Cyclic-prefix OFDM (CP-OFDM)

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The circulant channel matrix, *H*

- Received samples with cyclic prefix : $\overline{r} = \overline{x}\,H,$ where H is a *circulant matrix*.
- <u>MATLAB example</u>: $K = 8, \nu = 4$

```
c = [1 2 3 4 0 0 0 0];
H = gallery('circul',c)


H =
    1    2    3    4    0    0    0    0
    0    1    2    3    4    0    0    0
    0    0    1    2    3    4    0    0
    0    0    0    1    2    3    4    0
    0    0    0    0    1    2    3    4
    4    0    0    0    0    1    2    3
    3    4    0    0    0    0    1    2
    2    3    4    0    0    0    0    1
```

Singular value decomposition (SVD)

- The channel matrix can be "decomposed" as $\boxed{H = Q\Lambda Q^*}$
- DFT matrix[1]: $Q = \left(1/\sqrt{K}\right)\left[e^{-j(2\pi/K)nk}\right], 0 \le n,k \le K-1$
- Inverse DFT matrix: $Q^* = \left(1/\sqrt{K}\right)\left[e^{j(2\pi/K)nk}\right], 0 \le n,k \le K-1$

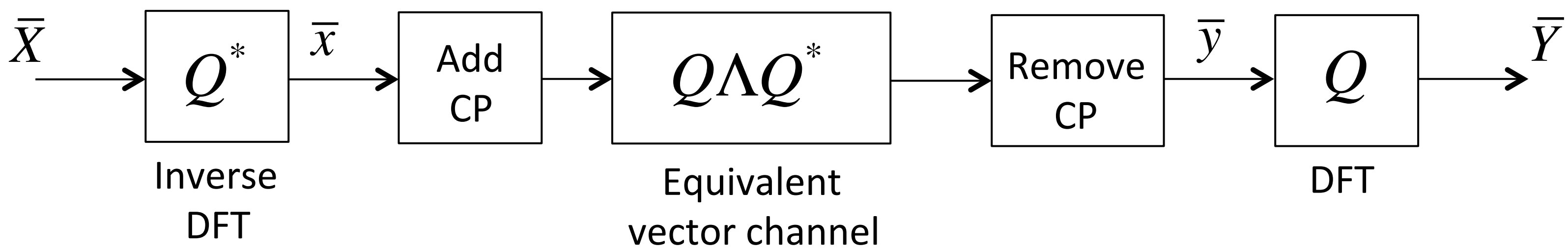
such that $QQ^* = Q^*Q = I_K,\ \Lambda = \mathrm{diag}\begin{pmatrix} C_0 & C_1 & \cdots & C_{K-1} \end{pmatrix},$

$$\begin{pmatrix} C_0 & C_1 & \cdots & C_{K-1} \end{pmatrix} = \begin{pmatrix} c_0 & c_1 & \cdots & c_{K-1} \end{pmatrix} Q$$

is the DFT of the *channel impulse response* (a vector of length K, obtained by appending $K - \nu$ zeros)

[1] DFT: Discrete Fourier transform

Equivalent vector channel of CP-OFDM



Since $QQ^* = Q^*Q = I_K$, ***K* parallel channels are created!**

$$\boxed{\bar{Y} = \Lambda \bar{X}, \quad Y_k = C_k X_k, \; 0 \le k \le K-1}$$

where $C_k = \left[C\left(f \right) \right]_{f=k/T}$