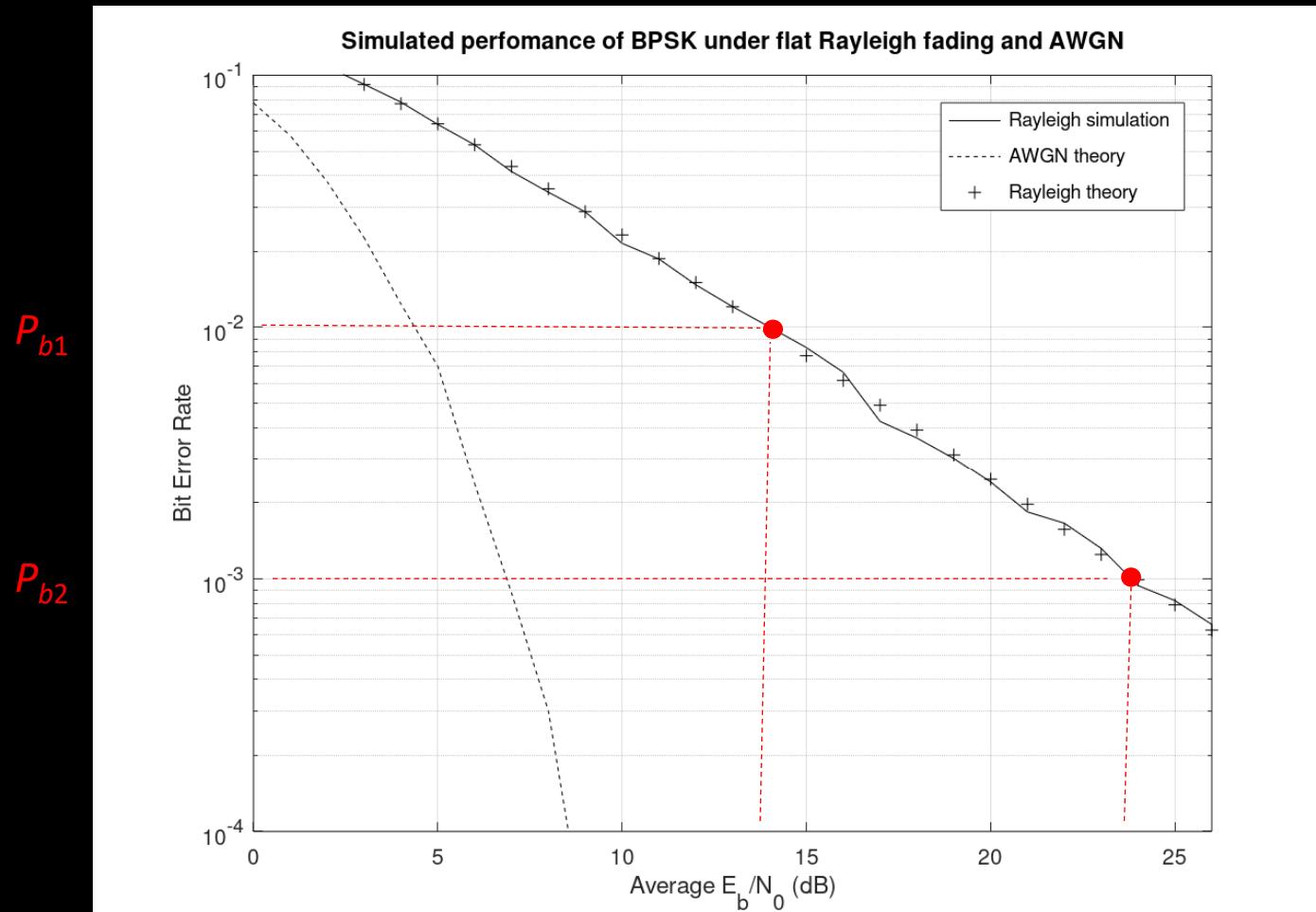
# Diversity order

- Note that the average bit error probability curve of BPSK (or any other modulation) under flat fading has linear characteristic with a “slope” equal to -1, where

$$\text{slope} = \frac{10 \log_{10}(P_{b2}/P_{b1})}{E_{s2}/N_0 \text{ (dB)} - E_{s1}/N_0 \text{ (dB)}}$$

- The integer  $D = -\text{slope}$  is in fact the **diversity order** of a modulation scheme under flat Rayleigh fading
- All modulation schemes studied so far have  $D = 1$
- More on this later

# BPSK: Diversity order $D = 1$



$E_s/N_0$  (dB)

$E_s/N_0$  (dB)

# Random phase: DPSK

- In cases where the carrier phase changes due to movement are relatively slow (compared to the bit rate), *differential BPSK* is a solution