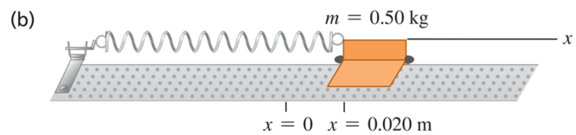


(a) The force exerted *on* the spring (shown by the vector F) has x-component F_x The force exerted *by* the spring has x-component F_x (b) A glider is attached to the same spring and allowed to oscillate.

IDENTIFY and SET UP Because the spring force (equal in magnitude to the stretching force) is proportional to the displacement, the motion is simple harmonic. We find k from Hooke's law, Eq. (14.3), and ω f and T from Eqs. (14.10), (14.11), and (14.12), respectively.

EXECUTE

a. When x the force the spring exerts on the spring balance is F_x From Eq. (14.3),

$$k \qquad \frac{F_x}{x} \qquad \underline{\qquad\qquad}$$

b. From Eqs. (14.10), (14.11), and (14.12), with m

$$\omega \qquad \sqrt{\frac{k}{m}}$$

$$f \qquad \frac{\omega}{\pi} \qquad \pi$$

$$T \qquad \frac{}{f}$$

EVALUATE The amplitude of the oscillation is 0.020 m, the distance that we pulled the glider before releasing it. In SHM the angular frequency, frequency, and period are all independent of the amplitude. Note that a period is usually stated in “seconds” rather than “seconds per cycle.”

KEY CONCEPT

The magnitude of the force exerted by an ideal spring equals the spring constant k times the distance the spring is stretched or compressed. An object of mass m attached to an ideal spring oscillates in simple harmonic motion, with its period, frequency, and angular frequency determined by the values of k and m

Video Tutor Solution: **Example 14.2**

Displacement, Velocity, and Acceleration in SHM

We still need to find the displacement x as a function of time for a harmonic oscillator. Equation (14.4) for an object in SHM along the x-axis is identical to Eq. (14.8) for the x-coordinate of the reference point in uniform circular motion with constant angular speed $\omega \qquad \overline{k \;\; m}$ Hence Eq. (14.5), $x \quad A \quad \theta$ describes the x-coordinate for both situations. If at t the phasor OQ makes an angle ϕ (the Greek letter phi) with the positive x-axis, then at any later time t this angle is θ ωt ϕ We substitute this into Eq. (14.5) to obtain

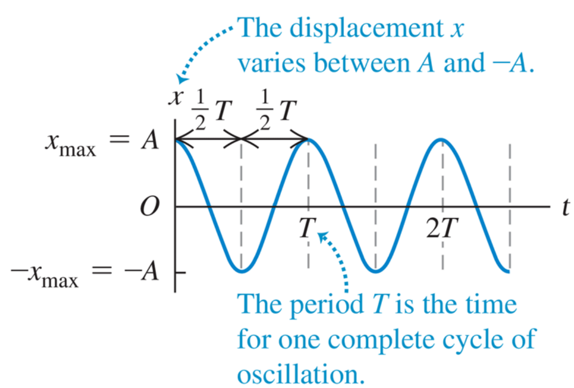
(14.13)

Displacement in simple harmonic motion as a function of time

$$\underbrace{x}_{\text{Displacement in simple harmonic motion as a function of time}} = \overbrace{A}^{\text{Amplitude}} \cos(\underbrace{\omega}_{\text{Angular frequency} = \sqrt{k/m}} \overbrace{t}^{\text{Time}} + \overbrace{\phi}^{\text{Phase angle}}) \qquad (14.13)$$

Figure 14.9 shows a graph of Eq. (14.13) for the particular case ϕ We could also have written Eq. (14.13) in terms of a sine function rather than a cosine by using the identity α α π *In simple harmonic motion the displacement is a periodic, sinusoidal function of time*. There are many other periodic functions, but none so simple as a sine or cosine function.

Figure 14.9



Graph of x versus t [see Eq. (14.13)] for simple harmonic motion. The case shown has ϕ

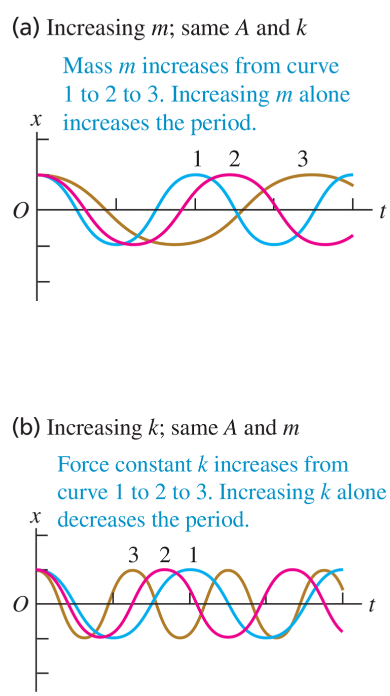
The value of the cosine function is always between and 1, so in Eq. (14.13), x is always between A and A This confirms that A is the amplitude of the motion.

The cosine function in Eq. (14.13) repeats itself whenever time t increases by one period T or when $\omega t \quad \phi$ increases by π radians. Thus, if we start at time t the time T to complete one cycle is

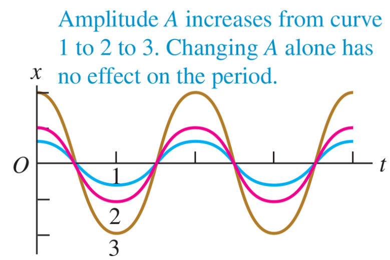
$$\omega T \qquad \sqrt{\frac{k}{m}}\,T \qquad \pi \qquad\qquad T \qquad \pi \qquad \sqrt{\frac{m}{k}}$$

which is just Eq. (14.12). Changing either m or k changes the period T (Figs. 14.10a and 14.10b), but T does not depend on the amplitude A (Fig. 14.10c).

Figure 14.10



(c) Increasing A; same k and m



Variations of simple harmonic motion. All cases shown have ϕ [see Eq. (14.13)].

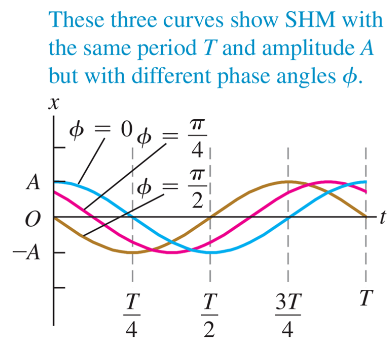
The constant ϕ in Eq. (14.13) is called the **phase angle**. It tells us at what point in the cycle the motion was at t (equivalent to where around the circle the point Q was at t We denote the displacement at t by x Putting t and x x in Eq. (14.13), we get

(14.14)

$$x \quad A \quad \phi$$

If ϕ then x A A and the object starts at its maximum positive displacement. If ϕ π then x A π A and the particle starts at its maximum *negative* displacement. If ϕ π then x A π and the particle is initially at the origin. Figure 14.11 shows the displacement x versus time for three different phase angles.

Figure 14.11



Variations of simple harmonic motion: same m k and A but different phase angles ϕ

We find the velocity v_x and acceleration a_x as functions of time for a harmonic oscillator by taking derivatives of Eq. (14.13) with respect to time:

(14.15)

$$v_x \quad \frac{dx}{dt} \quad \omega A \quad \omega t \quad \phi$$

(14.16)

$$a_x \quad \frac{dv_x}{dt} \quad \frac{d\ x}{dt} \quad \omega\ A \quad \omega t \quad \phi$$

The velocity v_x oscillates between

$$v \qquad \omega A \qquad v \qquad \omega A$$

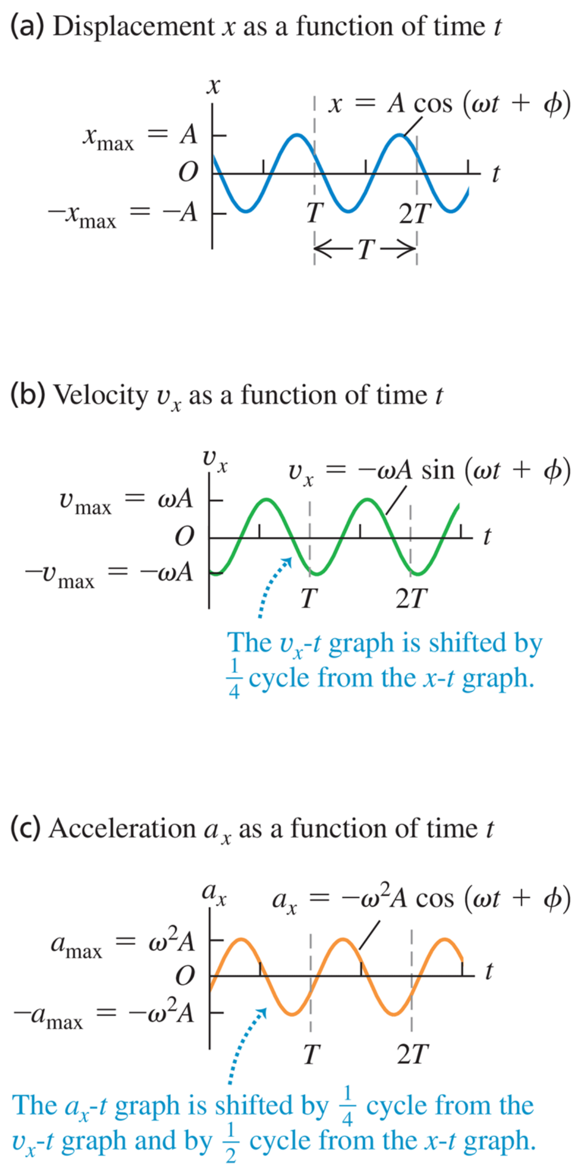
and the acceleration a_x oscillates between

$$a \qquad \omega\ A \qquad a \qquad \omega\ A$$

(Fig. 14.12). Comparing Eq. (14.16) with Eq. (14.13) and recalling that ω k m from Eq. (14.9), we see that

$$a_x \qquad \omega \; x \qquad \frac{k}{m}x$$

Figure 14.12



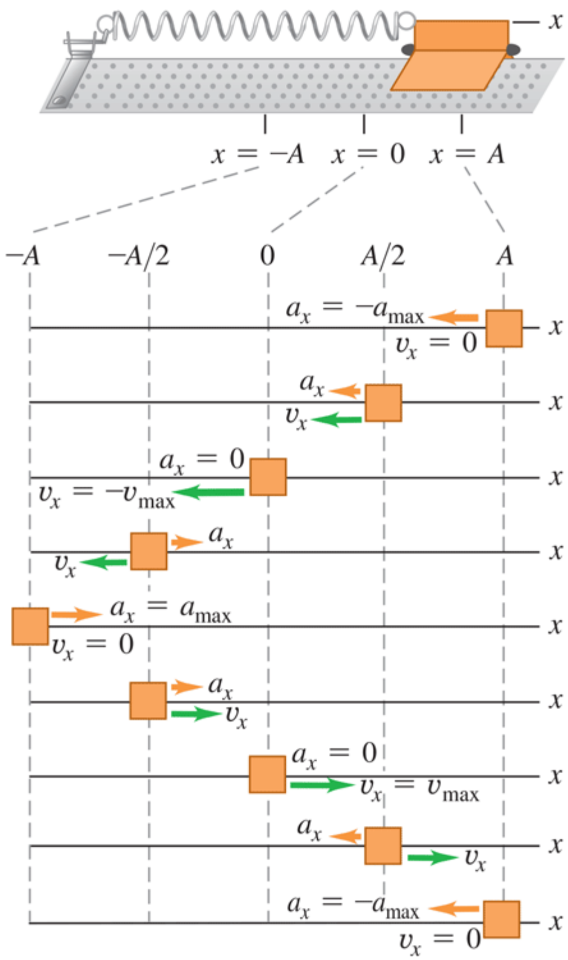
Graphs of (a) x versus t (b) v_x versus t and (c) a_x versus t for an object in SHM. For the motion depicted in these graphs, $\phi \quad \pi$

which is just Eq. (14.4) for simple harmonic motion. This confirms that Eq. (14.13) for x as a function of time is correct.

We actually derived Eq. (14.16) earlier in a geometrical way by taking the x-component of the acceleration vector of the reference point Q This was done in Fig. 14.6b and Eq. (14.7) (recall that θ ωt ϕ In the same way, we could have derived Eq. (14.15) by taking the x-component of the velocity vector of Q as shown in Fig. 14.6b. We'll leave the details for you to work out.

Note that the sinusoidal graph of displacement versus time (Fig. 14.12a) is shifted by one-quarter period from the graph of velocity versus time (Fig. 14.12b) and by one-half period from the graph of acceleration versus time (Fig. 14.12c). Figure 14.13 shows why this is so. When the object is passing through the equilibrium position so that x the velocity equals either v or v (depending on which way the object is moving) and the acceleration is zero. When the object is at either its most positive displacement, x A or its most negative displacement, x A the velocity is zero and the object is instantaneously at rest. At these points, the restoring force F_x kx and the acceleration of the object have their maximum magnitudes. At x A the acceleration is negative and equal to a At x A the acceleration is positive: a_x a

Figure 14.13



How x-velocity v_x and x-acceleration a_x vary during one cycle of SHM.

Here's how we can determine the amplitude A and phase angle ϕ for an oscillating object if we are given its initial displacement x and initial velocity v_x The initial velocity v_x is the velocity at time t putting v_x v_x and t in Eq. (14.15), we find

(14.17)

$$v_x \qquad \omega A \qquad \phi$$

To find ϕ we divide Eq. (14.17) by Eq. (14.14). This eliminates A and gives an equation that we can solve for ϕ

(14.18)

$$\frac{v_{\;x}}{x} \qquad \frac{\omega A \qquad \phi}{A \qquad \phi} \qquad \omega \qquad \phi$$

$$\phi \qquad \qquad \frac{v_{\;x}}{\omega x}$$

It is also easy to find the amplitude A if we are given x and $v_{\;x}$ We'll sketch the derivation, and you can fill in the details. Square Eq. (14.14); then divide Eq. (14.17) by ω square it, and add to the square of Eq. (14.14). The right side will be $A \qquad \phi \qquad \phi$ which is equal to A The final result is

(14.19)

$$A \qquad \sqrt{x \qquad \frac{v_{\;x}}{\omega}}$$

Note that when the object has both an initial displacement x and a nonzero initial velocity $v_{\;x}$ the amplitude A is *not* equal to the initial displacement. That's reasonable; if you start the object at a positive x but give it a positive velocity $v_{\;x}$ it will go *farther* than x before it turns and comes back, and so $A \quad x$

Problem-Solving Strategy 14.1 Simple Harmonic Motion I: Describing Motion

IDENTIFY *the relevant concepts*: An oscillating system undergoes simple harmonic motion (SHM) *only* if the restoring force is directly proportional to the displacement.

SET UP *the problem* using the following steps:

1. Identify the known and unknown quantities, and determine which are the target variables.

2. Distinguish between two kinds of quantities. *Properties of the system* include the mass m the force constant k and quantities derived from m and k such as the period T frequency f and angular frequency ω These are independent of *properties of the motion*, which describe how the system behaves when it is set into motion in a particular way; they include the amplitude A maximum velocity v and phase angle ϕ and values of x v_x and a_x at particular times.
3. If necessary, define an x-axis as in Fig. 14.13, with the equilibrium position at x

EXECUTE *the solution* as follows:

1. Use the equations given in Sections 14.1 and 14.2 to solve for the target variables.
2. To find the values of x v_x and a_x at particular times, use Eqs. (14.13), (14.15), and (14.16), respectively. If both the initial displacement x and initial velocity v_x are given, determine ϕ and A from Eqs. (14.18) and (14.19). If the object has an initial positive displacement x but zero initial velocity v_x then the amplitude is A x and the phase angle is ϕ If it has an initial positive velocity v_x but no initial displacement x the amplitude is A v_x ω and the phase angle is ϕ π Express all phase angles in *radians*.

EVALUATE *your answer*: Make sure that your results are consistent. For example, suppose you used x and v_x to find general expressions for x and v_x at time t If you substitute t into these expressions, you should get back the given values of x and v_x

Example 14.3 Describing SHM

WITH VARIATION PROBLEMS

We give the glider of Example 14.2 an initial displacement x and an initial velocity v_x (a) Find the period, amplitude, and phase angle of the resulting motion. (b) Write equations for the displacement, velocity, and acceleration as functions of time.

IDENTIFY and SET UP As in Example 14.2, the oscillations are SHM. We use equations from this section and the given values k m x and v_x to calculate the target variables A and ϕ and to obtain expressions for x v_x and a_x

EXECUTE

a. In SHM the period and angular frequency are *properties of the system* that depend on only k and m not on the amplitude, and so are the same as in Example 14.2 T and ω From Eq. (14.19), the amplitude is

$$A \quad \sqrt{x \quad \frac{v_x}{\omega}}$$

We use Eq. (14.18) to find the phase angle:

$$\phi \quad \frac{v_x}{\omega x}$$

b. The displacement, velocity, and acceleration at any time are given by Eqs. (14.13), (14.15), and (14.16), respectively. We substitute the values of A ω and ϕ into these equations:

$$x \qquad t$$
$$v_x \qquad t$$
$$a_x \qquad t$$

EVALUATE You can check the expressions for x and v_x by confirming that if you substitute t they yield x x and v_x v x

KEY CONCEPT

You can determine the amplitude and phase angle of a simple harmonic oscillation from the initial position, initial velocity, and angular frequency of the motion.

Video Tutor Solution: **Example 14.3**

Test Your Understanding of Section 14.2

A glider is attached to a spring as shown in Fig. 14.13. If the glider is moved to x and released from rest at time t it will oscillate with amplitude A and phase angle ϕ (a) Suppose instead that at t the glider is at x and is moving to the right in Fig. 14.13. In this situation is the amplitude greater than, less than, or equal to 0.10 m? Is the phase angle greater than, less than, or equal to zero? (b) Suppose instead that at t the glider is at x and is moving to the left in Fig. 14.13. In this situation is the amplitude greater than, less than, or equal to 0.10 m? Is the phase angle greater than, less than, or equal to zero?

14.3 Energy in Simple Harmonic Motion

We can learn even more about simple harmonic motion by using energy considerations. The only horizontal force on the object in SHM in Figs. 14.2 and 14.13 is the conservative force exerted by an ideal spring. The vertical forces do no work, so the total mechanical energy of the system is *conserved*. We also assume that the mass of the spring itself is negligible.

The kinetic energy of the object is $K = \frac{1}{2}mv^2$ and the potential energy of the spring is $U = \frac{1}{2}kx^2$, just as in Section 7.2. There are no nonconservative forces that do work, so the total mechanical energy $E = K + U$ is conserved:

(14.20)

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \text{constant}$$

(Since the motion is one-dimensional, $v^2 = v_x^2$.)

The total mechanical energy E is also directly related to the amplitude A of the motion. When the object reaches the point $x = A$, its maximum displacement from equilibrium, it momentarily stops as it turns back toward the equilibrium position. That is, when $x = A$ (or $-A$), $v_x = 0$. At this point the energy is entirely potential, and $E = \frac{1}{2}kA^2$. Because E is constant, it is equal to $\frac{1}{2}kA^2$ at any other point. Combining this expression with Eq. (14.20), we get

(14.21)

Total mechanical energy in simple harmonic motion

$$E = \frac{1}{2}\overset{\text{Mass}}{m}\underset{\text{Velocity}}{v_x}^2 + \frac{1}{2}\overset{\text{Force constant of restoring force}}{k}\underset{\text{Displacement}}{x}^2 = \frac{1}{2}k\underset{\text{Amplitude}}{A}^2 = \text{constant} \qquad (14.21)$$

We can verify this equation by substituting x and v_x from Eqs. (14.13) and (14.15) and using $\omega^2 = k/m$ from Eq. (14.9):

$$\begin{aligned} E &= \frac{1}{2}\,m_x^2 + \frac{1}{2}\,kx^2 = \frac{1}{2}\,m[-\omega A \sin(\omega t + \phi)]^2 + \frac{1}{2}\,k[A\cos(\omega t + \phi)]^2 \\ &= \frac{1}{2}kA^2 \sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi) = \frac{1}{2}kA^2 \end{aligned}$$

(Recall that $\sin^2\alpha + \cos^2\alpha = 1.$) Hence our expressions for displacement and velocity in SHM are consistent with energy conservation, as they must be.

We can use Eq. (14.21) to solve for the velocity v_x of the object at a given displacement x :

(14.22)

$$v_x = \pm\sqrt{\frac{k}{m}}\,\sqrt{A^2 - x^2}$$

The $\pm$ sign means that at a given value of x the object can be moving in either direction. For example, when $x = \pm A/2$,

$$v_x = \pm\sqrt{\frac{k}{m}}\,\sqrt{A^2 - \left(\pm\frac{A}{2}\right)^2} = \pm\sqrt{\frac{3}{4}}\,\sqrt{\frac{k}{m}}\,A$$

Equation (14.22) also shows that the *maximum* speed $v_{\max}$ occurs at $x = 0$. Using Eq. (14.10), $\omega = \sqrt{k/m}$, we find that

(14.23)

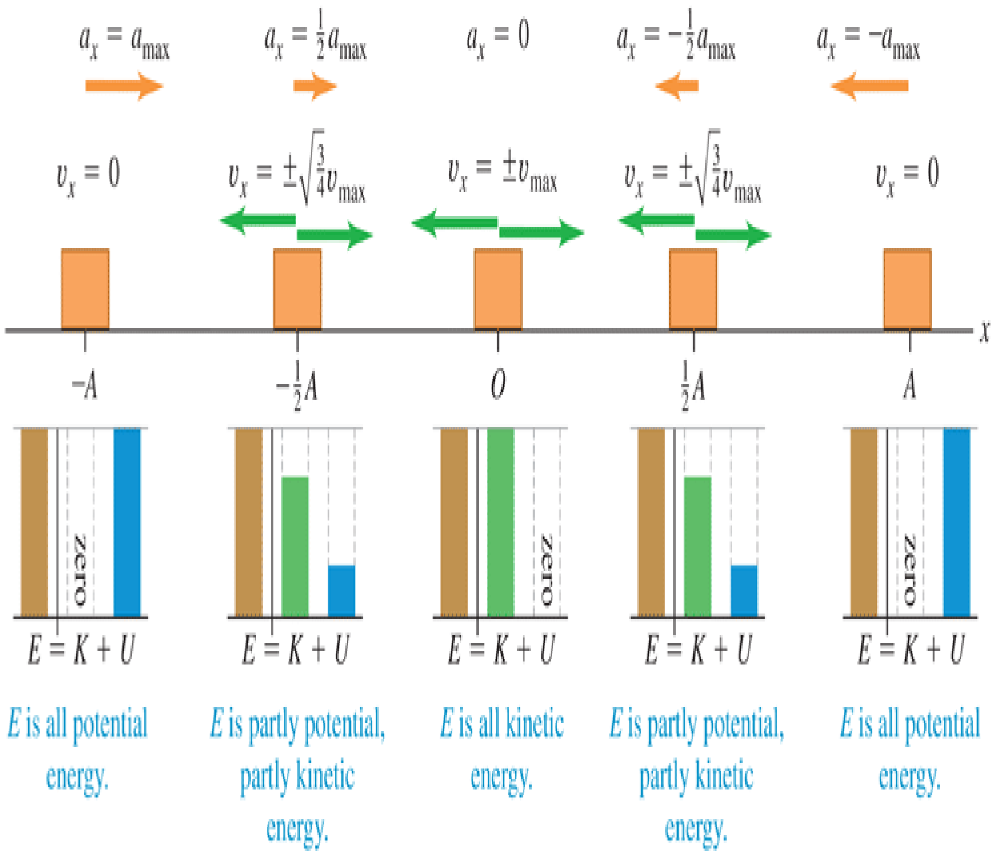
$$v_{\max} = \sqrt{\frac{k}{m}}\,A = \omega A$$

This agrees with Eq. (14.15): v_x oscillates between $-\omega A$ and $+\omega A$.

Interpreting *E*, *K*, and *U* in SHM

Figure 14.14 shows the energy quantities E, K, and U at $x = 0$, $x = \pm A/2$, and $x = \pm A$. Figure 14.15 (next page) is a graphical display of Eq. (14.21); energy (kinetic, potential, and total) is plotted vertically and the coordinate x is plotted horizontally. The parabolic curve in Fig. 14.15a represents the potential energy $U = \frac{1}{2}kx^2$. The horizontal line represents the total mechanical energy E, which is constant and does not vary with x. At any value of x between $-A$ and A, the vertical distance from the x-axis to the parabola is U; since $E = K + U$, the remaining vertical distance up to the horizontal line is K. Figure 14.15b shows both K and U as functions of x. The horizontal line for E intersects the potential-energy curve at $x = -A$ and $x = A$, so at these points the energy is entirely potential, the kinetic energy is zero, and the object comes momentarily to rest before reversing direction. As the object oscillates between $-A$ and A, the energy is continuously transformed from potential to kinetic and back again.

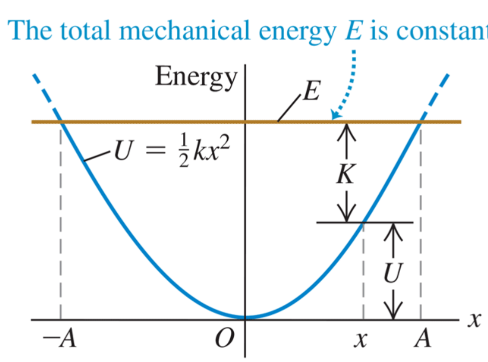
Figure 14.14



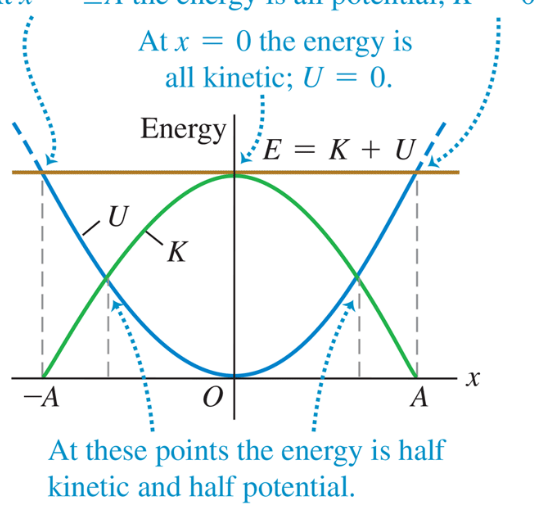
Graphs of E, K, and U versus displacement in SHM. The velocity of the object is *not* constant, so these images of the object at equally spaced positions are *not* equally spaced in time.

Figure 14.15

(a) The potential energy U and total mechanical energy E for an object in SHM as a function of displacement x



(b) The same graph as in (a), showing kinetic energy K as well



Kinetic energy K, potential energy U, and total mechanical energy E as functions of displacement for SHM. At each value of x the sum of the values of K and U equals the constant value of E. Can you show that the energy is half kinetic and half potential at $x = \pm\sqrt{\frac{1}{2}}A$?

Figure 14.15a shows the connection between the amplitude A and the corresponding total mechanical energy $E = \frac{1}{2}kA^2$. If we tried to make x greater than A (or less than $-A$), U would be greater than E, and K would have to be negative. But K can never be negative, so x can't be greater than A or less than $-A$.

Problem-Solving Strategy 14.2 Simple Harmonic Motion II: Energy

The SHM energy equation, Eq. (14.21), is a useful relationship among velocity, displacement, and total mechanical energy. If a problem requires you to relate displacement, velocity, and acceleration without reference to time, consider using Eq. (14.4) (from Newton's second law) or Eq. (14.21) (from energy conservation). Because Eq. (14.21) involves x^2 and $v_x{}^2$, you must infer the *signs* of x and v_x from the situation. For instance, if the object is moving from the equilibrium position toward the point of greatest positive displacement, then x is positive and v_x is positive.

Example 14.4 Velocity, acceleration, and energy in SHM

WITH VARIATION PROBLEMS

(a) Find the maximum and minimum velocities attained by the oscillating glider of Example 14.2. (b) Find the maximum and minimum accelerations. (c) Find the velocity v_x and acceleration a_x when the glider is halfway from its initial position to the equilibrium position $x = 0$. (d) Find the total mechanical energy, potential energy, and kinetic energy at this position.

IDENTIFY and SET UP The problem concerns properties of the motion at specified *positions*, not at specified *times*, so we can use the energy relationships of this section. Figure 14.13 shows our choice of x-axis. The maximum displacement from equilibrium is $A = 0.020$ m. We use Eqs. (14.22) and (14.4) to find v_x and a_x for a given x. We then use Eq. (14.21) for given x and v_x to find the total, potential, and kinetic energies E, U, and K.

EXECUTE

a. From Eq. (14.22), the velocity v_x at any displacement x is

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

The glider's maximum *speed* occurs when it is moving through $x = 0$:

$$v_{\max} = \sqrt{\frac{k}{m}} A = \sqrt{\frac{200 \text{ N/m}}{0.50 \text{ kg}}} (0.020 \text{ m}) = 0.40 \text{ m/s}$$

Its maximum and minimum (most negative) *velocities* are $+0.40$ m/s and -0.40 m/s, which occur when it is moving through $x = 0$ to the right and left, respectively.

b. From Eq. (14.4), $a_x = -(k/m)\, x$. The glider's maximum (most positive) acceleration occurs at the most negative value of x, $x = -A$:

$$a_{\max} = -\frac{k}{m}(-A) = -\frac{200 \text{ N/m}}{0.50 \text{ kg}}(-0.020 \text{ m}) = 8.0 \text{ m/s}^2$$

The minimum (most negative) acceleration is $a_{\min} = -8.0 \text{ m/s}^2$, which occurs at $x = +A = +0.020$ m.

c. The point halfway from $x = x_0 = A$ to $x = 0$ is $x = A/2 = 0.010$ m. From Eq. (14.22), at this point

$$v_x = -\sqrt{\frac{200 \text{ N/m}}{0.50 \text{ kg}}} \sqrt{(0.020 \text{ m})^2 - (0.010 \text{ m})^2} = -0.35 \text{ m/s}$$

We choose the negative square root because the glider is moving from $x = A$ toward $x = 0$. From Eq. (14.4),

$$a_x = -\frac{200 \text{ N/m}}{0.50 \text{ kg}}(0.010 \text{ m}) = -4.0 \text{ m/s}^2$$

Figure 14.14 shows the conditions at $x = 0, \ \pm A/2$, and $\pm A$.

d. The energies are

$$
\begin{aligned}
E &= \tfrac{1}{2}kA^2 = \tfrac{1}{2}\,(200\ \mathrm{N/m})(0.020\ \mathrm{m})^2 = 0.040\ \mathrm{J} \\
U &= \tfrac{1}{2}kx^2 = \tfrac{1}{2}\,(200\ \mathrm{N/m})(0.020\ \mathrm{m})^2 = 0.010\ \mathrm{J} \\
K &= \tfrac{1}{2}mv_x^2 = \tfrac{1}{2}\,(0.50\ \mathrm{kg})(-0.35\ \mathrm{m/s})^2 = 0.030\ \mathrm{J}
\end{aligned}
$$

EVALUATE At $x = A/2$, the total mechanical energy is one-fourth potential energy and three-fourths kinetic energy. You can confirm this by inspecting Fig. 14.15b.

KEY CONCEPT

In simple harmonic motion, the sum of kinetic energy and elastic potential energy is conserved. You can use this to find the velocity of an object in SHM as a function of its displacement.

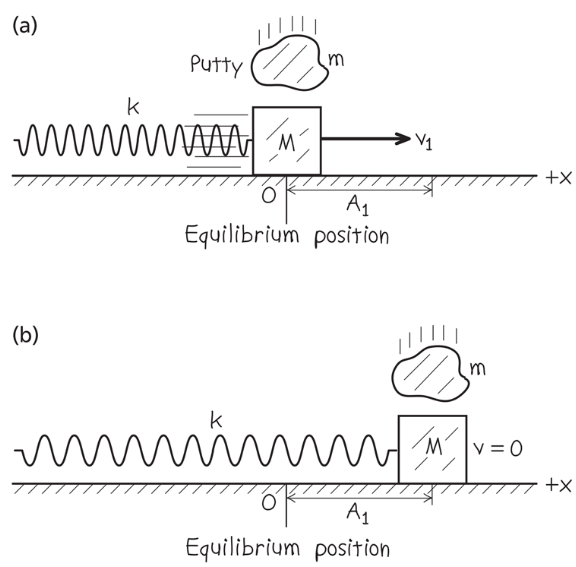
Video Tutor Solution: **Example 14.4**

Example 14.5 Energy and momentum in SHM

A block of mass M attached to a horizontal spring with force constant k is moving in SHM with amplitude A_1. As the block passes through its equilibrium position, a lump of putty of mass m is dropped from a small height and sticks to it. (a) Find the new amplitude and period of the motion. (b) Repeat part (a) if the putty is dropped onto the block when it is at one end of its path.

IDENTIFY and SET UP The problem involves the motion at a given position, not a given time, so we can use energy methods. Figure 14.16 shows our sketches. Before the putty falls, the total mechanical energy of the block–spring system is constant. In part (a), the putty–block collision is completely inelastic: The horizontal component of momentum is conserved, kinetic energy decreases, and the amount of mass that's oscillating increases. After the collision, the total mechanical energy remains constant at its new value. In part (b) the oscillating mass also increases, but the block isn't moving when the putty is added; there is effectively no collision at all, and no mechanical energy is lost. We find the amplitude A_2 after each collision from the final energy of the system by using Eq. (14.21) and conservation of momentum. The period T_2 after the collision is the same in both parts (a) and (b) because the final mass is the same; we find it by using Eq. (14.12).

Figure 14.16



Our sketches for this problem.

EXECUTE

a. Before the collision the total mechanical energy of the block and spring is $E_1 = \frac{1}{2}kA_1^2$. The block is at $x = 0$, so $U = 0$ and the energy is purely kinetic (Fig. 14.16a). If we let v_1 be the speed of the block at this point, then $E_1 = \frac{1}{2}kA_1^2 = \frac{1}{2}Mv_1^2$ and

$$v_1 = \sqrt{\frac{k}{M}}\,A_1$$

During the collision the *x*-component of momentum of the block–putty system is conserved. (Why?) Just before the collision this component is the sum of Mv_1 (for the block) and zero (for the putty). Just after the collision the block and putty move together with speed v_2, so their combined *x*-component of momentum is $(M+m)v_2$. From conservation of momentum,

$$Mv_1 + 0 = (M+m)v_2 \qquad \text{so} \qquad v_2 = \frac{M}{M+m}v_1$$

The collision lasts a very short time, so the block and putty are still at the equilibrium position just after the collision. The energy is still purely kinetic but is *less* than before the collision:

$$\begin{aligned} E_2 &= \frac{1}{2}(M+m)v_2^2 = \frac{1}{2}\frac{M^2}{M+m}v_1^2 \\ &= \frac{M}{M+m}\left(\frac{1}{2}Mv_1^2\right) = \left(\frac{M}{M+m}\right)E_1 \end{aligned}$$

Since $E_2 = \frac{1}{2}kA_2^2$, where A_2 is the amplitude after the collision,

$$\begin{aligned}\frac{1}{2}kA_2^2 &= \left(\frac{M}{M+m}\right)\frac{1}{2}kA_1^2 \\ A_2 &= A_1\sqrt{\frac{M}{M+m}}\end{aligned}$$

From Eq. (14.12), the period of oscillation after the collision is

$$T_2 = 2\pi\sqrt{\frac{M+m}{k}}$$

b. When the putty falls, the block is instantaneously at rest (Fig. 14.16b). The x-component of momentum is zero both before and after the collision. The block and putty have zero kinetic energy just before and just after the collision. The energy is all potential energy stored in the spring, so adding the putty has *no effect* on the total mechanical energy. That is, $E_2 = E_1 = \frac{1}{2}kA_1^2$, and the amplitude is unchanged: $A_2 = A_1$. The period is again $T_2 = 2\pi\sqrt{(M+m)/k}$.

EVALUATE Energy is lost in part (a) because the putty slides against the moving block during the collision, and energy is dissipated by kinetic friction. No energy is lost in part (b) because there is no sliding during the collision.

KEY CONCEPT

You can use conservation of total mechanical energy in simple harmonic motion to help solve problems that involve motion at a given position, not at a given time.

Video Tutor Solution: **Example 14.5**

Test Your Understanding of Section 14.3

(a) To double the total energy for a mass-spring system oscillating in SHM, by what factor must the amplitude increase? (i) 4; (ii) 2; (iii) $\sqrt{2} = 1.414$; (iv) $\sqrt[4]{2} = 1.189$. (b) By what factor will the frequency change due to this amplitude increase? (i) 4; (ii) 2; (iii) $\sqrt{2} = 1.414$; (iv) $\sqrt[4]{2} = 1.189$; (v) it does not change.

14.4 Applications of Simple Harmonic Motion

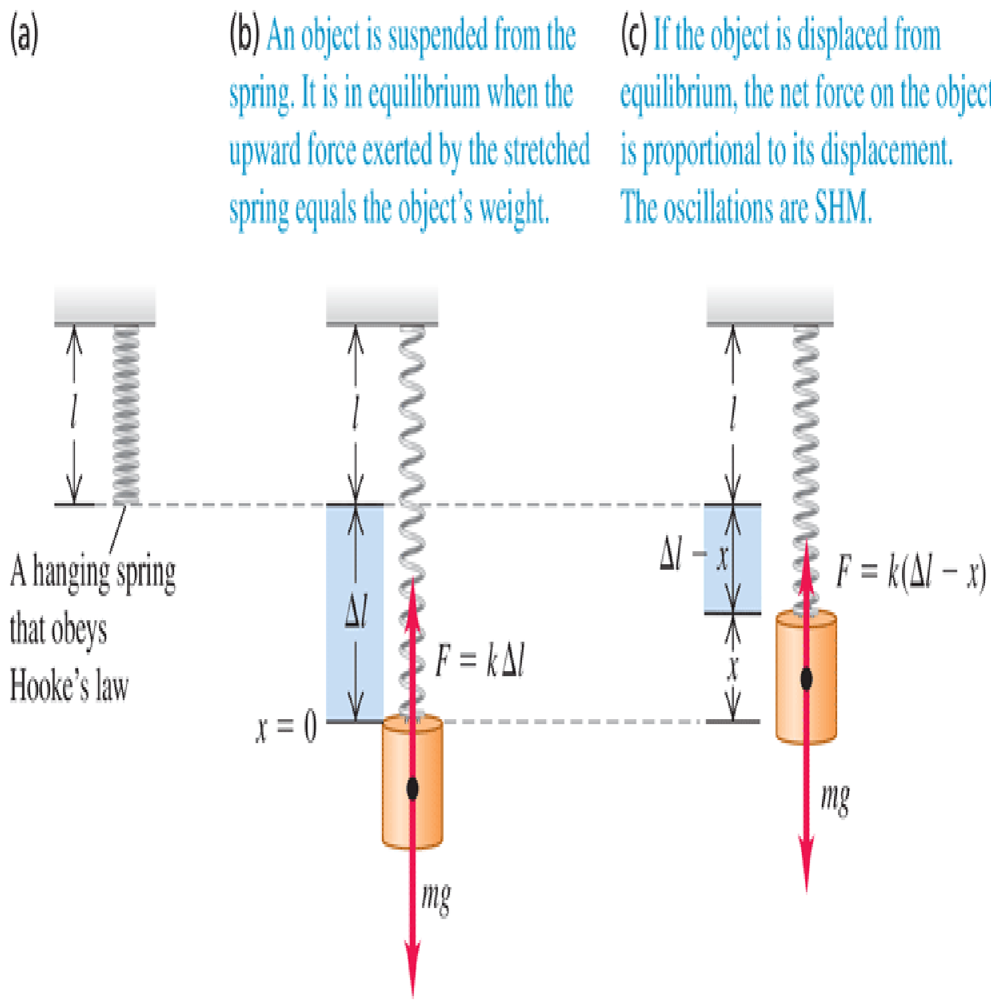
So far, we've looked at a grand total of *one* situation in which simple harmonic motion (SHM) occurs: an object attached to an ideal horizontal spring. But SHM can occur in any system in which there is a restoring force that is directly proportional to the displacement from equilibrium, as given by Eq. (14.3), $F_x = -kx$. The restoring force originates in different ways in different situations, so we must find the force constant k for each case by examining the net force on the system. Once this is done, it's straightforward to find the angular frequency ω, frequency f, and period T; we just substitute the value of k into Eqs. (14.10), (14.11), and (14.12), respectively. Let's use these ideas to examine several examples of simple harmonic motion.

Vertical SHM

Suppose we hang a spring with force constant k (Fig. 14.17a) and suspend from it an object with mass m. Oscillations will now be vertical; will they still be SHM? In Fig. 14.17b the object hangs at rest, in equilibrium. In this position the spring is stretched an amount Δl just great enough that the spring's upward vertical force $k\,\Delta l$ on the object balances its weight mg:

$$k\,\Delta l = mg$$

Figure 14.17



An object attached to a hanging spring.

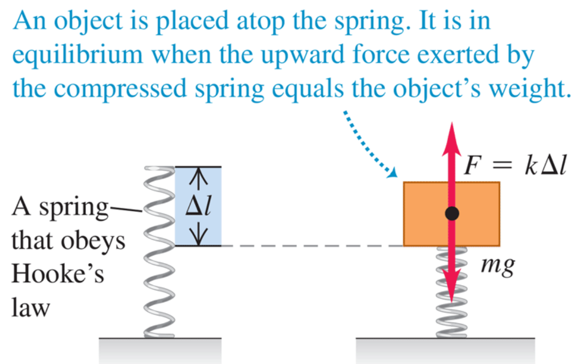
Take $x = 0$ to be this equilibrium position and take the positive x-direction to be upward. When the object is a distance x *above* its equilibrium position (Fig. 14.17c), the extension of the spring is $\Delta l - x$. The upward force it exerts on the object is then $k\left(\Delta l - x\right)$, and the net x-component of force on the object is

$$F_{\text{net}} = k\left(\Delta l - x\right) + \left(-mg\right) = -kx$$

that is, a net downward force of magnitude kx. Similarly, when the object is *below* the equilibrium position, there is a net upward force with magnitude kx. In either case there is a restoring force with magnitude kx.

If the object is set in vertical motion, it oscillates in SHM with the same angular frequency as though it were horizontal, $\omega = \sqrt{k/m}$. So vertical SHM doesn't differ in any essential way from horizontal SHM. The only real change is that the equilibrium position $x = 0$ no longer corresponds to the point at which the spring is unstretched. The same ideas hold if an object with weight mg is placed atop a compressible spring (Fig. 14.18) and compresses it a distance Δl.

Figure 14.18



If the weight mg compresses the spring a distance Δl, the force constant is $k = mg/\Delta l$ and the angular frequency for vertical SHM is $\omega = \sqrt{k/m}$—the same as if the object were suspended from the spring (see Fig. 14.17).

Example 14.6 Vertical SHM in an old car

The shock absorbers in an old car with mass 1000 kg are completely worn out. When a 980 N person climbs slowly into the car at its center of gravity, the car sinks 2.8 cm. The car (with the person aboard) hits a bump, and the car starts oscillating up and down in SHM. Model the car and person as a single object on a single spring, and find the period and frequency of the oscillation.

IDENTIFY and SET UP The situation is like that shown in Fig. 14.18. The compression of the spring when the person's weight is added tells us the force constant, which we can use to find the period and frequency (the target variables).

EXECUTE When the force increases by 980 N, the spring compresses an additional 0.028 m, and the x-coordinate of the car changes by -0.028 m. Hence the effective force constant (including the effect of the entire suspension) is

$$k = -\frac{F_x}{x} = -\frac{980 \text{ N}}{-0.028 \text{ m}} = 3.5 \times 10^4 \text{ kg/s}^2$$

The person's mass is $w/g = (980 \text{ N})/(9.8 \text{ m/s}^2) = 100 \text{ kg}$. The *total* oscillating mass is $m = 1000 \text{ kg} + 100 \text{ kg} = 1100 \text{ kg}$. The period T is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1100 \text{ kg}}{3.5 \times 10^4 \text{ kg/s}^2}} = 1.11 \text{ s}$$

The frequency is $f = 1/T = 1/(1.11 \text{ s}) = 0.90 \text{ Hz}$.

EVALUATE A persistent oscillation with a period of about 1 second makes for a very unpleasant ride. The purpose of shock absorbers is to make such oscillations die out (see Section 14.7).

KEY CONCEPT

The same equations apply to both horizontal and vertical simple harmonic motion. The only effect of gravity on vertical SHM is to change the equilibrium position of the oscillating object.

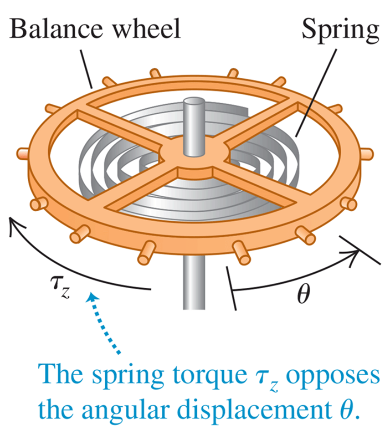
Video Tutor Solution: **Example 14.6**

Angular SHM

A mechanical watch keeps time based on the oscillations of a balance wheel (Fig. 14.19). The wheel has a moment of inertia I about its axis. A coil spring exerts a restoring torque τ_z that is proportional to the angular displacement θ from the equilibrium position. We write $\tau_z = -\kappa\theta$, where κ (the Greek letter kappa) is a constant called the *torsion constant*. Using the rotational analog of Newton's second law for a rigid body, $\sum \tau_z = I\alpha_z = I\, d^2\theta/dt^2$, Eq. (10.7), we find

$$-\kappa\theta = I\alpha \qquad \text{or} \qquad \frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$$

Figure 14.19



The balance wheel of a mechanical watch. The spring exerts a restoring torque that is proportional to the angular displacement θ, so the motion is angular SHM.

This equation is exactly the same as Eq. (14.4) for simple harmonic motion, with x replaced by θ and k/m replaced by κ/I. So we are dealing with a form of *angular* simple harmonic motion. The angular frequency ω and frequency f are given by Eqs. (14.10) and (14.11), respectively, with the same replacement:

(14.24)

$$\text{Angular simple harmonic motion} \rightarrow \overset{\text{Angular frequency}}{\omega} = \underbrace{\sqrt{\frac{\kappa}{I}} \quad \text{and} \quad \overset{\text{Frequency}}{f} = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}}_{\text{Torsion constant divided by moment of inertia}} \qquad (14.24)$$

The angular displacement θ as a function of time is given by

$$\theta = \mathrm{\theta} \cos(\omega t + \phi)$$

where θ (the capital Greek letter theta) plays the role of an angular amplitude.

It’s a good thing that the motion of a balance wheel *is* simple harmonic. If it weren’t, the frequency might depend on the amplitude, and the watch would run too fast or too slow as the spring ran down.

Vibrations of Molecules

The following discussion of the vibrations of molecules uses the binomial theorem. If you aren’t familiar with this theorem, you should read about it in the appropriate section of a math textbook.

When two atoms are separated by a few atomic diameters, they can exert attractive forces on each other. But if the atoms are so close that their electron shells overlap, the atoms repel each other. Between these limits, there can be an equilibrium separation distance at which two atoms form a *molecule*. If these atoms are displaced slightly from equilibrium, they will oscillate.

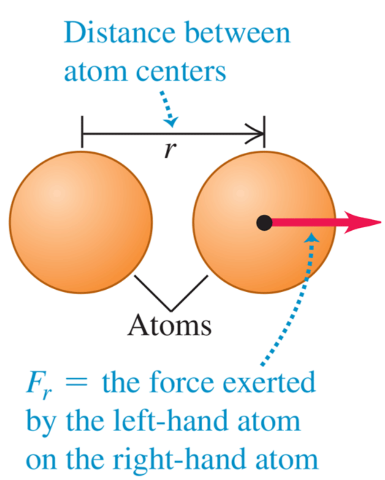
Let's consider one type of interaction between atoms called the *van der Waals interaction*. Our immediate task here is to study oscillations, so we won't go into the details of how this interaction arises. Let the center of one atom be at the origin and let the center of the other atom be a distance r away (Fig. 14.20a); the equilibrium distance between centers is $r = R_0$. Experiment shows that the van der Waals interaction can be described by the potential-energy function

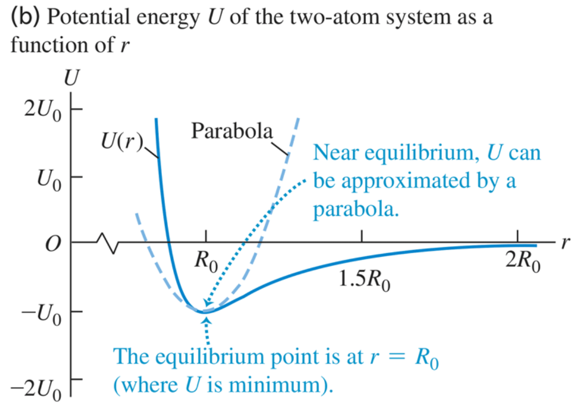
(14.25)

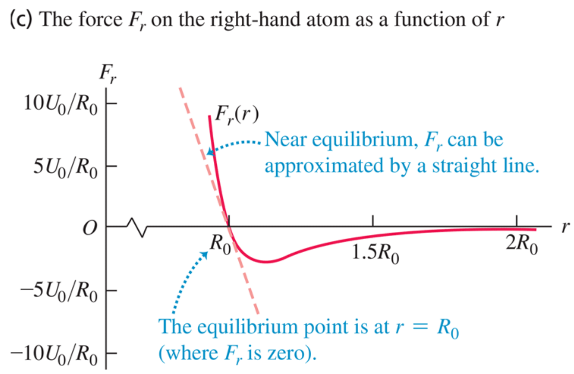
$$U = U_0 \left[\left(\frac{R_0}{r} \right)^{12} - 2 \left(\frac{R_0}{r} \right)^{6} \right]$$

Figure 14.20

(a) Two-atom system







(a) Two atoms with centers separated by r. (b) Potential energy U and (c) force F_r in the van der Waals interaction.

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where U_0 is a positive constant with units of joules. When the two atoms are very far apart, $U = 0$; when they are separated by the equilibrium distance $r = R_0$, $U = -U_0$. From Section 7.4, the force on the second atom is the negative derivative of Eq. (14.25):

(14.26)

$$F_r = -\frac{dU}{dr} = U_0\left[\frac{12R_0^{12}}{r^{13}} - 2\frac{6R_0^6}{r^7}\right] = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^7\right]$$

Figures 14.20b and 14.20c plot the potential energy and force, respectively. The force is positive for $r < R_0$ and negative for $r > R_0$, so it is a *restoring* force.

Let's examine the restoring force F_r in Eq. (14.26). We let x represent the displacement from equilibrium:

$$x = r - R_0 \qquad \text{so} \qquad r = R_0 + x$$

In terms of x, the force F_r in Eq. (14.26) becomes

(14.27)

$$\begin{aligned} F_r &= 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{R_0+x}\right)^{13} - \left(\frac{R_0}{R_0+x}\right)^{7}\right] \\ &= 12\frac{U_0}{R_0}\left[\frac{1}{(1+x/R_0)^{13}} - \frac{1}{(1+x/R_0)^{7}}\right] \end{aligned}$$

This looks nothing like Hooke's law, $F_x = -kx$, so we might be tempted to conclude that molecular oscillations cannot be SHM. But let us restrict ourselves to *small-amplitude* oscillations so that the absolute value of the displacement x is small in comparison to R_0 and the absolute value of the ratio x/R_0 is much less than 1. We can then simplify Eq. (14.27) by using the *binomial theorem*:

(14.28)

$$(1+u)^n = 1 + nu + \frac{n(n-1)}{2!}u^2 + \frac{n(n-1)(n-2)}{3!}u^3 + \cdots$$

If $|u|$ is much less than 1, each successive term in Eq. (14.28) is much smaller than the one it follows, and we can safely approximate $(1+u)^n$ by just the first two terms. In Eq. (14.27), U is replaced by x/R_0 and n equals -13 or -7, so

(14.29)

$$\begin{aligned}\frac{1}{(1+x/R_0)^{13}} &= (1+x/R_0)^{-13} \approx 1+(-13)\frac{x}{R_0}\\ \frac{1}{(1+x/R_0)^{7}} &= (1+x/R_0)^{-7} \approx 1+(-7)\frac{x}{R_0}\\ F_r \approx 12\frac{U_0}{R_0}\left[\left(1+(13)\frac{x}{R_0}\right)-\left(1+(-7)\frac{x}{R_0}\right)\right] &= -\left(\frac{72U_0}{R_0^2}\right)x\end{aligned}$$

This is just Hooke's law, with force constant $k = 72U_0/R_0^2$. (Note that k has the correct units, $\mathrm{J/m^2}$ or $\mathrm{N/m}$.) So oscillations of molecules bound by the van der Waals interaction can be simple harmonic motion, provided that the amplitude is small in comparison to R_0 so that the approximation $|x/R_0| \ll 1$ used in the derivation of Eq. (14.29) is valid.

You can also use the binomial theorem to show that the potential energy U in Eq. (14.25) can be written as $U \approx \frac{1}{2}kx^2 + C$, where $C = -U_0$ and k is again equal to $72U_0/R_0^2$. Adding a constant to the potential-energy function has no effect on the physics, so the system of two atoms is fundamentally no different from a mass attached to a horizontal spring for which $U = \frac{1}{2}kx^2$.

Example 14.7 Molecular vibration

Two argon atoms form the molecule Ar_2 as a result of a van der Waals interaction with $U_0 = 1.68 \times 10^{-21}$ J and $R_0 = 3.82 \times 10^{-10}$ m. Find the frequency of small oscillations of one Ar atom about its equilibrium position.

IDENTIFY and SET UP This is the situation shown in Fig. 14.20. Because the oscillations are small, we can use Eq. (14.29) to find the force constant k and Eq. (14.11) to find the frequency f of SHM.

EXECUTE From Eq. (14.29),

$$k = \frac{72U_0}{R_0^2} = \frac{72(1.68 \times 10^{-21}\ \text{J})}{(3.82 \times 10^{-10}\ \text{m})^2} = 0.829\ \text{J/m}^2 = 0.829\ \text{N/m}$$

(This force constant is comparable to that of a loose toy spring like a Slinky™.) From Appendix D, the average atomic mass of argon is $(39.948\ \text{u})(1.66 \times 10^{-27}\ \text{kg}/1\ \text{u}) = 6.63 \times 10^{-26}\ \text{kg}$.

From Eq. (14.11), if one atom is fixed and the other oscillates,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.829\ \text{N/m}}{6.63 \times 10^{-26}\ \text{kg}}} = 5.63 \times 10^{11}\ \text{Hz}$$

EVALUATE Our answer for f isn't quite right. If no net external force acts on the molecule, its center of mass (halfway between the atoms) doesn't accelerate, so *both* atoms must oscillate with the same amplitude in opposite directions. It turns out that we can account for this by replacing m with $m/2$ in our expression for f. This makes f larger by a factor of $\sqrt{2}$, so the correct frequency is $f = \sqrt{2}(5.63 \times 10^{11}\ \text{Hz}) = 7.96 \times 10^{11}\ \text{Hz}$. What's more, on the atomic scale we must use *quantum mechanics* rather than Newtonian mechanics to describe motion; happily, quantum mechanics also yields $f = 7.96 \times 10^{11}\ \text{Hz}$.

KEY CONCEPT

Molecular vibrations, as well as the vibrations of many other systems, are approximately simple harmonic if the oscillation amplitude is sufficiently small.

Video Tutor Solution: **Example 14.7**

Test Your Understanding of Section 14.4

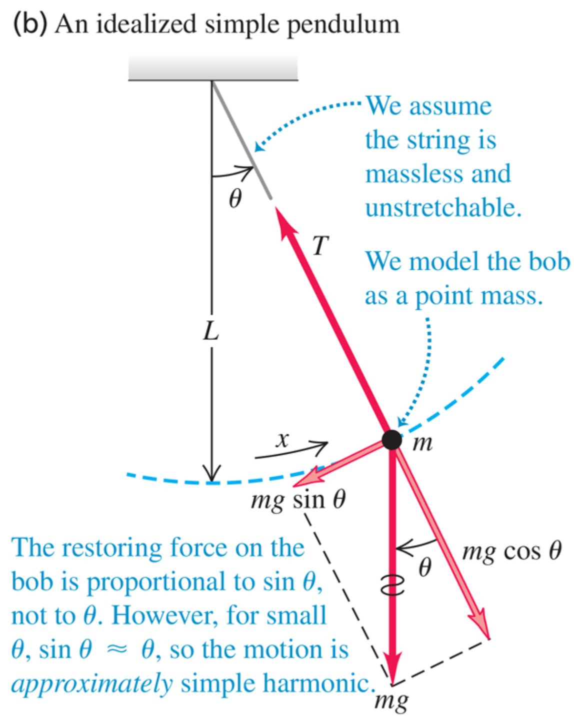
A block attached to a hanging ideal spring oscillates up and down with a period of 10 s on earth. If you take the block and spring to Mars, where the acceleration due to gravity is only about 40% as large as on earth, what will be the new period of oscillation? (i) 10 s; (ii) more than 10 s; (iii) less than 10 s.

14.5 The Simple Pendulum

A **simple pendulum** is an idealized model consisting of a point mass suspended by a massless, unstretchable string. When the point mass is pulled to one side of its straight-down equilibrium position and released, it oscillates about the equilibrium position. Familiar situations such as a wrecking ball on a crane's cable or a person on a swing (Fig. 14.21a) can be modeled as simple pendulums.

Figure 14.21

(a) A real pendulum



The dynamics of a simple pendulum.

The path of the point mass (sometimes called a *pendulum bob*) is not a straight line but the arc of a circle with radius L equal to the length of the string (Fig. 14.21b). We use as our coordinate the distance x measured along the arc. If the motion is simple harmonic, the restoring force must be directly proportional to x or (because $x = L\theta$) to θ. Is it?

Figure 14.21b shows the radial and tangential components of the forces on the mass. The restoring force F_θ is the tangential component of the net force:

(14.30)

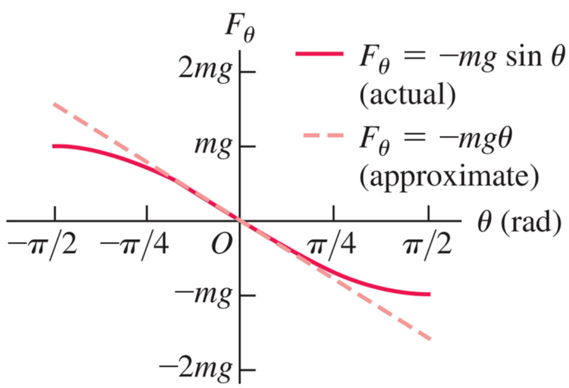
$$F_\theta = -mg \sin\theta$$

Gravity provides the restoring force F_θ; the tension T merely acts to make the point mass move in an arc. Since F_θ is proportional to $sin\ \theta$, not to θ, the motion is *not* simple harmonic. However, if angle θ is *small*, $\sin\theta$ is very nearly equal to θ in radians (Fig. 14.22). (When $\theta = 0.1$ rad, about 6°, $\sin\theta = 0.0998$. That's only 0.2% different.) With this approximation, Eq. (14.30) becomes

(14.31)

$$F_\theta = -mg\theta = -mg\frac{x}{L} = -\frac{mg}{L}x$$

Figure 14.22



For small angular displacements θ, the restoring force $F_\theta = -mg\sin\theta$ on a simple pendulum is approximately equal to $-mg\theta$; that is, it is approximately proportional to the displacement θ. Hence for small angles the oscillations are simple harmonic.

