

Error performance of binary modulation methods

EE 161: Digital Communication Systems
San José State University

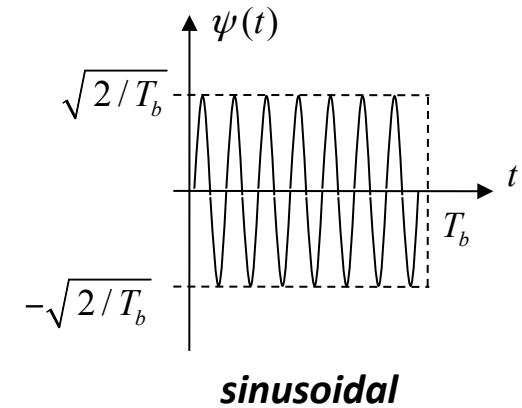
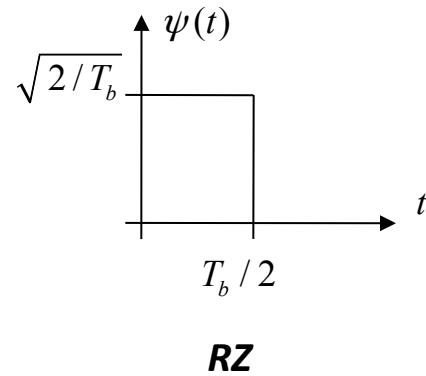
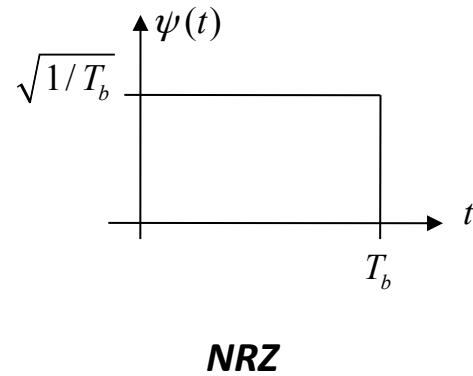
Average energy per bit

- Consider **polar mapping** with a pulse shape $\psi(t)$ such that

$$\psi(t) = \frac{g(t)}{\sqrt{E_g}}, \quad E_g = \int_0^{T_b} g^2(t) dt. \quad \boxed{E_\psi = \int_0^{T_b} \psi^2(t) dt = 1}$$

This type of pulse is known as a **unit-energy pulse**.

Examples of unit-energy pulses:



Polar mapping

- With **polar mapping**, the transmitted signal waveforms are

$$s_1(t) = +a\psi(t), \quad \text{if } B = 0$$

$$s_2(t) = -a\psi(t), \quad \text{if } B = 1$$

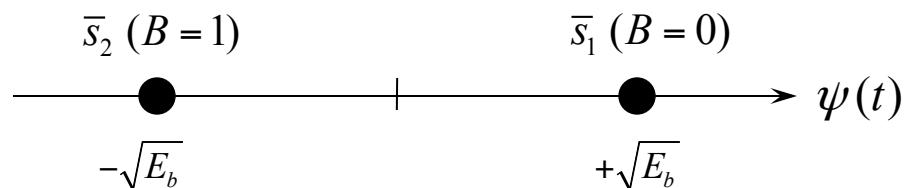
- Assuming equally likely bits, the **average energy per bit** E_b is

$$E_b = \frac{1}{2} E_{s1} + \frac{1}{2} E_{s2}$$

where

$$E_{s1} = E_{s2} = \int_0^{T_b} s_1^2(t) dt = (\pm a)^2 \int_0^{T_b} \psi^2(t) dt = a^2$$

Therefore, $E_b = a^2$ and $a = \sqrt{E_b}$ Geometric representation:



NOTE: ASK is also known as 2-PAM (or binary pulse amplitude modulation) and BPSK (or binary phase shift keying)

(Fig. 8.7) Signal constellation for polar mapping (ASK modulation for sinusoidal pulses)

