# MDCorr Notes

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# 1 FFT

The discrete Fourrier transform of a series $x_n$ with size $N$ is computed as

$$X_k = \sum_{n}^{N-1} x_n e^{-2\pi ikn/N}, \tag{1}$$

and its inverse,

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} e^{2\pi ikn/N}, \tag{2}$$

These transformations can be denoted,

$$\begin{aligned} X_k &= \mathcal{F}_N \left\{ x_n \right\} \\ x_x &= \mathcal{F}_N^{-1} \left\{ X_k \right\} \end{aligned} \tag{3}$$

where $N$ is the size of the tranform operation. Now suppose $N$ is divisible by some integer $a$. The sum can be decomposed into parts,

$$\begin{aligned} X_k &= \sum_{l=0}^{a} \sum_{m=0}^{N/a} x_{am+l} e^{-2\pi ik(am+l)/N} \\ &= \sum_{l=0}^{a} e^{-2\pi il/N} \sum_{m=0}^{N/a} x_{am+l} e^{-2\pi ik(am)/N} \\ &= \sum_{l=0}^{a} e^{-2\pi il/N} \sum_{m=0}^{N/a} x_{am+l} e^{-2\pi ikm/(N/a)} \\ \mathcal{F}_N \left\{ x_n \right\} &= \sum_{l=0}^{a} e^{-2\pi il/N} \mathcal{F}_{N/a} \left\{ x_{am+l} \right\} \end{aligned} \tag{4}$$

Similarly

$$\mathcal{F}_N^{-1} \left\{ X_k \right\} = \frac{1}{N} \sum_{l=0}^{a} e^{2\pi il/N} \mathcal{F}_{N/a}^{-1} \left\{ x_{am+l} \right\}. \tag{5}$$

Thus the Fourrier transform can thus be decomposed into the sum of smaller fourrier transforms. The runtime of this substep $s$ with size $n_s$ is

$$\begin{aligned} R_s\left(n_s\right) &= a_s n_s + a_s R_{s+1}\left(n_s/a_s\right) \\ &= a_s\left(n_s + R_{s+1}\left(n_s/a_s\right)\right) \\ &= a_s\left(n_s + a_{s+1}\left(n_{s+1} + R_{s+2}\left(n_{s+1}/a_{s+1}\right)\right)\right) \\ &= \dots \end{aligned} \tag{6}$$

This is because each $k \in 0, ..., n_s - 1$ must be evaluated. Assuming $a_s$ $R_{s+1}$ subproblems have already been performed, that leaves an $O(1)$ read operation from the child calculation and $a$ butterfly operations for each $k$. If $N$ has the prime factorization $a_s$, then there will be $|\boldsymbol{a}|$ layers, each with a cost of $n_s a_s$ for each step. The size of each layer is given by

$$n_s = N \prod_{r=0}^{s} a_r^{-1} \tag{7}$$

with $a_0 \equiv 1$. The composition of these runtimes is therefore,

$$\begin{aligned} R(N) &= \sum_{s=1}^{|\boldsymbol{a}|} a_s N \prod_{r=0}^{s} a_r^{-1} \\ &= N \sum_{s=1}^{|\boldsymbol{a}|} \prod_{r=0}^{s-1} a_r^{-1} \end{aligned} \tag{8}$$

Now suppose all $a_s = N/|\boldsymbol{a}| \equiv a$, then

$$R(N) = N \sum_{s=1}^{|\boldsymbol{a}|} a^{-(s-1)} \tag{9}$$