The restoring force is then proportional to the coordinate for small displacements, and the force constant is $k = mg/L$. From Eq. (14.10) the angular frequency ω of a simple pendulum with small amplitude is

(14.32)

Angular frequency of simple pendulum, small amplitude

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}} \qquad (14.32)$$

Acceleration due to gravity (g); Pendulum length (L); Pendulum mass (cancels) (m)

The corresponding frequency and period relationships are

(14.33)

$$\underbrace{f}_{\text{Frequency of simple pendulum, small amplitude}} = \frac{\overbrace{\omega}^{\text{Angular frequency}}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\overbrace{g}^{\text{Acceleration due to gravity}}}{\underbrace{L}_{\text{Pendulum length}}}} \qquad (14.33)$$

(14.34)

$$\underbrace{T}_{\text{Period of simple pendulum, small amplitude}} = \frac{2\pi}{\underbrace{\omega}_{\text{Angular frequency}}} = \frac{1}{\underbrace{f}_{\text{Frequency}}} = 2\pi\sqrt{\frac{\overbrace{L}^{\text{Pendulum length}}}{\underbrace{g}_{\text{Acceleration due to gravity}}}} \qquad (14.34)$$

These expressions don't involve the *mass* of the particle. That's because the gravitational restoring force is proportional to m, so the mass appears on *both* sides of $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ and cancels out. (The same physics explains why objects of different masses fall with the same acceleration in a vacuum.) For small oscillations, the period of a pendulum for a given value of g is determined entirely by its length.

Equations (14.32), (14.33) and (14.34) tell us that a long pendulum (large L) has a longer period than a shorter one. Increasing g increases the restoring force, causing the frequency to increase and the period to decrease.

The motion of a pendulum is only *approximately* simple harmonic. When the maximum angular displacement Θ (amplitude) is not small, the departures from simple harmonic motion can be substantial. In general, the period T is given by

(14.35)

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1^2}{2^2}\sin^2\frac{\Theta}{2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}\sin^4\frac{\Theta}{2} + \cdots\right)$$

We can compute T to any desired degree of precision by taking enough terms in the series. You can confirm that when $\Theta = 15^\circ$, the true period is longer than that given by the approximate Eq. (14.34) by less than 0.5%.

A pendulum is a useful timekeeper because the period is *very nearly* independent of amplitude, provided that the amplitude is small. Thus, as a pendulum clock runs down and the amplitude of the swings decreases a little, the clock still keeps very nearly correct time.

Example 14.8 A simple pendulum

WITH VARIATION PROBLEMS

Find the period and frequency of a simple pendulum 1.000 m long at a location where $g = 9.800\ \mathrm{m/s^2}$.

IDENTIFY and SET UP This is a simple pendulum, so we can use Eq. (14.34) to determine the pendulum's period T from its length and Eq. (14.1) to find the frequency f from T.

EXECUTE From Eqs. (14.34) and (14.1),

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.000\ \mathrm{m}}{9.800\ \mathrm{m/s^2}}} = 2.007\ \mathrm{s}$$

and

$$f = \frac{1}{T} = \frac{1}{2.007\ \text{s}} = 0.4982\ \text{Hz}$$

EVALUATE The period is almost exactly 2 s. When the metric system was established, the second was *defined* as half the period of a 1 m simple pendulum. This was a poor standard, however, because the value of g varies from place to place. We discussed more modern time standards in Section 1.3.

KEY CONCEPT

The period of a simple pendulum depends only on the length of the pendulum and the value of g, not on the mass of the pendulum bob.

Video Tutor Solution: **Example 14.8**

Test Your Understanding of Section 14.5

When an object oscillating on a horizontal spring passes through its equilibrium position, its acceleration is zero (see Fig. 14.2b). When the bob of an oscillating simple pendulum passes from left to right through its equilibrium position, is its acceleration (i) zero; (ii) to the left; (iii) to the right; (iv) upward; or (v) downward?

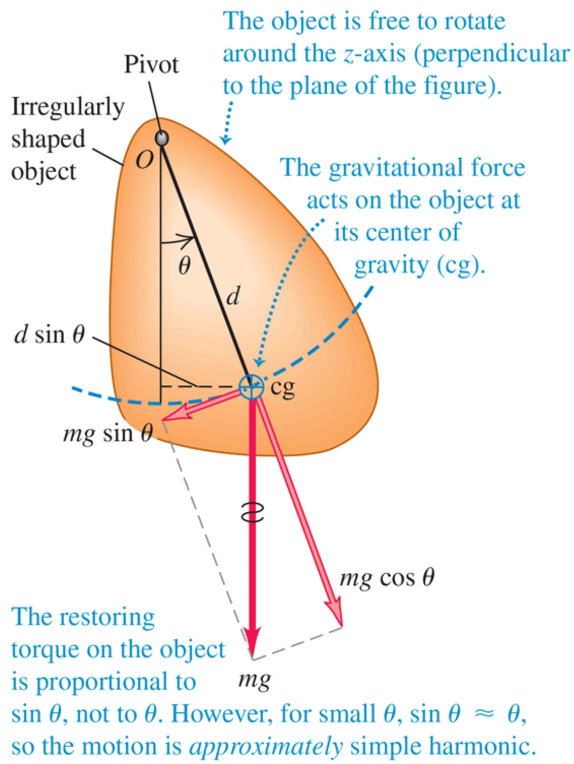
14.6 The Physical Pendulum

A **physical pendulum** is any *real* pendulum that uses an extended object, in contrast to the idealized *simple* pendulum with all of its mass concentrated at a point. Figure 14.23 shows an object of irregular shape pivoted so that it can turn without friction about an axis through point O. In equilibrium the center of gravity (cg) is directly below the pivot; in the position shown, the object is displaced from equilibrium by an angle θ, which we use as a coordinate for the system. The distance from O to the center of gravity is d, the moment of inertia of the object about the axis of rotation through O is I, and the total mass is m. When the object is displaced as shown, the weight mg causes a restoring torque

(14.36)

$$\tau_z = -(mg)(d \sin \theta)$$

Figure 14.23



Dynamics of a physical pendulum.

The negative sign shows that the restoring torque is clockwise when the displacement is counterclockwise, and vice versa.

When the object is released, it oscillates about its equilibrium position. The motion is not simple harmonic because the torque τ_z is proportional to $\sin\theta$ rather than to θ itself. However, if θ is small, we can approximate $\sin\theta$ by θ in radians, just as we did in analyzing the simple pendulum. Then the motion is *approximately* simple harmonic. With this approximation,

$$\tau_z = -(mgd)\theta$$

From Section 10.2, the equation of motion is $\sum \tau_z = I\alpha_z$, so

(14.37)

$$\begin{aligned} -(mgd)\theta &= I\alpha_z = I\frac{d^2\theta}{dt^2} \\ \frac{d^2\theta}{dt^2} &= -\frac{mgd}{I}\theta \end{aligned}$$

Comparing this with Eq. (14.4), we see that the role of (k/m) for the spring-mass system is played here by the quantity (mgd/I). Thus the angular frequency is

(14.38)

Angular frequency of physical pendulum, small amplitude

Mass; Acceleration due to gravity; Distance from rotation axis to center of gravity; Moment of inertia

$$\omega = \sqrt{\frac{mgd}{I}} \qquad (14.38)$$

The frequency f is $1/2\pi$ times this, and the period T is $1/f$:

(14.39)

Period of physical pendulum, small amplitude

Moment of inertia; Distance from rotation axis to center of gravity; Mass; Acceleration due to gravity

$$T = 2\pi\sqrt{\frac{I}{mgd}} \qquad (14.39)$$

Equation (14.39) is the basis of a common method for experimentally determining the moment of inertia of an object with a complicated shape. First locate the center of gravity by balancing the object. Then suspend the object so that it is free to oscillate about an axis, and measure the period T of small-amplitude oscillations. Finally, use Eq. (14.39) to calculate the moment of inertia I of the object about this axis from T, the object's mass m, and the distance d from the axis to the center of gravity. Biomechanics researchers use this method to find the moments of inertia of an animal's limbs. This information is important for analyzing how an animal walks, as we'll see in the second of the two following examples.

Example 14.9 Physical pendulum versus simple pendulum

WITH VARIATION PROBLEMS

If the object in Fig. 14.23 is a uniform rod with length L, pivoted at one end, what is the period of its motion as a pendulum?

IDENTIFY and SET UP Our target variable is the oscillation period T of a rod that acts as a physical pendulum. We find the rod's moment of inertia in Table 9.2, and then determine T from Eq. (14.39).

EXECUTE ? The moment of inertia of a uniform rod about an axis through one end is $I = \frac{1}{3}ML^2$. The distance from the pivot to the rod's center of gravity is $d = L/2$. Then from Eq. (14.39),

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{MgL/2}} = 2\pi\sqrt{\frac{2L}{3g}}$$

EVALUATE If the rod is a meter stick ($L = 1.00\text{ m}$) and $g = 9.80\text{ m/s}^2$, then

$$T = 2\pi\sqrt{\frac{2(1.00\text{ m})}{3(9.80\text{ m/s}^2)}} = 1.64\text{ s}$$

The period is smaller by a factor of $\sqrt{\frac{2}{3}} = 0.816$ than that of a simple pendulum of the same length (see Example 14.8). The rod's moment of inertia around one end, $I = \frac{1}{3}ML^2$, is one-third that of the simple pendulum, and the rod's cg is half as far from the pivot as that of the simple pendulum. You can show that, taken together in Eq. (14.39), these two differences account for the factor $\sqrt{\frac{2}{3}}$ by which the periods differ.

KEY CONCEPT

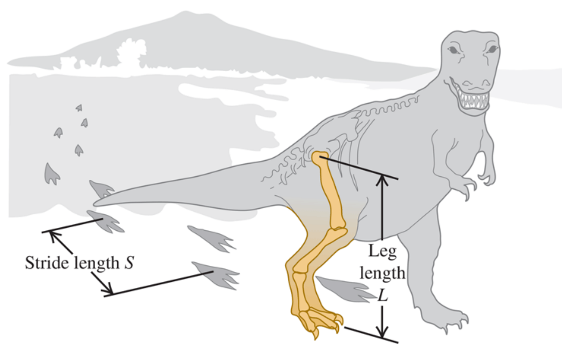
The period of a physical pendulum depends on the value of g, the distance from the pivot to the physical pendulum's center of gravity, and how the mass is distributed within the physical pendulum.

Video Tutor Solution: **Example 14.9**

Example 14.10 *Tyrannosaurus rex* and the physical pendulum

All walking animals, including humans, have a natural walking pace—a number of steps per minute that is more comfortable than a faster or slower pace. Suppose that this pace corresponds to the oscillation of the leg as a physical pendulum. (a) How does this pace depend on the length L of the leg from hip to foot? Treat the leg as a uniform rod pivoted at the hip joint. (b) Fossil evidence shows that *T. rex*, a two-legged dinosaur that lived about 65 million years ago, had a leg length $L = 3.1$ m and a stride length $S = 4.0$ m (the distance from one footprint to the next print of the same foot; see Fig. 14.24). Estimate the walking speed of *T. rex*.

Figure 14.24



The walking speed of *Tyrannosaurus rex* can be estimated from leg length L and stride length S.

IDENTIFY and SET UP Our target variables are (a) the relationship between walking pace and leg length L and (b) the walking speed of *T. rex*. We treat the leg as a physical pendulum, with a period of oscillation as found in Example 14.9. We can find the walking speed from the period and the stride length.

EXECUTE

a. From Example 14.9 the period of oscillation of the leg is $T = 2\pi\sqrt{2L/3g}$, which is proportional to $\sqrt{L}$. Each step takes half of one period, so the walking pace (in steps per second) equals twice the oscillation frequency $f = 1/T$ and is proportional to $1/\sqrt{L}$. The longer the leg, the slower the pace.

b. In our model, T. *rex* traveled one stride length S in a time

$$T = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2\,(3.1\text{ m})}{3(9.8\text{ m/s}^2)}} = 2.9\text{ s}$$

so its walking speed was

$$v = \frac{S}{T} = \frac{4.0\text{ m}}{2.9\text{ s}} = 1.4\text{ m/s} = 5.0\text{ km/h} = 3.1\text{ mi}/h$$

This is roughly the walking speed of an adult human.

EVALUATE A uniform rod isn't a very good model for a leg. The legs of many animals, including both *T. rex* and humans, are tapered; there is more mass between hip and knee than between knee and foot. The center of mass is therefore less than $L/2$ from the hip; a reasonable guess would be about $L/4$. The moment of inertia is therefore *considerably* less than $ML^2/3$—say, $ML^2/15$. Use the analysis of Example 14.9 with these corrections; you'll get a shorter oscillation period and an even greater walking speed for *T. rex*.

KEY CONCEPT

You can apply the idea of a physical pendulum to many oscillating systems, including the swinging limbs of humans and animals.

Video Tutor Solution: **Example 14.10**

Test Your Understanding of Section 14.6

The center of gravity of a simple pendulum of mass m and length L is located at the pendulum bob, a distance L from the pivot point. The center of gravity of a uniform rod of the same mass m and length $2L$ pivoted at one end is also a distance L from the pivot point. Compared to the period of the simple pendulum, is the period of this uniform rod (i) longer; (ii) shorter; or (iii) the same?

14.7 Damped Oscillations

The idealized oscillating systems we have discussed so far are frictionless. There are no nonconservative forces, the total mechanical energy is constant, and a system set into motion continues oscillating forever with no decrease in amplitude.

Real-world systems always have some dissipative forces, however, and oscillations die out with time unless we replace the dissipated mechanical energy (Fig. 14.25). A mechanical pendulum clock continues to run because potential energy stored in the spring or a hanging weight system replaces the mechanical energy lost due to friction in the pivot and the gears. But eventually the spring runs down or the weights reach the bottom of their travel. Then no more energy is available, and the pendulum swings decrease in amplitude and stop.

Figure 14.25

A swinging bell left to itself will eventually stop oscillating due to damping forces (air resistance and friction at the point of suspension).

The decrease in amplitude caused by dissipative forces is called **damping** (*not* "dampening"), and the corresponding motion is called **damped**

oscillation. The simplest case is a simple harmonic oscillator with a frictional damping force that is directly proportional to the *velocity* of the oscillating object. This behavior occurs in friction involving viscous fluid flow, such as in shock absorbers or sliding between oil-lubricated surfaces. We then have an additional force on the object due to friction, $F_x = -bv_x$, where $v_x = dx/dt$ is the velocity and b is a constant that describes the strength of the damping force. The negative sign shows that the force is always opposite in direction to the velocity. The *net* force on the object is then

(14.40)

$$\sum F_x = -kx - bv_x$$

and Newton's second law for the system is

(14.41)

$$-kx - bv_x = ma_x \qquad \text{or} \qquad -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

Equation (14.41) is a differential equation for x; it's the same as Eq. (14.4), the equation for the acceleration in SHM, but with the added term $-bdx/dt$. We won't go into how to solve this equation; we'll just present the solution. If the damping force is relatively small, the motion is described by

(14.42)

Initial amplitude
Damping constant
Mass
Time
Displacement of oscillator, little damping

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi) \qquad (14.42)$$

Angular frequency of damped oscillations
Phase angle

The angular frequency of these damped oscillations is given by

(14.43)

Angular frequency of oscillator, little damping

Force constant of restoring force

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \qquad (14.43)$$

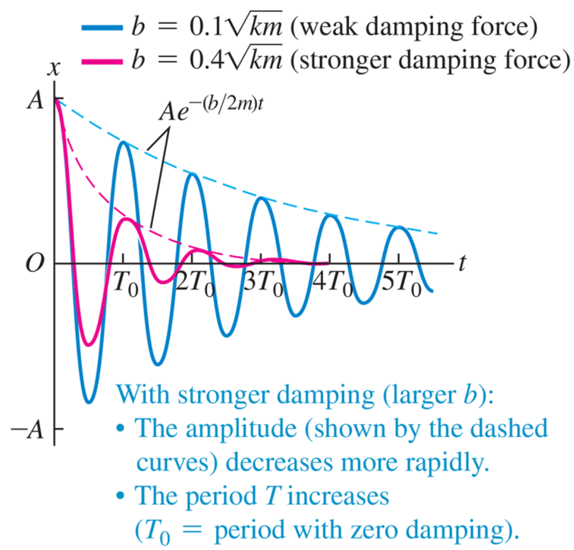
Damping constant

Mass

You can verify that Eq. (14.42) is a solution of Eq. (14.41) by calculating the first and second derivatives of x, substituting them into Eq. (14.41), and checking whether the left and right sides are equal.

The motion described by Eq. (14.42) differs from the undamped case in two ways. First, the amplitude $Ae^{-(b/2m)t}$ is not constant but decreases with time because of the exponential factor $e^{-(b/2m)t}$. Figure 14.26 is a graph of Eq. (14.42) for $\phi = 0$; the larger the value of b, the more quickly the amplitude decreases.

Figure 14.26



Graph of displacement versus time for an oscillator with little damping [see Eq. (14.42)] and with phase angle $\phi = 0$. The curves are for two values of the damping constant b.

CAUTION When frequencies are imaginary Note that when there is overdamping and b is greater than $2\sqrt{km}$, the argument of the square root in Eq. (14.43) is negative and the angular frequency of oscillation ω is an imaginary number. This is a mathematical clue that there is *no* oscillation in this case.

Second, the angular frequency $\omega\prime$, given by Eq. (14.43), is no longer equal to $\omega = \sqrt{k/m}$ but is somewhat smaller. It becomes zero when b becomes so large that

(14.44)

$$\frac{k}{m} - \frac{b^2}{4m^2} = 0 \qquad \text{or} \qquad b = 2\sqrt{km}$$

When Eq. (14.44) is satisfied, the condition is called **critical damping**. The system no longer oscillates but returns to its equilibrium position without oscillation when it is displaced and released.

If b is greater than $2\sqrt{km}$, the condition is called **overdamping.** Again there is no oscillation, but the system returns to equilibrium more slowly than with critical damping. For the overdamped case the solutions of Eq. (14.41) have the form

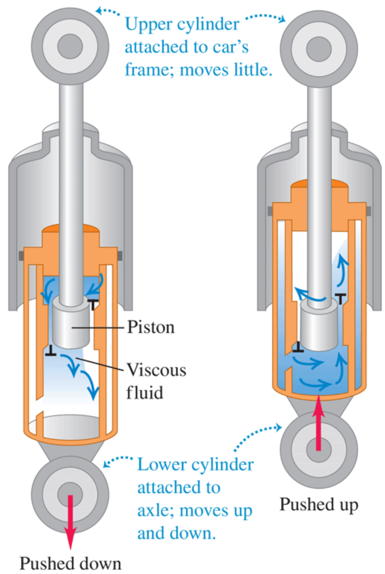
$$x = C_1 e^{-a_1 t} + C_2 e^{-a_2 t}$$

where C_1 and C_2 are constants that depend on the initial conditions and a_1 and a_2 are constants determined by m, k, and b.

When b is less than the critical value, as in Eq. (14.42), the condition is called **underdamping**. The system oscillates with steadily decreasing amplitude.

In a vibrating tuning fork or guitar string, it is usually desirable to have as little damping as possible. By contrast, damping plays a beneficial role in the oscillations of an automobile's suspension system. The shock absorbers provide a velocity-dependent damping force so that when the car goes over a bump, it doesn't continue bouncing forever (Fig. 14.27). For optimal passenger comfort, the system should be critically damped or slightly underdamped. Too much damping would be counterproductive; if the suspension is overdamped and the car hits a second bump just after the first one, the springs in the suspension will still be compressed somewhat from the first bump and will not be able to fully absorb the impact.

Figure 14.27



An automobile shock absorber. The viscous fluid causes a damping force that depends on the relative velocity of the two ends of the unit.

Energy in Damped Oscillations

In damped oscillations the damping force is nonconservative; the total mechanical energy of the system is not constant but decreases continuously, approaching zero after a long time. To derive an expression for the rate of change of energy, we first write an expression for the total mechanical energy E at any instant:

$$E = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2$$

To find the rate of change of this quantity, we take its time derivative:

$$\frac{dE}{dt} = m v_x \frac{dv_x}{dt} + k x \frac{dx}{dt}$$

But $dv_x/dt = a_x$ and $dx/dt = v_x$, so

$$\frac{dE}{dt} = v_x \left(m a_x + k x \right)$$

From Eq. (14.41), $ma_x + kx = -b\,dx/dt = -bv_x$, so

(14.45)

$$\frac{dE}{dt} = v_x\,(-bv_x) = -bv_x^2 \text{ (damped oscillations)}$$

The right side of Eq. (14.45) is negative whenever the oscillating object is in motion, whether the x-velocity v_x is positive or negative. This shows that as the object moves, the energy decreases, though not at a uniform rate. The term $-bv_x^2 = (-bv_x)v_x$ (force times velocity) is the rate at which the damping force does (negative) work on the system (that is, the damping *power*). This equals the rate of change of the total mechanical energy of the system.

Similar behavior occurs in electric circuits containing inductance, capacitance, and resistance. There is a natural frequency of oscillation, and the resistance plays the role of the damping constant b. We'll study these circuits in detail in Chapters 30 and 31.

Test Your Understanding of Section 14.7

An airplane is flying in a straight line at a constant altitude. If a wind gust strikes and raises the nose of the airplane, the nose will bob up and down until the airplane eventually returns to its original attitude. Are these oscillations (i) undamped; (ii) underdamped; (iii) critically damped; or (iv) overdamped?

14.8 Forced Oscillations and Resonance

Video Tutor Demo: Vibrating Rods

A damped oscillator left to itself will eventually stop moving. But we can maintain a constant-amplitude oscillation by applying a force that varies with time in a periodic way. As an example, consider your cousin Throckmorton on a playground swing. You can keep him swinging with constant amplitude by giving him a push once each cycle. We call this additional force a **driving force**.

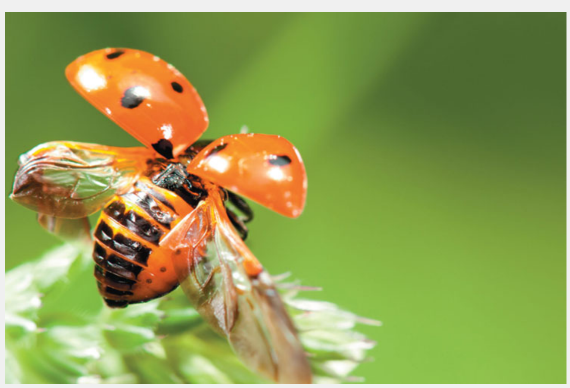
Damped Oscillation with a Periodic Driving Force

If we apply a periodic driving force with angular frequency ω_{d} to a damped harmonic oscillator, the motion that results is called a **forced oscillation** or a *driven oscillation*. It is different from the motion that occurs when the system is simply displaced from equilibrium and then left alone, in which case the system oscillates with a **natural angular frequency** ω' determined by m, k, and b, as in Eq. (14.43). In a forced oscillation, however, the angular frequency with which the mass oscillates is equal to the driving angular frequency ω_d. This does *not* have to be equal to the natural angular frequency ω'. If you grab the ropes of Throckmorton's swing, you can force the swing to oscillate with any frequency you like.

BIO Application

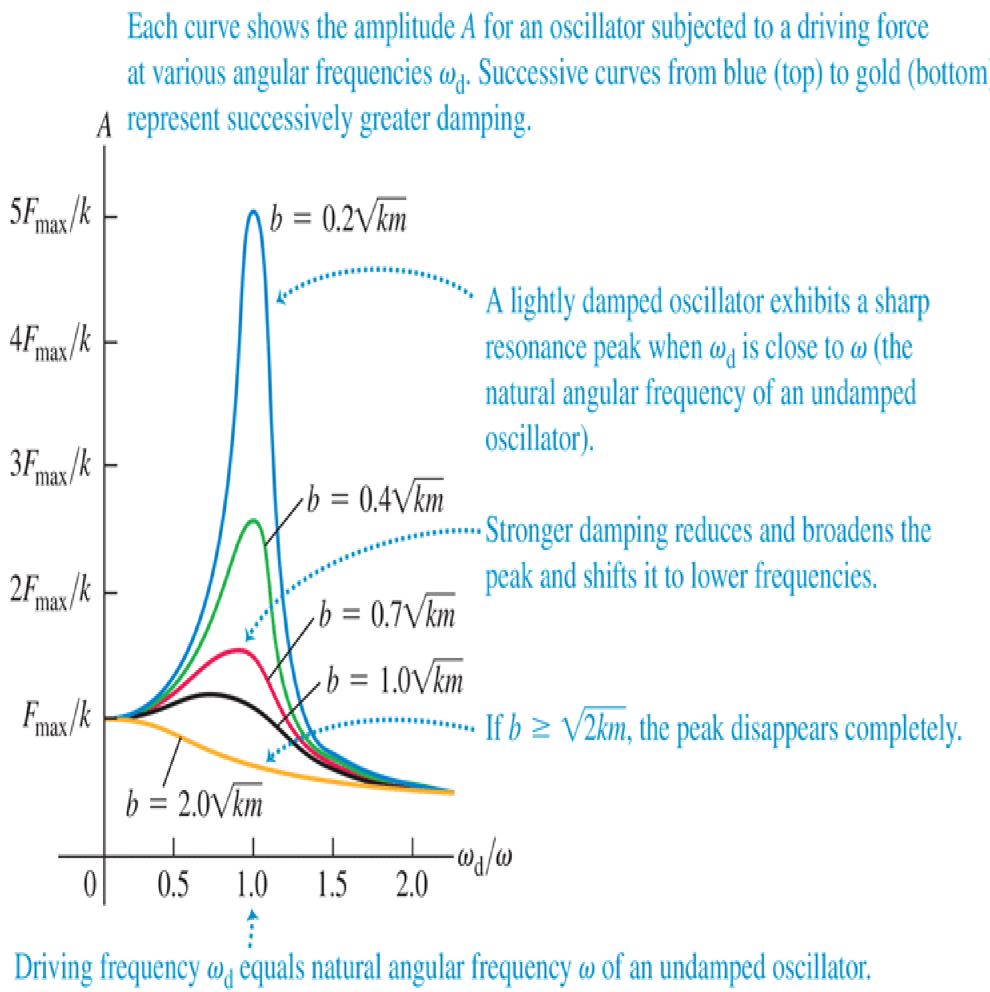
Forced Oscillations

This lady beetle (or "ladybug," family Coccinellidae) flies by means of a forced oscillation. Unlike the wings of birds, this insect's wings are extensions of its exoskeleton. Muscles attached to the inside of the exoskeleton apply a periodic driving force that deforms the exoskeleton rhythmically, causing the attached wings to beat up and down. The oscillation frequency of the wings and exoskeleton is the same as the frequency of the driving force.



Suppose we force the oscillator to vibrate with an angular frequency ω_d that is nearly *equal* to the angular frequency $\omega\prime$ it would have with no driving force. What happens? The oscillator is naturally disposed to oscillate at $\omega = \omega\prime$, so we expect the amplitude of the resulting oscillation to be larger than when the two frequencies are very different. Detailed analysis and experiment show that this is just what happens. The easiest case to analyze is a *sinusoidally* varying force—say, $F(t) = F_{\max} \cos \omega_d t$. If we vary the frequency ω_d of the driving force, the amplitude of the resulting forced oscillation varies in an interesting way (Fig. 14.28). When there is very little damping (small b), the amplitude goes through a sharp peak as the driving angular frequency ω_d nears the natural oscillation angular frequency $\omega\prime$. When the damping is increased (larger b), the peak becomes broader and smaller in height and shifts toward lower frequencies.

Figure 14.28



Graph of the amplitude A of forced oscillation as a function of the angular frequency ω_d of the driving force. The horizontal axis shows the ratio of ω_d to the angular frequency $\omega = \sqrt{k/m}$ of an undamped oscillator. Each curve has a different value of the damping constant b.

Using more differential equations than we're ready for, we could find an expression for the amplitude A of the forced oscillation as a function of the driving angular frequency. Here is the result:

(14.46)

Maximum value of driving force

Amplitude of a forced oscillator

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}} \qquad (14.46)$$

Damping constant

Force constant of restoring force

Mass

Driving angular frequency

When $k - m\omega_d^2 = 0$, the first term under the radical is zero, so A has a maximum near $\omega_d = \sqrt{k/m}$. The height of the curve at this point is proportional to $1/b$; the less damping, the higher the peak. At the low-frequency extreme, when $\omega_d = 0$, we get $A = F_{max}/k$. This corresponds to a *constant* force F_{max} and a constant displacement $A = F_{max}/k$ from equilibrium, as we might expect.

Resonance and Its Consequences

The peaking of the amplitude at driving frequencies close to the natural frequency of the system is called **resonance**. Physics is full of examples of resonance; building up the oscillations of a child on a swing by pushing with a frequency equal to the swing's natural frequency is one. A vibrating rattle in a car that occurs only at a certain engine speed is another example. Inexpensive loudspeakers often have an annoying boom or buzz when a musical note coincides with the natural frequency of the speaker cone or housing. In Chapter 16 we'll study examples of resonance that involve sound. Resonance also occurs in electric circuits, as we'll see in Chapter 31; a tuned circuit in a radio receiver responds strongly to waves with frequencies near its natural frequency. This phenomenon lets us select one radio station and reject other stations.

BIO Application

Canine Resonance

Unlike humans, dogs have no sweat glands and so must pant in order to cool down. The frequency at which a dog pants is very close to the resonant frequency of its respiratory system. This causes the maximum amount of air inflow and outflow and so minimizes the effort that the dog must exert to cool itself.

Resonance in mechanical systems can be destructive. A company of soldiers once destroyed a bridge by marching across it in step; the frequency of their steps was close to a natural frequency of the bridge, and the resulting oscillation had large enough amplitude to tear the bridge apart. Ever since, marching soldiers have been ordered to break step before crossing a bridge. Some years ago, vibrations of the engines of a particular type of airplane had just the right frequency to resonate with the natural frequencies of its wings. Large oscillations built up, and occasionally the wings fell off.

Test Your Understanding of Section 14.8

When driven at a frequency near its natural frequency, an oscillator with very little damping has a much greater response than the same oscillator with more damping. When driven at a frequency that is much higher or lower than the natural frequency, which oscillator will have the greater response: (i) the one with very little damping or (ii) the one with more damping?

Chapter 14 Summary

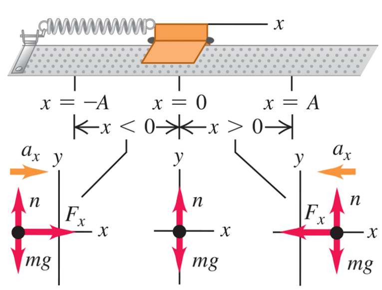
Periodic motion: Periodic motion is motion that repeats itself in a definite cycle. It occurs whenever an object has a stable equilibrium position and a restoring force that acts when the object is displaced from equilibrium. Period T is the time for one cycle. Frequency f is the number of cycles per unit time. Angular frequency ω is 2π times the frequency. (See Example 14.1.)

(14.1)

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

(14.2)

$$\omega = 2\pi f = \frac{2\pi}{T}$$



Simple harmonic motion: If the restoring force F_x in periodic motion is directly proportional to the displacement x, the motion is called simple harmonic motion (SHM). In many cases this condition is satisfied if the displacement from equilibrium is small.
The angular frequency, frequency, and period in SHM do not depend on the amplitude but on only the mass m and force constant

k. The displacement, velocity, and acceleration in SHM are sinusoidal functions of time; the amplitude A and phase angle ϕ of the oscillation are determined by the initial displacement and velocity of the object. (See Examples 14.2, 14.3, 14.6, and 14.7.)

(14.3)

$$F_x = -kx$$

(14.4)

$$a_x = \frac{F_x}{m} = -\frac{k}{m}\,x$$

(14.10)

$$\omega = \sqrt{\frac{k}{m}}$$

(14.11)

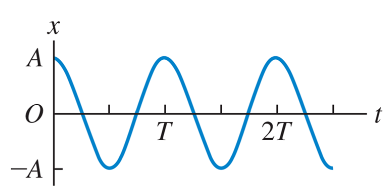
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

(14.12)

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

(14.13)

$$x = A\cos(\omega t + \phi)$$

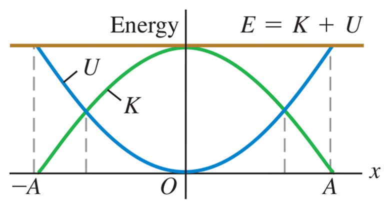


Energy in simple harmonic motion: Energy is conserved in SHM. The total energy can be expressed in terms of the force constant k and amplitude A. (See Examples 14.4 and 14.5.)

(14.21)

$$E = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2$$

$$= \frac{1}{2} k A^2 = \text{constant}$$

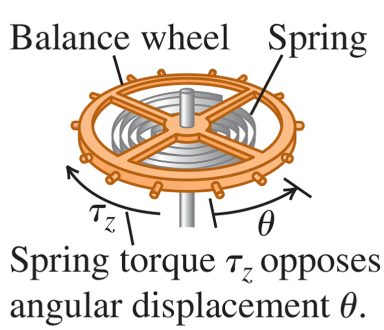


Angular simple harmonic motion: In angular SHM, the frequency and angu Energy in simple harmonic motion lar frequency are related to the moment of inertia I and the torsion constant κ.

(14.24)

$$\omega = \sqrt{\frac{\kappa}{I}} \quad \text{and}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{\kappa}{I}}$$



Simple pendulum: A simple pendulum consists of a point mass m at the end of a massless string of length L. Its motion is approximately simple harmonic for sufficiently small amplitude; the angular frequency, frequency, and period then depend on only g and L, not on the mass or amplitude. (See Example 14.8.)

(14.32)

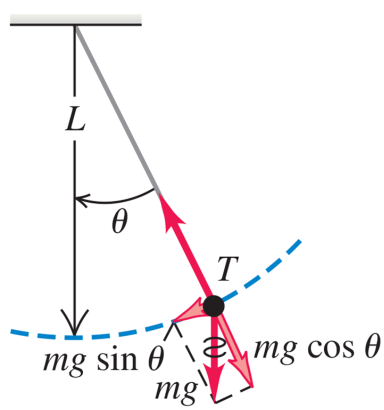
$$\omega = \sqrt{\frac{g}{L}}$$

(14.33)

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

(14.34)

$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$



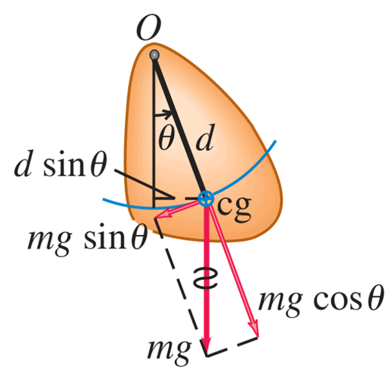
Physical pendulum: A physical pendulum is any object suspended from an axis of rotation. The angular frequency and period for small-amplitude oscillations are independent of amplitude but depend on the mass m, distance d from the axis of rotation to the center of gravity, and moment of inertia I about the axis. (See Examples 14.9 and 14.10.)

(14.38)

$$\omega = \sqrt{\frac{mgd}{I}}$$

(14.39)

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$



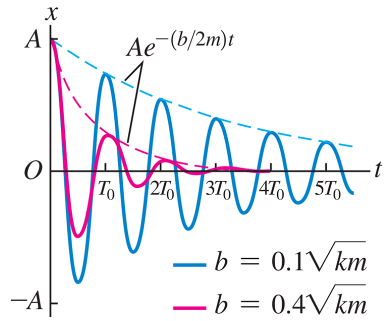
Damped oscillations: When a force $F_x = -bv_x$ is added to a simple harmonic oscillator, the motion is called a damped oscillation. If $b < 2\sqrt{km}$ (called underdamping), the system oscillates with a decaying amplitude and an angular frequency ω' that is lower than it would be without damping. If $b = 2\sqrt{km}$ (called critical damping) or $b > 2\sqrt{km}$ (called overdamping), when the system is displaced it returns to equilibrium without oscillating.

(14.42)

$$x = Ae^{-(b/2m)t}\cos\left(\omega' t + \phi\right)$$

(14.43)

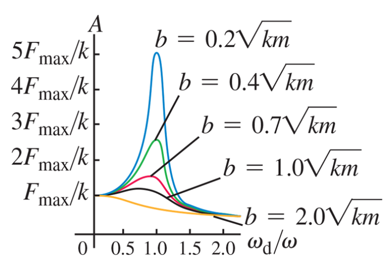
$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$



Forced oscillations and resonance: When a sinusoidally varying driving force is added to a damped harmonic oscillator, the resulting motion is called a forced oscillation or driven oscillation. The amplitude is a function of the driving frequency ω_d and reaches a peak at a driving frequency close to the natural frequency of the system. This behavior is called resonance.

(14.46)

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$



Guided Practice: Periodic Motion

For assigned homework and other learning materials, go to
Mastering Physics.

Key Example Variation Problems

Be sure to review Examples 14.2 and 14.3 (Section 14.2) before attempting these problems.

VP14.3.1 A glider of mass 0.400 kg is placed on a frictionless, horizontal air track. One end of a horizontal spring is attached to the glider, and the other end is attached to the end of the track. When released, the glider oscillates in SHM with frequency 4.15 Hz. (a) Find the period and angular frequency of the motion. (b) Find the force constant k of the spring. (c) Find the magnitude of the force that the spring exerts on the glider when the spring is stretched by 0.0200 m.

VP14.3.2 A hockey puck attached to a horizontal spring oscillates on a frictionless, horizontal surface. The spring has force constant 4.50 N/m and the oscillation period is 1.20 s. (a) What is the mass of the puck? (b) During an oscillation, the acceleration of the puck has maximum magnitude $1.20\ \mathrm{m/s^2}$. What is the amplitude of the oscillation?

VP14.3.3 The piston of a gasoline engine oscillates in SHM with frequency 50.0 Hz. At one point in the cycle the piston is 0.0300 m from equilibrium and moving at 12.4 m/s. (a) What is the amplitude of the motion? (b) What is the maximum speed the piston attains during its oscillation?

VP14.3.4 A cat is sleeping on a platform that oscillates from side to side in SHM. The combined mass of the cat and platform is 5.00 kg, and the force constant of the horizontal spring attached to the platform that makes it oscillate is 185 N/m. Ignore friction. (a) What is the frequency of the oscillation? (b) The cat will wake up if the acceleration of the platform is greater than 1.52 m/s^2. What is the maximum amplitude of oscillation that will allow the cat to stay asleep?

Be sure to review Example 14.4 (Section 14.3) before attempting these problems.

VP14.4.1 A hockey puck oscillates on a frictionless, horizontal track while attached to a horizontal spring. The puck has mass 0.150 kg and the spring has force constant 8.00 N/m. The maximum speed of the puck during its oscillation is 0.350 m/s. (a) What is the amplitude of the oscillation? (b) What is the total mechanical energy of the oscillation? (c) What are the potential energy and the kinetic energy of the puck when the displacement of the glider is 0.0300 m?

VP14.4.2 A block of mass 0.300 kg attached to a horizontal spring oscillates on a frictionless surface. The oscillation has amplitude 0.0440 m, and total mechanical energy $E = 6.00 \times 10^{-2}$J. Find (a) the force constant of the spring and (b) the block's speed when the potential energy equals exactly $E/2$.

VP14.4.3 A glider attached to a horizontal spring oscillates on a horizontal air track. The total mechanical energy of the oscillation is 4.00×10^{-3} J, the amplitude of the oscillation is 0.0300 m, and the maximum speed of the glider is 0.125 m/s. (a) What are the force constant of the spring and the mass of the glider? (b) What is the maximum acceleration of the glider? (c) What is the magnitude of the glider's acceleration when the potential energy equals 3.00×10^{-3} J?

VP14.4.4 An object is undergoing SHM with amplitude A. For what values of the displacement is the kinetic energy equal to (a) $\frac{1}{3}$ of the total mechanical energy; (b) $\frac{4}{5}$ of the total mechanical energy?

Be sure to review Examples 14.8 and 14.9 (Sections 14.5 and 14.6) before attempting these problems.

VP14.9.1 On an alien planet, a simple pendulum of length 0.500 m has oscillation frequency 0.609 Hz. Find (a) the period of the pendulum and (b) the acceleration due to gravity on this planet's surface.

VP14.9.2 What must be the length of a simple pendulum if its oscillation frequency is to be equal to that of an air-track glider of mass 0.350 kg attached to a spring of force constant 8.75 N/m?

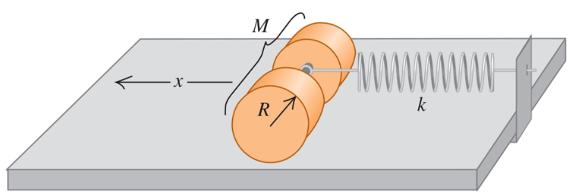
VP14.9.3 At a bicycle repair shop, a bicycle tire of mass M and radius R is suspended from a peg on the wall. The moment of inertia of the tire around the peg is $2MR^2$. If the tire is displaced from equilibrium and starts swinging back and forth, what will be its frequency of oscillation?

VP14.9.4 A rod has length 0.900 m and mass 0.600 kg and is pivoted at one end. The rod is *not* uniform; the center of mass of the rod is not at its center but is 0.500 m from the pivot. The period of the rod's motion as a pendulum is 1.59 s. What is the moment of inertia of the rod around the pivot?

Bridging Problem: Oscillating and Rolling

Two uniform, solid cylinders of radius R and total mass M are connected along their common axis by a short, light rod and rest on a horizontal tabletop (Fig. 14.29). A frictionless ring at the rod's center is attached to a spring of force constant k; the spring's other end is fixed. The cylinders are pulled to the left a distance x, stretching the spring, then released from rest. Due to friction between the tabletop and the cylinders, the cylinders roll without slipping as they oscillate. Show that the motion of the center of mass of the cylinders is simple harmonic, and find its period.

Figure 14.29



Rolling cylinders attached to a spring.

Solution Guide

IDENTIFY and SET UP

1. What condition must be satisfied for the motion of the center of mass of the cylinders to be simple harmonic?
2. Which equations should you use to describe (a) the translational and rotational motions of the cylinders; (b) Which equation

should you use to describe the condition that the cylinders roll without slipping? (**HINT:** See Section 10.3.)

3. Sketch the situation and choose a coordinate system. List the unknown quantities and decide which is the target variable.

EXECUTE

4. Draw a free-body diagram for the cylinders when they are displaced a distance x from equilibrium.
5. Solve the equations to find an expression for the acceleration of the center of mass of the cylinders. What does this expression tell you?
6. Use your result from step 5 to find the period of oscillation of the center of mass of the cylinders.

EVALUATE

7. What would be the period of oscillation if there were no friction and the cylinders didn't roll? Is this period larger or smaller than your result from step 6? Is this reasonable?

Video Tutor Solution: **Chapter 14 Bridging Problem**

Questions/Exercises/Problems: Periodic Motion

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

Discussion Questions

Q14.1 An object is moving with SHM of amplitude A on the end of a spring. If the amplitude is doubled, what happens to the total distance the object travels in one period? What happens to the period? What happens to the maximum speed of the object? Discuss how these answers are related.

Q14.2 Think of several examples in everyday life of motions that are, at least approximately, simple harmonic. In what respects does each differ from SHM?

Q14.3 Does a tuning fork or similar tuning instrument undergo SHM? Why is this a crucial question for musicians?

Q14.4 A box containing a pebble is attached to an ideal horizontal spring and is oscillating on a friction-free air table. When the box has reached its maximum distance from the equilibrium point, the pebble is suddenly lifted out vertically without disturbing the box. Will the following characteristics of the motion increase, decrease, or remain the same in the subsequent motion of the box? Justify each answer.
(a) Frequency; (b) period; (c) amplitude; (d) the maximum kinetic energy of the box; (e) the maximum speed of the box.

Q14.5 If a uniform spring is cut in half, what is the force constant of each half? Justify your answer. How would the frequency of SHM using a half-spring differ from the frequency using the same mass and the entire spring?

Q14.6 A glider is attached to a fixed ideal spring and oscillates on a horizontal, friction-free air track. A coin rests atop the glider and oscillates with it. At what points in the motion is the friction force on the coin greatest? The least? Justify your answers.

Q14.7 Two identical gliders on an air track are connected by an ideal spring. Could such a system undergo SHM? Explain. How would the period compare with that of a single glider attached to a spring whose other end is rigidly attached to a stationary object? Explain.

Q14.8 You are captured by Martians, taken into their ship, and put to sleep. You awake some time later and find yourself locked in a small room with no windows. All the Martians have left you with is your digital watch, your school ring, and your long silver-chain necklace. Explain how you can determine whether you are still on earth or have been transported to Mars.

Q14.9 The system shown in Fig. 14.17 is mounted in an elevator. What happens to the period of the motion (does it increase, decrease, or remain the same) if the elevator (a) accelerates upward at $5.0\ \text{m/s}^2$; (b) moves upward at a steady $5.0\ \text{m/s}$; (c) accelerates downward at $5.0\ \text{m/s}^2$? Justify your answers.

Q14.10 If a pendulum has a period of 2.5 s on earth, what would be its period in a space station orbiting the earth? If a mass hung from a vertical spring has a period of 5.0 s on earth, what would its period be in the space station? Justify your answers.

Q14.11 A simple pendulum is mounted in an elevator. What happens to the period of the pendulum (does it increase, decrease, or remain the same) if the elevator (a) accelerates upward at 5.0 m/s^2; (b) moves upward at a steady 5.0 m/s ; (c) accelerates downward at 5.0 m/s^2; (d) accelerates downward at 9.8 m/s^2 ? Justify your answers.

Q14.12 What should you do to the length of the string of a simple pendulum to (a) double its frequency; (b) double its period; (c) double its angular frequency?

Q14.13 If a pendulum clock is taken to a mountaintop, does it gain or lose time, assuming it is correct at a lower elevation? Explain.

Q14.14 When the amplitude of a simple pendulum increases, should its period increase or decrease? Give a qualitative argument; do not rely on Eq. (14.35). Is your argument also valid for a physical pendulum?

Q14.15 Why do short dogs (like Chihuahuas) walk with quicker strides than do tall dogs (like Great Danes)?

Q14.16 At what point in the motion of a simple pendulum is the string tension greatest? Least? In each case give the reasoning behind your answer.

Q14.17 Could a standard of time be based on the period of a certain standard pendulum? What advantages and disadvantages would such a standard have compared to the actual present-day standard discussed in Section 1.3?

Q14.18 For a simple pendulum, clearly distinguish between ω (the angular speed) and ω (the angular frequency). Which is constant and which is variable?

Q14.19 In designing structures in an earthquake-prone region, how should the natural frequencies of oscillation of a structure relate to typical earthquake frequencies? Why? Should the structure have a large or small amount of damping?

Exercises

Section 14.1 Describing Oscillation

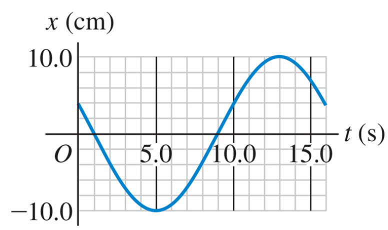
14.1 • BIO (a) **Music**. When a person sings, his or her vocal cords vibrate in a repetitive pattern that has the same frequency as the note that is sung. If someone sings the note B flat, which has a frequency of 466 Hz, how much time does it take the person's vocal cords to vibrate through one complete cycle, and what is the angular frequency of the cords? (b) **Hearing**. When sound waves strike the eardrum, this membrane vibrates with the same frequency as the sound. The highest pitch that young humans can hear has a period of 50.0 μs. What are the frequency and angular frequency of the vibrating eardrum for this sound? (c) **Vision**. When light having vibrations with angular frequency ranging from 2.7×10^{15} rad/s to 4.7×10^{15} rad/s strikes the retina of the eye, it stimulates the receptor cells there and is perceived as visible light. What are the limits of the period and frequency of this light? (d) **Ultrasound**. High-frequency sound waves (ultrasound) are used to probe the interior of the body, much as x rays do. To detect small objects such as tumors, a frequency of around 5.0 MHz is used. What are the period and angular frequency of the molecular vibrations caused by this pulse of sound?

14.2 • If an object on a horizontal, frictionless surface is attached to a spring, displaced, and then released, it will oscillate. If it is displaced 0.120 m from its equilibrium position and released with zero initial speed, then after 0.800 s its displacement is found to be 0.120 m on the opposite side, and it has passed the equilibrium position once during this interval. Find (a) the amplitude; (b) the period; (c) the frequency.

14.3 • The tip of a tuning fork goes through 440 complete vibrations in 0.500 s. Find the angular frequency and the period of the motion.

14.4 • The displacement of an oscillating object as a function of time is shown in Fig. E14.4. What are (a) the frequency; (b) the amplitude; (c) the period; (d) the angular frequency of this motion?

Figure E14.4



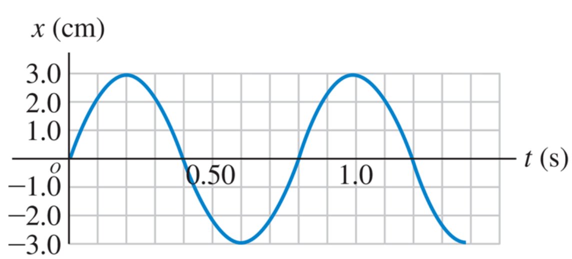
14.5 •• A machine part is undergoing SHM with a frequency of 4.00 Hz and amplitude 1.80 cm. How long does it take the part to go from $x = 0$ to $x = -1.80$ cm?

Section 14.2 Simple Harmonic Motion

14.6 • You are pushing your nephew on a playground swing. The swing seat is suspended from a horizontal bar by two light chains. Based on your experience with swings, estimate the length of each chain. Treat the motion of the child as that of a simple pendulum and assume that for safety the amplitude of the motion is kept small. You give your nephew a light push each time he reaches his closest distance from you. How much time elapses between your pushes?

14.7 • A 2.40 kg ball is attached to an unknown spring and allowed to oscillate. Figure E14.7 shows a graph of the ball's position x as a function of time t. What are the oscillation's (a) period, (b) frequency, (c) angular frequency, and (d) amplitude? (e) What is the force constant of the spring?

Figure E14.7



14.8 •• In a physics lab, you attach a 0.200 kg air-track glider to the end of an ideal spring of negligible mass and start it oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 2.60 s. Find the spring's force constant.

14.9 • When an object of unknown mass is attached to an ideal spring with force constant 120 N/m, it is found to vibrate with a frequency of 6.00 Hz. Find (a) the period of the motion; (b) the angular frequency; (c) the mass of the object.

14.10 • When a 0.750 kg mass oscillates on an ideal spring, the frequency is 1.75 Hz. What will the frequency be if 0.220 kg are (a) added to the original mass and (b) subtracted from the original mass? Try to solve this problem *without* finding the force constant of the spring.

14.11 •• An object is undergoing SHM with period 0.900 s and amplitude 0.320 m. At $t = 0$ the object is at $x = 0.320$ m and is instantaneously at rest. Calculate the time it takes the object to go (a) from $x = 0.320$ m to $x = 0.160$ m and (b) from $x = 0.160$ m to $x = 0$.

14.12 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the block is at $x = 0.280$ m, the acceleration of the block is $-5.30\ \mathrm{m/s^2}$. What is the frequency of the motion?

14.13 • A 2.00 kg, frictionless block is attached to an ideal spring with force constant 300 N/m. At $t = 0$ the spring is neither stretched nor compressed and the block is moving in the negative direction at 12.0 m/s. Find (a) the amplitude and (b) the phase angle. (c) Write an equation for the position as a function of time.

14.14 •• Repeat Exercise 14.13, but assume that at $t = 0$ the block has velocity -4.00 m/s and displacement $+0.200$ m **away from equilibrium.**

14.15 •• A block of mass m is undergoing SHM on a horizontal, frictionless surface while attached to a light, horizontal spring. The spring has force constant k, and the amplitude of the SHM is A. The block has $v = 0$, and $x = +A$ at $t = 0$.

It first reaches $x = 0$ when $t = T/4$, where T is the period of the motion. (a) In terms of T, what is the time t when the block first reaches $x = A/2$? (b) The block has its maximum speed when $t = T/4$. What is the value of t when the speed of the block first reaches the value $v_{\max}/2$? (c) Does $v = v_{\max}/2$ when $x = A/2$?

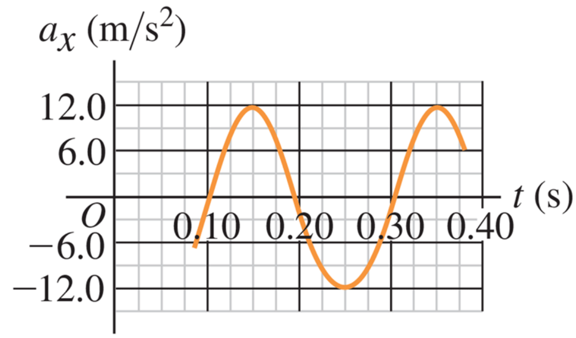
14.16 •• A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the amplitude of the motion is 0.090 m, it takes the block 2.70 s to travel from $x = 0.090$ m to $x = -0.090$ m. If the amplitude is doubled, to 0.180 m, how long does it take the block to travel (a) from $x = 0.180$ m to $x = -0.180$ m and (b) from $x = 0.090$ m to $x = -0.090$ m?

14.17 • BIO **Weighing Astronauts.** This procedure has been used to "weigh" astronauts in space: A 42.5 kg chair is attached to a spring and allowed to oscillate. When it is empty, the chair takes 1.30 s to make one complete vibration. But with an astronaut sitting in it, with her feet off the floor, the chair takes 2.54 s for one cycle. What is the mass of the astronaut?

14.18 • A 0.400 kg object undergoing SHM has $a_x = -1.80 \text{ m/s}^2$ when $x = 0.300$ m. What is the time for one oscillation?

14.19 • On a frictionless, horizontal air track, a glider oscillates at the end of an ideal spring of force constant 2.50 N/cm. The graph in Fig. E14.19 shows the acceleration of the glider as a function of time. Find (a) the mass of the glider; (b) the maximum displacement of the glider from the equilibrium point; (c) the maximum force the spring exerts on the glider.

Figure E14.19



14.20 • A 0.500 kg mass on a spring has velocity as a function of time given by $v_x(t) = -(3.60 \text{ cm/s}) \sin[(4.71 \text{ rad/s})t - (\pi/2)]$. What are (a) the period; (b) the amplitude; (c) the maximum acceleration of the mass; (d) the force constant of the spring?

14.21 • A 1.50 kg mass on a spring has displacement as a function of time given by

$$x(t) = (7.40 \text{ cm}) \cos[(4.16 \text{ rad/s})t - 2.42]$$

Find (a) the time for one complete vibration; (b) the force constant of the spring; (c) the maximum speed of the mass; (d) the maximum force on the mass; (e) the position, speed, and acceleration of the mass at $t = 1.00$ s; (f) the force on the mass at that time.

14.22 • BIO **Weighing a Virus.** In February 2004, scientists at Purdue University used a highly sensitive technique to measure the mass of a vaccinia virus (the kind used in smallpox vaccine). The procedure involved measuring the frequency of oscillation of a tiny sliver of silicon (just 30 nm long) with a laser, first without the virus and then after the virus had attached itself to the silicon. The difference in mass caused a change in the frequency. We can model such a process as a mass on a spring. (a) Show that the ratio of the frequency with the virus attached (f_{S+V}) to the frequency without the virus (f_S) is given by $f_{S+V}/f_S = 1/\sqrt{1+(m_V/m_S)}$, where m_V is the mass of the virus and m_S is the mass of the silicon sliver. Notice that it is *not* necessary to know or measure the force constant of the spring. (b) In some data, the silicon sliver has a mass of 2.10×10^{-16}g and a frequency of 2.00×10^{15} Hz without the virus and 2.87×10^{14} Hz with the virus. What is the mass of the virus, in grams and in femtograms?

14.23 •• **CALC** The *jerk* is defined to be the time rate of change of the acceleration. (a) If the velocity of an object undergoing SHM is given by $v_x = -\omega A \sin(\omega t)$, what is the equation for the x-component of the jerk as a function of time? (b) What is the value of x for the object when the x-component of the jerk has its largest positive value? (c) What is x when the x-component of the jerk is most negative? (d) When it is zero? (e) If v_x equals $-0.040\ \mathrm{s}^2$ times the x-component of the jerk for all t, what is the period of the motion?

Section 14.3 Energy in Simple Harmonic Motion

14.24 •• For the oscillating object in Fig. E14.4, what are (a) its maximum speed and (b) its maximum acceleration?

14.25 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.165 m. The maximum speed of the block is 3.90 m/s. What is the maximum magnitude of the acceleration of the block?

14.26 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.250 m and the period is 3.20 s. What are the speed and acceleration of the block when $x = 0.160$ m?

14.27 • A 0.150 kg toy is undergoing SHM on the end of a horizontal spring with force constant $k = 300$ N/m. When the toy is 0.0120 m from its equilibrium position, it is observed to have a speed of 0.400 m/s. What are the toy's (a) total energy at any point of its motion; (b) amplitude of motion; (c) maximum speed during its motion?

14.28 •• A harmonic oscillator has angular frequency ω and amplitude A. (a) What are the magnitudes of the displacement and velocity when the elastic potential energy is equal to the kinetic energy? (Assume that $U = 0$ at equilibrium.) (b) How often does this occur in each cycle? What is the time between occurrences? (c) At an instant when the displacement is equal to $A/2$, what fraction of the total energy of the system is kinetic and what fraction is potential?

14.29 • A 0.500 kg glider, attached to the end of an ideal spring with force constant $k = 450$ N/m, undergoes SHM with an amplitude of 0.040 m. Compute (a) the maximum speed of the glider; (b) the speed of the glider when it is at

$x = -0.015$ m; (c) the magnitude of the maximum acceleration of the glider; (d) the acceleration of the glider at $x = -0.015$ m; (e) the total mechanical energy of the glider at any point in its motion.

14.30 •• A block of mass m is undergoing SHM on a horizontal, frictionless surface while attached to a light, horizontal spring. The spring has force constant k, and the amplitude of the motion of the block is A. (a) The average speed is the total distance traveled by the block divided by the time it takes it to travel this distance. Calculate the average speed for one cycle of the SHM. (b) How does the average speed for one cycle compare to the maximum speed v_{max}? (c) Is the average speed more or less than half the maximum speed? Based on your answer, does the block spend more time while traveling at speeds greater than $v_{\text{max}}/2$ or less than $v_{\text{max}}/2$?

14.31 •• A block of mass m is undergoing SHM on a horizontal, frictionless surface while it is attached to a light, horizontal spring that has force constant k. The amplitude of the SHM of the block is A. What is the distance $|x|$ of the block from its equilibrium position when its speed v is half its maximum speed v_{max}? Is this distance larger or smaller than $A/2$?

14.32 •• A block with mass $m = 0.300$ kg is attached to one end of an ideal spring and moves on a horizontal frictionless surface. The other end of the spring is attached to a wall. When the block is at $x = +0.240$ m, its acceleration is $a_x = -12.0\ \text{m/s}^2$ and its velocity is $v_x = +4.00$ m/s. What are (a) the spring's force constant k; (b) the amplitude of the motion; (c) the maximum speed of the block during its motion; and (d) the maximum magnitude of the block's acceleration during its motion?

14.33 •• You are watching an object that is moving in SHM. When the object is displaced 0.600 m to the right of its equilibrium position, it has a velocity of 2.20 m/s to the right and an acceleration of 8.40 m/s^2 to the left. How much farther from this point will the object move before it stops momentarily and then starts to move back to the left?

14.34 •• A mass is oscillating with amplitude A at the end of a spring. How far (in terms of A) is this mass from the equilibrium position of the spring when the elastic potential energy equals the kinetic energy?

14.35 • A 2.00 kg frictionless block attached to an ideal spring with force constant 315 N/m is undergoing simple harmonic motion. When the block has displacement $+0.200$ m, it is moving in the negative x-direction with a speed of 4.00 m/s. Find (a) the amplitude of the motion; (b) the block's maximum acceleration; and (c) the maximum force the spring exerts on the block.

Section 14.4 Applications of Simple Harmonic Motion

14.36 • A proud deep-sea fisherman hangs a 65.0 kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.180 m. (a) Find the force constant of the spring. The fish is now pulled down 5.00 cm and released. (b) What is the period of oscillation of the fish? (c) What is the maximum speed it will reach?

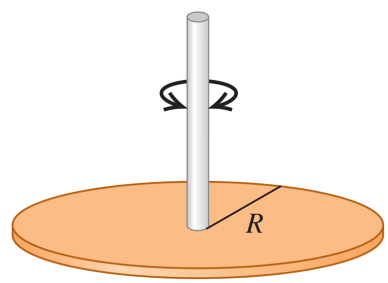
14.37 • A 175 g glider on a horizontal, frictionless air track is attached to a fixed ideal spring with force constant 155 N/m. At the instant you make measurements on the glider, it is moving at 0.815 m/s and is 3.00 cm from its equilibrium point. Use *energy conservation* to find (a) the amplitude of the motion and (b) the maximum speed of the glider. (c) What is the angular frequency of the oscillations?