Unipolar mapping

- With *unipolar mapping*, the transmitted signal waveforms are

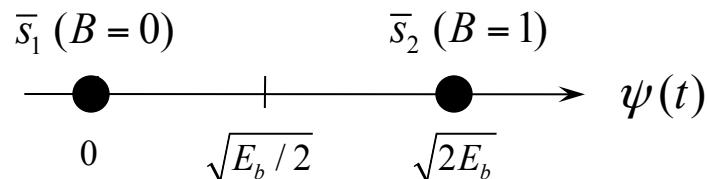
$$s_1(t) = 0, \quad \text{if } B = 0$$

$$s_2(t) = +a\psi(t), \quad \text{if } B = 1$$

- This gives

$$E_b = \frac{1}{2} E_{s2} = \frac{1}{2} a^2$$

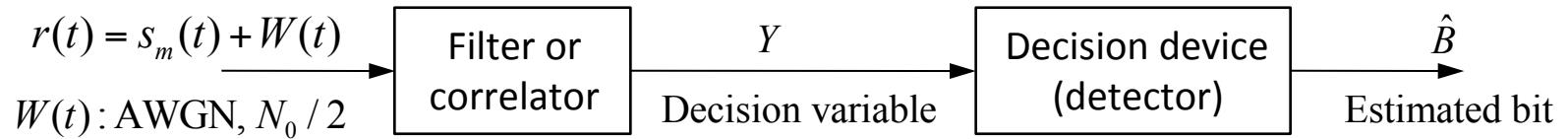
Therefore, $a = \sqrt{2E_b}$



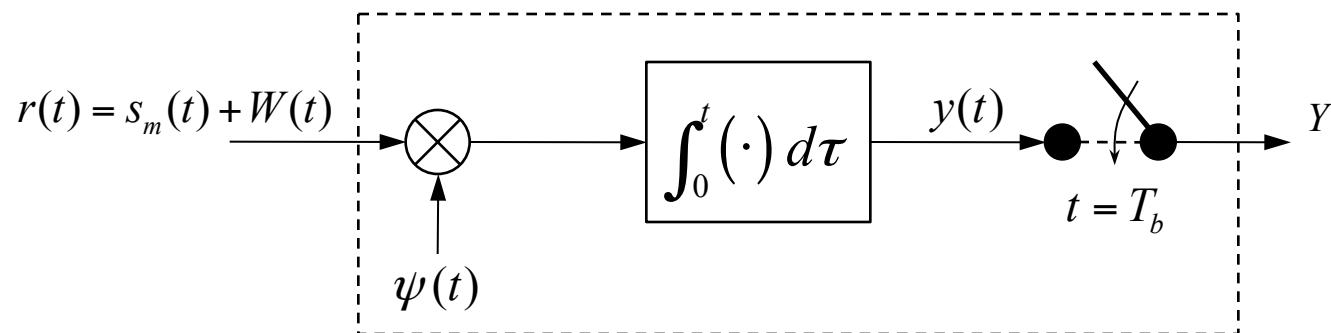
Signal constellation for unipolar mapping (OOK modulation)

Optimum receiver

- Receiver architecture for polar/unipolar mapping:



- Correlator structure:



(Fig. 8.17 of textbook)

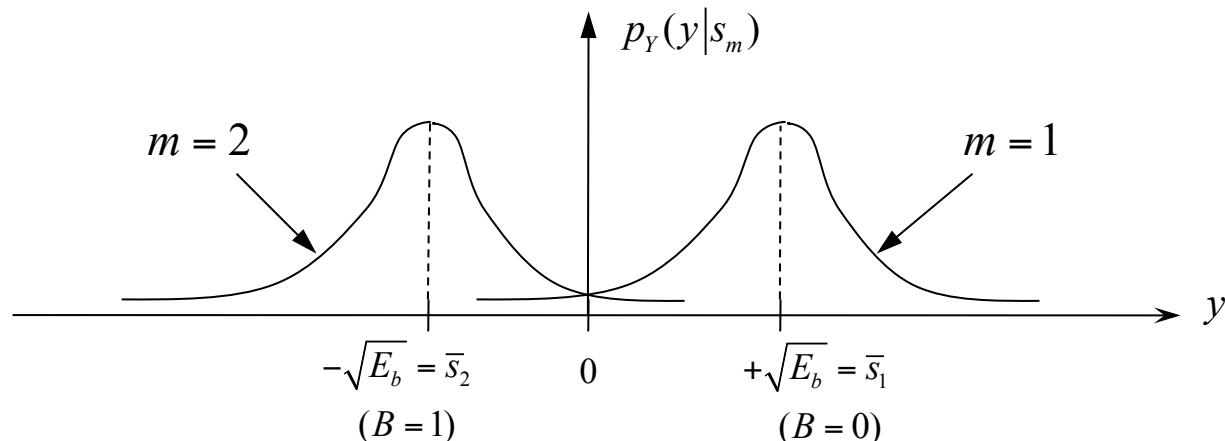
- An alternative structure, known as ***matched filter*** will be studied later

