UNIVERSIDADE FEDERAL DO CEARÁ
CENTRO DE TECNOLOGIA
DEPARTAMENTO DE ENGENHARIA DE TELEINFORMÁTICA

Atividade 02 Cálculo Numérico

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Q1

a) $\begin{cases} 20x_1 + 7x_2 + 9x_3 = 16 \\ 7x_1 + 30x_2 + 8x_3 = 38 \\ 9x_1 + 8x_2 + 30x_2 = 38 \end{cases}$

$$\left[\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 7 & 30 & 8 & 38 \\ 9 & 8 & 30 & 38 \end{array}\right]$$

$L_2 = L_2 - \frac{7}{20} L_1$

$$\left(\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & \frac{551}{20} & \frac{97}{20} & \frac{162}{5} \\ 9 & 8 & 30 & 38 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & \frac{551}{20} & \frac{97}{20} & \frac{162}{5} \\ 0 & \frac{97}{20} & \frac{519}{20} & \frac{154}{5} \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & \frac{551}{20} & \frac{97}{20} & \frac{162}{5} \\ 0 & 0 & \frac{13828}{551} & \frac{13828}{551} \end{array}\right)$$

$L_3 = L_3 - \frac{9}{20} L_3$ $\qquad$ $L_3 = L_3 - \left(\frac{97}{551}\right) L_2$

$$\begin{cases} 20x_1 + 7x_2 + 9x_3 = 16 \\ \frac{551}{20} x_2 + \frac{97}{20} x_3 = \frac{162}{5} \\ \frac{13828}{551} x_3 = \frac{13828}{551} \end{cases}$$

$\underline{x_3 = 1}$

$\Rightarrow \frac{551}{20} x_2 + \frac{97}{20} = \frac{162}{5}$

$\frac{551}{20} x_2 = \frac{551}{20} \Rightarrow \underline{x_2 = 1}$

$20x_1 + 7 + 9 = 16$

$\underline{x_1 = 0}$

Solução: $x = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Q1

b) $\left(\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 7 & 30 & 8 & 38 \\ 9 & 8 & 30 & 38 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & \frac{551}{20} & \frac{97}{20} & \frac{162}{5} \\ 9 & 8 & 30 & 38 \end{array}\right) = \left(\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & \frac{551}{20} & \frac{97}{20} & \frac{162}{5} \\ 0 & \frac{97}{20} & \frac{519}{20} & \frac{154}{5} \end{array}\right)$

$L_2 = L_2 - \frac{7}{20} L_1$ $\quad$ $L_3 = L_3 = \frac{9}{20} L_3$ $\quad$ $L_3 = L_3 - \frac{97}{551} L_2$

$$\left(\begin{array}{ccc|c} 20 & 7 & 9 & 16 \\ 0 & \frac{551}{20} & \frac{97}{20} & \frac{162}{5} \\ 0 & 0 & \frac{13828}{551} & \frac{13828}{551} \end{array}\right) \quad \begin{cases} 20x_1 + 7x_2 + 9x_3 = 16 \\ \frac{551}{20} x_2 + \frac{97}{20} x_3 = \frac{162}{5} \\ \frac{13828}{551} x_3 = \frac{13828}{551} \end{cases}$$

Como os pivôs não precisaram ser trocados, a resolução ficou igual o item a).

solução : $x = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Q2

a) $\begin{pmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{pmatrix} \Rightarrow \begin{pmatrix} 20 & 7 & 9 \\ 0 & \frac{551}{20} & \frac{97}{20} \\ 0 & \frac{97}{20} & \frac{519}{20} \end{pmatrix} \Rightarrow \begin{pmatrix} 20 & 7 & 9 \\ 0 & \frac{551}{20} & \frac{97}{20} \\ 0 & 0 & \frac{13828}{551} \end{pmatrix}$

