# Digital Communication Systems over Bandlimited Channels: Intersymbol Interference

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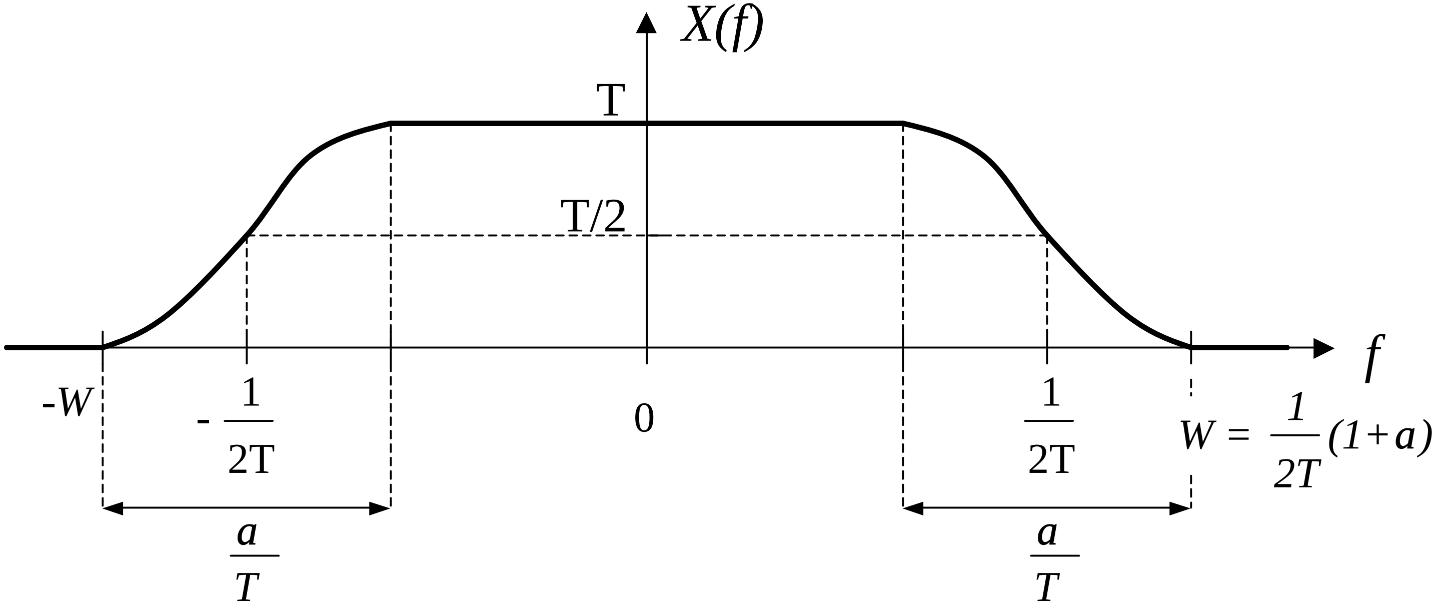
# Intersymbol interference (ISI)

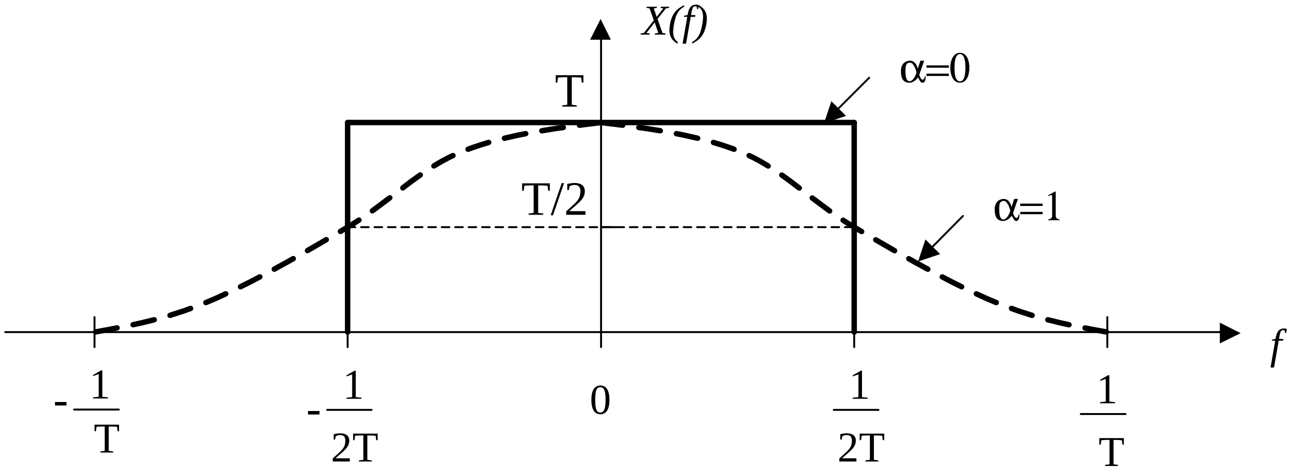
- Nyquist criterion for ISI-free data transmission:

