# MATCHED FILTER

EE161 S'12

2/14/12

## Matched filter (p. 398 of textbook)

A filter whose impulse response is given by

$$\boxed{h(t) = s(T-t)},$$

→ where $s(t)$ is confined to the interval $(0, T)$ (assumed to be) ← ??

is called the matched filter to the signal $s(t)$.

Response of the MF to $s(t)$:

$$y(t) = \int_0^t s(\tau)\, h(t-\tau)\, d\tau \quad \left(= s(t) * h(t)\right)$$

$$= \int_0^t s(\tau)\, s(T-(t-\tau))\, d\tau$$

at $t = T$:

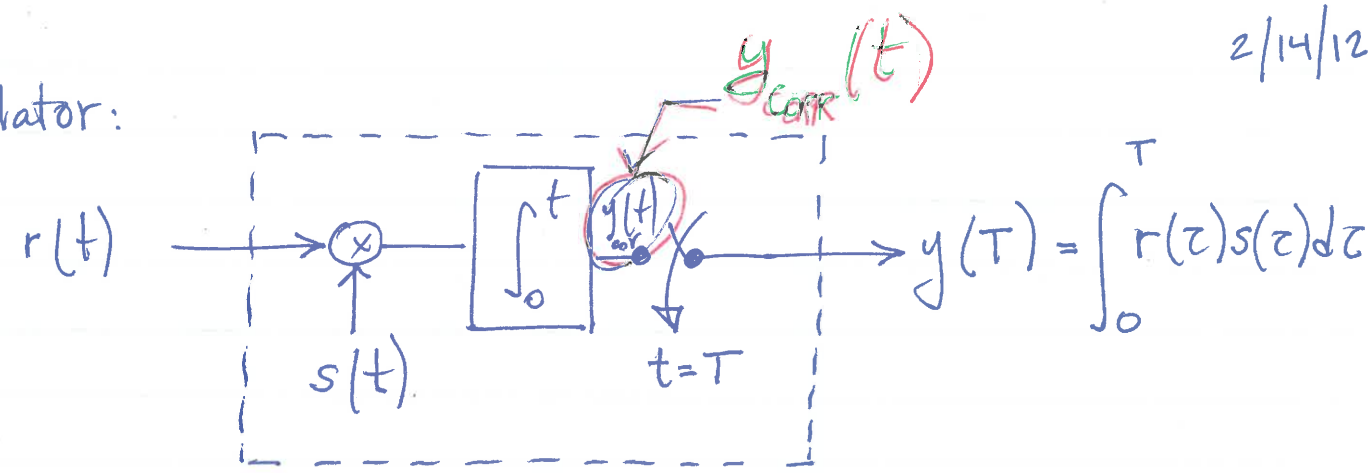
$$y(T) = \int_0^T s^2(\tau)\, d\tau = E_s$$

the autocorrelation (energy) of $s(t)$.

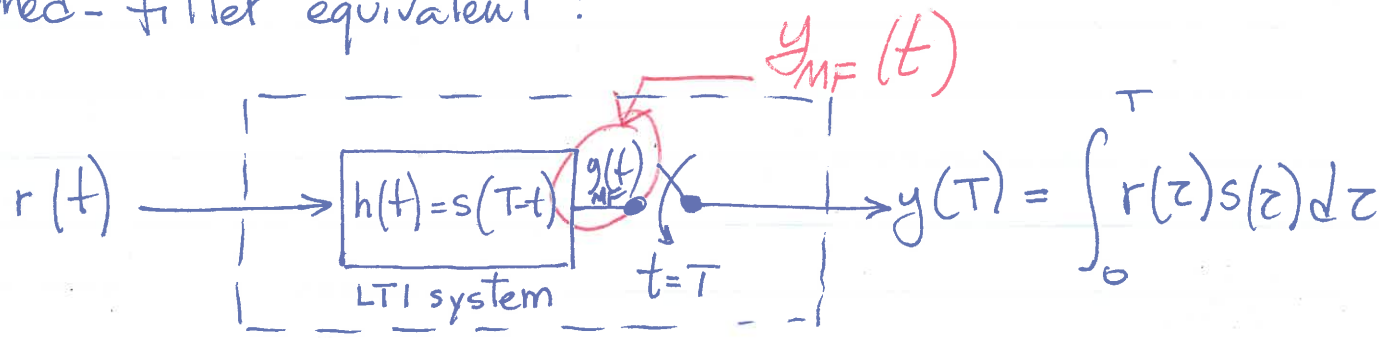
Same output as a correlator.