Decision rule

- The output of the correlator is a bi-modal **Gaussian random variable**

$$Y = \int_0^{T_b} r(\tau) \psi(\tau) d\tau = \bar{s}_m + N$$

where N is a sample of AWGN with $\sigma_N^2 = N_0 / 2$ and conditional PDF's:



(Fig. 8.18 of textbook: Decision variable PDF's for polar mapping)

- Decision rule:

$$\hat{B} = \begin{cases} 0, & Y \geq 0 \\ 1, & Y < 0 \end{cases}$$

Probability of a bit error

$$P_b = P[\hat{B} \neq B] = \frac{1}{2} P[\hat{B} \neq 0 | B = 0] + \frac{1}{2} P[\hat{B} \neq 1 | B = 1]$$

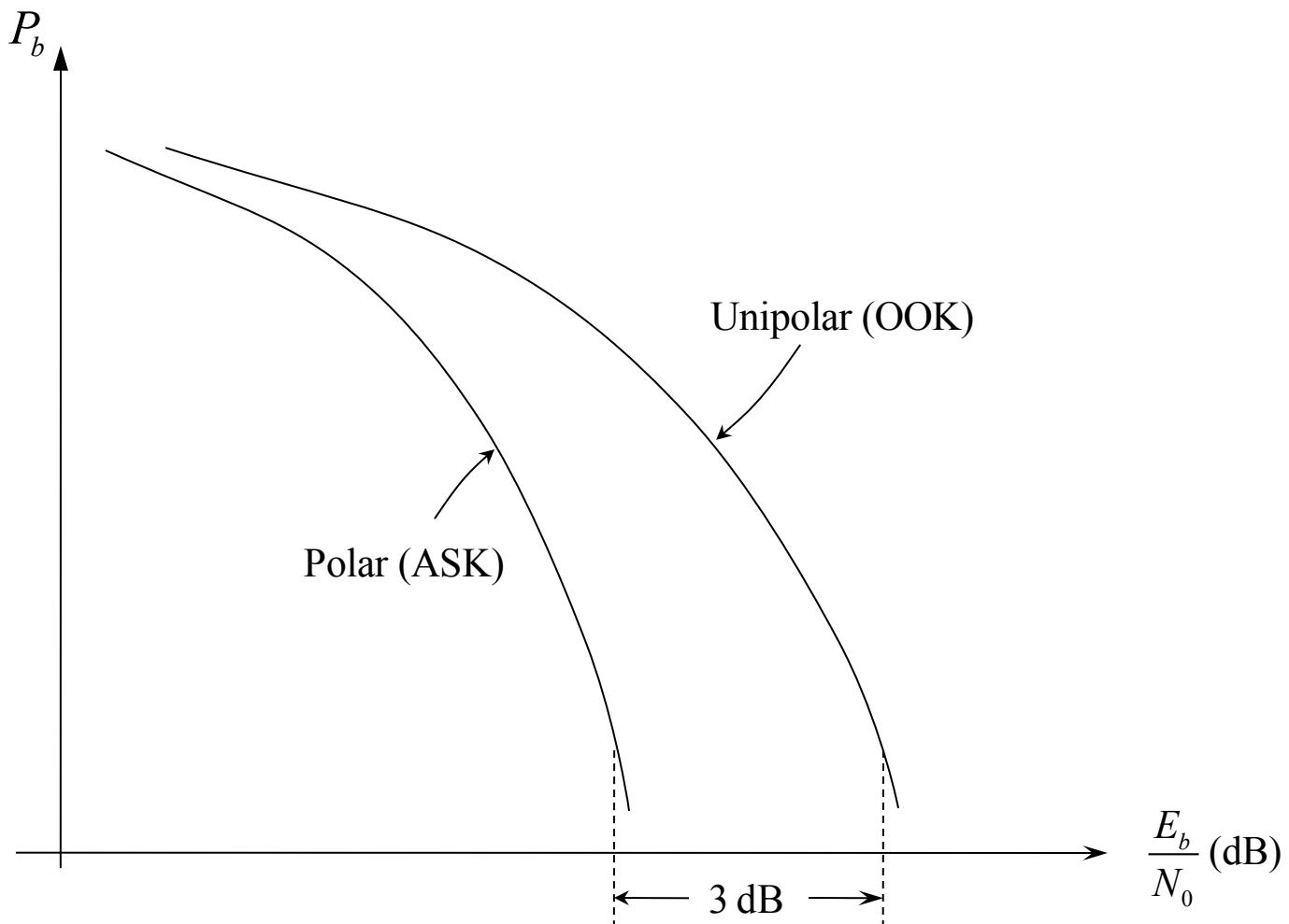
- By symmetry:

$$\begin{aligned} P_b &= P[\hat{B} \neq 1 | B = 1] = P[\hat{B} = 0 | B = 1] = P[Y \geq 0 | B = 1] \\ &= Q\left(\frac{0 - (-\sqrt{E_b})}{\sqrt{N_0 / 2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \end{aligned}$$

- For unipolar mapping:

$$\begin{aligned} P_b &= P[\hat{B} \neq 0 | B = 0] = P[\hat{B} = 1 | B = 0] = P[Y \geq \sqrt{E_b / 2} | B = 0] \\ &= Q\left(\frac{\sqrt{E_b / 2}}{\sqrt{N_0 / 2}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \end{aligned}$$

Error performance of ASK and OOK



Binary orthogonal modulation

- In this method the transmitted pulses are

$$s_1(t) = \sqrt{E_b} \psi_1(t), \quad \text{if } B = 0$$

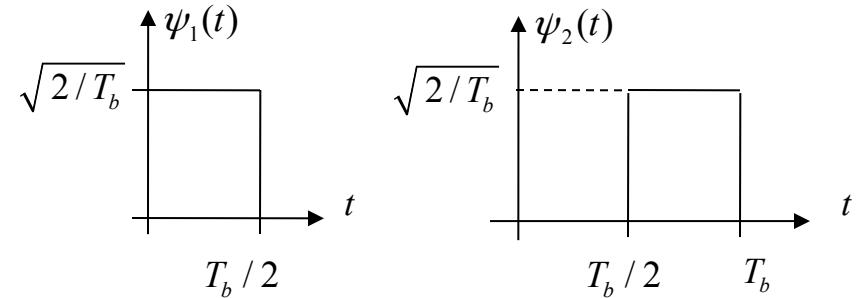
$$s_2(t) = \sqrt{E_b} \psi_2(t), \quad \text{if } B = 1$$

with

$$\int_0^{T_b} \psi_1(t) \psi_2(t) dt = 0, \quad (\text{orthogonal})$$

Examples:

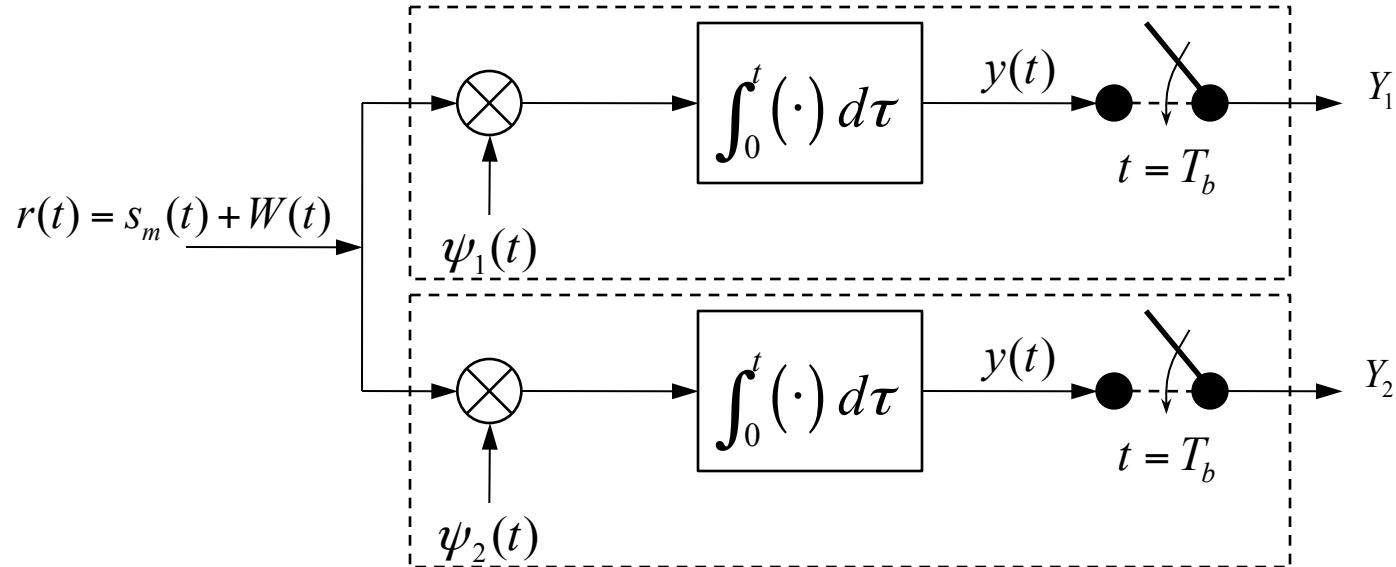
- Pulse position modulation (2-PPM):



- Binary frequency-shift keying (BFSK):

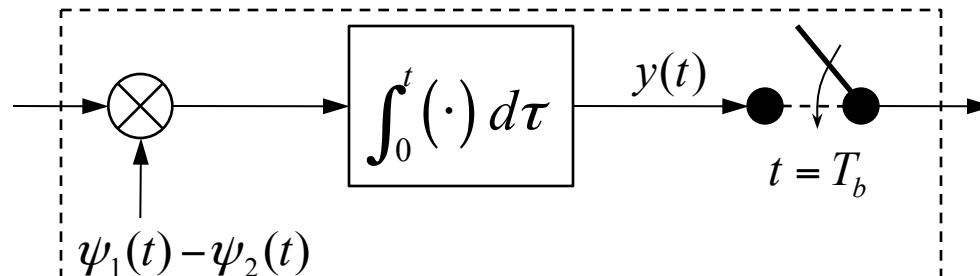
$$\psi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t), \quad \psi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t), \quad f_1 - f_2 = \frac{m}{2T_b}, \quad m : \text{integer}$$

Optimum receiver for BFSK



(Fig. 8.20 of textbook)

- Equivalent:

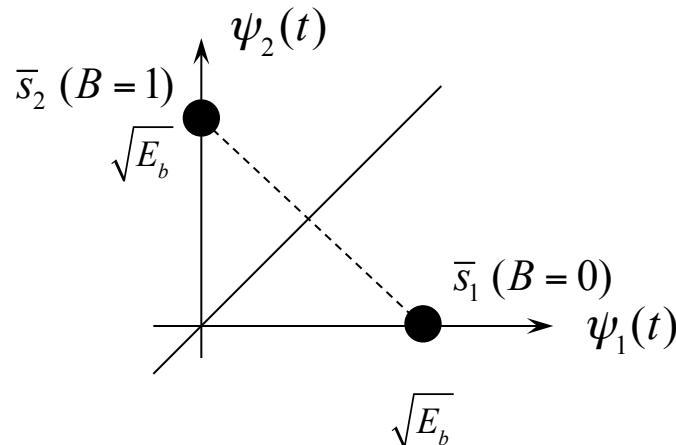


(Fig. 8.22 of textbook)

$$\begin{aligned}
 Y &= Y_1 - Y_2 \\
 \text{Gaussian with} \\
 \mu_Y &= \pm\sqrt{E_b} \\
 \sigma_Y^2 &= 2\left(\frac{N_0}{2}\right) = N_0
 \end{aligned}$$