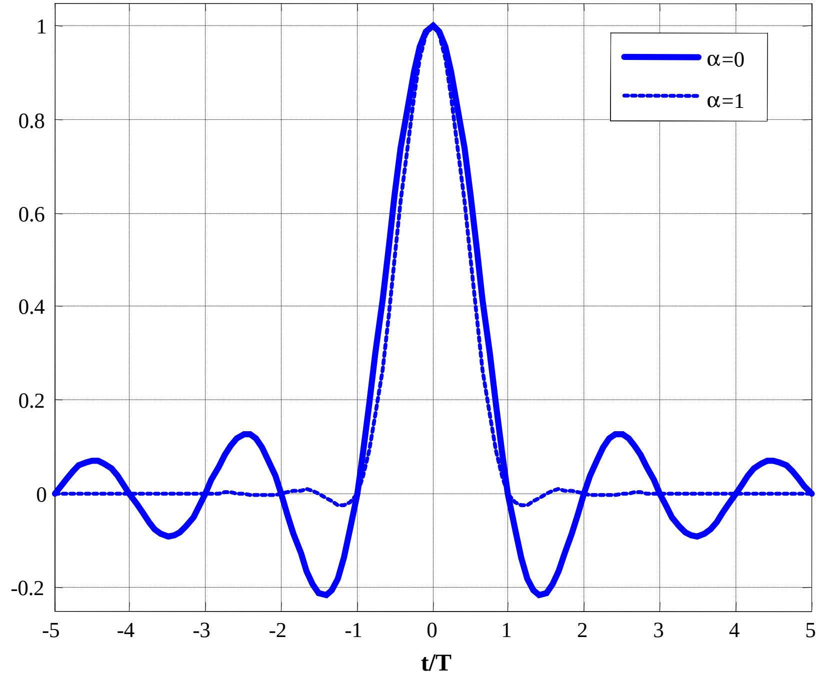
$$X_d(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(f - \frac{n}{T}) = 1$$

- ISI can be removed using a ***raised-cosine spectrum*** *X(f)* with ***rolloff factor a***



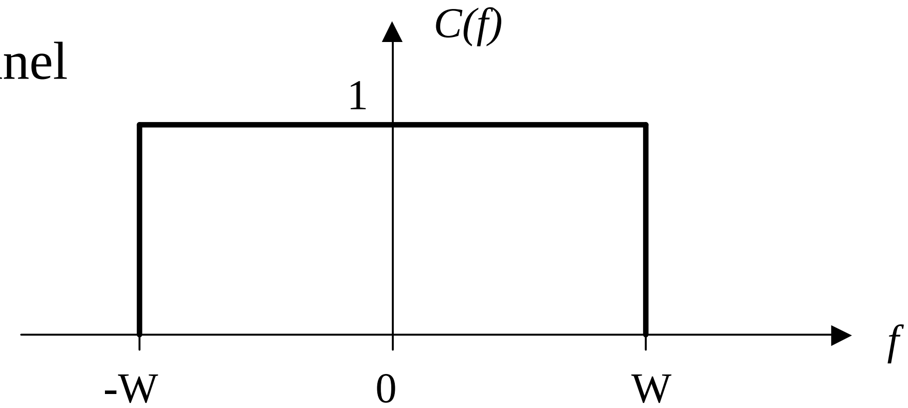
# Raised-cosine spectrum



# Pulse shaping and matched filter design

Ideal bandlimited Channel Transfer function:




- Then

$$X(f) = G_T(f)C(f)G_R(f) = G_T(f)G_R(f), \quad |f| \le W.$$

- Pulse shaping and matched filters can be chosen as ***square-root raised cosine (SRRC) filters*** with

$$|G_T(f)| = |G_R(f)| = \sqrt{X(f)}$$

# Square-Root Raised Cosine (SRRC) Filters

- Rolloff factor, α

$$\boxed{W = \frac{B}{2} = \frac{1}{2T}(1+\alpha)}$$

- Impulse response

$$\boxed{h(t) = \frac{1}{\sqrt{T}} \frac{\sin[(1-\alpha)\pi t/T] + (4\alpha t/T)\cos[(1+\alpha)\pi t/T]}{\pi t/T\,[1-(4\alpha t/T)^2]}}$$

**Note:**

$X(f) \xleftrightarrow{F} x(t) = h(t) * h(t)$, **where $X(f)$ is a raised-cosine spectrum**

# Eye diagram – A simple illustrative example

Consider binary signaling with polar mapping (antipodal), whereby a bit sequence $\{b_k\}$ is received as

$$y(t) = \sum_{k=-\infty}^{\infty} a_k x(t - kT),$$

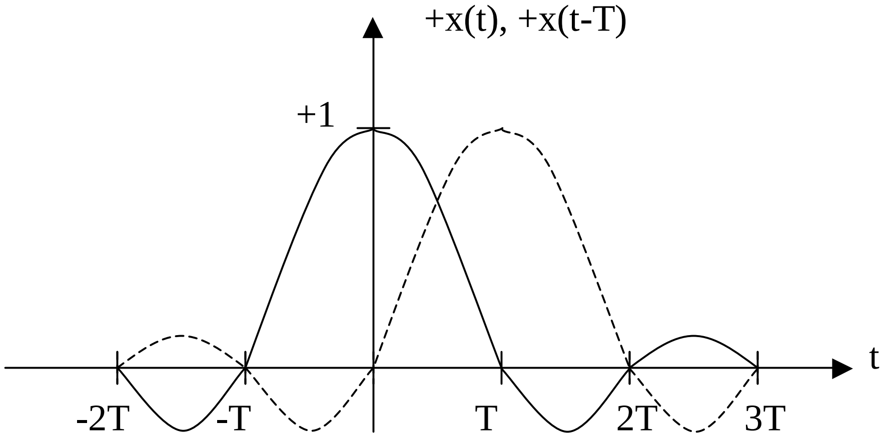
where $a_k=+1$ if $b_k=0$, $a_k=-1$ if $b_k=1$, and

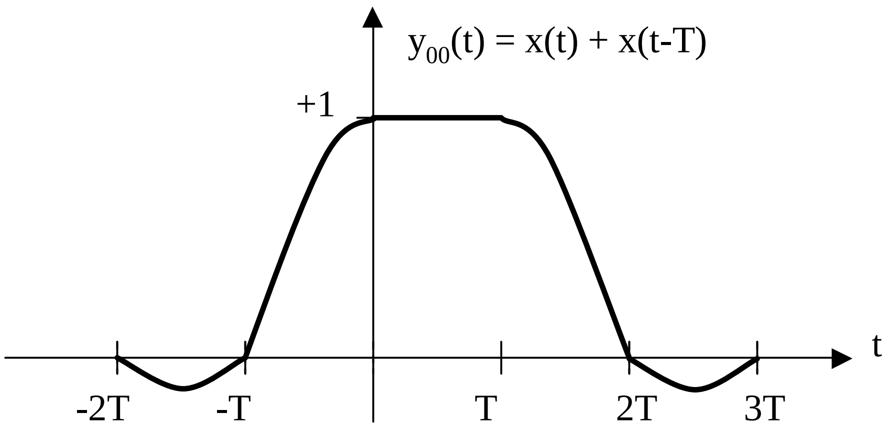
$$x(t) = \mathrm{sinc}\left(\frac{t}{T}\right)$$

is the overall impulse response of a binary communication system. For illustration purposes, let a pair of bits $\{b_0,b_1\}$, be transmitted and assume that no further transmission occurs before or after these bits.

