EE 160: Principles of Communication Systems
Experiment 2: Spectra of sampled signals

I. INTRODUCTION

a. Objectives

i. Study sampling with a periodic gating signal in both the time and the frequency domains
ii. Reconstruct a sampled lowpass signal from its samples using a filter
iii. Study the effects of aliasing
iv. Implement a frequency mixer using sampling

b. Required reading

i. Proakis and Salehi, section 7.1
ii. The theory of bandpass sampling, IEEE paper in web page.

c. List of parts

i. Spectrum analyzer preamplifier
   1. 1 MΩ resistor
   2. One section of the LF347 (or equivalent) quad op amp IC

ii. Lowpass filter
   1. Four 2 kΩ resistors
   2. One 10 kΩ resistor
   3. One 33 kΩ resistor
   4. One 0.015 μF capacitor
   5. One 6800 pF capacitor
   6. One 1800 pF (or 2000 pF) capacitor
   7. One 220 pF capacitor
   8. One LF347 (or equivalent) quad op amp IC

NOTES:

(1) For best results, keep all component leads short, especially in the **lowpass filter**.
(2) You will continue to use this circuit in the next two experiments. **DO NOT DISMANTLE YOUR CIRCUIT AFTER THIS EXPERIMENT**.

REMINDER: BUILD THE CIRCUIT IN FIGURE 2.1 BEFORE YOU COME TO THE LAB

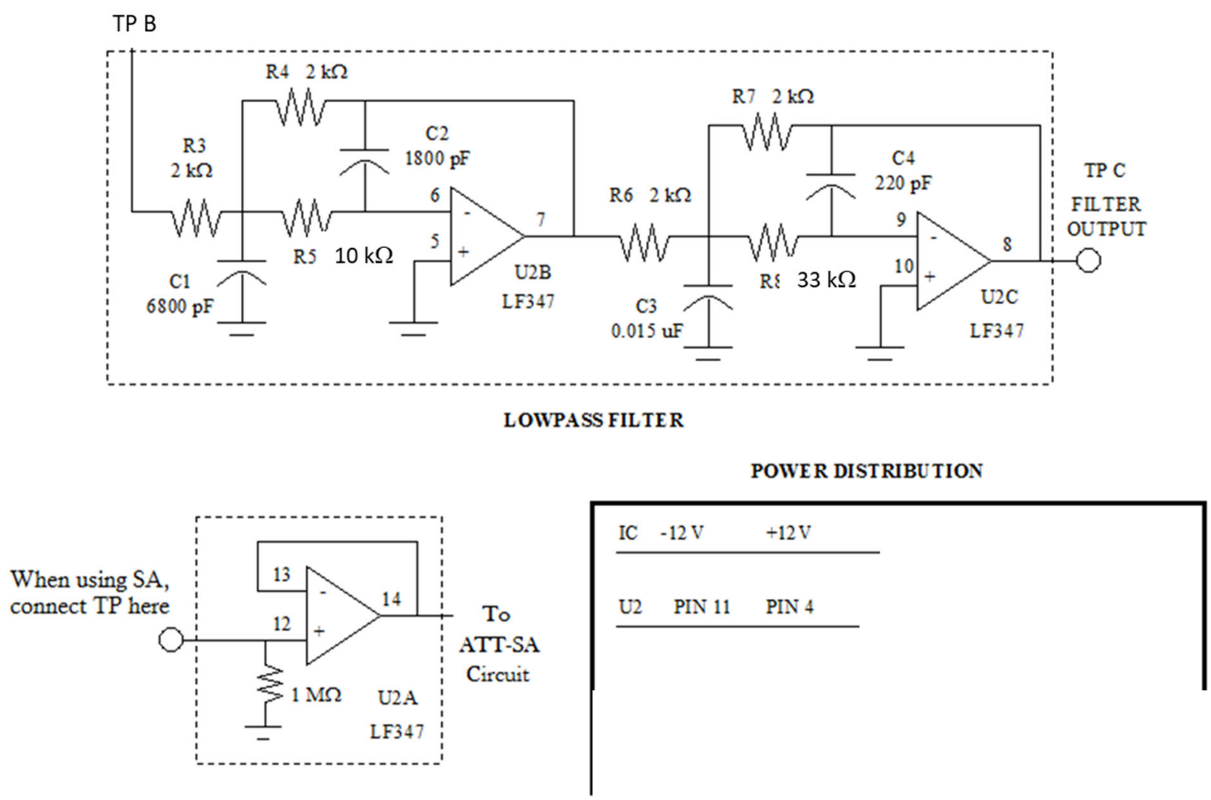

Figure 2.1: Circuit diagram for Experiment 2

II. THEORY

A periodic *gating signal* p(t) can be represented by a periodic train of rectangular pulses of period T and duty cycle $\alpha=\tau/T$. This signal has Fourier series coefficients $p_n=\alpha\ \mathrm{sinc}(n\alpha)$.

Sampling an input signal x(t) by the periodic gate function p(t) is obtained by a simple multiplication in the time domain resulting in the sampled signal $x_s(t)=x(t)p(t)$. Therefore it may be represented in the frequency domain by its Fourier transform

$$X_s(f) = \sum_{n=-\infty}^{\infty} p_n X(f - \mathrm{nf}_0),$$

where $f_0=1/T$ is the sampling frequency of the periodic gate function. Thus, the sampled spectrum can be expressed as an infinite number of copies of X(f) that have been scaled by p_n and shifted by multiples of the sampling frequency f_0.

Suppose that x(t) is band-limited, so that its spectrum X(f) is entirely contained within the range $|f|\leq B<f_0/2$. Then, x(t) may be faithfully recovered from $x_s(t)$ by an ideal low pass filter (LPF) with bandwidth $f_0/2$. In the context of sampling, this LPF is known as a reconstruction filter.

