The concept of diversity and diversity combining

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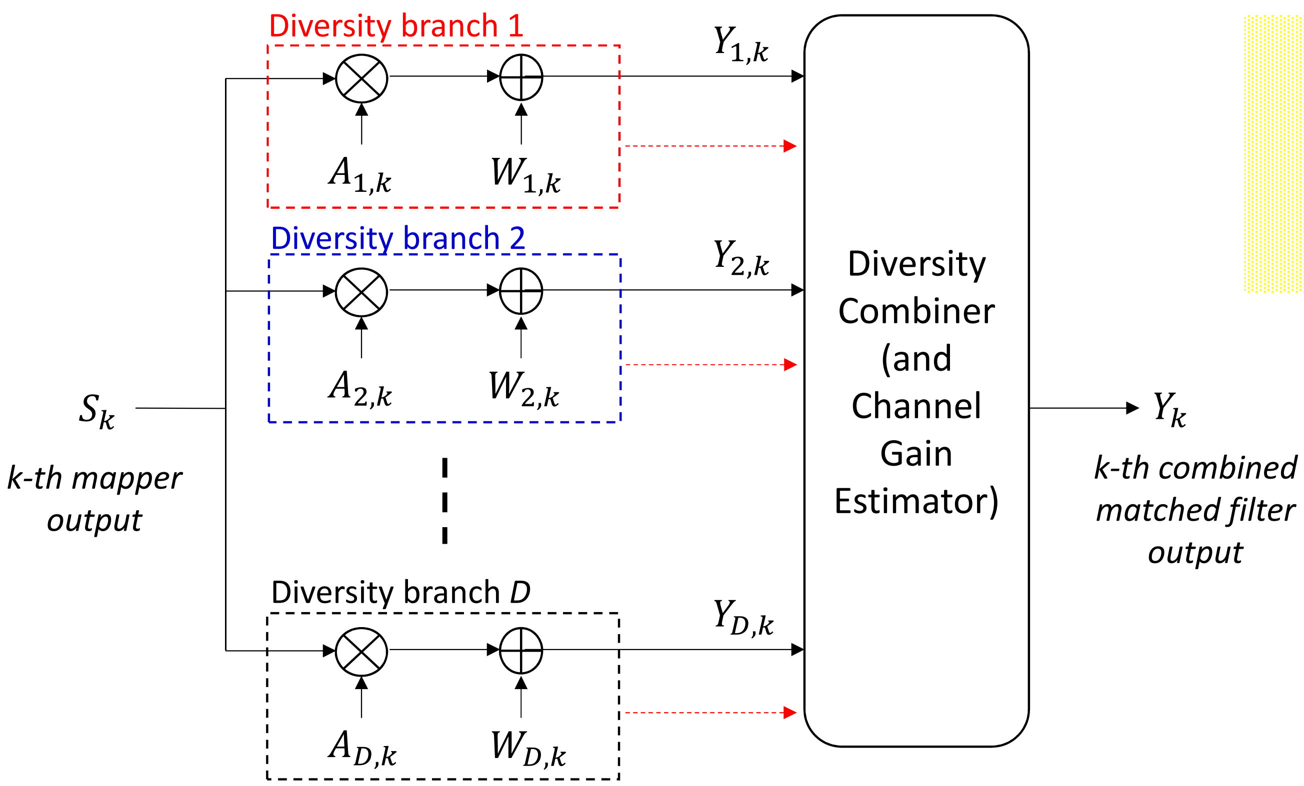
Diversity

- Diversity in wireless communications is the process of creating *D* statistically independent (flat Rayleigh) fading channels
- Each channel is referred to as a ***diversity branch***
- *D* is known as the ***diversity order*** of the system
- In general, each independent channel can have different types of fading
  - In this class focus is on ***slow flat Rayleigh fading***

Complex baseband model with diversity



Each diversity branch *i* has

$A_{i,k}$: Channel gain (complex)

$W_{i,k}$: AWGN sample (complex)

$Y_{i,k}$: MF output (complex)

$$\boldsymbol{A_{i,k} = \alpha_{i,k} e^{-j\varphi_{i,k}}}$$

(as in textbook)

Signal energy under Rayleigh fading

- The *instantaneous* signal energy (or power)-to-noise ratio,

$$\Gamma = \frac{E_S}{N_0} |A|^2,$$

is an exponential r.v. of parameter equal to the average signal energy-to-noise ratio γ_0, i.e., its pdf is given by

$$\boxed{p_\Gamma(\gamma) = (1/\gamma_0) \exp(-\gamma/\gamma_0), \quad \gamma>0,}$$

where $\gamma_0 = (E_s / N_0)\, \mathrm{E}\{A^2\}$, and A is the Rayleigh fading amplitude.