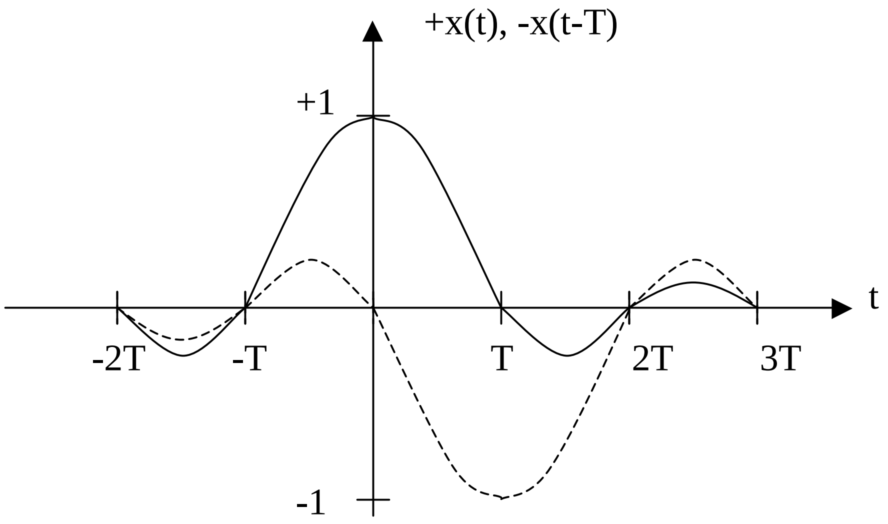
We now examine the four possible pairs of transmitted bits.

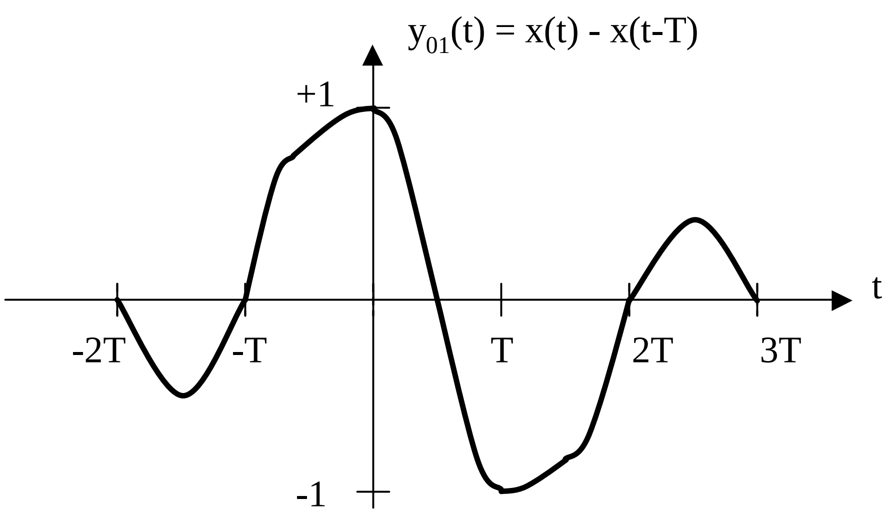
# Case 1: $\{b_0,b_1\}=\{0,0\}$.



Case 2: $\{b_0,b_1\}=\{0,1\}$.



Case 3: $\{b_0,b_1\}=\{1,0\}$.

