SIT718 - ASSIGNMENT 3

Rajeshkumar Mourya
rmourya@deakin.edu.au
218615876

Question 1

1. Reason for LP

a. We have two decision variables in the case study, Product A and Product B
b. Objective of the case study is to minimize the cost of the beverage, which is mix of product A and B, suggesting additive nature of the A and B and we do not see any other mathematically operations involved i.e. nature of the problem is linear
c. We have conditions under which we need to minimize the cost, i.e. constraints are involved.
d. Considering these points, Linear programming model is suitable to find the minimum cost required for the given constraints.

2. Formulation of LP

1. Let x be Product A variable and y for product B and z is our minimum cost
2. Constraints:
    a. At least 4.5 litres of Orange per 100 litres: $6x + 4y >= 4.5 * 100$
    b. At least 5 litres of Mango per 100 litres: $4x + 6y >= 5 * 100$
    c. At most 6 litres of Lime per 100 litres: $3x + 8y <= 6 * 100$
    d. Minimum 100 litres of beverage per week: $x + y >= 100$
3. Cost per litre for A is 5$ and cost per litre for B is 6$

Minimization function: $100 * z = 5x + 6y$ -> **min $z = 0.05x + 0.06y$**
Constraint 1: **$6x + 4y >= 450$** i.e. a line with **$y = -1.5x + 112.5$**
Constraint 2: **$4x + 6y >= 500$** i.e. a line with **$y = -0.667x + 83.333$**
Constraint 3: **$3x + 8y <= 600$** i.e. a line with **$y = -0.375x + 75$**
Constraint 4: **$x + y >= 100$** i.e. a line with **$y = -x + 100$**
$x>= 0$ and $y >=0$ since amount of concentrate cannot be non-negative

3. Graphical method (zweigmedia.com, 2013)

Enter the linear programming problem here:

- ( ) Maximize
- (•) Minimize

$z =$ 0.05x+0.06y subject to the constraints:

( ) Show only the region defined by the following contraints:

```
6x+4y>=450
4x+6y>=500
3x+8y<=600
x+y>=100
x>=0
y>=0
```

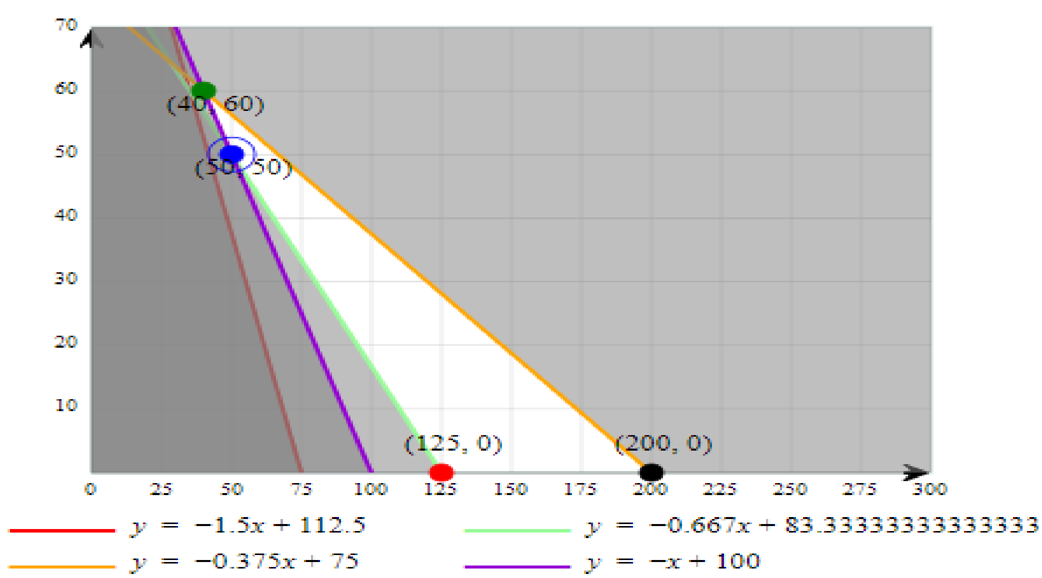
LP Examples | Graphing Examples | Solve

Rounding: 3 decimal places Fraction Mode

Erase Everything

The solution will appear below.

