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Correcting low-frequency variability bias in stochastic weather generators☆

James W. Hansen [a,*], Theodoros Mavromatis [b]

[a] *International Research Institute for Climate Prediction, P.O. Box 1000, Palisades, NY 10964-8000, USA*

[b] *Agricultural and Biological Engineering Department, University of Florida, P.O. Box 110570, Gainesville, FL 32611-0570, USA*

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Abstract

Stochastic weather generators used with agricultural simulation models tend to under predict interannual variability of generated climate, often resulting in distortion of simulated agricultural or hydrological variables. This study presents a stochastic weather generator that attempts to improve interannual variability characteristics by perturbing monthly parameters using a low-frequency stochastic model, and evaluates the effectiveness of the low-frequency component on interannual variability of generated monthly climate and simulated crop variables. Effectiveness of the low-frequency correction was tested by comparing results based on observed weather sequences to those generated from the same underlying stochastic model without and with the low-frequency component. For monthly precipitation and maximum and minimum temperatures at 25 locations in the continental USA, the low-frequency correction reduced total error and eliminated negative bias of interannual variability, and reduced the number of station-months with significant differences between observed and generated interannual variability, but over-represented variability of precipitation frequency. For 11 crop scenarios, the low-frequency correction reduced the number of instances in which mean simulated yields and development times differed for observed and generated weather, and improved all measures of interannual variability of simulated yields and development times. We conclude that the approach presented here to disaggregate and separately model the high- and low-frequency components of weather variability can effectively address the negative bias of interannual variability of monthly climatic means found in some stochastic weather generators, and improve crop simulation applications of stochastically-generated weather. Further refinement is needed to better represent interannual variability of both precipitation occurrence and intensity processes, and to rectify over-correction of interannual temperature variability.

Keywords: Crop simulation models; Stochastic process; Precipitation; Temperature

1. Introduction

Stochastic weather generators used most frequently with agricultural and ecological simulation models tend to under predict interannual variability of generated sequences of precipitation (Gregory et al., 1993; Jones and Thornton, 1993; Katz and Parlange, 1998; Wilks, 1999) and other variables (Mearns et al., 1996; Semenov et al., 1998; Mavromatis and Hansen, 2001). The use of generated sequences of weather data generally results in under-prediction of variability, and sometimes biased prediction of the mean values, of output of agricultural or hydrological

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* Corresponding author. Fax: +1-914-680-4864.
E-mail addresses: jhansen@iri.ldeo.columbia.edu (J.W. Hansen), theo@agen.ufl.edu (T. Mavromatis).

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Nomenclature

K	daily clearness defined as R_s/R_0
p_{ijk}	transition probability of precipitation occurrence defined as $\mathbf{P}\{\mathbf{w_t}=\mathbf{k}\mid\mathbf{w_{t-1}}=\mathbf{j},\ \mathbf{w_{t-2}}=\mathbf{i}\}$
q_{ij}	joint probability of precipitation occurrence defined as $\mathbf{P}\{\mathbf{w_t}=\mathbf{j},\ \mathbf{w_{t-1}}=\mathbf{i}\}$
R_s, R_0	daily global solar irradiance ($\mathrm{MJ\,m^{-2}}$) at the earth's surface and outside its atmosphere
T_{max}, T_{min}	daily maximum and minimum temperature (°C)
w	state of daily precipitation occurrence ($w=1$) or non-occurrence ($w=0$)
$W(T)$	mean number of wet days in a T-day (e.g. 1 month) period
x	logit-transformed relative daily clearness
y_{ij}, $\bar{y}_i$, $\bar{Y}$	observation for day j of month i, mean of month i, and mean across all years
$\bar{Y}$, S	mean and standard deviation of a meteorological variable computed from observations
z', Δ_z	perturbed value of parameter z, and degree of shift ($z'=z+\Delta_z$)
Greek letters	
β_1, β_2, α	means of component exponential distributions within a hyper-exponential distribution, and their mixing probability
ϵ_t, e_t	independent, auto- and cross-correlated standard Gaussian deviates sampled at time t
π	probability of daily precipitation occurrence
ρ	1 month lag auto-correlation
μ, σ	mean and standard deviation of a meteorological variable implied by a parameterized model
Subscripts	
C	denotes correction variability that must be added to parameterized model to match observed low-frequency variability
T, H, L	total, high- and low-frequency components of variability

simulation models (Richardson, 1985; Jones and Thornton, 1993; Semenov and Porter, 1995; Mearns et al., 1996; Mavromatis and Jones, 1998).

Large-scale atmospheric circulation patterns such as the El Niño-Southern Oscillation (ENSO) induce low-frequency (i.e. interannual) local climatic responses in many regions (Kiladis and Diaz, 1989; Ropelewski and Halpert, 1996; Trenberth, 1997). These large-scale fluctuations interact with high-frequency (i.e. daily) weather variability associated with synoptic- to meso-scale atmospheric processes. For an observed sequence of a years of n_i observations, $i=1,\dots,a$ of daily values of a meteorological variable, y, for a given calendar month, the total variance of y

$$S_{\mathrm{T}}^2=\frac{\sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij}-\bar{Y})^2}{\sum_{i=1}^{a}(n_i-1)} \tag{1}$$

can be partitioned into high-frequency variance

$$S_{\mathrm{H}}^2=\frac{\sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_i)^2}{\sum_{i=1}^{a}n_i-a} \tag{2}$$

