Optimal Control Squatting Problems

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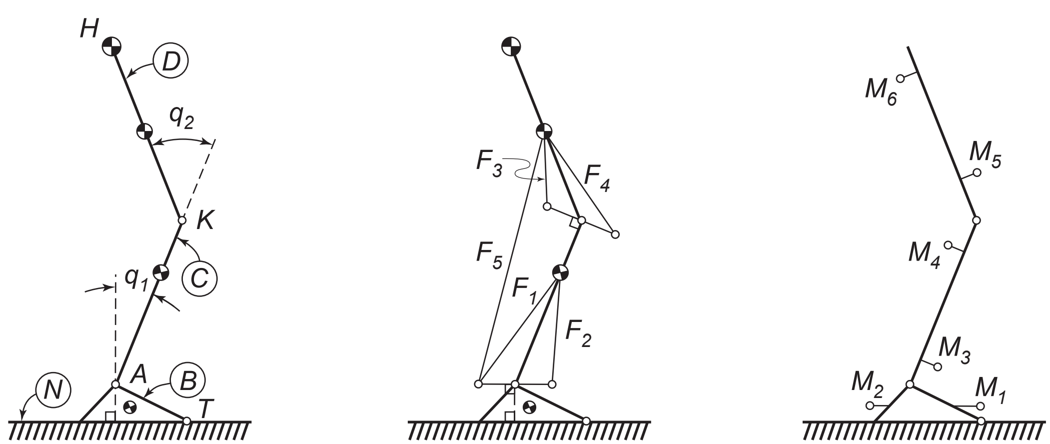
Original Squatting Model

Computational Biomechanics Laboratory



Reference Frames, Points, and Generalized Coordinates | Muscle Forces and Lines of Action | Surface Marker Locations

Original Optimal Control Problem

$$\min \sum e_i^2$$

subject to

Dynamic constraints:

$$\frac{da}{dt} = f_1(e,a) \qquad \frac{du}{dt} = f_2(a,q,u)$$

$$\frac{dq}{dt} = u$$

Bound constraints:

$$0 \le e \le 1$$

$$0 \le a \le 1$$

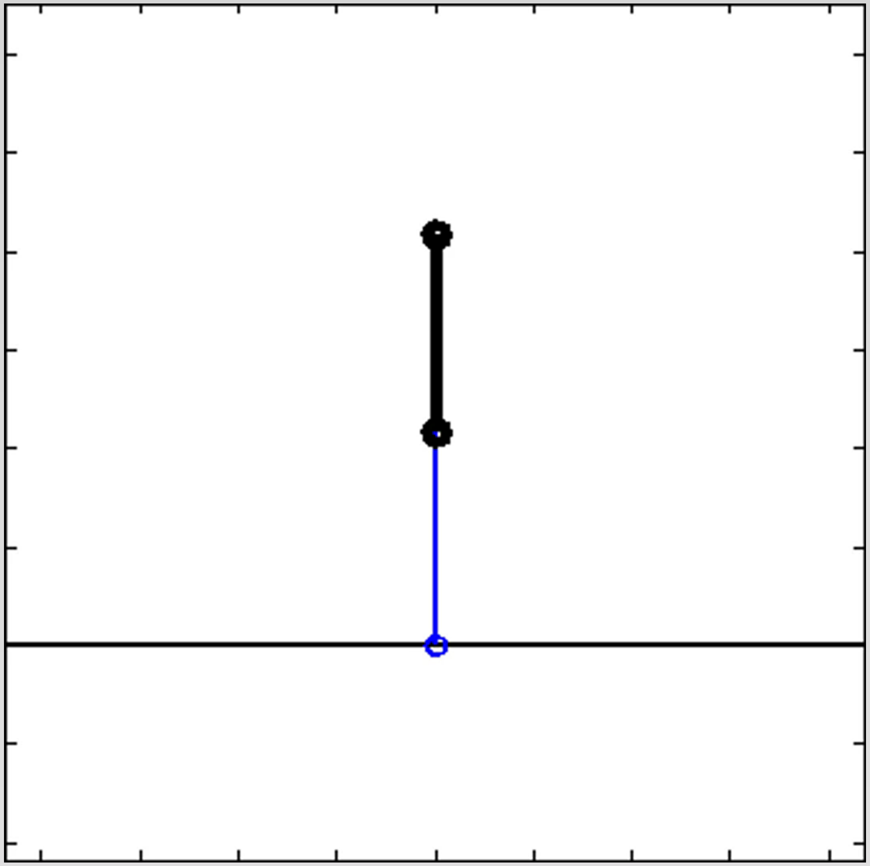
Terminal constraints:

$$t = t_f / 2 : q_1 = 30^\circ \ , q_2 = 60^\circ$$

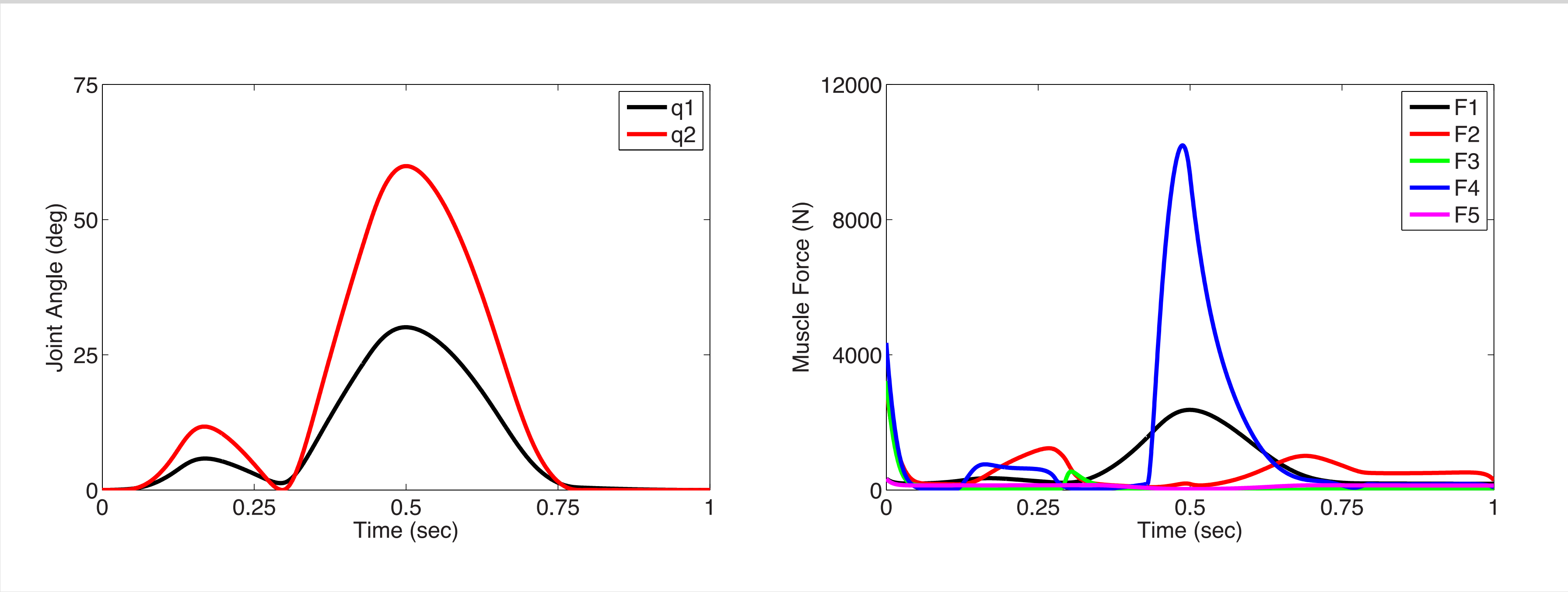
$$t = t_f : q_1 = 0^\circ, q_2 = 0^\circ$$