$L_2 = L_2 - \frac{7}{20} L_1$

$L_3 = L_3 - \frac{9}{20} L_1$

$L_3 = L_3 - \frac{97}{551} L_2$

matriz U

matriz $L = \begin{pmatrix} 1 & 0 & 0 \\ 7/20 & 1 & 0 \\ 9/20 & \frac{97}{551} & 1 \end{pmatrix}$

I) $Ly = b$ $\begin{cases} y_1 = 16 \\ \frac{7}{20} y_1 + y_2 = 38 \\ \frac{9}{20} y_1 + \frac{97}{551} y_2 + y_3 = 38 \end{cases} \Rightarrow$

$\underline{y_1 = 16}$

$\frac{7}{20}(16) + y_2 = 38$

$\frac{112}{20} + y_2 = 38$

$y_2 = 38 - \frac{112}{20}$

$\underline{y_2 = \frac{162}{5}}$

$\frac{9}{20}(16) + \frac{97}{551}\left(\frac{162}{5}\right) + y_3 = 38$

$y_3 = 38 - \frac{72}{10} + \frac{15714}{2755} \Rightarrow \underline{y_3 = \frac{13828}{551}}$

II) $Ux = y$ $\begin{cases} 20x_1 + 7x_2 + 9x_3 = 16 \\ \frac{551}{20} x_2 + 97/20\, x_3 = 162/5 \\ \frac{13828}{551} x_3 = \frac{13828}{551} \end{cases} \Rightarrow$

$\underline{x_3 = 1}$

$x_2 = \left(\frac{162}{5} - \frac{97}{20}\right)\frac{20}{551}$

$x_2 = 1$

$x_1 = \frac{16 - 7 - 9}{20}$

$\underline{x_1 = 0}$

solução $x = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(23) b)

$$\left(\begin{array}{ccc|ccc} 20 & 7 & 9 & 1 & 0 & 0 \\ 7 & 30 & 8 & 0 & 1 & 0 \\ 9 & 8 & 30 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 7 & 30 & 8 & 0 & 1 & 0 \\ 9 & 8 & 30 & 0 & 0 & 1 \end{array}\right)$$

$L_1 = L_1/20$

$L_2 = L_2 - 7L_1$

$$\left(\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & \frac{551}{20} & \frac{97}{20} & \frac{-7}{20} & 1 & 0 \\ 9 & 8 & 30 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & \frac{551}{20} & 97/20 & -7/20 & 1 & 0 \\ 0 & 97/20 & 519/20 & -9/20 & 0 & 1 \end{array}\right)$$

$L_3 = L_3 - 9L_1$

$L_2 = L_2/(551/20)$

$$\left(\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & 1 & 97/551 & -7/551 & \frac{20}{551} & 0 \\ 0 & 97/20 & \frac{519}{20} & -9/20 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & 1 & 97/551 & -7/551 & \frac{20}{551} & 0 \\ 0 & 0 & \frac{13828}{551} & \frac{-214}{551} & \frac{-97}{551} & 1 \end{array}\right)$$

$L_3 = L_3 - \frac{97}{20} L_2$

$L_3 = L_3 / \left(\frac{13828}{551}\right)$

$$\left(\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & 1 & 97/551 & \frac{-7}{551} & \frac{20}{551} & 0 \\ 0 & 0 & 1 & \frac{-107}{6914} & \frac{-97}{13828} & \frac{551}{13828} \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 7/20 & 9/20 & 1/20 & 0 & 0 \\ 0 & 1 & 0 & \frac{-69}{6914} & \frac{519}{13828} & \frac{-97}{13828} \\ 0 & 0 & 1 & \frac{-107}{6914} & \frac{-97}{13828} & \frac{551}{13828} \end{array}\right)$$