2/14/12

Correlator:

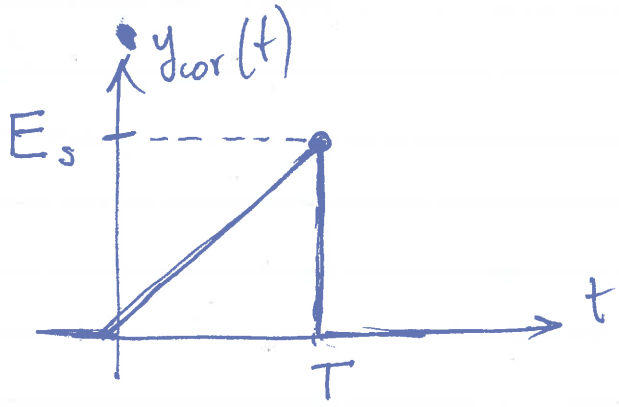



Matched-filter equivalent:

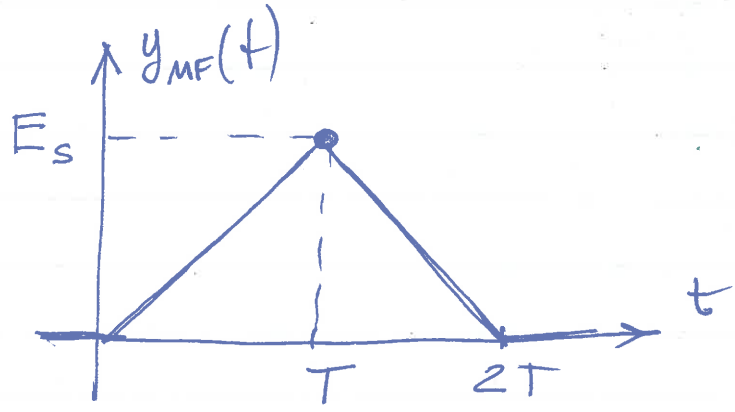



Their result at $t=T$ is the same, even though responses $y(t)$ are different.

Example 1: NRZ pulse. (Input $s(t)$ same as reference $s(t)$.)




