

Optimal Control Problems

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June&July, 2019

REU Summer 2019

Outline

Introduction

Examples

Literature

- ▶ Books:
Liberzon [Lib12] (online preview and free preliminary PDF copy are available here
<http://liberzon.csl.illinois.edu/publications.html>
(accessed June 20, 2019)),
Macki and Strauss [MS82],
Betts [Bet10], classic text by Bryson and Ho [BH75], Gerds [Ger12].
- ▶ Articles that focus in the history of optimal control: [PB94], [PP09], [PP12], [Sar00], [SW97].
- ▶ Software:
Pyomo [HLW⁺17] Pyomo.DAE [NSW⁺18]

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Additional examples can be found e.g., in

- ▶ Betts' book [Bet10],
- ▶ the example section of the GPOPS-II web-page <http://www.gpops2.com> (accessed June 20, 2019),
- ▶ Vanderbei's article [Van01],
- ▶ the Pyomo.DAE [NSW⁺18] example folder <https://github.com/Pyomo/pyomo/tree/master/examples/dae> (accessed June 20, 2019),
- ▶ and the books by Seierstad and Sydsæter [St87], Sethi [Set19], Weber [Web11] with focus on economic applications.

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