In practice, LPF's have smooth transfer functions and are described by their passband frequency f_p, below which the filter's attenuation is minimal, and its stopband frequency f_s, above which the filter's attenuation is high. In order to use such a filter to recover x(t) from $x_s(t)$, the original signal x(t) must be bandlimited to $|f|\leq B<\min(f_p, f_0- f_s)$. The condition $B\leq f_p$ ensures that the copy of X(f) centered at f=0 falls entirely within the filter's passband; the condition $B\leq f_0- f_s$ ensures that the two adjacent copies of X(f) centered at $f=\pm f_0$ fall entirely within the filter's stopband.

When the spectrum of x(t) extends beyond $|f|<\min(f_p, f_0- f_s)$, some of the power in the copies of X(f) centered at $f=\pm f_0$ appears at the output of the lowpass filter. This leakage of power from higher frequency images is called aliasing. In most cases, aliasing is undesirable because it distorts the recovered signal.

In some cases, controlled aliasing can be used to obtain a desired effect. Suppose that x(t) is now a narrow bandwidth ("bandpass") signal centered at a frequency well above $f_0/2$. In order to process this signal with inexpensive circuitry, we may wish to convert x(t) to an equivalent signal with a much lower center frequency. This process is also known as <u>down-conversion</u>. For example, suppose that x(t) is a narrowband signal limited to the frequency range $|f - 210\ \mathrm{kHz}| < 1$ kHz. Sampling x(t) using a 10% duty cycle 100 kHz periodic gate signal and processing the sampled signal through a low pass filter with $f_p > 20$ kHz and $f_s < 80$ kHz. The filtered sampled signal will be frequency-shifted copy of the bandpass signal occupying the range $|f - 10\mathrm{kHz}| < 10$ kHz. This sampling process uses controlled aliasing to mix or heterodyne two signals in order to obtain the sum or difference of the two input frequencies. In the sample, we are effectively mixing the 210 kHz bandpass signal with the second harmonic (200 kHz) of the periodic gate signal to obtain a difference signal at 10 kHz. What would happen if we used a 105 kHz periodic gate signal?

