

Some Properties of the Fourier Series

(Problem 2.41)

9/13/11

If x_n are the FSC of $x(t)$ and $y_n \leftrightarrow x_n$
 y_n then $y_n \leftrightarrow y(t)$

(a) For $y(t) = x(t-t_0)$ we have

$$y_n = x_n e^{-j 2\pi \frac{n}{T_0} t_0}$$

$$\rightarrow |y_n| = |x_n|$$

(b) For $y(t) = \frac{d}{dt} x(t)$ we have

$$y_n = j 2\pi \frac{n}{T_0} x_n$$

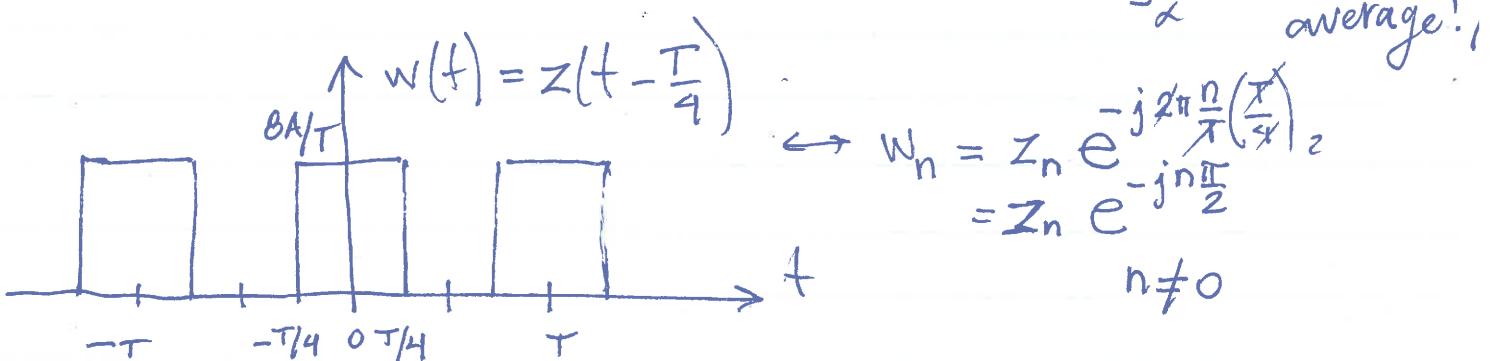
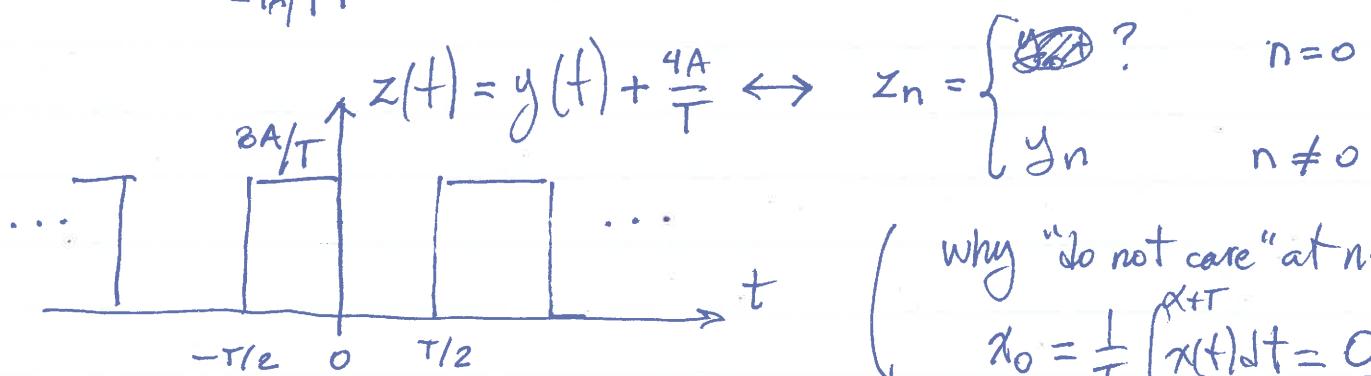
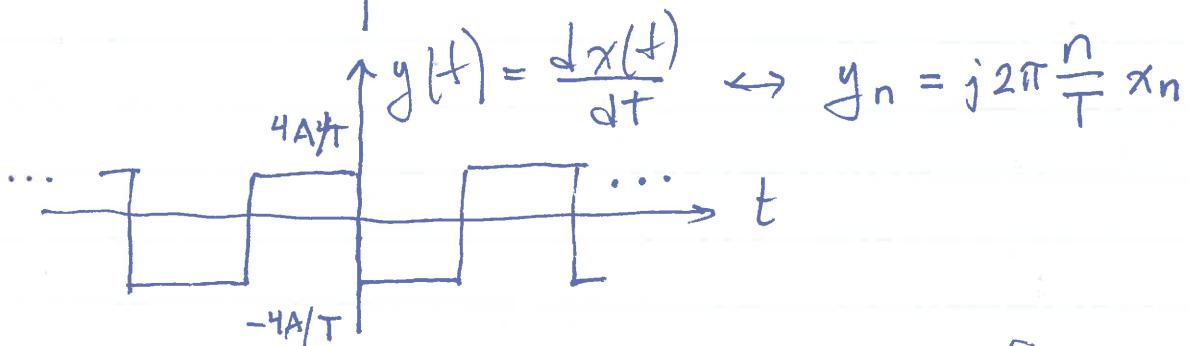
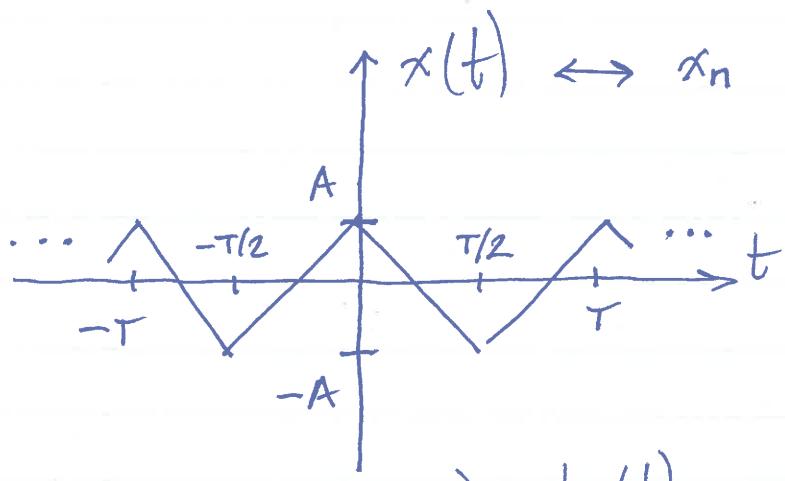
(*) I do not like T_0 ...