Two phases

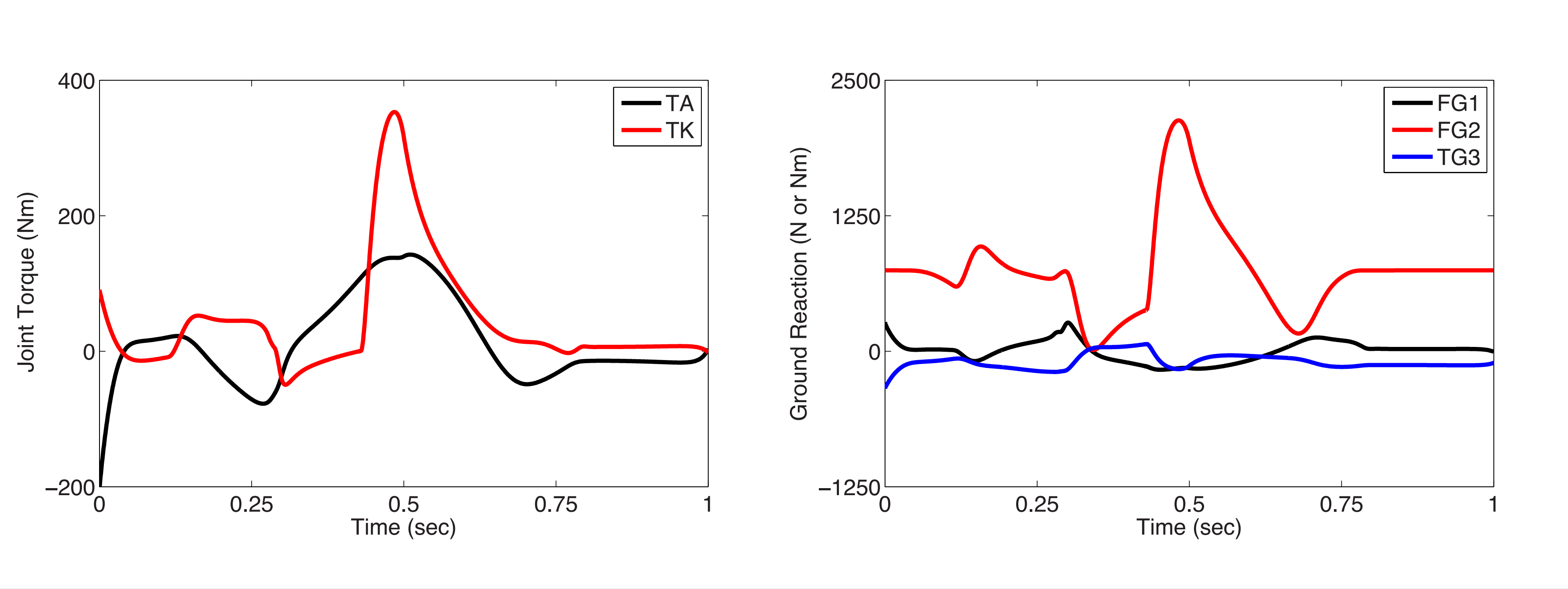
Original Optimal Control Solution



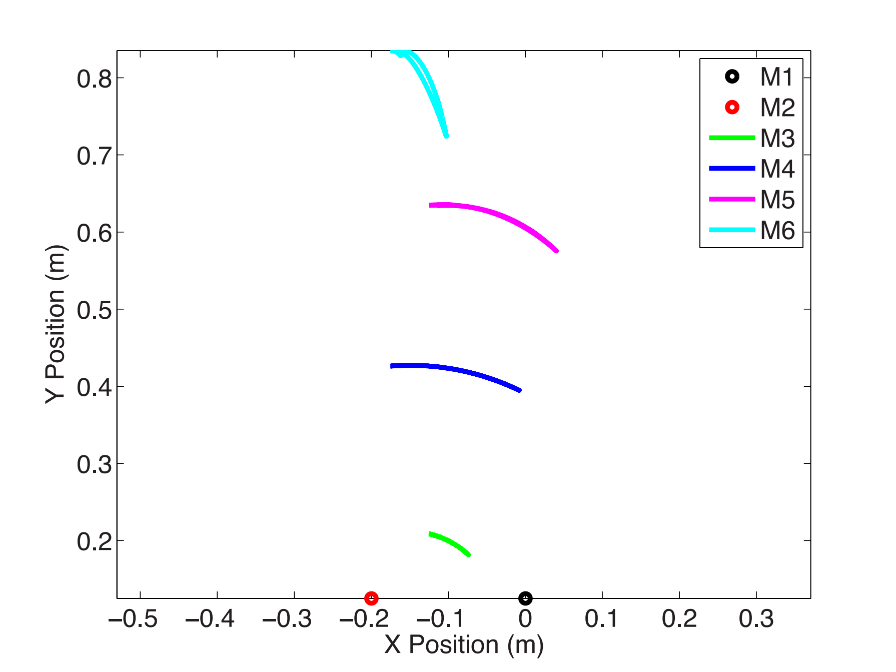
Original Optimal Control Solution



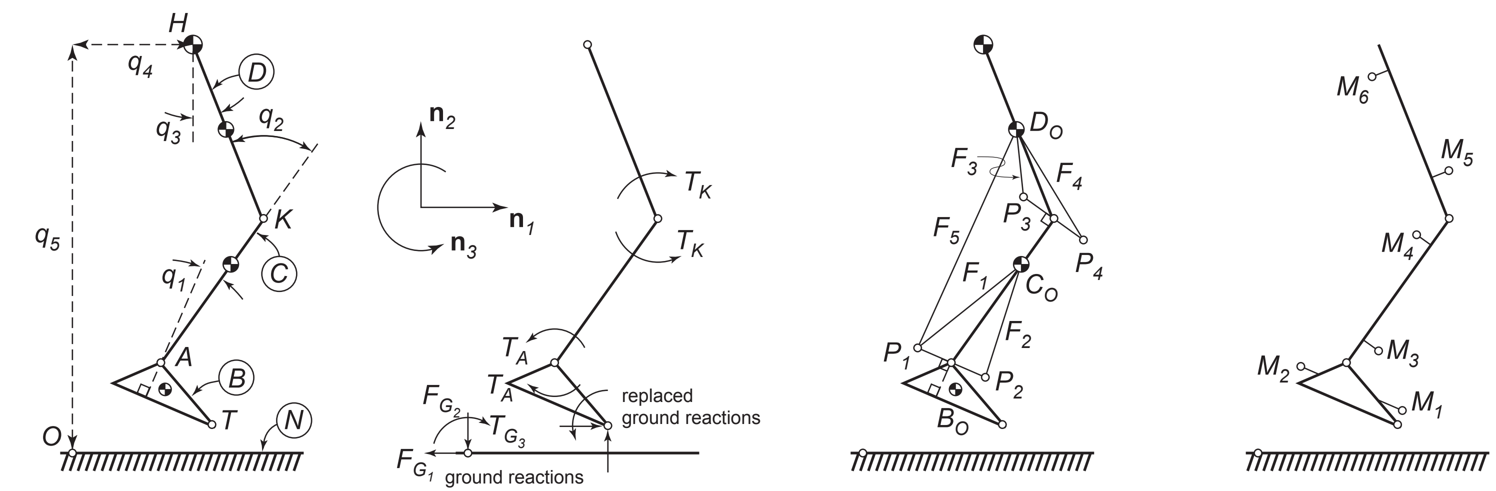
Original Optimal Control Solution



Original Optimal Control Solution



Expanded Squatting Model



Reference Frames, Points, and Generalized Coordinates

Net Forces and Torques

Muscle Forces and Lines of Action

Surface Marker Locations

Benchmark Optimal Control Problems

Computational Biomechanics Laboratory

Benchmark Problem 1: Dynamic muscle force estimation

- Dynamics: Activation and contraction
- Controls: Muscle excitations, normalized muscle velocity, reserve torques
- Inputs: Joint torques, muscle-tendon kinematics
- Cost function: Minimize effort and reserve torques

Benchmark Problem 2: Dynamically consistent IK analysis

- Dynamics: Skeletal
- Controls: Joint torques
- Inputs: Marker positions, ground reactions
- Cost function: Minimize marker tracking errors

Benchmark Problem 1

$$\min \sum e_i^2 + cT_{Res_j}^2$$

subject to

Dynamic constraints:

$$\frac{da}{dt} = f_1(e,a)$$

$$\frac{d\tilde{\ell}^M}{dt} = w \cdot \frac{v_{\max}^M}{\ell_o^M} \text{ (i.e., } w = \tilde{v}^M\text{)}$$

Bound constraints:

$$0 \le e \le 1$$

$$0 \le a \le 1$$

$$-1 \le w \le 1$$

Path constraints:

$$T_{Exp_j} - (T_{Mod_j} + T_{Res_j}) = 0$$

$$f_3(w, \tilde{\ell}^M, a) = 0$$

Benchmark Problem 2

$$\min \sum (m_{xExp_i} - m_{xMod_i})^2 + (m_{yExp_i} - m_{yMod_i})^2$$

subject to

Dynamic constraints:

$$\frac{du}{dt} = f_2(T_j, q, u)$$

$$\frac{dq}{dt} = u$$