of days within each month in the series, and low-frequency variance

$$S_{\mathrm{L}}^2=\frac{1}{a-1}\left(\left(\sum_{i=1}^{a}n_i-1\right)S_{\mathrm{T}}^2-\left(\sum_{i=1}^{a}n_i-a\right)S_{\mathrm{H}}^2\right) \tag{3}$$

of monthly means among years. Specifying parameter values by calendar month, and defining low-frequency variability as the variability of monthly means among years are based here primarily on convention and convenience. Many stochastic daily weather generators that are used in conjunction with agricultural and environmental simulation models represent all observed variability (i.e. S_{T}^2) with high-frequency, short-memory stochastic processes that cannot distinguish low-frequency climate variations such as those induced by ENSO. This view of the problem suggests that one potential approach to improving the interannual variability characteristics might be to perturb monthly parameters using a low-frequency stochastic model. This has been attempted, for example, by conditioning parameters on indices of large-scale atmospheric circulation (Katz and Parlange, 1993; Grondona et al., 2000), and by perturbing parameters using a low-frequency stochastic process (Jones and Thornton, 1993).

This paper describes one implementation of an approach toward correcting low-frequency variability bias, and examines effects of a low-frequency stochastic component on interannual variability of generated monthly climate and simulated crop variables. We first describe a stochastic weather generator and its modification using a scheme for perturbing monthly parameter values. We then compare the abilities of the basic model, and its implementation with the low-frequency correction scheme, to reproduce distributions of monthly climatic means and simulated crop yields for locations in the continental US. In a follow-up study by Mavromatis and Hansen (2001), interannual variability of generated monthly climate for 12 locations in several countries and simulated crop responses from 10 crop-location combinations resulting from the disaggregated variance weather generator described here generally compared favorably to results from three other stochastic generators commonly-used in crop simulation applications.

2. Model description

2.1. Aggregated variance model (Model 1)

The high-frequency component of the weather generator is similar in many respects to the widely-used generator, WGEN (Richardson and Wright, 1984), but incorporates several improvements. It first simulates the occurrence of precipitation with a two-state, hybrid second-order Markov chain. When a wet day is simulated, precipitation amount is sampled from a hyper-exponential distribution. Maximum and minimum daily temperatures, and a transformation of daily solar irradiance are sampled from Gaussian distributions conditioned on the occurrence of precipitation. The solar irradiance component is based on the transformation proposed by Hansen (1999). A more detailed description follows.

2.1.1. Precipitation occurrence

A second-order Markov chain can substantially improve the modeled distribution of dry period durations relative to the more commonly-used first-order precipitation occurrence model for some locations and seasons (Gates and Tong, 1976; Chin, 1977; Eidsvik, 1980; Katz and Parlange, 1998; Wilks, 1999). Jones and Thornton (1993) argued that higher-order temporal dependence is the norm in the tropics. Because the first-order model usually captures the distribution of wet period durations as well as higher-order models (Racsko et al., 1991; Wilks, 1999), Stern and Coe (1984) proposed using a Markov chain of hybrid-order that would simulate precipitation occurrence with a first-order chain if the previous day was wet, and a higher-order chain if the previous day was dry. To obtain kth-order temporal dependence for dry sequences, the hybrid-order Markov chain requires only $k + 1$ parameters, compared to 2^k for the full Markov chain. For 30 stations within the continental US, the Bayesian information criterion (BIC) selected the first-order model most often, followed by the second-order hybrid model in climates with stronger temporal dependence of dry days, particularly in the western states (Wilks, 1999).

Precipitation occurrence in the weather generator described here is based on a two-state, second-order hybrid Markov chain. It considers only the first-order transition probability, p_{11}, when the preceding day was wet, and second-order transition probabilities, p_{001} and p_{101}, when the preceding day is dry.

2.1.2. Precipitation amount

Comparisons have shown that a probability mixture of two exponential distributions usually fits observed distributions of daily precipitation amounts better than more commonly-used alternatives, such as the single exponential and gamma (Richardson, 1982a; Woolhiser and Roldán, 1982; Foufoula-Georgiou and Lettenmaier, 1987; Wilks, 1999). Furthermore, interannual variability of monthly precipitation is strongly influenced by rare extreme events. By better representing these extreme events, the three-parameter mixed exponential (also known as a hyper-exponential, e.g. Whitt (1981)) distribution improves interannual variability of monthly precipitation total relative to the more frequently-used two-parameter gamma distribution (Wilks, 1999).

When the Markov model simulates a wet day, the amount of precipitation is sampled from a hyper-exponential distribution

$$f(x) = \alpha \frac{\exp(-x/\beta_1)}{\beta_1} + (1-\alpha) \frac{\exp(-x/\beta_2)}{\beta_2} \quad (4)$$

with means β_1 and β_2, mixing probability α, mean

$$\mu_I = \alpha\beta_1 + (1-\alpha)\beta_2 \quad (5)$$

and variance

$$\sigma_I^2 = 2\alpha\beta_1^2 + 2(1-\alpha)\beta_2^2 - \mu_I^2 \quad (6)$$

2.1.3. Temperatures and solar irradiance

Stochastic generation of daily minimum (T_{min}) and maximum temperatures (T_{max}) and global solar irradiance (R_s) each involve sampling from a mixture of two Gaussian distributions, conditioned on the occurrence of precipitation. Auto- and cross-correlation among generated T_{max}, T_{min} and R_s are imposed by sampling standard Gaussian deviates from a trivariate, auto-regressive process (Matalas, 1967), as implemented in WGEN (Richardson and Wright, 1984), with correlation coefficients (Richardson, 1982b) that are held constant across location and season.