14.38 •• A uniform, solid metal disk of mass 6.50 kg and diameter 24.0 cm hangs in a horizontal plane, supported at its center by a vertical metal wire. You find that it requires a horizontal force of 4.23 N tangent to the rim of the disk to turn it by 3.34°, thus twisting the wire. You now remove this force and release the disk from rest. (a) What is the torsion constant for the metal wire? (b) What are the frequency and period of the torsional oscillations of the disk? (c) Write the equation of motion for $\theta(t)$ for the disk.

14.39 • A thrill-seeking cat with mass 4.00 kg is attached by a harness to an ideal spring of negligible mass and oscillates vertically in SHM. The amplitude is 0.050 m, and at the highest point of the motion the spring has its natural unstretched length. Calculate the elastic potential energy of the spring (take it to be zero for the unstretched spring), the kinetic energy of the cat, the gravitational potential energy of the system relative to the lowest point of the motion, and the sum of these three energies when the cat is (a) at its highest point; (b) at its lowest point; (c) at its equilibrium position.

14.40 • A thin metal disk with mass 2.00×10^{-3} kg and radius 2.20 cm is attached at its center to a long fiber (Fig. E14.40). The disk, when twisted and released, oscillates with a period of 1.00 s. Find the torsion constant of the fiber.

Figure E14.40



14.41 •• A certain alarm clock ticks four times each second, with each tick representing half a period. The balance wheel consists of a thin rim with radius 0.55 cm, connected to the balance shaft by thin spokes of negligible mass. The total mass of the balance wheel is 0.90 g. (a) What is the moment of inertia of the balance wheel about its shaft? (b) What is the torsion constant of the coil spring (Fig. 14.19)?

14.42 •• You want to find the moment of inertia of a complicated machine part about an axis through its center of mass. You suspend it from a wire along this axis. The wire has a torsion constant of 0.450 N·m/rad. You twist the part a small amount about this axis and let it go, timing 165 oscillations in 265 s. What is its moment of inertia?

Section 14.5 The Simple Pendulum

14.43 •• You pull a simple pendulum 0.240 m long to the side through an angle of 3.50° and release it. (a) How much time does it take the pendulum bob to reach its highest speed? (b) How much time does it take if the pendulum is released at an angle of 1.75° instead of 3.50°?

14.44 •• Equation (14.35) shows that the equation $T = 2\pi\sqrt{L/g}$ for the period of a simple pendulum is an approximation that is accurate only when the angular displacement Θ of the pendulum is small. For what value of Θ is $T = 2\pi\sqrt{L/g}$ in error by 2.0%? In your calculation consider only the first correction term in Eq. (14.35).

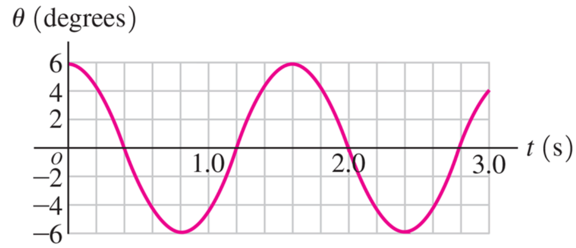
14.45 • A building in San Francisco has light fixtures consisting of small 2.35 kg bulbs with shades hanging from the ceiling at the end of light, thin cords 1.50 m long. If a minor earthquake occurs, how many swings per second will these fixtures make?

14.46 • **A Pendulum on Mars**. A certain simple pendulum has a period on the earth of 1.60 s. What is its period on the surface of Mars, where $g = 3.71\ \mathrm{m/s^2}$?

14.47 • After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 50.0 cm. She finds that the pendulum makes 100 complete swings in 136 s. What is the value of g on this planet?

14.48 •• In the laboratory, a student studies a pendulum by graphing the angle θ that the string makes with the vertical as a function of time t, obtaining the graph shown in Fig. E14.48. (a) What are the period, frequency, angular frequency, and amplitude of the pendulum's motion? (b) How long is the pendulum? (c) Is it possible to determine the mass of the bob?

Figure E14.48



14.49 •• A small sphere with mass m is attached to a massless rod of length L that is pivoted at the top, forming a simple pendulum. The pendulum is pulled to one side so that the rod is at an angle θ from the vertical, and released from rest. (a) In a diagram, show the pendulum just after it is released. Draw vectors representing the *forces* acting on the small sphere and the *acceleration* of the sphere. Accuracy counts! At this point, what is the linear acceleration of the sphere? (b) Repeat part (a) for the instant when the pendulum rod is at an angle $\theta/2$ from the vertical. (c) Repeat part (a) for the instant when the pendulum rod is vertical. At this point, what is the linear speed of the sphere?

Section 14.6 The Physical Pendulum

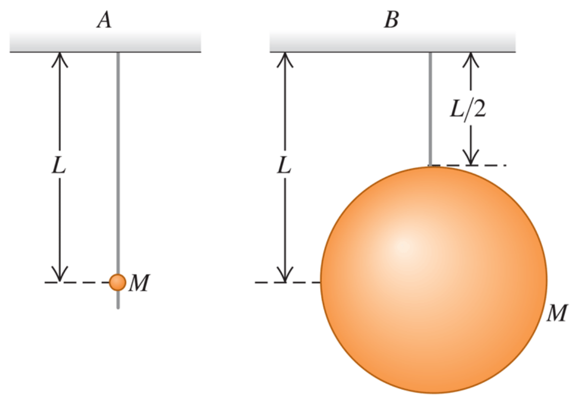
14.50 •• We want to hang a thin hoop on a horizontal nail and have the hoop make one complete small-angle oscillation each 2.0 s. What must the hoop's radius be?

14.51 • Two pendulums have the same dimensions (length L) and total mass (m). Pendulum A is a very small ball swinging at the end of a uniform massless bar. In pendulum B, half the mass is in the ball and half is in the uniform bar. Find the period of each pendulum for small oscillations. Which one takes longer for a swing?

14.52 •• A 1.80 kg monkey wrench is pivoted 0.250 m from its center of mass and allowed to swing as a physical pendulum. The period for small-angle oscillations is 0.940 s. (a) What is the moment of inertia of the wrench about an axis through the pivot? (b) If the wrench is initially displaced 0.400 rad from its equilibrium position, what is the angular speed of the wrench as it passes through the equilibrium position?

14.53 •• The two pendulums shown in Fig. E14.53 each consist of a uniform solid ball of mass M supported by a rigid massless rod, but the ball for pendulum A is very tiny while the ball for pendulum B is much larger. Find the period of each pendulum for small displacements. Which ball takes longer to complete a swing?

Figure E14.53



14.54 •• CP A holiday ornament in the shape of a hollow sphere with mass $M = 0.015$ kg and radius $R = 0.050$ m is hung from a tree limb by a small loop of wire attached to the surface of the sphere. If the ornament is displaced a small distance and released, it swings back and forth as a physical pendulum with negligible friction. Calculate its period. (**HINT:** Use the parallel-axis theorem to find the moment of inertia of the sphere about the pivot at the tree limb.)

Section 14.7 Damped Oscillations

14.55 • An object is moving in damped SHM, and the damping constant can be varied. If the angular frequency of the motion is ω when the damping constant is zero, what is the angular frequency, expressed in terms of ω, when the damping constant is one-half the critical damping value?

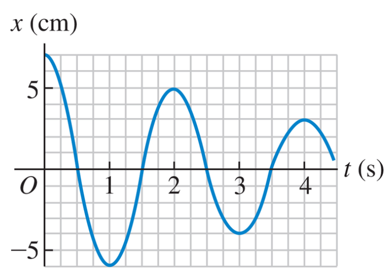
14.56 •• A 50.0 g hard-boiled egg moves on the end of a spring with force constant $k = 25.0$ N/m. Its initial displacement is 0.300 m. A damping force $F_x = -bv_x$ acts on the egg, and the amplitude of the motion decreases to 0.100 m in 5.00 s. Calculate the magnitude of the damping constant b.

14.57 • An unhappy 0.300 kg rodent, moving on the end of a spring with force constant $k = 2.50$ N/m, is acted on by a damping force $F_x = -bv_x$. (a) If the constant b has the value

0.900 kg/s, what is the frequency of oscillation of the rodent? (b) For what value of the constant b will the motion be critically damped?

14.58 •• A mass is vibrating at the end of a spring of force constant 225 N/m. Figure E14.58 shows a graph of its position x as a function of time t. (a) At what times is the mass not moving? (b) How much energy did this system originally contain? (c) How much energy did the system lose between $t = 1.0$ s and $t = 4.0$ s? Where did this energy go?

Figure E14.58



Section 14.8 Forced Oscillations and Resonance

14.59 • A sinusoidally varying driving force is applied to a damped harmonic oscillator of force constant k and mass m. If the damping constant has a value b_1, the amplitude is A_1 when the driving angular frequency equals $\sqrt{k/m}$. In terms of A_1, what is the amplitude for the same driving frequency and the same driving force amplitude $F_{\max}$, if the damping constant is (a) $3b_1$ and (b) $b_1/2$?

14.60 •• Equation (14.46) and Fig14.28. describe a damped and driven oscillator. (a) For a damping constant $b = 0.20\sqrt{km}$, confirm that the amplitude A is $5F_{\max}/k$ when $\omega_{\mathrm{d}} = \omega$, where $\omega = \sqrt{k/m}$ is the natural angular frequency. (b) Repeat part (a) for $b = 0.40\sqrt{km}$, and confirm that the amplitude A is $2.5F_{\max}/k$ when $\omega_{\mathrm{d}} = \omega$. (c) As a measure of the width of the resonance peak, calculate A when $\omega_{\mathrm{d}} = \omega/2$ for $b = 0.20\sqrt{km}$ and for $b = 0.40\sqrt{km}$. In each case, what is the ratio of the amplitude for $\omega_{\mathrm{d}} = \omega$ to the amplitude for $\omega_{\mathrm{d}} = \omega/2$? For which value of the damping constant does the amplitude increase by the larger factor?

Problems

14.61 •• Object A has mass m_A and is in SHM on the end of a spring with force constant k_A. Object B has mass m_B and is in SHM on the end of a spring with force constant k_B. The amplitude A_A for object A is twice the amplitude A_B for the motion of object B. Also, $m_B = 4m_A$ and $k_A = 9k_B$. (a) What is the ratio of the maximum speeds of the two objects, $v_{\max,A}/v_{\max,B}$? (b) What is the ratio of their maximum accelerations, $a_{\max,A}/a_{\max,B}$?

14.62 ••• An object is undergoing SHM with period 0.300 s and amplitude 6.00 cm. At $t = 0$ the object is instantaneously at rest at $x = 6.00$ cm. Calculate the time it takes the object to go from $x = 6.00$ cm to $x = -1.50$ cm.

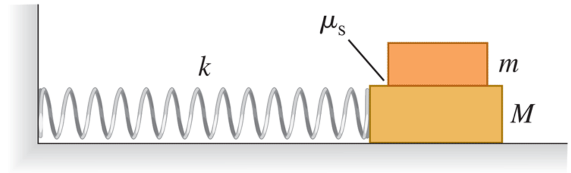
14.63 •• An object is undergoing SHM with period 1.200 s and amplitude 0.600 m. At $t = 0$ the object is at $x = 0$ and is moving in the negative x-direction. How far is the object from the equilibrium position when $t = 0.480$ s?

14.64 • Four passengers with combined mass 250 kg compress the springs of a car with worn-out shock absorbers by 4.00 cm when they get in. Model the car and passengers as a single object on a single ideal spring. If the loaded car has a period of vibration of 1.92 s, what is the period of vibration of the empty car?

14.65 •• An object with mass m is moving in SHM. It has amplitude A_1 and total mechanical energy E_1 when the spring has force constant k_1. You want to quadruple the total mechanical energy, so $E_2 = 4E_1$, and halve the amplitude, so $A_2 = A_1/2$, by using a different spring, one with force constant k_2. (a) How is k_2 related to k_1? (b) What effect will the change in spring constant and amplitude have on the maximum speed of the moving object?

14.66 •• **CP** A block with mass M rests on a frictionless surface and is connected to a horizontal spring of force constant k. The other end of the spring is attached to a wall (Fig. P14.66). A second block with mass m rests on top of the first block. The coefficient of static friction between the blocks is μ_s. Find the *maximum* amplitude of oscillation such that the top block will not slip on the bottom block.

Figure P14.66



14.67 •• A block with mass m is undergoing SHM on a horizontal, frictionless surface while attached to a light, horizontal spring that has force constant k. You use motion sensor equipment to measure the maximum speed of the block during its oscillations. You repeat the measurement for the same spring and blocks of different masses while keeping the amplitude A at a constant value of 12.0 cm. You plot your data as $v^2{}_{\text{max}}$ versus 1/m and find that the data lie close to a straight line that has slope $8.62 \text{ N} \cdot m$. What is the force constant k of the spring?

14.68 •• CP Consider the system of two blocks and a spring shown in Fig. P14.66. The horizontal surface is frictionless, but there is static friction between the two blocks. The spring has force constant $k = 150 \text{ N/m}$. The masses of the two blocks are $m = 0.500 \text{ kg}$ and $M = 4.00 \text{ kg}$. You set the blocks into motion by releasing block M with the spring stretched a distance d from equilibrium. You start with small values of d, and then repeat with successively larger values. For small values of d, the blocks move together in SHM. But for larger values of d the top block slips relative to the bottom block when the bottom block is released. (a) What is the period of the motion of the two blocks when d is small enough to have no slipping? (b) The largest value d can have and there be no slipping is $d = 8.8$ cm. What is the coefficient of static friction μ_s between the surfaces of the two blocks?

14.69 ••• A 1.50 kg, horizontal, uniform tray is attached to a vertical ideal spring of force constant 185 N/m and a 275 g metal ball is in the tray. The spring is below the tray, so it can oscillate up and down. The tray is then pushed down to point A, which is 15.0 cm below the equilibrium point, and released from rest. (a) How high above point A will the tray be when the metal ball leaves the tray? (**HINT:** This does *not* occur when the ball and tray reach their maximum speeds.) (b) How much time elapses between releasing the system at point A and the ball leaving the tray? (c) How fast is the ball moving just as it leaves the tray?

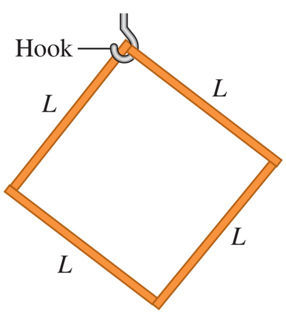
14.70 • **CP** A 10.0 kg mass is traveling to the right with a speed of 2.00 m/s on a smooth horizontal surface when it collides with and sticks to a second 10.0 kg mass that is initially at rest but is attached to one end of a light, horizontal spring with force constant 170.0 N/m. The other end of the spring is fixed to a wall to the right of the second mass. (a) Find the frequency, amplitude, and period of the subsequent oscillations. (b) How long does it take the system to return the first time to the position it had immediately after the collision?

14.71 ••• An apple weighs 1.00 N. When you hang it from the end of a long spring of force constant 1.50 N/m and negligible mass, it bounces up and down in SHM. If you stop the bouncing and let the apple swing from side to side through a small angle, the frequency of this simple pendulum is half the bounce frequency. (Because the angle is small, the back-and-forth swings do not cause any appreciable change in the length of the spring.) What is the unstretched length of the spring (with the apple removed)?

14.72 ••• **CP** **SHM of a Floating Object**. An object with height h, mass M, and a uniform cross-sectional area A floats upright in a liquid with density ρ. (a) Calculate the vertical distance from the surface of the liquid to the bottom of the floating object at equilibrium. (b) A downward force with magnitude F is applied to the top of the object. At the new equilibrium position, how much farther below the surface of the liquid is the bottom of the object than it was in part (a)? (Assume that some of the object remains above the surface of the liquid.) (c) Your result in part (b) shows that if the force is suddenly removed, the object will oscillate up and down in SHM. Calculate the period of this motion in terms of the density ρ of the liquid, the mass M, and the cross-sectional area A of the object. You can ignore the damping due to fluid friction (see Section 14.7).

14.73 •• **CP** A square object of mass m is constructed of four identical uniform thin sticks, each of length L, attached together. This object is hung on a hook at its upper corner (Fig. P14.73). If it is rotated slightly to the left and then released, at what frequency will it swing back and forth?

Figure P14.73



14.74 ••• An object with mass 0.200 kg is acted on by an elastic restoring force with force constant 10.0 N/m. (a) Graph elastic potential energy U as a function of displacement x over a range of x from -0.300 m to $+0.300$ m. On your graph, let 1 cm $=$ 0.05 J vertically and 1 cm $=$ 0.05 m horizontally. The object is set into oscillation with an initial potential energy of 0.140 J and an initial kinetic energy of 0.060 J. Answer the following questions by referring to the graph. (b) What is the amplitude of oscillation? (c) What is the potential energy when the displacement is one-half the amplitude? (d) At what displacement are the kinetic and potential energies equal? (e) What is the value of the phase angle ϕ if the initial velocity is positive and the initial displacement is negative?

14.75 • **CALC** A 2.00 kg bucket containing 10.0 kg of water is hanging from a vertical ideal spring of force constant 450 N/m and oscillating up and down with an amplitude of 3.00 cm. Suddenly the bucket springs a leak in the bottom such that water drops out at a steady rate of 2.00 g/s. When the bucket is half full, find (a) the period of oscillation and (b) the rate at which the period is changing with respect to time. Is the period getting longer or shorter? (c) What is the shortest period this system can have?

14.76 •• Quantum mechanics is used to describe the vibrational motion of molecules, but analysis using classical physics gives some useful insight. In a classical model the vibrational motion can be treated as SHM of the atoms connected by a spring. The two atoms in a diatomic molecule vibrate about their center of mass, but in the molecule HI, where one atom is much more massive than the other, we can treat the hydrogen atom as oscillating in SHM while the iodine atom remains at rest. (a) A classical estimate of the vibrational frequency is $f = 7 \times 10^{13}$ Hz. The mass of a hydrogen atom differs little from the mass of a proton. If the HI molecule is modeled as two atoms connected by a spring, what is the force constant of the spring? (b) The vibrational energy of the molecule is measured to be about 5×10^{-20} J. In the classical model, what is the maximum speed of the H atom during its SHM? (c) What is the amplitude of the vibrational motion? How does your result compare to the equilibrium distance between the two atoms in the HI molecule, which is about 1.6×10^{-10} m?

14.77 •• A 5.00 kg partridge is suspended from a pear tree by an ideal spring of negligible mass. When the partridge is pulled down 0.100 m below its equilibrium position and released, it vibrates with a period of 4.20 s. (a) What is its speed as it passes through the equilibrium position? (b) What is its acceleration when it is 0.050 m above the equilibrium position? (c) When it is moving upward, how much time is required for it to move from a point 0.050 m below its equilibrium position to a point 0.050 m above it? (d) The motion of the partridge is stopped, and then it is removed from the spring. How much does the spring shorten?

14.78 •• A slender rod of length 80.0 cm and mass 0.400 kg has its center of gravity at its geometrical center. But its density is not uniform; it increases by the same amount from the center of the rod out to either end. You want to determine the moment of inertia I_{cm} of the rod for an axis perpendicular to the rod at its center, but you don't know its density as a function of distance along the rod, so you can't use an integration method to calculate I_{cm}. Therefore, you make the following measurements: You suspend the rod about an axis that is a distance d (measured in meters) above the center of the rod and measure the period T (measured in seconds) for small-amplitude oscillations about the axis. You repeat this for several values of d. When you plot your data as $T^2 - 4\pi^2 d/g$ versus $1/d$, the data lie close to a straight line that has slope $0.320\ \mathrm{m{\cdot}s^2}$. What is the value of I_{cm} for the rod?

14.79 •• CP **SHM of a Butcher's Scale**. A spring of negligible mass and force constant $k = 400\ \mathrm{N/m}$ is hung vertically, and a 0.200 kg pan is suspended from its lower end. A butcher drops a 2.2 kg steak onto the pan from a height of 0.40 m. The steak makes a totally inelastic collision with the pan and sets the system into vertical SHM. What are (a) the speed of the pan and steak immediately after the collision; (b) the amplitude of the subsequent motion; (c) the period of that motion?

14.80 •• A 40.0 N force stretches a vertical spring 0.250 m. (a) What mass must be suspended from the spring so that the system will oscillate with a period of 1.00 s? (b) If the amplitude of the motion is 0.050 m and the period is that specified in part (a), where is the object and in what direction is it moving 0.35 s after it has passed the equilibrium position, moving downward? (c) What force (magnitude and direction) does the spring exert on the object when it is 0.030 m below the equilibrium position, moving upward?

14.81 •• **Don't Miss the Boat**. While on a visit to Minnesota ("Land of 10,000 Lakes"), you sign up to take an excursion around one of the larger lakes. When you go to the dock where the 1500 kg boat is tied, you find that the boat is bobbing up and down in the waves, executing simple harmonic motion with amplitude 20 cm. The boat takes 3.5 s to make one complete up-and-down cycle. When the boat is at its highest point, its deck is at the same height as the stationary dock. As you watch the boat bob up and down, you (mass 60 kg) begin to feel a bit woozy, due in part to the previous night's dinner of lutefisk. As a result, you refuse to board the boat unless the level of the boat's deck is within 10 cm of the dock level. How much time do you have to board the boat comfortably during each cycle of up-and-down motion?

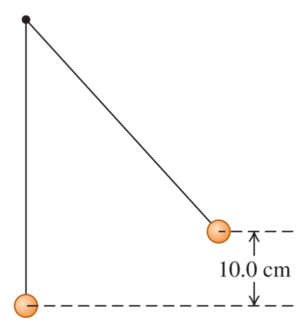
14.82 • **CP** An interesting, though highly impractical example of oscillation is the motion of an object dropped down a hole that extends from one side of the earth, through its center, to the other side. With the assumption (not realistic) that the earth is a sphere of uniform density, prove that the motion is simple harmonic and find the period. [*Note*: The gravitational force on the object as a function of the object's distance r from the center of the earth was derived in Example 13.10 (Section 13.6). The motion is simple harmonic if the acceleration a_x and the displacement from equilibrium x are related by Eq. (14.8), and the period is then $T = 2\pi/\omega$.]

14.83 ••• **CP** A rifle bullet with mass 8.00 g and initial horizontal velocity 280 m/s strikes and embeds itself in a block with mass 0.992 kg that rests on a frictionless surface and is attached to one end of an ideal spring. The other end of the spring is attached to the wall. The impact compresses the spring a maximum distance of 15.0 cm. After the impact, the block moves in SHM. Calculate the period of this motion.

14.84 ••• **CP** Two uniform solid spheres, each with mass $M = 0.800$ kg and radius $R = 0.0800$ m, are connected by a short, light rod that is along a diameter of each sphere and are at rest on a horizontal tabletop. A spring with force constant $k = 160$ N/m has one end attached to the wall and the other end attached to a frictionless ring that passes over the rod at the center of mass of the spheres, which is midway between the centers of the two spheres. The spheres are each pulled the same distance from the wall, stretching the spring, and released. There is sufficient friction between the tabletop and the spheres for the spheres to roll without slipping as they move back and forth on the end of the spring. Show that the motion of the center of mass of the spheres is simple harmonic and calculate the period.

14.85 • **CP** In Fig. P14.85 the upper ball is released from rest, collides with the stationary lower ball, and sticks to it. The strings are both 50.0 cm long. The upper ball has mass 2.00 kg, and it is initially 10.0 cm higher than the lower ball, which has mass 3.00 kg. Find the frequency and maximum angular displacement of the motion after the collision.

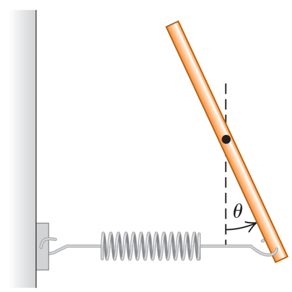
Figure P14.85



14.86 •• **The Silently Ringing Bell.** A large, 34.0 kg bell is hung from a wooden beam so it can swing back and forth with negligible friction. The bell's center of mass is 0.60 m below the pivot. The bell's moment of inertia about an axis at the pivot is 18.0 kg·m^2. The clapper is a small, 1.8 kg mass attached to one end of a slender rod of length L and negligible mass. The other end of the rod is attached to the inside of the bell; the rod can swing freely about the same axis as the bell. What should be the length L of the clapper rod for the bell to ring silently—that is, for the period of oscillation for the bell to equal that of the clapper?

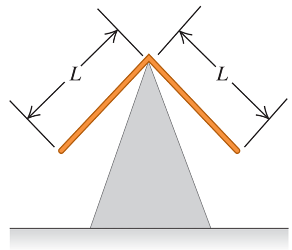
14.87 •• CALC A slender, uniform, metal rod with mass M is pivoted without friction about an axis through its midpoint and perpendicular to the rod. A horizontal spring with force constant k is attached to the lower end of the rod, with the other end of the spring attached to a rigid support. If the rod is displaced by a small angle Θ from the vertical (Fig. P14.87) and released, show that it moves in angular SHM and calculate the period. (**HINT:** Assume that the angle Θ is small enough for the approximations $\sin\Theta \approx \Theta$ and $\cos\Theta \approx 1$ to be valid. The motion is simple harmonic if $d^2\theta/dt^2 = -\omega^2\theta$, and the period is then $T = 2\pi/\omega$.)

Figure P14.87



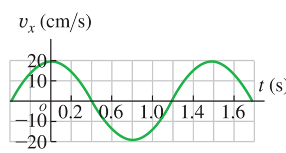
14.88 ••• Two identical thin rods, each with mass m and length L, are joined at right angles to form an L-shaped object. This object is balanced on top of a sharp edge (Fig. P14.88). If the L-shaped object is deflected slightly, it oscillates. Find the frequency of oscillation.

Figure P14.88



14.89 •• **DATA** A mass m is attached to a spring of force constant 75 N/m and allowed to oscillate. Figure P14.89 shows a graph of its velocity component v_x as a function of time t. Find (a) the period, (b) the frequency, and (c) the angular frequency of this motion. (d) What is the amplitude (in cm), and at what times does the mass reach this position? (e) Find the maximum acceleration magnitude of the mass and the times at which it occurs. (f) What is the value of m?

Figure P14.89



14.90 •• **DATA** You hang various masses m from the end of a vertical, 0.250 kg spring that obeys Hooke's law and is tapered, which means the diameter changes along the length of the spring. Since the mass of the spring is not negligible, you must replace m in the equation $T = 2\pi\sqrt{m/k}$ with $m + m_{\text{eff}}$, where m_{eff} is the effective mass of the oscillating spring. (See Challenge Problem 14.93.) You vary the mass m and measure the time for 10 complete oscillations, obtaining these data:

m (kg)	0.100	0.200	0.300	0.400	0.500
Time (s)	8.7	10.5	12.2	13.9	15.1

(a) Graph the square of the period T versus the mass suspended from the spring, and find the straight line of best fit. (b) From the slope of that line, determine the force constant of the spring. (c) From the vertical intercept of the line, determine the spring's effective mass. (d) What fraction is m_{eff} of the spring's mass? (e) If a 0.450 kg mass oscillates on the end of the spring, find its period, frequency, and angular frequency.

14.91 ••• **DATA** Experimenting with pendulums, you attach a light string to the ceiling and attach a small metal sphere to the lower end of the string. When you displace the sphere 2.00 m to the left, it nearly touches a vertical wall; with the string taut, you release the sphere from rest. The sphere swings back and forth as a simple pendulum, and you measure its period T. You repeat this act for strings of various lengths L, each time starting the motion with the sphere displaced 2.00 m to the left of the vertical position of the string. In each case the sphere's radius is very small compared with L. Your results are given in the table:

L (m)	12.00	10.00	8.00	6.00	5.00	4.00	3.00	2.50	2.30
T (s)	6.96	6.36	5.70	4.95	4.54	4.08	3.60	3.35	3.27

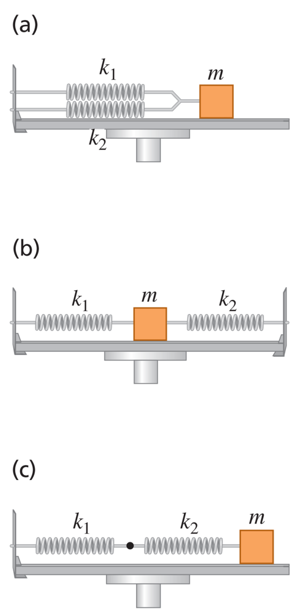
(a) For the five largest values of L, graph T^2 versus L. Explain why the data points fall close to a straight line. Does the slope of this line have the value you expected? (b) Add the remaining data to your graph. Explain why the data start to deviate from the straight-line fit as L decreases. To see this effect more clearly, plot T/T_0 versus L, where

$T_0 = 2\pi\sqrt{L/g}$ and $g = 9.80\ \mathrm{m/s}^2$. (c) Use your graph of T/T_0 versus L to estimate the angular amplitude of the pendulum (in degrees) for which the equation $T = 2\pi\sqrt{L/g}$ is in error by 5%.

Challenge Problems

14.92 ••• **The Effective Force Constant of Two Springs.** Two springs with the same unstretched length but different force constants k_1 and k_2 are attached to a block with mass m on a level, frictionless surface. Calculate the effective force constant k_{eff} in each of the three cases (a), (b), and (c) depicted in Fig. P14.92. (The effective force constant is defined by $\sum F_x = -k_{\text{eff}}\, x$.) (d) An object with mass m, suspended from a uniform spring with a force constant k, vibrates with a frequency f_1. When the spring is cut in half and the same object is suspended from one of the halves, the frequency is f_2. What is the ratio f_1/f_2?

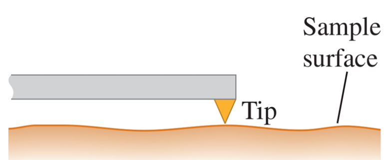
Figure P14.92



14.93 ••• CALC **A Spring with Mass.** The preceding problems in this chapter have assumed that the springs had negligible mass. But of course no spring is completely massless. To find the effect of the spring's mass, consider a spring with mass M, equilibrium length L_0, and spring constant k. When stretched or compressed to a length L, the potential energy is $\frac{1}{2}kx^2$, where $x = L - L_0$. (a) Consider a spring, as described above, that has one end fixed and the other end moving with speed v. Assume that the speed of points along the length of the spring varies linearly with distance l from the fixed end. Assume also that the mass M of the spring is distributed uniformly along the length of the spring. Calculate the kinetic energy of the spring in terms of M and v. (**HINT:** Divide the spring into pieces of length dl; find the speed of each piece in terms of l, v, and L; find the mass of each piece in terms of dl, M, and L; and integrate from 0 to L. The result is *not* $\frac{1}{2}Mv^2$, since not all of the spring moves with the same speed.) (b) Take the time derivative of the conservation of energy equation, Eq. (14.21), for a mass m moving on the end of a *massless* spring. By comparing your results to Eq. (14.8), which defines ω, show that the angular frequency of oscillation is $\omega = \sqrt{k/m}$. (c) Apply the procedure of part (b) to obtain the angular frequency of oscillation ω of the spring considered in part (a). If the *effective mass* M' of the spring is defined by $\omega = \sqrt{k/M'}$, what is M' in terms of M?

MCAT-Style Passage Problems

BIO **"Seeing" Surfaces at the Nanoscale.** One technique for making images of surfaces at the nanometer scale, including membranes and biomolecules, is dynamic atomic force microscopy. In this technique, a small tip is attached to a cantilever, which is a flexible, rectangular slab supported at one end, like a diving board. The cantilever vibrates, so the tip moves up and down in simple harmonic motion. In one operating mode, the resonant frequency for a cantilever with force constant k=1000 N/m is 100 kHz. As the oscillating tip is brought within a few nanometers of the surface of a sample (as shown in the figure), it experiences an attractive force from the surface. For an oscillation with a small amplitude (typically, 0.050 nm), the force F that the sample surface exerts on the tip varies linearly with the displacement x of the tip, $|F| = k_{\text{surf}}\, x$, where k_{surf} is the effective force constant for this force. The net force on the tip is therefore $(k + k_{\text{surf}})x$, and the frequency of the oscillation changes slightly due to the interaction with the surface. Measurements of the frequency as the tip moves over different parts of the sample's surface can provide information about the sample.



14.94 If we model the vibrating system as a mass on a spring, what is the mass necessary to achieve the desired resonant frequency when the tip is not interacting with the surface? (a) 25 ng; (b) 100 ng; (c) 2.5 μg; (d) 100 μg.

14.95 In the model of Problem 14.94, what is the total mechanical energy of the vibration when the tip is not interacting with the surface? (a) 1.2×10^{-18} J; (b) 1.2×10^{-16} J; (c) 1.2×10^{-9} J; (d) 5.0×10^{-8} J.

14.96 By what percentage does the frequency of oscillation change if $k_{\text{surf}} = 5$ N/m? (a) 0.1%; (b) 0.2%; (c) 0.5%; (d) 1.0%.

Answers: Periodic Motion

Chapter Opening Question ?

(i) The back-and-forth motion of a leg during walking is like a physical pendulum, for which the oscillation period is $T = 2\pi\sqrt{I/mgd}$ [see Eq. (14.39)]. In this expression I is the moment of inertia of the pendulum, m is its mass, and d is the distance from the rotation axis to the pendulum center of mass. I is proportional to m, so the mass cancels out of this expression for T. Hence only the dimensions of the leg matter. (See Examples 14.9 and 14.10.)

Test Your Understanding

14.1 (a) $x < 0$, (b) $x > 0$, (c) $x < 0$, (d) $x > 0$, (e) $x > 0$, (f) $x = 0$ Figure 14.2 shows that both the net x-component of force F_x and the x-acceleration a_x are positive when $x < 0$ (so the object is displaced to the left and the spring is compressed), while both F_x and a_x are negative when $x > 0$ (so the object is displaced to the right and the spring is stretched). Hence x and a_x always have *opposite* signs. This is true whether the object is moving to the right ($v_x > 0$), to the left ($v_x < 0$), or not at all ($v_x = 0$), since the force exerted by the spring depends on only whether it is compressed or stretched and by what distance. This explains the answers to (a) through (e). If the acceleration is zero as in (f), the net force must also be zero and so the spring must be relaxed; hence $x = 0$.

14.2 (a) $A > 0.10\ m$, $\phi < 0$; (b) $A > 0.10\ m$, $\phi > 0$ In both situations the initial ($t = 0$) x-velocity v_{0x} is nonzero, so from Eq. (14.19) the amplitude $A = \sqrt{{x_0}^2 + ({v_{0x}}^2/\omega^2)}$ is greater than the initial x-coordinate $x_0 = 0.10$ m. From Eq. (14.18) the phase angle is $\phi = \arctan(-v_{0x}/\omega x_0)$, which is positive if the quantity $-v_{0x}/\omega x_0$ (the argument of the arctangent function) is positive and negative if $-v_{0x}/\omega x_0$ is negative. In part (a) both x_0 and v_{0x} are positive, so $-v_{0x}/\omega x_0 < 0$ and $\phi < 0$. In part (b) x_0 is positive and v_{0x} is negative, so $-v_{0x}/\omega x_0 > 0$ and $\phi > 0$.

14.3 (a) **(iii)**, (b) **(v)** To increase the total energy $E = \frac{1}{2}kA^2$ by a factor of 2, the amplitude A must increase by a factor of $\sqrt{2}$. Because the motion is SHM, changing the amplitude has no effect on the frequency.

14.4 **(i)** The oscillation period of an object of mass m attached to a hanging spring of force constant k is given by $T = 2\pi\sqrt{m/k}$, the same expression as for an object attached to a horizontal spring. Neither m nor k changes when the apparatus is taken to Mars, so the period is unchanged. The only difference is that in equilibrium, the spring will stretch a shorter distance on Mars than on earth due to the weaker gravity.

14.5 **(iv)** Just as for an object oscillating on a spring, at the equilibrium position the *speed* of a pendulum bob is instantaneously not changing (this is where the speed is maximum, so its derivative at this time is zero). But the *direction* of motion is changing because the pendulum bob follows a circular path. Hence the bob must have a component of acceleration perpendicular to the path and toward the center of the circle (see Section 3.4). To cause this acceleration at the equilibrium position when the string is vertical, the upward tension force at this position must be greater than the weight of the bob. This causes a net upward force on the bob and an upward acceleration toward the center of the circular path.

14.6 **(i)** The period of a physical pendulum is given by Eq. (14.39), $T = 2\pi\sqrt{I/mgd}$. The distance $d = L$ from the pivot to the center of gravity is the same for both the rod and the simple pendulum, as is the mass m. Thus for any displacement angle θ the same restoring torque acts on both the rod and the simple pendulum. However, the rod has a greater moment of inertia:

$I_{\text{rod}} = \dfrac{1}{3}m(2L)^2 = \dfrac{4}{3}mL^2$ and $I_{\text{simple}} = mL^2$ (all the mass of the pendulum is a distance L from the pivot). Hence the rod has a longer period.

14.7 **(ii)** The oscillations are underdamped with a decreasing amplitude on each cycle of oscillation, like those graphed in Fig. 14.26. If the oscillations were undamped, they would continue indefinitely with the same amplitude. If they were critically damped or overdamped, the nose would not bob up and down but would return smoothly to the original equilibrium attitude without overshooting.

14.8 **(i)** Figure 14.28 shows that the curve of amplitude versus driving frequency moves upward at *all* frequencies as the value of the damping constant b is decreased. Hence for fixed values of k and m, the oscillator with the least damping (smallest value of b) will have the greatest response at any driving frequency.

Key Example Variation Problems

VP14.3.1 **a.** $T = 0.241$ s, $\omega = 26.1$ rad/s

b. 272 N/m

c. 5.44 N

VP14.3.2 **a.** 0.164 kg

b. 0.0438 m

VP14.3.3 **a.** 0.0496 m

b. 15.6 m/s

VP14.3.4 **a.** 0.968 Hz

b. 4.11×10^{-2} m

VP14.4.1 **a.** 0.0479 m

b. 9.19×10^{-3} J

c. $U = 3.60 \times 10^{-3}$ J, $K = 5.59 \times 10^{-3}$ J

VP14.4.2 **a.** 62.0 N/m

b. $0.447\ \mathrm{m/s}$

VP14.4.3 **a.** $k =\ 8.89\ \mathrm{N/m},\ m = 0.512\ \mathrm{kg}$

b. $0.521\ \mathrm{m/s^2}$

c. $0.451\ \mathrm{m/s^2}$

VP14.4.4 **a.** $x\ = \pm A\sqrt{2/3}$

b. $x\ = \pm A\sqrt{1/5}$

VP14.9.1 **a.** $1.64\ \mathrm{s}$

b. $7.32\ \mathrm{m/s^2}$

VP14.9.2 $0.392\ \mathrm{m}$

VP14.9.3 $\frac{1}{2\pi}\sqrt{\frac{g}{2R}}$

VP14.9.4 $0.188\ \mathrm{kg{\cdot}m^2}$

Bridging Problem

$T = 2\pi\sqrt{3M/2k}$

Part II: Waves/Acoustics

Chapter 15
Mechanical Waves

? When an earthquake strikes, the news of the event travels through the body of the earth in the form of seismic waves. Which aspects of a seismic wave determine how much power is carried by the wave: (i) the amplitude; (ii) the frequency; (iii) both the amplitude and the frequency; or (iv) neither the amplitude nor the frequency?

Learning Outcomes

In this chapter, you'll learn...

15.1 What is meant by a mechanical wave, and the different varieties of mechanical waves.

15.2 How to use the relationship among speed, frequency, and wavelength for a periodic wave.

15.3 How to interpret and use the mathematical expression for a sinusoidal periodic wave.

15.4 How to calculate the speed of waves on a rope or string.

15.5 How to calculate the rate at which a mechanical wave transports energy.

15.6 What happens when mechanical waves overlap and interfere.

15.7 The properties of standing waves on a string, and how to analyze these waves.

15.8 How stringed instruments produce sounds of specific frequencies.

You'll need to review...

8.1 The impulse–momentum theorem.

14.1, 14.2 Periodic motion and simple harmonic motion.

Ripples on a pond, musical sounds, seismic tremors triggered by an earthquake—all these are *wave* phenomena. Waves can occur whenever a system is disturbed from equilibrium and when the disturbance can travel, or *propagate*, from one region of the system to another. As a wave propagates, it carries energy. The energy in light waves from the sun warms the surface of our planet; the energy in seismic waves can crack our planet's crust.

This chapter and the next are about mechanical waves—waves that travel within some material called a *medium*. (Chapter 16 is concerned with sound, an important type of mechanical wave.) We'll begin this chapter by deriving the basic equations for describing waves, including the important special case of *sinusoidal* waves in which the wave pattern is a repeating sine or cosine function. To help us understand waves in general, we'll look at the simple case of waves that travel on a stretched string or rope.

Waves on a string are important in music. When a musician strums a guitar or bows a violin, she makes waves that travel in opposite directions along the instrument's strings. What happens when these oppositely directed waves overlap is called *interference*. We'll discover that sinusoidal waves can occur on a guitar or violin string only for certain special frequencies, called *normal-mode frequencies*, determined by the properties of the string. These normal-mode frequencies determine the pitch of the musical sounds that a stringed instrument produces. (In the next chapter we'll find that interference also helps explain the pitches of *wind* instruments such as pipe organs.)

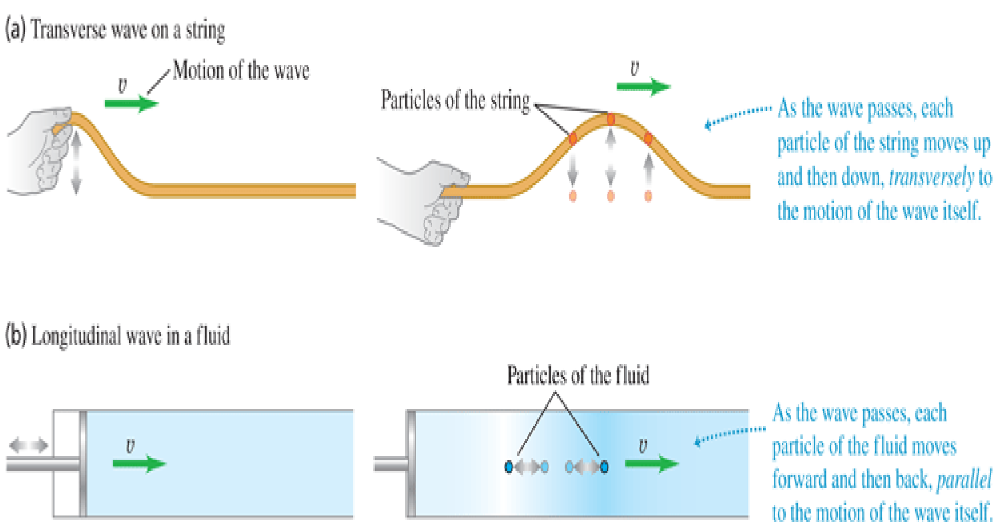
Not all waves are mechanical in nature. *Electromagnetic* waves—including light, radio waves, infrared and ultraviolet radiation, and x rays—can propagate even in empty space, where there is *no* medium. We'll explore these and other nonmechanical waves in later chapters.

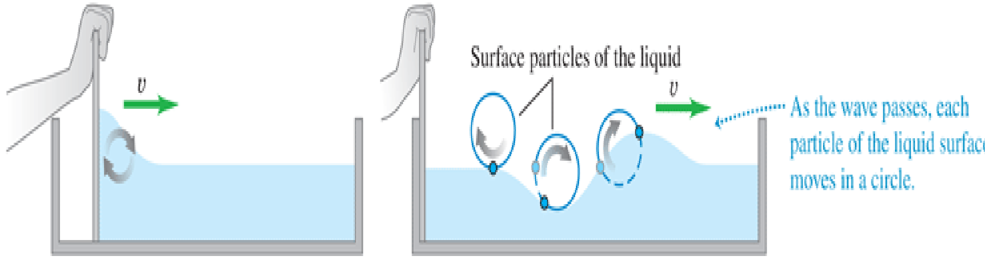
15.1 Types of Mechanical Waves

A **mechanical wave** is a disturbance that travels through some material or substance called the **medium** for the wave. As the wave travels through the medium, the particles that make up the medium undergo displacements of various kinds, depending on the nature of the wave.

Figure 15.1 shows three varieties of mechanical waves. In Fig. 15.1a the medium is a string or rope under tension. If we give the left end a small upward shake or wiggle, the wiggle travels along the length of the string. Successive sections of string go through the same motion that we gave to the end, but at successively later times. Because the displacements of the medium are perpendicular or *transverse* to the direction of travel of the wave along the medium, this is called a **transverse wave.**

Figure 15.1





Three ways to make a wave that moves to the right. (a) The hand moves the string up and then returns, producing a transverse wave. (b) The piston moves to the right, compressing the gas or liquid, and then returns, producing a longitudinal wave. (c) The board moves to the right and then returns, producing a combination of longitudinal and transverse waves.

In Fig. 15.1b the medium is a liquid or gas in a tube with a rigid wall at the right end and a movable piston at the left end. If we give the piston a single back-and-forth motion, displacement and pressure fluctuations travel down the length of the medium. This time the motions of the particles of the medium are back and forth along the *same* direction that the wave travels. We call this a **longitudinal wave.**

In Fig. 15.1c the medium is a liquid in a channel, such as water in an irrigation ditch or canal. When we move the flat board at the left end forward and back once, a wave disturbance travels down the length of the channel. In this case the displacements of the water have *both* longitudinal and transverse components.

Each of these systems has an equilibrium state. For the stretched string it is the state in which the system is at rest, stretched out along a straight line. For the fluid in a tube it is a state in which the fluid is at rest with uniform pressure. And for the liquid in a trough it is a smooth, level water surface. In each case the wave motion is a disturbance from equilibrium that travels from one region of the medium to another. And in each case forces tend to restore the system to its equilibrium position when it is displaced, just as the force of gravity tends to pull a pendulum toward its straight-down equilibrium position when it is displaced.

BIO Application

Waves on a Snake's Body

A snake moves itself along the ground by producing waves that travel backward along its body from its head to its tail. The waves remain stationary with respect to the ground as they push against the ground, so the snake moves forward.

These examples have three things in common. First, in each case the disturbance travels or *propagates* with a definite speed through the medium. This speed is called the speed of propagation, or simply the **wave speed.** Its value is determined in each case by the mechanical properties of the medium. We'll use the symbol for wave speed. (The wave speed is *not* the same as the speed with which particles move when they are disturbed by the wave. We'll return to this point in Section 15.3.) Second, the medium itself does not travel through space; its individual particles undergo back-and-forth or up-and-down motions around their equilibrium positions. The overall pattern of the wave disturbance is what travels. Third, to set any of these systems into motion, we have to put in energy by doing mechanical work on the system. The wave motion transports this energy from one region of the medium to another. *Waves transport energy, but not matter, from one region to another* (Fig. 15.2).

Figure 15.2

"Doing the wave" at a sports stadium is an example of a mechanical wave: The disturbance propagates through the crowd, but there is no transport of matter (none of the spectators moves from one seat to another).

Test Your Understanding of Section 15.1

What type of wave is “the wave” shown in Fig. 15.2? (i) Transverse; (ii) longitudinal; (iii) a combination of transverse and longitudinal.

15.2 Periodic Waves

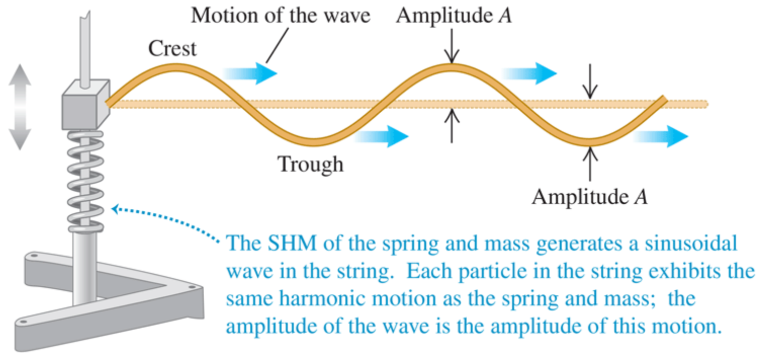
The transverse wave on a stretched string in Fig. 15.1a is an example of a *wave pulse.* The hand exerts a transverse force that shakes the string up and down just once, producing a single "wiggle," or pulse, that travels along the length of the string. The tension in the string restores its straight-line shape once the pulse has passed.

A more interesting situation develops when we give the free end of the string a repetitive, or *periodic,* motion. (You should review the discussion of periodic motion in Chapter 14 before going ahead.) Each particle in the string undergoes periodic motion as the wave propagates, and we have a **periodic wave.**

Periodic Transverse Waves

Suppose we move one end of the string up and down with *simple harmonic motion* (SHM) as in Fig. 15.3, with amplitude A, frequency f, angular frequency $\omega = 2\pi f$, and period $T = 1/f = 2\pi/\omega$. The wave that results is a symmetric sequence of *crests* and *troughs.* As we'll see, periodic waves with SHM are particularly easy to analyze; we call them **sinusoidal waves.** It turns out that *any* periodic wave can be represented as a combination of sinusoidal waves. So this kind of wave motion is worth special attention.

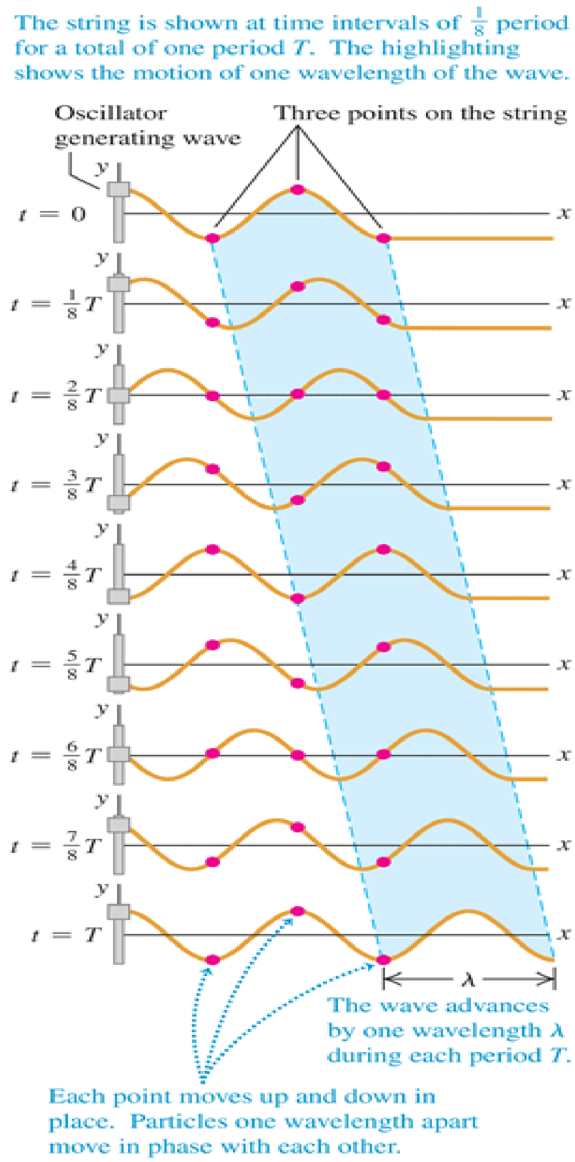
Figure 15.3



A block of mass m attached to a spring undergoes simple harmonic motion, producing a sinusoidal wave that travels to the right on the string. (In a real-life system a driving force would have to be applied to the block to replace the energy carried away by the wave.)

In Fig. 15.3 the wave is a *continuous succession* of transverse sinusoidal disturbances. Figure 15.4 shows the shape of a part of the string near the left end at time intervals of $\frac{1}{8}$ of a period, for a total time of one period. The wave shape advances steadily toward the right, as indicated by the highlighted area. As the wave moves, any point on the string (any of the red dots, for example) oscillates up and down about its equilibrium position with simple harmonic motion. *When a sinusoidal wave passes through a medium, every particle in the medium undergoes simple harmonic motion with the same frequency.*

Figure 15.4



A sinusoidal transverse wave traveling to the right along a string. The vertical scale is exaggerated.

CAUTION Wave motion vs. particle motion Don't confuse the motion of the *transverse wave* along the string and the motion of a *particle* of the string. The wave moves with constant speed v *along* the length of the string, while the motion of the particle is simple harmonic and *transverse* (perpendicular) to the length of the string.

For a periodic wave, the shape of the string at any instant is a repeating pattern. The **wavelength** λ (the Greek letter lambda) of the wave is the distance from one crest to the next, or from one trough to the next, or from any point to the corresponding point on the next repetition of the wave shape. The wave pattern travels with constant speed v and advances a distance of one wavelength λ in a time interval of one period T. So the wave speed is $v = \lambda/T$ or, because $f = 1/T$ from Eq. (14.1),

(15.1)

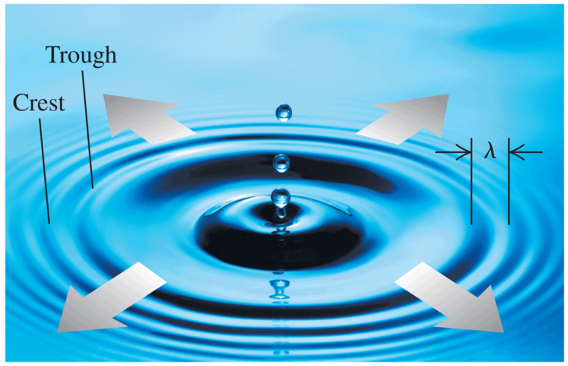
For a **periodic wave:**

$$\overset{\text{Wave speed}}{v} = \overset{\text{Wavelength}}{\lambda}\underset{\text{Frequency}}{f} \qquad (15.1)$$

The speed of propagation equals the product of wavelength and frequency. The frequency is a property of the *entire* periodic wave because all points on the string oscillate with the same frequency f.

Waves on a string propagate in just one dimension (in Fig. 15.4, along the x-axis). But the ideas of frequency, wavelength, and amplitude apply equally well to waves that propagate in two or three dimensions. Figure 15.5 shows a wave propagating in two dimensions on the surface of a tank of water. As with waves on a string, the wavelength is the distance from one crest to the next, and the amplitude is the height of a crest above the equilibrium level.

Figure 15.5



A series of drops falling into water produces a periodic wave that spreads radially outward. The wave crests and troughs are concentric circles. The wavelength λ is the radial distance between adjacent crests or adjacent troughs.

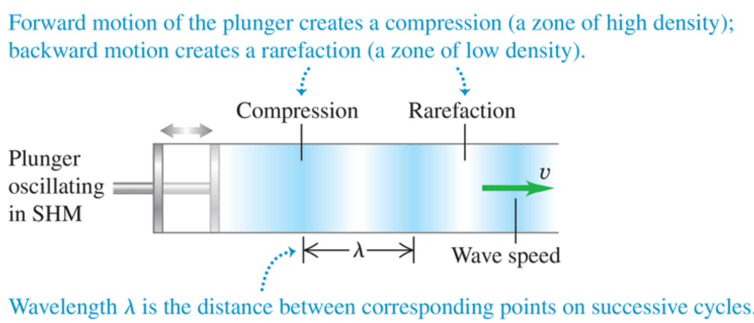
In many important situations including waves on a string, the wave speed v is determined entirely by the mechanical properties of the medium. In this case, increasing f causes λ to decrease so the product $v = \lambda f$ remains the same, and waves of *all* frequencies propagate with the same wave speed. In this chapter we'll consider *only* waves of this kind. (In later chapters we'll study the propagation of light waves in transparent materials where the wave speed depends on frequency; this turns out to be the reason raindrops create a rainbow.)

Periodic Longitudinal Waves

To understand the mechanics of a periodic *longitudinal* wave, consider a long tube filled with a fluid, with a piston at the left end as in Fig. 15.1b. If we push the piston in, we compress the fluid near the piston, increasing the pressure in this region. This region then pushes against the neighboring region of fluid, and so on, and a wave pulse moves along the tube.

Now suppose we move the piston back and forth in SHM along a line parallel to the axis of the tube (Fig. 15.6). This motion forms regions in the fluid where the pressure and density are greater or less than the equilibrium values. We call a region of increased density a *compression*; a region of reduced density is a *rarefaction*. Figure 15.6 shows compressions as darkly shaded areas and rarefactions as lightly shaded areas. The wavelength is the distance from one compression to the next or from one rarefaction to the next.

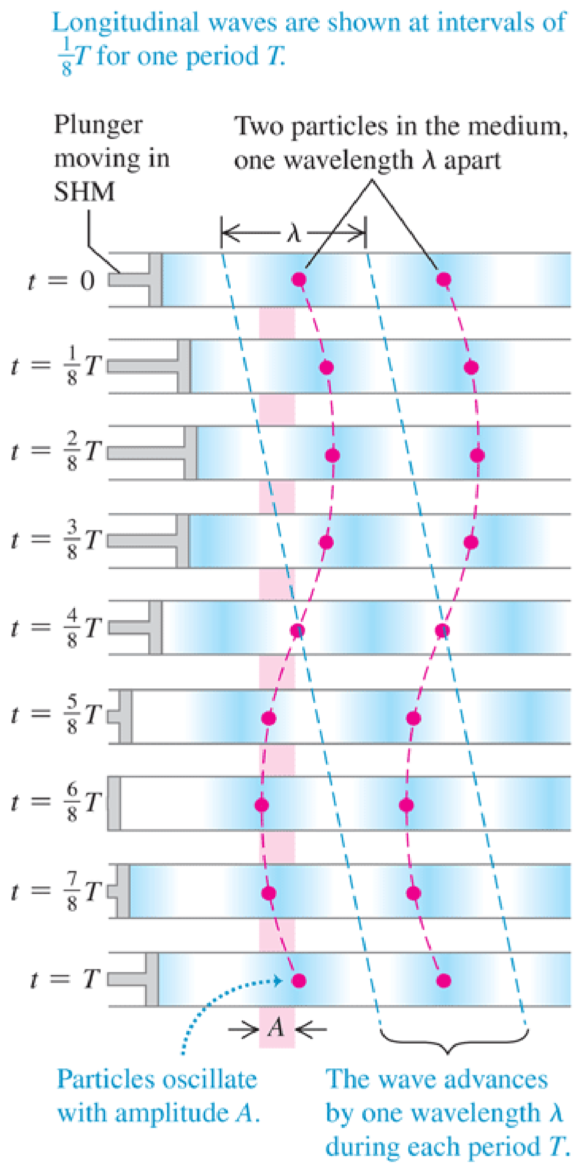
Figure 15.6



Using an oscillating piston to make a sinusoidal longitudinal wave in a fluid.

Figure 15.7 shows the wave propagating in the fluid-filled tube at time intervals of $\frac{1}{8}$ of a period, for a total time of one period. The pattern of compressions and rarefactions moves steadily to the right, just like the pattern of crests and troughs in a sinusoidal transverse wave (compare Fig. 15.4). Each particle in the fluid oscillates in SHM parallel to the direction of wave propagation (that is, left and right) with the same amplitude A and period T as the piston. The particles shown by the two red dots in Fig. 15.7 are one wavelength apart, and so oscillate in phase with each other.

Figure 15.7



A sinusoidal longitudinal wave traveling to the right in a fluid. The wave has the same amplitude A and period T as the oscillation of the piston.

Just like the sinusoidal transverse wave shown in Fig. 15.4, in one period T the longitudinal wave in Fig. 15.7 travels one wavelength λ to the right. Hence the fundamental equation $v = \lambda f$ holds for longitudinal waves as well as for transverse waves, and indeed for *all* types of periodic waves. Just as for transverse waves, in this chapter and the next we'll consider only situations in which the speed of longitudinal waves does not depend on the frequency.

Example 15.1 Wavelength of a musical sound

WITH VARIATION PROBLEMS

Sound waves are longitudinal waves in air. The speed of sound depends on temperature; at $20^\circ\mathrm{C}$ it is $344\ \mathrm{m}/s$ $(1130\ \mathrm{ft/s})$. What is the wavelength of a sound wave in air at $20^\circ\mathrm{C}$ if the frequency is $262\ \mathrm{Hz}$ (the approximate frequency of middle C on a piano)?