$L_2 = L_2 - \left(\frac{97}{551}\right) L_3$

$L_1 = L_1 - \frac{9}{20} L_3$

$$\left(\begin{array}{ccc|ccc} 1 & 7/20 & 0 & \frac{7877}{138280} & \frac{873}{276560} & \frac{-4959}{276560} \\ 0 & 1 & 0 & \frac{-69}{6914} & \frac{519}{13828} & \frac{-97}{13828} \\ 0 & 0 & 1 & \frac{-107}{6914} & \frac{-97}{13828} & \frac{551}{13828} \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{209}{3457} & \frac{-69}{6914} & \frac{-107}{6914} \\ 0 & 1 & 0 & \frac{-69}{6914} & \frac{519}{13828} & \frac{-97}{13828} \\ 0 & 0 & 1 & \frac{-107}{6914} & \frac{-97}{13828} & \frac{551}{13828} \end{array}\right)$$

$L_1 = L_1 - \frac{7}{20}(L_2)$

Q3 a)

$$\begin{pmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{pmatrix} \Rightarrow \begin{pmatrix} 20 & 7 & 9 \\ 0 & \frac{551}{20} & 97/20 \\ 9 & 8 & 30 \end{pmatrix} \Rightarrow \begin{pmatrix} 20 & 7 & 9 \\ 0 & \frac{551}{20} & 97/20 \\ 0 & 97/20 & 519/20 \end{pmatrix}$$

$L_2 = L_2 - \frac{7}{20} L_1$ $\qquad L_3 = L_3 - \frac{9}{20} L_3$ $\qquad L_3 = L_3 - \left(\frac{97}{551}\right) L_2$

$$A = \begin{pmatrix} 20 & 7 & 9 \\ 0 & \frac{551}{20} & 97/20 \\ 0 & 0 & \frac{13828}{551} \end{pmatrix} \qquad \det A = 20 \cdot \frac{551}{20} \cdot \frac{13828}{551}$$

$$\underline{\det A = 13828}$$

Q4 a) $\begin{cases} 10d_1 + 2d_2 + 2d_3 = 28 \\ d_1 + 10d_2 + 2d_3 = 7 \\ 2d_1 - 7d_2 - 10d_3 = -17 \end{cases}$

$10 > 2 + 2$ ✓
$10 > 1 + 2$ ✓
$-7 > -10 + 2$ ✓

logo o critério de linhas é satisfeito

b) $\begin{bmatrix} 10 & 2 & 2 \\ 1 & 10 & 2 \\ 2 & -7 & -10 \end{bmatrix}$ $\beta_1 = \dfrac{2+2}{10} = 0,4$; $\beta_2 = \dfrac{1 \cdot 0,4 + 2}{10} = 0,24$

$\beta_3 = \dfrac{2 \cdot 0,4 + (-7) \cdot 0,24}{-10} = 0,088$

Como $\beta = 0,4 < 1$ o sistema irá convergir.

Q4 c) $\begin{bmatrix} 10 & 2 & 2 \\ 1 & 10 & 2 \\ 2 & -7 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 28 \\ 7 \\ -17 \end{bmatrix}$ $\quad \varepsilon = 5 \cdot 10^{-1} = 0.5$

$x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$x_1 = \frac{1}{10}\left(-2x_2 - 2x_3 + 28\right)$$
$$x_2 = \frac{1}{10}\left(-x_1 - 2x_3 + 7\right)$$
$$x_3 = -\frac{1}{10}\left(-2x_1 + 7x_2 - 17\right)$$

partindo de $x^{(0)}$ temos:

$$x_1 = \frac{1}{10}\left(-2(0) - 2(0) + 28\right)$$
$$x_2 = \frac{1}{10}\left(-(0) - 2(0) + 7\right)$$
$$x_3 = -\frac{1}{10}\left(-2(0) + 7(0) - 27\right)$$

$$x_1 = 2{,}8$$
$$x_2 = 0{,}7$$
$$x_3 = 2{,}7$$

$\therefore \quad x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ e $x^{(1)} = \begin{pmatrix} 2{,}8 \\ 0{,}7 \\ 2{,}7 \end{pmatrix}$

$$|x_1^{(1)} - x_1^{(0)}| = 2{,}8$$
$$|x_2^{(1)} - x_2^{(0)}| = 0{,}7$$
$$|x_3^{(1)} - x_3^{(0)}| = 2{,}7$$