Several stochastic generators sample R_s from a truncated mixture of Gaussian distributions conditioned on precipitation occurrence (e.g. Richardson and Wright, 1984; Hanson et al., 1993; Pickering et al., 1994; Wallis and Griffiths, 1995). However, the truncated Gaussian is a rather poor representation of the distribution of observed R_s on either wet or dry days (Liu and Jordan, 1960; Olseth and Skartveit, 1984; Hansen, 1999). Solar irradiance at the top of the earth's atmosphere (H_O) is a nearly-deterministic function of latitude and time of year. The daily clearness index, defined as $K = H/H_O$, captures the stochastic component of solar irradiance due to varying atmospheric conditions (primarily clouds, aerosols and water vapor) (Iqbal, 1983). Hansen (1999) showed that a logit transformation of K, re-scaled between its upper and lower limits, reduced departures from normality at all of ten US locations for dry days and at most of the locations on wet days. Based on Akaike's (1974) information criterion and goodness-of-fit tests, the resulting logit-transformed conditional Gaussian model fit observed data better than the truncated conditional Gaussian model that is commonly-used in stochastic generators. The weather generator described here implements the solar irradiance model of Hansen (1999), with the lower limit of K set at 0.03 and its upper limit estimated empirically.

2.2. Disaggregated variance model (Model 2)

The disaggregated variance model (Model 2) is a modified version of Model 1 with different standard deviations for sampling temperatures and solar irradiance, and a low-frequency component designed to correct any negative bias of interannual variability of monthly climatic means. Low-frequency variability of a given meteorological variable, y is imposed by perturbing its mean in month t by $\Delta_{y,t}$ sampled stochastically from a first-order auto-regressive process

$$e_{y,t} = \rho_{y,m} e_{y,t-1} + \varepsilon_t \sqrt{1-\rho_{y,m}^2}, \quad \Delta_{y,t} = S_{C,y} e_{y,t} \quad (7)$$

where $S_{C,y}^2$ is a low-frequency correction variance, $\rho_{y,m}$ the 1 month lag auto-correlation of monthly means for calendar month m, and $\varepsilon \sim N(0,1)$ is an independent random deviate. In general, S_C^2 is calculated as the non-negative difference between observed interannual variability S_L^2 of monthly means and the interannual variability σ_L^2 implied by the parameterized high-frequency stochastic model. The second term in Eq. (7) corrects inflation of the asymptotic variance of the generated series due to auto-correlation (Pankratz, 1983). A contemporaneous multivariate auto-regressive process, obtained by sampling ε_t for each variable from a multivariate Gaussian distribution, accounts for cross-correlation of monthly means among the four variables. A Cholesky decomposition is used to sample multivariate Gaussian random deviates with the appropriate cross-correlation (Dagpunar, 1988). Auto- and cross-correlation are calculated for each variable in each calendar month from the observed time series of monthly means.

2.2.1. Temperatures and irradiance

Daily maximum and minimum temperature, and logit-transformed daily relative clearness (x) are each sampled from a mixture of two Gaussian distributions parameterized separately for dry ($w = 0$) and wet

($w = 1$) days. The resulting mixed Gaussian distribution

$$f(y) = (1-\pi) f_0(y|\bar{Y}_0, S_0) + \pi f_1(y|\bar{Y}_1, S_1) \tag{8}$$

has variance (Katz, 1996)

$$\sigma_{\mathrm{H}}^2 = (1-\pi)\sigma_0^2 + \pi\sigma_1^2 + \pi(1-\pi)(\mu_1 - \mu_0)^2 \tag{9}$$

The appropriate standard deviation for each wet state w used in Eq. (9) is calculated centered on the monthly mean of each year in the series (Eq. (2)). Interannual variability of mean monthly values is related to daily variability by approximately (Mearns et al., 1997)

$$\sigma_{\mathrm{L}}^2 \approx \frac{\sigma_{\mathrm{H}}^2}{N} \frac{1+\rho_1}{1-\rho_1} \tag{10}$$

where N is the number of days in the month and ρ_1 the first-order auto-correlation of daily observations. Auto-correlation of daily values is a function of both the multivariate auto-regressive process (Matalas, 1967) used to generated daily temperatures and irradiance, and the auto-correlation of precipitation occurrence (Katz, 1996)

$$\rho_1 = \frac{\phi_1\{(1-\pi)\sigma_0^2) + \pi(1-\pi)(d_1(\mu_1-\mu_0)^2 - \phi_1(1-d_1)(\sigma_1-\sigma_0)^2\}}{\sigma_{\mathrm{H}}^2} \tag{11}$$

Correction variance S_{C}^2 is then calculated as the difference, $S_{\mathrm{L}}^2 - \sigma_{\mathrm{L}}^2$.

2.2.2. Precipitation

Let $R(T)$ represent total precipitation, and $W(T)$ the number of days with rain in T days. The variance of $R(T)$

$$\sigma_{R(T)}^2 = T\pi\sigma_{\mathrm{I}}^2 + \sigma_{W(T)}^2 \mu_{\mathrm{I}}^2 \tag{12}$$

is a function both of the occurrence and intensity processes (Katz and Parlange, 1993). Because no simple analytical expression of $\sigma_{W(T)}^2$ associated with the hybrid-order Markov occurrence process is available, $\sigma_{W(T)}^2$ is calculated from the distribution of $W(T)$ obtained from an iterative algorithm described in Katz and Parlange (1998) and adapted from an implemented graciously provided by Katz (personal communication).

