SIT718 Real world Analytics
Assessment Task 3
Total Marks = 100, Weighting - 30%

Your final submission should consist of:

1. "name-report.pdf": A pdf file (created in any word processor) with up to 8 pages, containing the solutions of the questions, labelled with your name;

2. "name-code.R": Two codes combined in one with your R file, labelled with yourname.R, with lp models for Questions 2 and Questions 3.

Your assignment **will not be assessed** if we cannot reproduce your results with your R code.

Reference style: Harvard.

1. A food factory is making a beverage for a customer from mixing two different existing products A and B. The compositions of A and B and prices ($/L) are given as follows,

	Amount (L) in /100 L of A and B			
	Lime	Orange	Mango	Cost ($/L)
A	3	6	4	5
B	8	4	6	6

The customer requires that there must be at least 4.5 Litres (L) Orange and at least 5 Litres of Mango concentrate per 100 Litres of the beverage respectively, but no more than 6 Litres of Lime concentrate per 100 Litres of beverage. The customer needs at least 100 Litres of the beverage per week.

a) Explain why a linear programming model would be suitable for this case study.

[5 marks]

b) Formulate a Linear Programming (LP) model for the factory that minimises the total cost of producing the beverage while satisfying all constraints.

[10 marks]

c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution on the graph. Annotate all lines on your graph. What is the minimal cost for the product?

[10 marks]

Note: you can use graphical solvers available online but make sure that your graph is clear, all variables involved are clearly represented and annotated, and each line is clearly marked and related to the corresponding equation.

d) Is there a range for the cost ($) of A that can be changed without affecting the optimum solution obtained above?

[5 marks]

2. A factory makes three products called Spring, Autumn, and Winter, from three materials containing Cotton, Wool and Silk. The following table provides details on the sales price, production cost and purchase cost per ton of products and materials respectively.

	Sales price	Production cost		Purchase price
Spring	$60	$5	Cotton	$30
Autumn	$55	$4	Wool	$45
Winter	$60	$5	Silk	$50

The maximal demand (in tons) for each product, the minimum cotton and wool proportion in each product is as follows:

	Demand	min Cotton proportion	min Wool proportion
Spring	4800	55%	30%
Autumn	3000	45%	40%
Winter	3500	30%	50%

a) Formulate an LP model for the factory that maximises the profit, while satisfying the demand and the cotton and wool proportion constraints.

[10 Marks]

b) Solve the model using R/R Studio. Find the optimal profit and optimal values of the decision variables.

[10 Marks]

Hints:
1. Let $x_{ij} \geq 0$ be a decision variable that denotes the number of tons of products j for $j \in \{1 = Spring, 2 = Autumn, 3 = Winter\}$ to be produced from Materials $i \in \{C=Cotton, W=Wool, S=Silk\}$.

2. The proportion of a particular type of Material in a particular type of Product can be calculated as:
e.g., the proportion of Cotton in product Spring is given by: $\dfrac{x_{C1}}{x_{C1} + x_{W1} + x_{S1}}$.

3. Helen and David are playing a game by putting chips in two piles (each player has two piles P1 and P2), respectively. Helen has 6 chips and David has 4 chips. Each player places his/her chips in his/her two piles, then compare the number of chips in his/her two piles with that of the other player's two piles. Note that once a chip is placed in one pile it cannot be moved to another pile. There are four comparisons including Helen's P1 vs David's P1, Helen's P1 vs David's P2, Helen's P2 vs David's P1, and Helen's P2 vs David's P2. For each comparison, the player with more chips in the pile will score 1 point (the opponent will lose 1 point). If the number of chips is the same in the two piles, then nobody will score any points from this comparison. The final score of the game is the sum score over the four comparisons. For example, if Helen puts 5 and 1 chips in her P1 and P2, David puts 3 and 1 chips in his P1 and P2, respectively. Then Helen will get 1 (5 vs 3) + 1 (5 vs 1) - 1 (1 vs 3) + 0 (1 vs 1) = 1 as her final score, and David will get his final score of -1.

(a) Give reasons why/how this game can be described as a two-players-zero-sum game.

[5 Marks]

(b) Formulate the payoff matrix for the game.

[5 Marks]

(c) Explain what is a saddle point. Verify: does the game have a saddle point?

[5 Marks]

(d) Construct a linear programming model for each player in this game;

[5 Marks]

(e) Produce an appropriate code to solve the linear programming model in part (c).

[5 Marks]

(f) Solve the game for David using the linear programming model you constructed in part (d). Interpret your solution.

[5 Marks]

[Hint: To record the number of chips in each pile for each player you may use the notation (i, j), where i is the number of chips in P1 and j is the number of chips in P2, for example (2,4) means two chips in P1 and four chips in P2.]

4. Supposing there are three players, each player is given a bag and asked to contribute in his own money with one of the three amount $\{\$0, \$3, \$6\}$. A referee collects all the money from the three bags and then doubles the amount using additional money. Finally, each player share the whole money equally. For example, if both Players 1 and 2 put $0 and Player 3 puts $3, then the referee adds another $3 so that the total becomes $6. After that, each player will obtain $2 at the end. Every player want to maximise his profit, but he does not know the amount contributed from other players. [Hint: profit = money he obtained - money he contributed.]

(a) Compute the profits of each player under all strategy combinations and make the payoff matrix for the three players. [Hint: you can create multiple payoff tables to demonstrate the strategy combinations. The referee is not a player and should not be in the payoff table.]

[**10 Marks**]

(b) Find the Nash equilibrium of this game. What are the profits at this equilibrium? Explain your reason clearly.

[**10 Marks**]