Correlator

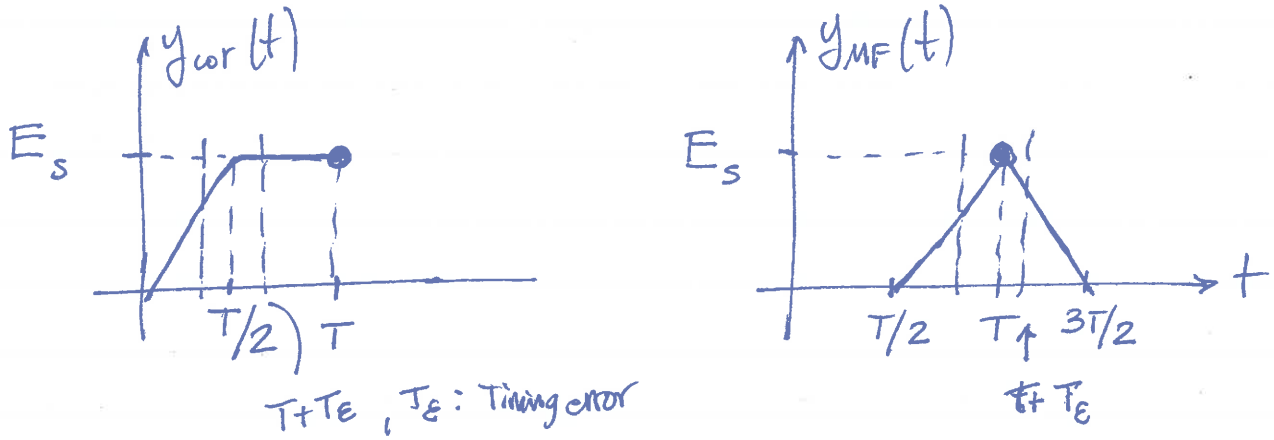



Matched filter

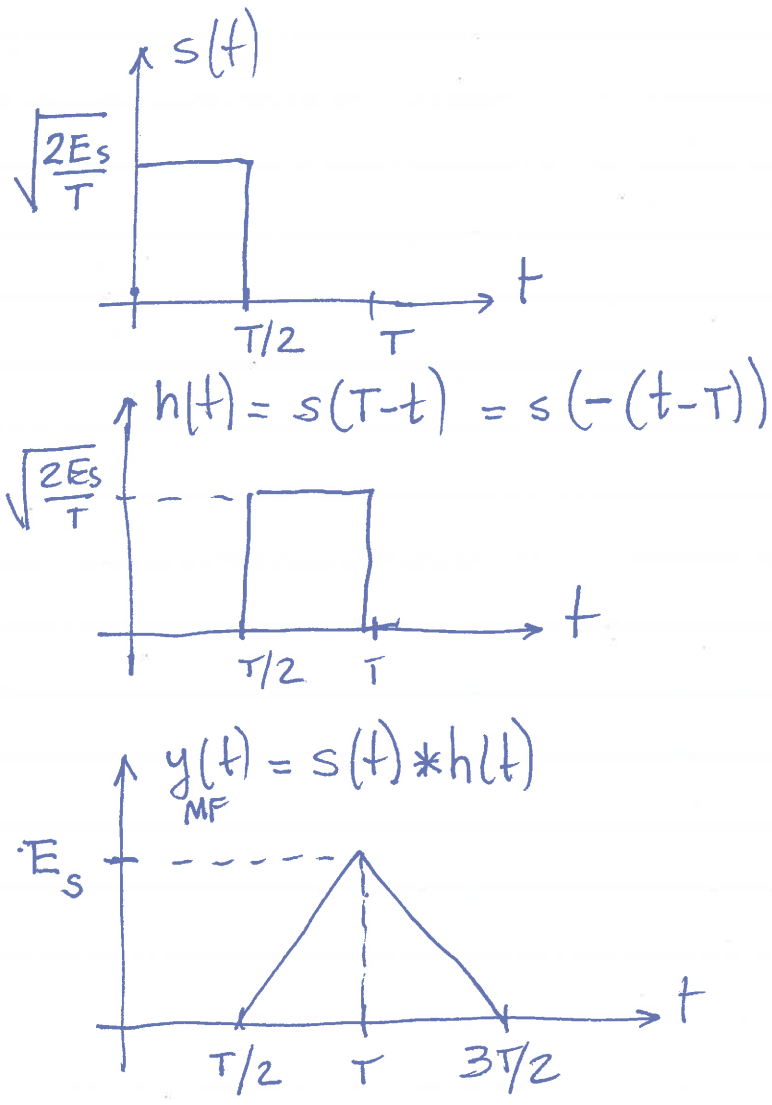
2/14/12

Generally:
MF is less sensitive to timing errors.

Example 2: RZ pulse




Note:



2/14/12

Properties of the matched filter:

(1) Energy detector

$$y(T) = [s(t) * h(t)]_{t=T} = E_s$$

where $E_s = \int_0^T s^2(\tau)\,d\tau$, is the energy of $s(t)$.

(2) Maximizes the output signal-to-noise ratio (SNR).

$$\left(\frac{S}{N}\right)_o = \frac{2E_s}{N_o}$$

Maximum output SNR of an MF.