Because $\sigma_{R(T)}^2$ is a function of the mean and variance of the intensity process, and of the properties of the occurrence process, several options exist for imposing additional low-frequency variability to the overall high-frequency model. Like Jones and Thornton (1993), we chose to correct any low-frequency variability bias by stochastically re-sampling π each month. Re-arranging Eq. (12) and substituting the observed $S_{R(T)}^2$ for the model-derived $\sigma_{R(T)}^2$ yields the low-frequency variability of $W(T)$

$$S_{W(T),\mathrm{O}}^2 = \frac{S_{S(T)}^2 - T\pi\sigma_{\mathrm{I}}^2}{\mu_{\mathrm{I}}^2} \tag{13}$$

required for the model to reproduce the observed interannual variability of $S(T)$. The residual variability that must be added to π to match the observed precipitation variability is obtained as the difference between the required variance of $W(T)$ (Eq. (13)) and the calculated variance of the parameterized Markov occurrence model, re-scaled from T days to 1 day

$$S_{\pi,\mathrm{C}}^2 = \left(\frac{S_{W(T),\mathrm{O}}^2 - \sigma_{W(T),\mathrm{M}}^2}{T}\right)^{1/2} \tag{14}$$

The Gaussian distribution used to impose low-frequency variability for $T_{\max}$, $T_{\min}$ and R_{s} is unbounded, whereas π is bounded between 0 and 1. To avoid bounds violations and provide a more realistic distribution, the Gaussian deviate is transformed to a deviate from a beta distribution. Parameters of the beta distribution are calculated as

$$\alpha_1 = \frac{(1-\pi)\pi^2}{\sigma_{\pi,\mathrm{C}}^2} - \pi \tag{15}$$

$$\alpha_2 = \frac{\alpha_1(1-\pi)}{\pi} \tag{16}$$

to give a mean of π and standard deviation of $\sigma_{\pi,\mathrm{C}}$. A Gaussian deviate obtained from Eq. (7) is first transformed to a uniform distribution

$$u = \frac{1 + \mathrm{erf}(e/\sqrt{2})}{2} \tag{17}$$

then transformed using a numerical approximation of the inverse $\beta(\alpha_1, \alpha_2)$ distribution to obtain Δ_π.

The procedure for perturbing π is further complicated by the hybrid second-order Markov chain occurrence model. It involves adjusting the three transition probabilities in a manner that preserves first- and second-order persistence of the Markov process. The following derivation is based on expansion of relationships given in appendix B of Katz and Parlange (1998). Let

$$q_{i,j} \equiv \mathbf{P}\{\mathbf{w_t} = \mathbf{j}, \mathbf{w_{t-1}} = \mathbf{i}\} = \pi_i p_{i,j} \tag{18}$$

define the probability of an i, j sequence, where $\pi_1 \equiv \pi$ and $\pi_0 \equiv 1 - \pi$. Then p_{01} can be obtained from

$$q_{10} = \pi(1 - p_{101}) \tag{19}$$

$$q_{00} = 1 - \pi - q_{10} \tag{20}$$

$$q_{01} = q_{00} p_{001} + q_{00} p_{001} \tag{21}$$

$$p_{01} = \frac{q_{01}}{1 - \pi} \tag{22}$$

allowing us to calculate first- and second-order persistence

$$d_1 = p_{11} - p_{01} \tag{23}$$

$$d_2 = p_{101} - p_{001} \tag{24}$$

Before generating precipitation for a given month, π is perturbed by adjusting the transition probabilities such that d_1 and d_2 are preserved

$$\pi' = \pi + \Delta_\pi \tag{25}$$

$$p'_{01} = \pi'(1 - d_1) \tag{26}$$

$$p'_{11} = p'_{01} + d_1 \tag{27}$$

$$q'_{01} = p'_{01}(1 - \pi') \tag{28}$$

$$q'_{10} = \pi'(1 - p'_{11}) \tag{29}$$

$$q'_{00} = 1 - \pi' - q'_{10} \tag{30}$$

$$p'_{001} = \frac{q'_{01} - q'_{10} d_2}{q'_{00} + q'_{10}} \tag{31}$$

$$p'_{101} = P'_{001} + d_2 \tag{32}$$

3. Methods

3.1. Weather data

Comparisons were based on long-term daily precipitation and minimum and maximum temperature data from the continental United States. Twenty-four stations (Table 1) were selected from the US Historical Climatology Network (USHCN, Karl et al., 1990) to provide reasonably uniform spatial coverage and represent a diversity of climates. The USHCN is a high-quality set of stations selected to avoid urban heat bias and other potential sources of temporal inconsistency. Although not part of the USHCN, Grand Island data from the NCDC Summary of the Day were included because the length and completeness of the series were consistent with other locations, and because of the availability of data and authors' experience with crop simulation analyses for this site. Daily solar irradiance used as input to crop simulation models are a mixture of measured and modeled data from the Solar and Meteorological Surface Observation Network (SAMSON) data base (NREL, 1992).

3.2. Parameter estimation and stochastic generation

Parameter estimation from observed daily data is straightforward for transition probabilities of the hybrid-order precipitation occurrence process, and means of $T_{\max}$, $T_{\min}$ and x (i.e. logit-transformed daily clearness) conditioned on w. Standard deviations of daily $T_{\max}$, $T_{\min}$ and x conditioned on w are calculated centered on the long-term monthly mean (Eq. (1)) for Model 1, and centered on the mean of each month in the time series (Eq. (2)) for Model 2. An iterative maximum likelihood algorithm for mixtures of distributions (Agha and Ibrahim, 1984) was adapted to estimate parameters of the hyper-exponential precipitation intensity process (β_1, β_2 and α), with initial estimates obtained by the method of moments (Rider, 1960). For consistency with the generation process, auto- and cross-correlation coefficients for the low-frequency component of Model 2 were calculated from time series of monthly means of $T_{\max}$, $T_{\min}$ and x estimated from daily observations standardized with means and standard deviations for each calendar month conditioned on w. All parameters were estimated for each calendar month.