IDENTIFY and SET UP This problem involves Eq. (15.1), $v = \lambda f$, which relates wave speed v, wavelength λ, and frequency f for a periodic wave. The target variable is the wavelength λ. We are given $v = 344\ \mathrm{m}/s$ and $f = 262\ \mathrm{Hz} = 262\ \mathrm{s}^{-1}$.

EXECUTE We solve Eq. (15.1) for λ:

$$\lambda = \frac{v}{f} = \frac{344\ \mathrm{m/s}}{262\ \mathrm{Hz}} = \frac{344\ \mathrm{m/s}}{262\ \mathrm{s}^{-1}} = 1.31\ \mathrm{m}$$

EVALUATE The speed v of sound waves does *not* depend on the frequency. Hence $\lambda = v/f$ says that wavelength changes in inverse proportion to frequency. As an example, high (soprano) C is two octaves above middle C. Each octave corresponds to a factor of 2 in frequency, so the frequency of high C is four times that of middle C: $f = 4(262\ \mathrm{Hz}) = \ 1048\ \mathrm{Hz}$. Hence the *wavelength* of high C is *one-fourth* as large: $\lambda = (1.31\ \mathrm{m})/4 = 0.328\ \mathrm{m}$.

KEY CONCEPT

The product of a wave's wavelength and frequency has the same value no matter what the frequency is. This product equals the wave speed.

Video Tutor Solution: **Example 15.1**

Test Your Understanding of Section 15.2

If you double the wavelength of a wave on a particular string without changing the physical properties of the string, what happens to the wave speed v and the frequency f? (i) v doubles and f is unchanged; (ii) v is unchanged and f doubles; (iii) v becomes one-half as great and f is unchanged; (iv) v is unchanged and f becomes one-half as great; (v) none of these.

15.3 Mathematical Description of a Wave

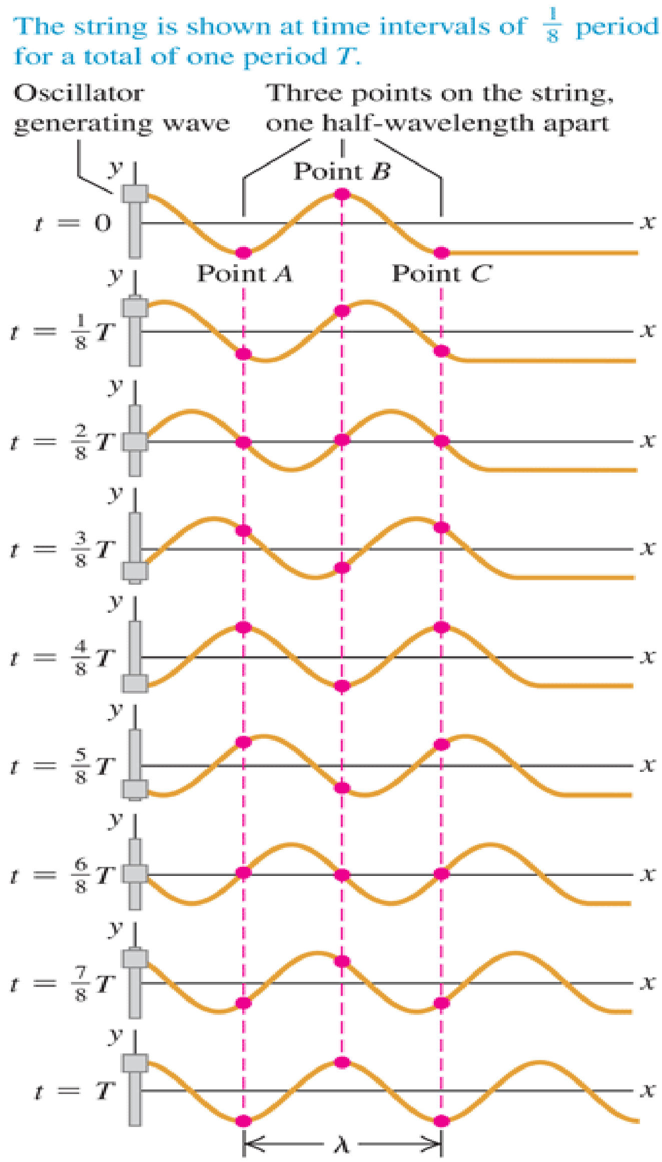
Many characteristics of periodic waves can be described by using the concepts of wave speed, amplitude, period, frequency, and wavelength. Often, though, we need a more detailed description of the positions and motions of individual particles of the medium at particular times during wave propagation.

As a specific example, let's look at waves on a stretched string. If we ignore the sag of the string due to gravity, the equilibrium position of the string is along a straight line. We take this to be the x-axis of a coordinate system. Waves on a string are *transverse;* during wave motion a particle with equilibrium position x is displaced some distance y in the direction perpendicular to the x-axis. The value of y depends on which particle we are talking about (that is, y depends on x) and also on the time t when we look at it. Thus y is a *function* of both x and t; $y = y(x,\ t)$. We call $y(x,\ t)$ the **wave function** that describes the wave. If we know this function for a particular wave motion, we can use it to find the displacement (from equilibrium) of any particle at any time. From this we can find the velocity and acceleration of any particle, the shape of the string, and anything else we want to know about the behavior of the string at any time.

Wave Function for a Sinusoidal Wave

Let's see how to determine the form of the wave function for a sinusoidal wave. Suppose a sinusoidal wave travels from left to right (the direction of increasing x) along the string, as in Fig. 15.8. Every particle of the string oscillates in simple harmonic motion with the same amplitude and frequency. But the oscillations of particles at different points on the string are *not* all in step with each other. The particle at point B in Fig. 15.8 is at its maximum positive value of y at $t = 0$ and returns to $y = 0$ at $t = \frac{2}{8}T$; these same events occur for a particle at point A or point C at $t = \frac{4}{8}T$ and $t = \frac{6}{8}T$, exactly one half-period later. For any two particles of the string, the motion of the particle on the right (in terms of the wave, the "downstream" particle) lags behind the motion of the particle on the left by an amount proportional to the distance between the particles.

Figure 15.8



Tracking the oscillations of three points on a string as a sinusoidal wave propagates along it.

Hence the cyclic motions of various points on the string are out of step with each other by various fractions of a cycle. We call these differences *phase differences*, and we say that the *phase* of the motion is different for different points. For example, if one point has its maximum positive displacement at the same time that another has its maximum negative displacement, the two are a half-cycle out of phase. (This is the case for points A and B, or points B and C.)

Suppose that the displacement of a particle at the left end of the string $(x = 0)$, where the wave originates, is given by

(15.2)

$$y\,(x = 0,\ t) = A\ \cos\,\omega t = A\ \cos\,2\pi f t$$

That is, the particle oscillates in SHM with amplitude A, frequency f, and angular frequency $\omega = 2\pi f$. The notation $y\,(x = 0,\ t)$ reminds us that the motion of this particle is a special case of the wave function $y\,(x,\ t)$ that describes the entire wave. At $t = 0$ the particle at $x = 0$ is at its maximum positive displacement $(y = A)$ and is instantaneously at rest (because y is a maximum).

The wave disturbance travels from $x = 0$ to some point x to the right of the origin in an amount of time given by x/v, where v is the wave speed. So the motion of point x at time t is the same as the motion of point $x = 0$ at the earlier time $t - x/v$. Hence we can find the displacement of point x at time t by simply replacing t in Eq. (15.2) by $(t - x/v)$:

$$y\,(x,\ t) = A\ \cos\left[\omega\left(t - \frac{x}{v}\right)\right]$$

Because $cos\,(-\theta) = \cos\theta$ we can rewrite the wave function as

(15.3)

$$\underbrace{y(x,t)}_{\text{Wave function for a sinusoidal wave propagating in }+x\text{-direction}} = \overbrace{A}^{\text{Amplitude}}\cos\left[\underbrace{\omega}_{\text{Angular frequency }=\,2\pi f}\left(\frac{\overset{\text{Position}}{x}}{\underset{\text{Wave speed}}{v}} - \overset{\text{Time}}{t}\right)\right] \qquad (15.3)$$

The displacement $y(x,\ t)$ is a function of both the location x of the point and the time t. We could make Eq. (15.3) more general by allowing for different values of the phase angle, as we did for SHM in Section 14.2, but for now we omit this.

We can rewrite the wave function given by Eq. (15.3) in several different but useful forms. We can express it in terms of the period $T = 1/f$ and the wavelength $\lambda = v/f = 2\pi v/\omega$:

(15.4)

Wave function for a sinusoidal wave propagating in $+x$-direction

$$y(x,t) = \overset{\text{Amplitude}}{A}\cos\left[2\pi\left(\frac{\overset{\text{Position}}{x}}{\underset{\text{Wavelength}}{\lambda}} - \frac{\overset{\text{Time}}{t}}{\underset{\text{Period}}{T}}\right)\right] \qquad (15.4)$$

It's convenient to define a quantity k, called the **wave number:**

(15.5)

$$k = \frac{2\pi}{\lambda} \qquad \text{(wave number)}$$

Substituting $\lambda = 2\pi/k$ and $f = \omega/2\pi$ into Eq. (15.1), $v = \lambda f$, gives

(15.6)

$$\omega = vk \qquad \text{(periodic wave)}$$

We can then rewrite Eq. (15.4) as

(15.7)

Wave function for a sinusoidal wave propagating in $+x$-direction

$$y(x,t) = \overset{\text{Amplitude}}{A}\cos(\underset{\text{Wave number} = 2\pi/\lambda}{k}\overset{\text{Position}}{x} - \underset{\text{Angular frequency} = 2\pi f}{\omega}\overset{\text{Time}}{t}) \tag{15.7}$$

Which of these various forms for the wave function $y(x,\ t)$ we use in any specific problem is a matter of convenience. Note that ω has units rad/s, so for unit consistency in Eqs. (15.6) and (15.7) the wave number k must have the units rad/m. (Warning: Some textbooks define the wave number as $1/\lambda$ rather than $2\pi/\lambda$.)

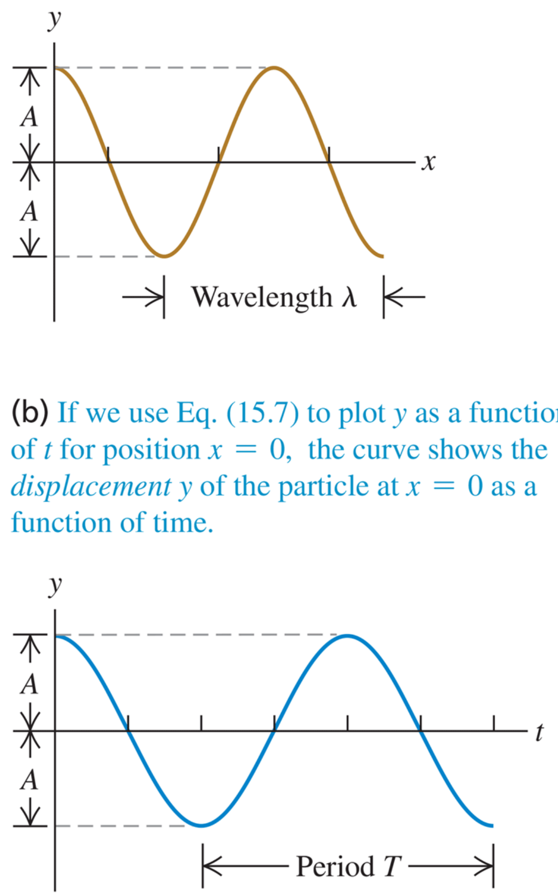
CAUTION Amplitude is independent of wavelength or frequency We learned in Section 15.2 that the frequency and wavelength of a wave are closely related: Increasing the frequency decreases the wavelength, and decreasing the frequency increases the wavelength. The amplitude of a wave, however, does not depend on the frequency or the wavelength. Changing the frequency or wavelength has no effect on the amplitude, and changing the wave amplitude by itself has no effect on the frequency or wavelength.

Graphing the Wave Function

Figure 15.9a graphs the wave function $y(x,\ t)$ as a function of x for a specific time t. This graph gives the displacement y of a particle from its equilibrium position as a function of the coordinate x of the particle. If the wave is a transverse wave on a string, the graph in Fig. 15.9a represents the shape of the string at that instant, like a flash photograph of the string. In particular, at time $t = 0$,

$$y(x,\ t = 0) = A\cos kx = A\cos 2\pi\frac{x}{\lambda}$$

Figure 15.9



(a) If we use Eq. (15.7) to plot y as a function of x for time $t = 0$, the curve shows the *shape* of the string at $t = 0$.

(b) If we use Eq. (15.7) to plot y as a function of t for position $x = 0$, the curve shows the *displacement* y of the particle at $x = 0$ as a function of time.

Two graphs of the wave function $y(x, t)$ in Eq. (15.7). (a) Graph of displacement y versus coordinate x at time $t = 0$. (b) Graph of displacement y versus time t at coordinate $x = 0$. The vertical scale is exaggerated in both (a) and (b).

Figure 15.9b is a graph of the wave function versus time t for a specific coordinate x. This graph gives the displacement y of the particle at x as a function of time; that is, it describes the motion of that particle. At position $x = 0$,

$$y(x=0,\ t) = A\cos(-\omega t) = A\cos\omega t = A\cos 2\pi\frac{t}{T}$$

This is consistent with our original statement about the motion at $x = 0$, Eq. (15.2).

CAUTION Wave graphs Although they may look the same, Figs. 15.9a and 15.9b are *not* identical. Figure 15.9a is a picture of the shape of the string at $t = 0$, while Fig. 15.9b is a graph of the displacement y of a particle at $x = 0$ as a function of time.

More on the Wave Function

We can modify Eqs. (15.3), (15.4), (15.5), (15.6) and (15.7) to represent a wave traveling in the *negative* x-direction. In this case the displacement of point x at time t is the same as the motion of point $x = 0$ at the *later* time $(t + x/v)$, so in Eq. (15.2) we replace t by $(t + x/v)$. For a wave traveling in the negative x-direction,

(15.8)

$$y(x,\ t) = A\cos\left[\omega\left(\frac{x}{v}+t\right)\right] = A\cos\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)\right] = A\cos(kx+\omega t)$$

$$\text{(sinusoidal wave moving in } -x\text{-direction)}$$

In the expression $y(x,\ t) = A\cos(kx \pm \omega t)$ for a wave traveling in the $-x$- or $+x$-direction, the quantity $(kx \pm \omega t)$ is called the **phase.** It plays the role of an angular quantity (always measured in radians) in Eq. (15.7) or (15.8), and its value for any values of x and t determines what part of the sinusoidal cycle is occurring at a particular point and time. For a crest (where $y = A$ and the cosine function has the value 1), the phase could be 0, $\pm 2\pi,\ \pm 4\pi$, and so on; for a trough (where $y = -A$ and the cosine has the value -1), it could be $\pm\pi,\ \pm 3\pi,\ \pm 5\pi$, and so on.

The wave speed is the speed with which we have to move along with the wave to keep alongside a point of a given phase, such as a particular crest of a wave on a string. For a wave traveling in the $+x$-direction, that means $kx - \omega t =$constant. Taking the derivative with respect to t, we find $k\, dx/dt = \omega$, or

$$\frac{dx}{dt} = \frac{\omega}{k}$$

Comparing this with Eq. (15.6), we see that dx/dt is equal to the speed v of the wave. Because of this relationship, v is sometimes called the *phase velocity* of the wave. (*Phase speed* would be a better term.)

Problem-Solving Strategy 15.1 Mechanical Waves

IDENTIFY *the relevant concepts:* As always, identify the target variables; these may include mathematical *expressions* (for example, the wave function for a given situation). Note that wave problems fall into two categories. *Kinematics* problems, concerned with describing wave motion, involve wave speed v, wavelength λ (or wave number k), frequency f (or angular frequency ω), and amplitude A. They may also involve the position, velocity, and acceleration of individual particles in the medium. *Dynamics* problems also use concepts from Newton's laws. Later in this chapter we'll encounter problems that involve the relationship of wave speed to the mechanical properties of the medium.

SET UP *the problem* using the following steps:

1. List the given quantities. Sketch graphs of y versus x (like Fig. 15.9a) and of y versus t (like Fig. 15.9b), and label them with known values.

2. Identify useful equations. These may include Eq. (15.1) ($v = \lambda f$), Eq. (15.6) ($\omega = vk$), and Eqs. (15.3), (15.4), and (15.7), which express the wave function in various forms. From the wave function, you can find the value of y at any point (value of x) and at any time t.
3. If you need to determine the wave speed v and don't know both λ and f, you may be able to use a relationship between v and the mechanical properties of the system. (In the next section we'll develop this relationship for waves on a string.)

EXECUTE *the solution:* Solve for the unknown quantities using the equations you've identified. To determine the wave function from Eq. (15.3), (15.4), or (15.7), you must know A and any two of v, λ, and f (or v, k, and ω).

EVALUATE *your answer:* Confirm that the values of v, f, and λ (or v, ω, and k) agree with the relationships given in Eq. (15.1) or (15.6). If you've calculated the wave function, check one or more special cases for which you can predict the results.

Example 15.2 Wave on a clothesline

WITH VARIATION PROBLEMS

Cousin Throckmorton holds one end of the clothesline taut and wiggles it up and down sinusoidally with frequency 2.00 Hz and amplitude 0.075 m. The wave speed on the clothesline is $v = 12.0\ \text{m/s}$. At $t = 0$ Throcky's end has maximum positive displacement and is instantaneously at rest. Assume that no wave bounces back from the far end. (a) Find the wave amplitude A, angular frequency ω, period T, wavelength λ, and wave number k. (b) Write a wave function describing the wave. (c) Write equations for the displacement, as a function of time, of Throcky's end of the clothesline and of a point 3.00 m from that end.

IDENTIFY and SET UP This is a kinematics problem about the clothesline's wave motion. Throcky produces a sinusoidal wave that propagates along the clothesline, so we can use all of the expressions of this section. In part (a) our target variables are A, ω, T, λ, and k. We use the relationships $\omega = 2\pi f$, $f = 1/T$, $v = \lambda f$, and $k = 2\pi/\lambda$. In parts (b) and (c) our target "variables" are expressions for displacement, which we'll obtain from an appropriate equation for the wave function. We take the positive x-direction to be the direction in which the wave propagates, so either Eq. (15.4) or (15.7) will yield the desired expression. A photograph of the clothesline at time $t = 0$ would look like Fig. 15.9a, with the maximum displacement at $x = 0$ (the end that Throcky holds).

EXECUTE (a) The wave amplitude and frequency are the same as for the oscillations of Throcky's end of the clothesline, $A = 0.075$ m and $f = 2.00$ Hz. Hence

$$\begin{aligned}\omega &= 2\pi f = \left(2\pi\,\frac{\text{rad}}{\text{cycle}}\right)\left(2.00\,\frac{\text{cycles}}{\text{s}}\right)\\ &= 4.00\pi\ \text{rad/s} = 12.6\ \text{rad}/s\end{aligned}$$

The period is $T = 1/f = 0.500$ s, and from Eq. (15.1),

$$\lambda = \frac{v}{f} = \frac{12.0\ \text{m/s}}{2.00\ \text{s}^{-1}} = 6.00\ \text{m}$$

We find the wave number from Eq. (15.5) or (15.6):

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\ \text{rad}}{6.00\ \text{m}} = 1.05\ \text{rad/m}$$

or

$$k = \frac{\omega}{v} = \frac{4.00\pi \text{ rad/s}}{12.0 \text{ m/s}} = 1.05 \text{ rad/m}$$

(b) We write the wave function using Eq. (15.4) and the values of A, T, and λ from part (a):

$$\begin{aligned} y(x,\ t) &= A \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \\ &= (0.075 \text{ m}) \cos(12.6 \text{ rad/s})t \\ &= (0.075 \text{ m}) \cos[\pi - (12.6 \text{ rad/s})t] \end{aligned}$$

We can also get this same expression from Eq. (15.7) by using the values of ω and k from part (a).

(c) We can find the displacement as a function of time at $x = 0$ and $x = +3.00$ m by substituting these values into the wave function from part (b):

$$\begin{aligned} y(x = 0,\ t) &= (0.075 \text{ m}) \cos 2\pi \left(\frac{0}{6.00 \text{ m}} - \frac{t}{0.500 \text{ s}} \right) \\ &= (0.075 \text{ m}) \cos(12.6 \text{ rad/s})t \\ y(x = +3.00 \text{ m},\ t) &= (0.075 \text{ m}) \cos\ 2\pi \left(\frac{3.00 \text{ m}}{6.00 \text{ m}} - \frac{t}{0.500 \text{ s}} \right) \\ &= (0.075 \text{ m}) \cos[\pi - (12.6 \text{ rad/s})t] \\ &= -(0.075 \text{ m}) \cos(12.6 \text{ rad/s})t \end{aligned}$$

EVALUATE In part (b), the quantity $(1.05 \text{ rad/m})x - (12.6 \text{ rad/s})t$ is the *phase* of a point x on the string at time t. The two points in part (c) oscillate in SHM with the same frequency and amplitude, but their oscillations differ in phase by $(1.05 \text{ rad/m})(3.00 \text{ m}) = 3.15\ rad = \pi$ radians within rounding error—that is, one half-cycle—because the points are separated by one half-wavelength: $\lambda/2 = (6.00 \text{ m})/2 = 3.00$ m. Thus, while a graph of y versus t for the point at $x = 0$ is a cosine curve (like Fig. 15.9b), a graph of y versus t for the point $x = 3.00$ m is a *negative* cosine curve (the same as a cosine curve shifted by one half-cycle).

Using the expression for $y(x = 0,\ t)$ in part (c), can you show that the end of the string at $x = 0$ is instantaneously at rest at $t = 0$, as stated at the beginning of this example? (*Hint:* Calculate the y-velocity at this point by taking the derivative of y with respect to t.)

KEY CONCEPT

The wave function of a wave describes the displacement from equilibrium of the wave medium. It gives this displacement at any position in the medium and at any time.

Video Tutor Solution: **Example 15.2**

Particle Velocity and Acceleration in a Sinusoidal Wave

From the wave function we can get an expression for the transverse velocity of any *particle* in a transverse wave. We call this v_y to distinguish it from the wave propagation speed v. To find the transverse velocity v_y at a particular point x, we take the derivative of the wave function $y(x,\ t)$ with respect to t, keeping x constant. If the wave function is

$$y(x,\ t) = A\cos(kx - \omega t)$$

then

(15.9)

$$v_y(x,\ t) = \frac{\partial y(x,\ t)}{\partial t} = \omega A\sin(kx - \omega t)$$

The ∂ in this expression is a modified d, used to remind us that $y(x,\ t)$ is a function of *two* variables and that we are allowing only one (t) to vary. The other (x) is constant because we are looking at a particular point on the string. This derivative is called a *partial derivative.* If you haven't yet encountered partial derivatives in your study of calculus, don't fret; it's a simple idea.

Equation (15.9) shows that the transverse velocity of a particle varies with time, as we expect for simple harmonic motion. The maximum particle speed is ωA; this can be greater than, less than, or equal to the wave speed v, depending on the amplitude and frequency of the wave.

The *acceleration* of any particle is the *second* partial derivative of $y(x,\ t)$ with respect to t:

(15.10)

$$\begin{aligned} a_y(x,\ t) = \frac{\partial^2 y(x,\ t)}{\partial t^2} &= -\omega^2 A\cos(kx - \omega t) \\ &= -\omega^2 y(x,\ t) \end{aligned}$$

The acceleration of a particle equals $-\omega^2$ times its displacement, which is the result we obtained in Section 14.2 for simple harmonic motion.

We can also compute partial derivatives of $y(x,\ t)$ with respect to x, holding t constant. The first partial derivative $\partial y(x,\ t)/\partial x$ is the *slope* of the string at point x and at time t. The second partial derivative with respect to x tells us the *curvature* of the string:

(15.11)

$$\frac{\partial^2 y(x,\ t)}{\partial x^2} = -k^2 A \cos(kx - \omega t) = -k^2 y(x,\ t)$$

From Eqs. (15.10) and (15.11) and the relationship $\omega = vk$ we see that

$$\frac{\partial^2 y(x,\ t)/\partial t^2}{\partial^2 y(x,\ t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2 \quad \text{and}$$

(15.12)

Wave equation involves second partial derivatives of wave function:

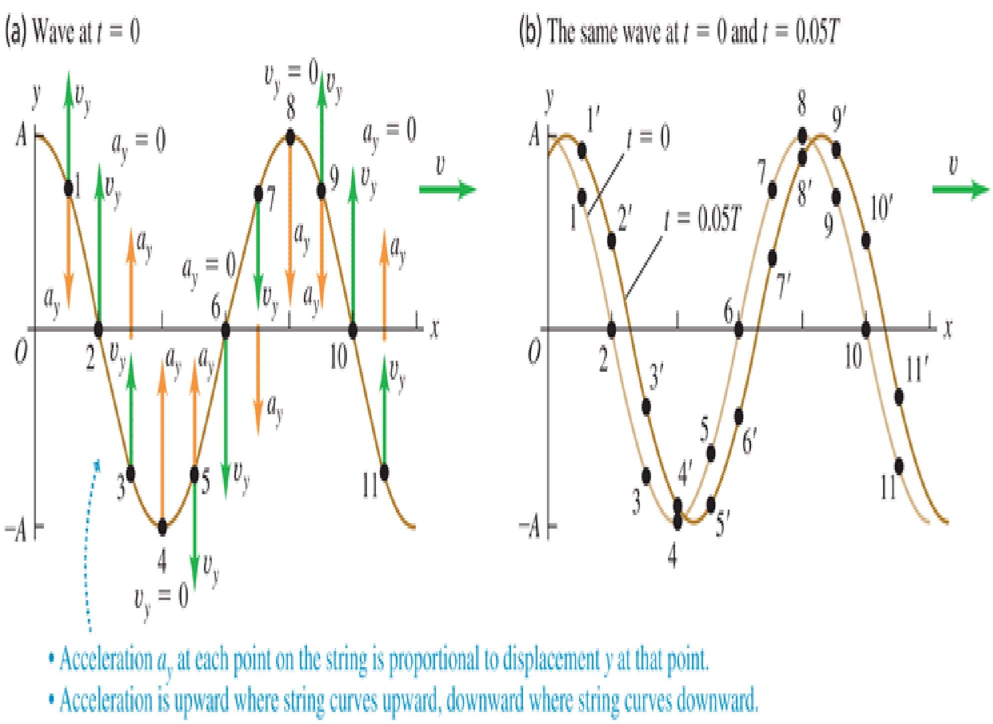
$$\overbrace{\frac{\partial^2 y(x,t)}{\partial x^2}}^{\text{Second partial derivative with respect to } x} = \frac{1}{\underbrace{v^2}_{\text{Wave speed}}} \underbrace{\frac{\partial^2 y(x,t)}{\partial t^2}}_{\text{Second partial derivative with respect to } t} \qquad (15.12)$$

We've derived Eq. (15.12) for a wave traveling in the positive x-direction. You can show that the wave function for a sinusoidal wave propagating in the *negative* x-direction, $y(x,\ t) = A\cos(kx + \omega t)$, also satisfies this equation.

Equation (15.12), called the **wave equation,** is one of the most important equations in all of physics. Whenever it occurs, we know that a disturbance can propagate as a wave along the x-axis with wave speed v. The disturbance need not be a sinusoidal wave; we'll see in the next section that *any* wave on a string obeys Eq. (15.12), whether the wave is periodic or not. In Chapter 32 we'll find that electric and magnetic fields satisfy the wave equation; the wave speed will turn out to be the speed of light, which will lead us to the conclusion that light is an electromagnetic wave.

Figure 15.10a (next page) shows the transverse velocity v_y and transverse acceleration a_y, given by Eqs. (15.9) and (15.10), for several points on a string as a sinusoidal wave passes along it. At points where the string has an upward curvature $\left(\partial^2 y/\partial x^2 > 0\right)$, the acceleration is positive $\left(a_y = \partial^2 y/\partial t^2 > 0\right)$; this follows from the wave equation, Eq. (15.12). For the same reason the acceleration is negative $\left(a_y = \partial^2 y/\partial t^2 < 0\right)$ at points where the string has a downward curvature $\left(\partial^2 y/\partial x^2 < 0\right)$, and the acceleration is zero $\left(a_y = \partial^2 y/\partial t^2 = 0\right)$ at points of inflection where the curvature is zero $\left(\partial^2 y/\partial x^2 = 0\right)$. Remember that v_y and a_y are the *transverse* velocity and acceleration of points on the string; these points move along the y-direction, not along the propagation direction of the wave. Figure 15.10b shows these motions for several points on the string.

Figure 15.10



(a) Another view of the wave at $t = 0$ in Fig. 15.9a. The vectors show the transverse velocity v_y and transverse acceleration a_y at several points on the string.(b) From $t = 0$ to $\mathrm{t} = 0.05T$, a particle at point 1 is displaced to point 1′, a particle at point 2 is displaced to point 2′, and so on.

For *longitudinal* waves, the wave function $y(x,\ t)$ still measures the displacement of a particle of the medium from its equilibrium position. The difference is that for a longitudinal wave, this displacement is *parallel* to the *x*-axis instead of perpendicular to it. We'll discuss longitudinal waves in detail in Chapter 16.

Test Your Understanding of Section 15.3

Figure 15.8 shows a sinusoidal wave of period T on a string at times 0, $\frac{1}{8}T$, $\frac{2}{8}T$, $\frac{3}{8}T$, $\frac{4}{8}T$, $\frac{5}{8}T$, $\frac{6}{8}T$, $\frac{7}{8}T$, and T. (a) At which time is point A on the string moving upward with maximum speed? (b) At which time does point B on the string have the greatest upward acceleration? (c) At which time does point C on the string have a downward acceleration and a downward velocity?

15.4 Speed of a Transverse Wave

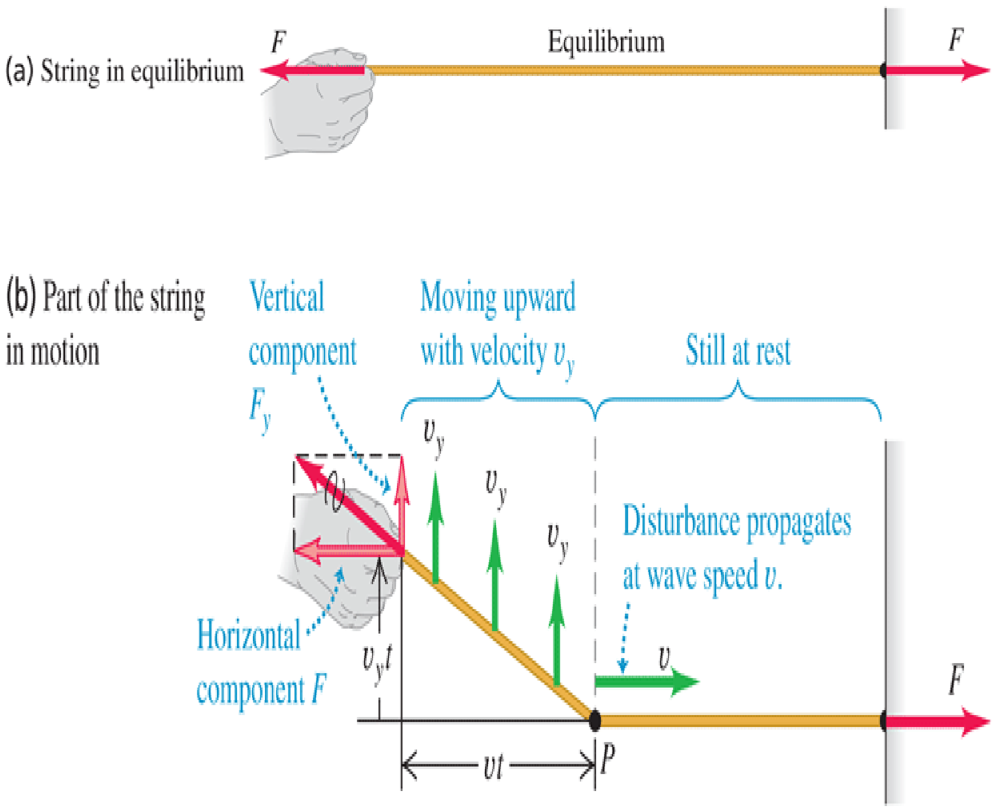
One of the key properties of any wave is the wave *speed*. Light waves in air have a much greater speed of propagation than do sound waves in air (3.00×10^8 m/s versus 344 m/s); that's why you see the flash from a bolt of lightning before you hear the clap of thunder. In this section we'll see what determines the speed of propagation of one particular kind of wave: transverse waves on a string. The speed of these waves is important to understand because it is an essential part of analyzing stringed musical instruments, as we'll discuss later in this chapter. Furthermore, the speeds of many kinds of mechanical waves turn out to have the same basic mathematical expression as does the speed of waves on a string.

What determines the speed of transverse waves on a string are the *tension* in the string and its *mass per unit length* (also called *linear mass density*). Increasing the tension also increases the restoring forces that tend to straighten the string when it is disturbed, thus increasing the wave speed. Increasing the mass per unit length makes the motion more sluggish, and so decreases the wave speed. We'll develop the exact relationship among wave speed, tension, and mass per unit length by two different methods. The first is simple in concept and considers a specific wave shape; the second is more general but also more formal.

Wave Speed on a String: First Method

We consider a perfectly flexible string (Fig. 15.11). In the equilibrium position the tension is F and the linear mass density (mass per unit length) is μ. (When portions of the string are displaced from equilibrium, the mass per unit length decreases a little, and the tension increases a little.) We ignore the weight of the string so that when the string is at rest in the equilibrium position, the string forms a perfectly straight line as in Fig. 15.11a.

Figure 15.11



Propagation of a transverse wave on a string.

Starting at time $t = 0$, we apply a constant upward force F_y at the left end of the string. We might expect that the end would move with constant acceleration; that would happen if the force were applied to a *point* mass. But here the effect of the force F_y is to set successively more and more mass in motion. The wave travels with constant speed v, so the division point P between moving and nonmoving portions moves with the same constant speed v (Fig. 15.11b).

Figure 15.11b shows that all particles in the moving portion of the string move upward with constant *velocity* v_y, not constant acceleration. To see why this is so, we note that the *impulse* of the force F_y up to time t is $F_y t$. According to the impulse–momentum theorem (see Section 8.1), the impulse is equal to the change in the total transverse component of momentum of the moving part of the string. Because the system started with zero transverse momentum, this is equal to the total transverse momentum mv_y at time t:

(15.13)

$$\begin{aligned} \text{Transverse impulse} &= \text{Transverse momentum} \\ \mathrm{F}_y t &= m v_y \end{aligned}$$

The total momentum thus must increase proportionately with time. But since the division point P moves with constant speed, the length of string that is in motion and hence the total mass m in motion are also proportional to the time t that the force has been acting. So the *change* of momentum must be associated entirely with the increasing amount of mass in motion, not with an increasing velocity of an individual mass element. That is, mv_y changes because m, not v_y, changes.

At time t, the left end of the string has moved up a distance $v_y t$, and the boundary point P has advanced a distance vt. The total force at the left end of the string has components F and F_y. Why F? There is no motion in the direction along the length of the string, so there is no unbalanced horizontal force. Therefore F, the magnitude of the horizontal component, does not change when the string is displaced. In the displaced position the tension is $\left(F^2 + {F_y}^2\right)^{1/2}$; this is greater than F, so the string stretches somewhat.

To derive an expression for the wave speed v, we note that in Fig. 15.11b the right triangle whose vertex is at P, with sides $v_y t$ and vt, is similar to the right triangle whose vertex is at the position of the hand, with sides F_y and F. Hence

$$\frac{F_y}{F} = \frac{v_y\, t}{vt} \qquad F_y = F\frac{v_y}{v}$$

and

$$\text{Transverse impulse} = F_y\, t = F\frac{v_y}{v}\, t$$

The mass m of the moving portion of the string is the product of the mass per unit length μ and the length vt, or μvt. The transverse momentum is the product of this mass and the transverse velocity v_y :

$$\text{Transverse momentum} = mv_y = (\mu vt)v_y$$

Substituting these into Eq. (15.13), we obtain

$$F\frac{v_y}{v}\, t = \mu vtv_y$$

We solve this for the wave speed v:

(15.14)

$$\text{Speed of a transverse wave on a string} \rightarrow v = \sqrt{\frac{\overset{\leftarrow\ \text{Tension in string}}{F}}{\underset{\leftarrow\ \text{Mass per unit length}}{\mu}}} \tag{15.14}$$

Equation (15.14) confirms that the wave speed v increases when the tension F increases but decreases when the mass per unit length μ increases (Fig. 15.12).

Figure 15.12

These transmission cables have a relatively large amount of mass per unit length (μ) and a low tension (F). If the cables are disturbed—say, by a bird landing on them—transverse waves will travel along them at a slow speed $v = \sqrt{F/\mu}$.

Note that v_y does not appear in Eq. (15.14); thus the wave speed doesn't depend on v_y. Our calculation considered only a very special kind of pulse, but we can consider *any* shape of wave disturbance as a series of pulses with different values of v_y. So even though we derived Eq. (15.14) for a special case, it is valid for *any* transverse wave motion on a string, including the sinusoidal and other periodic waves we discussed in Section 15.3. Note also that the wave speed doesn't depend on the amplitude or frequency of the wave, in accordance with our assumptions in Section 15.3.

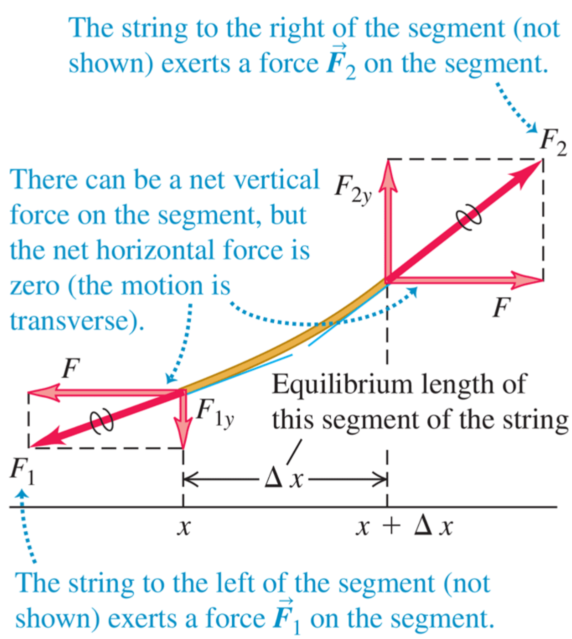
Wave Speed on a String: Second Method

Here is an alternative derivation of Eq. (15.14). If you aren't comfortable with partial derivatives, it can be omitted. We apply Newton's second law, $\sum \vec{F} = m\vec{a}$, to a small segment of string whose length in the equilibrium position is Δx (Fig. 15.13). The mass of the segment is $m = \mu\, \Delta x$. The x-components of the forces have equal magnitude F and add to zero because the motion is transverse and there is no component of acceleration in the x-direction. To obtain F_{1y} and F_{2y}, we note that the ratio F_{1y}/F is equal in magnitude to the *slope* of the string at point x and that F_{2y}/F is equal to the slope at point $x + \Delta x$. Taking proper account of signs, we find

(15.15)

$$\frac{F_{1y}}{F} = -\left(\frac{\partial y}{\partial x}\right)_x \qquad \frac{F_{2y}}{F} = \left(\frac{\partial y}{\partial x}\right)_{x+\Delta x}$$

Figure 15.13



Free-body diagram for a segment of string. The force at each end of the string is tangent to the string at the point of application.

The notation reminds us that the derivatives are evaluated at points x and $x + \Delta x$, respectively. From Eq. (15.15) we find that the net y-

component of force is

(15.16)

$$F_y = F_{1y} + F_{2y} = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

We now equate F_y from Eq. (15.16) to the mass $\mu\ \Delta x$ times the y-component of acceleration $\partial^2 y/\partial t^2$:

(15.17)

$$F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right] = \mu\ \Delta x \frac{\partial^2 y}{\partial t^2}$$

or, dividing Eq. (15.17) by $F\Delta x$,

(15.18)

$$\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\Delta x} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

We now take the limit as $\Delta x \to 0$. In this limit, the left side of Eq. (15.18) becomes the derivative of $\partial y/\partial x$ with respect to x (at constant t) —that is, the *second* (partial) derivative of y with respect to x:

(15.19)

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

Now, Eq. (15.19) has exactly the same form as the *wave equation*, Eq. (15.12), that we derived at the end of Section 15.3. That equation and Eq. (15.19) describe the very same wave motion, so they must be identical. Comparing the two equations, we see that for this to be so, we must have

$$v = \sqrt{\frac{F}{\mu}}$$

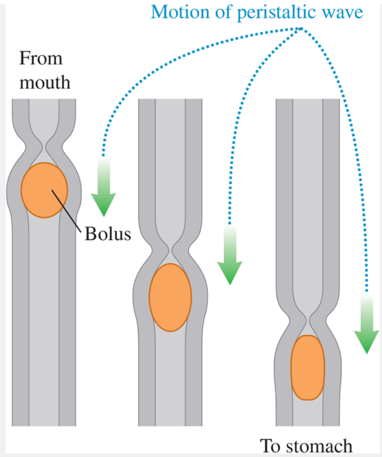
which is the same expression as Eq. (15.14).

In going through this derivation, we didn't make any special assumptions about the shape of the wave. Since our derivation led us to rediscover Eq. (15.12), the wave equation, we conclude that the wave equation is valid for waves on a string that have *any* shape.

BIO Application

Eating and Transverse Waves

Swallowing food causes peristalsis, in which a transverse wave propagates down your esophagus. The wave is a radial contraction of the esophagus that pushes the bolus (the mass of swallowed food) toward the stomach. Unlike the speed of waves on a uniform string, the speed of this peristaltic wave is not constant: It averages about 3 cm/s in the upper esophagus, about 5 cm/s in the mid-esophagus, and about 2.5 cm/s in the lower esophagus.



The Speed of Mechanical Waves

Equation (15.14) gives the wave speed for only the special case of mechanical waves on a stretched string or rope. Remarkably, it turns out that for many types of mechanical waves, including waves on a string, the expression for wave speed has the same general form:

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

To interpret this expression, let's look at the now-familiar case of waves on a string. The tension F in the string plays the role of the restoring force; it tends to bring the string back to its undisturbed, equilibrium configuration. The mass of the string—or, more properly, the linear mass density μ—provides the inertia that prevents the string from returning instantaneously to equilibrium. Hence we have $v = \sqrt{F/\mu}$ for the speed of waves on a string.

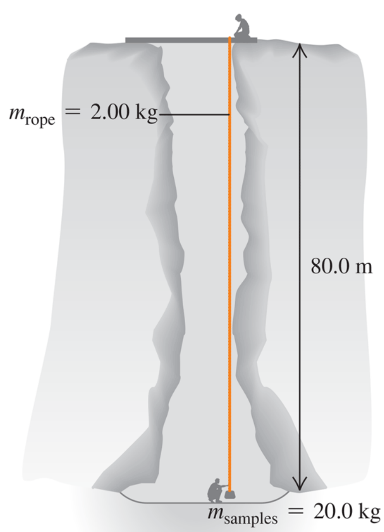
In Chapter 16 we'll see a similar expression for the speed of sound waves in a gas. Roughly speaking, the gas pressure provides the force that tends to return the gas to its undisturbed state when a sound wave passes through. The inertia is provided by the density, or mass per unit volume, of the gas.

Example 15.3 Calculating wave speed

WITH VARIATION PROBLEMS

One end of a 2.00 kg rope is tied to a support at the top of a mine shaft 80.0 m deep (Fig. 15.14). The rope is stretched taut by a 20.0 kg box of rocks attached at the bottom. (a) A geologist at the bottom of the shaft signals to a colleague at the top by jerking the rope sideways. What is the speed of a transverse wave on the rope? (b) If a point on the rope is in transverse SHM with $f = 2.00$ Hz, how many cycles of the wave are there in the rope's length?

Figure 15.14



Using transverse waves to send signals along a vertical rope.

IDENTIFY and SET UP In part (a) we can find the wave speed (our target variable) by using the *dynamic* relationship $v = \sqrt{F/\mu}$ [Eq.

(15.14)]. In part (b) we find the wavelength from the *kinematic* relationship $v = f\lambda$; from that we can find the target variable, the number of wavelengths that fit into the rope's 80.0 m length. We'll assume that the rope is massless (even though its weight is 10% that of the box), so that the box alone provides the tension in the rope.

EXECUTE The tension in the rope due to the box is

$$F = m_{\text{box}}\, g = (20.0 \text{ kg})(9.80 \text{ m/s}^2) = 196 \text{ N}$$

and the rope's linear mass density is

$$\mu = \frac{m_{\text{rope}}}{L} = \frac{2.00 \text{ kg}}{80.0 \text{ m}} = 0.0250 \text{ kg}/m$$

Hence, from Eq. (15.14), the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{196 \text{ N}}{0.0250 \text{ kg/m}}} = 88.5 \text{ m}/s$$

(b) From Eq. (15.1), the wavelength is

$$\lambda = \frac{v}{f} = \frac{88.5 \text{ m/s}}{2.00 \text{ s}^{-1}} = 44.3 \text{ m}$$

There are $(80.0 \text{ m})/(44.3 \text{ m}) = 1.81$ wavelengths (that is, cycles of the wave) in the rope.

EVALUATE Because of the rope's weight, its tension is greater at the top than at the bottom. Hence both the wave speed and the wavelength increase as a wave travels up the rope. If you take account of this, can you verify that the wave speed at the top of the rope is $92.9 \text{ m}/s$?

KEY CONCEPT

The speed of a transverse wave on a string is determined by the tension in the string and the string's linear mass density (mass per unit length). For a given string, the wave speed is independent of frequency or wavelength.

Video Tutor Solution: Example 15.3

Test Your Understanding of Section 15.4

The six strings of a guitar are the same length and under nearly the same tension, but they have different thicknesses. On which string do waves travel the fastest? (i) The thickest string; (ii) the thinnest string; (iii) the wave speed is the same on all strings.

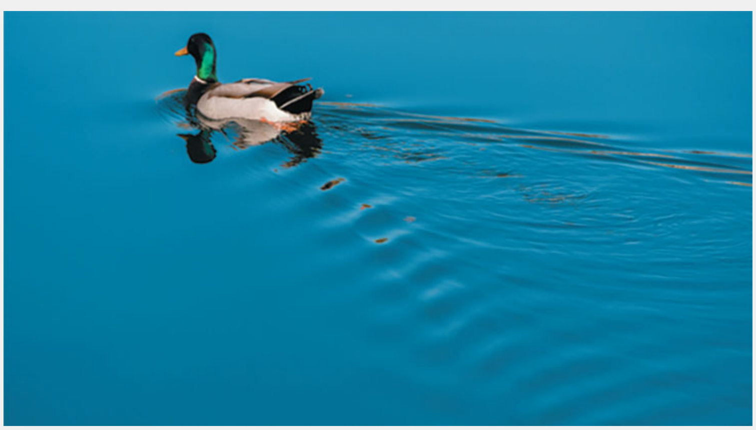
15.5 Energy in Wave Motion

Every wave motion has *energy* associated with it. The energy we receive from sunlight and the destructive effects of ocean surf and earthquakes bear this out. To produce any of the wave motions we have discussed in this chapter, we have to apply a force to a portion of the wave medium; the point where the force is applied moves, so we do *work* on the system. As the wave propagates, each portion of the medium exerts a force and does work on the adjoining portion. In this way a wave can transport energy from one region of space to another.

BIO Application

Surface Waves and the Swimming Speed of Ducks

When a duck swims, it necessarily produces waves on the surface of the water. The faster the duck swims, the larger the wave amplitude and the more power the duck must supply to produce these waves. The maximum power available from their leg muscles limits the maximum swimming speed of ducks to only about $0.7\ \mathrm{m/s}\ (2.5\ \mathrm{km/h} = 1.6\ \mathrm{mi/h})$.

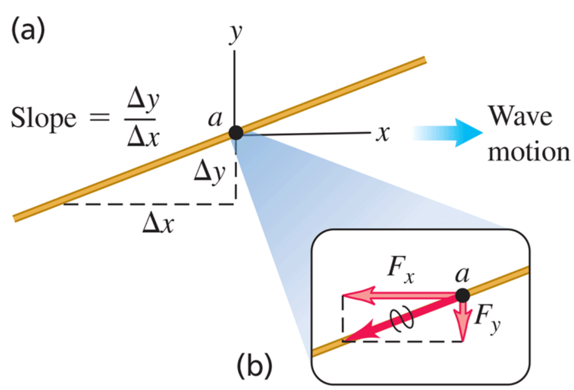


As an example, let's look again at transverse waves on a string. How is energy transferred from one portion of the string to another? Picture a wave traveling from left to right (the positive x-direction) past a point a on the string (Fig. 15.15a). The string to the left of point a exerts a force on the string to the right of it, and vice versa. In Fig. 15.15b we show the components F_x and F_y of the force that the string to the left of a exerts on the string to the right of a. As in Figs. 15.11 and 15.13, the magnitude of the horizontal component F_x equals the tension F in the undisturbed string. Note that F_y/F is equal to the negative of the *slope* of the string at a, and this slope is also given by $\partial y/\partial x$. Putting these together, we have

(15.20)

$$F_y(x,\ t) = -F\frac{\partial y(x,\ t)}{\partial x}$$

Figure 15.15



(a) Point a on a string carrying a wave from left to right.(b) The components of the force exerted on the part of the string to the right of point a by the part of the string to the left of point a.

We need the negative sign because F_y is negative when the slope is positive (as in Fig. 15.15b). We write the vertical force as $F_y(x,\ t)$ as a reminder that its value may be different at different points along the string and at different times.

When point a moves in the y-direction, the force F_y does *work* on this point and therefore transfers energy into the part of the string to the right of a. The corresponding power P (rate of doing work) at the point a is the transverse force $F_y(x,\ t)$ at a times the transverse velocity $v_y(x,\ t) = \partial y(x,\ t)/\partial t$ of that point:

(15.21)

$$P(x,\ t) = F_y(x,\ t)v_y(x,\ t) = -F\frac{\partial y(x,\ t)}{\partial x}\ \frac{\partial y(x,\ t)}{\partial t}$$

This power is the *instantaneous* rate at which energy is transferred along the string at position x and time t. Note that energy is transferred only at points where the string has a nonzero slope ($\partial y/\partial x$ is nonzero), so that the tension force has a transverse component, and where the string has a nonzero transverse velocity ($\partial y/\partial t$ is nonzero) so that the transverse force can do work.

Equation (15.21) is valid for *any* wave on a string, sinusoidal or not. For a sinusoidal wave with wave function given by Eq. (15.7), we have

(15.22)

$$\begin{aligned} y(x,\ t) &= A\cos(kx - \omega t) \\ \frac{\partial y(x,\ t)}{\partial x} &= -kA\sin(kx - \omega t) \\ \frac{\partial y(x,\ t)}{\partial t} &= \omega A\sin(kx - \omega t) \\ P(x,\ t) &= Fk\omega A^2\sin^2(kx - \omega t) \end{aligned}$$

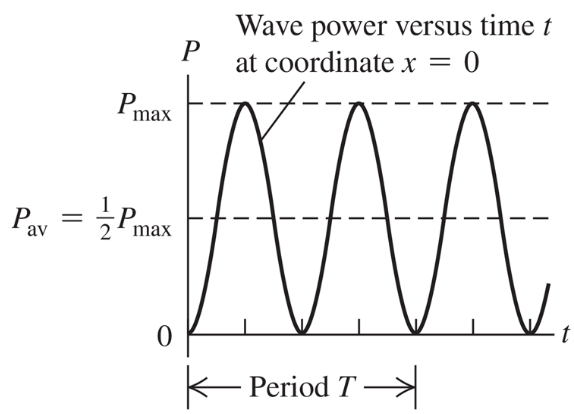
By using the relationships $\omega = vk$ and $v^2 = F/\mu$, we can also express Eq. (15.22) in the alternative form

(15.23)

$$P(x,\ t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$$

The $\sin^2$ function is never negative, so the instantaneous power in a sinusoidal wave is either positive (so that energy flows in the positive x-direction) or zero (at points where there is no energy transfer). Energy is never transferred in the direction opposite to the direction of wave propagation (Fig. 15.16).

Figure 15.16



The instantaneous power $P(x,\ t)$ in a sinusoidal wave as given by Eq. (15.23), shown as a function of time at coordinate $x = 0$. The power is never negative, which means that energy never flows opposite to the direction of wave propagation.

The maximum value of the instantaneous power $P(x,\ t)$ occurs when the $\sin^2$ function has the value unity:

(15.24)

$$P_{max} = \sqrt{\mu F}\,\omega^2 A^2$$

The *average* value of the $\sin^2$ function, averaged over any whole number of cycles, is $\frac{1}{2}$. Hence we see from Eq. (15.23) that the *average* power P_{av} is just one-half the maximum instantaneous power P_{max} (Fig. 15.16):

(15.25)

$$\underbrace{P_{av}}_{\text{Average power, sinusoidal wave on a string}} = \frac{1}{2}\sqrt{\underbrace{\mu}_{\text{Mass per unit length}}\underbrace{F}_{\text{Tension in string}}}\;\overbrace{\omega}^{\text{Wave angular frequency}}{}^{2}\,\overbrace{A}^{\text{Wave amplitude}}{}^{2} \qquad (15.25)$$

? The average rate of energy transfer is proportional to the square of the amplitude and to the square of the frequency. This proportionality is a general result for mechanical waves of all types, including seismic waves (see the photo that opens this chapter). For a mechanical wave, the rate of energy transfer quadruples if the frequency is doubled (for the same amplitude) or if the amplitude is doubled (for the same frequency).

Electromagnetic waves turn out to be a bit different. While the average rate of energy transfer in an electromagnetic wave is proportional to the square of the amplitude, just as for mechanical waves, it is independent of the value of ω.

Example 15.4 Power in a wave

WITH VARIATION PROBLEMS

(a) In Example 15.2 (Section 15.3), at what maximum rate does Throcky put energy into the clothesline? That is, what is his maximum instantaneous power? The linear mass density of the clothesline is $\mu = 0.250\ \text{kg/m}$, and Throcky applies tension $F = 36.0\ \text{N}$. (b) What is his average power? (c) As Throcky tires, the amplitude decreases. What is the average power when the amplitude is 7.50 mm?

IDENTIFY and SET UP In part (a) our target variable is the *maximum instantaneous* power P_{max}, while in parts (b) and (c) it is the *average* power. For part (a) we'll use Eq. (15.24), and for parts (b) and (c) we'll use Eq. (15.25); Example 15.2 gives us all the needed quantities.

EXECUTE (a) From Eq. (15.24),

$$\begin{aligned} P_{\text{max}} &= \sqrt{\mu F}\omega^2 A^2 \\ &= \sqrt{(0.250\,\text{kg/m})\,(36.0\,\text{N})}(4.00\pi\text{rad/s})^2(0.075\,\text{m})^2 \\ &= 2.66\ \text{W} \end{aligned}$$

(b) From Eqs. (15.24) and (15.25), the average power is one-half of the maximum instantaneous power, so

$$P_{\text{av}} = \frac{1}{2}P_{\text{max}} = \frac{1}{2}(2.66\ \text{W}) = 1.33\ \text{W}$$

(c) The new amplitude is $\frac{1}{10}$ of the value we used in parts (a) and (b). From Eq. (15.25), the average power is proportional to A^2, so the new average power is

$$P_{\text{av}} = \left(\frac{1}{10}\right)^2(1.33\ \text{W}) = 0.0133\ \text{W} = 13.3\ \text{mW}$$

EVALUATE Equation (15.23) shows that P_{max} occurs when $\sin^2(kx - \omega t) = 1$. At any given position x, this happens twice per period of the wave—once when the sine function is equal to $+1$, and once when it's equal to -1. The *minimum* instantaneous power is zero; this occurs when $\sin^2(kx - \omega t) = 0$, which also happens twice per period.

Can you confirm that the given values of μ and F give the wave speed mentioned in Example 15.2?

KEY CONCEPT

The power (energy per second) carried by a wave on a string depends on the string tension and the linear mass density, as well as the frequency and amplitude of the wave. If the wave is sinusoidal, the average power equals one-half of the maximum instantaneous power.

Video Tutor Solution: Example 15.4

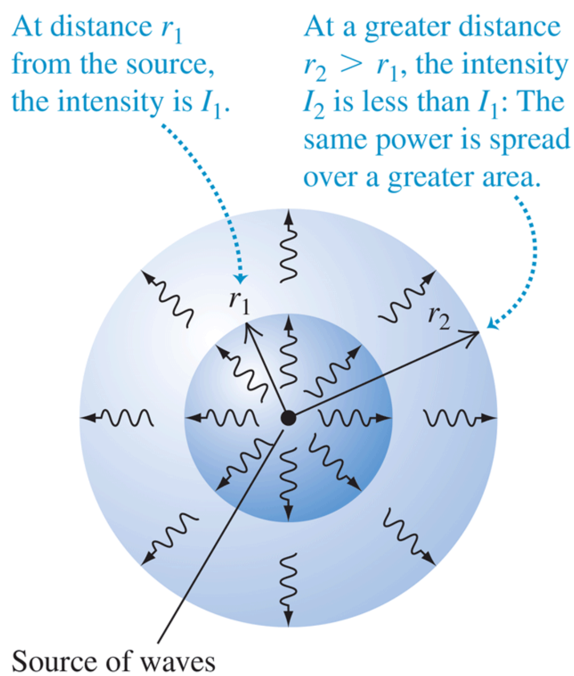
Wave Intensity

Waves on a string carry energy in one dimension (along the direction of the string). But other types of waves, including sound waves in air and seismic waves within the earth, carry energy across all three dimensions of space. For waves of this kind, we define the **intensity** (denoted by I) to be *the time average rate at which energy is transported by the wave, per unit* *area,* across a surface perpendicular to the direction of propagation. Intensity I is average power per unit area and is usually measured in watts per square meter $(\mathrm{W/m^2})$.

If waves spread out equally in all directions from a source, the intensity at a distance r from the source is inversely proportional to r^2 (Fig. 15.17). This result, called the *inverse-square law for intensity,* follows directly from energy conservation. If the power output of the source is P, then the average intensity I_1 through a sphere with radius r_1 and surface area $4\pi_1^2$ is

$$4\pi {r_1}^2 I_1 = 4\pi {r_2}^2 I_2$$

Figure 15.17



The greater the distance from a wave source, the greater the area over which the wave power is distributed and the smaller the wave intensity.

$$I_1 = \frac{P}{4\pi {r_1}^2}$$

A similar expression gives the average intensity I_2 through a sphere with a different radius r_2. If no energy is absorbed between the two spheres, the power P must be the same for both, and

(15.26)

Inverse-square law for intensity:
Intensity is inversely proportional to the square of the distance from source.

Intensity at point 1
Intensity at point 2

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \qquad (15.26)$$

Distance from source to point 2
Distance from source to point 1

Example 15.5 The inverse-square law

WITH VARIATION PROBLEMS

A siren on a tall pole radiates sound waves uniformly in all directions. At a distance of 15.0 m from the siren, the sound intensity is $0.250\ \mathrm{W/m^2}$. At what distance is the intensity $0.010\ \mathrm{W/m^2}$?

IDENTIFY and SET UP Because sound is radiated uniformly in all directions, we can use the inverse-square law, Eq. (15.26). At $r_1 = 15.0\ \mathrm{m}$ the intensity is $I_1 = 0.250\ \mathrm{W/m^2}$, and the target variable is the distance r_2 at which the intensity is $I_2 = 0.010\ \mathrm{W/m^2}$.

EXECUTE We solve Eq. (15.26) for r_2 :

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (15.0\ \mathrm{m})\sqrt{\frac{0.250\ \mathrm{W/m^2}}{0.010\ \mathrm{W/m^2}}} = 75.0\ \mathrm{m}$$

EVALUATE As a check on our answer, note that r_2 is five times greater than r_1. By the inverse-square law, the intensity I_2 should be $1/5^2 = 1/25$ as great as I_1, and indeed it is.

By using the inverse-square law, we've assumed that the sound waves travel in straight lines away from the siren. A more realistic solution, which is beyond our scope, would account for the reflection of sound waves from the ground.

KEY CONCEPT

If waves emitted from a source spread out equally in all directions, the wave intensity is inversely proportional to the square of the distance from the source.