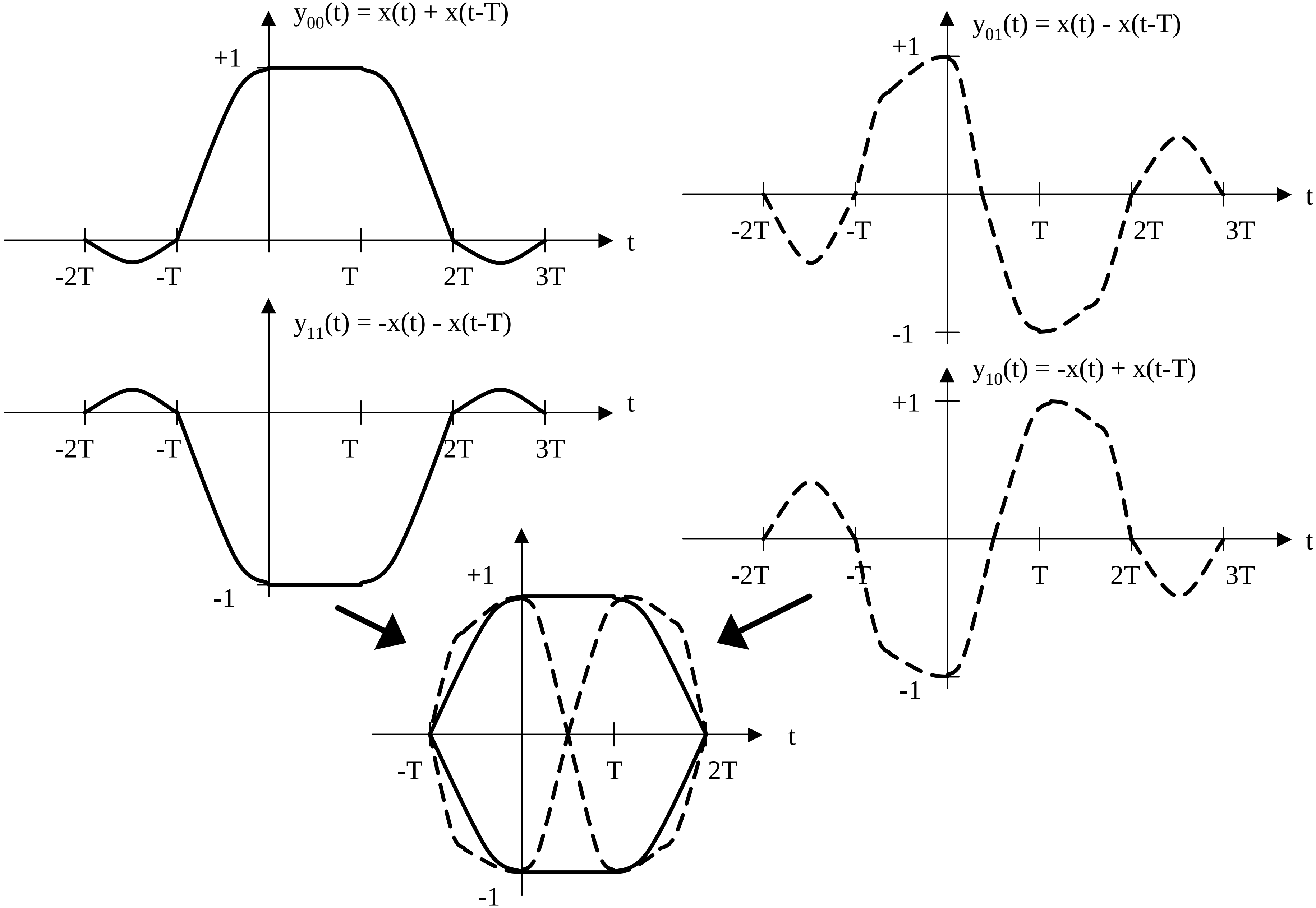
$$y_{10}(t) = -y_{01}(t)$$

Case 4: $\{b_0,b_1\}=\{1,1\}$.

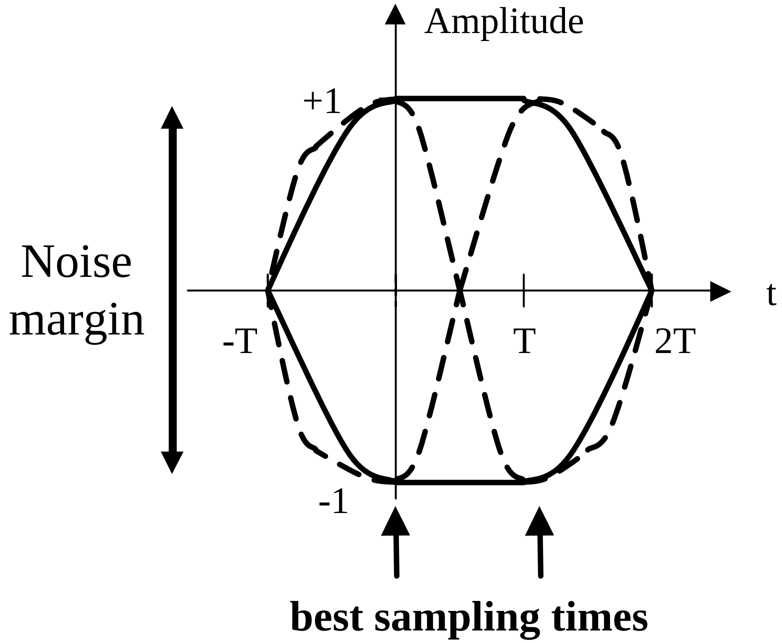
$$y_{11}(t) = -y_{00}(t)$$

An eye diagram can be constructed by overlapping the signals $y_{ij}(t)$, $\{i,j\}=\{0,1\}^2$, in the interval [-T, 2T] (for illustrative purposes only.)