(c) For $y(t) = x(t) + a$, a : constant

$$y_n = x_n, \quad n \neq 0$$

$$y_0 = x_0 + a$$

Example: FSC of triangular waveform. Use result of train of rectangular pulses with $\tau/T_0 = 1/2$.



We know
that

$$w_n = \frac{8A}{T} \left[\frac{1}{Z} \operatorname{sinc}\left(\frac{n}{Z}\right) \right]$$

(Amplitude scaling:
 $\alpha x(t) \Leftrightarrow \alpha x_n$)

(why "do not care" at $n=0$?
 $x_0 = \frac{1}{T} \int_{-\infty}^{\infty} x(t) dt = \underline{0}$
 average!)

$$\begin{aligned} w_n &= z_n e^{-j 2\pi \frac{n}{Z} \left(\frac{T}{4} \right)} \\ &= z_n e^{-j n \frac{\pi}{2}} \end{aligned}$$

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Back substitution:

$$z_n = w_n e^{jn\frac{\pi}{2}} = \frac{4A}{T} \operatorname{sinc}\left(\frac{n}{2}\right) e^{jn\frac{\pi}{2}}$$
$$= y_n, \quad n \neq 0.$$

$$\rightarrow x_n = \frac{y_n}{j 2\pi \frac{n}{T}} = \frac{\frac{2A}{T} \operatorname{sinc}\left(\frac{n}{2}\right) e^{jn\frac{\pi}{2}}}{j 2\pi \frac{n}{T}}$$

$$x_n = \frac{2A}{j\pi n} \operatorname{sinc}\left(\frac{n}{2}\right) e^{jn\frac{\pi}{2}}$$

or

$$x_n = \frac{2A}{\pi n} \operatorname{sinc}\left(\frac{n}{2}\right) e^{j\frac{\pi}{2}(n-1)}$$

$$\left(j = e^{j\frac{\pi}{2}}\right)$$