**EE161 — Spring 2025** **San José State University**

## Solution of Homework # 6

1. A systematic binary linear (5,2,3) code

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

   (a) The parity submatrix is

$$P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

   Therefore,

$$H = \begin{pmatrix} P^\top I_{n-k} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

   (b) Encoding table:

| $\bar{B}$ | $\bar{c}$ |
|---|---|
| 00 | 00000 |
| 01 | 01111 |
| 10 | 10011 |
| 11 | 11100 |

   (c) Hard-decision decoding lookup table:

| $\bar{s}$ | $\bar{e}$ |
|---|---|
| 000 | 00000 |
| 011 | 10000 |
| 111 | 01000 |
| 100 | 00100 |
| 010 | 00010 |
| 001 | 00001 |

   (d) Received vector is $\bar{r} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \end{pmatrix}$.

      i. Syndrome:

$$\bar{s} = \bar{r}H^\top = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

      ii. From the table in part (c), $\bar{e} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix}$.

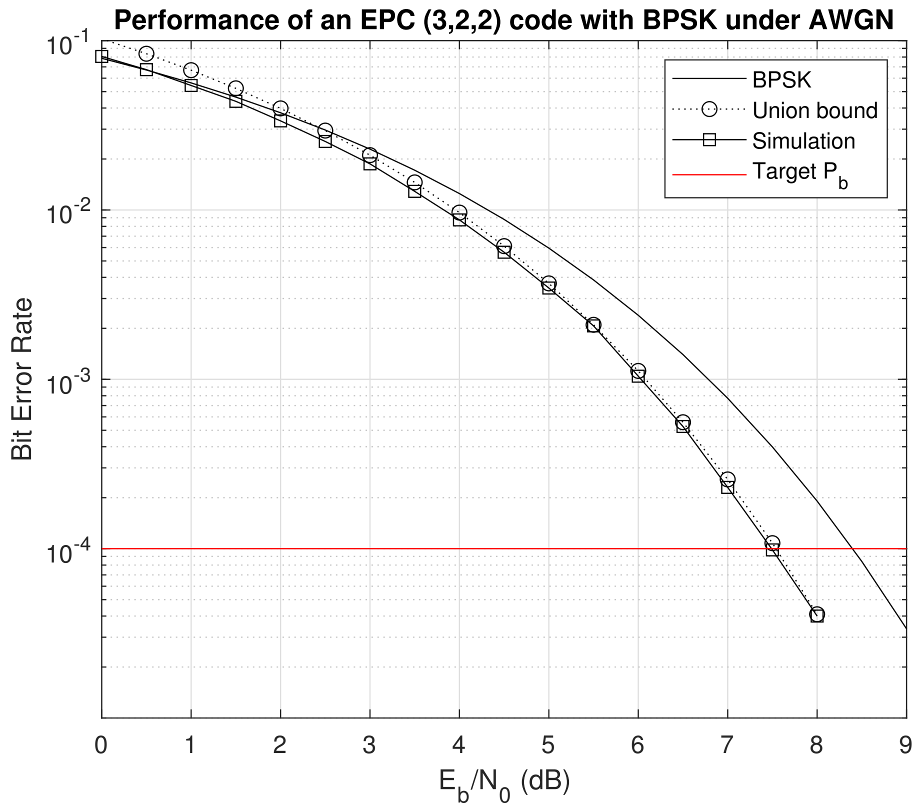
      iii. Estimate codeword:

$$\hat{c} = \bar{r} \oplus \bar{e} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

      The first two bits are the information bits and thus $\hat{B} = \begin{pmatrix} 0 & 1 \end{pmatrix}$.

2. <u>Maximum-likelihood (soft-decision) decoding of a binary EPC (3,2,2) code</u>

(a) Simulation results:




(b) Based on the simulations, at BER $= 10^{-4}$, the coding gain is $G \approx$ <u>0.9 dB</u>. Forney's RCG value is <u>0.93 dB</u> which is a very good approximation.