III. INSTRUMENTS AND MATERIAL

III.1 Oscilloscope techniques

This experiment will test your command of the scope's trigger controls. When you first look at a sampled sine wave on the scope, you may see only a series of periodic gate pulses with blurred amplitudes. This display will occur whenever the scope is synchronized to the sampling pulses p(t), but not to the sampled signal x(t). By carefully adjusting the scope's trigger controls, you should be able to see the "envelope" of the sampled signal, although the periodic gate pulse edges may be now blurred. With further tweaking of both the scope controls and the function generator frequency, you should be able to synchronize the scope to both the sampled signal and the sampling clock. In this way, you can get a clear display of the sampled signal $x_s(t)$.

III.2 Lowpass filter

The low pass (LP) filter design was obtained by using standard component values in a fourth-order Chebyshev (Type I) OP AMP filter with a cutoff frequency $f_c = 5.7$ kHz.

III.3 Spectrum analyzer (SA) pre-amplifier

The 1K input impedance of the ATT-SA is too low for some of the measurements required by this experiment. U2D is used as a unity-gain buffer amplifier to increase the analyzer's input impedance to 1MΩ. Use the preamplifier for all spectrum analyzer measurements in this and the next two experiments.

IV. PRE-LAB WORK

IV.1 Clock signals spectra

Compute and plot the **one-sided** power (line) spectrum P_n ($0 \leq n \leq 20$) of the periodic gate function p(t), for duty cycles 0.1, 0.2 and 0.5. Make sure that you use a dB power scale. Also, remember $P_0 = |c_0|^2$, and $P_n = 2|c_n|^2$ for $n \geq 1$.

IV.2 Sampling of bandpass signals

Let $x(t)=2\cos(2\pi f_0 t)$ and $p(t)$ be a periodic train of unit-amplitude rectangular pulses of fundamental frequency $f_s=3f_0/2$ and a 50\% duty cycle.

(1) Compute and sketch carefully the spectrum $X_s(f)$ of the sampled signal $x_s(t)=x(t)p(t)$ in the range $[-2f_s,2f_s]$.

(2) Show that a sharp lowpass filter with cutoff frequency W, such that $f_0/2 < W < f_0$, with input x_s(t) will produce at its output a sinusoidal signal of fundamental frequency f_1 such that $f_1=f_0/2$. Thus demonstrate that the bandpass sampling theorem can be used to down-convert a bandpass signal.

IV.3 Lowpass filter

Verify the design and functionality of the op amp lowpass filter as follows: First download the two LTspice models exp2_LPF_modified.asc and LF347.MOD from the webpage in the same directory in the PC and then run the model exp2_LPF_modified.asc, after modifying it with resistors and capacitors values measured with the digital multimeter. Upon completion of the simulation, move the mouse and click on the label Vout in the schematic diagram to display the amplitude and phase Bode plots. Print (and save as PDF file) the resulting figure. You will need these Bode plots in order to complete your report.

IV.3 Matlab models

(1) Low-pass sampling

Download the Matlab Simulink model *sampling_lowpass_sinusoidal.mdl* from the web page of the lab. The model simulates sampling of a sine waveform of frequency 20 kHz using a periodic train of rectangular pulses of fundamental frequency f_s =100 kHz and duty cycle 50%. The sampled waveform is recovered with a lowpass filter of cutoff frequency 44 kHz. Run the model and print or sketch the spectra and waveforms. These serve as a reference when the first part of the experiment is performed.

(2) Sampling for downconversion

Modify the model of part (1) as follows: Set the sine waveform to a fundamental frequency value of $f_0=2f_s/3=66.67$ kHz. Verify that at the output of the filter there is a sinusoidal signal of fundamental frequency $f_1= f_s/3= f_0/2$ (approx. 33.33 kHz).

V. MEASUREMENTS

V.1 Circuit check

Using the function generator and oscilloscope, check the low filter's frequency response. Connect the generator to the LPF input (TP B). In particular, verify that the filter has a lowpass response such that:

(1) The gain is approximately within -2 dB and 0 dB from DC to at least 4 kHz.

(2) The stopband attenuation is at least 20 dB at 20 kHz.

Then, measure and record the following parameters of the filter:

(1) The gain at 1 and 1 kHz

(2) The -3 dB, -20 dB, and -40 dB bandwidths.