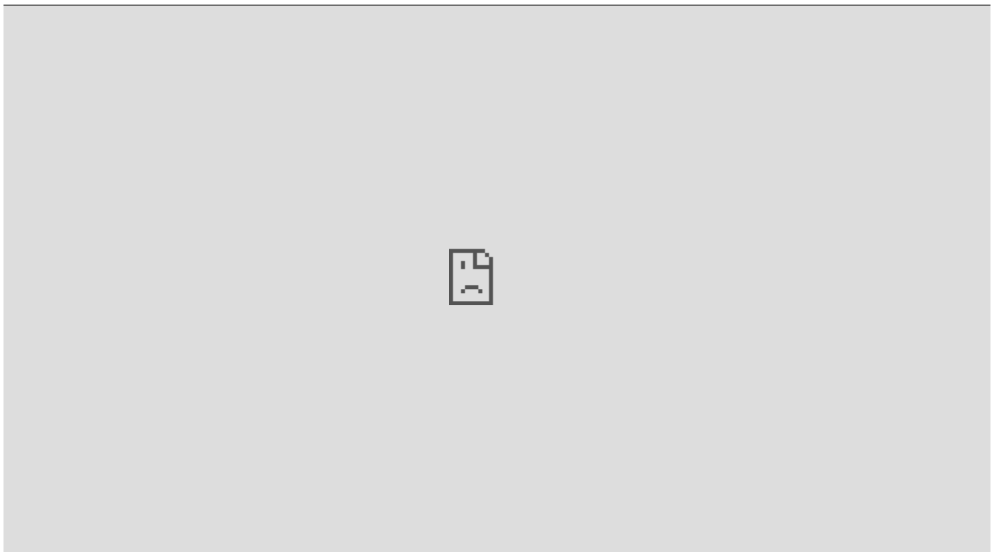
Video Tutor Solution: **Example 15.5**

Test Your Understanding of Section 15.5

Four identical strings each carry a sinusoidal wave of frequency 10 Hz. The string tension and wave amplitude are different for different strings. Rank the following strings in order from highest to lowest value of the average wave power: (i) tension 10 N, amplitude 1.0 mm; (ii) tension 40 N, amplitude 1.0 mm; (iii) tension 10 N, amplitude 4.0 mm; (iv) tension 20 N, amplitude 2.0 mm.

15.6 Wave Interference, Boundary Conditions, and Superposition

Video Tutor Demo: **Out-of-Phase Speakers**



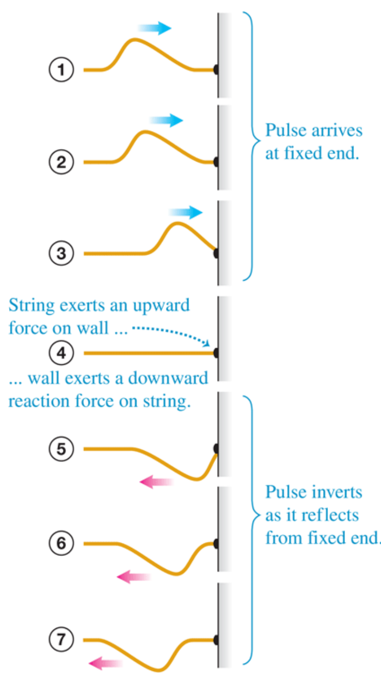
Up to this point we've been discussing waves that propagate continuously in the same direction. But when a wave strikes the boundaries of its medium, all or part of the wave is *reflected.* When you yell at a building wall or a cliff face some distance away, the sound wave is reflected from the rigid surface and you hear an echo. When you flip the end of a rope whose far end is tied to a rigid support, a pulse travels the length of the rope and is reflected back to you. In both cases, the initial and reflected waves overlap in the same region of the medium. We use the term **interference** to refer to what happens when two or more waves pass through the same region at the same time.

As a simple example of wave reflections and the role of the boundary of a wave medium, let's look again at transverse waves on a stretched string.

What happens when a wave pulse or a sinusoidal wave arrives at the *end* of the string?

If the end is fastened to a rigid support as in Fig. 15.18, it is a *fixed* end that cannot move. The arriving wave exerts a force on the support (drawing 4 in Fig. 15.18); the reaction to this force, exerted by the support on the string, "kicks back" on the string and sets up a reflected pulse or wave traveling in the reverse direction (drawing 7). The reflected pulse moves in the opposite direction from the initial, or *incident*, pulse, and its displacement is also opposite.

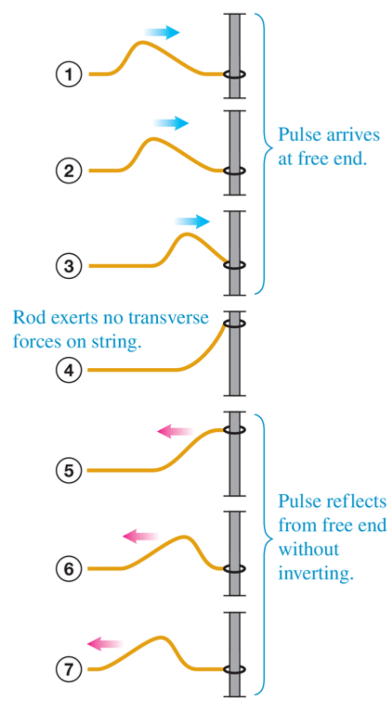
Figure 15.18



Reflection of a wave pulse at a fixed end of a string. Time increases from top to bottom.

The opposite situation from an end that is held stationary is a *free* end, one that is perfectly free to move in the direction perpendicular to the length of the string. For example, the string might be tied to a light ring that slides on a frictionless rod perpendicular to the string, as in Fig. 15.19. The ring and rod maintain the tension but exert no transverse force. When a wave arrives at this free end, the ring slides along the rod. The ring reaches a maximum displacement, and both it and the string come momentarily to rest, as in drawing 4 in Fig. 15.19. But the string is now stretched, giving increased tension, so the free end of the string is pulled back down, and again a reflected pulse is produced (drawing 7). As for a fixed end, the reflected pulse moves in the opposite direction from the initial pulse, but now the direction of the displacement is the same as for the initial pulse. The conditions at the end of the string, such as a rigid support or the complete absence of transverse force, are called **boundary conditions.**

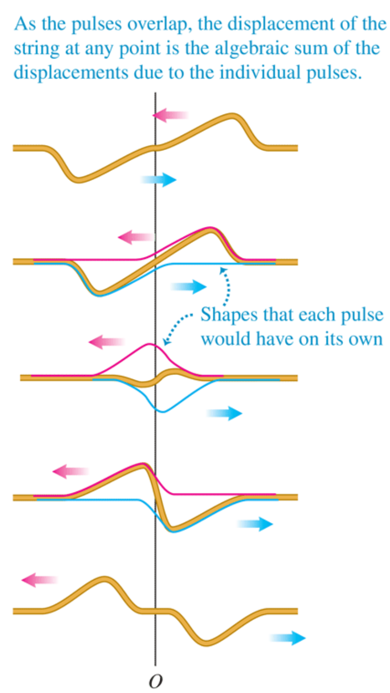
Figure 15.19



Reflection of a wave pulse at a free end of a string. Time increases from top to bottom. (Compare to Fig. 15.18.)

The formation of the reflected pulse is similar to the overlap of two pulses traveling in opposite directions. Figure 15.20 (next page) shows two pulses with the same shape, one inverted with respect to the other, traveling in opposite directions. As the pulses overlap and pass each other, the total displacement of the string is the *algebraic sum* of the displacements at that point in the individual pulses. Because these two pulses have the same shape, the total displacement at point O in the middle of the figure is zero at all times. Thus the motion of the left half of the string would be the same if we cut the string at point O, threw away the right side, and held the end at O fixed. The two pulses on the left side then correspond to the incident and reflected pulses, combining so that the total displacement at O is *always* zero. For this to occur, the reflected pulse must be inverted relative to the incident pulse, just as for reflection from the fixed end in Fig. 15.18.

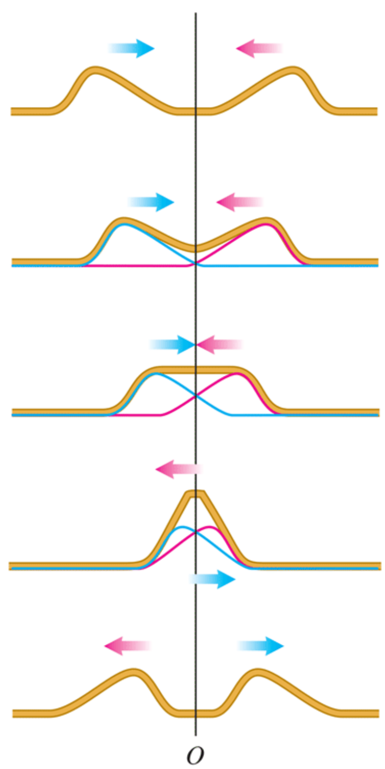
Figure 15.20



Overlap of two wave pulses—one right side up, one inverted—traveling in opposite directions. Time increases from top to bottom.

Figure 15.21 shows two pulses with the same shape, traveling in opposite directions but *not* inverted relative to each other. The displacement at point O in the middle of the figure is not zero, but the slope of the string at this point is always zero. According to Eq. (15.20), this corresponds to the absence of any transverse force at this point. In this case the motion of the left half of the string would be the same as if we cut the string at point O and attached the end to a frictionless sliding ring (Fig. 15.19) that maintains tension without exerting any transverse force. In other words, this situation corresponds to reflection of a pulse at a free end of a string at point O. In this case the reflected pulse is *not* inverted.

Figure 15.21



Overlap of two wave pulses—both right side up—traveling in opposite directions. Time increases from top to bottom. Compare to Fig. 15.20.

The Principle of Superposition

Combining the displacements of the separate pulses at each point to obtain the actual displacement is an example of the **principle of superposition:** When two waves overlap, the actual displacement of any point on the string at any time is obtained by adding the displacement the point would have if only the first wave were present and the displacement it would have if only the second wave were present. In other words, the wave function $y(x,\ t)$ for the resulting motion is obtained by *adding* the two wave functions for the two separate waves:

(15.27)

Principle of superposition:

Wave functions of two overlapping waves

$$y(x, t) = y_1(x, t) + y_2(x, t) \qquad (15.27)$$

Wave function of combined wave = sum of individual wave functions

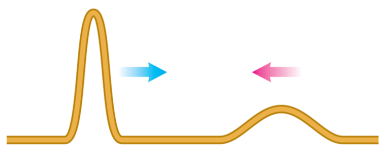
Mathematically, this additive property of wave functions follows from the form of the wave equation, Eq. (15.12) or (15.19), which every physically possible wave function must satisfy. Specifically, the wave equation is *linear;* that is, it contains the function $y(x,\ t)$ only to the first power (there are no terms involving $y(x,\ t)^2$, $y(x,\ t)^{1/2}$, etc.). As a result, if any two functions $y_1(x,\ t)$ and $y_2(x,\ t)$ satisfy the wave equation separately, their sum $y_1(x,\ t) + y_2(x,\ t)$ also satisfies it and is therefore a physically possible motion. Because this principle depends on the linearity of the wave equation and the corresponding linear-combination property of its solutions, it is also called the *principle of linear superposition*. For some physical systems, such as a medium that does not obey Hooke's law, the wave equation is *not* linear; this principle does not hold for such systems.

The principle of superposition is of central importance in all types of waves. When a friend talks to you while you are listening to music, you can distinguish the speech and the music from each other. This is precisely because the total sound wave reaching your ears is the algebraic sum of the wave produced by your friend's voice and the wave produced by the speakers of your stereo. If two sound waves did *not* combine in this simple linear way, the sound you would hear in this situation would be a hopeless jumble. Superposition also applies to electromagnetic waves (such as light).

Test Your Understanding of Section 15.6

Figure 15.22 shows two wave pulses with different shapes traveling in different directions along a string. Make a series of sketches like Fig. 15.21 showing the shape of the string as the two pulses approach, overlap, and then pass each other.

Figure 15.22



Two wave pulses with different shapes.

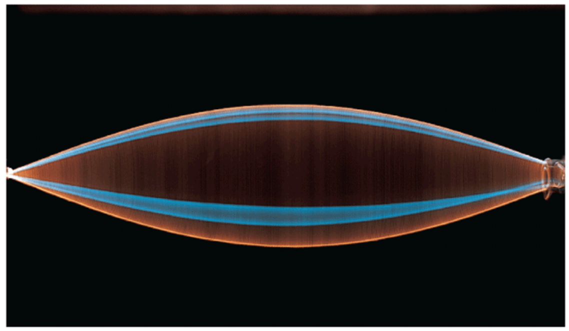
15.7 Standing Waves on a String

We've looked at the reflection of a wave *pulse* on a string when it arrives at a boundary point (either a fixed end or a free end). Now let's consider what happens when a *sinusoidal* wave on a string is reflected by a fixed end. We'll again approach the problem by considering the superposition of two waves propagating through the string, one representing the incident wave and the other representing the wave reflected at the fixed end.

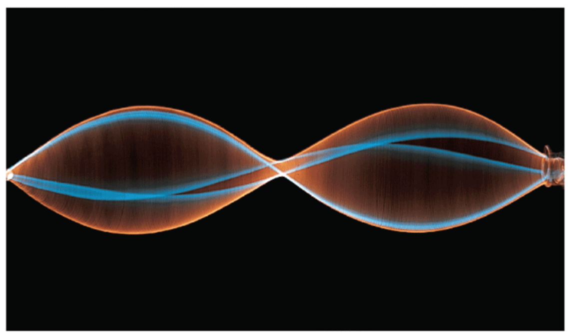
Figure 15.23 shows a string that is fixed at its left end. Its right end is moved up and down in simple harmonic motion to produce a wave that travels to the left; the wave reflected from the fixed end travels to the right. The resulting motion when the two waves combine no longer looks like two waves traveling in opposite directions. The string appears to be subdivided into segments, as in the time-exposure photographs of Figs. 15.23a, 15.23b, 15.23c, and 15.23d. Figure 15.23e shows two instantaneous shapes of the string in Fig. 15.23b. Let's compare this behavior with the waves we studied in Sections 15.1 through 15.5. In a wave that travels along the string, the amplitude is constant and the wave pattern moves with a speed equal to the wave speed. Here, instead, the wave pattern remains in the same position along the string and its amplitude fluctuates. There are particular points called **nodes** (labeled N in Fig. 15.23e) that never move at all. Midway between the nodes are points called **antinodes** (labeled A in Fig. 15.23e) where the amplitude of motion is greatest. Because the wave pattern doesn't appear to be moving in either direction along the string, it is called a **standing wave.** (To emphasize the difference, a wave that *does* move along the string is called a **traveling wave.**)

Figure 15.23

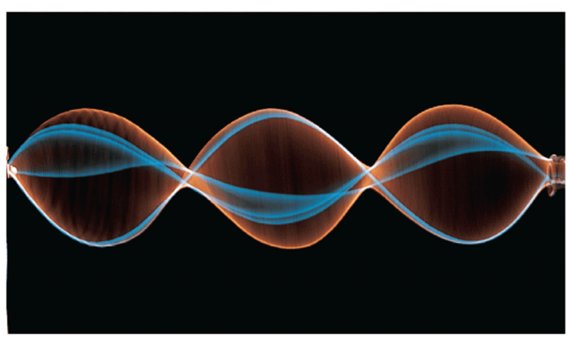
(a) String is one-half wavelength long.



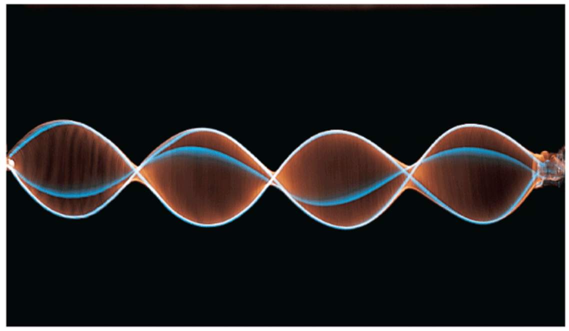
(b) String is one wavelength long.



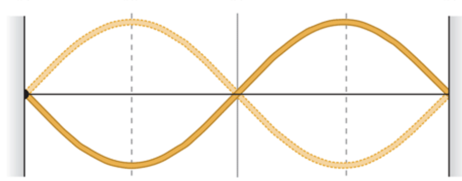
(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



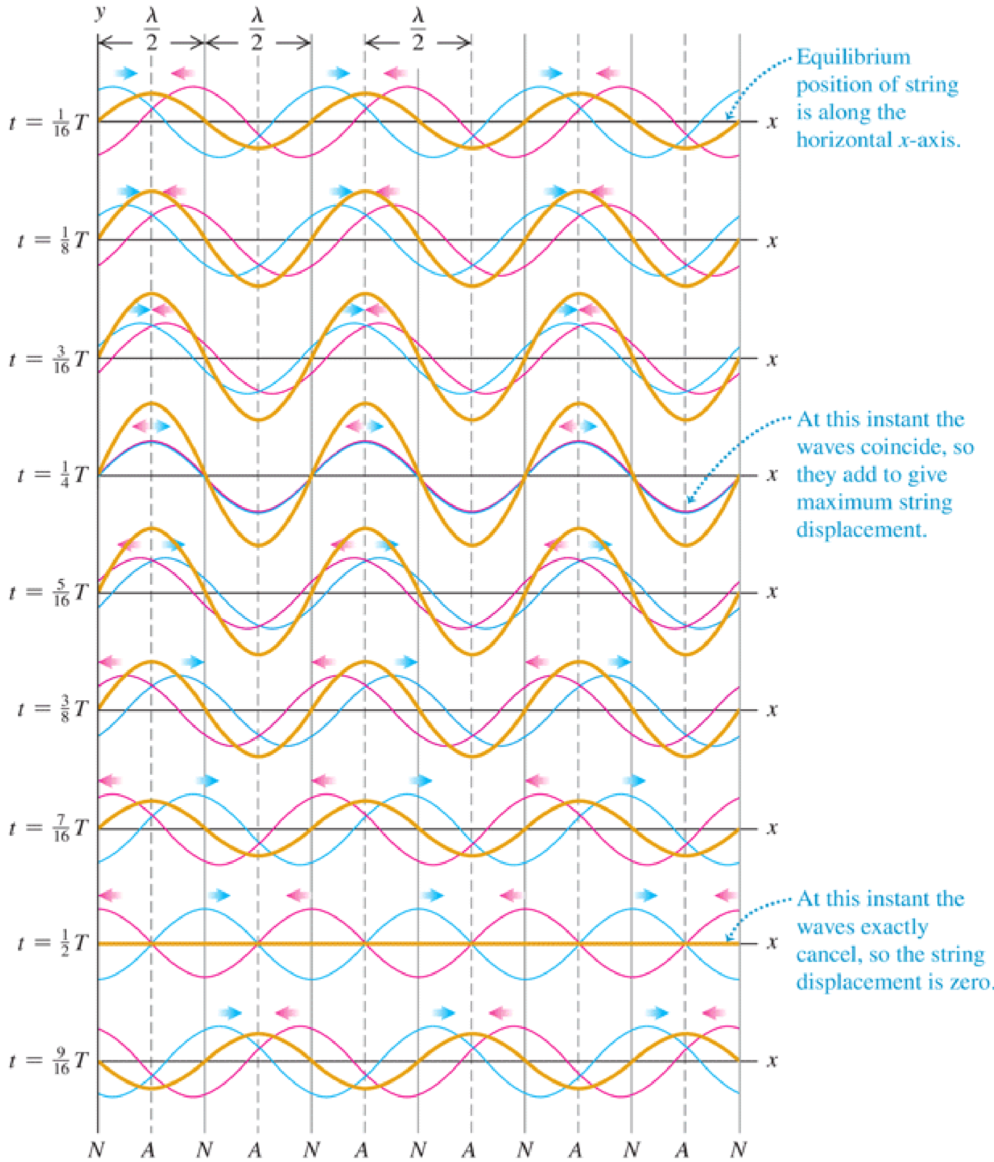
(e) The shape of the string in (b) at two different instants



(a)–(d) Time exposures of standing waves in a stretched string. From (a) to (d), the frequency of oscillation of the right-hand end increases and the wavelength of the standing wave decreases. (e) The extremes of the motion of the standing wave in part (b), with nodes at the center and at the ends. The right-hand end of the string moves very little compared to the antinodes and so is essentially a node.

The principle of superposition explains how the incident and reflected waves combine to form a standing wave. In Fig. 15.24 (next page) the red curves show a wave traveling to the left. The blue curves show a wave traveling to the right with the same propagation speed, wavelength, and amplitude. The waves are shown at nine instants, $\frac{1}{16}$ of a period apart. At each point along the string, we add the displacements (the values of y) for the two separate waves; the result is the total wave on the string, shown in gold.

Figure 15.24



Formation of a standing wave. A wave traveling to the left (red curves) combines with a wave traveling to the right (blue curves) to form a standing wave (gold curves).

At certain instants, such as $t = \frac{1}{4}T$, the two wave patterns are exactly in phase with each other, and the shape of the string is a sine curve with twice the amplitude of either individual wave. At other instants, such as $t = \frac{1}{2}T$, the two waves are exactly out of phase with each other, and the total wave at that instant is zero. The resultant displacement is *always* zero at those places marked N at the bottom of Fig. 15.24. These are the *nodes.* At a node the displacements of the two waves in red and blue are always equal and opposite and cancel each other out. This cancellation is called **destructive interference.** Midway between the nodes are the points of *greatest* amplitude, or the *antinodes,* marked A. At the antinodes the displacements of the two waves in red and blue are always identical, giving a large resultant displacement; this phenomenon is called **constructive interference.** We can see from the figure that the distance between successive nodes or between successive antinodes is one half-wavelength, or $\lambda/2$.

We can derive a wave function for the standing wave of Fig. 15.24 by adding the wave functions $y_1(x,\ t)$ and $y_2(x,\ t)$ for two waves with equal amplitude, period, and wavelength traveling in opposite directions. Here $y_1(x,\ t)$ (the red curves in Fig. 15.24) represents an incoming, or *incident,* wave traveling to the left along the $+x$-axis, arriving at the point $x=0$ and being reflected; $y_2(x,\ t)$ (the blue curves in Fig. 15.24) represents the *reflected* wave traveling to the right from $x=0$. We noted in Section 15.6 that the wave reflected from a fixed end of a string is inverted, so we give a negative sign to one of the waves:

$$\begin{aligned} y_1(x,\ t) &= -A\cos(kx+\omega t) \quad \text{(incident wave traveling to the left)} \\ y_2(x,\ t) &= A\cos(kx-\omega t) \quad \text{(reflected wave traveling to the right)} \end{aligned}$$

The change in sign corresponds to a shift in *phase* of 180° or π radians. At $x=0$ the motion from the reflected wave is $A\cos\omega t$ and the motion from the incident wave is $-A\cos\omega t$, which we can also write as $A\cos(\omega t+\pi)$. From Eq. (15.27), the wave function for the standing wave is the sum of the individual wave functions:

$$y(x,\ t) = y_1(x,\ t) + y_2(x,\ t) = A[-\cos(kx+\omega t) + \cos(kx-\omega t)]$$

We can rewrite each of the cosine terms by using the identities for the cosine of the sum and difference of two angles: $\cos{(a \pm b)} = \cos{a} \cos{b} \mp \sin{a} \sin{b}$. Applying these and combining terms, we obtain the wave function for the standing wave:

$$y\left(x,\ t\right) = y_1\left(x,\ t\right) + y_2\left(x,\ t\right) = \left(2A\ \sin{kx}\right)\ \sin{\omega t} \qquad \text{or}$$

(15.28)

Standing wave on a string, fixed end at $x = 0$:

$$\overbrace{y(x,t)}^{\text{Wave function}} = (\overbrace{A_{SW}}^{\text{Standing-wave amplitude}} \sin \underbrace{k}_{\text{Wave number}}\underbrace{x}_{\text{Position}})\ \sin \underbrace{\omega}_{\text{Angular frequency}} \underbrace{t}_{\text{Time}} \qquad (15.28)$$

The standing-wave amplitude A_{SW} is twice the amplitude A of either of the original traveling waves: $A_{SW} = 2A$.

Equation (15.28) has two factors: a function of x and a function of t. The factor $A_{SW}\ \sin{kx}$ shows that at each instant the shape of the string is a sine curve. But unlike a wave traveling along a string, the wave shape stays in the same position, oscillating up and down as described by the $\sin{\omega t}$ factor. This behavior is shown by the gold curves in Fig. 15.24. Each point in the string still undergoes simple harmonic motion, but all the points between any successive pair of nodes oscillate *in phase*. This is in contrast to the phase differences between oscillations of adjacent points that we see with a traveling wave.

We can use Eq. (15.28) to find the positions of the nodes; these are the points for which $\sin{kx} = 0$, so the displacement is *always* zero. This occurs when $kx = 0,\ \pi,\ 2\pi,\ 3\pi,\ \ldots$, or, using $k = 2\pi/\lambda$,

(15.29)

$$\begin{aligned} x &= 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots \quad \text{(nodes of a standing wave on} \\ &= 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots \quad \text{a string, fixed end at } x = 0) \end{aligned}$$

In particular, there is a node at $x = 0$, as there should be, since this point is a fixed end of the string.

A standing wave, unlike a traveling wave, *does not* transfer energy from one end to the other. The two waves that form it would individually carry equal amounts of power in opposite directions. There is a local flow of energy from each node to the adjacent antinodes and back, but the *average* rate of energy transfer is zero at every point. If you use the wave function of Eq. (15.28) to evaluate the wave power given by Eq. (15.21), you will find that the average power is zero.

Problem-Solving Strategy 15.2 Standing Waves

IDENTIFY *the relevant concepts:* Identify the target variables. Then determine whether the problem is purely *kinematic* (involving only such quantities as wave speed v, wavelength λ, and frequency f) or whether *dynamic* properties of the medium (such as F and μ for transverse waves on a string) are also involved.

SET UP *the problem* using the following steps:

1. Sketch the shape of the standing wave at a particular instant. This will help you visualize the nodes (label them N) and antinodes (A). The distance between adjacent nodes (or antinodes) is $\lambda/2$; the distance between a node and the adjacent antinode is $\lambda/4$.
2. Choose the equations you'll use. The wave function for the standing wave, like Eq. (15.28), is often useful.

3. You can determine the wave speed if you know λ and f (or, equivalently, $k = 2\pi/\lambda$ and $\omega = 2\pi f$) or if you know the relevant properties of the medium (for a string, F and μ).

EXECUTE *the solution:* Solve for the target variables. Once you've found the wave function, you can find the displacement y at any point x and at any time t. You can find the velocity and acceleration of a particle in the medium by taking the first and second partial derivatives of y with respect to time.

EVALUATE *your answer:* Compare your numerical answers with your sketch. Check that the wave function satisfies the boundary conditions (for example, the displacement should be zero at a fixed end).

Example 15.6 Standing waves on a guitar string

WITH VARIATION PROBLEMS

A guitar string lies along the x-axis when in equilibrium. The end of the string at $x = 0$ (the bridge of the guitar) is fixed. A sinusoidal wave with amplitude $A = 0.750 \text{ mm} = 7.50 \times 10^{-4}$ m and frequency $f = 440$ Hz, corresponding to the red curves in Fig. 15.24, travels along the string in the $-x$-direction at 143 m/s. It is reflected from the fixed end, and the superposition of the incident and reflected waves forms a standing wave. (a) Find the equation giving the displacement of a point on the string as a function of position and time. (b) Locate the nodes. (c) Find the amplitude of the standing wave and the maximum transverse velocity and acceleration.

IDENTIFY and SET UP This is a *kinematics* problem (see Problem-Solving Strategy 15.1 in Section 15.3). The target variables are:

in part (a), the wave function of the standing wave; in part (b), the locations of the nodes; and in part (c), the maximum displacement y, transverse velocity v_y, and transverse acceleration a_y. Since there is a fixed end at $x = 0$, we can use Eqs. (15.28) and (15.29) to describe this standing wave. We'll need the relationships $\omega = 2\pi f$, $v = \omega/k$, and $v = \lambda f$.

EXECUTE (a) The standing-wave amplitude is $A_{\mathrm{SW}} = 2A\ = 1.50 \times 10^{-3}$ m (twice the amplitude of either the incident or reflected wave). The angular frequency and wave number are

$$\begin{aligned}\omega &= 2\pi f = (2\pi\ \mathrm{rad})(440\ \mathrm{s}^{-1}) = 2760\ \mathrm{rad/s}\\ k &= \frac{\omega}{v} = \frac{2760\ \mathrm{rad/s}}{143\ \mathrm{m/s}} = 19.3\ \mathrm{rad/m}\end{aligned}$$

Equation (15.28) then gives

$$\begin{aligned}y(x,\ t) &= (A_{\mathrm{SW}}\ kx)\ \sin\ \omega t\\ &= \left[\left(1.50 \times 10^{-3}\mathrm{m}\right)\ \sin\left(19.3\ \mathrm{rad/m}\right)x\right] \sin\left(2760\ \mathrm{rad/s}\right)t\end{aligned}$$

(b) From Eq. (15.29), the positions of the nodes are $x = 0$, $\lambda/2$, λ, $3\lambda/2,\ \ldots$. The wavelength is $\lambda = v/f = (143\ \mathrm{m/s})/(440\ \mathrm{Hz})\ = 0.325$ m, so the nodes are at $x = 0, 0.163\ \mathrm{m}, 0.325\ \mathrm{m},\ 0.488\ \mathrm{m},\ \ldots$.

(c) From the expression for $y(x,\ t)$ in part (a), the maximum displacement from equilibrium is $A_{\mathrm{SW}} = 1.50 \times 10^{-3}\ \mathrm{m}\ =\ 1.50\ \mathrm{mm}$. This occurs at the *antinodes,* which are midway between adjacent nodes (that is, at $x = 0.081\ \mathrm{m},\ 0.244\ \mathrm{m},\ 0.406\ \mathrm{m},\ \ldots$).

For a particle on the string at any point x, the transverse (y-) velocity is

$$\begin{aligned} v_y(x,\ t) &= \tfrac{\partial y(x,\, t)}{\partial t} \\ &= \begin{array}{l} \left[\left(1.50 \times 10^{-3}\mathrm{m}\right)\ \sin\left(19.3\ \mathrm{rad/m}\right)x\right] \\ \times \left[(2760\ \mathrm{rad/s})\ \cos(2760\ \mathrm{rad/s})t\right] \end{array} \\ &= \left[(4.15\ \mathrm{m/s})\ \sin(19.3\ \mathrm{rad/m})x\right] \cos(2760\ \mathrm{rad/s})t \end{aligned}$$

At an antinode, $\sin(19.3\ \mathrm{rad/m})x = \pm 1$ and the transverse velocity varies between $+4.15\ \mathrm{m}/s$ and $-4.15\ \mathrm{m/s}$. As is always the case in SHM, the maximum velocity occurs when the particle is passing through the equilibrium position ($y = 0$).

The transverse acceleration $a_y(x,\ t)$ is the *second* partial derivative of $y(x,\ t)$ with respect to time. You can show that

$$\begin{aligned} a_y(x,\ t) &= \frac{\partial v_y(x,\ t)}{\partial t} = \frac{\partial^2 y(x,\ t)}{\partial t^2} \\ &= \left[\left(-1.15 \times 10^4\ \mathrm{m/s^2}\right) \sin(19.3\ \mathrm{rad/m})x\right] \sin\left(2760\ \mathrm{rad/s}\right)t \end{aligned}$$

At the antinodes, the transverse acceleration varies between $+1.15 \times 10^4\ \mathrm{m/s^2}$ and $-1.15 \times 10^4\ \mathrm{m/s^2}$.

EVALUATE The maximum transverse velocity at an antinode is quite respectable (about 15 km/h, or $9.3\ \mathrm{mi}/h$). But the maximum transverse acceleration is tremendous, 1170 times the acceleration due to gravity! Guitar strings are actually fixed at *both* ends; we'll see the consequences of this in the next section.

KEY CONCEPT

A sinusoidal standing wave is the superposition of two sinusoidal waves of the same amplitude and frequency traveling in opposite directions. The wave nodes, which are spaced one half-wavelength apart, are points where the displacement in the standing wave is always zero.

Video Tutor Solution: **Example 15.6**

Test Your Understanding of Section 15.7

Suppose the frequency of the standing wave in Example 15.6 were doubled from 440 Hz to 880 Hz. Would all of the nodes for $f = 440\ \text{Hz}$ also be nodes for $f = 880\ \text{Hz}$? If so, would there be additional nodes for $f = 880\ \text{Hz}$? If not, which nodes are absent for $f = 880\ \text{Hz}$?

15.8 Normal Modes of a String

When we described standing waves on a string rigidly held at one end, as in Fig. 15.23, we made no assumptions about the length of the string or about what was happening at the other end. Let's now consider a string of a definite length L, rigidly held at *both* ends. Such strings are found in many musical instruments, including pianos, violins, and guitars. When a guitar string is plucked, a wave is produced in the string; this wave is reflected and re-reflected from the ends of the string, making a standing wave. This standing wave on the string in turn produces a sound wave in the air, with a frequency determined by the properties of the string. This is what makes stringed instruments so useful in making music.

To understand a standing wave on a string fixed at both ends, we first note that the standing wave must have a node at *both* ends of the string. We saw in the preceding section that adjacent nodes are one half-wavelength ($\lambda/2$) apart, so the length of the string must be $\lambda/2$, or $2\,(\lambda/2)$, or $3\,(\lambda/2)$, or in general some integer number of half-wavelengths:

(15.30)

$$L = n\frac{\lambda}{2} \quad (n = 1,\ 2,\ 3,\ \ldots) \quad \text{(string fixed at both ends)}$$

That is, if a string with length L is fixed at both ends, a standing wave can exist only if its wavelength satisfies Eq. (15.30).

Solving this equation for λ and labeling the possible values of λ as λ_n, we find

(15.31)

$$\lambda_n = \frac{2L}{n} \quad (n = 1,\ 2,\ 3,\ \ldots) \quad \text{(string fixed at both ends)}$$

Waves can exist on the string if the wavelength is *not* equal to one of these values, but there cannot be a steady wave pattern with nodes and antinodes, and the total wave cannot be a standing wave. Equation (15.31) is illustrated by the standing waves shown in Figs. 15.23a, 15.23b, 15.23c, and 15.23d; these represent $n = 1,\ 2,\ 3,$ and 4, respectively.

Corresponding to the series of possible standing-wave wavelengths λ_n is a series of possible standing-wave frequencies f_n, each related to its corresponding wavelength by $f_n = v/\lambda_n$. The smallest frequency f_1 corresponds to the largest wavelength (the $n = 1$ case), $\lambda_1 = 2L$:

(15.32)

$$f_1 = \frac{v}{2L} \quad \text{(string fixed at both ends)}$$

This is called the **fundamental frequency.** The other standing-wave frequencies are $f_2 = 2v/2L$, $f_3 = 3v/2L$, and so on. These are all integer multiples of f_1, such as $2f_1$, $3f_1$, $4f_1$, and so on. We can express *all* the frequencies as

(15.33)

Wave speed

Fundamental frequency = $v/2L$

Standing-wave frequencies, string fixed at both ends:

$$f_n = n\frac{v}{2L} = nf_1 \quad (n = 1, 2, 3, \ldots) \qquad (15.33)$$

Length of string

These frequencies are called **harmonics,** and the series is called a **harmonic series.** Musicians sometimes call f_2, f_3, and so on **overtones;** f_2 is the second harmonic or the first overtone, f_3 is the third harmonic or the second overtone, and so on. The first harmonic is the same as the fundamental frequency (Fig. 15.25).

Figure 15.25

Each string of a violin naturally oscillates at its harmonic frequencies, producing sound waves in the air with the same frequencies.

For a string with fixed ends at $x = 0$ and $x = L$, the wave function $y(x,\ t)$ of the nth standing wave is given by Eq. (15.28) (which satisfies the condition that there is a node at $x = 0$), with $\omega = \omega_n = 2\pi f_n$ and $k = k_n = 2\pi/\lambda_n$:

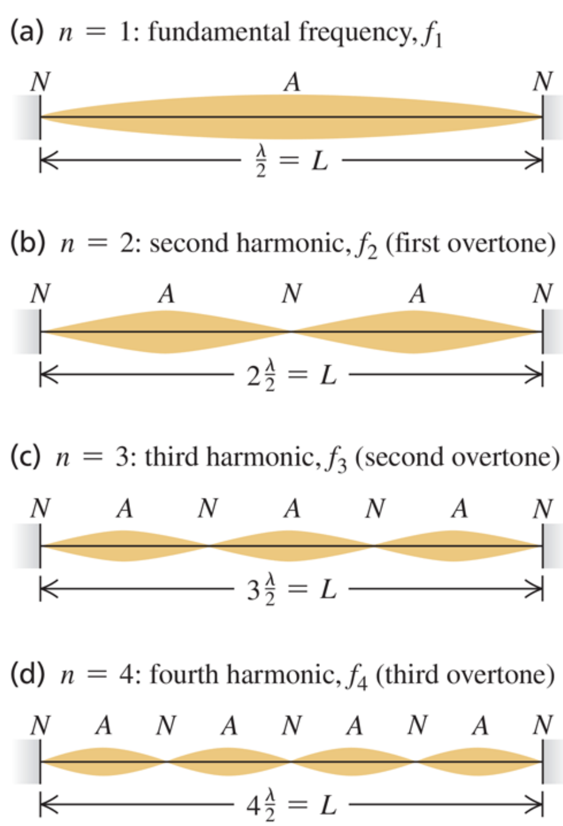
(15.34)

$$y_n(x,\ t) = A_{\mathrm{SW}} \sin k_n x \sin \omega_n t$$

You can confirm that this wave function has nodes at both $x = 0$ and $x = L$.

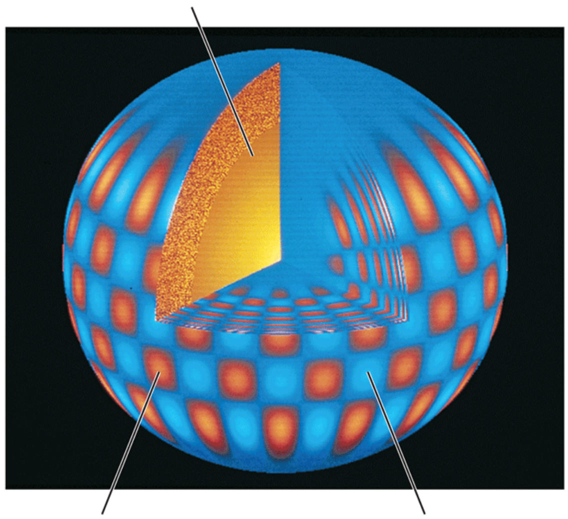
A **normal mode** of an oscillating system is a motion in which all particles of the system move sinusoidally with the same frequency. For a system made up of a string of length L fixed at both ends, each of the frequencies given by Eq. (15.33) corresponds to a possible normal-mode pattern. Figure 15.26 shows the first four normal-mode patterns and their associated frequencies and wavelengths; these correspond to Eq. (15.34) with $n = 1, 2, 3,$ and 4. By contrast, a harmonic oscillator, which has only one oscillating particle, has only one normal mode and one characteristic frequency. The string fixed at both ends has infinitely many normal modes $(n = 1,\ 2,\ 3,\ \ldots)$ because it is made up of a very large (effectively infinite) number of particles. More complicated oscillating systems also have infinite numbers of normal modes, though with more complex normal-mode patterns (Fig. 15.27,).

Figure 15.26



The first four normal modes of a string fixed at both ends. (Compare these to the photographs in Fig. 15.23.)

Figure 15.27



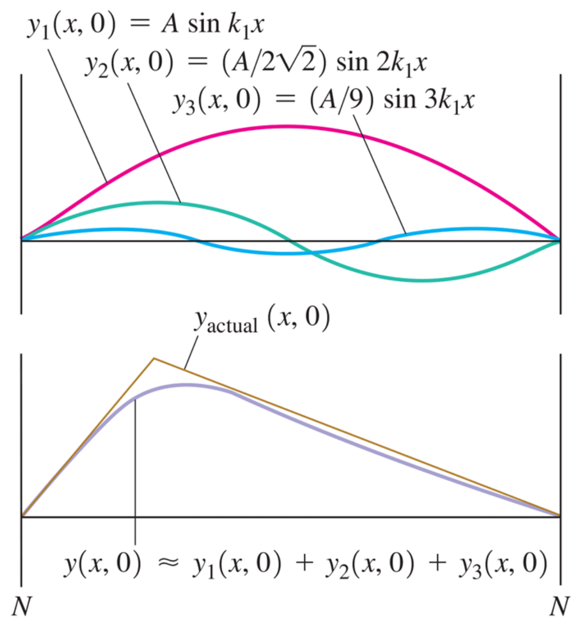
Astronomers have discovered that the sun oscillates in several different normal modes. This computer simulation shows one such mode.

Complex Standing Waves

If we could displace a string so that its shape is the same as one of the normal-mode patterns and then release it, it would vibrate with the frequency of that mode. Such a vibrating string would displace the surrounding air with the same frequency, producing a traveling sinusoidal sound wave that your ears would perceive as a pure tone. But when a string is struck (as in a piano) or plucked (as is done to guitar strings), the shape of the displaced string is *not* one of the patterns in Fig. 15.26. The motion is therefore a combination or *superposition* of many normal modes. Several simple harmonic motions of different frequencies are present simultaneously, and the displacement of any point on the string is the superposition of the displacements associated with the individual modes. The sound produced by the vibrating string is likewise a superposition of traveling sinusoidal sound waves, which you perceive as a rich, complex tone with the fundamental frequency f_1 . The standing wave on the string and the traveling sound wave in the air have similar **harmonic content** (the extent to which frequencies higher than the fundamental are present). The harmonic content depends on how the string is initially set into motion. If you pluck the strings of an acoustic guitar in the normal location over the sound hole, the sound that you hear has a different harmonic content than if you pluck the strings next to the fixed end on the guitar body.

It is possible to represent every possible motion of the string as some superposition of normal-mode motions. Finding this representation for a given vibration pattern is called *harmonic analysis.* The sum of sinusoidal functions that represents a complex wave is called a *Fourier series.* Figure 15.28 shows how a standing wave that is produced by plucking a guitar string of length L at a point $L/4$ from one end can be represented as a combination of sinusoidal functions.

Figure 15.28



When a guitar string is plucked (pulled into a triangular shape) and released, a standing wave results. The standing wave is well represented (except at the sharp maximum point) by the sum of just three sinusoidal functions. Including additional sinusoidal functions further improves the representation.

Standing Waves and String Instruments

From Eq. (15.32), the fundamental frequency of a vibrating string is $f_1 = v/2L$. The speed v of waves on the string is determined by Eq. (15.14), $v = \sqrt{F/\mu}$. Combining these equations, we find

(15.35)

Fundamental frequency, string fixed at both ends

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} \qquad (15.35)$$

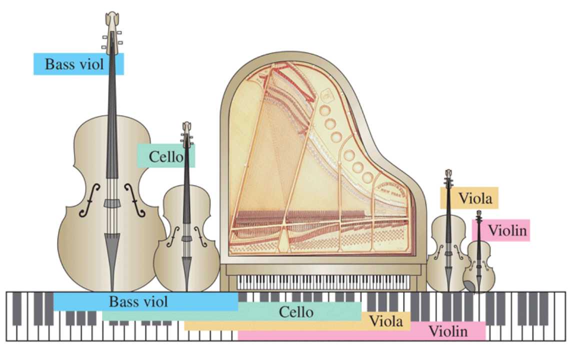
Tension in string (F)

Mass per unit length (μ)

Length of string (L)

This is also the fundamental frequency of the sound wave created in the surrounding air by the vibrating string. The inverse dependence of frequency on length L is illustrated by the long strings of the bass (low-frequency) section of the piano or the bass viol compared with the shorter strings of the treble section of the piano or the violin (Fig. 15.29). The pitch of a violin or guitar is usually varied by pressing a string against the fingerboard with the fingers to change the length L of the vibrating portion of the string. Increasing the tension F increases the wave speed v and thus increases the frequency (and the pitch). All string instruments are "tuned" to the correct frequencies by varying the tension; you tighten the string to raise the pitch. Finally, increasing the mass per unit length μ decreases the wave speed and thus the frequency. The lower notes on a steel guitar are produced by thicker strings, and one reason for winding the bass strings of a piano with wire is to obtain the desired low frequency from a relatively short string.

Figure 15.29



Comparing the range of a concert grand piano to the ranges of a bass viol, a cello, a viola, and a violin. In all cases, longer strings produce bass notes and shorter strings produce treble notes.

Wind instruments such as saxophones and trombones also have normal modes. As for stringed instruments, the frequencies of these normal modes determine the pitch of the musical tones that these instruments produce. We'll discuss these instruments and many other aspects of sound in Chapter 16.

Example 15.7 A giant bass viol

WITH VARIATION PROBLEMS

In an attempt to get your name in *Guinness World Records*, you build a bass viol with strings of length 5.00 m between fixed points. One string, with linear mass density 40.0 g/m, is tuned to a 20.0 Hz fundamental frequency (the lowest frequency that the human ear can hear). Calculate (a) the tension of this string, (b) the frequency and wavelength on the string of the second harmonic, and (c) the frequency and wavelength on the string of the second overtone.

IDENTIFY and SET UP In part (a) the target variable is the string tension F; we'll use Eq. (15.35), which relates F to the known values $f_1 = 20.0\text{ Hz}$, $L = 5.00\text{ m}$, and $\mu = 40.0\text{ g}/m$. In parts (b) and (c) the target variables are the frequency and wavelength of a given harmonic and a given overtone. We determine these from the given length of the string and the fundamental frequency, using Eqs. (15.31) and (15.33).

EXECUTE (a) We solve Eq. (15.35) for F:

$$\begin{aligned} F &= 4\mu L^2 f_1^2 = 4(40.0 \times 10^{-3}\,\text{kg/m})(5.00\,\text{m})^2(20.0\,\text{s}^{-1})^2 \\ &= 1600\,\text{N} = 360\,\text{lb} \end{aligned}$$

(b) From Eqs. (15.33) and (15.31), the frequency and wavelength of the second harmonic ($n = 2$) are

$$\begin{aligned} f_2 &= 2f_1 = 2(20.0\text{ Hz}) = 40.0\text{ Hz} \\ \lambda_2 &= \frac{2L}{2} = \frac{2\,(5.00\,\text{m})}{2} = 5.00\,\text{m} \end{aligned}$$

(c) The second overtone is the "second tone over" (above) the fundamental—that is, $n = 3$. Its frequency and wavelength are

$$\begin{aligned} f_3 &= 3f_1 = 3(20.0\ \text{Hz}) = 60.0\ \text{Hz} \\ \lambda_3 &= \frac{2L}{3} = \frac{2\,(5.00\ \text{m})}{3} = 3.33\ \text{m} \end{aligned}$$

EVALUATE The string tension in a real bass viol is typically a few hundred newtons; the tension in part (a) is a bit higher than that. The wavelengths in parts (b) and (c) are equal to the length of the string and two-thirds the length of the string, respectively, which agrees with the drawings of standing waves in Fig. 15.26.

KEY CONCEPT

A string with its ends fixed vibrates in a type of standing wave called a normal mode when all of its particles move sinusoidally with the same frequency. The frequencies of oscillation of the normal modes are integer multiples of a minimum normal-mode frequency, called the fundamental frequency.

Video Tutor Solution: **Example 15.7**

Example 15.8 From waves on a string to sound waves in air

WITH VARIATION PROBLEMS

What are the frequency and wavelength of the sound waves produced in the air when the string in Example 15.7 is vibrating at its fundamental frequency? The speed of sound in air at $20^\circ\mathrm{C}$ is $344\ \mathrm{m/s}$.

IDENTIFY and SET UP Our target variables are the frequency and wavelength for the *sound wave* produced by the bass viol string. The frequency of the sound wave is the same as the fundamental frequency f_1 of the standing wave, because the string forces the surrounding air to vibrate at the same frequency. The wavelength of the sound wave is $\lambda_{1(\mathrm{sound})} = v_{sound}/f_1$.

EXECUTE We have $f = f_1 = 20.0\ \mathrm{Hz}$, so

$$\lambda_{1(\mathrm{sound})} = \frac{v_{\mathrm{sound}}}{f_1} = \frac{344\ \mathrm{m/s}}{20.0\ \mathrm{Hz}} = 17.2\ \mathrm{m}$$

EVALUATE In Example 15.7, the wavelength of the fundamental on the string was $\lambda_{1(\mathrm{string})} = 2L = 2(5.00\ \mathrm{m}) = 10.0\ \mathrm{m}$. Here $\lambda_{1(\mathrm{sound})} = 17.2\ \mathrm{m}$ is greater than that by the factor of $17.2/10.0 = 1.72$. This is as it should be: Because the frequencies of the sound wave and the standing wave are equal, $\lambda = v/f$ says that the wavelengths in air and on the string are in the same ratio as the corresponding wave speeds; here $v_{\mathrm{sound}} = 344\ \mathrm{m/s}$ is greater than $v_{\mathrm{string}} = (10.0\ \mathrm{m})(20.0\ \mathrm{Hz}) = 200\ \mathrm{m/s}$ by just the factor 1.72.

KEY CONCEPT

A string vibrating at a certain frequency produces sound waves of the same frequency in the surrounding air. The standing wave on the string and the sound wave will have different wavelengths, however, if the speed of waves on the string does not equal the speed of sound.

Video Tutor Solution: **Example 15.8**

Test Your Understanding of Section 15.8

While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point. Which normal modes *cannot* be present on the string while you are touching it in this way?

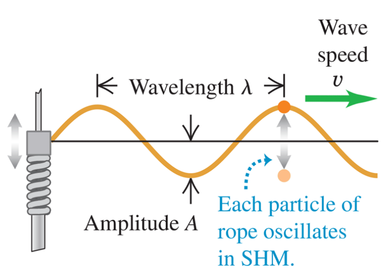
Chapter 15 Summary

Waves and their properties: A wave is any disturbance that propagates from one region to another. A mechanical wave travels within some material called the medium. The wave speed v depends on the type of wave and the properties of the medium.

In a periodic wave, the motion of each point of the medium is periodic with frequency f and period T. The wavelength λ is the distance over which the wave pattern repeats, and the amplitude A is the maximum displacement of a particle in the medium. The product of λ and f equals the wave speed. A sinusoidal wave is a special periodic wave in which each point moves in simple harmonic motion. (See Example 15.1.)

(15.1)

$$v = \lambda f$$



Wave functions and wave dynamics: The wave function $y(x,\ t)$ describes the displacements of individual particles in the medium. Equations (15.3), (15.4), and (15.7) give the wave equation for a sinusoidal wave traveling in the $+x$-direction. If the wave is moving in the $-x$-direction, the minus signs in the cosine functions are replaced by plus signs. (See Example 15.2.)

The wave function obeys a partial differential equation called the wave equation, Eq. (15.12).

The speed of transverse waves on a string depends on the tension F and mass per unit length μ. (See Example 15.3.)

(15.3)

$$y(x,\ t) = A \cos\left[\omega\left(\frac{x}{v} - t\right)\right]$$

(15.4)

$$y(x,\ t) = A \cos 2\pi\left[\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

(15.7)

$$y(x,\ t) = A \cos(kx - \omega t)$$

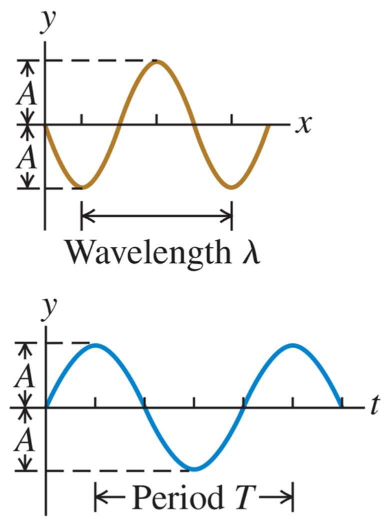
where $k = 2\pi/\lambda$ and $\omega = 2\pi f = vk$

(15.12)

$$\frac{\partial^2 y(x,\ t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,\ t)}{\partial t^2}$$

(15.14)

$$v = \sqrt{\frac{F}{\mu}} \text{ (waves on a string)}$$



Wave power: Wave motion conveys energy from one region to another. For a sinusoidal mechanical wave, the average power P_{av} is proportional to the square of the wave amplitude and the square of the frequency. For waves that spread out in three dimensions, the wave intensity I is inversely proportional to the square of the distance from the source. (See Examples 15.4 and 15.5.)

(15.25)

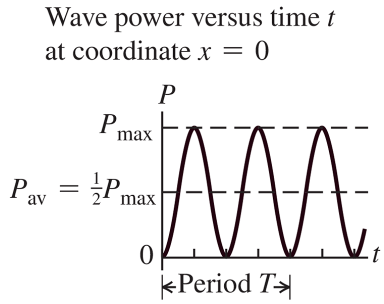
$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

(average power, sinusoidal wave)

(15.26)

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

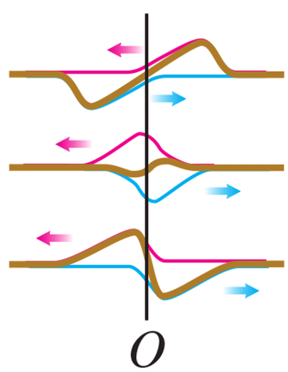
(inverse-square law for intensity)



Wave superposition: A wave reflects when it reaches a boundary of its medium. At any point where two or more waves overlap, the total displacement is the sum of the displacements of the individual waves (principle of superposition).

(15.27)

$$y(x,\ t) = y_1(x,\ t) + y_2(x,\ t)$$
(principle of superposition)



Standing waves on a string: When a sinusoidal wave is reflected from a fixed or free end of a stretched string, the incident and reflected waves combine to form a standing sinusoidal wave with nodes and antinodes. Adjacent nodes are spaced a distance $\lambda/2$ apart, as are adjacent antinodes. (See Example 15.6.)

When both ends of a string with length L are held fixed, standing waves can occur only when L is an integer multiple of $\lambda/2$. Each frequency with its associated vibration pattern is called a normal mode. (See Examples 15.7 and 15.8.)

(15.28)

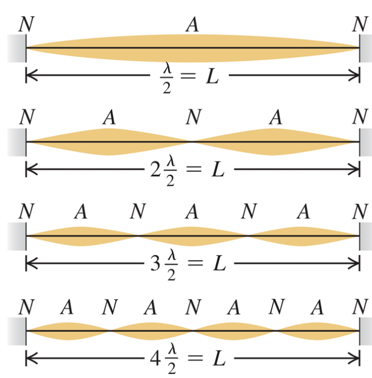
$$y(x,\ t) = (A_{\mathrm{SW}}\ kx)\ \sin \omega t$$
$$\text{(standing wave on a string, fixed end at } x = 0\text{)}$$

(15.33)

$$f_n = n\frac{v}{2L} = nf_1\ (n = 1,\ 2,\ 3,\ \ldots)$$

(15.35)

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$
$$\text{(string fixed at both ends)}$$



Guided Practice: Mechanical Waves

For assigned homework and other learning materials, go to **Mastering Physics**.

Key Example Variation Problems

Be sure to review Examples 15.1, 15.2, and 15.3 (Sections 15.2, 15.3, and 15.4) before attempting these problems.

VP15.3.1 A boat is at anchor outside a harbor. A steady sinusoidal ocean wave makes the boat bob up and down with a period of 5.10 s and an amplitude of 1.00 m. The wave has wavelength 30.5 m. For this wave, what are (a) the frequency, (b) the wave speed, (c) the angular frequency, and (d) the wave number?

VP15.3.2 Sound waves in the thin Martian atmosphere travel at m 245 m/s. (a) What are the period and wavelength of a 125 Hz sound wave in the Martian atmosphere? (b) What are the frequency and angular frequency of a sound wave in the Martian atmosphere that has wavelength 3.00 m?

VP15.3.3 You are testing a mountain climbing rope that has a linear mass density of 0.0650 kg/m. The rope is held horizontal and is under a tension of 8.00×10^2 N to simulate the stress of supporting a mountain climber's weight. (a) What is the speed of transverse waves on this rope? (b) You oscillate one end of the rope up and down in SHM with frequency 25.0 Hz and amplitude 5.00 mm. What is the wavelength of the resulting waves on the rope? (c) At $t = 0$ the end you are oscillating is at its maximum positive displacement and is instantaneously at rest. Write an equation for the displacement as a function of time at a point 2.50 m from that end. Assume that no wave bounces back from the other end.

VP15.3.4 The tension in a long string is 25.0 N. You oscillate one end of the string up and down with frequency 45.0 Hz. When this end is at its maximum upward displacement, the nearest point that is at its maximum negative displacement is 0.400 m down the string. What are (a) the speed of waves on the string and (b) the linear mass density of the string?

Be sure to review Examples 15.4 and 15.5 (Section 15.5) before attempting these problems.

VP15.5.1 An athlete exerts a tension of 6.00×10^2 N on one end of a horizontal rope that has length 50.0 m and mass 2.50 kg. The other end is tied to a post. If she wiggles the rope with period 0.575 s and amplitude 3.00 cm, what are (a) the angular frequency of the oscillation and (b) the average rate at which energy is transferred along the rope?

VP15.5.2 A length of piano wire (mass density 5.55×10^{-4} kg/m) is under 185 N of tension. A sinusoidal wave of frequency 256 Hz carries a maximum power of 5.20 W along the wire. What is the amplitude of this wave?

VP15.5.3 A portable audio speaker has a power output of 8.00 W. (a) If the speaker emits sound equally in all directions, what is the sound intensity at a distance of 2.00 m from the speaker? (b) At what distance from the speaker is the intensity equal to 0.045 W/m^2?

VP15.5.4 The "ears" of a frog are two circular membranes located behind the frog's eyes. In one species of frog each membrane is 0.500 cm in radius. If a source of sound has a power output of 2.50×10^{-6} W, emits sound equally in all directions, and is located 1.50 m from the frog, how much sound energy arrives at one of the membranes each second?

Be sure to review Examples 15.6, 15.7, and 15.8 (Sections 15.7 and 15.8) before attempting these problems.

VP15.8.1 For a standing wave on a string, the distance between nodes is 0.125 m, the frequency is 256 Hz, and the amplitude is 1.40×10^{-3} m. What are (a) the speed of waves on this string, (b) the maximum transverse velocity at an antinode, and (c) the maximum transverse acceleration at an antinode?

VP15.8.2 The G string of a guitar has a fundamental frequency of 196 Hz. The linear mass density of the string is 2.29×10^{-3} kg/m, and the length of string that is free to vibrate (between the nut and bridge of the guitar) is 0.641 m. What are (a) the speed of waves on the G string and (b) the tension in this string?

VP15.8.3 A cable is stretched between two posts 3.00 m apart. The speed of waves on this cable is 96.0 m/s, and the tension in the cable is 175 N. If a standing wave on this cable has five antinodes, what are (a) the wavelength of the standing wave, (b) the frequency of the standing wave, and (c) the linear mass density of the cable?

VP15.8.4 One of the strings on a musical instrument is 0.500 m in length and has linear mass density 1.17×10^{-3} kg/m. The second harmonic on this string has frequency 512 Hz. (a) What is the tension in the string? (b) The speed of sound in air at 20°C is 344 m/s. If the string is vibrating at its fundamental frequency, what is the wavelength of the sound wave that the string produces in air?

Bridging Problem: Waves on a Rotating Rope

A uniform rope with length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. You can ignore the force of gravity on the rope. (a) At a point on the rope a distance r from the end that is held, what is the tension F? (b) What is the speed of transverse waves at this point? (c) Find the time required for a transverse wave to travel from one end of the rope to the other.

Solution Guide

IDENTIFY and SET UP

1. Draw a sketch of the situation and label the distances r and L. The tension in the rope will be different at different values of r. Do you see why? Where on the rope do you expect the tension to be greatest? Where do you expect it will be least?
2. Where on the rope do you expect the wave speed to be greatest? Where do you expect it will be least?
3. Think about the portion of the rope that is farther out than r from the end that is held. What forces act on this portion? (Remember that you can ignore gravity.) What is the mass of this portion? How far is its center of mass from the rotation axis?