Note: We use $\gamma_0 = \bar{\Gamma}$ in these notes

Outage probability under slow flat Rayleigh fading

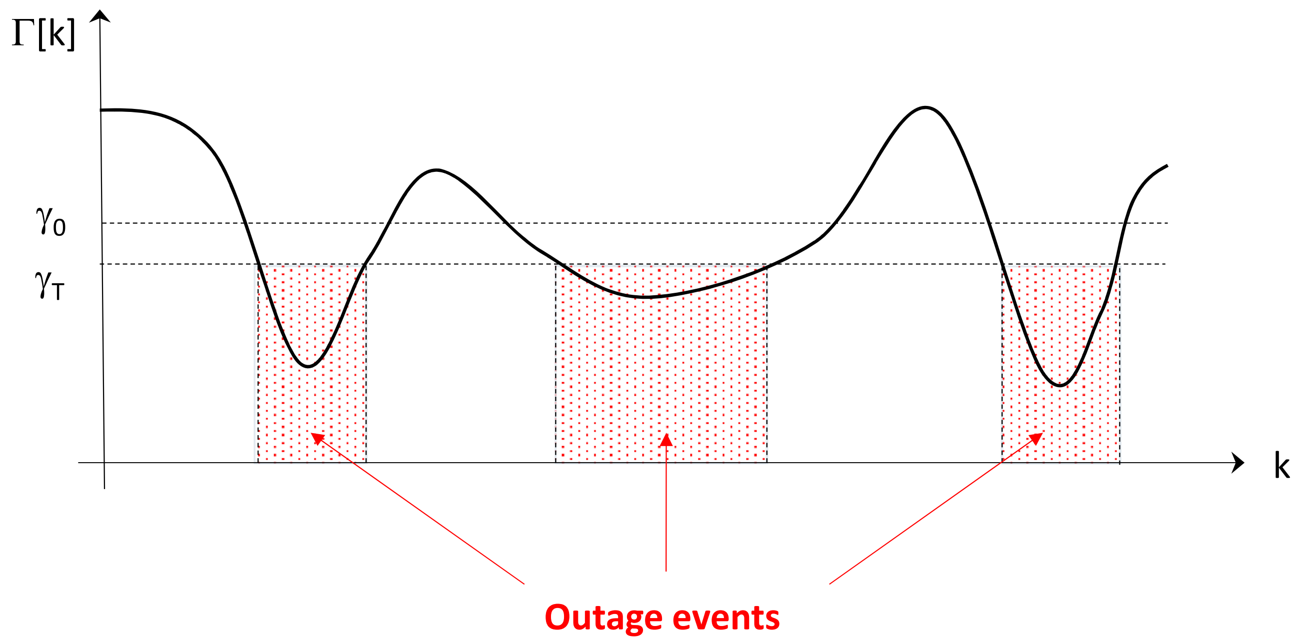
- The outage probability is defined as the probability that Γ falls below a threshold γ_T on the signal energy required to achieve a maximum average bit error probability:

$$\boxed{P[\,\Gamma \le \gamma_T\,] = F_{\mathrm{T}(\gamma_T)} = 1 - \exp\left(-\frac{\gamma_T}{\gamma_0}\right)}$$