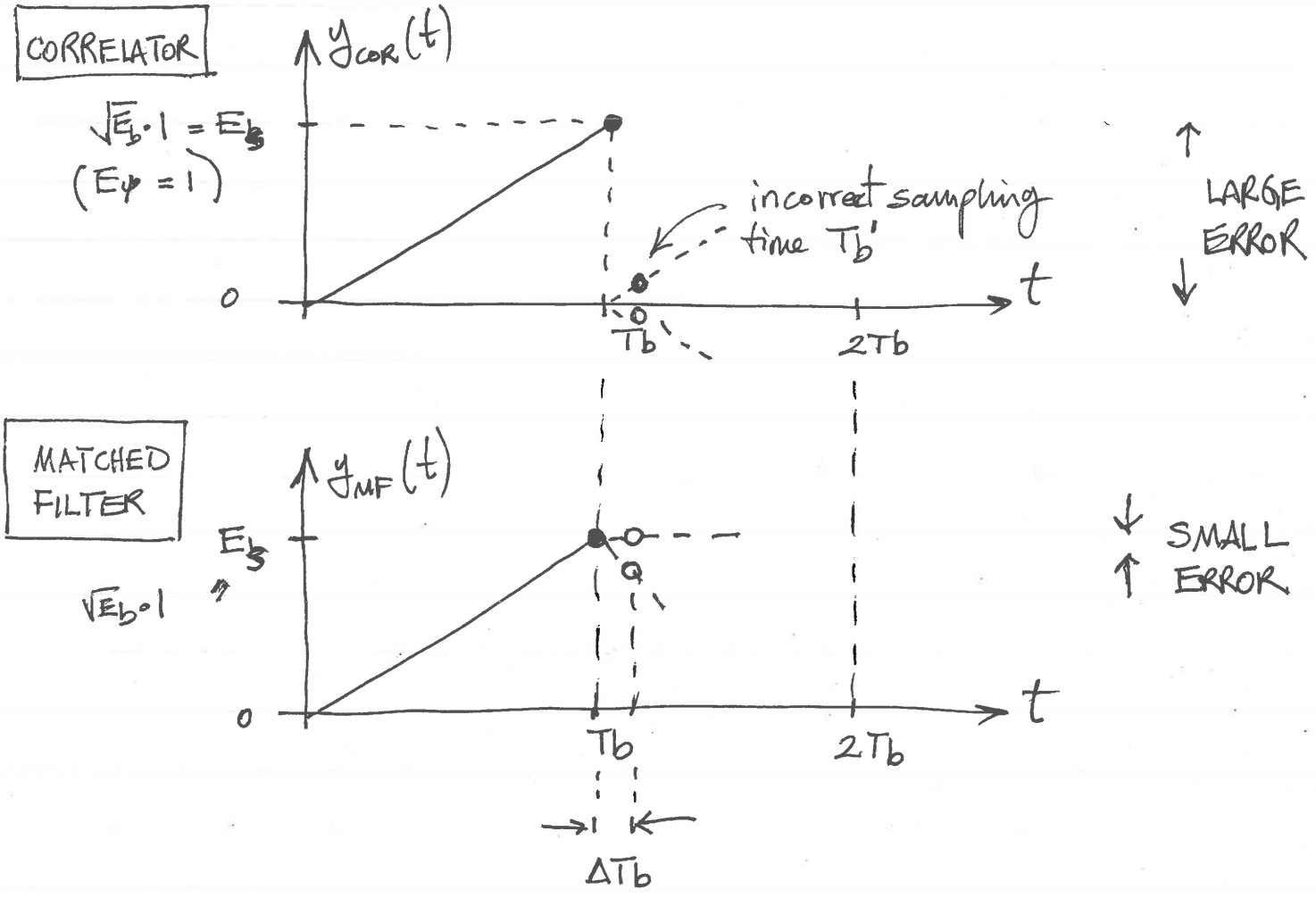
where $\frac{N_o}{2} = \sigma^2$ is the variance of the AWGN sample.

* Correlator : Only works for finite-duration pulses

MF : Works for any energy-type signal!!

# Matched filter implementation advantage

Robust against timing errors. For example, one bit sent by NRZ pulse. (polar mapping)




$T_b' = T_b + \Delta T_b$ sampling time at <u>receiver</u>.

$\Delta T_b$ : Timing error due to <u>different clocks</u>
(one at transmitter and one at receiver)