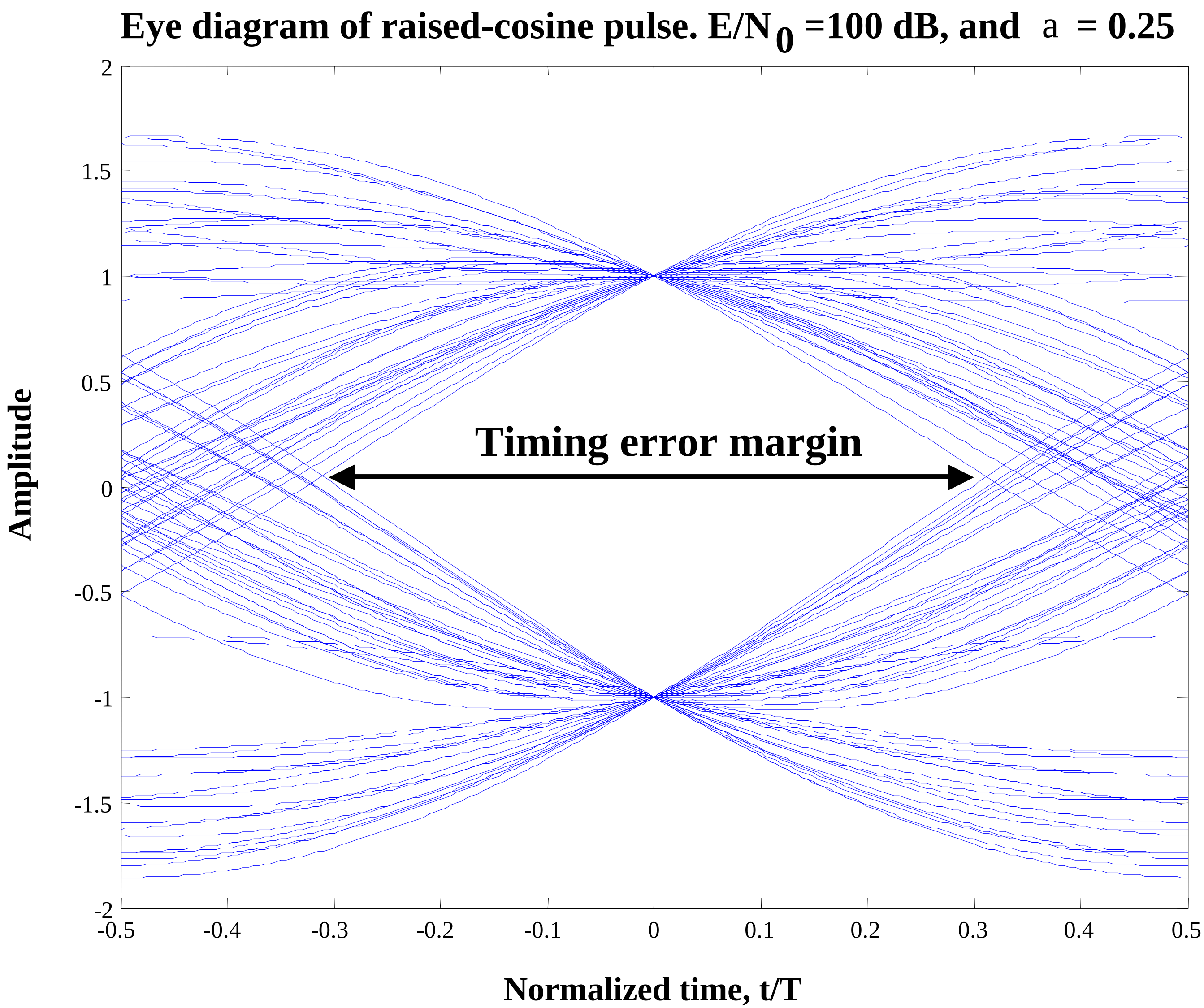
**In a laboratory, the eye diagram is obtained by measuring a long information bit sequence with an oscilloscope triggered at the symbol rate 1/T (or a small sub-multiple of it) and sufficient persistency**