Vertex	Lines through vertex	Value of objective
(50, 50)	$4x + 6y = 500$ $x + y = 100$	5.5 Minimum
(125, 0)	$4x + 6y = 500$ $y = 0$	6.25
(40, 60)	$3x + 8y = 600$ $x + y = 100$	5.6
(200, 0)	$3x + 8y = 600$ $y = 0$	10



Xmin: 0 Xmax: 300

Ymin: 0 Ymax: 70

Gridlines X: 25 Y: 10

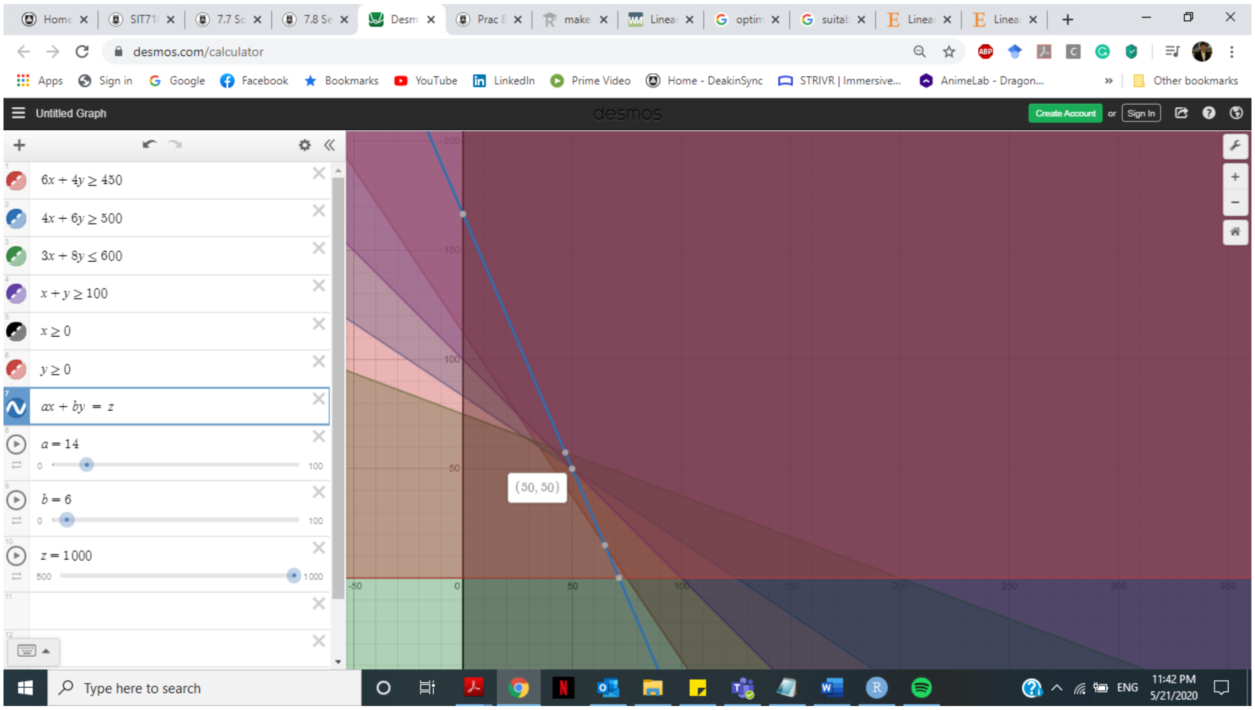
Graph Show vertex coordinates

(Waner, 2013)

optimal solution will be x= 50, y = 50, z = 5.5
Minimum cost per litre will be 5.5 $ or 550 $ per 100 litres

