

Some Properties of the Fourier Series

(Problem 2.41)

9/13/11

If x_n are the FSC of $x(t)$ and $y_n \leftrightarrow y(t)$ then $x_n \leftrightarrow x(t)$ and $y_n \leftrightarrow y(t)$

(a) For $y(t) = x(t - t_0)$ we have

$$y_n = x_n e^{-j 2\pi \frac{n}{T_0} t_0} \rightarrow |y_n| = |x_n|$$

(b) For $y(t) = \frac{d}{dt} x(t)$ we have

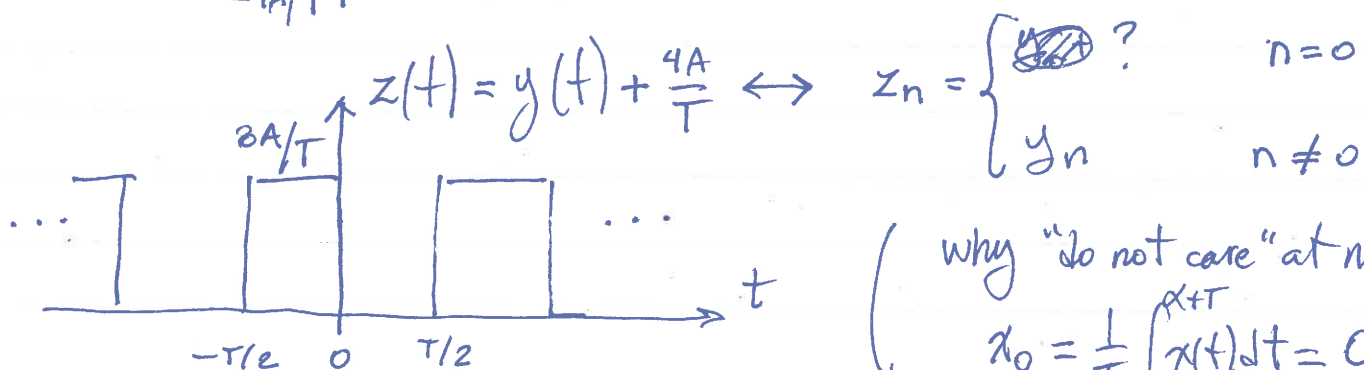
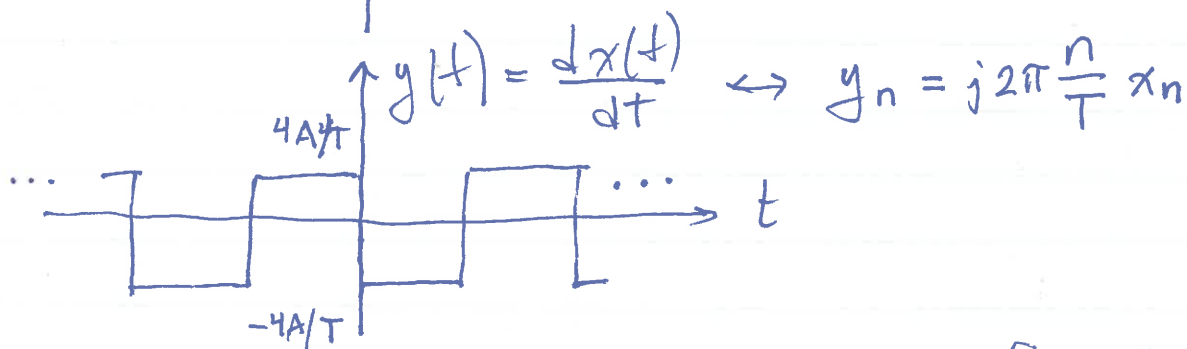
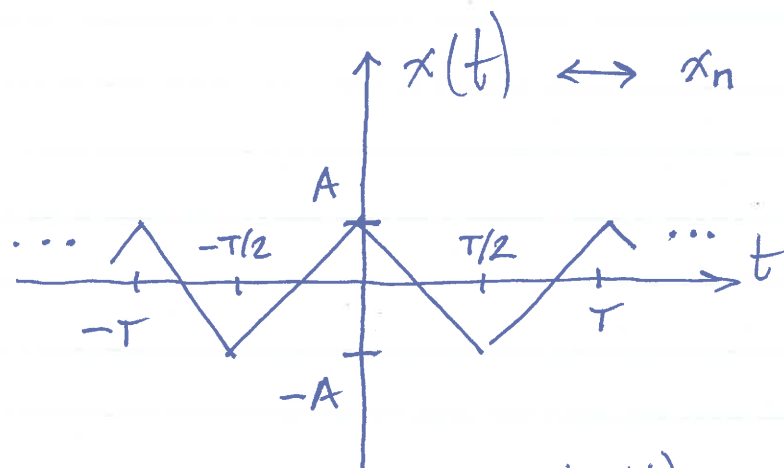
$$y_n = j 2\pi \frac{n}{T_0} x_n \quad (*) \quad \text{I do not like } T_0 \dots$$

(c) For $y(t) = x(t) + a$, a : constant

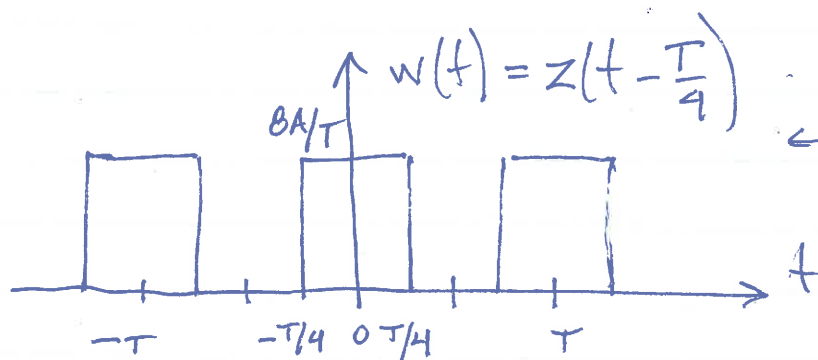
$$y_n = x_n, \quad n \neq 0$$

$$y_0 = x_0 + a$$

Example: FSC of triangular waveform. Use result of train of rectangular pulses with $\tau/T_0 = 1/2$.



Why "do not care" at $n=0$?
 $x_0 = \frac{1}{T} \int_{-\infty}^{\infty} x(t) dt = \underline{\underline{0}}$
 average!



We know that

$$w_n = \frac{8A}{T} \left[\frac{1}{2} \text{sinc}\left(\frac{n}{2}\right) \right]$$

(Amplitude scaling:
 $ax(t) \leftrightarrow ax_n$)

2b/2

9/9/14

Back substitution:

$$z_n = W_n e^{jn\frac{\pi}{2}} = \frac{4A}{T} \operatorname{sinc}\left(\frac{n}{2}\right) e^{jn\frac{\pi}{2}}$$

$$= y_n, \quad n \neq 0.$$

$$\rightarrow x_n = \frac{y_n}{j 2\pi \frac{n}{T}} = \frac{\cancel{\frac{24A}{T}} \operatorname{sinc}\left(\frac{n}{2}\right) e^{jn\frac{\pi}{2}}}{j \cancel{4\pi} \frac{n}{\cancel{T}}}$$

$$x_n = \frac{2A}{j\pi n} \operatorname{sinc}\left(\frac{n}{2}\right) e^{jn\frac{\pi}{2}}$$

or

$$x_n = \frac{2A}{\pi n} \operatorname{sinc}\left(\frac{n}{2}\right) e^{j\frac{\pi}{2}(n-1)}$$

$$(j = e^{j\frac{\pi}{2}})$$