Table 1
Weather stations used in the study

Station	Latitude (°N)	Longitude (°E)	Elevation (m)	Precipitation (mm per year)	Start date[a]	Percent complete[b]
Aberdeen, ID	42.95	−112.83	4404	224	May 1914	99.1
Apalachicola, FL	29.73	−85.03	19	1444	October 1903	99.3
Atlantic City, NJ	39.38	−74.43	10	1026	January 1901	100.0
Belle Glade, FL	26.65	−80.63	15	1412	May 1924	99.3
Burlington, VT	44.47	−73.15	330	841	January 1901	100.0
Charleston, SC	32.78	−79.93	10	1141	January 1901	100.0
Cheyenne, WY	41.15	−104.82	6124	390	January 1901	100.0
Cut Bank, MO	48.60	−112.37	3837	300	January 1904	94.0
Dickinson, ND	46.88	−102.80	2459	410	January 1901	99.0
Durango, CO	37.28	−107.88	6591	489	January 1901	97.9
Grand Island, NE	41.00	−98.30	561	632	January 1901	99.5
Greenville, MS	33.38	−91.02	132	1338	January 1920	98.9
Hallettsville, TX	29.47	−96.95	275	946	January 1901	98.1
Harrisburg, IL	37.73	−88.52	365	1117	January 1901	97.2
Medicine Lodge, KS	37.28	−98.58	1500	660	January 1901	97.1
Mesa, AZ	33.42	−111.80	1235	213	January 1901	90.8
Minneapolis, MN	44.88	−93.22	834	692	January 1901	100.0
Napa, CA	38.28	−122.27	35	616	January 1901	97.5
Portland, OR	45.45	−122.15	748	2095	February 1932	99.7
Rogersville, TN	36.42	−82.98	1355	1125	January 1901	99.0
San Jon, NM	35.12	−103.33	4229	416	June 1907	97.6
Spokane, WA	47.63	−117.53	2355	403	January 1901	100.0
Tifton, GA	31.50	−83.53	380	1197	April 1922	99.3
Tustin Irvine Ranch, CA	33.73	−117.78	118	327	December 1927	97.9
Winnemucca, NV	40.90	−117.80	4299	211	January 1901	100.0

[a] Records terminate in August 1997, except for Grand Island (December 1995) and Durango (February 1991).
[b] Sum of valid observations of rain, T_{max}, T_{min}/(3 × number of days in record).

For each station in Table 1, each of the two weather generator models was parameterized using all valid observations. Stochastic daily weather sequences were then generated for the set of months represented in the observed record.

3.3. Crop simulation

Harvest yields, and time to anthesis and harvest maturity were simulated in response to historic and generated weather for given soil parameters and initial conditions, cultivars and crop management scenarios. The crop models included in version 3.5 of the Decision Support System for Agrotechnology Transfer (DSSAT) (Jones et al., 1998) were used: CERES (Ritchie et al., 1998) for maize and wheat, and CROPGRO (Boote et al., 1998) for soybean. Inputs to these models include daily weather data (minimum and maximum temperature, precipitation and solar irradiance), soil properties, initial soil water content, cultivar characteristics, planting date and spatial arrangement, and N fertilizer management (Hunt and Boote, 1998).

The study considered 11 crop-location combinations (Table 2). We obtained representative soil characteristics, cultivars and representative management from published studies or researchers familiar with each region. Each crop-location scenario was simulated as a continuous sequence with carryover of soil conditions. The models simulated the daily soil water balance assuming rainfed production. Each simulation was initialized, with soil water at 60% of water-holding capacity, 1 month before the first planting date. For maize and wheat, N components of the soil and crop models were disabled on the assumption that sufficient fertilizer is used to avoid N stress. For soybean, soil N dynamics and biological

Table 2
Crop simulation scenarios used to evaluate weather generators, and mean and standard deviation of grain yields and days to harvest maturity simulated with observed weather data.

Code	Station	Crop	Cultivar	Planting		Source	Simulation results			
				Date	Density (m^{-2})		Yield ($Mg\,ha^{-1}$)		Maturity (days)	
							Mean	*S*	Mean	*S*
A	Atlantic City	Soybean	HS93-4118	26 June	30.0	Northern Trial data set	2.71	0.47	109	2.5
B	Cut Bank	Spring wheat	Manitou	13 April	193.0	Lanning et al. (2000)	1.30	0.90	102	7.2
C	Cut Bank	Winter wheat	Newton	10 September	161.0	Berg et al. (2000)	1.91	1.21	310	7.0
D	Grand Island	Maize	Pioneer 3382	1 May	5.2	Tsventsinskaya (2001)	6.09	3.39	122	8.2
E	Grand Island	Winter wheat	Newton	15 September	180.0	Tsventsinskaya (2001)	2.74	1.11	274	5.7
F	Minneapolis	Maize	Pioneer 3720	25 April	5.9	Sheaffer et al. (1999)	9.24	3.18	132	13.1
G	Minneapolis	Soybean	IA 2036	20 May	38.7	Orf et al. (1999)	2.61	0.99	135	4.4
H	Minneapolis	Winter wheat	Newton	1 September	270.0	Anderson and Bush (1999)	3.38	0.81	301	5.7
I	Tifton	Maize	McCurdy 6714	15 March	6.7	Jones et al. (2000)	10.23	2.22	124	4.5
J	Tifton	Soybean	Stonewall	15 May	34.0	Jones et al. (2000)	2.55	0.82	157	1.2
K	Tifton	Winter wheat	Florida 302	15 November	330.0	Jones et al. (2000)	2.84	0.34	177	8.0

N fixation were simulated with no added fertilizer N. Crop simulations were repeated using synthetic weather sequences produced by each weather generator for the same number of years as the observed sequences.