4. List the unknown quantities and decide which are your target variables.

EXECUTE

5. Draw a free-body diagram for the portion of the rope that is farther out than r from the end that is held.
6. Use your free-body diagram to help you determine the tension in the rope at distance r.
7. Use your result from step 6 to find the wave speed at distance r.
8. Use your result from step 7 to find the time for a wave to travel from one end to the other. (*Hint:* The wave speed is $v = dr/dt$, so the time for the wave to travel a distance dr along the rope is $dt = dr/v$. Integrate this to find the total time. See Appendix B.)

EVALUATE

9. Do your results for parts (a) and (b) agree with your expectations from steps 1 and 2? Are the units correct?
10. Check your result for part (a) by considering the net force on a small segment of the rope at distance r with length dr and mass $dm = (m/L)dr$. [*Hint:* The tension forces on this segment are $F(r)$ on one side and $F(r + dr)$ on the other side. You will get an equation for dF/dr that you can integrate to find F as a function of r.]

Video Tutor Solution: **Chapter 15 Bridging Problem**

Questions/Exercises/Problems: Mechanical Waves

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

Discussion Questions

Q15.1 Two waves travel on the same string. Is it possible for them to have (a) different frequencies; (b) different wavelengths; (c) different speeds; (d) different amplitudes; (e) the same frequency but different wavelengths? Explain your reasoning.

Q15.2 Under a tension F, it takes 2.00 s for a pulse to travel the length of a taut wire. What tension is required (in terms of F) for the pulse to take 6.00 s instead? Explain how you arrive at your answer.

Q15.3 What kinds of energy are associated with waves on a stretched string? How could you detect such energy experimentally?

Q15.4 The amplitude of a wave decreases gradually as the wave travels along a long, stretched string. What happens to the energy of the wave when this happens?

Q15.5 For the wave motions discussed in this chapter, does the speed of propagation depend on the amplitude? What makes you say this?

Q15.6 The speed of ocean waves depends on the depth of the water; the deeper the water, the faster the wave travels. Use this to explain why ocean waves crest and "break" as they near the shore.

Q15.7 Is it possible to have a longitudinal wave on a stretched string? Why or why not? Is it possible to have a transverse wave on a steel rod? Again, why or why not? If your answer is yes in either case, explain how you would create such a wave.

Q15.8 For transverse waves on a string, is the wave speed the same as the speed of any part of the string? Explain the difference between these two speeds. Which one is constant?

Q15.9 The four strings on a violin have different thicknesses, but are all under approximately the same tension. Do waves travel faster on the thick strings or the thin strings? Why? How does the fundamental vibration frequency compare for the thick versus the thin strings?

Q15.10 A sinusoidal wave can be described by a cosine function, which is negative just as often as positive. So why isn't the average power delivered by this wave zero?

Q15.11 Two strings of different mass per unit length μ_1 and μ_2 are tied together and stretched with a tension F. A wave travels along the string and passes the discontinuity in μ. Which of the following wave properties will be the same on both sides of the discontinuity, and which will change: speed of the wave; frequency; wavelength? Explain the physical reasoning behind each answer.

Q15.12 A long rope with mass m is suspended from the ceiling and hangs vertically. A wave pulse is produced at the lower end of the rope, and the pulse travels up the rope. Does the speed of the wave pulse change as it moves up the rope, and if so, does it increase or decrease? Explain.

Q15.13 In a transverse wave on a string, the motion of the string is perpendicular to the length of the string. How, then, is it possible for energy to move along the length of the string?

Q15.14 Energy can be transferred along a string by wave motion. However, in a standing wave on a string, no energy can ever be transferred past a node. Why not?

Q15.15 Can a standing wave be produced on a string by superposing two waves traveling in opposite directions with the same frequency but different amplitudes? Why or why not? Can a standing wave be produced by superposing two waves traveling in opposite directions with different frequencies but the same amplitude? Why or why not?

Q15.16 If you stretch a rubber band and pluck it, you hear a (somewhat) musical tone. How does the frequency of this tone change as you stretch the rubber band further? (Try it!) Does this agree with Eq. (15.35) for a string fixed at both ends? Explain.

Q15.17 A musical interval of an *octave* corresponds to a factor of 2 in frequency. By what factor must the tension in a guitar or violin string be increased to raise its pitch one octave? To raise it two octaves? Explain your reasoning. Is there any danger in attempting these changes in pitch?

Q15.18 By touching a string lightly at its center while bowing, a violinist can produce a note exactly one octave above the note to which the string is tuned—that is, a note with exactly twice the frequency. Why is this possible?

Q15.19 As we discussed in Section 15.1, water waves are a combination of longitudinal and transverse waves. Defend the following statement: “When water waves hit a vertical wall, the wall is a node of the longitudinal displacement but an antinode of the transverse displacement."

Q15.20 Violins are short instruments, while cellos and basses are long. In terms of the frequency of the waves they produce, explain why this is so.

Q15.21 What is the purpose of the frets on a guitar? In terms of the frequency of the vibration of the strings, explain their use.

Exercises

Section 15.2 Periodic Waves

15.1 • The speed of sound in air at 20°C is 344 m/s. (a) What is the wavelength of a sound wave with a frequency of 784 Hz, corresponding to the note G_5 on a piano, and how many milliseconds does each vibration take? (b) What is the wavelength of a sound wave one octave higher (twice the frequency) than the note in part (a)?

15.2 • BIO **Ultrasound Imaging.** Sound having frequencies above the range of human hearing (about 20,000 Hz) is called *ultrasound*. Waves above this frequency can be used to penetrate the body and to produce images by reflecting from surfaces. In a typical ultrasound scan, the waves travel through body tissue with a speed of 1500 m/s. For a good, detailed image, the wavelength should be no more than 1.0 mm. What frequency sound is required for a good scan?

15.3 • **Tsunami!** On December 26, 2004, a great earthquake occurred off the coast of Sumatra and triggered immense waves (tsunami) that killed more than 200,000 people. Satellites observing these waves from space measured 800 km from one wave crest to the next and a period between waves of 1.0 hour. What was the speed of these waves in m/s and in km/h? Does your answer help you understand why the waves caused such devastation?

15.4 •• A fisherman notices that his boat is moving up and down periodically, owing to waves on the surface of the water. It takes 2.5 s for the boat to travel from its highest point to its lowest, a total distance of 0.53 m. The fisherman sees that the wave crests are spaced 4.8 m apart. (a) How fast are the waves traveling? (b) What is the amplitude of each wave? (c) If the total vertical distance traveled by the boat were 0.30 m but the other data remained the same, how would the answers to parts (a) and (b) change?

15.5 • BIO (a) **Audible wavelengths.** The range of audible frequencies is from about 20 Hz to 20,000 Hz. What is the range of the wavelengths of audible sound in air? (b) **Visible light.** The range of visible light extends from 380 nm to 750 nm. What is the range of visible frequencies of light? (c) **Brain surgery.** Surgeons can remove brain tumors by using a cavitron ultrasonic surgical aspirator, which produces sound waves of frequency 23 kHz. What is the wavelength of these waves in air? (d) **Sound in the body.** What would be the wavelength of the sound in part (c) in bodily fluids in which the speed of sound is 1480 m/s but the frequency is unchanged?

Section 15.3 Mathematical Description of a Wave

15.6 • A small bead of mass 4.00 g is attached to a horizontal string. Transverse waves of amplitude $A = 0.800$ cm and frequency $f = 20.0$ Hz are set up on the string. Assume the mass of the bead is small enough that the bead doesn't alter the wave motion. During the wave motion, what is the maximum vertical force that the string exerts on the bead?

15.7 • Transverse waves on a string have wave speed 8.00 m/s, amplitude 0.0700 m, and wavelength 0.320 m. The waves travel in the $-x$-direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement. (a) Find the frequency, period, and wave number of these waves. (b) Write a wave function describing the wave. (c) Find the transverse displacement of a particle at $x = 0.360$ m at time $t = 0.150$ s. (d) How much time must elapse from the instant in part (c) until the particle at $x = 0.360$ m next has maximum upward displacement?

15.8 • A certain transverse wave is described by

$$y(x,\ t) = (6.50\ \text{mm})\ \cos 2\pi\left(\frac{x}{28.0\ \text{cm}} - \frac{t}{0.0360\ \text{s}}\right)$$

Determine the wave's (a) amplitude; (b) wavelength; (c) frequency; (d) speed of propagation; (e) direction of propagation.

15.9 • **CALC** Which of the following wave functions satisfies the wave equation, Eq. (15.12)? (a) $y\,(x,\ t) = A\cos\,(kx + \omega t)$; (b) $y\,(x,\ t) = A\sin\,(kx + \omega t)$; (c) $y\,(x,\ t) = A(\cos kx + \cos \omega t)$. (d) For the wave of part (b), write the equations for the transverse velocity and transverse acceleration of a particle at point x.

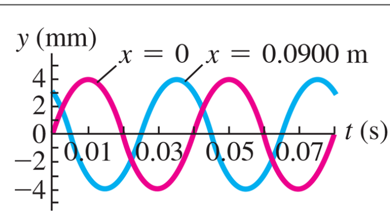
15.10 • A water wave traveling in a straight line on a lake is described by the equation

$$y(x,\ t) = (2.75\ \text{cm})\cos(0.410\ \text{rad/cm}\ x + 6.20\ \text{rad/s}\ t)$$

where y is the displacement perpendicular to the undisturbed surface of the lake. (a) How much time does it take for one complete wave pattern to go past a fisherman in a boat at anchor, and what horizontal distance does the wave crest travel in that time? (b) What are the wave number and the number of waves per second that pass the fisherman? (c) How fast does a wave crest travel past the fisherman, and what is the maximum speed of his cork floater as the wave causes it to bob up and down?

15.11 • A sinusoidal wave is propagating along a stretched string that lies along the x-axis. The displacement of the string as a function of time is graphed in Fig. E15.11 for particles at $x = 0$ and at $x = 0.0900$ m. (a) What is the amplitude of the wave? (b) What is the period of the wave? (c) You are told that the two points $x = 0$ and $x = 0.0900$ m are within one wavelength of each other. If the wave is moving in the $+x$-direction, determine the wavelength and the wave speed. (d) If instead the wave is moving in the $-x$-direction, determine the wavelength and the wave speed. (e) Would it be possible to determine definitively the wavelengths in parts (c) and (d) if you were not told that the two points were within one wavelength of each other? Why or why not?

Figure E15.11



15.12 •• CALC **Speed of Propagation vs. Particle Speed.** (a) Show that Eq. (15.3) may be written as

$$y(x,\ t) = A\cos\left[\frac{2\pi}{\lambda}\ (x - vt)\right]$$

(b) Use $y(x,\ t)$ to find an expression for the transverse velocity v_y of a particle in the string on which the wave travels. (c) Find the maximum speed of a particle of the string. Under what circumstances is this equal to the propagation speed v? Less than v? Greater than v?

15.13 •• A transverse wave on a string has amplitude 0.300 cm, wavelength 12.0 cm, and speed 6.00 cm/s. It is represented by $y(x,\ t)$ as given in Exercise 15.12. (a) At time $t = 0$, compute y at 1.5 cm intervals of x (that is, at $x = 0,\ x = 1.5$ cm, $x = 3.0$ cm, and so on) from $x = 0$ to $x = 12.0$ cm. Graph the results. This is the shape of the string at time $t = 0$. (b) Repeat the calculations for the same values of x at times $t = 0.400$ s and $t = 0.800$ s. Graph the shape of the string at these instants. In what direction is the wave traveling?

Section 15.4 Speed of a Transverse Wave

15.14 • A musical novice learns that doubling the fundamental increases the pitch by one octave and decides to do that to a guitar string. What factor increase in tension would be necessary? Do you think this would be a good idea?

15.15 • One end of a horizontal rope is attached to a prong of an electrically driven tuning fork that vibrates the rope transversely at 120 Hz. The other end passes over a pulley and supports a 1.50 kg mass. The linear mass density of the rope is 0.0480 kg/m. (a) What is the speed of a transverse wave on the rope? (b) What is the wavelength? (c) How would your answers to parts (a) and (b) change if the mass were increased to 3.00 kg?

15.16 • With what tension must a rope with length 2.50 m and mass 0.120 kg be stretched for transverse waves of frequency 40.0 Hz to have a wavelength of 0.750 m?

15.17 •• The upper end of a 3.8-m-long steel wire is fastened to the ceiling, and a 54 kg object is suspended from the lower end of the wire. You observe that it takes a transverse pulse 0.049 s to travel from the bottom to the top of the wire. What is the mass of the wire?

15.18 •• A 1.50 m string of weight 0.0125 N is tied to the ceiling at its upper end, and the lower end supports a weight W. Ignore the very small variation in tension along the length of the string that is produced by the weight of the string. When you pluck the string slightly, the waves traveling up the string obey the equation

$$y\,(x,\ t) = (8.50\ \mathrm{mm})\ \cos(172\ \mathrm{rad/m\ x} - 4830\ \mathrm{rad/s\ t})$$

Assume that the tension of the string is constant and equal to W. (a) How much time does it take a pulse to travel the full length of the string? (b) What is the weight W? (c) How many wavelengths are on the string at any instant of time? (d) What is the equation for waves traveling *down* the string?

15.19 • A thin, 75.0 cm wire has a mass of 16.5 g. One end is tied to a nail, and the other end is attached to a screw that can be adjusted to vary the tension in the wire. (a) To what tension (in newtons) must you adjust the screw so that a transverse wave of wavelength 3.33 cm makes 625 vibrations per second? (b) How fast would this wave travel?

15.20 •• A heavy rope 6.00 m long and weighing 29.4 N is attached at one end to a ceiling and hangs vertically. A 0.500 kg mass is suspended from the lower end of the rope. What is the speed of transverse waves on the rope at the (a) bottom of the rope, (b) middle of the rope, and (c) top of the rope? (d) Is the tension in the middle of the rope the average of the tensions at the top and bottom of the rope? Is the wave speed at the middle of the rope the average of the wave speeds at the top and bottom? Explain.

Section 15.5 Energy in Wave Motion

15.21 •• In Example 15.4 the average power that Throckmorton puts into the clothesline is small, about 1 W. How much better can you do? Assume a rope that has linear mass density 0.500 kg/m, so 1 m of the rope weighs about 1 lb. The rope is long and is attached to a post at one end. You hold the other end of the rope in your hand and supply sinusoidal wave pulses by moving your arm up and down. Estimate the amplitude of the pulses to be the length of your arm. Estimate the maximum tension you can supply to the rope by pulling on it horizontally, and estimate the time for you to complete each pulse. (a) Ignoring any effects from reflection of the pulses from the other end of the rope, what average power can you supply to the rope? (b) You try to increase your power output by halving the amplitude so you can double the frequency of the pulses. What change in P_{av} does this produce?

15.22 •• A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. A wave with frequency 120.0 Hz and amplitude 1.6 mm travels along the wire. (a) Calculate the average power carried by the wave. (b) What happens to the average power if the wave amplitude is halved?

15.23 • A horizontal wire is stretched with a tension of 94.0 N, and the speed of transverse waves for the wire is 406 m/s. What must the amplitude of a traveling wave of frequency 69.0 Hz be for the average power carried by the wave to be 0.365 W?

15.24 •• **Threshold of Pain.** You are investigating the report of a UFO landing in an isolated portion of New Mexico, and you encounter a strange object that is radiating sound waves uniformly in all directions. Assume that the sound comes from a point source and that you can ignore reflections. You are slowly walking toward the source. When you are 7.5 m from it, you measure its intensity to be 0.11 $\mathrm{W/m^2}$. An intensity of 1.0 W/m^2 is often used as the "threshold of pain." How much closer to the source can you move before the sound intensity reaches this threshold?

15.25 •• A jet plane at takeoff can produce sound of intensity 10.0 W/m^2 at 30.0 m away. But you prefer the tranquil sound of normal conversation, which is 1.0 $\mu\mathrm{W/m^2}$. Assume that the plane behaves like a point source of sound. (a) What is the closest distance you should live from the airport runway to preserve your peace of mind? (b) What intensity from the jet does your friend experience if she lives twice as far from the runway as you do? (c) What power of sound does the jet produce at takeoff?

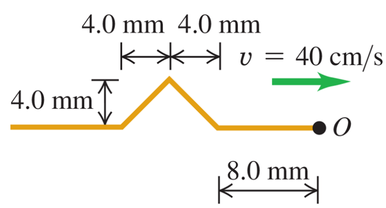
15.26 • A fellow student with a mathematical bent tells you that the wave function of a traveling wave on a thin rope is $y(x,\ t)\ =\ (2.30\ \mathrm{mm})\cos[(6.98\ \mathrm{rad/m})x + (742\ \mathrm{rad/s})t]$. Being more practical, you measure the rope to have a length of 1.35 m and a mass of 0.00338 kg. You are then asked to determine the following: (a) amplitude; (b) frequency; (c) wavelength; (d) wave speed; (e) direction the wave is traveling; (f) tension in the rope; (g) average power transmitted by the wave.

15.27 • **Energy Output.** By measurement you determine that sound waves are spreading out equally in all directions from a point source and that the intensity is 0.026 W/m^2 at a distance of 4.3 m from the source. (a) What is the intensity at a distance of 3.1 m from the source? (b) How much sound energy does the source emit in one hour if its power output remains constant?

Section 15.6 Wave Interference, Boundary Conditions, and Superposition

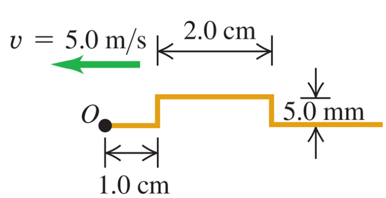
15.28 • **Reflection.** A wave pulse on a string has the dimensions shown in Fig. E15.28 at $t = 0$. The wave speed is 40 cm/s. (a) If point O is a fixed end, draw the total wave on the string at $t = 15$ ms, 20 ms, 25 ms, 30 ms, 35 ms, 40 ms, and 45 ms. (b) Repeat part (a) for the case in which point O is a free end.

Figure E15.28



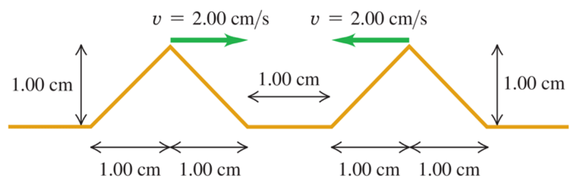
15.29 • **Reflection.** A wave pulse on a string has the dimensions shown in Fig. E15.29 at $t = 0$. The wave speed is 5.0 m/s. (a) If point O is a fixed end, draw the total wave on the string at $t = 1.0$ ms, 2.0 ms, 3.0 ms, 4.0 ms, 5.0 ms, 6.0 ms, and 7.0 ms. (b) Repeat part (a) for the case in which point O is a free end.

Figure E15.29



15.30 • **Interference of Triangular Pulses.** Two triangular wave pulses are traveling toward each other on a stretched string as shown in Fig. E15.30. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at $t = 0$. Sketch the shape of the string at $t = 0.250$ s, t $= 0.500$ s, t $= 0.750$ s, t $= 1.000$ s, and $t = 1.250$ s.

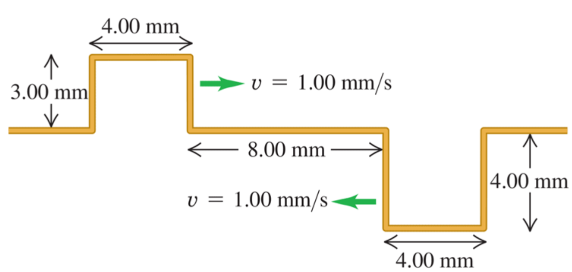
Figure E15.30



15.31 • Suppose that the left-traveling pulse in Exercise 15.30 is *below* the level of the unstretched string instead of above it. Make the same sketches that you did in that exercise.

15.32 •• **Interference of Rectangular Pulses.** Figure E15.32 shows two rectangular wave pulses on a stretched string traveling toward each other. Each pulse is traveling with a speed of 1.00 mm/s and has the height and width shown in the figure. If the leading edges of the pulses are 8.00 mm apart at $t = 0$, sketch the shape of the string at $t = 4.00$ s, $t = 6.00$ s, and $t = 10.0$ s.

Figure E15.32



Section 15.7 Standing Waves on a String

Section 15.8 Normal Modes of a String

15.33 •• For a violin, estimate the length of the portions of the strings that are free to vibrate. (a) The frequency of the note played by the open E5 string vibrating in its fundamental standing wave is 659 Hz. Use your estimate of the length to calculate the wave speed for the transverse waves on the string. (b) The vibrating string produces sound waves in air with the same frequency as that of the string. Use 344 m/s for the speed of sound in air and calculate the wavelength of the E5 note in air. Which is larger: the wavelength on the string or the wavelength in air? (c) Repeat parts (a) and (b) for a bass viol, which is typically played by a person standing up. Start your calculation by estimating the length of the bass viol string that is free to vibrate. The G2 string produces a note with frequency 98 Hz when vibrating in its fundamental standing wave.

15.34 •• CALC Adjacent antinodes of a standing wave on a string are 15.0 cm apart. A particle at an antinode oscillates in simple harmonic motion with amplitude 0.850 cm and period 0.0750 s. The string lies along the $+x$-axis and is fixed at $x = 0$. (a) How far apart are the adjacent nodes? (b) What are the wavelength, amplitude, and speed of the two traveling waves that form this pattern? (c) Find the maximum and minimum transverse speeds of a point at an antinode. (d) What is the shortest distance along the string between a node and an antinode?

15.35 • Standing waves on a wire are described by Eq. (15.28), with $A_{\mathrm{SW}} = 2.50$ mm, and $k = 0.750\pi$ rad/m. The left end of the wire is at $x = 0$. At what distances from the left end are (a) the nodes of the standing wave and (b) the antinodes of the standing wave?

15.36 • A 1.50-m-long rope is stretched between two supports with a tension that makes the speed of transverse waves 62.0 m/s. What are the wavelength and frequency of (a) the fundamental; (b) the second overtone; (c) the fourth harmonic?

15.37 • A wire with mass 40.0 g is stretched so that its ends are tied down at points 80.0 cm apart. The wire vibrates in its fundamental mode with frequency 60.0 Hz and with an amplitude at the antinodes of 0.300 cm. (a) What is the speed of propagation of transverse waves in the wire? (b) Compute the tension in the wire. (c) Find the maximum transverse velocity and acceleration of particles in the wire.

15.38 • A piano tuner stretches a steel piano wire with a tension of 800 N. The steel wire is 0.400 m long and has a mass of 3.00 g. (a) What is the frequency of its fundamental mode of vibration? (b) What is the number of the highest harmonic that could be heard by a person who is capable of hearing frequencies up to 10,000 Hz?

15.39 • **CALC** A thin, taut string tied at both ends and oscillating in its third harmonic has its shape described by the equation $y(x,\ t)\ = (5.60\ \text{ cm})\ \sin\,[(0.0340\ \text{ rad}\,/\,\text{cm})x]\ \sin\,[(50.0\ \text{ rad}\,/\,\text{s})t]$, where the origin is at the left end of the string, the x-axis is along the string, and the y-axis is perpendicular to the string. (a) Draw a sketch that shows the standing-wave pattern. (b) Find the amplitude of the two traveling waves that make up this standing wave. (c) What is the length of the string? (d) Find the wavelength, frequency, period, and speed of the traveling waves. (e) Find the maximum transverse speed of a point on the string. (f) What would be the equation $y(x,\ t)$ for this string if it were vibrating in its eighth harmonic?

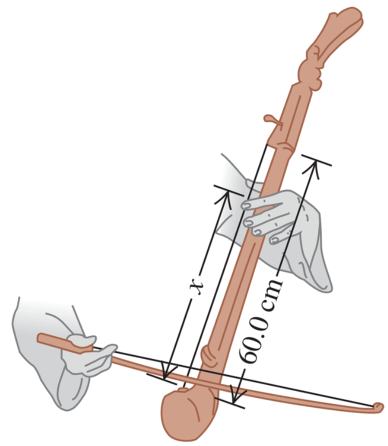
15.40 • The wave function of a standing wave is $y(x,\ t)\ =\ (4.44\ \text{mm})\sin[(32.5\ \text{rad/m})x]\ \ \sin[(754\ \text{rad/s})t]$. For the two traveling waves that make up this standing wave, find the (a) amplitude; (b) wavelength; (c) frequency; (d) wave speed; (e) wave functions. (f) From the information given, can you determine which harmonic this is? Explain.

15.41 • Standing waves are produced on a string that is held fixed at both ends. The tension in the string is kept constant. (a) For the second overtone standing wave the node-to-node distance is 8.00 cm. What is the length of the string? (b) What is the node-to-node distance for the fourth harmonic standing wave?

15.42 •• One string of a certain musical instrument is 75.0 cm long and has a mass of 8.75 g. It is being played in a room where the speed of sound is 344 m/s. (a) To what tension must you adjust the string so that, when vibrating in its second overtone, it produces sound of wavelength 0.765 m? (Assume that the breaking stress of the wire is very large and isn't exceeded.) (b) What frequency sound does this string produce in its fundamental mode of vibration?

15.43 • The portion of the string of a certain musical instrument between the bridge and upper end of the finger board (that part of the string that is free to vibrate) is 60.0 cm long, and this length of the string has mass 2.00 g. The string sounds an A_4 note (440 Hz) when played. (a) Where must the player put a finger (what distance x from the bridge) to play a D_5 note (587 Hz)? (See Fig. E15.43.) For both the A_4 and D_5 notes, the string vibrates in its fundamental mode. (b) Without retuning, is it possible to play a G_4 note (392 Hz) on this string? Why or why not?

Figure E15.43



15.44 • (a) A horizontal string tied at both ends is vibrating in its fundamental mode. The traveling waves have speed v, frequency f, amplitude A, and wavelength λ. Calculate the maximum transverse velocity and maximum transverse acceleration of points located at (i) $x = \lambda/2$, (ii) $x = \lambda/4$, and (iii) $x = \lambda/8$, from the left-hand end of the string. (b) At each of the points in part (a), what is the amplitude of the motion? (c) At each of the points in part (a), how much time does it take the string to go from its largest upward displacement to its largest downward displacement?

Problems

15.45 •• A sinusoidal wave with wavelength 0.400 m travels along a string. The maximum transverse speed of a point on the string is 3.00 m/s and the maximum transverse acceleration is $8.50 \times 10^4\ \text{m/s}^2$. What are the propagation speed v and the amplitude A of the wave?

15.46 • A transverse wave on a rope is given by

$$y(x,\ t) = (0.750\ \text{cm}) \cos \pi\left[(0.400\ \text{cm}^{-1})x + (250\ \text{s}^{-1})t\right]$$

(a) Find the amplitude, period, frequency, wavelength, and speed of propagation. (b) Sketch the shape of the rope at these values of t: 0, 0.0005 s, 0.0010 s. (c) Is the wave traveling in the $+x$- or $-x$-direction? (d) The mass per unit length of the rope is 0.0500 kg/m. Find the tension. (e) Find the average power of this wave.

15.47 • **CALC** A transverse sine wave with an amplitude of 2.50 mm and a wavelength of 1.80 m travels from left to right along a long, horizontal, stretched string with a speed of 36.0 m/s. Take the origin at the left end of the undisturbed string. At time $t = 0$ the left end of the string has its maximum upward displacement. (a) What are the frequency, angular frequency, and wave number of the wave? (b) What is the function $y(x,\ t)$ that describes the wave? (c) What is $y(t)$ for a particle at the left end of the string? (d) What is $y(t)$ for a particle 1.35 m to the right of the origin? (e) What is the maximum magnitude of transverse velocity of any particle of the string? (f) Find the transverse displacement and the transverse velocity of a particle 1.35 m to the right of the origin at time $t = 0.0625$ s.

15.48 •• **CP** A 1750 N irregular beam is hanging horizontally by its ends from the ceiling by two vertical wires (A and B), each 1.25 m long and weighing 0.290 N. The center of gravity of this beam is one-third of the way along the beam from the end where wire A is attached. If you pluck both strings at the same time at the beam, what is the time delay between the arrival of the two pulses at the ceiling? Which pulse arrives first? (Ignore the effect of the weight of the wires on the tension in the wires.)

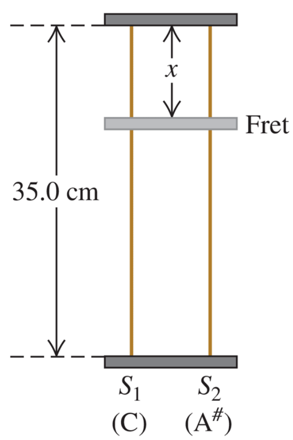
15.49 ••• **CP** One end of a light uniform rod is attached to a wall by a frictionless hinge. The rod is held in a horizontal position by a wire that runs from the other end of the rod to the wall. The wire has length 2.00 m and makes an angle of 30.0 $^\circ$ with the rod. A block with mass m is suspended by a light rope attached to the middle of the rod. The transverse fundamental standing wave on the wire has frequency f. The mass m is varied and for each value the frequency is measured. You plot f^{1} versus m and find that your data lie close to a straight line with slope $20.4\ \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$. What is the mass of the wire? Assume that the change in the length of the wire when the tension changes is small enough to neglect.

15.50 •• **Weightless Ant.** An ant with mass m is standing peacefully on top of a horizontal, stretched rope. The rope has mass per unit length μ and is under tension F. Without warning, Cousin Throckmorton starts a sinusoidal transverse wave of wavelength λ propagating along the rope. The motion of the rope is in a vertical plane. What minimum wave amplitude will make the ant become momentarily weightless? Assume that m is so small that the presence of the ant has no effect on the propagation of the wave.

15.51 •• You must determine the length of a long, thin wire that is suspended from the ceiling in the atrium of a tall building. A 2.00-cm-long piece of the wire is left over from its installation. Using an analytical balance, you determine that the mass of the spare piece is 14.5 μg. You then hang a 0.400 kg mass from the lower end of the long, suspended wire. When a small-amplitude transverse wave pulse is sent up that wire, sensors at both ends measure that it takes the wave pulse 26.7 ms to travel the length of the wire. (a) Use these measurements to calculate the length of the wire. Assume that the weight of the wire has a negligible effect on the speed of the transverse waves. (b) Discuss the accuracy of the approximation made in part (a).

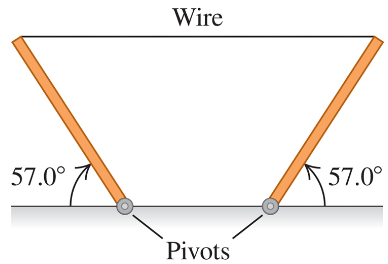
15.52 •• **Music.** You are designing a two-string instrument with metal strings 35.0 cm long, as shown in Fig. P15.52. Both strings are under the *same tension.* String S_1 has a mass of 8.00 g and produces the note middle C (frequency 262 Hz) in its fundamental mode. (a) What should be the tension in the string? (b) What should be the mass of string S_2 so that it will produce A-sharp (frequency 466 Hz) as its fundamental? (c) To extend the range of your instrument, you include a fret located just under the strings but not normally touching them. How far from the upper end should you put this fret so that when you press S_1 tightly against it, this string will produce C-sharp (frequency 277 Hz) in its fundamental? That is, what is x in the figure? (d) If you press S_2 against the fret, what frequency of sound will it produce in its fundamental?

Figure P15.52



15.53 ••• CP A 5.00 m, 0.732 kg wire is used to support two uniform 235 N posts of equal length (Fig. P15.53). Assume that the wire is essentially horizontal and that the speed of sound is 344 m/*s*. A strong wind is blowing, causing the wire to vibrate in its 5th overtone. What are the frequency and wavelength of the sound this wire produces?

Figure P15.53



15.54 ••• CP You are exploring a newly discovered planet. The radius of the planet is 7.20×10^7 m. You suspend a lead weight from the lower end of a light string that is 4.00 m long and has mass 0.0280 kg. You measure that it takes 0.0685 s for a transverse pulse to travel from the lower end to the upper end of the string. On the earth, for the same string and lead weight, it takes 0.0390 s for a transverse pulse to travel the length of the string. The weight of the string is small enough that you ignore its effect on the tension in the string. Assuming that the mass of the planet is distributed with spherical symmetry, what is its mass?

15.55 •• For a string stretched between two supports, two successive standing-wave frequencies are 525 Hz and 630 Hz. There are other standing-wave frequencies lower than 525 Hz and higher than 630 Hz. If the speed of transverse waves on the string is 384 m/s, what is the length of the string? Assume that the mass of the wire is small enough for its effect on the tension in the wire to be ignored.

15.56 •• Transverse standing waves are produced on a string that has length 0.800 m and is held fixed at each end. Each standing-wave pattern has a node at the fixed ends plus additional nodes along the length of the string. You measure the frequencies f_n for standing waves that have n of these nodes along their length. The tension in the string is kept constant. You plot n versus f_n and find that your data lie close to a straight line that has slope 7.30×10^{-3} s. What is the speed of transverse waves on the string?

15.57 • • • CP A 1.80-m-long uniform bar that weighs 638 N is suspended in a horizontal position by two vertical wires that are attached to the ceiling. One wire is aluminum and the other is copper. The aluminum wire is attached to the left-hand end of the bar, and the copper wire is attached 0.40 m to the left of the right-hand end. Each wire has length 0.600 m and a circular cross section with radius 0.280 mm. What is the fundamental frequency of transverse standing waves for each wire?

15.58 • • CALC A transverse standing wave is set up on a string that is held fixed at both ends. The amplitude of the standing wave at an antinode is 1.80 mm and the speed of propagation of transverse waves on the string is 260 m/s. The string extends along the x-axis, with one of the fixed ends at $x = 0$, so that there is a node at $x = 0$. The smallest value of x where there is an antinode is $x = 0.150$ m. (a) What is the maximum transverse speed of a point on the string at an antinode? (b) What is the maximum transverse speed of a point on the string at $x = 0.075$ m?

15.59 • • A horizontal wire is tied to supports at each end and vibrates in its second-overtone standing wave. The tension in the wire is 5.00 N, and the node-to-node distance in the standing wave is 6.28 cm. (a) What is the length of the wire? (b) A point at an antinode of the standing wave on the wire travels from its maximum upward displacement to its maximum downward displacement in 8.40 ms. What is the wire's mass?

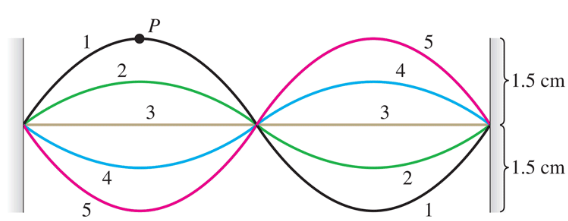
15.60 • • • CP A vertical, 1.20 m length of 18 gauge (diameter of 1.024 mm) copper wire has a 100.0 N ball hanging from it. (a)

What is the wavelength of the third harmonic for this wire? (b) A 500.0 N ball now *replaces* the original ball. What is the change in the wavelength of the third harmonic caused by replacing the light ball with the heavy one? (*Hint:* See Table 11.1 for Young's modulus.)

15.61 ••• A sinusoidal transverse wave travels on a string. The string has length 8.00 m and mass 6.00 g. The wave speed is 30.0 m/s, and the wavelength is 0.200 m. (a) If the wave is to have an average power of 50.0 W, what must be the amplitude of the wave? (b) For this same string, if the amplitude and wavelength are the same as in part (a), what is the average power for the wave if the tension is increased such that the wave speed is doubled?

15.62 •• A vibrating string 50.0 cm long is under a tension of 1.00 N. The results from five successive stroboscopic pictures are shown in Fig. P15.62. The strobe rate is set at 5000 flashes per minute, and observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between. (a) Find the period, frequency, and wavelength for the traveling waves on this string. (b) In what normal mode (harmonic) is the string vibrating? (c) What is the speed of the traveling waves on the string? (d) How fast is point P moving when the string is in (i) position 1 and (ii) position 3? (e) What is the mass of this string?

Figure P15.62



15.63 ••• **CP** A 1.005 m chain consists of small spherical beads, each with a mass of 1.00 g and a diameter of 5.00 mm, threaded on an elastic strand with negligible mass such that adjacent beads are separated by a center-to-center distance of 10.0 mm. There are beads at each end of the chain. The strand has a spring constant of 28.8 N/m. The chain is stretched horizontally on a frictionless tabletop to a length of 1.50 m, and the beads at both ends are fixed in place. (a) What is the linear mass density of the chain? (b) What is the tension in the chain? (c) With what speed would a pulse travel down the chain? (d) The chain is set vibrating and exhibits a standing-wave pattern with four antinodes. What is the frequency of this motion? (e) If the beads are numbered sequentially from 1 to 101, what are the numbers of the five beads that remain motionless? (f) The 13th bead has a maximum speed of 7.54 m/s. What is the amplitude of that bead's motion? (g) If $x_0 = 0$ corresponds to the center of the 1st bead and $x_{101} = 1.50$ m corresponds to the center of the 101st bead, what is the position x_n of the nth bead? (h) What is the maximum speed of the 30th bead?

15.64 •• A strong string of mass 3.00 g and length 2.20 m is tied to supports at each end and is vibrating in its fundamental mode. The maximum transverse speed of a point at the middle of the string is 9.00 9.00 m/s. The tension in the string is 330 N. (a) What is the amplitude of the standing wave at its antinode? (b) What is the magnitude of the maximum transverse acceleration of a point at the antinode?

15.65 •• A thin string 2.50 m in length is stretched with a tension of 90.0 N between two supports. When the string vibrates in its first overtone, a point at an antinode of the standing wave on the string has an amplitude of 3.50 cm and a maximum transverse speed of 28.0 m/s. (a) What is the string's mass?

(b) What is the magnitude of the maximum transverse acceleration of this point on the string?

15.66 ••• **CALC** A guitar string is vibrating in its fundamental mode, with nodes at each end. The length of the segment of the string that is free to vibrate is 0.386 m. The maximum transverse acceleration of a point at the middle of the segment is $8.40 \times 10^3 \text{ m/s}^2$ and the maximum transverse velocity is 3.80 m/s. (a) What is the amplitude of this standing wave? (b) What is the wave speed for the transverse traveling waves on this string?

15.67 ••• A uniform cylindrical steel wire, 55.0 cm long and 1.14 mm in diameter, is fixed at both ends. To what tension must it be adjusted so that, when vibrating in its first overtone, it produces the note D-sharp of frequency 311 Hz? Assume that it stretches an insignificant amount. (*Hint:* See Table 12.1.)

15.68 •• A string with both ends held fixed is vibrating in its third harmonic. The waves have a speed of 192 m/s and a frequency of 240 Hz. The amplitude of the standing wave at an antinode is 0.400 cm. (a) Calculate the amplitude at points on the string a distance of (i) 40.0 cm; (ii) 20.0 cm; and (iii) 10.0 cm from the left end of the string. (b) At each point in part (a), how much time does it take the string to go from its largest upward displacement to its largest downward displacement? (c) Calculate the maximum transverse velocity and the maximum transverse acceleration of the string at each of the points in part (a).

15.69 ••• **CP** A large rock that weighs 164.0 N is suspended from the lower end of a thin wire that is 3.00 m long. The density of the rock is 3200 kg/m^3. The mass of the wire is small enough that its effect on the tension in the wire can be ignored. The upper end of the wire is held fixed. When the rock is in air, the fundamental frequency for transverse standing waves on the wire is 42.0 Hz. When the rock is totally submerged in a liquid, with the top of the rock just below the surface, the fundamental frequency for the wire is 28.0 Hz. What is the density of the liquid?

15.70 •• (a) Estimate the tension you would need to apply to a standard small rubber band to stretch it between your fingers to a doubled length of 10 cm. (b) Such a rubber band has a typical mass of 0.10 g. Use this value to estimate the mass density of the stretched rubber band in SI units. (c) Use your estimated values to determine the expected frequency of vibration when the string is plucked. (d) Is this result realistic?

15.71 •• **Tuning an Instrument.** A musician tunes the C-string of her instrument to a fundamental frequency of 65.4 Hz. The vibrating portion of the string is 0.600 m long and has a mass of 14.4 g. (a) With what tension must the musician stretch it? (b) What percent increase in tension is needed to increase the frequency from 65.4 Hz to 73.4 Hz, corresponding to a rise in pitch from C to D?

15.72 • **Holding Up Under Stress.** A string or rope will break apart if it is placed under too much tensile stress [see Eq. (11.8)]. Thicker ropes can withstand more tension without breaking because the thicker the rope, the greater the cross-sectional area and the smaller the stress. One type of steel has density $7800\ \mathrm{kg/m^3}$ and will break if the tensile stress exceeds $7.0 \times 10^8\ \mathrm{N/m^2}$. You want to make a guitar string from 4.0 g of this type of steel. In use, the guitar string must be able to withstand a tension of 900 N without breaking. Your job is to determine (a) the maximum length and minimum radius the string can have; (b) the highest possible fundamental frequency of standing waves on this string, if the entire length of the string is free to vibrate.

15.73 •• DATA In your physics lab, an oscillator is attached to one end of a horizontal string. The other end of the string passes over a frictionless pulley. You suspend a mass M from the free end of the string, producing tension Mg in the string. The oscillator produces transverse waves of frequency f on the string. You don't vary this frequency during the experiment, but you try strings with three different linear mass densities μ. You also keep a fixed distance between the end of the string where the oscillator is attached and the point where the string is in contact with the pulley's rim. To produce standing waves on the string, you vary M; then you measure the node-to-node distance d for each standing-wave pattern and obtain the following data:

String	A	A	B	B	C
μ (g/cm)	0.0260	0.0260	0.0374	0.0374	0.0482
M (g)	559	249	365	207	262
d (cm)	48.1	31.9	32.0	24.2	23.8

(a) Explain why you obtain only certain values of d. (b) Graph μd^2 (in $\mathrm{kg \cdot m}$) versus M (in kg). Explain why the data plotted this way should fall close to a straight line. (c) Use the slope of the best straight-line fit to the data to determine the frequency f of the waves produced on the string by the oscillator. Take $g = 9.80\ \mathrm{m/s^2}$. (d) For string A ($\mu = 0.0260\ \mathrm{g/cm}$), what value of M (in grams) would be required to produce a standing wave with a node-to-node distance of 24.0 cm? Use the value of f that you calculated in part (c).

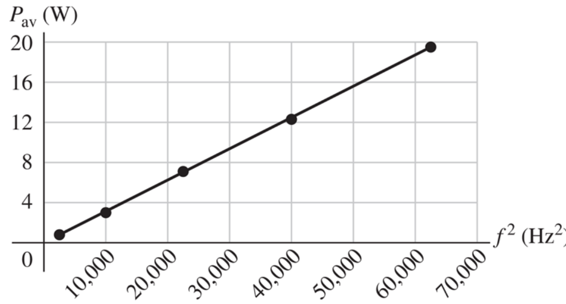
15.74 •• DATA *Scale length* is the length of the part of a guitar string that is free to vibrate. A standard value of scale length for an acoustic guitar is 25.5 in. The frequency of the fundamental standing wave on a string is determined by the string's scale length, tension, and linear mass density. The standard frequencies f to which the strings of a six-string guitar are tuned are given in the table:

String	E2	A2	D3	G3	B3	E4
f **(Hz)**	82.4	110.0	146.8	196.0	246.9	329.6

Assume that a typical value of the tension of a guitar string is 78.0 N (although tension varies somewhat for different strings). (a) Calculate the linear mass density μ(in g/cm) for the E2, G3, and E4 strings. (b) Just before your band is going to perform, your G3 string breaks. The only replacement string you have is an E2. If your strings have the linear mass densities calculated in part (a), what must be the tension in the replacement string to bring its fundamental frequency to the G3 value of 196.0 Hz?

15.75 •• DATA You are measuring the frequency dependence of the average power P_{av} transmitted by traveling waves on a wire. In your experiment you use a wire with linear mass density 3.5 g/m. For a transverse wave on the wire with amplitude 4.0 mm, you measure P_{av} (in watts) as a function of the frequency f of the wave (in Hz). You have chosen to plot P_{av} as a function of f^2 (Fig. P15.75). (a) Explain why values of P_{av} plotted versus f^2 should be well fit by a straight line. (b) Use the slope of the straight-line fit to the data shown in Fig. P15.75 to calculate the speed of the waves. (c) What angular frequency ω would result in $P_{av} = 10.0$ W?

Figure P15.75

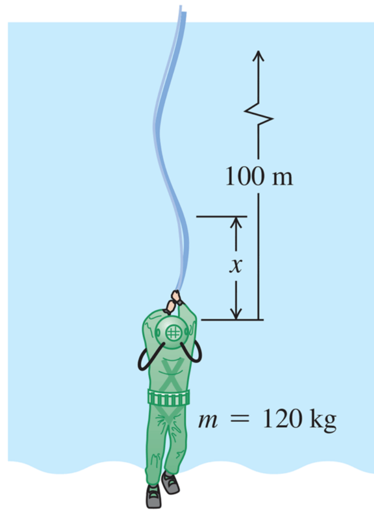


Challenge PROBLEMS

15.76 •• A rectangular neoprene sheet has width $W = 1.00$ m and length $L = 4.00$ m. The two shorter edges are affixed to rigid steel bars that are used to stretch the sheet taut and horizontal. The force applied to either end of the sheet is $F = 81.0$ N. The sheet has a total mass $M = 4.00$ kg. The left edge of the sheet is wiggled vertically in a uniform sinusoidal motion with amplitude $A = 10.0$ cm and frequency $f = 1.00$ Hz. This sends waves spanning the width of the sheet rippling from left to right. The right side of the sheet moves upward and downward freely as these waves complete their traversal. (a) Use a two-dimensional generalization of the discussion in Section 15.4 to derive an expression for the velocity with which the waves move along the sheet in terms of generic values of W, L, F, M, f, and A. What is the value of this speed for the specified choices of these parameters? (b) If the positive x-axis is oriented rightward and the steel bars are parallel to the y-axis, the height of the sheet may be characterized as $z(x,\ y) = A\sin(kx - \omega t)$. What is the value of the wave number k? (c) Write down an expression with generic parameters for the rate of rightward energy transfer by the slice of sheet at a given value of x at generic time t. (d) The power at $x = 0$ is supplied by the agent wiggling the left bar upward and downward. How much energy is supplied each second by that agent? Express your answer in terms of generic parameters and also as a specific energy for the given parameters.

15.77 ••• **CP CALC** A deep-sea diver is suspended beneath the surface of Loch Ness by a 100-m-long cable that is attached to a boat on the surface (Fig. P15.77). The diver and his suit have a total mass of 120 kg and a volume of 0.0800 m^3. The cable has a diameter of 2.00 cm and a linear mass density of $\mu = 1.10$ kg/m. The diver thinks he sees something moving in the murky depths and jerks the end of the cable back and forth to send transverse waves up the cable as a signal to his companions in the boat. (a) What is the tension in the cable at its lower end, where it is attached to the diver? Do not forget to include the buoyant force that the water (density 1000 kg/m^3) exerts on him. (b) Calculate the tension in the cable a distance x above the diver. In your calculation, include the buoyant force on the cable. (c) The speed of transverse waves on the cable is given by $v = \sqrt{F/\mu}$ [Eq. (15.14)]. The speed therefore varies along the cable, since the tension is not constant. (This expression ignores the damping force that the water exerts on the moving cable.) Integrate to find the time required for the first signal to reach the surface.

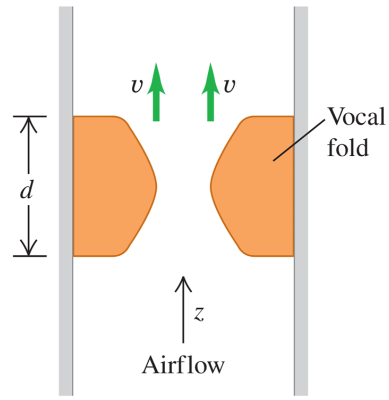
Figure P15.77



MCAT-Style Passage Problems

BIO Waves on Vocal Folds. In the larynx, sound is produced by the vibration of the *vocal folds* (also called “vocal cords”). The accompanying figure is a cross section of the vocal tract at one instant in time. Air flows upward (in the $+z$-direction) through the vocal tract, causing a transverse wave to propagate vertically upward along the surface of the vocal folds. In a typical adult male, the thickness of the vocal folds in the direction of airflow is $d = 2.0$ mm. High-speed photography shows that for a frequency of vibration of $f = 125$ Hz, the wave along the surface of the vocal folds travels upward at a speed of $v = 375$ cm$/s$. Use t for time, z for displacement in the $+z$-direction, and λ for wavelength.

Figure P15.79

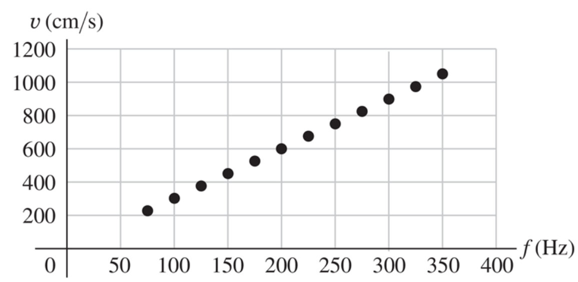


15.78 What is the wavelength of the wave that travels on the surface of the vocal folds when they are vibrating at frequency f? (a) 2.0 mm; (b) 3.3 mm; (c) 0.50 cm; (d) 3.0 cm.

15.79 Which of these is a possible mathematical description of the wave in Problem 15.78? (a) $A \sin[2\pi f\,(t + z/v)]$; (b) $A \sin[2\pi f\,(t - z/v)]$; (c) $A \sin(2\pi ft)\cos(2\pi z/\lambda)$; (d) $A \sin(2\pi ft)\ \sin(2\pi z/\lambda)$.

15.80 The wave speed is measured for different vibration frequencies. A graph of the wave speed as a function of frequency (Fig. P15.80) indicates that as the frequency increases, the wavelength (a) increases; (b) decreases; (c) doesn't change; (d) becomes undefined.

Figure P15.80



Answers: Mechanical Waves

Chapter Opening Question ?

(iii) The power of a mechanical wave depends on both its amplitude and its frequency [see Eq. (15.25)].

Test Your Understanding

15.1 **(i)** The "wave" travels horizontally from one spectator to the next along each row of the stadium, but the displacement of each spectator is vertically upward. Since the displacement is perpendicular to the direction in which the wave travels, the wave is transverse.

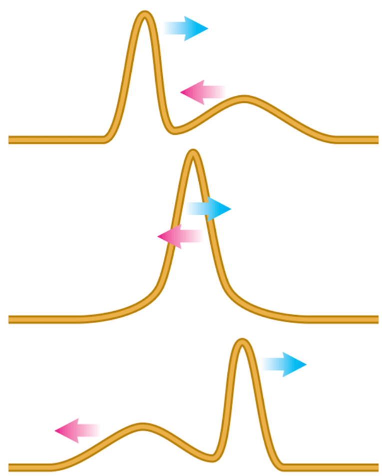
15.2 **(iv)** The speed of waves on a string, v, does not depend on the wavelength. We can rewrite the relationship $v = \lambda f$ as $f = v/\lambda$, which tells us that if the wavelength λ doubles, the frequency f becomes one-half as great.

15.3 **(a)** $\frac{2}{8}T$, **(b)** $\frac{4}{8}T$, **(c)** $\frac{5}{8}T$ Since the wave is sinusoidal, each point on the string oscillates in simple harmonic motion (SHM). Hence we can apply all of the ideas from Chapter 14 about SHM to the wave depicted in Fig. 15.8. (a) A particle in SHM has its maximum speed when it is passing through the equilibrium position ($y = 0$ in Fig. 15.8). The particle at point A is moving upward through this position at $t = \frac{2}{8}T$. In vertical SHM the greatest *upward* acceleration occurs when a particle is at its maximum *downward* displacement. This occurs for the particle at point B at $t = \frac{4}{8}T$. A particle in vertical SHM has a *downward* acceleration when its displacement is *upward*. The particle at C has an upward displacement and is moving downward at $t = \frac{5}{8}T$.

15.4 **(ii)** The relationship $v = \sqrt{F/\mu}$ [Eq. (15.14)] says that the wave speed is greatest on the string with the smallest linear mass density. This is the thinnest string, which has the smallest amount of mass m and hence the smallest linear mass density $\mu = m/L$ (all strings are the same length).

15.5 **(iii), (iv), (ii), (i)** Equation (15.25) says that the average power in a sinusoidal wave on a string is $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$. All four strings are identical, so all have the same mass, length, and linear mass density μ. The frequency f is the same for each wave, as is the angular frequency $\omega = 2\pi f$. Hence the average wave power for each string is proportional to the square root of the string tension F and the square of the amplitude A. Compared to string (i), the average power in each string is (ii) $\sqrt{4} = 2$ times greater; (iii) $4^2 = 16$ times greater; and (iv) $\sqrt{2}\,(2)^2 = 4\sqrt{2}$ times greater.

15.6



15.7 **Yes, yes** Doubling the frequency makes the wavelength half as large. Hence the spacing between nodes (equal to $\lambda/2$) is also half as large. There are nodes at all of the previous positions, but there is also a new node between every pair of old nodes.

15.8 $n = 1,\ 3,\ 5, \ldots$ When you touch the string at its center, you are producing a node at the center. Hence only standing waves with a node at $x = L/2$ are allowed. From Figure 15.26 you can see that the normal modes $n = 1,\ 3,\ 5,\ \ldots$cannot be present.

Key Example Variation Problems

VP15.3.1 **a.** 0.196 Hz

b. 5.98 m/s

c. 1.23 rad/s

d. 0.206 m^{-1}

VP15.3.2 **a.** $T = 8.00 \times 10^{-3}\text{s}$, $\lambda = 1.96\text{ m}$

b. $f = 81.7\text{ Hz}$, $\omega = 513\text{ rad/s}$

VP15.3.3 **a.** 111 m/s

b. 4.44 m

c. $y(x = +2.50\text{ m}, t) = (5.00\text{ mm})$

$\cos\ [(\ 3.54\text{rad}) - (157\text{ rad/s})t]$

VP15.3.4 **a.** 36.0 m/s

b. 0.0193 kg/m

VP15.5.1 **a.** $10.9\text{ rad}/s$

b. 0.294 W

VP15.5.2 $2.50 \times 10^{-3}\text{ m}$

VP15.5.3 **a.** 0.159 W/m^2

b. 3.76 m

VP15.5.4 $6.94 \times 10^{-12}\text{ J}$

VP15.8.1 **a.** 64.0 m/s

b. 2.25 m/s

c. $3.62 \times 10^3\text{ m}/s^2$

VP15.8.2 **a.** 251 m/s

b. 145 N

VP15.8.3 **a.** 1.20 m

b. 80.0 Hz

c. $1.90 \times 10^{-2}\text{kg/m}$

VP15.8.4 **a.** 76.7 N

b. 1.34 m

Bridging Problem

(a) $F(r) = \dfrac{m\omega^2}{2L}\left(L^2 - r^2\right)$

(b) $v(r) = \omega\sqrt{\dfrac{L^2 - r^2}{2}}$

(c) $\dfrac{\pi}{\omega\sqrt{2}}$

Chapter 16
Sound and Hearing

?The sound from a horn travels more slowly on a cold winter day high in the mountains than on a warm summer day at sea level. This is because at high elevations in winter, the air has lower (i) pressure; (ii) density; (iii) humidity; (iv) temperature; (v) mass per mole.

Learning Outcomes

In this chapter, you'll learn...

16.1 How to describe a sound wave in terms of either particle displacements or pressure fluctuations.

16.2 How to calculate the speed of sound waves in different materials.

16.3 How to calculate the intensity of a sound wave.

16.4 What determines the particular frequencies of sound produced by an organ or a flute.

16.5 How resonance occurs in musical instruments.

16.6 What happens when sound waves from different sources overlap.

16.7 How to describe what happens when two sound waves of slightly different frequencies are combined.

16.8 Why the pitch of a siren changes as it moves past you.

16.9 Why an airplane flying faster than sound produces a shock wave.

You'll need to review...

6.4 Power.

8.1 The impulse–momentum theorem.

11.4 Bulk modulus and Young's modulus.

12.2 Gauge pressure and absolute pressure.

14.8 Forced oscillations and resonance.

15.1, 15.2, 15.3, 15.4, 15.5, 15.6, 15.7, 15.8 Mechanical waves.

Of all the mechanical waves that occur in nature, the most important in our everyday lives are longitudinal waves in a medium—usually air—called *sound* waves. The reason is that the human ear is tremendously sensitive and can detect sound waves even of very low intensity. The ability to hear an unseen nocturnal predator was essential to the survival of our ancestors, so it is no exaggeration to say that we humans owe our existence to our highly evolved sense of hearing.

In Chapter 15 we described mechanical waves primarily in terms of displacement; however, because the ear is primarily sensitive to changes in pressure, it's often more appropriate to describe sound waves in terms of *pressure* fluctuations. We'll study the relationships among displacement, pressure fluctuation, and intensity and the connections between these quantities and human sound perception.

When a source of sound or a listener moves through the air, the listener may hear a frequency different from the one emitted by the source. This is the Doppler effect, which has important applications in medicine and technology.

16.1 Sound Waves

The most general definition of **sound** is a longitudinal wave in a medium. Our main concern is with sound waves in air, but sound can travel through any gas, liquid, or solid. You may be all too familiar with the propagation of sound through a solid if your neighbor's stereo speakers are right next to your wall.

The simplest sound waves are sinusoidal waves, which have definite frequency, amplitude, and wavelength. The human ear is sensitive to waves in the frequency range from about 20 to 20,000 Hz, called the **audible range**, but we also use the term "sound" for similar waves with frequencies above **(ultrasonic)** and below **(infrasonic)** the range of human hearing.

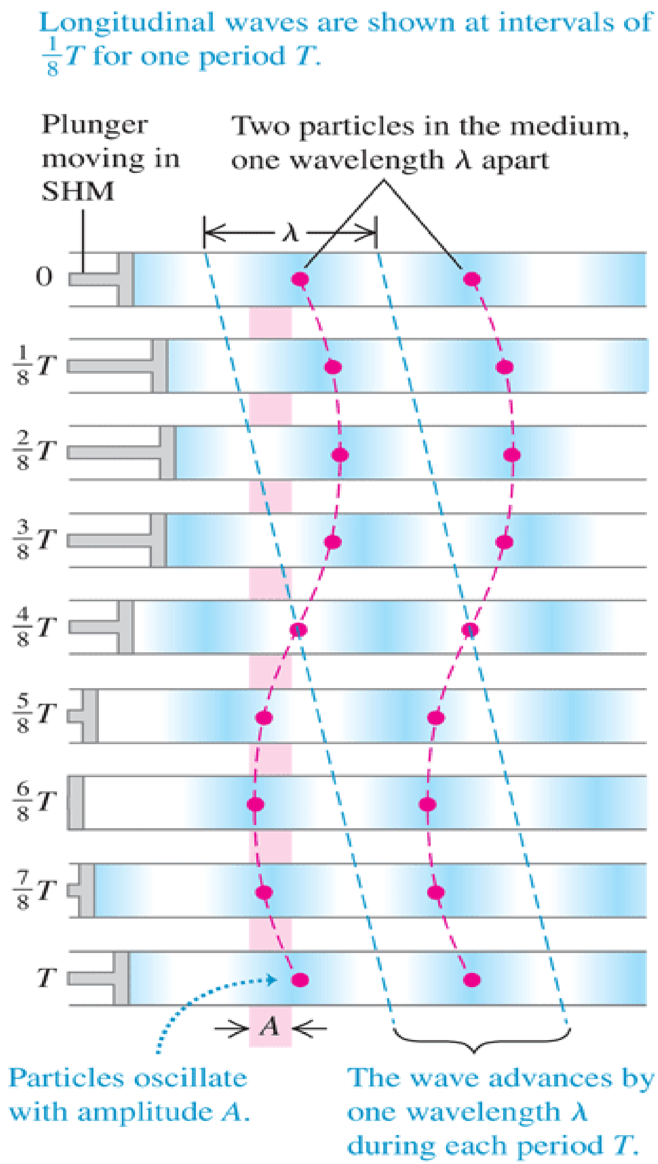
Sound waves usually travel outward in all directions from the source of sound, with an amplitude that depends on the direction and distance from the source. We'll return to this point in the next section. For now, we concentrate on the idealized case of a sound wave that propagates in the positive x-direction only. As we discussed in Section 15.3, for such a wave, the wave function $y(x,\ t)$ gives the instantaneous displacement y of a particle in the medium at position x at time t. If the wave is sinusoidal, we can express it by using Eq. (15.7):

(16.1)

$$y(x,\ t) = A\cos(kx - \omega t) \quad \begin{array}{l}\text{(sound wave propagating}\\ \text{in the } +x \text{ direction)}\end{array}$$

In a longitudinal wave the displacements are *parallel* to the direction of travel of the wave, so distances x and y are measured parallel to each other, not perpendicular as in a transverse wave. The amplitude A is the maximum displacement of a particle in the medium from its equilibrium position (Fig. 16.1). Hence A is also called the **displacement amplitude.**

Figure 16.1



A sinusoidal longitudinal wave traveling to the right in a fluid. (Compare to Fig. 15.7.)