4. Sensitivity analysis

We can have Cost of A varied between 0 $ to 14 $ and still the optimal solution will be (50,50), anything greater than 14 will then disturb the optimal solution



(Desmos, 2020)

Question 2

1. Formulation of LP model

Let **Xij** be the decision variable for number of tons of product **j** where material is **i**, such that

i -> {c = cotton, w = Wool, s = Silk}
j -> {1 = Spring, 2 = Autumn, 3 = Winter}

Type of material

Cotton ->	Xc1 + Xc2 + Xc3
Wool ->	Xw1 + Xw2 + Xw3
Silk ->	Xs1 + Xs2 + Xs3

Type of product

Spring -> $X_{c1} + X_{w1} + X_{s1}$
Autumn -> $X_{c2} + X_{w2} + X_{s2}$
Winter -> $X_{c3} + X_{w3} + X_{s3}$

Sales -> 60 (Spring) + 55 (Autumn) + 60 (Winter)
Production cost -> 5 (Spring) + 4 (Autumn) + 5 (Winter)
Purchase cost -> 30 (Cotton) + 45 (Wool) + 50 (Silk)

Total profit = Sales – Production cost – Purchase cost
Total profit = 60 (Spring) + 55 (Autumn) + 60 (Winter) – [5 (Spring) + 4 (Autumn) + 5 (Winter)]

- [30 (Cotton) + 45 (Wool) + 50 (Silk)]

Total profit = 55(Spring) + 51(Autumn) + 55(Winter) - [30 (Cotton) + 45 (Wool) + 50 (Silk)]

$$\text{Max } Z = 55 (X_{c1} + X_{w1} + X_{s1}) + 51 (X_{c2} + X_{w2} + X_{s2}) + 55 (X_{c3} + X_{w3} + X_{s3}) - 30 (X_{c1}+ X_{c2}+X_{c3}) -45(X_{w1}+X_{w2}+X_{w3}) - 50(X_{s1}+X_{s2}+X_{s3})$$

$$\textbf{Max } z = 25X_{c1} + 21 X_{c2} + 25X_{c3} + 10X_{w1} + 6X_{w2} + 10X_{w3} + 5X_{s1} + X_{s2} + 5X_{s3}$$

Demand Constraints:

Spring: $X_{c1} + X_{w1} + X_{s1} <= 4800$
Autumn: $X_{c2} + X_{w2} + X_{s2} <= 3000$
Winter: $X_{c3} + X_{w3} + X_{s3} <= 3500$

Proportion constraints for Cotton:

Spring: $X_{c1}/(X_{c1}+X_{w1}+X_{s1}) >= 0.55$ ->
$0.45X_{c1} - 0.55X_{w1} - 0.55X_{s1} >= 0$
Autumn: $0.55X_{c2} - 0.45X_{w2} - 0.45X_{s2} >= 0$
Winter: $0.7X_{c3} - 0.3X_{w3} - 0.3X_{s3} >= 0$

Proportion constraints for Wool:

Spring: $-0.3X_{c1} + 0.7X_{w1} - 0.3X_{s1} >= 0$
Autumn: $-0.4X_{c2} + 0.6X_{w2} -0.4X_{s2} >= 0$
Winter: $-0.5X_{c3} + 0.5X_{w3} - 0.5X_{s3} >= 0$

2. Solution

Mapping matrix:

	Index	1	2	3	4	5	6	7	8	9		
	Variable	X										
		c1	c2	c3	w1	w2	w3	s1	s2	s3		
	Obj	25	21	25	10	6	10	5	1	5		
											Operator	**total**
Constraints	1	1			1			1			<=	4800
	2		1			1			1		<=	3000