3.4. Analyses

To evaluate the low-frequency correction scheme described above, we compared historical monthly sequences of precipitation total and frequencies and monthly mean T_{max} and T_{min} to sequences generated both without (Model 1) and with the low-frequency correction (Model 2). Solar irradiance was excluded from analysis because it was generally either absent, derived from proxy data, or measured for relatively short periods at the locations used in this study. The same procedure was applied to crop simulation output derived from observed weather, and synthetic weather produced by each weather generator model.

To test the hypothesis of equal interannual variability between observed and generated series, Levene's (1960) method was applied when the samples were normally distributed. However, when a Kolmogorov–Smirnov single-sample test (Conover, 1999) indicated significant non-normality of either sample, the Brown and Forsythe (1974) modification of Levene's test was applied, which is more robust than Levene's with respect to departures from normality. Departures from normality were frequent only for precipitation frequencies and amounts. An independent t-test was used to test the hypothesis of equal central tendency when the assumptions of normality and equality of variances held. If tests of either assumption failed, the non-parametric Mann and Whitney (1947) Rank Sum test was used instead. Finally, the non-parametric Kolmogorov–Smirnov two-sample test (Conover, 1999) was used to test the hypothesis of identical distributions. Although the Kolmogorov–Smirnov test is generally less powerful than the tests used to evaluate equality of central tendency and variability, it can identify differences in other characteristics (e.g. skewness) of the two distributions.

In addition to statistical hypothesis tests, the performance of the weather generators was characterized with the help of two descriptive statistics that indicate the degree of consistency between results based on observed and generated weather. Root-mean-squared-error (RMSE)

$$\mathrm{RMSE} = \left(\frac{1}{n}\sum_{i=1}^{n}(y_{\mathrm{G},i} - y_{\mathrm{O},i})^2\right)^{1/2}$$

and mean bias error expressed either on an absolute (mean T_{max} and T_{min})

$$\mathrm{MBE} = \frac{1}{n}\sum_{i=1}^{n}(y_{\mathrm{G},i} - y_{\mathrm{O},i}) \tag{33}$$

or percent basis (all other variables)

$$\mathrm{MBE}\,(\%) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_{\mathrm{G},i}}{y_{\mathrm{O},i}} - 1\right) \times 100 \tag{34}$$

were determined for means and interannual standard deviations of each monthly climate and simulated crop variable considered, where y is either the mean or standard deviation of the variable, and i is an observation (station-month for meteorological variables or crop scenario for crop variables) from a total of n. Subscripts refer to observed (O) or generated (G) weather.

4. Results and discussion

4.1. Generated monthly climate

Model 1 reproduced mean values of monthly climate variables with little apparent bias, acceptable RMSE (Table 3), and small proportion ($\leq 3\%$) of station-months for which generated and observed results differed significantly (Table 4). Although Model 2 reduced the number of station-months in which mean precipitation total differed significantly from observed, yet it produced a small positive bias of mean monthly precipitation. Because Model 2 generated a similar percent increase of mean number of wet days, we speculate that the transformation to a beta distribution implemented in the low-frequency correction of π may have slightly increased mean precipitation frequency. Increased RMSE of mean precipitation amount and frequency (Table 3) probably reflect the positive mean bias associated with Model 2. Model 2 reduced the number of station-months in which mean temperatures differed significantly from observed (Table 4).

Table 3
Mean bias error (MBE) and root-mean-squared-error (RMSE) of mean and interannual standard deviations of generated monthly climate and simulated crop response

Variable	Mean				Standard deviation			
	MBE[a]		RMSE		MBE[a]		RMSE	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
Generated monthly climate ($n = 300$)								
Rain (mm)	0.65%	2.53%	4.67	5.27	−11.88%	1.06%	10.04	5.22
Wet days (days)	−1.05%	3.11%	0.356	0.495	−6.21%	21.12%	0.428	1.003
T_{max} (°C)	0.070	0.044	0.251	0.248	−15.18%	2.26%	0.472	0.239
T_{min} (°C)	−0.044	−0.023	0.220	0.251	−6.53%	7.46%	0.372	0.234
Simulated crop response ($n = 11$)								
Yield (Mg ha^{-1})	−2.60%	0.47%	0.227	0.318	−12.03%	1.10%	0.255	0.070
Anthesis (days)	0.21%	−0.20%	1.294	1.016	−6.97%	2.79%	0.977	0.710
Maturity (days)	0.09%	−0.23%	1.038	1.075	−13.00%	−1.62%	1.354	1.011

[a] Absolute bias (°C, Eq. (33)) for mean T_{max} and T_{min}. Percent bias (Eq. (34)) for all other variables.

Table 4
Number of calendar months in which central tendency, interannual variability and distribution of generated climate variables differed significantly from observed ($n = 300$).