Sound Waves as Pressure Fluctuations

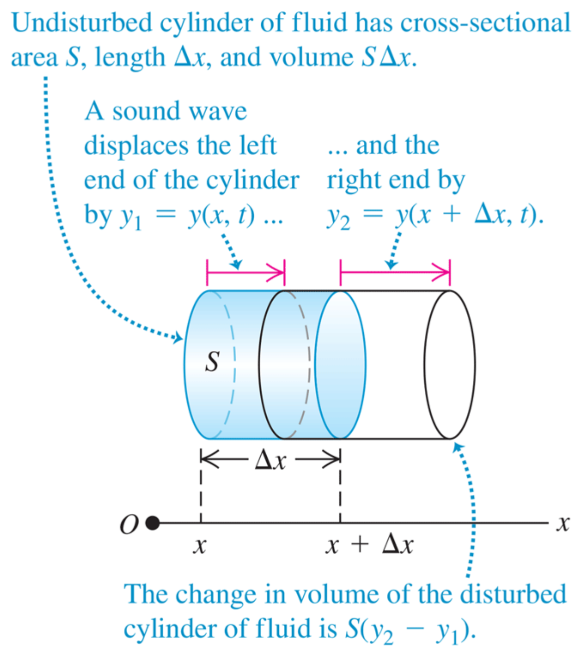
We can also describe sound waves in terms of variations of *pressure* at various points. In a sinusoidal sound wave in air, the pressure fluctuates sinusoidally above and below atmospheric pressure p_a with the same frequency as the motions of the air particles. The human ear operates by sensing such pressure variations. A sound wave entering the ear canal exerts a fluctuating pressure on one side of the eardrum; the air on the other side of the eardrum, vented to the outside by the Eustachian tube, is at atmospheric pressure. The pressure difference on the two sides of the eardrum sets it into motion. Microphones and similar devices also usually sense pressure differences, not displacements.

Let $p(x,\ t)$ be the instantaneous pressure fluctuation in a sound wave at any point x at time t. That is, $p(x,\ t)$ is the amount by which the pressure *differs* from normal atmospheric pressure p_a. Think of $p(x,\ t)$ as the *gauge pressure* defined in Section 12.2; it can be either positive or negative. The *absolute* pressure at a point is then $p_a + p(x,\ t)$.

To see the connection between the pressure fluctuation $p(x,\ t)$ and the displacement $y(x,\ t)$ in a sound wave propagating in the $+x$-direction, consider an imaginary cylinder of a wave medium (gas, liquid, or solid) with cross-sectional area S and its axis along the direction of propagation (Fig. 16.2). When no sound wave is present, the cylinder has length Δx and volume $V = S\,\Delta x$, as shown by the shaded volume in Fig. 16.2. When a wave is present, at time t the end of the cylinder that is initially at x is displaced by $y_1 = y(x,\ t)$, and the end that is initially at $x + \Delta x$ is displaced by $y_2 = y(x + \Delta x,\ t)$; this is shown by the red lines. If $y_2 > y_1$, as shown in Fig. 16.2, the cylinder's volume has increased, which causes a decrease in pressure. If $y_2 < y_1$, the cylinder's volume has decreased and the pressure has increased. If $y_2 = y_1$, the cylinder is simply shifted to the left or right; there is no volume change and no pressure fluctuation.

The pressure fluctuation depends on the *difference* between the displacements at neighboring points in the medium.

Figure 16.2



As a sound wave propagates along the x-axis, the left and right ends undergo different displacements y_1 and y_2 .

Quantitatively, the change in volume ΔV of the cylinder is

$$\Delta V = S\,(y_2 - y_1) = S[y\,(x + \Delta x,\ t) - y\,(x,\ t)]$$

In the limit as $\Delta x \to 0$, the fractional change in volume dV/V (volume change divided by original volume) is

(16.2)

$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S[y\,(x + \Delta x,\ t) - y\,(x,\ t)]}{S\ \ \Delta x} = \frac{\partial y\,(x,\ t)}{\partial x}$$

The fractional volume change is related to the pressure fluctuation by the bulk modulus B, which by definition [Eq. (11.3)] is $B = -p(x,\ t)/(dV/V)$ (see Section 11.4). Solving for $p(x,\ t)$, we have

(16.3)

$$p(x,\ t) = -B\frac{\partial y(x,\ t)}{\partial x}$$

The negative sign arises because when $\partial y(x,\ t)/\partial x$ is positive, the displacement is greater at $x + \Delta x$ than at x, corresponding to an increase in volume, a decrease in pressure, and a negative pressure fluctuation.

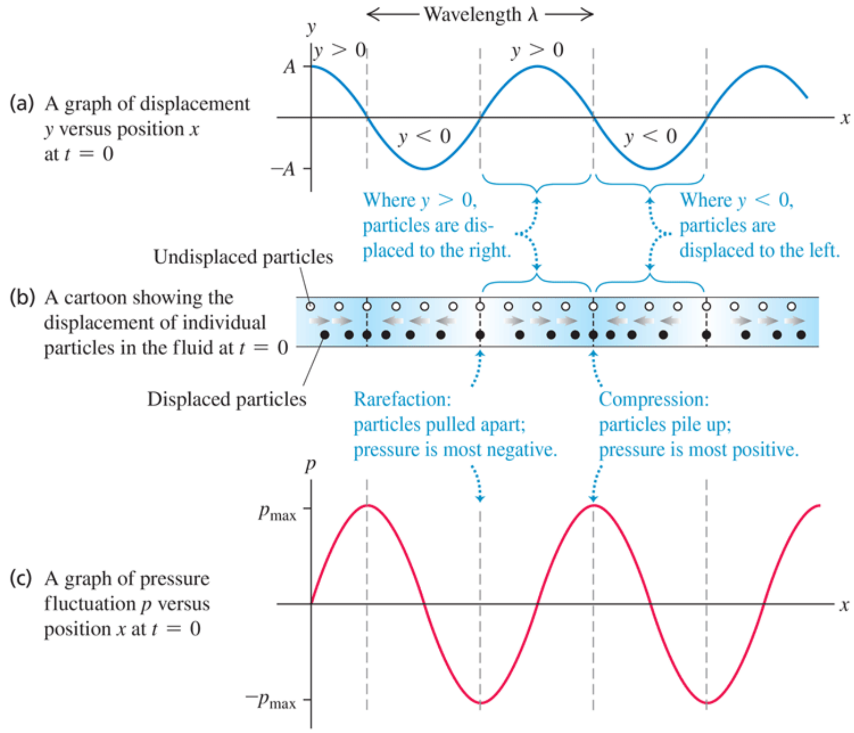
When we evaluate $\partial y(x,\ t)/\partial x$ for the sinusoidal wave of Eq. (16.1), we find

(16.4)

$$p(x,\ t) = BkA\sin(kx - \omega t)$$

Figure 16.3 shows $y(x,\ t)$ and $p(x,\ t)$ for a sinusoidal sound wave at $t = 0$. It also shows how individual particles of the wave are displaced at this time. While $y(x,\ t)$ and $p(x,\ t)$ describe the same wave, these two functions are one-quarter cycle out of phase: At any time, the displacement is greatest where the pressure fluctuation is zero, and vice versa. In particular, note that the compressions (points of greatest pressure and density) and rarefactions (points of lowest pressure and density) are points of *zero* displacement.

Figure 16.3



Three ways to describe a sound wave.

Equation (16.4) shows that the quantity BkA represents the maximum pressure fluctuation. We call this the **pressure amplitude,** denoted by $p_{\max}$:

(16.5)

Bulk modulus of medium

Pressure amplitude, sinusoidal sound wave

$$p_{\max} = BkA \qquad (16.5)$$

Displacement amplitude

Wave number $= 2\pi/\lambda$

Waves of shorter wavelength λ (larger wave number $k = 2\pi/\lambda$) have greater pressure variations for a given displacement amplitude because the maxima and minima are squeezed closer together. A medium with a large value of bulk modulus B is less compressible and so requires a greater pressure amplitude for a given volume change (that is, a given displacement amplitude).

CAUTION Graphs of a sound wave The graphs in Fig. 16.3 show the wave at only *one* instant of time. Because the wave is propagating in the $+x$-direction, as time goes by the wave patterns described by the functions $y(x,\ t)$ and $p(x,\ t)$ move to the right at the wave speed $v = \omega/k$. The particles, by contrast, simply oscillate back and forth in simple harmonic motion as shown in Fig. 16.1.

Example 16.1 Amplitude of a sound wave in air

In a sinusoidal sound wave of moderate loudness, the maximum pressure variations are about 3.0×10^{-2} Pa above and below atmospheric pressure. Find the corresponding maximum displacement if the frequency is 1000 Hz. In air at normal atmospheric pressure and density, the speed of sound is 344 m/s and the bulk modulus is 1.42×10^{5} Pa.

IDENTIFY and SET UP This problem involves the relationship between two ways of describing a sound wave: in terms of displacement and in terms of pressure. The target variable is the displacement amplitude A. We are given the pressure amplitude p_{max}, wave speed v, frequency f, and bulk modulus B. Equation (16.5) relates the target variable A to p_{max}. We use $\omega = vk$ [Eq. (15.6)] to determine the wave number k from v and the angular frequency $\omega = 2\pi f$.

EXECUTE From Eq. (15.6),

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi\ \text{rad})(1000\ \text{Hz})}{344\ \text{m/s}} = 18.3\ \text{rad/m}$$

Then from Eq. (16.5), the maximum displacement is

$$A = \frac{p_{\max}}{Bk} = \frac{3.0 \times 10^{-2}\ \text{Pa}}{(1.42 \times 10^{5}\ \text{Pa})(18.3\ \text{rad/m})} = 1.2 \times 10^{-8}\ \text{m}$$

EVALUATE This displacement amplitude is only about $\frac{1}{100}$ the size of a human cell. The ear actually senses pressure fluctuations; it detects these minuscule displacements only indirectly.

KEY CONCEPT

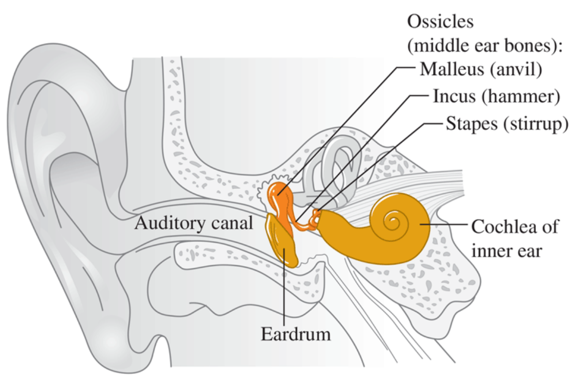
In a sound wave, the pressure amplitude (maximum pressure fluctuation) and displacement amplitude (maximum displacement of a particle in the medium) are proportional to each other. The proportionality constant depends on the wavelength of the sound and the bulk modulus of the medium.

Video Tutor Solution: **Example 16.1**

Example 16.2 Amplitude of a sound wave in the inner ear

A sound wave that enters the human ear sets the eardrum into oscillation, which in turn causes oscillation of the *ossicles*, a chain of three tiny bones in the middle ear (Fig. 16.4). The ossicles transmit this oscillation to the fluid (mostly water) in the inner ear; there the fluid motion disturbs hair cells that send nerve impulses to the brain with information about the sound. The area of the moving part of the eardrum is about 43 mm^2, and that of the stapes (the smallest of the ossicles) where it connects to the inner ear is about 3.2 mm^2. For the sound in Example 16.1, determine (a) the pressure amplitude and (b) the displacement amplitude of the wave in the fluid of the inner ear, in which the speed of sound is $1500\ \text{m/s}$.

Figure 16.4



The anatomy of the human ear. The middle ear is the size of a small marble; the ossicles (incus, malleus, and stapes) are the smallest bones in the human body.

IDENTIFY and SET UP Although the sound wave here travels in liquid rather than air, the same principles and relationships among the properties of the wave apply. We can ignore the mass of the tiny ossicles (about $58\ \text{mg} = 5.8 \times 10^{-5}\ \text{kg}$), so the force they exert on the inner-ear fluid is the same as that exerted on the eardrum and ossicles by the incident sound wave. (In Chapters 4 and 5 we used the same idea to say that the tension is the same at either end of a massless rope.) Hence the pressure amplitude in the inner ear, $p_{\text{max (inner ear)}}$, is greater than in the outside air, $p_{\text{max (air)}}$, because the same force is exerted on a smaller area (the area of the stapes versus the area of the eardrum). Given $p_{\text{max (inner ear)}}$, we find the displacement amplitude $A_{\text{inner ear}}$ from Eq. (16.5).

EXECUTE (a) From the area of the eardrum and the pressure amplitude in air found in Example 16.1, the maximum force exerted by the sound wave in air on the eardrum is $F_{\text{max}} = p_{\text{max (air)}} S_{\text{eardrum}}$. Then

$$\begin{aligned} p_{\text{max (inner ear)}} &= \frac{F_{\text{max}}}{S_{\text{stapes}}} = p_{\text{max (air)}} \frac{S_{\text{eardrum}}}{S_{\text{stapes}}} \\ &= (3.0 \times 10^{-2}\ \text{Pa}) \frac{43\ \text{mm}^2}{3.2\ \text{mm}^2} = 0.40\ \text{Pa} \end{aligned}$$

(b) To find the maximum displacement $A_{\text{inner ear}}$, we use $A = p_{\text{max}}/Bk$ as in Example 16.1. The inner-ear fluid is mostly water, which has a much greater bulk modulus B than air. From Table 11.2 the compressibility of water (unfortunately also called k) is $45.8 \times 10^{-11}\ \text{Pa}^{-1}$, so $B_{\text{fluid}} = 1/(45.8 \times 10^{-11}\ \text{Pa}^{-1}) = 2.18 \times 10^9\ \text{Pa}$.

The wave in the inner ear has the same angular frequency ω as the wave in the air because the air, eardrum, ossicles, and inner-ear fluid all oscillate together (see Example 15.8 in Section 15.8). But because the wave speed v is greater in the inner ear than in the air (1500 m/s versus 344 m/s), the wave number $k = \omega/v$ is smaller. Using the value of ω from Example 16.1, we find

$$k_{\text{inner ear}} = \frac{\omega}{v_{\text{inner ear}}} = \frac{(2\pi\ \text{rad})(1000\ \text{Hz})}{1500\ \text{m/s}} = 4.2\ \text{rad/m}$$

Putting everything together, we have

$$A_{\text{inner ear}} = \frac{p_{\text{max (inner ear)}}}{B_{\text{fluid}}\,k_{\text{inner ear}}} = \frac{0.40\ \text{Pa}}{(2.18 \times 10^{9}\ \text{Pa})(4.2\ \text{rad/m})}$$
$$= 4.4 \times 10^{-11}\ \text{m}$$

EVALUATE In part (a) we see that the ossicles increase the pressure amplitude by a factor of $(43\ \text{mm}^2)/(3.2\ \text{mm}^2) = 13$. This amplification helps give the human ear its great sensitivity.

The displacement amplitude in the inner ear is even smaller than in the air. But *pressure* variations within the inner-ear fluid are what set the hair cells into motion, so what matters is that the pressure amplitude is larger in the inner ear than in the air.

KEY CONCEPT

When a sound wave travels from one medium into a different medium, the wave frequency and angular frequency remain the same. The wave number and wavelength can change, however, as can the pressure amplitude and displacement amplitude.

Video Tutor Solution: **Example 16.2**

Perception of Sound Waves

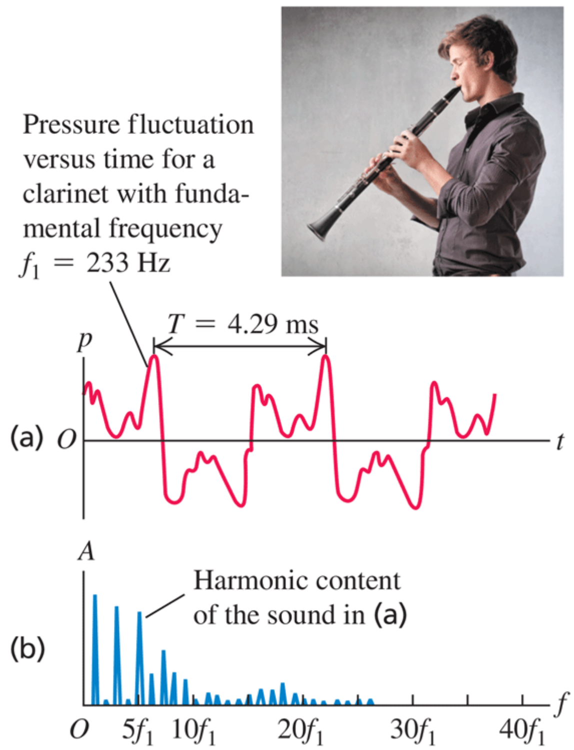
The physical characteristics of a sound wave are directly related to the perception of that sound by a listener. For a given frequency, the greater the pressure amplitude of a sinusoidal sound wave, the greater the perceived **loudness.** The relationship between pressure amplitude and loudness is not a simple one, and it varies from one person to another. One important factor is that the ear is not equally sensitive to all frequencies in the audible range. A sound at one frequency may seem louder than one of equal pressure amplitude at a different frequency. At 1000 Hz the minimum pressure amplitude that can be perceived with normal hearing is about 3×10^{-5} Pa; to produce the same loudness at 200 Hz or 15,000 Hz requires about 3×10^{-4} Pa. Perceived loudness also depends on the health of the ear. Age usually brings a loss of sensitivity at high frequencies.

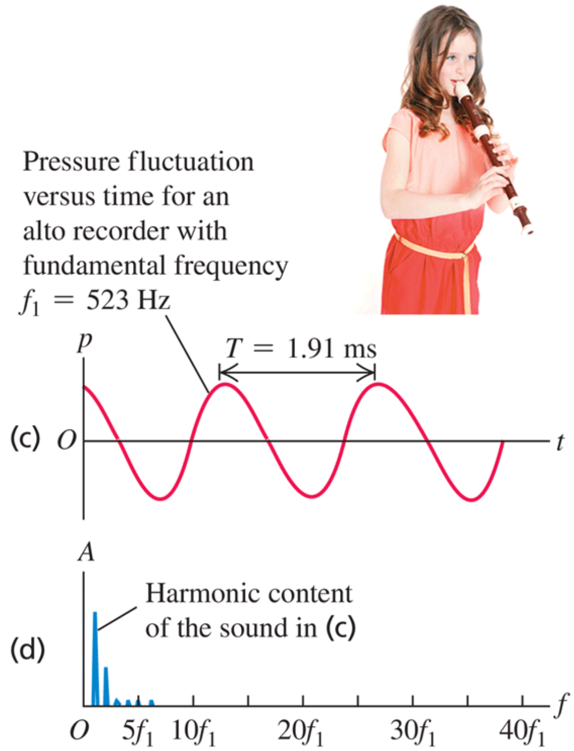
The frequency of a sound wave is the primary factor in determining the **pitch** of a sound, the quality that lets us classify the sound as "high" or "low." The higher the frequency of a sound (within the audible range), the higher the pitch that a listener will perceive. Pressure amplitude also plays a role in determining pitch. When a listener compares two sinusoidal sound waves with the same frequency but different pressure amplitudes, the one with the greater pressure amplitude is usually perceived as louder but also as slightly lower in pitch.

Musical sounds have wave functions that are more complicated than a simple sine function. Figure 16.5a shows the pressure fluctuation in the sound wave produced by a clarinet. The pattern is so complex because the column of air in a wind instrument like a clarinet vibrates at a fundamental frequency and at many harmonics at the same time. (In Section 15.8, we described this same behavior for a string that has been plucked, bowed, or struck. We'll examine the physics of wind instruments in Section 16.4.) The sound wave produced in the surrounding air has a similar amount of each harmonic—that is, a similar *harmonic content*.

Figure 16.5b shows the harmonic content of the sound of a clarinet. The mathematical process of translating a pressure-time graph like Fig. 16.5a into a graph of harmonic content like Fig. 16.5b is called *Fourier analysis*.

Figure 16.5





Different representations of the sound of (a), (b) a clarinet and (c), (d) an alto recorder. (Graphs adapted from R.E. Berg and D.G. Stork, *The Physics of Sound*, Prentice-Hall, 1982.)

Two tones produced by different instruments might have the same fundamental frequency (and thus the same pitch) but sound different because of different harmonic content. The difference in sound is called **timbre** and is often described in subjective terms such as reedy, mellow, and tinny. A tone that is rich in harmonics, like the clarinet tone in Figs. 16.5a and 16.5b, usually sounds thin and "reedy," while a tone containing mostly a fundamental, like the alto recorder tone in Figs. 16.5c and 16.5d, is more mellow and flutelike. The same principle applies to the human voice, which is another wind instrument; the vowels "a" and "e" sound different because of differences in harmonic content.

Another factor in determining tone quality is the behavior at the beginning (*attack*) and end (*decay*) of a tone. A piano tone begins with a thump and then dies away gradually. A harpsichord tone, in addition to having different harmonic content, begins much more quickly with a click, and the higher harmonics begin before the lower ones. When the key is released, the sound also dies away much more rapidly with a harpsichord than with a piano. Similar effects are present in other musical instruments.

Unlike the tones made by musical instruments, **noise** is a combination of *all* frequencies, not just frequencies that are integer multiples of a fundamental frequency. (An extreme case is “white noise,” which contains equal amounts of all frequencies across the audible range.) Examples include the sound of the wind and the hissing sound you make in saying the consonant “s.”

BIO Application

Hearing Loss from Amplified Sound

Due to exposure to highly amplified music, many young musicians have suffered permanent ear damage and have hearing typical of persons 65 years of age. Headphones for personal music players used at high volume pose similar threats to hearing. Be careful!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Test Your Understanding of Section 16.1

You use an electronic signal generator to produce a sinusoidal sound wave in air. You then increase the frequency of the wave from 100 Hz to 400 Hz while keeping the pressure amplitude constant. What effect does this have on the displacement amplitude of the sound wave? (i) It becomes four times greater; (ii) it becomes twice as great; (iii) it is unchanged; (iv) it becomes $\frac{1}{2}$ as great; (v) it becomes $\frac{1}{4}$ as great.

16.2 Speed of Sound Waves

We found in Section 15.4 that the speed v of a transverse wave on a string depends on the string tension F and the linear mass density μ : $v = \sqrt{F/\mu}$. What, we may ask, is the corresponding expression for the speed of sound waves in a gas or liquid? On what properties of the medium does the speed depend?

We can make an educated guess about these questions by remembering a claim that we made in Section 15.4: For mechanical waves in general, the expression for the wave speed is of the form

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

A sound wave in a bulk fluid causes compressions and rarefactions of the fluid, so the restoring-force term in the above expression must be related to how difficult it is to compress the fluid. This is precisely what the bulk modulus B of the medium tells us. According to Newton's second law, inertia is related to mass. The "massiveness" of a bulk fluid is described by its density, or mass per unit volume, ρ. Hence we expect that the speed of sound waves should be of the form $v = \sqrt{B/\rho}$.

To check our guess, we'll derive the speed of sound waves in a fluid in a pipe. This is a situation of some importance, since all musical wind instruments are pipes in which a longitudinal wave (sound) propagates in a fluid (air) (Fig. 16.6). Human speech works on the same principle; sound waves propagate in your vocal tract, which is an air-filled pipe connected to the lungs at one end (your larynx) and to the outside air at the other end (your mouth). The steps in our derivation are completely parallel to those we used in Section 15.4 to find the speed of transverse waves.

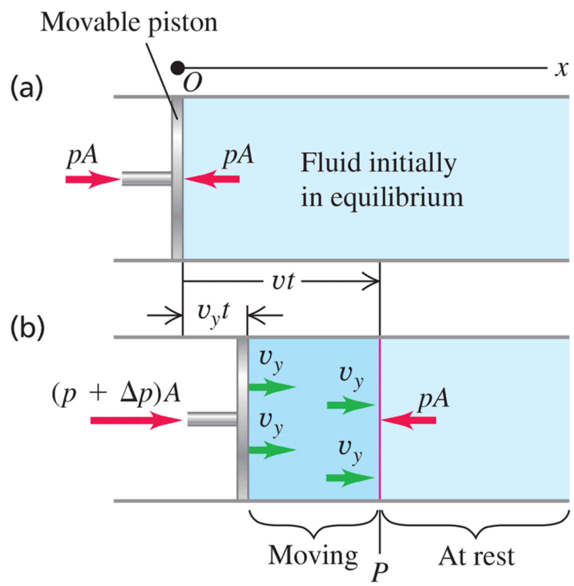
Figure 16.6

When a wind instrument like this French horn is played, sound waves propagate through the air within the instrument's pipes. The properties of the sound that emerges from the large bell depend on the speed of these waves.

Speed of Sound in a Fluid

Figure 16.7 shows a fluid with density ρ in a pipe with cross-sectional area A. In equilibrium (Fig. 16.7a), the fluid is at rest and under a uniform pressure p. We take the x-axis along the length of the pipe. This is also the direction in which we make a longitudinal wave propagate, so the displacement y is also measured along the pipe, just as in Section 16.1 (see Fig. 16.2).

Figure 16.7



A sound wave propagating in a fluid confined to a tube. (a) Fluid in equilibrium. (b) A time t after the piston begins moving to the right at speed v_y, the fluid between the piston and point P is in motion. The speed of sound waves is v.

At time $t = 0$ we start the piston at the left end moving toward the right with constant speed v_y. This initiates a wave motion that travels to the right along the length of the pipe, in which successive sections of fluid begin to move and become compressed at successively later times.

Figure 16.7b shows the fluid at time t. All portions of fluid to the left of point P are moving to the right with speed v_y, and all portions to the right of P are still at rest. The boundary between the moving and stationary portions travels to the right with a speed equal to the speed of propagation or wave speed v. At time t the piston has moved a distance $v_y t$, and the boundary has advanced a distance vt. As with a transverse disturbance in a string, we can compute the speed of propagation from the impulse–momentum theorem.

The quantity of fluid set in motion in time t originally occupied a section of the cylinder with length vt, cross-sectional area A, volume vtA, and mass $\rho v t A$. Its longitudinal momentum (that is, momentum along the length of the pipe) is

$$\text{Longitudinal momentum} = (\rho v t A) v_y$$

Next we compute the increase of pressure, Δp, in the moving fluid. The original volume of the moving fluid, Avt, has decreased by an amount $Av_y t$. From the definition of the bulk modulus B, Eq. (11.13) in Section 11.5,

$$B = \frac{-\text{Pressure change}}{\text{Fractional volume change}} = \frac{-\Delta p}{-Av_y t / Avt} \quad \text{and} \quad \Delta p = B\frac{v_y}{v}$$

The pressure in the moving fluid is $p + \Delta p$, and the force exerted on it by the piston is $(p + \Delta p)A$. The net force on the moving fluid (see Fig. 16.7b) is $\Delta p A$, and the longitudinal impulse is

$$\text{Longitudinal impulse} = \Delta p A t = B\frac{v_y}{v} A t$$

Because the fluid was at rest at time $t = 0$, the change in momentum up to time t is equal to the momentum at that time. Applying the impulse–momentum theorem (see Section 8.1), we find

(16.6)

$$B\frac{v_y}{v} A t = \rho v t A v_y$$

When we solve this expression for v, we get

(16.7)

$$\underbrace{v}_{\text{Speed of a longitudinal wave in a fluid}} = \sqrt{\frac{\overbrace{B}^{\text{Bulk modulus of fluid}}}{\underbrace{\rho}_{\text{Density of fluid}}}} \qquad (16.7)$$

which agrees with our educated guess.

While we derived Eq. (16.7) for waves in a pipe, it also applies to longitudinal waves in a bulk fluid, including sound waves traveling in air or water.

Speed of Sound in a Solid

When a longitudinal wave propagates in a *solid* rod or bar, the situation is somewhat different. The rod expands sideways slightly when it is compressed longitudinally, while a fluid in a pipe with constant cross section cannot move sideways. Using the same kind of reasoning that led us to Eq. (16.7), we can show that the speed of a longitudinal pulse in the rod is given by

(16.8)

$$\underbrace{v}_{\text{Speed of a longitudinal wave in a solid rod}} = \sqrt{\frac{\overbrace{Y}^{\text{Young's modulus of rod material}}}{\underbrace{\rho}_{\text{Density of rod material}}}} \tag{16.8}$$

We defined Young's modulus in Section 11.4.

CAUTION Solid rods vs. bulk solids Equation (16.8) applies to only rods whose sides are free to bulge and shrink a little as the wave travels. It does not apply to longitudinal waves in a *bulk* solid because sideways motion in any element of material is prevented by the surrounding material. The speed of longitudinal waves in a bulk solid depends on the density, the bulk modulus, and the *shear* modulus.

Note that Eqs. (16.7) and (16.8) are valid for sinusoidal and other periodic waves, not just for the special case discussed here.

Table 16.1 lists the speed of sound in several bulk materials. Sound waves travel more slowly in lead than in aluminum or steel because lead has a lower bulk modulus and shear modulus and a higher density.

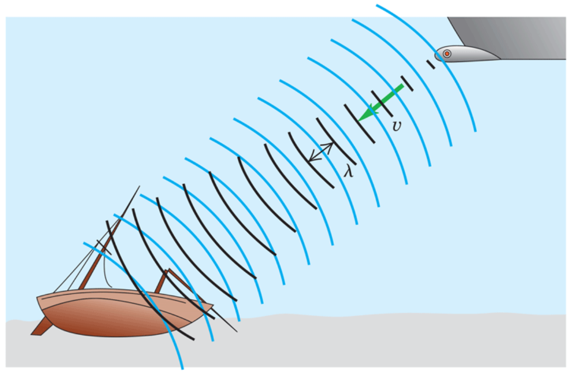
Table 16.1 **Speed of Sound in Various Bulk Materials**

Material	Speed of Sound (m/s)
Gases	
Air (20°C)	344
Helium (20°C)	999
Hydrogen (20°C)	1330
Liquids	
Liquid helium (4 K)	211
Mercury (20°C)	1451
Water (0°C)	1402
Water (20°C)	1482
Water (100°C)	1543
Solids	
Aluminum	6420
Lead	1960
Steel	5941

Example 16.3 Wavelength of sonar waves

A ship uses a sonar system (Fig. 16.8) to locate underwater objects. Find the speed of sound waves in water using Eq. (16.7), and find the wavelength of a 262 Hz wave.

Figure 16.8



A sonar system uses underwater sound waves to detect and locate submerged objects.

IDENTIFY and SET UP Our target variables are the speed and wavelength of a sound wave in water. In Eq. (16.7), we use the density of water, $\rho = 1.00 \times 10^3\ \mathrm{kg/m^3}$, and the bulk modulus of water, which we find from the compressibility (see Table 11.2). Given the speed and the frequency $f = 262\ \mathrm{Hz}$, we find the wavelength from $v = f\lambda$.

EXECUTE In Example 16.2, we used Table 11.2 to find $B = 2.18 \times 10^9\ \mathrm{Pa}$. Then

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \times 10^9\ \mathrm{Pa}}{1.00 \times 10^3\ \mathrm{kg/m^3}}} = 1476\ \mathrm{m/s}$$

and

$$\lambda = \frac{v}{f} = \frac{1476\ \mathrm{m/s}}{262\ \mathrm{s^{-1}}} = 5.64\ \mathrm{m}$$

EVALUATE The calculated value of v agrees well with the value in Table 16.1. Water is denser than air (ρ is larger) but is also much more incompressible (B is much larger), and so the speed $v = \sqrt{B/\rho}$ is greater than the $344\ \mathrm{m/s}$ speed of sound in air at ordinary temperatures. The relationship $\lambda = v/f$ then says that a sound wave in water must have a longer wavelength than a wave of the same frequency in air. Indeed, we found in Example 15.1 (Section 15.2) that a 262 Hz sound wave in air has a wavelength of only 1.31 m.

KEY CONCEPT

The speed of sound waves in a fluid depends on the fluid's bulk modulus and density. A sound wave of a given frequency has a longer wavelength in a medium that has a faster sound speed.

Video Tutor Solution: **Example 16.3**

Dolphins emit high-frequency sound waves (typically 100,000 Hz) and use the echoes for guidance and for hunting. The corresponding wavelength in water is 1.48 cm. With this high-frequency "sonar" system they can sense objects that are roughly as small as the wavelength (but not much smaller). *Ultrasonic imaging* in medicine uses the same principle; sound waves of very high frequency and very short wavelength, called *ultrasound*, are scanned over the human body, and the "echoes" from interior organs are used to create an image. With ultrasound of frequency $5\ \text{MHz} = 5 \times 10^6\ \text{Hz}$, the wavelength in water (the primary constituent of the body) is 0.3 mm, and features as small as this can be discerned in the image (Fig. 16.9). Ultrasound is more sensitive than x rays in distinguishing various kinds of tissues and does not have the radiation hazards associated with x rays.

Figure 16.9

This three-dimensional image of a fetus in the womb was made using a sequence of ultrasound scans. Each individual scan reveals a two-dimensional "slice" through the fetus; many such slices were then combined digitally. Ultrasound imaging is also used to study heart valve action and to detect tumors.

Speed of Sound in a Gas

Most of the sound waves that we encounter propagate in air. To use Eq. (16.7) to find the speed of sound waves in air, we note that the bulk modulus of a gas depends on pressure: The greater the pressure applied to compress a gas, the more it resists further compression and hence the greater the bulk modulus. (That's why specific values of the bulk modulus for gases are not given in Table 11.1.) The expression for the bulk modulus of a gas for use in Eq. (16.7) is

(16.9)

$$B = \gamma p_0$$

where p_0 is the equilibrium pressure of the gas. The quantity γ (the Greek letter gamma) is called the *ratio of heat capacities*. It is a dimensionless number that characterizes the thermal properties of the gas. (We'll learn more about this quantity in Chapter 19.) As an example, the ratio of heat capacities for air is $\gamma = 1.40$. At normal atmospheric pressure $p_0 = 1.013 \times 10^5$ Pa, so $B = (1.40)(1.013 \times 10^5 \text{ Pa}) = 1.42 \times 10^5$ Pa. This value is minuscule compared to the bulk modulus of a typical solid (see Table 11.1), which is approximately 10^{10} to 10^{11} Pa. This shouldn't be surprising: It's simply a statement that air is far easier to compress than steel.

The density ρ of a gas also depends on the pressure, which in turn depends on the temperature. It turns out that the ratio B/ρ for a given type of ideal gas does *not* depend on the pressure at all, only the temperature. From Eq. (16.7), this means that the speed of sound in a gas is fundamentally a function of temperature T:

(16.10)

$$\underbrace{v}_{\text{Speed of sound in an ideal gas}} = \sqrt{\frac{\overbrace{\gamma}^{\text{Ratio of heat capacities}}\overbrace{R}^{\text{Gas constant}}\underbrace{T}_{\text{Absolute temperature}}}{\underbrace{M}_{\text{Molar mass}}}} \qquad (16.10)$$

This expression incorporates several quantities that we'll study in Chapters 17, 18, and 19. The temperature T is the *absolute* temperature in kelvins (K), equal to the Celsius temperature plus 273.15; thus 20.00°C corresponds to $T = 293.15$ K. The quantity M is the *molar mass*, or mass per mole of the substance of which the gas is composed. The *gas constant* R has the same value for all gases. The current best numerical value of R is

$$R = 8.3144598\ (48)\ \text{J/mol} \cdot \text{K}$$

which for practical calculations we can write as 8.314 J/mol·K.

? For any particular gas, γ, R, and M are constants, and the wave speed is proportional to the square root of the absolute temperature. We'll see in Chapter 18 that Eq. (16.10) is almost identical to the expression for the average speed of molecules in an ideal gas. This shows that sound speeds and molecular speeds are closely related.

Example 16.4 Speed of sound in air

Find the speed of sound in air at $T = 20^\circ\text{C}$, and find the range of wavelengths in air to which the human ear (which can hear frequencies in the range of 20–20,000 Hz) is sensitive. The mean molar mass for air (a mixture of mostly nitrogen and oxygen) is $M = 28.8 \times 10^{-3}\ \text{kg/mol}$ and the ratio of heat capacities is $\gamma = 1.40$.

IDENTIFY and SET UP We use Eq. (16.10) to find the sound speed from γ, T, and M, and we use $v = f\lambda$ to find the wavelengths corresponding to the frequency limits. Note that in Eq. (16.10) temperature T *must* be expressed in kelvins, not Celsius degrees.

EXECUTE At $T = 20^\circ\text{C} = 293\ \text{K}$, we find

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.40)(8.314\ \text{J/mol·K})(293\ \text{K})}{28.8 \times 10^{-3}\ \text{kg/mol}}} = 344\ \text{m/s}$$

Using this value of v in $\lambda = v/f$, we find that at 20°C the frequency $f = 20\ \text{Hz}$ corresponds to $\lambda = 17\ \text{m}$ and $f = 20,000\ \text{Hz}$ to $\lambda = 1.7\ \text{cm}$.

EVALUATE Our calculated value of v agrees with the measured sound speed at $T = 20^\circ\text{C}$.

KEY CONCEPT

The speed of sound in a gas is determined by the temperature of the gas, its molar mass, and its ratio of heat capacities.

Video Tutor Solution: **Example 16.4**

A gas is actually composed of molecules in random motion, separated by distances that are large in comparison with their diameters. The vibrations that constitute a wave in a gas are superposed on the random thermal motion. At atmospheric pressure, a molecule travels an average distance of about 10^{-7} m between collisions, while the displacement amplitude of a faint sound may be only 10^{-9} m. We can think of a gas with a sound wave passing through as being comparable to a swarm of bees; the swarm as a whole oscillates slightly while individual insects move about within the swarm, apparently at random.

Test Your Understanding of Section 16.2

Mercury is 13.6 times denser than water. Based on Table 16.1, at 20°C which of these liquids has the greater bulk modulus? (i) Mercury; (ii) water; (iii) both are about the same; (iv) not enough information is given to decide.

16.3 Sound Intensity

Traveling sound waves, like all other traveling waves, transfer energy from one region of space to another. In Section 15.5 we introduced the *wave intensity* I, equal to the time average rate at which wave energy is transported per unit area across a surface perpendicular to the direction of propagation. Let's see how to express the intensity of a sound wave in a fluid in terms of the displacement amplitude A or pressure amplitude p_{max}.

Let's consider a sound wave propagating in the $+x$-direction so that we can use our expressions from Section 16.1 for the displacement $y(x,\ t)$ [Eq. (16.1)] and pressure fluctuation $p(x,\ t)$ [Eq. (16.4)]. In Section 6.4 we saw that power equals the product of force and velocity [see Eq. (6.18)]. So the power per unit area in this sound wave equals the product of $p(x,\ t)$ (force per unit area) and the *particle* velocity $v_y\ (x,\ t)$, which is the velocity at time t of that portion of the wave medium at coordinate x. Using Eqs. (16.1) and (16.4), we find

$$\begin{aligned} v_y\ (x,\ t) &= \frac{\partial y(x,\ t)}{\partial t} = \omega A \sin(kx - \omega t) \\ p(x,\ t) v_y(x,\ t) &= [BkA \sin(kx - \omega t)]\,[\omega A \sin(kx - \omega t)] \\ &= B\omega k A^2 \sin^2(kx - \omega t) \end{aligned}$$

CAUTION Wave velocity vs. particle velocity Remember that the velocity of the wave as a whole is *not* the same as the particle velocity. While the wave continues to move in the direction of propagation, individual particles in the wave medium merely slosh back and forth, as shown in Fig. 16.1. Furthermore, the maximum speed of a particle of the medium can be very different from the wave speed.

The intensity is the time average value of the power per unit area $p(x,\ t)v_y(x,\ t)$. For any value of x the average value of the function $\sin^2(kx - \omega t)$ over one period $T = 2\pi/\omega$ is $\frac{1}{2}$, so

(16.11)

$$I = \frac{1}{2}\,B\omega kA^2$$

Using the relationships $\omega = vk$ and $v = \sqrt{B/\rho}$, we can rewrite Eq. (16.11) as

(16.12)

Intensity of a sinusoidal sound wave in a fluid

$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 \qquad (16.12)$$

Angular frequency $= 2\pi f$

Displacement amplitude

Density of fluid

Bulk modulus of fluid

It is usually more useful to express I in terms of the pressure amplitude p_{max} . Using Eqs. (16.5) and (16.12) and the relationship $\omega = vk$, we find

(16.13)

$$I = \frac{\omega {p_{\text{max}}}^2}{2Bk} = \frac{v {p_{\text{max}}}^2}{2B}$$

By using the wave speed relationship $v = \sqrt{B/\rho}$, we can also write Eq. (16.13) in the alternative forms

(16.14)

$$\underset{\text{Intensity of a sinusoidal sound wave in a fluid}}{I} = \frac{\overset{\text{Pressure amplitude}}{p_{\max}}{}^{2}}{2\underset{\text{Density of fluid}}{\rho}\underset{\text{Wave speed}}{v}} = \frac{\overset{\text{Pressure amplitude}}{p_{\max}}{}^{2}}{2\sqrt{\underset{\text{Density of fluid}}{\rho}\underset{\text{Bulk modulus of fluid}}{B}}} \tag{16.14}$$

You should verify these expressions. Comparison of Eqs. (16.12) and (16.14) shows that sinusoidal sound waves of the same intensity but different frequency have different displacement amplitudes A but the *same* pressure amplitude $p_{\max}$. This is another reason it is usually more convenient to describe a sound wave in terms of pressure fluctuations, not displacement.

The *total* average power carried across a surface by a sound wave equals the product of the intensity at the surface and the surface area, if the intensity over the surface is uniform. The total average sound power emitted by a person speaking in an ordinary conversational tone is about 10^{-5} W, while a loud shout corresponds to about 3×10^{-2} W. If all the residents of New York City were to talk at the same time, the total sound power would be about 100 W, equivalent to the electric power requirement of a medium-sized light bulb. On the other hand, the power required to fill a large auditorium or stadium with loud sound is considerable (see Example 16.7).

If the sound source emits waves in all directions equally, the intensity decreases with increasing distance r from the source according to the inverse-square law (Section 15.5): The intensity is proportional to $1/r^2$. The intensity can be increased by confining the sound waves to travel in the desired direction only (Fig. 16.10), although the $1/r^2$ law still applies.

Figure 16.10

By cupping your hands like this, you direct the sound waves emerging from your mouth so that they don't propagate to the sides. Hence you can be heard at greater distances.

The inverse-square relationship also does not apply indoors because sound energy can reach a listener by reflection from the walls and ceiling. Indeed, part of the architect's job in designing an auditorium is to tailor these reflections so that the intensity is as nearly uniform as possible over the entire auditorium.

Problem-Solving Strategy 16.1 Sound Intensity

IDENTIFY *the relevant concepts:* The relationships between the intensity and amplitude of a sound wave are straightforward. Other quantities are involved in these relationships, however, so it's particularly important to decide which is your target variable.

SET UP *the problem* using the following steps:

1. Sort the physical quantities into categories. Wave properties include the displacement and pressure amplitudes A and p_{max}. The frequency f can be determined from the angular frequency ω, the wave number k, or the wavelength λ. These quantities are related through the wave speed v,

which is determined by properties of the medium (B and ρ for a liquid, and γ, T, and M for a gas).

2. List the given quantities and identify the target variables. Find relationships that take you where you want to go.

EXECUTE *the solution:* Use your selected equations to solve for the target variables. Express the temperature in kelvins (Celsius temperature plus 273.15) to calculate the speed of sound in a gas.

EVALUATE *your answer:* If possible, use an alternative relationship to check your results.

Example 16.5 Intensity of a sound wave in air

WITH VARIATION PROBLEMS

Find the intensity of the sound wave in Example 16.1, with $p_{\max} = 3.0 \times 10^{-2}$ Pa. Assume the temperature is 20°C so that the density of air is $\rho = 1.20\ \mathrm{kg/m^3}$ and the speed of sound is $v = 344\ \mathrm{m/s}$.

IDENTIFY and SET UP Our target variable is the intensity I of the sound wave. We are given the pressure amplitude $p_{\max}$ of the wave as well as the density ρ and wave speed v for the medium. We can determine I from $p_{\max}$, ρ, and v from Eq. (16.14).

EXECUTE From Eq. (16.14),

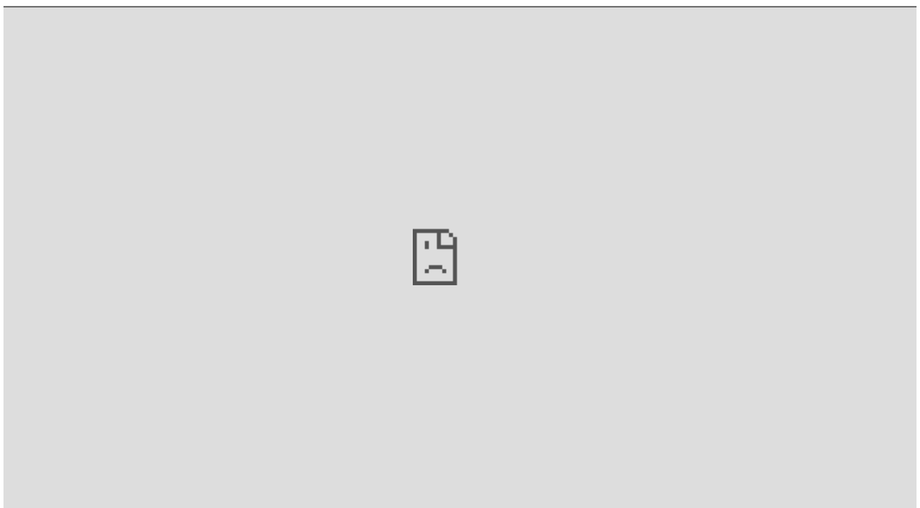
$$\begin{aligned} I &= \frac{{p_{\max}}^2}{2\rho v} = \frac{(3.0\times10^{-2}\ \mathrm{Pa})^2}{2(1.20\ \mathrm{kg/m^3})(344\ \mathrm{m/s})} \\ &= 1.1 \times 10^{-6}\ \mathrm{J/(s\cdot m^2)} = 1.1 \times 10^{-6}\ \mathrm{W/m^2} \end{aligned}$$

EVALUATE This seems like a very low intensity, but it is well within the range of sound intensities encountered on a daily basis. A very loud sound wave at the threshold of pain has a pressure amplitude of about 30 Pa and an intensity of about 1 $\mathrm{W/m^2}$. The pressure amplitude of the faintest sound wave that can be heard is about 3×10^{-5} Pa, and the corresponding intensity is about $10^{-12}\ \mathrm{W/m^2}$. (Try these values of $p_{\max}$ in Eq. (16.14) to check that the corresponding intensities are as we have stated.)

KEY CONCEPT

The intensity (power per unit area) of a sound wave is proportional to the square of the pressure amplitude of the wave. The proportionality constant depends on the density of the medium and the speed of sound in the medium.

Video Tutor Solution: **Example 16.5**



Example 16.6 Same intensity, different frequencies

WITH VARIATION PROBLEMS

What are the pressure and displacement amplitudes of a 20 Hz sound wave with the same intensity as the 1000 Hz sound wave of

Examples 16.1 and 16.5?

IDENTIFY and SET UP In Examples 16.1 and 16.5 we found that for a 1000 Hz sound wave with $p_{\text{max}} = 3.0 \times 10^{-2}$ Pa, A = 1.2×10^{-8} m and $I = 1.1 \times 10^{-6}$ W/m^2. Our target variables are p_{max} and A for a 20 Hz sound wave of the same intensity I. We can find these using Eqs. (16.14) and (16.12), respectively.

EXECUTE We can rearrange Eqs. (16.14) and (16.12) as $p_{\text{max}}{}^2 = 2I\sqrt{\rho B}$ and $\omega^2 A^2 = 2I/\sqrt{\rho B}$, respectively. These tell us that for a given sound intensity I in a given medium (constant ρ and B), the quantities p_{max} and ωA (or, equivalently, fA) are *constants* that don't depend on frequency. From the first result we immediately have $p_{\text{max}} = 3.0 \times 10^{-2}$ Pa for $f = 20$ Hz, the same as for f =1000 Hz. If we write the second result as $f_{20}A_{20} = f_{1000}A_{1000}$, we have

$$
\begin{aligned}
A_{20} &= \frac{f_{1000}}{f_{20}} A_{1000} \\
&= \frac{1000 \text{ Hz}}{20 \text{ Hz}} (1.2 \times 10^{-8} \text{ m}) = 6.0 \times 10^{-7} \text{ m} = 0.60\ \mu\text{m}
\end{aligned}
$$

EVALUATE Our result reinforces the idea that pressure amplitude is a more convenient description of a sound wave and its intensity than displacement amplitude.

KEY CONCEPT

If two sound waves in a given medium have the same intensity but different frequencies, the wave with the higher frequency has the greater *displacement* amplitude. The two waves have the same *pressure* amplitude, however.

Video Tutor Solution: **Example 16.6**

Example 16.7 “Play it loud!”

WITH VARIATION PROBLEMS

For an outdoor concert we want the sound intensity to be $1\ \mathrm{W/m^2}$ at a distance of 20 m from the speaker array. If the sound intensity is uniform in all directions, what is the required average acoustic power output of the array?

IDENTIFY, SET UP, and EXECUTE This example uses the definition of sound intensity as power per unit area. The total power is the target variable; the area in question is a hemisphere centered on the speaker array. We assume that the speakers are on the ground and that none of the acoustic power is directed into the ground, so the acoustic power is uniform over a hemisphere 20 m in radius. The surface area of this hemisphere is $\frac{1}{2}\ (4\pi)(20\ \mathrm{m})^2$, or about $2500\ \mathrm{m}^2$. The required power is the product of this area and the intensity: $(1\ \mathrm{W/m^2})(2500\ \mathrm{m}^2)\ = 2500\ \mathrm{W} = 2.5\ \mathrm{kW}$.

EVALUATE The electrical power input to the speaker would need to be considerably greater than 2.5 kW, because speaker efficiency is not very high (typically a few percent for ordinary speakers, and up to 25% for horn-type speakers).

KEY CONCEPT

To find the acoustic power output of a source of sound, multiply the area over which the emitted sound wave is distributed by the average intensity of the sound over that area.

Video Tutor Solution: **Example 16.7**

The Decibel Scale

Because the ear is sensitive over a broad range of intensities, a *logarithmic* measure of intensity called **sound intensity level** is often used:

(16.15)

Intensity of sound wave

Sound intensity level

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0} \qquad (16.15)$$

Reference intensity $= 10^{-12}\ \text{W/m}^2$

Logarithm to base 10

The chosen reference intensity I_0 in Eq. (16.15) is approximately the threshold of human hearing at 1000 Hz. Sound intensity levels are expressed in **decibels,** abbreviated dB. A decibel is $\frac{1}{10}$ of a *bel,* a unit named for Alexander Graham Bell (the inventor of the telephone). The bel is inconveniently large for most purposes, and the decibel is the usual unit of sound intensity level.

If the intensity of a sound wave equals I_0 or $10^{-12}\ \mathrm{W/m^2}$, its sound intensity level is $\beta = 0$ dB. An intensity of $1\ \mathrm{W/m^2}$ corresponds to 120 dB. Table 16.2 gives the sound intensity levels of some familiar sounds. You can use Eq. (16.15) to check the value of β given for each intensity in the table.

Table 16.2 Sound Intensity Levels from Various Sources (Representative Values)

Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, $I\ (\mathrm{W/m^2})$
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

Because the ear is not equally sensitive to all frequencies in the audible range, some sound-level meters weight the various frequencies unequally. One such scheme leads to the so-called dBA scale; this scale deemphasizes the low and very high frequencies, where the ear is less sensitive.

Example 16.8 Temporary—or permanent—hearing loss

WITH VARIATION PROBLEMS

A 10 min exposure to 120 dB sound will temporarily shift your threshold of hearing at 1000 Hz from 0 dB up to 28 dB. Ten years of exposure to 92 dB sound will cause a *permanent* shift to 28 dB. What sound intensities correspond to 28 dB and 92 dB?

IDENTIFY and SET UP We are given two sound intensity levels β; our target variables are the corresponding intensities. We can solve Eq. (16.15) to find the intensity I that corresponds to each value of β.

EXECUTE We solve Eq. (16.15) for I by dividing both sides by 10 dB and using the relationship $10^{\log x} = x$:

$$I = I_0 10^{\beta/(10\,dB)}$$

For $\beta = 28$ dB and $\beta = 92$ dB, the exponents are $\beta/(10\text{ dB})$ = 2.8 and 9.2, respectively, so that

$$\begin{aligned} I_{28\text{ dB}} &= (10^{-12}\text{ W/m}^2)10^{2.8} = 6.3 \times 10^{-10}\text{ W/m}^2 \\ I_{92\text{ dB}} &= (10^{-12}\text{ W/m}^2)10^{9.2} = 1.6 \times 10^{-3}\text{ W/m}^2 \end{aligned}$$

EVALUATE If your answers are a factor of 10 too large, you may have entered 10×10^{-12} in your calculator instead of 1×10^{-12}. Be careful!

KEY CONCEPT

The sound intensity level (in decibels, or dB) is a logarithmic measure of the intensity of a sound wave. Adding 10 dB to the sound intensity level corresponds to multiplying the intensity by a factor of 10.

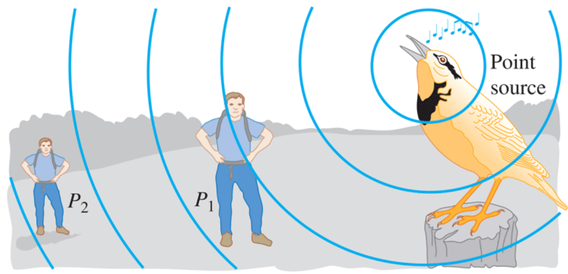
Video Tutor Solution: **Example 16.8**

Example 16.9 A bird sings in a meadow

WITH VARIATION PROBLEMS

Consider an idealized bird (treated as a point source) that emits constant sound power, with intensity obeying the inverse-square law (Fig. 16.11). If you move twice the distance from the bird, by how many decibels does the sound intensity level drop?

Figure 16.11



When you double your distance from a point source of sound, by how much does the sound intensity level decrease?

IDENTIFY and SET UP The decibel scale is logarithmic, so the *difference* between two sound intensity levels (the target variable) corresponds to the *ratio* of the corresponding intensities, which is determined by the inverse-square law. We label the two points P_1 and P_2 (Fig. 16.11). We use Eq. (16.15), the definition of sound intensity level, at each point. We use Eq. (15.26), the inverse-square law, to relate the intensities at the two points.

EXECUTE The difference $\beta_2 - \beta_1$ between any two sound intensity levels is related to the corresponding intensities by

$$\begin{aligned}\beta_2 - \beta_1 &= (10\text{ dB}) \quad \log\frac{I_2}{I_0} - \log\frac{I_1}{I_0} \\ &= (10\text{ dB})[(\log I_2 - \log I_0) - (\log I_1 - \log I_0)] \\ &= (10\text{ dB}) \log\frac{I_2}{I_1}\end{aligned}$$

For this inverse-square-law source, Eq. (15.26) yields $I_2/I_1 = r_1^2/r_2^2 = \frac{1}{4}$, so

$$\beta_2 - \beta_1 = (10\text{ dB}) \log\frac{I_2}{I_1} = (10\text{ dB}) \log\frac{1}{4} = -6.0\text{ dB}$$

EVALUATE Our result is negative, which tells us (correctly) that the sound intensity level is less at P_2 than at P_1. The 6 dB difference doesn't depend on the sound intensity level at P_1; *any* doubling of the distance from an inverse-square-law source reduces the sound intensity level by 6 dB.

Note that the perceived *loudness* of a sound is not directly proportional to its intensity. For example, most people interpret an increase of 8 dB to 10 dB in sound intensity level (corresponding to increasing intensity by a factor of 6 to 10) as a doubling of loudness.

KEY CONCEPT

The *difference* between the sound intensity levels of two sounds is proportional to the logarithm of the *ratio* of the intensities of those sounds.

Video Tutor Solution: **Example 16.9**

Test Your Understanding of Section 16.3

You double the intensity of a sound wave in air while leaving the frequency unchanged. (The pressure, density, and temperature of the air remain unchanged as well.) What effect does this have on the displacement amplitude, pressure amplitude, bulk modulus, sound speed, and sound intensity level?

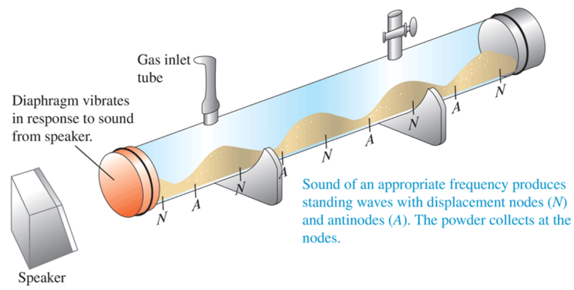
16.4 Standing Sound Waves and Normal Modes

When longitudinal (sound) waves propagate in a fluid in a pipe, the waves are reflected from the ends in the same way that transverse waves on a string are reflected at its ends. The superposition of the waves traveling in opposite directions again forms a standing wave. Just as for transverse standing waves on a string (see Section 15.7), standing sound waves in a pipe can be used to create sound waves in the surrounding air. This is the principle of the human voice as well as many musical instruments, including woodwinds, brasses, and pipe organs.

Transverse waves on a string, including standing waves, are usually described only in terms of the displacement of the string. But, as we have seen, sound waves in a fluid may be described either in terms of the displacement of the fluid or in terms of the pressure variation in the fluid. To avoid confusion, we'll use the terms **displacement node** and **displacement antinode** to refer to points where particles of the fluid have zero displacement and maximum displacement, respectively.

We can demonstrate standing sound waves in a column of gas using an apparatus called a Kundt's tube (Fig. 16.12). A horizontal glass tube a meter or so long is closed at one end and has a flexible diaphragm at the other end that can transmit vibrations. A nearby loudspeaker is driven by an audio oscillator and amplifier; this produces sound waves that force the diaphragm to vibrate sinusoidally with a frequency that we can vary. The sound waves within the tube are reflected at the other, closed end of the tube. We spread a small amount of light powder uniformly along the bottom of the tube. As we vary the frequency of the sound, we pass through frequencies at which the amplitude of the standing waves becomes large enough for the powder to be swept along the tube at those points where the gas is in motion. The powder therefore collects at the displacement nodes (where the gas is not moving). Adjacent nodes are separated by a distance equal to $\lambda/2$.

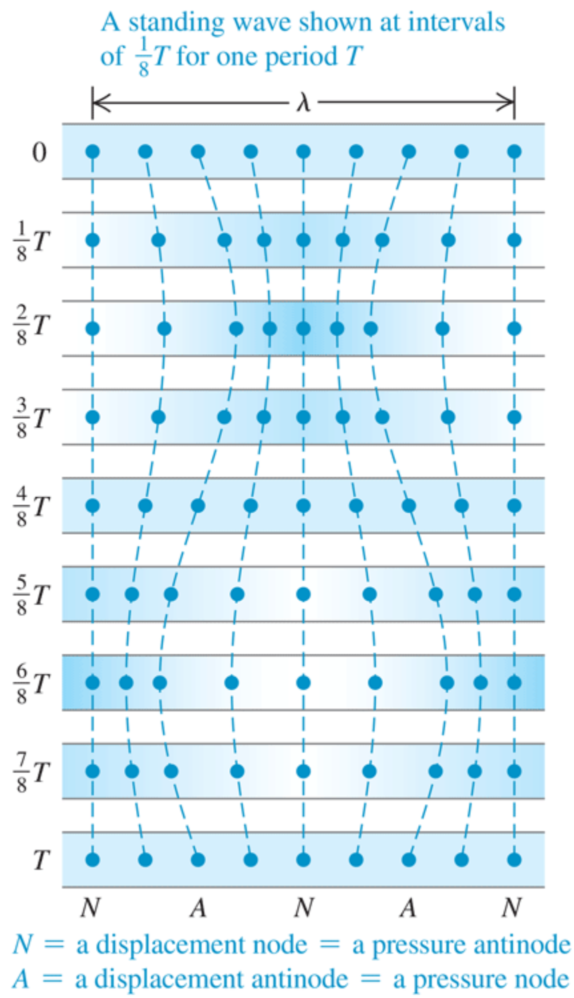
Figure 16.12



Demonstrating standing sound waves using a Kundt's tube. The blue shading represents the density of the gas at an instant when the gas pressure at the displacement nodes is a maximum or a minimum.