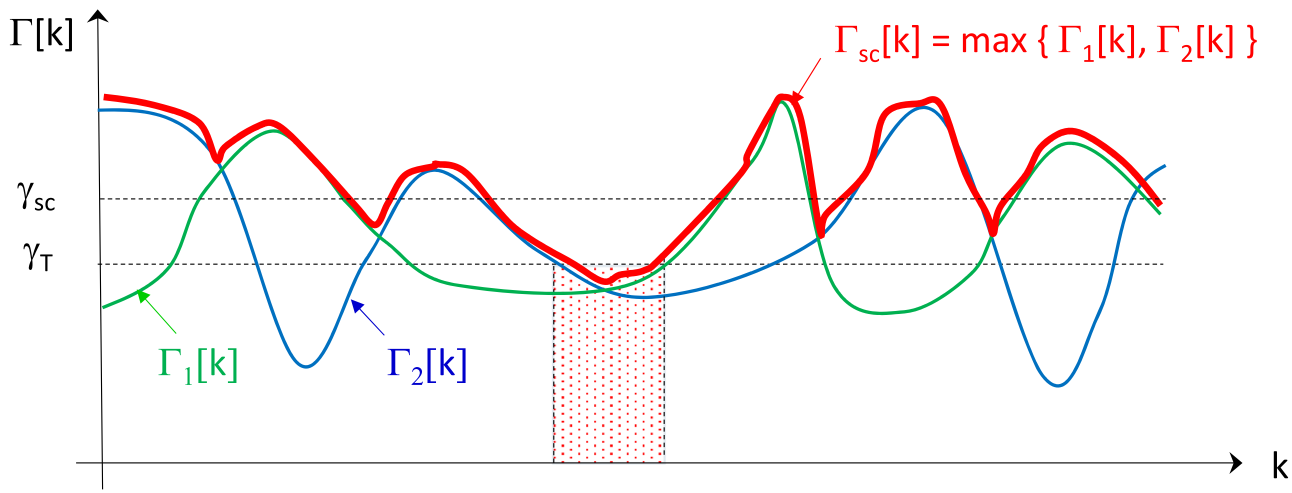
- How to set the threshold?
  - Under slow fading conditions, the threshold γ_T is determined based on the mapping (e.g., PAM, PSK, QAM) and the average bit error probability as a function of γ_T under AWGN (Q-function)

Whiteboard

Illustration of outage events



Selection combining: Two-branch (*D* = 2) example



- The selection combiner choses the diversity branch with the *largest value* of Γ:

$$\Gamma_{sc} = \max \{ \Gamma_1, \Gamma_2 \}$$

- The average γ_{sc} increases
- The variance decreases

$$\rightarrow \boxed{P[\Gamma_{sc} \leq \gamma_T] = \left(1 - \exp\left(-\frac{\gamma_T}{\gamma_0}\right)\right)^2 < P[\Gamma \leq \gamma_T]}$$

Diversity combining techniques

- Complex baseband model: Operate on *matched-filter (or correlator) outputs*. Let the gain of each branch be $A_i = \alpha_i e^{-j\varphi_i}$
- Three popular techniques to combine the outputs:

1. Selection combining (SC): $\boxed{Y_{\mathrm{SC}} = \arg\max\{|Y_i|\}}$ **(Worst performance)**

2. Equal-gain combining (EGC): $\boxed{Y_{\mathrm{EGC}} = \sum_{i=1}^{D} Y_i e^{j\varphi_i}}$

3. Maximal-ratio combining (MRC): $\boxed{Y_{\mathrm{MRC}} = \sum_{i=1}^{D} \alpha_i Y_i e^{j\varphi_i}}$ **(Best performance)**

D-branch diversity performance

- Remarkably, once *D* diversity branches are created, **all** diversity combining techniques achieve the same diversity order *D* !!!

- That is, the average bit error probability is inversely proportional to the average signal energy-to-noise ratio, E_s/N_0, raised to the *D*th power:

$$\boxed{P_b \approx \frac{K_D}{(4\gamma_0)^D},}$$

where K_D depends on the specific combining technique.

