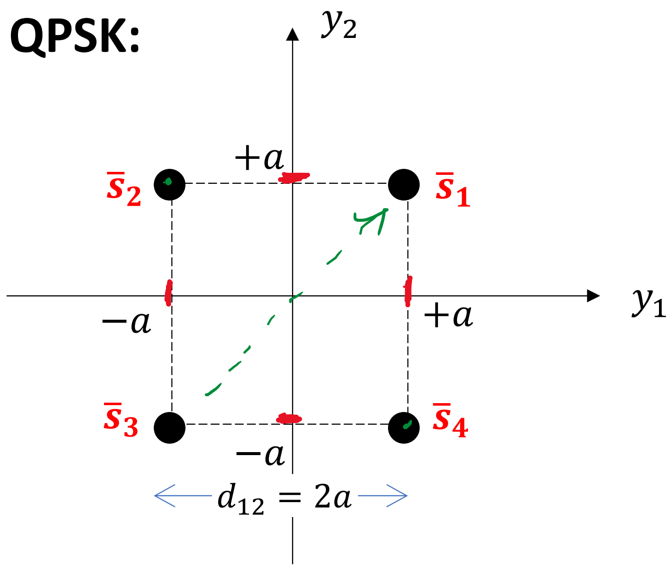
Simplified analysis using pairwise error probability and nearest neighbors

QPSK:



*Notice that each signal point has two points at distance d_{12}. In other words, the **number of nearest neighbors** is*

$$\boxed{\mathrm{NN} = 2}$$

Also, each symbol (signal) carries $\boxed{\ell = 2}$ ***bits per symbol***

(1) Pairwise error probability:

$$P_{12} = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{(2a)^2}{2N_0}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

(2) Symbol error probability:

$$P_e \approx \mathrm{NN} \cdot P_{12} = 2\, P_{12} = 2\, Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

(3) Bit error probability (Gray labeling):

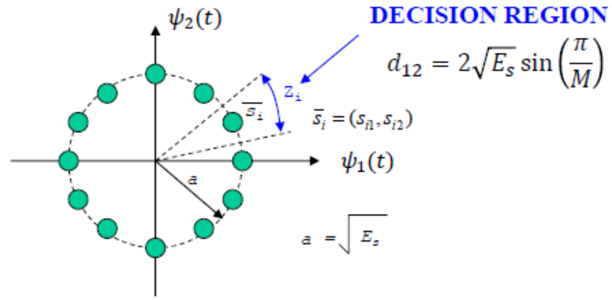
$$P_b = \frac{P_e}{\ell} = \frac{2}{2}\, Q\left(\sqrt{\frac{E_s}{N_0}}\right) = \boxed{Q\left(\sqrt{\frac{E_s}{N_0}}\right)}$$

NN approximation

This and subsequent slides are in lecture note “6c_NN_approximation.pdf”

M-ary Phase-Shift Keying (M-PSK)

Signal constellation:



Average probability of a bit error:

$$\boxed{P_b \approx Q\left(\sqrt{\frac{2E_s}{N_0}\sin^2\left(\frac{\pi}{M}\right)}\right)} = Q\left(\sqrt{\frac{2E_b}{N_0}\ell\sin^2\left(\frac{\pi}{M}\right)}\right)$$

$$\boxed{P_b = \frac{2}{\ell}\, Q\left(\sqrt{\frac{2E_s}{N_0}\sin^2\left(\frac{\pi}{M}\right)}\right), \qquad M = 2^\ell}$$

NN approximation with Gray labeling

NN approximation for *M*-PAM

$$\boxed{P_b = \frac{(M-2)2+2}{\ell M}\, Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right), \qquad M = 2^{\ell}}$$

Verification for 4-PAM: $M = 4, \ell$=2:

$$P_b = \frac{(4-2)2+2}{2(4)}\, Q\left(\sqrt{\frac{6E_s}{(4^2-1)N_0}}\right) = \frac{6}{8}\, Q\left(\sqrt{\frac{6E_s}{15N_0}}\right) = \boxed{\frac{3}{4}\, Q\left(\sqrt{\frac{2E_s}{5N_0}}\right)}$$

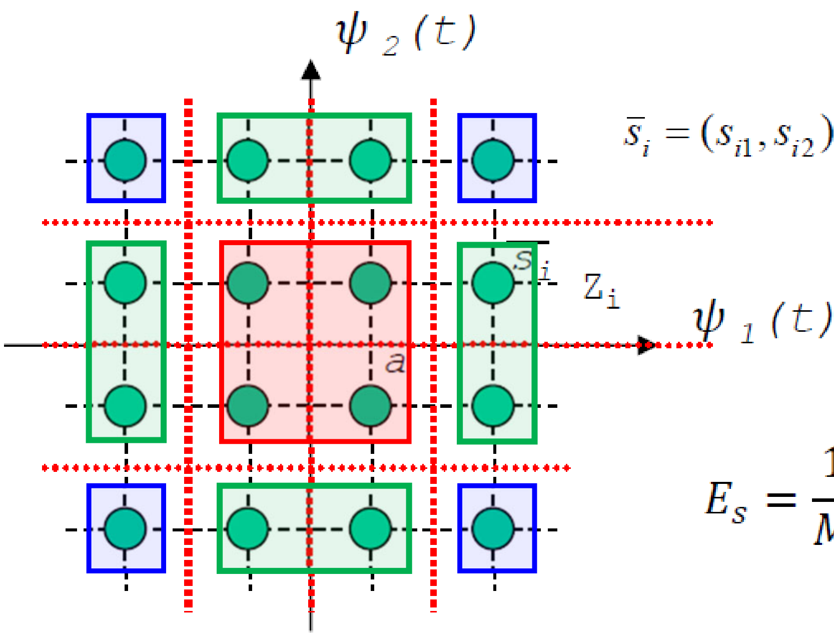
NN approximation for *M*-QAM, *M* > 4

$$\boxed{P_b = \frac{(4)2 + 4\left(\sqrt{M} - 2\right)3 + \left((\sqrt{M} - 2)^2\right)4}{\ell M}\, Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right), \qquad M = 2^{\ell}}$$

For 16-QAM: $M = 16, \ell$=4:

$$P_b = \frac{(4)2 + 4(4-2)3 + ((4-2)^2)4}{64}\, Q\left(\sqrt{\frac{3E_s}{(16-1)N_0}}\right) = \boxed{\frac{3}{4} Q\left(\sqrt{\frac{E_s}{5N_0}}\right)}$$

16-QAM: Average number of nearest points, NN



$$E_s = \frac{1}{M}\sum_{i=1}^{M} |\bar{s}_i|^2$$

: 4 corner points, 2 neighbors ea.

: $4\left(\sqrt{M}-2\right) = 8$ wall points, 3 neighbors ea.

: $\left(\sqrt{M}-2\right)^2 = 4$ inner points, 4 neighbors ea.

$$\boxed{\boldsymbol{NN = \frac{(4)2 + (8)3 + (4)4}{16} = \frac{48}{16} = 3}}$$

General method

(1) Compute the average number of signal points, NN, at the minimum pairwise distance d_{12}

(2) Using the number of bits/symbol $\ell = \log_2(M)$, compute the approximation

$$\boxed{P_b \approx \frac{\mathrm{NN}}{\ell} P_{12} = \frac{\mathrm{NN}}{\ell}\, Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)}$$

Gray labeling

$$\boxed{P_b \approx \frac{\mathrm{NN}}{2} P_{12} = \frac{\mathrm{NN}}{2}\, Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)}$$

Natural labeling