$$d_r = \frac{2{,}8}{2{,}8} = 1 > \varepsilon$$

iteração 2:

$$x_1^{(2)} = \frac{1}{10}\left(-2(0{,}7) - 2(2{,}7) + 28\right) = 2{,}12$$
$$x_2^{(2)} = \frac{1}{10}\left(-(2{,}8) - 2(2{,}7) + 7\right) = -0{,}12$$
$$x_3^{(2)} = -\frac{1}{10}\left(-2(2{,}8) + 7(0{,}7) - 17\right) = 1{,}77$$

$$|x_1^{(2)} - x_1^{(1)}| = 0{,}68$$
$$|x_2^{(2)} - x_2^{(1)}| = 0{,}58$$
$$|x_3^{(2)} - x_3^{(1)}| = 0{,}93$$

$$d_r = \frac{0{,}93}{2{,}12} = 0{,}44 < \varepsilon$$

logo a solução é $x^{(2)} = \begin{pmatrix} 2{,}12 \\ -0{,}12 \\ 1{,}77 \end{pmatrix}$

Jogador 2 ganhou

(Q4) d) $\begin{bmatrix} 10 & 2 & 2 \\ 1 & 10 & 2 \\ 2 & -7 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 28 \\ 7 \\ -17 \end{bmatrix}$ $\quad x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\varepsilon = 0.5$

$x_1 = \frac{1}{10}(-2x_2 - 2x_3 + 28)$

$x_2 = \frac{1}{10}(-x_1 - 2x_3 + 7) \Rightarrow$

$x_3 = -\frac{1}{10}(2x_1 + 7x_2 - 17)$

$x_1^{(1)} = \frac{1}{10}(-2(0) - 2(0) + 28) =$

$x_2^{(1)} = \frac{1}{10}(-28 - 2(0) + 7) =$

$x_3^{(1)} = -\frac{1}{10}(-2(28) + 7(-4) - 17) =$

$=$

$x_1^{(1)} = 2{,}8$

$x_2^{(1)} = -2{,}1$

$x_3^{(1)} = +3{,}73$

$x^{(1)} = \begin{bmatrix} 2{,}8 \\ -2{,}1 \\ 3{,}73 \end{bmatrix}$

$|x^{(1)} - x^{(0)}| = \begin{bmatrix} 2{,}8 \\ -2{,}1 \\ 3{,}73 \end{bmatrix}$

$dr = \frac{3{,}73}{3{,}73} = 1 > \varepsilon$

iteração 2:

$x_1^{(2)} = \frac{1}{10}(-2(2{,}8) - 2(-2{,}1) + 28) = 2{,}66$

$x_2^{(2)} = \frac{1}{10}(-2{,}66 - 2(3{,}73) + 7) = -0{,}312$

$x_3^{(2)} = -\frac{1}{10}(-2(2{,}66) + 7(-0{,}312) - 17) = 2{,}45$

$|x^{(2)} - x^{(1)}| = \begin{bmatrix} 0{,}14 \\ 1{,}79 \\ 1{,}28 \end{bmatrix} \quad \therefore \quad dr = \frac{1{,}79}{2{,}66} = 0{,}67 > \varepsilon$

iteração 3:

$x_1^{(3)} = \frac{1}{10}(-2(2{,}66) - 2(-0{,}312) + 28) = 2{,}33$

$x_2^{(3)} = \frac{1}{10}(-2{,}33 - 2(2{,}45) + 7) = 0{,}44$

$x_3^{(3)} = -\frac{1}{10}(-2(2{,}33) + 7(0{,}44) - 17) = +1{,}8$

$|x^{(3)} - x^{(2)}| = \begin{bmatrix} 0{,}33 \\ 0{,}128 \\ 0{,}52 \end{bmatrix}$

$dr = \frac{0{,}52}{2{,}33} = 0{,}22 < \varepsilon$ logo $x^{(3)}$ é solução

$\boxed{\text{Jogador 2 venceu!}}$