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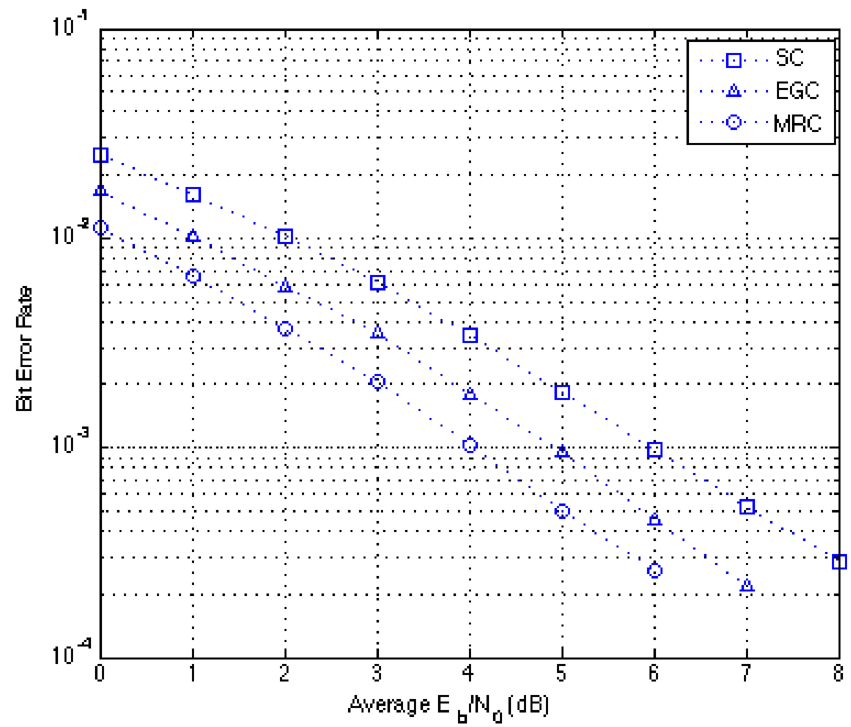
MATLAB simulation: BPSK with 4-branch diversity



Improvement in *post-combiner* average E_s / N_0 [1]

- SC:

$$\boxed{\frac{\gamma_{\text{SC}}}{\gamma_0} = \sum_{i=1}^{D} \frac{1}{i}}$$

- EGC:

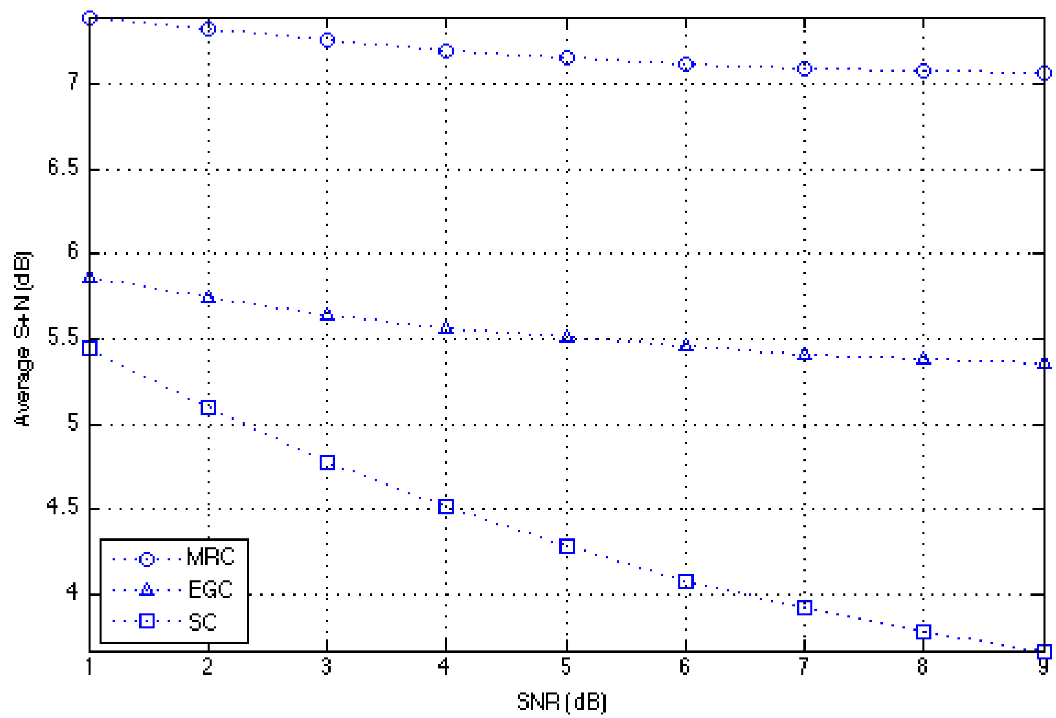
$$\boxed{\frac{\gamma_{\text{EGC}}}{\gamma_0} = 1 + \frac{\pi}{4}(D - 1)}$$

- MRC:

$$\boxed{\frac{\gamma_{\text{MRC}}}{\gamma_0} = D}$$

Whiteboard

MATLAB simulation: BPSK, 4-branch diversity (cont.)



Reference

1. P.M. Shankar, *Introduction to Wireless Systems*, Wiley, 2002.