Error performance of BFSK

- Signal constellation:



- Decision rule:

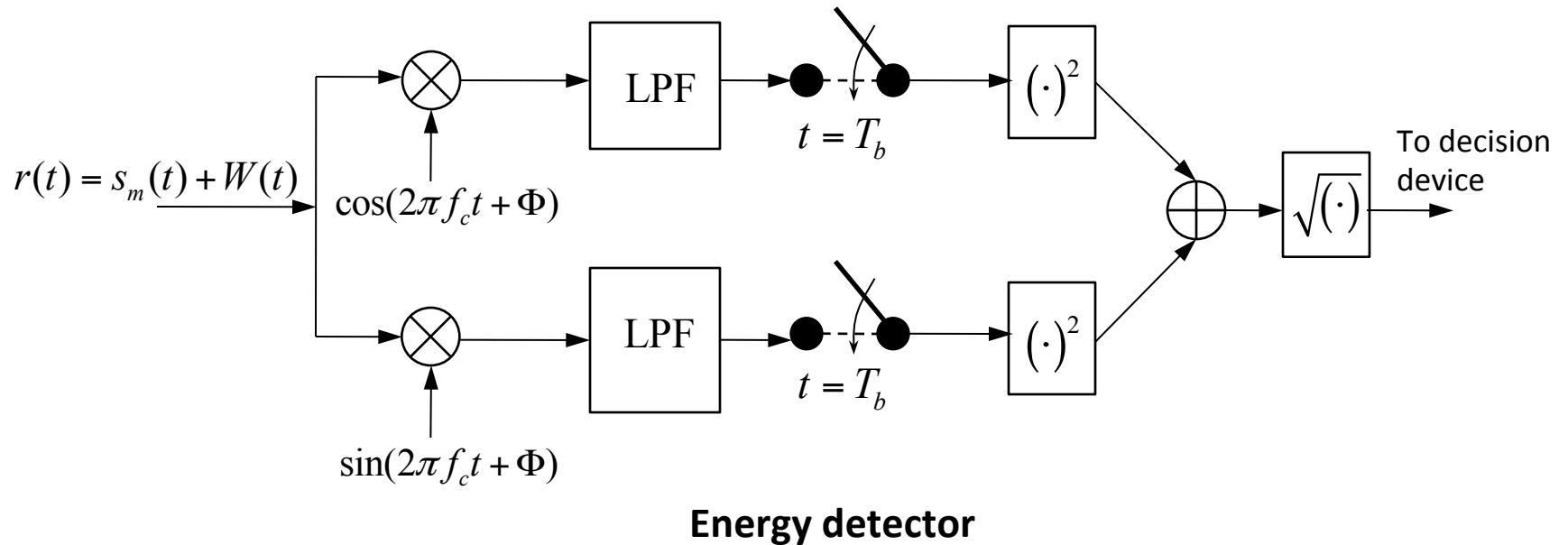
$$\hat{B} = \begin{cases} 0, & Y \geq 0 \quad (Y_1 \geq Y_2) \\ 1, & Y < 0 \quad (Y_1 < Y_2) \end{cases}$$

- Probability of a bit error:

$$\begin{aligned} P_b &= P[\hat{B} \neq 1 | B = 1] = P[\hat{B} = 0 | B = 1] = P[Y \leq 0 | B = 1] \\ &= Q\left(\frac{0 - (-\sqrt{E_b})}{\sqrt{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \end{aligned}$$

- Same as OOK modulation!

Non-coherent detection of OOK



- Performance is only marginally inferior to coherent BFSK (less than 1 dB)¹
 - This is one of the modulation methods used in the wireless experiments 5 and 6 of the EE160 lab

¹ See <http://www.engr.sjsu.edu/rmorelos/ee252s08/solhomework3.pdf>