$y_{00}(t) = x(t) + x(t-T)$
+1
t
-2T
-T
T
2T
3T
$y_{11}(t) = -x(t) - x(t-T)$
t
-2T
-T
T
2T
3T
-1
$y_{01}(t) = x(t) - x(t-T)$
+1
t
-2T
-T
T
2T
3T
-1
$y_{10}(t) = -x(t) + x(t-T)$
+1
t
-2T
-T
T
2T
3T
-1
+1
t
-T
T
2T
-1

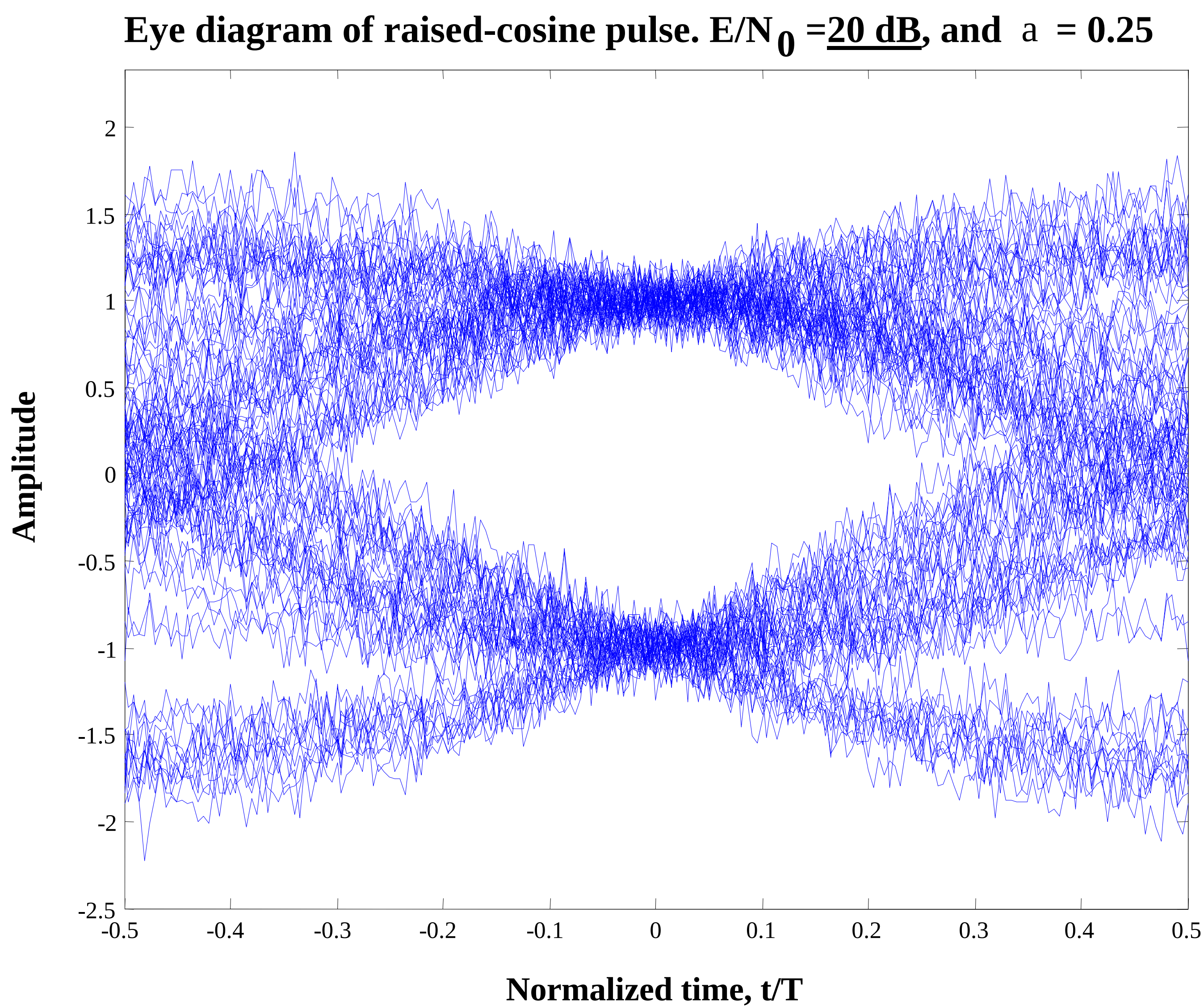