Take any other measurements that you made need to make a Bode magnitude plot of the filter, in the range from 1 kHz to 150 kHz.

V.2 Periodic gate sampling of sinusoidal waveforms

Use the signal generator (SG) to produce a sine waveform of 1 kHz and 5 V_{pp}. Verify the amplitudes of the signal using the oscilloscope. (You need to set the output impedance of the SG to *Hi Z*.) Periodic gate sampling (or practical sampling) is performed in this experiment by an amplitude modulated waveform with a square waveform (duty cycle 50%) as a carrier.

Use the amplitude modulation function of the SG, changing the carrier shape to square as follows: In the 33220A SG, press the *Mod* button, choose 20 kHz as *AM frequency* and Square as *shape*. In the 33500B SG, press the *Modulate* button, choose 20 kHz as *AM frequency,* Square as *shape* and set *Modulate* ON. Keep the signal amplitude constant for all measurements in this section. Then make and record the following measurements:

(1) Observe the SG output on the scope. Record the waveform and its amplitude.

(2) Observe the SG output on the spectrum analyzer (with the ATT-SA circuit connected, as in experiment 1). Sketch the spectrum and label a few of the lines with their amplitudes.

(3) Connect the SG output to the filter TP B. Repeat steps (1) and (2) with the filtered signal at TP C.

In making these and subsequent measurements, you may find it useful to monitor both TP B and TP C simultaneously on the two oscilloscope channels.

V.3 Periodic gate sampling of a triangular waveform

Change the waveform in the SG to a 2 kHz 5 $V_{p\text{-}p}$ triangular wave (symmetry of 50%). Then:

(1) Observe the signal on both the oscilloscope and the spectrum analyzer. Sketch the waveform (time domain) and spectrum (frequency between 2 kHz and 25 kHz).

(2) Repeat step (1) for the signal at the LP filter's output (TP C). Is the reconstructed signal a triangular wave?

(3) On the scope, lower the sweep rate until you can see 20 to 40 cycles of the filtered signal at TP C. Note the "wavy" amplitude pattern. Change the function generator frequency near 2.5 kHz to see how this amplitude pattern changes. What causes these uneven amplitude patterns?

V.4 Aliasing and frequency mixing

(1) Step the triangular waveform frequency in 2 kHz increments from 2 kHz to 20 kHz. Measure and record the frequencies of the filtered waveform (TP C) at each setting. Take note of the patterns that develop at TP C as you pass 5 kHz and 15 kHz.

(2) Sweep the frequency slowly from 4.8 kHz to 5.2 kHz as you watch 50 or more cycles of the filtered output (TP C) on the scope. Notice the amplitude patterns that develop. Select a pattern; label key time and amplitude points. Record the frequencies of the triangular waveform and of the AM frequency sampler clock that produce this pattern. Measure the frequency of the filtered signal (TP C).

V.5 Down-conversion by sampling

Use the signal generator to produce a sinusoidal waveform of amplitude equal to 5 V_{pp} and fundamental frequency equal to $f_0=2f_s/3=6.67$ kHz, where f_s is the clock frequency (10 kHz).

Observe the output of the lowpass filter using the spectrum analyzer and the oscilloscope to verify that it is a sinusoidal signal of fundamental frequency $f_1= f_s/3= f_0/2$ (approx. 3.33 kHz).

VI. ANALYSIS OF RESULTS

- Present all the measurements of your lowpass filter clearly in tables and graphs. In particular, include Bode plots (from 10 Hz to 15 kHz) of the amplitude of the op amp lowpass filter, based on both your measurements and the LTspice model.
- Discuss the behavior of the filtered signal (TP C) for the sampled triangular wave. Is the output distorted? Why?
- Explain the relationship between the measured input (TP A) and output (TP C) frequencies. For one combination of input and output frequencies, show how aliasing arises in (a) the time domain and (b) the frequency domain.
- Explain the selected amplitude pattern measured in step (2) of section V.4.
- How do the results from section V.5 compare to the theoretical computations from the pre-lab work in section IV.2 and the Matlab simulation in section IV.3? When comparing, account for the factor of 10 in the frequency values used in the experiment.