	3			1			1			1	<=	3500
	4	0.45			-0.55			-0.55			>=	0
	5		0.55			-0.45			-0.45		>=	0
	6			0.7			-0.3			-0.3	>=	0
	7	-0.3			0.7			-0.3			>=	0
	8		-0.4			0.6			-0.4		>=	0
	9			-0.5			0.5			-0.5	>=	0

Optimal Solution:

Z/Max. profit = 204650

Xc1 = 3360 **Xc2 = 1800** **Xc3 = 1750**

Xw1 = 1440 **Xw2 = 1200** **Xw3 = 1750**

Xs1 = 0 **Xs2 = 0** **Xs3 = 0**

The optimal solution values suggest, by eliminating usage of silk in the products, profits can be maximized and proportions constraints for cotton and wool are satisfied.

Question 3

1. Two player – sum – zero game

a. There are only two players involved, Helen and David
b. For payment, if Helen scores 1 point off the comparison of piles, David gets -1, i.e. the player score can be interpreted as Score (Helen) = - Score (David) and the sum of payoffs for Helen and David is zero
c. That's why it can be considered as two player zero sum game

2. Payoff matrix

Helen has 6 chips and David has 4 chips, the strategies they can use to place the chips in piles p1 and p2 can be given as:

For Helen: if she places C number of chips in pile 1, other pile will have 6-C chips (c = [0, 6])
For David: if he places D number of chips in pile 1, other pile will have 4-D chips (D = [0, 4])

Using these strategies and calculation of final score as provided in the Q.3, payload matrix can be formulated as:

David →	(0,4)	(1,3)	(2,2)	(3,1)	(4,0)	
Helen ↓						Si ↓
(0,6)	1	0	0	0	1	0
(1,5)	2	1	0	1	2	0
(2,4)	1	2	2	2	1	1
(3,3)	0	2	4	2	0	0
(4,2)	1	2	2	2	1	1
(5,1)	2	1	0	1	2	0
(6,0)	1	0	0	0	1	0
Ti →	2	2	4	2	2	

3. Saddle point analysis

A solution with set of strategies Ai and Bj for player 1 and player 2 respectively where no player can do better than the other player, we call it saddle point.

This means if Max (Si) = Min (Ti) in the payoff matrix, then we have a saddle point and pure strategy where player 1 and player 2 can do nothing better than strategy Ai and Bj respectively.

Although, observing Si and Ti values in payoff matrix above, we get L = 1 and U = 2
As L < U, we do not have any saddle point for the game and mixed strategies will be applied to find the value of game.

Value of game lies between [1,2] (inclusive)

4. Linear programming model

a. Helen's game:

Helen has 7 strategies that can be mixed: x1, x2, x3, x4, x5, x6, x7
If David chooses his mix of strategies (y1, y2, y3, y4, y5), we can formulate the LP as

Max z = v
Such that
v – (x1 + 2x2+ x3 + x5 + 2x6 + x7) <= 0
v – (x2 + 2x3 + 2x4 + 2x5 + x6) <= 0
v – (2x3 + 4x4 + 2x5) <= 0
v – (x2 + 2x3 + 2x4 + 2x5 + x6) <= 0
v – (x1 + 2x2 + x3 + x5 + 2x6 + x7) <= 0

x1 + x2 + x3 + x4 + x5 + x6 + x7 = 1
x1, x2, x3, x4, x5, x6, x7 >= 0
v u.r.s.

b. David's game:

David has 4 strategies that can be mixed: y1, y2, y3, y4
If Helen chooses her mix of strategies (x1, x2, x3, x4, x5, x6, x7), we can formulate the LP for David's game as:

Min w = v
Such that
v – (y1 + y5) >= 0
v – (2y1 + y2 + y4 + 2y5) >= 0
v – (y1 + 2y2 + 2y3 + 2y4 + y5) >= 0
v – (2y2 + 4y3 + 2y4) >= 0

v – (y1 + 2y2+ 2y3 +2y4 + y5) >= 0
v – (2y1 + y2 + y4 + 2y5) >= 0
v – (y1 + y5) >= 0

y1 + y2 + y3 + y4 + y5 = 1
y1, y2, y3, y4, y5 >= 0
v u.r.s.