Variable	Central tendency		Variability		Distribution	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
Rain amount	7	5	49	12	9	6
Number of wet days	2	9	31	118	3	15
T_{max}	8	0	113	13	13	2
T_{min}	6	2	78	22	7	7

Model 1 produced substantial negative interannual variability bias for all generated climate variables (Table 3, Figs. 1–4). Model 2 corrected the negative MBE and reduced RMSE of the standard deviation of total precipitation (Table 3, Fig. 1), and reduced the number of station-months in which generated precip-

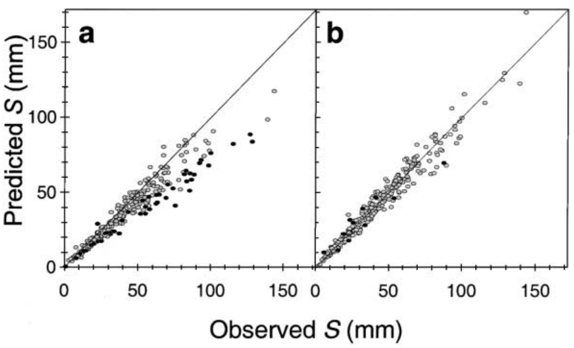

Fig. 1. Standard deviations, among years, of monthly precipitation total observed and generated by (a) Model 1 and (b) Model 2, $n = 300$ station-months. Solid symbols indicate significant ($P < 0.05$) differences between observed and generated standard deviations.

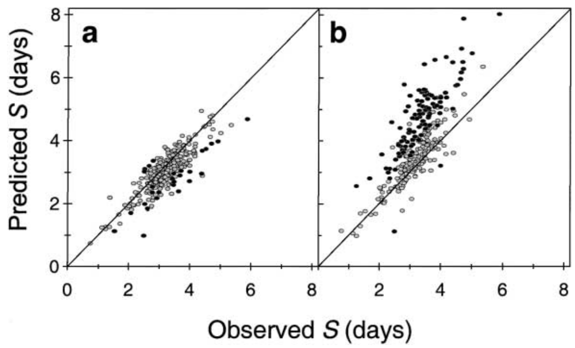

Fig. 2. Standard deviations, among years, of monthly numbers of wet days observed and generated by (a) Model 1 and (b) Model 2, $n = 300$ station-months. Solid symbols indicate significant ($P < 0.05$) differences between observed and generated standard deviations.

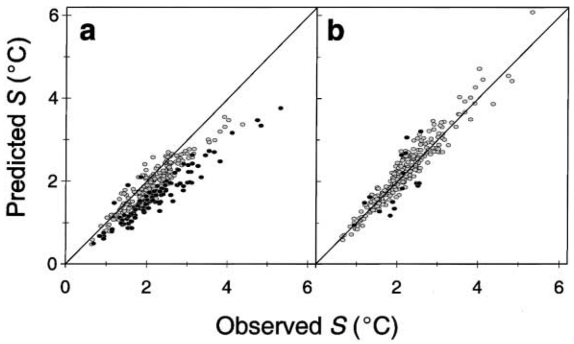

Fig. 3. Standard deviations, among years, of monthly mean T_{max} observed and generated by (a) Model 1 and (b) Model 2, $n = 300$ station-months. Solid symbols indicate significant ($P < 0.05$) differences between observed and generated standard deviations.

itation variability differed significantly from observed (Table 4). However, the improvements in interannual variability of generated precipitation total came at the expense of substantial positive bias, increased total error (RMSE) of interannual variability of precipitation frequency (Fig. 2), and increase in the number of station-months in which generated variability of precipitation frequency differed significantly from observed (Table 4). The low-frequency correction implemented in Model 2 corrects interannual variability of total precipitation by increasing variability of the occurrence process. However, these results suggest that some of the variability of total precipitation unexplained by Model 1 is due to variability of the intensity process.

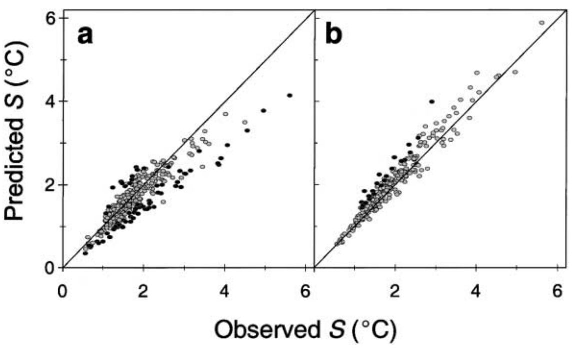

Fig. 4. Standard deviations, among years, of monthly mean T_{min} observed and generated by (a) Model 1 and (b) Model 2, $n = 300$ station-months. Solid symbols indicate significant ($P < 0.05$) differences between observed and generated standard deviations.

Model 2 corrected observed negative bias and reduced total error of interannual variability of temperatures resulting from Model 1 (Table 3, Figs. 3 and 4), and substantially reduced the number of station-months in which variability of generated temperatures differed significantly from observed (Table 4). However, Model 2 appeared to overcorrect the negative bias of Model 1, particularly for T_{min}, resulting in higher interannual variability than observed. The observed over-correction of interannual variability bias is likely due to the variability of π imposed by Model 2 to correct the interannual variability of monthly precipitation. Because Eq. (9) incorrectly assumes that π is constant, it under-represents the actual variability of T_{max} and T_{min} generated before the low-frequency correction is applied.

4.2. Simulated crop response

Both versions of the weather generator reproduced mean simulated crop yields quite well (Table 3). Only spring wheat at Cut Bank showed significantly-lower mean yield with Model 1 than with observed weather; Model 2 eliminated that yield under-prediction (Table 5). However, Model 2 generated a higher RMSE of mean yield than Model 1 (Table 5). Mean times to anthesis and maturity showed very small MBE with Model 1, and little apparent difference with Model 2. However, RMSE of mean simulated time to anthesis was slightly lower for Model 2 than for Model 1. Significant differences in mean simulated time to anthesis (5 versus 2) and maturity (4 versus 2) occurred in more crop scenarios with weather from Model 1 than from Model 2 (Table 5).