# Eye diagram



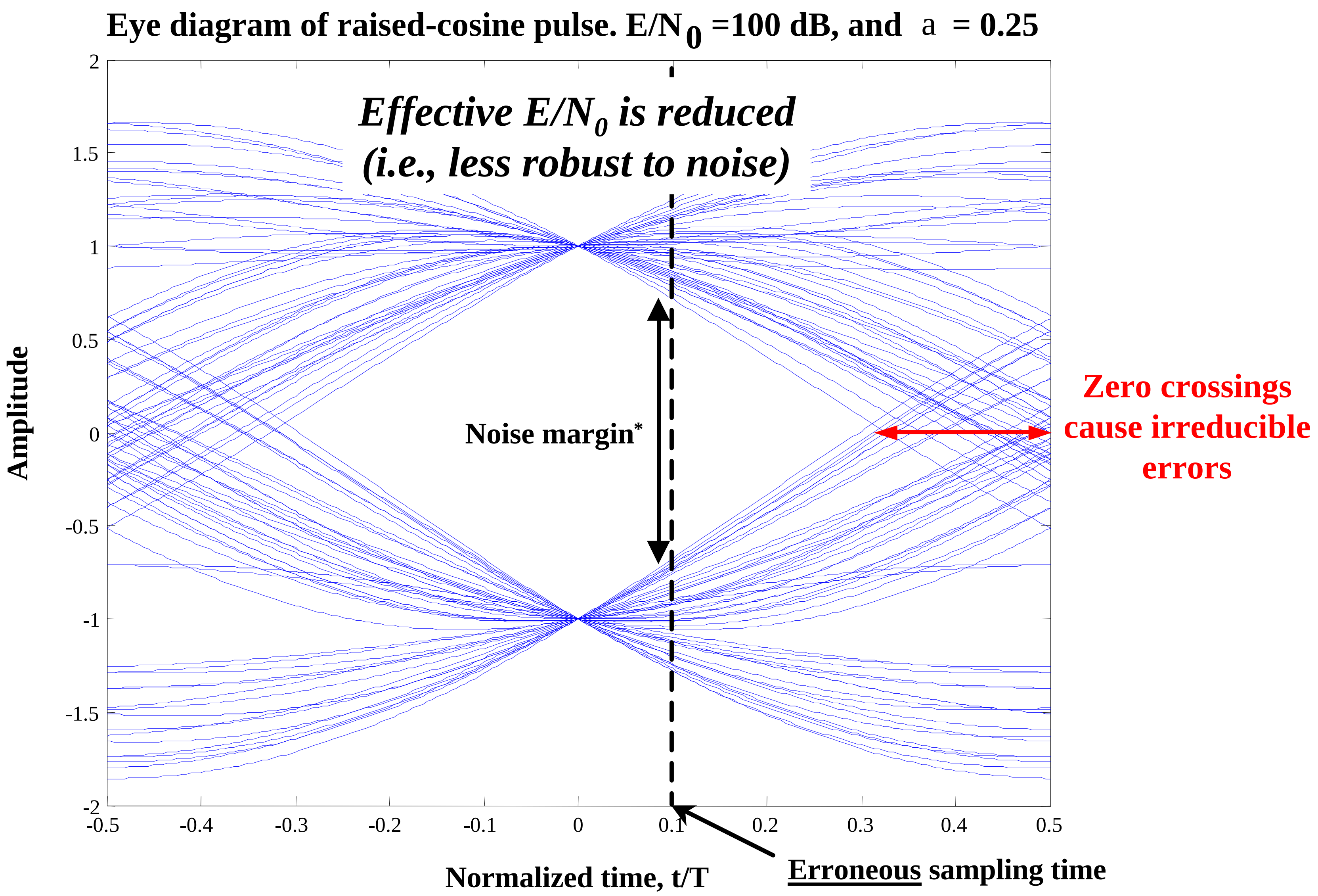
# Matlab simulation: BPSK example



# Matlab simulation: BPSK with AWGN



# Matlab simulation: BPSK with a timing error



# Matlab simulation: BPSK with AWGN and timing error