5. Please refer R file named "rajesh-code. R" submitted separately in the assignment folder
6. Solution for David's game

Value of the game is: 1.333333 or 4/3
Probability distribution of David's strategies:
y1 = 0.666667 or 2/3 y2 = 0 y3 = 0.333333 or 1/3
y4 = 0 y5 = 0
That is, to get best payoff, David should use strategy y1 2/3 times and y3 1/3 times.

Question 4

1. The payoff matrix

First, we will create a table for money obtained by players, as the total amount contributed is doubled by the referee and then equally divided among players, we get out matrix as:

Money Obtained										
		Player 3								
		0 $			3 $			6 $		
		Player 2			Player 2			Player 2		
		0 $	3 $	6 $	0 $	3 $	6 $	0 $	3 $	6 $
Player 1	0 $	0	2	4	2	4	6	4	6	8
	3 $	2	4	6	4	6	8	6	8	10
	6 $	4	6	8	6	8	10	8	10	12

To create profit payoff table, as mentioned in the question,
profit = money obtained – money contributed, the profit payoff matrix is formulated as below

Profit Payoff Matrix										
		Player 3								
		0 $			3 $			6 $		
		Player 2			Player 2			Player 2		
		0 $	3 $	6 $	0 $	3 $	6 $	0 $	3 $	6 $
Player 1	0 $	(0, 0, 0)	(2, -1, 2)	(4, -2, 4)	(2, 2, -1)	(4, 1, 1)	(6, 0, 3)	(4, 4, -2)	(6, 3, 0)	(8, 2, 2)
	3 $	(-1, 2, 2)	(1, 1, 4)	(3, 0, 6)	(1, 4, 1)	(3, 3, 3)	(5, 2, 5)	(3, 6, 0)	(5, 5, 2)	(7, 4, 4)
	6 $	(-2, 4, 4)	(0, 3, 6)	(2, 2, 8)	(0, 6, 3)	(2, 5, 5)	(4, 4, 7)	(2, 8, 2)	(4, 7, 4)	(6, 6, 6)

(Straffin, 1993; 3 person problems Game theory 2016)

2. Nash equilibrium for profit

Nash equilibrium for the game is at (0,0,0) where there is no player will put any money and there will be zero profit. If any player tries to change his/her strategy from this point, it will result in decrease in the profit. Ex. At (0,0,0), if any player changes amount invested, it will result in negative profit, i.e. they will lose either 1 or 2 dollars of money they contributed.

In every other scenario, at least one of the players have incentive to increase or decrease profit margin through change of strategy Ex. At (6,6,6), every player is receiving double the money they contributed, although both player 1 and player 2, can change the money contributed and maximize their profit (from 6 to 7). Provided the players do not know how much other players are contributing and there is no mention of coalitions, each player will refrain from contributing any money expecting other players might put contribute and they will receive profits without any contribution from own money (Straffin, 1993).

References

Waner,Stephan 2013, *Finite mathematics Utility: Linear programming grapher (two variables),* retrieved 21 May 2020, <https://www.zweigmedia.com/utilities/lpg/index.html?lang=en>

Desmos.com 2020, *Desmos|Graphing Calculator*, retrieved 21 May 2020, <https://www.desmos.com/calculator>

3 person problems Game theory 2016, Youtube, Mike Clark, 8 December, retrieved 24 May 2020, <https://www.youtube.com/watch?v=eES1WtG34Dg>

Straffin, PD 1993, *Game Theory and Strategy*, New Mathematical Library, Mathematical Association of America, Washington, viewed 24 May 2020, <http://search.ebscohost.com.ezproxy-b.deakin.edu.au/login.aspx?direct=true&AuthType=ip,sso&db=e000xww&AN=453286&site=ehost-live&scope=site>.