Model 1 showed substantial negative interannual variability bias of simulated yields (Table 3, Fig. 5). Model 2 corrected the bias and substantially reduced RMSE of the standard deviation of yields. Similarly, Model 2 corrected bias and reduced total error of interannual variability of simulated time to anthesis and maturity relative to Model 1 (Figs. 6 and 7). We speculate that under-representation of interannual variability of climate generated by Model 1 induced the observed biases of both mean crop simulation results and their variability. Because development time is largely a function of accumulated temperature, negative interannual variability bias of generated temperature minima and maxima likely influenced

Table 5
Crop simulation scenarios in which central tendency, variability and distribution of simulated response to observed and generated weather differed significantly ($n = 11$). Letters refer to scenario codes defined in Table 2

Variable	Central tendency		Variability		Distribution	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
Yield	B		A, B, C			G
Anthesis	A, E, G, H, J	A, E	G		A, E	
Maturity	A, E, G, J	E, J	E, G		A, E, G, J	J

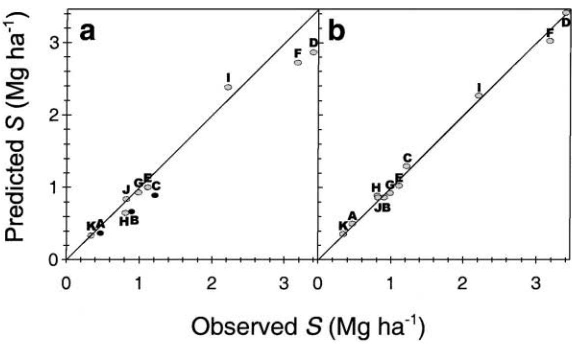

Fig. 5. Standard deviations, among years, of crop yields simulated using observed weather and weather generated by (a) Model 1 and (b) Model 2, $n = 300$ station-months. Solid symbols indicate significant ($P < 0.05$) differences of standard deviations resulting from observed and generated weather data.

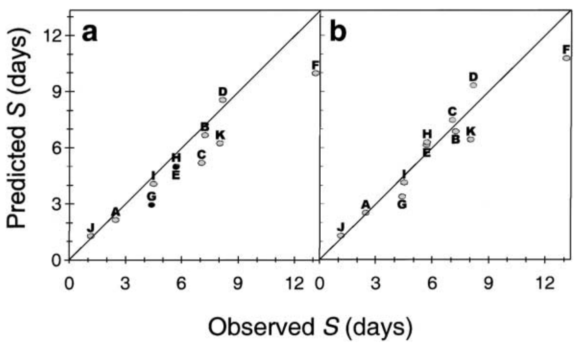

Fig. 7. Standard deviations, among years, of time to maturity simulated using observed weather and weather generated by (a) Model 1 and (b) Model 2, $n = 300$ station-months. Solid symbols indicate significant ($P < 0.05$) differences of standard deviations resulting from observed and generated weather data.

the negative variability biases observed for time to anthesis and development. Distortion of the interannual variability of monthly precipitation total likely played a major role in biasing variability of simulated yields. For all three crop variables, Model 2 corrected every instance, where variability of simulated results differed significantly between observed weather data and Model 1 (Table 5, Figs. 5–7).

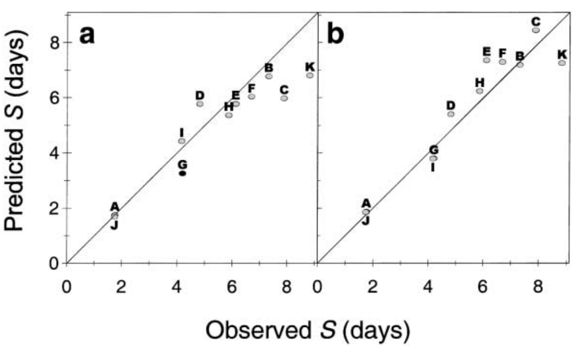

Fig. 6. Standard deviations, among years, of time to anthesis simulated using observed weather and weather generated by (a) Model 1 and (b) Model 2, $n = 300$ station-months. Solid symbols indicate significant ($P < 0.05$) differences of standard deviations resulting from observed and generated weather data.

5. Conclusions

Model 1 described in this paper includes enhancements shown in other studies to improve interannual variability of generated climate relative to other commonly-used aggregated variance models (Wilks, 1999). Yet, like many other stochastic generators, it clearly under-represented interannual variability of monthly precipitation total and mean temperatures at the stations analyzed. By most measures, the low-frequency correction applied in Model 2 improved interannual variability of generated climate and improved distributions of crop simulation results. For applications in which interannual variability is critical, a correction of this type would should improve results relative to the simpler aggregated variance models frequently employed. Results of the crop simulation scenarios considered in this study support such a conclusion. However, implications of exaggerating interannual variability of precipitation occurrence in order to correct interannual variability of precipitation amounts have not been fully explored. A more complicated alternative, perturbing the parameters of the precipitation intensity distribution, would yield more realistic interannual variability of precipitation frequency. Such an approach would presumably eliminate the small positive bias observed for interannual variability of generated temperatures and solar irradiance. If variability of both precipitation frequency and intensity are shown to be critical for some applications, then additional effort may be warranted to quantify and represent within stochastic weather generators the relative contribution of each to interannual variability of precipitation total.

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