Figure 16.13 shows the motions of nine different particles within a gas-filled tube in which there is a standing sound wave. A particle at a displacement node (N) does not move, while a particle at a displacement antinode (A) oscillates with maximum amplitude. Note that particles on opposite sides of a displacement node vibrate in opposite phase. When these particles approach each other, the gas between them is compressed and the pressure rises; when they recede from each other, there is an expansion and the pressure drops. Hence at a displacement *node* the gas undergoes the maximum amount of compression and expansion, and the variations in pressure and density above and below the average have their maximum value. By contrast, particles on opposite sides of a displacement *antinode* vibrate *in phase*; the distance between the particles is nearly constant, and there is *no* variation in pressure or density at a displacement antinode.

Figure 16.13



In a standing sound wave, a displacement node N is a pressure antinode (a point where the pressure fluctuates the most) and a displacement antinode A is a pressure node (a point where the pressure does not fluctuate at all).

We use the term **pressure node** to describe a point in a standing sound wave at which the pressure and density do not vary and the term **pressure antinode** to describe a point at which the variations in pressure and density are greatest. Using these terms, we can summarize our observations as follows:

> A pressure node is always a displacement antinode, and a pressure antinode is always a displacement node.

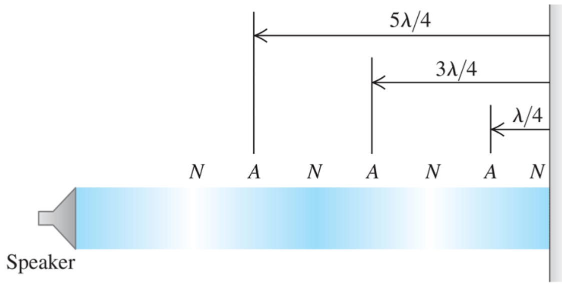
Figure 16.12 depicts a standing sound wave at an instant at which the pressure variations are greatest; the blue shading shows that the density and pressure of the gas have their maximum and minimum values at the displacement nodes.

When reflection takes place at a *closed* end of a pipe (an end with a rigid barrier or plug), the displacement of the particles at this end must always be zero, analogous to a fixed end of a string. Thus a closed end of a pipe is a displacement node and a pressure antinode; the particles do not move, but the pressure variations are maximum. An *open* end of a pipe is a pressure node because it is open to the atmosphere, where the pressure is constant. Because of this, an open end is always a displacement *antinode,* in analogy to a free end of a string; the particles oscillate with maximum amplitude, but the pressure does not vary. (The pressure node actually occurs somewhat beyond an open end of a pipe. But if the diameter of the pipe is small in comparison to the wavelength, which is true for most musical instruments, this effect can safely be ignored.) Thus longitudinal sound waves are reflected at the closed and open ends of a pipe in the same way that transverse waves in a string are reflected at fixed and free ends, respectively.

Conceptual Example 16.10 The sound of silence

A directional loudspeaker directs a sound wave of wavelength λ at a wall (Fig. 16.14). At what distances from the wall could you stand and hear no sound at all?

Figure 16.14



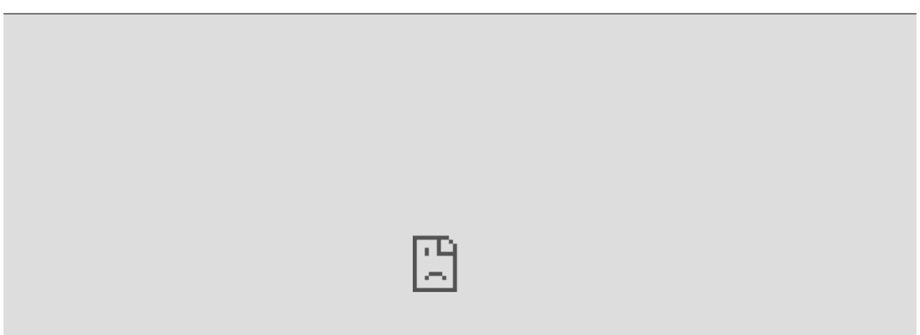
When a sound wave is directed at a wall, it interferes with the reflected wave to create a standing wave. The N's and A's are *displacement* nodes and antinodes.

SOLUTION Your ear detects pressure variations in the air; you'll therefore hear no sound if your ear is at a *pressure node*, which is a displacement antinode. The wall is at a displacement node; the distance from any node to an adjacent antinode is $\lambda/4$, and the distance from one antinode to the next is $\lambda/2$ (Fig. 16.14). Hence the displacement antinodes (pressure nodes), at which no sound will be heard, are at distances $d = \lambda/4$, $d = \lambda/4 + \lambda/2 = 3\lambda/4$, $d = 3\lambda/4 + \lambda/2 = 5\lambda/4$, ... from the wall. If the loudspeaker is not highly directional, this effect is hard to notice because of reflections of sound waves from the floor, ceiling, and other walls.

KEY CONCEPT

In a standing sound wave, a pressure node is a displacement antinode, and vice versa. The sound is loudest at a pressure antinode; there is no sound at a pressure node.

Video Tutor Solution: **Example 16.10**



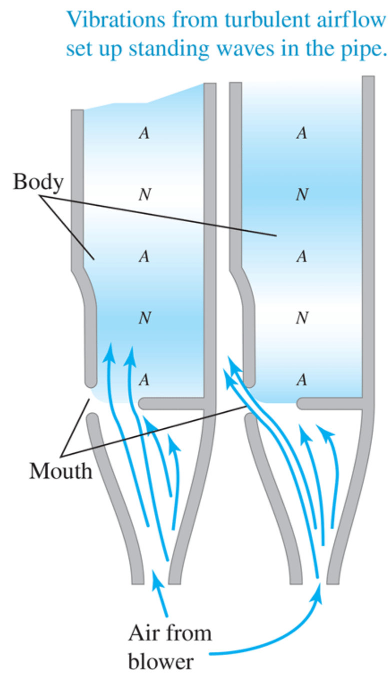
Organ Pipes and Wind Instruments

The most important application of standing sound waves is the production of musical tones. Organ pipes are one of the simplest examples (Fig. 16.15). Air is supplied by a blower to the bottom end of the pipe (Fig. 16.16). A stream of air emerges from the narrow opening at the edge of the horizontal surface and is directed against the top edge of the opening, which is called the *mouth* of the pipe. The column of air in the pipe is set into vibration, and there is a series of possible normal modes, just as with the stretched string. The mouth acts as an open end; it is a pressure node and a displacement antinode. The other end of the pipe (at the top in Fig. 16.16) may be either open or closed.

Figure 16.15

Organ pipes of different sizes produce tones with different frequencies.

Figure 16.16



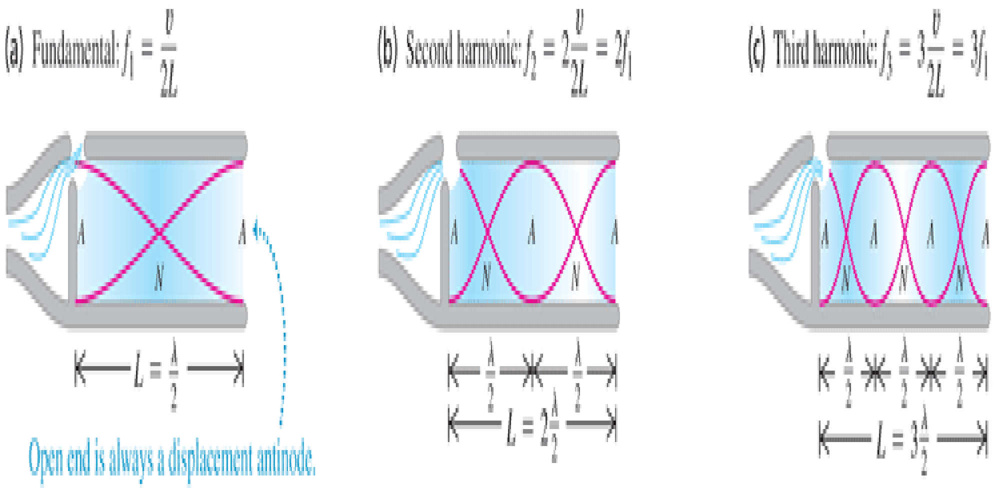
Cross sections of an organ pipe at two instants one half-period apart. The N's and A's are *displacement* nodes and antinodes; as the blue shading shows, these are points of maximum pressure variation and zero pressure variation, respectively.

In Fig. 16.17, both ends of the pipe are open, so both ends are pressure nodes and displacement antinodes. An organ pipe that is open at both ends is called an *open pipe*. The fundamental frequency f_1 corresponds to a standing-wave pattern with a displacement antinode at each end and a displacement node in the middle (Fig. 16.17a). The distance between adjacent antinodes is always equal to one half-wavelength, and in this case that is equal to the length L of the pipe; $\lambda/2 = L$. The corresponding frequency, obtained from the relationship $f = v/\lambda$, is

(16.16)

$$f_1 = \frac{v}{2L} \qquad \text{(open pipe)}$$

Figure 16.17



A cross section of an open pipe showing the first three normal modes. The shading indicates the pressure variations. The red curves are graphs of the displacement along the pipe axis at two instants separated in time by one half-period. The N's and A's are the *displacement* nodes and antinodes; interchange these to show the *pressure* nodes and antinodes.

Figures 16.17b and 16.17c show the second and third harmonics; their vibration patterns have two and three displacement nodes, respectively. For these, a half-wavelength is equal to $L/2$ and $L/3$, respectively, and the frequencies are twice and three times the fundamental, respectively: $f_2 = 2f_1$ and $f_3 = 3f_1$. For *every* normal mode of an open pipe the length L must be an integer number of half-wavelengths, and the possible wavelengths λ_n are given by

(16.17)

$$L = n\frac{\lambda_n}{2} \qquad \text{or} \qquad \lambda_n = \frac{2L}{n} \qquad (n = 1,\ 2,\ 3,\ \ldots) \qquad \text{(open pipe)}$$

The corresponding frequencies f_n are given by $f_n = v/\lambda_n$, so all the normal-mode frequencies for a pipe that is open at both ends are given by

(16.18)

Standing waves, open pipe:

$$\overbrace{f_n}^{\text{Frequency of } n\text{th harmonic } (n = 1, 2, 3, \ldots)} = \frac{n\overbrace{v}^{\text{Speed of sound in pipe}}}{2\underbrace{L}_{\text{Length of pipe}}} \qquad (16.18)$$

The value $n = 1$ corresponds to the fundamental frequency, $n = 2$ to the second harmonic (or first overtone), and so on. Alternatively, we can say

(16.19)

$$f_n = nf_1 \qquad (n = 1,\ 2,\ 3,\ \ldots) \qquad \text{(open pipe)}$$

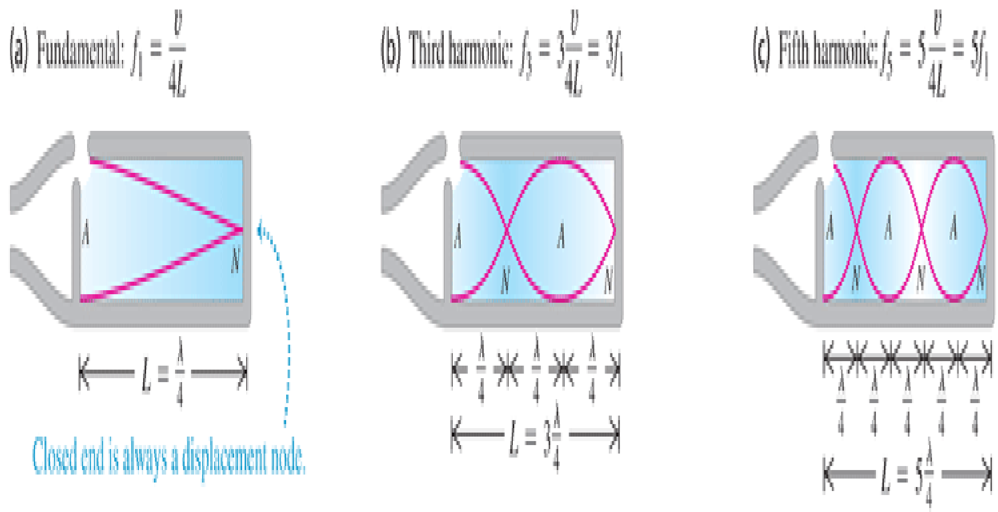
with f_1 given by Eq. (16.16).

Figure 16.18 shows a *stopped pipe:* It is open at the left end but closed at the right end. The left (open) end is a displacement antinode (pressure node), but the right (closed) end is a displacement node (pressure antinode). Figure 16.18a shows the lowest-frequency mode; the length of the pipe is the distance between a node and the adjacent antinode, or a quarter-wavelength ($L = \lambda_1/4$). The fundamental frequency is $f_1 = v/\lambda_1$, or

(16.20)

$$f_1 = \frac{v}{4L} \qquad \text{(stopped pipe)}$$

Figure 16.18



A cross section of a stopped pipe showing the first three normal modes as well as the *displacement* nodes and antinodes. Only odd harmonics are possible.

This is one-half the fundamental frequency for an *open* pipe of the same length. In musical language, the *pitch* of a closed pipe is one octave lower (a factor of 2 in frequency) than that of an open pipe of the same length. Figure 16.18b shows the next mode, for which the length of the pipe is *three-quarters* of a wavelength, corresponding to a frequency $3f_1$. For Fig. 16.18c, $L = 5\lambda/4$ and the frequency is $5f_1$. The possible wavelengths are given by

(16.21)

$$L = n\frac{\lambda_n}{4} \qquad \text{or} \qquad \lambda_n = \frac{4L}{n} \qquad (n = 1,\ 3,\ 5,\ \ldots) \qquad \text{(stopped pipe)}$$

The normal-mode frequencies are given by $f_n = v/\lambda_n$, or

(16.22)

Standing waves, stopped pipe:

Frequency of nth harmonic ($n = 1, 3, 5, \ldots$)

$$f_n = \frac{nv}{4L} \tag{16.22}$$

v: Speed of sound in pipe

L: Length of pipe

or

(16.23)

$$f_n = nf_1 \qquad (n = 1,\ 3,\ 5,\ \ldots) \qquad \text{(stopped pipe)}$$

with f_1 given by Eq. (16.20). We see that the second, fourth, and all *even* harmonics are missing. In a stopped pipe, the fundamental frequency is $f_1 = v/4L$, and only the odd harmonics in the series $(3f_1,\ 5f_1,\ \ldots)$ are possible.

A final possibility is a pipe that is closed at *both* ends, with displacement nodes and pressure antinodes at both ends. This wouldn't be of much use as a musical instrument because the vibrations couldn't get out of the pipe.

Example 16.11 A tale of two pipes

WITH VARIATION PROBLEMS

On a day when the speed of sound is 344 m/s, the fundamental frequency of a particular stopped organ pipe is 220 Hz. (a) How long is this pipe? (b) The second *overtone* of this pipe has the same wavelength as the third *harmonic* of an *open* pipe. How long is the open pipe?

IDENTIFY and SET UP This problem uses the relationship between the length and normal-mode frequencies of open pipes (Fig. 16.17) and stopped pipes (Fig. 16.18). In part (a), we determine the length of the stopped pipe from Eq. (16.22). In part (b), we must determine the length of an open pipe, for which Eq. (16.18) gives the frequencies.

EXECUTE (a) For a stopped pipe $f_1 = v/4L$, so

$$L_{\text{stopped}} = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(220 \text{ s}^{-1})} = 0.391 \text{ m}$$

(b) The frequency of the second overtone of a stopped pipe (the *third* possible frequency) is $f_5 = 5f_1 = 5(220 \text{ Hz}) = 1100 \text{ Hz}$. If the wavelengths for the two pipes are the same, the frequencies are also the same. Hence the frequency of the third harmonic of the open pipe, which is at $3f_1 = 3\,(v/2L)$, equals 1100 Hz. Then

$$1100 \text{ Hz} = 3\left(\frac{344 \text{ m/s}}{2L_{\text{open}}}\right) \qquad \text{and} \qquad L_{\text{open}} = 0.469 \text{ m}$$

EVALUATE The 0.391 m stopped pipe has a fundamental frequency of 220 Hz; the *longer* (0.469 m) open pipe has a *higher* fundamental frequency, $(1100 \text{ Hz})/3 = 367 \text{ Hz}$. This is not a contradiction, as you can see if you compare Figs. 16.17a and 16.18a.

KEY CONCEPT

For a pipe open at both ends (an "open pipe"), the normal-mode frequencies of a standing sound wave include both even and odd multiples of the pipe's fundamental frequency. For a pipe open at one end and closed at the other (a "stopped pipe"), the only normal-mode frequencies are the odd multiples of the pipe's fundamental frequency. The fundamental frequency of a stopped pipe is half that of an open pipe of the same length.

Video Tutor Solution: **Example 16.11**

In an organ pipe in actual use, several modes are always present at once; the motion of the air is a superposition of these modes. This situation is analogous to a string that is struck or plucked, as in Fig. 15.28. Just as for a vibrating string, a complex standing wave in the pipe produces a traveling sound wave in the surrounding air with a harmonic content similar to that of the standing wave. A very narrow pipe produces a sound wave rich in higher harmonics; a fatter pipe produces mostly the fundamental mode, heard as a softer, more flutelike tone. The harmonic content also depends on the shape of the pipe's mouth.

We have talked about organ pipes, but this discussion is also applicable to other wind instruments. The flute and the recorder are directly analogous. The most significant difference is that those instruments have holes along the pipe. Opening and closing the holes with the fingers changes the effective length L of the air column and thus changes the pitch. Any individual organ pipe, by comparison, can play only a single note. The flute and recorder behave as *open* pipes, while the clarinet acts as a *stopped* pipe (closed at the reed end, open at the bell).

Equations (16.18) and (16.22) show that the frequencies of any wind instrument are proportional to the speed of sound v in the air column inside the instrument. As Eq. (16.10) shows, v depends on temperature; it increases when temperature increases. Thus the pitch of all wind instruments rises with increasing temperature. An organ that has some of its pipes at one temperature and others at a different temperature is bound to sound out of tune.

Test Your Understanding of Section 16.4

If you connect a hose to one end of a metal pipe and blow compressed air into it, the pipe produces a musical tone. If instead you blow compressed helium into the pipe at the same pressure and temperature, will the pipe produce (i) the same tone, (ii) a higher-pitch tone, or (iii) a lower-pitch tone?

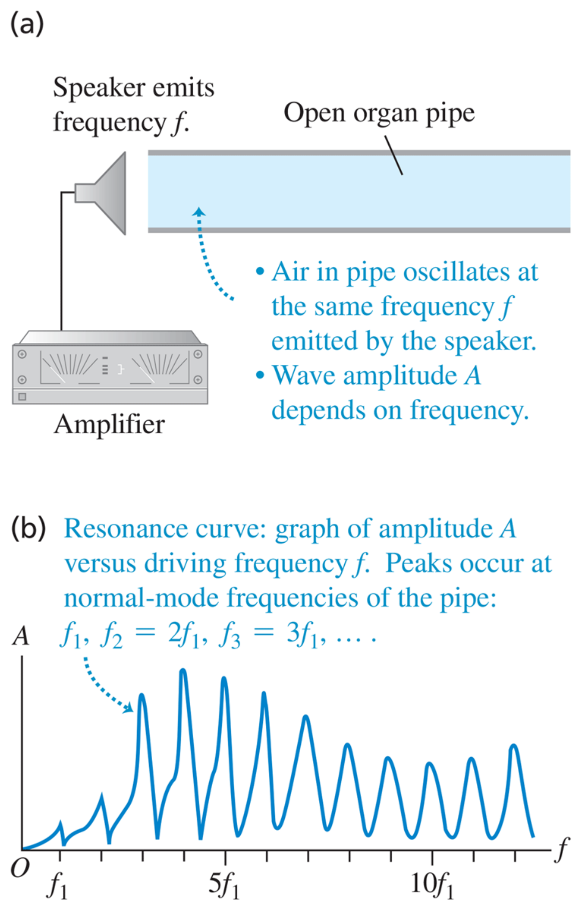
16.5 Resonance and Sound

Many mechanical systems have normal modes of oscillation. As we have seen, these include columns of air (as in an organ pipe) and stretched strings (as in a guitar; see Section 15.8). In each mode, every particle of the system oscillates with simple harmonic motion at the same frequency as the mode. Air columns and stretched strings have an infinite series of normal modes, but the basic concept is closely related to the simple harmonic oscillator, discussed in Chapter 14, which has only a single normal mode (that is, only one frequency at which it oscillates after being disturbed).

Suppose we apply a periodically varying force to a system that can oscillate. The system is then forced to oscillate with a frequency equal to the frequency of the applied force (called the *driving frequency*). This motion is called a *forced oscillation*. We talked about forced oscillations of the harmonic oscillator in Section 14.8, including the phenomenon of mechanical **resonance.** A simple example of resonance is pushing Cousin Throckmorton on a swing. The swing is a pendulum; it has only a single normal mode, with a frequency determined by its length. If we push the swing periodically with this frequency, we can build up the amplitude of the motion. But if we push with a very different frequency, the swing hardly moves at all.

Resonance also occurs when a periodically varying force is applied to a system with many normal modes. In Fig. 16.19a an open organ pipe is placed next to a loudspeaker that emits pure sinusoidal sound waves of frequency f, which can be varied by adjusting the amplifier. The air in the pipe is forced to vibrate with the same frequency f as the *driving force* provided by the loudspeaker. In general the amplitude of this motion is relatively small, and the air inside the pipe will not move in any of the normal-mode patterns shown in Fig. 16.17. But if the frequency f of the force is close to one of the normal-mode frequencies, the air in the pipe moves in the normal-mode pattern for that frequency, and the amplitude can become quite large. Figure 16.19b shows the amplitude of oscillation of the air in the pipe as a function of the driving frequency f. This **resonance curve** of the pipe has peaks where f equals the normal-mode frequencies of the pipe. The detailed shape of the resonance curve depends on the geometry of the pipe.

Figure 16.19



(a) The air in an open pipe is forced to oscillate at the same frequency as the sinusoidal sound waves coming from the loudspeaker. (b) The resonance curve of the open pipe graphs the amplitude of the standing sound wave in the pipe as a function of the driving frequency.

If the frequency of the force is precisely *equal* to a normal-mode frequency, the system is in resonance, and the amplitude of the forced oscillation is maximum. If there were no friction or other energy-dissipating mechanism, a driving force at a normal-mode frequency would continue to add energy to the system, the amplitude would increase indefinitely, and the peaks in the resonance curve of Fig. 16.19b would be infinitely high. But in any real system there is always some dissipation of energy, or damping, as we discussed in Section 14.8; the amplitude of oscillation in resonance may be large, but it cannot be infinite.

BIO Application

Resonance and the Sensitivity of the Ear

The auditory canal of the human ear (see Fig. 16.4) is an air-filled pipe open at one end and closed at the other (eardrum) end. The canal is about 2.5 cm=0.025 m long, so it has a resonance at its fundamental frequency $f_1 = v/4 = (344\ \mathrm{m/s})/[4(0.025\ \mathrm{m})] = 3440\ \mathrm{Hz}$. The resonance means that a sound at this frequency produces a strong oscillation of the eardrum. That's why your ear is most sensitive to sounds near 3440 Hz.

The “sound of the ocean” you hear when you put your ear next to a large seashell is due to resonance. The noise of the outside air moving past the seashell is a mixture of sound waves of almost all audible frequencies, which forces the air inside the seashell to oscillate. The seashell behaves like an organ pipe, with a set of normal-mode frequencies; hence the inside air oscillates most strongly at those frequencies, producing the seashell’s characteristic sound. To hear a similar phenomenon, uncap a full bottle of your favorite beverage and blow across the open top. The noise is provided by your breath blowing across the top, and the “organ pipe” is the column of air inside the bottle above the surface of the liquid. If you take a drink and repeat the experiment, you’ll hear a lower tone because the “pipe” is longer and the normal-mode frequencies are lower.

Resonance also occurs when a stretched string is forced to oscillate (see Section 15.8). Suppose that one end of a stretched string is held fixed while the other is given a transverse sinusoidal motion with small amplitude, setting up standing waves. If the frequency of the driving mechanism is *not* equal to one of the normal-mode frequencies of the string, the amplitude at the antinodes is fairly small. However, if the frequency is equal to any one of the normal-mode frequencies, the string is in resonance, and the amplitude at the antinodes is very much larger than that at the driven end. The driven end is not precisely a node, but it lies much closer to a node than to an antinode when the string is in resonance. The photographs of standing waves in Fig. 15.23 were made this way, with the left end of the string fixed and the right end oscillating vertically with small amplitude.

It is easy to demonstrate resonance with a piano. Push down the damper pedal (the right-hand pedal) so that the dampers are lifted and the strings are free to vibrate, and then sing a steady tone into the piano. When you stop singing, the piano seems to continue to sing the same note. The sound waves from your voice excite vibrations in the strings that have natural frequencies close to the frequencies (fundamental and harmonics) present in the note you sang.

A more spectacular example is a singer breaking a wine glass with her amplified voice. A good-quality wine glass has normal-mode frequencies that you can hear by tapping it. If the singer emits a loud note with a frequency corresponding exactly to one of these normal-mode frequencies, large-amplitude oscillations can build up and break the glass (Fig. 16.20).

Figure 16.20

The frequency of the sound from this trumpet exactly matches one of the normal-mode frequencies of the goblet. The resonant vibrations of the goblet have such large amplitude that the goblet tears itself apart.

Example 16.12 An organ–guitar duet

WITH VARIATION PROBLEMS

A stopped organ pipe is sounded near a guitar, causing one of the strings to vibrate with large amplitude. We vary the string tension until we find the maximum amplitude. The string is 80% as long as the pipe. If both pipe and string vibrate at their fundamental frequency, calculate the ratio of the wave speed on the string to the speed of sound in air.

IDENTIFY and SET UP The large response of the string is an example of resonance. It occurs because the organ pipe and the guitar string have the same fundamental frequency. If we let the subscripts a and s stand for the air in the pipe and the string, respectively, the condition for resonance is $f_{1\mathrm{a}} = f_{1\mathrm{s}}$. Equation (16.20) gives the fundamental frequency for a stopped pipe, and Eq. (15.32) gives the fundamental frequency for a guitar string held at both ends. These expressions involve the wave speed in air (v_a) and on the string (v_s) and the lengths of the pipe and string. We are given that $L_\mathrm{s} = 0.80L_\mathrm{a}$; our target variable is the ratio $v_\mathrm{s}/v_\mathrm{a}$.

EXECUTE From Eqs. (16.20) and (15.32), $f_{1\mathrm{a}} = v_\mathrm{a}/4L_\mathrm{a}$ and $f_{1\mathrm{s}} = v_\mathrm{s}/2L_\mathrm{s}$. These frequencies are equal, so

$$\frac{v_\mathrm{a}}{4L_\mathrm{a}} = \frac{v_\mathrm{s}}{2L_\mathrm{s}}$$

Substituting $L_\mathrm{s} = 0.80L_\mathrm{a}$ and rearranging, we get v_s/v_a = 0.40.

EVALUATE As an example, if the speed of sound in air is 344 m/s, the wave speed on the string is $(0.40)(344\ \mathrm{m/s}) = 138\ \mathrm{m/s}$. Note that while the standing waves in the pipe and on the string have the same frequency, they have different *wavelengths* $\lambda = v/f$ because the two media have different wave speeds v. Which standing wave has the greater wavelength?

KEY CONCEPT

If you force or drive a mechanical system (such as a guitar string or the air in a pipe) to vibrate at a frequency f, the system will oscillate with maximum amplitude (or *resonate*) if f equals one of the normal-mode frequencies of the system.

Video Tutor Solution: **Example 16.12**

Test Your Understanding of Section 16.5

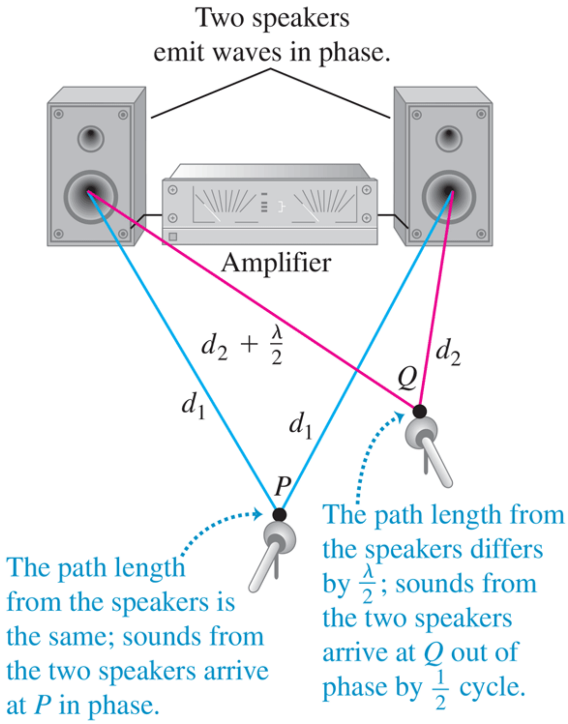
A stopped organ pipe of length L has a fundamental frequency of 220 Hz. For which of the following organ pipes will there be a resonance if a tuning fork of frequency 660 Hz is sounded next to the pipe? (There may be more than one correct answer.) (i) A stopped organ pipe of length L; (ii) a stopped organ pipe of length $2L$; (iii) an open organ pipe of length L; (iii) an open organ pipe of length $2L$.

16.6 Interference of Waves

Wave phenomena that occur when two or more waves overlap in the same region of space are grouped under the heading *interference*. As we have seen, standing waves are a simple example of an interference effect: Two waves traveling in opposite directions in a medium can combine to produce a standing-wave pattern with nodes and antinodes that do not move.

Figure 16.21 shows an example of another type of interference that involves waves that spread out in space. Two speakers, driven in phase by the same amplifier, emit identical sinusoidal sound waves with the same constant frequency. We place a microphone at point P in the figure, equidistant from the speakers. Wave crests emitted from the two speakers at the same time travel equal distances and arrive at point P at the same time; hence the waves arrive in phase, and there is constructive interference. The total wave amplitude that we measure at P is twice the amplitude from each individual wave.

Figure 16.21



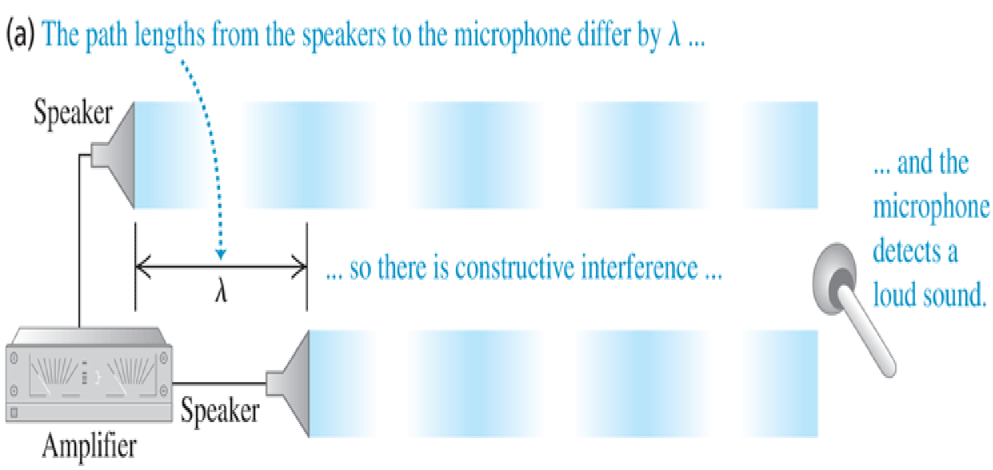
Two speakers driven by the same amplifier. Constructive interference occurs at point P, and destructive interference occurs at point Q.

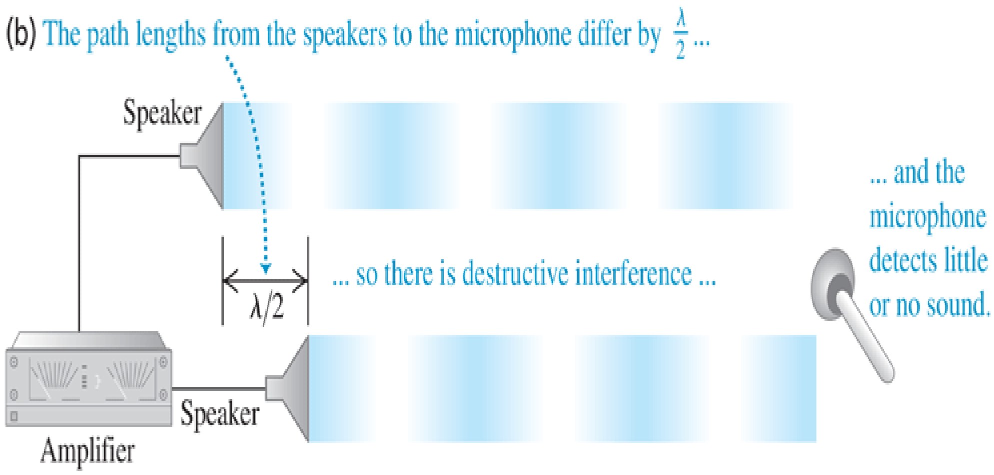
Now let's move the microphone to point Q, where the distances from the two speakers to the microphone differ by a half-wavelength. Then the two waves arrive a half-cycle out of step, or *out of phase;* a positive crest from one speaker arrives at the same time as a negative crest from the other. Destructive interference takes place, and the amplitude measured by the microphone is much *smaller* than when only one speaker is present. If the amplitudes from the two speakers are equal, the two waves cancel each other out completely at point Q, and the total amplitude there is zero.

CAUTION Interference and traveling waves The total wave in Fig. 16.21 is a *traveling* wave, not a standing wave. In a standing wave there is no net flow of energy in any direction; by contrast, in Fig. 16.21 there *is* an overall flow of energy from the speakers into the surrounding air, characteristic of a traveling wave. The interference between the waves from the two speakers simply causes the energy flow to be *channeled* into certain directions (for example, toward P) and away from other directions (for example, away from Q). You can see another difference between Fig. 16.21 and a standing wave by considering a point, such as Q, where destructive interference occurs. Such a point is *both* a displacement node *and* a pressure node because there is no wave at all at this point. In a standing wave, a pressure node is a displacement antinode, and vice versa.

Constructive interference occurs wherever the distances traveled by the two waves differ by a whole number of wavelengths, 0, λ λ λ then the waves arrive at the microphone in phase (Fig. 16.22a). If the distances from the two speakers to the microphone differ by any half-integer number of wavelengths, λ λ λ the waves arrive at the microphone out of phase and there will be destructive interference (Fig. 16.22b). In this case, little or no sound energy flows toward the microphone. The energy instead flows in other directions, to where constructive interference occurs.

Figure 16.22



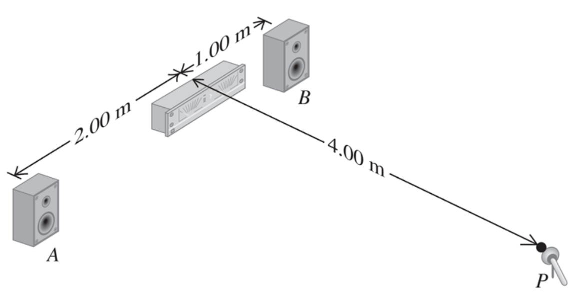


Two speakers driven by the same amplifier, emitting waves in phase. Only the waves directed toward the microphone are shown, and they are separated for clarity. (a) Constructive interference occurs when the path difference is 0, λ λ λ (b) Destructive interference occurs when the path difference is λ λ λ

Example 16.13 Loudspeaker interference

Two small loudspeakers, A and B (Fig. 16.23), are driven by the same amplifier and emit pure sinusoidal waves in phase. (a) For what frequencies does constructive interference occur at point P? (b) For what frequencies does destructive interference occur? The speed of sound is s

Figure 16.23



What sort of interference occurs at P?

IDENTIFY and SET UP The nature of the interference at P depends on the difference d in path lengths from point A to P and from point B to P. We calculate the path lengths using the Pythagorean theorem. Constructive interference occurs when d equals a whole number of wavelengths, while destructive interference occurs when d is a half-integer number of wavelengths. To find the corresponding frequencies, we use $v \quad f\lambda$

EXECUTE The A-to-P distance is

and the B-to-P distance is

The path difference is d

(a) Constructive interference occurs when $d \quad \lambda \quad \lambda$ or d $v \ f \quad v \ f \quad nv \ f$ So the possible frequencies are

$$f_n \qquad \frac{nv}{d} \qquad n\overline{\qquad\qquad} \qquad n$$

(b) Destructive interference occurs when $d \quad \lambda \quad \lambda \quad \lambda$ or $d \quad v \quad f \quad v \quad f \quad v \quad f$ The possible frequencies are

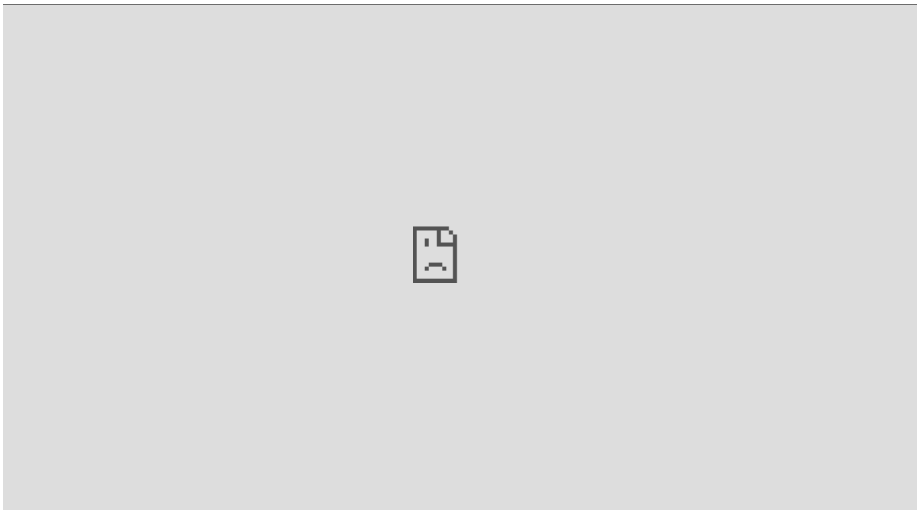
$$f_n \qquad \frac{nv}{d} \qquad n\overline{\qquad\qquad} \qquad n$$

EVALUATE As we increase the frequency, the sound at point P alternates between large and small (near zero) amplitudes, with maxima and minima at the frequencies given above. This effect may not be strong in an ordinary room because of reflections from the walls, floor, and ceiling.

KEY CONCEPT

Two sound waves of the same frequency interfere *constructively* at a certain point if the waves arrive there in phase. If the waves arrive at that point out of phase, they interfere *destructively*.

Video Tutor Solution: **Example 16.13**



Interference is the principle behind active noise-reduction headsets, which are used in loud environments such as airplane cockpits (Fig. 16.24). A microphone on the headset detects outside noise, and the headset circuitry replays the noise inside the headset shifted in phase by one half-cycle. This phase-shifted sound interferes destructively with the sounds that enter the headset from outside, so the headset wearer experiences very little unwelcome noise.

Figure 16.24

This aviation headset uses destructive interference to minimize the amount of noise from wind and propellers that reaches the wearer's ears.

Test Your Understanding of Section 16.6

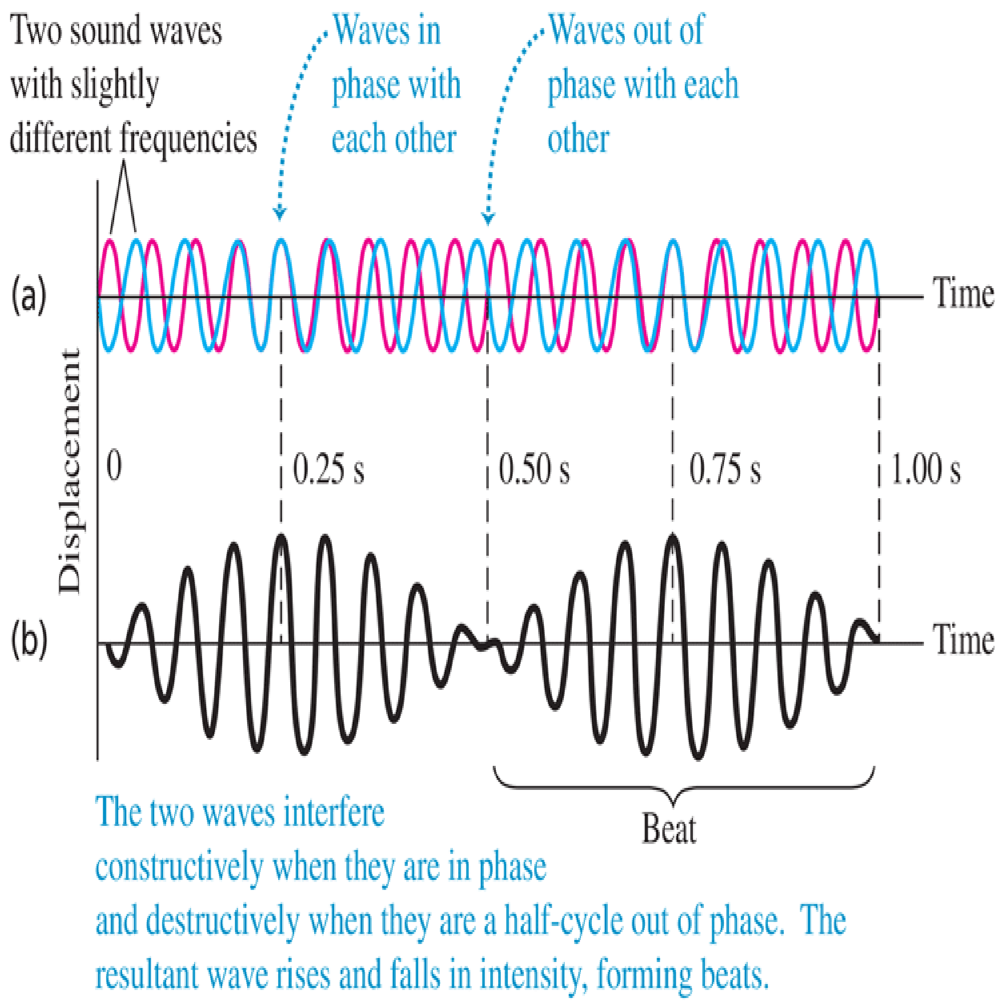
Suppose that speaker A in Fig. 16.23 emits a sinusoidal sound wave of frequency 500 Hz and speaker B emits a sinusoidal sound wave of frequency 1000 Hz. What sort of interference will there be between these two waves? (i) Constructive interference at various points, including point P, and destructive interference at various other points; (ii) destructive interference at various points, including point P, and constructive interference at various points; (iii) neither (i) nor (ii).

16.7 Beats

In Section 16.6 we talked about *interference* effects that occur when two different waves with the same frequency overlap in the same region of space. Now let's look at what happens when we have two waves with equal amplitude but slightly different frequencies. This occurs, for example, when two tuning forks with slightly different frequencies are sounded together, or when two organ pipes that are supposed to have exactly the same frequency are slightly "out of tune."

Consider a particular point in space where the two waves overlap. In Fig. 16.25a we plot the displacements of the individual waves at this point as functions of time. The total length of the time axis represents 1 second, and the frequencies are 16 Hz (blue graph) and 18 Hz (red graph). Applying the principle of superposition, we add the two displacement functions to find the total displacement function. The result is the graph of Fig. 16.25b. At certain times the two waves are in phase; their maxima coincide and their amplitudes add. But at certain times (like $t = 0.50$ s in Fig. 16.25) the two waves are exactly *out* of phase. The two waves then cancel each other, and the total amplitude is zero.

Figure 16.25



Beats are fluctuations in amplitude produced by two sound waves of slightly different frequency, here 16 Hz and 18 Hz. (a) Individual waves. (b) Resultant wave formed by superposition of the two waves. The beat frequency is 18 Hz − 16 Hz = 2 Hz.

The resultant wave in Fig. 16.25b looks like a single sinusoidal wave with an amplitude that varies from a maximum to zero and back. In this example the amplitude goes through two maxima and two minima in 1 second, so the frequency of this amplitude variation is 2 Hz. The amplitude variation causes variations of loudness called **beats,** and the frequency with which the loudness varies is called the **beat frequency.** In this example the beat frequency is the *difference* of the two frequencies. If the beat frequency is a few hertz, we hear it as a waver or pulsation in the tone.

We can prove that the beat frequency is *always* the difference of the two frequencies f_a and f_b. Suppose f_a is larger than f_b; the corresponding periods are T_a and T_b, with $T_a < T_b$. If the two waves start out in phase at time $t = 0$, they are again in phase when the first wave has gone through exactly one more cycle than the second. This happens at a value of t equal to T_{beat}, the *period* of the beat. Let n be the number of cycles of the first wave in time T_{beat}; then the number of cycles of the second wave in the same time is $(n - 1)$, and we have the relationships

$$T_{\text{beat}} = nT_a \qquad \text{and} \qquad T_{\text{beat}} = (n-1)T_b$$

Eliminating n between these two equations, we find

$$T_{\text{beat}} = \frac{T_a T_b}{T_b - T_a}$$

The reciprocal of the beat period is the beat *frequency*, $f_{\text{beat}} = 1/T_{\text{beat}}$, so

$$f_{\text{beat}} = \frac{T_b - T_a}{T_a T_b} = \frac{1}{T_a} - \frac{1}{T_b}$$

and finally

(16.24)

Beat frequency for waves a and b

$$f_{\text{beat}} = f_a - f_b \qquad (16.24)$$

Frequency of wave a

Frequency of wave b (lower than f_a)

As claimed, the beat frequency is the difference of the two frequencies.

CAUTION Beat frequency tells you only the difference in frequency between two sound waves The beat frequency for two sound waves always equals the higher frequency minus the lower frequency, so its value alone doesn't tell you which wave frequency is higher. For example, if you hear a beat frequency of 1 Hz and you know one of the sounds has frequency 256 Hz, the frequency of the other sound could be either 257 Hz or 255 Hz.

An alternative way to derive Eq. (16.24) is to write functions to describe the curves in Fig. 16.25a and then add them. Suppose that at a certain position the two waves are given by $y_a(t) = A \sin 2\pi f_a t$ and $y_b(t) = -A \sin 2\pi f_b t$. We use the trigonometric identity

$$\sin a - \sin b = 2 \sin \tfrac{1}{2}(a - b) \cos \tfrac{1}{2}(a + b)$$

We can then express the total wave $y(t) = y_a(t) + y_b(t)$ as

$$y_a(t) + y_b(t) = [2A \sin \frac{1}{2}(2\pi)(f_a - f_b)t] \cos \frac{1}{2}(2\pi)(f_a + f_b)t$$

The amplitude factor (the quantity in brackets) varies slowly with frequency $\frac{1}{2}(f_a - f_b)$. The cosine factor varies with a frequency equal to the *average* frequency $\frac{1}{2}(f_a + f_b)$. The *square* of the amplitude factor, which is proportional to the intensity that the ear hears, goes through two maxima and two minima per cycle. So the beat frequency f_{beat} that is heard is twice the quantity $\frac{1}{2}(f_a - f_b)$, or just $f_a - f_b$, in agreement with Eq. (16.24).

Beats between two tones can be heard up to a beat frequency of about 6 or 7 Hz. Two piano strings or two organ pipes differing in frequency by 2 or 3 Hz sound wavery and "out of tune," although some organ stops contain two sets of pipes deliberately tuned to beat frequencies of about 1 to 2 Hz for a gently undulating effect. Listening for beats is an important technique in tuning all musical instruments. *Avoiding* beats is part of the task of flying a multiengine propeller airplane (Fig. 16.26).

Figure 16.26

If the two propellers on this airplane are not precisely synchronized, the pilots, passengers, and listeners on the ground will hear beats as loud, annoying, throbbing sounds. On some airplanes the propellers are synched electronically; on others the pilot does it by ear, like tuning a piano.

At frequency differences greater than about 6 or 7 Hz, we no longer hear individual beats, and the sensation merges into one of *consonance* or *dissonance,* depending on the frequency ratio of the two tones. In some cases the ear perceives a tone called a *difference tone,* with a pitch equal to the beat frequency of the two tones. For example, if you listen to a whistle that produces sounds at 1800 Hz and 1900 Hz when blown, you'll hear not only these tones but also a much lower 100 Hz tone.

Test Your Understanding of Section 16.7

One tuning fork vibrates at 440 Hz, while a second tuning fork vibrates at an unknown frequency. When both tuning forks are sounded simultaneously, you hear a tone that rises and falls in intensity three times per second. What is the frequency of the second tuning fork? (i) 434 Hz; (ii) 437 Hz; (iii) 443 Hz; (iv) 446 Hz; (v) either 434 Hz or 446 Hz; (vi) either 437 Hz or 443 Hz.

16.8 The Doppler Effect

When a car approaches you with its horn sounding, the pitch seems to drop as the car passes. This phenomenon, first described by the 19th-century Austrian scientist Christian Doppler, is called the **Doppler effect.** When a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. A similar effect occurs for light and radio waves; we'll return to this later in this section.

To analyze the Doppler effect for sound, we'll work out a relationship between the frequency shift and the velocities of source and listener relative to the medium (usually air) through which the sound waves propagate. To keep things simple, we consider only the special case in which the velocities of both source and listener lie along the line joining them. Let v_{S} and v_{L} be the velocity components along this line for the source and the listener, respectively, relative to the medium. We choose the positive direction for both v_{S} and v_{L} to be the direction from the listener L to the source S. The speed of sound relative to the medium, v, is always considered positive.

Moving Listener and Stationary Source

Let's think first about a listener L moving with velocity v_{L} toward a stationary source S (Fig. 16.27). The source emits a sound wave with frequency f_{S} and wavelength $\lambda = v/f_{\mathrm{S}}$. The figure shows four wave crests, separated by equal distances λ. The wave crests approaching the moving listener have a speed of propagation *relative to the listener* of $(v + v_{\mathrm{L}})$. So the frequency f_{L} with which the crests arrive at the listener's position (that is, the frequency the listener hears) is

(16.25)

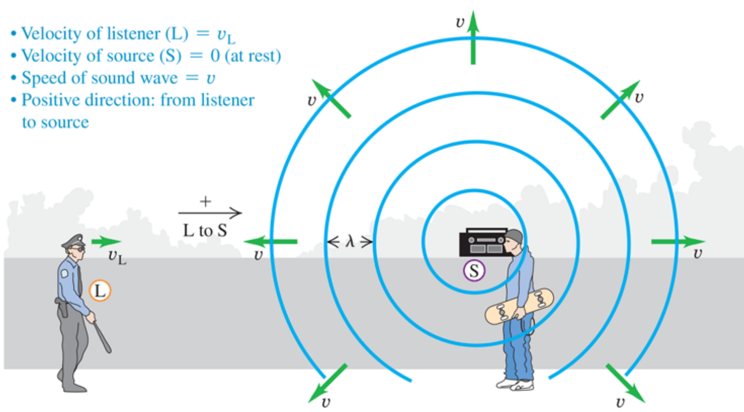
$$f_{\mathrm{L}} = \frac{v + v_{\mathrm{L}}}{\lambda} = \frac{v + v_{\mathrm{L}}}{v/f_{\mathrm{S}}}$$

or

(16.26)

$$f_{\mathrm{L}} = \left(\frac{v + v_{\mathrm{L}}}{v}\right) f_{\mathrm{S}} = \left(1 + \frac{v_{\mathrm{L}}}{v}\right) f_{\mathrm{S}} \quad \text{(moving listener, stationary source)}$$

Figure 16.27



A listener moving toward a stationary source hears a frequency that is higher than the source frequency. This is because the relative speed of listener and wave is greater than the wave speed v.

So a listener moving toward a source ($v_{\mathrm{L}} > 0$), as in Fig. 16.27, hears a higher frequency (higher pitch) than does a stationary listener. A listener moving away from the source ($v_{\mathrm{L}} < 0$) hears a lower frequency (lower pitch).

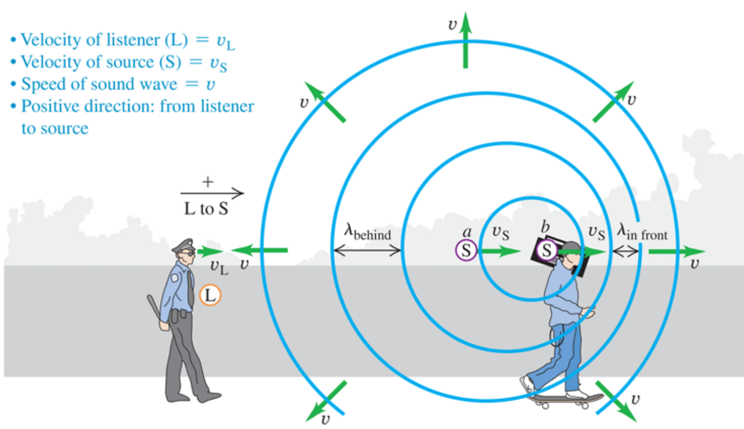
Moving Source and Moving Listener

Now suppose the source is also moving, with velocity v_S (Fig. 16.28). The wave speed relative to the wave medium (air) is still v; it is determined by the properties of the medium and is not changed by the motion of the source. But the wavelength is no longer equal to v/f_S. Here's why. The time for emission of one cycle of the wave is the period $T = 1/f_S$. During this time, the wave travels a distance v T$= v/f_S$ and the source moves a distance $v_S T = v_S/f_S$. The wavelength is the distance between successive wave crests, and this is determined by the *relative* displacement of source and wave. As Fig. 16.28 shows, this is different in front of and behind the source. In the region to the right of the source in Fig. 16.28 (that is, in front of the source), the wavelength is

(16.27)

$$\lambda_{\text{in front}} = \frac{v}{f_S} - \frac{v_S}{f_S} = \frac{v - v_S}{f_S} \quad \text{(wavelength in front of a moving source)}$$

Figure 16.28



Wave crests emitted by a source moving from a to b are crowded together in front of the source (to the right of this source) and stretched out behind it (to the left of this source).

In the region to the left of the source (that is, behind the source), it is

(16.28)

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S} \quad \text{(wavelength behind a moving source)}$$

The waves in front of and behind the source are compressed and stretched out, respectively, by the motion of the source.

To find the frequency heard by the listener behind the source, we substitute Eq. (16.28) into the first form of Eq. (16.25):

$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{(v + v_S)/f_S}$$

(16.29)

Doppler effect for moving listener L and moving source S:

Frequency heard by listener

Velocity of listener (+ if from L toward S, − if opposite)

$$f_L = \frac{v + v_L}{v + v_S} f_S \qquad (16.29)$$

Frequency emitted by source

Speed of sound

Velocity of source (+ if from L toward S, − if opposite)

Although we derived it for the particular situation shown in Fig. 16.28, Eq. (16.29) includes *all* possibilities for motion of source and listener (relative to the medium) along the line joining them. If the listener happens to be at rest in the medium, v_L is zero. When both source and listener are at rest or have the same velocity relative to the medium, $v_L = v_S$ and $f_L = f_S$. Whenever the direction of the source or listener velocity is opposite to the direction from the listener toward the source (which we have defined as positive), the corresponding velocity to be used in Eq. (16.29) is negative.

As an example, the frequency heard by a listener at rest ($v_L = 0$) is $f_L = [v/(v + v_S)] f_S$. If the source is moving toward the listener (in the negative direction), then $v_S < 0$, $f_L > f_S$, and the listener hears a higher frequency than that emitted by the source. If instead the source is moving away from the listener (in the positive direction), then $v_S > 0$, $f_L < f_S$, and the listener hears a lower frequency. This explains the change in pitch that you hear from the siren of an ambulance as it passes you (Fig. 16.29).

Figure 16.29

The Doppler effect explains why the siren on a fire engine or ambulance has a high pitch ($f_L > f_S$) when it is approaching you ($v_S < 0$) and a low pitch ($f_L < f_S$) when it is moving away ($v_S > 0$).

Problem-Solving Strategy 16.2 Doppler Effect

IDENTIFY *the relevant concepts:* The Doppler effect occurs whenever the source of waves, the wave detector (listener), or both are in motion.

SET UP *the problem* using the following steps:

1. Establish a coordinate system, with the positive direction from the listener toward the source. Carefully determine the

signs of all relevant velocities. A velocity in the direction from the listener toward the source is positive; a velocity in the opposite direction is negative. All velocities must be measured relative to the air in which the sound travels.

2. Use consistent subscripts to identify the various quantities: S for source and L for listener.
3. Identify which unknown quantities are the target variables.

EXECUTE *the solution* as follows:

1. Use Eq. (16.29) to relate the frequencies at the source and the listener, the sound speed, and the velocities of the source and the listener according to the sign convention of step 1. If the source is moving, you can find the wavelength measured by the listener using Eq. (16.27) or (16.28).
2. When a wave is reflected from a stationary or moving surface, solve the problem in two steps. In the first, the surface is the "listener"; the frequency with which the wave crests arrive at the surface is f_L. In the second, the surface is the "source," emitting waves with this same frequency f_L. Finally, determine the frequency heard by a listener detecting this new wave.

EVALUATE *your answer:* Is the *direction* of the frequency shift reasonable? If the source and the listener are moving toward each other, $f_L > f_S$; if they are moving apart, $f_L < f_S$. If the source and the listener have no relative motion, $f_L = f_S$.

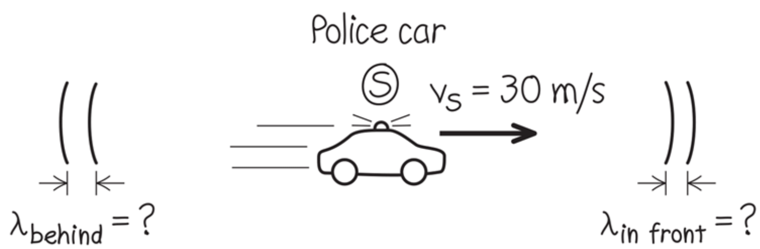
Example 16.14 Doppler effect I: Wavelengths

WITH VARIATION PROBLEMS

A police car's siren emits a sinusoidal wave with frequency $f_S = 300$ Hz. The speed of sound is 340 m/s and the air is still. (a) Find the wavelength of the waves if the siren is at rest. (b) Find the wavelengths of the waves in front of and behind the siren if it is moving at 30 m/s.

IDENTIFY and SET UP In part (a) there is no Doppler effect because neither source nor listener is moving with respect to the air; $v = \lambda f$ gives the wavelength. Figure 16.30 shows the situation in part (b): The source is in motion, so we find the wavelengths using Eqs. (16.27) and (16.28) for the Doppler effect.

Figure 16.30



Our sketch for this problem.

EXECUTE (a) When the source is at rest,

$$\lambda = \frac{v}{f_S} = \frac{340 \text{ m/s}}{300 \text{ Hz}} = 1.13 \text{ m}$$

(b) From Eq. (16.27), in front of the siren

$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S} = \frac{340 \text{ m/s} - 30 \text{ m/s}}{300 \text{ Hz}} = 1.03 \text{ m}$$

From Eq. (16.28), behind the siren

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S} = \frac{340 \text{ m/s} + 30 \text{ m/s}}{300 \text{ Hz}} = 1.23 \text{ m}$$

EVALUATE The wavelength is shorter in front of the siren and longer behind it, as we expect.

KEY CONCEPT

If a source of sound is moving through still air, a listener behind the source hears a sound of increased wavelength. A listener in front of the source hears a sound of decreased wavelength.

Video Tutor Solution: Example 16.14

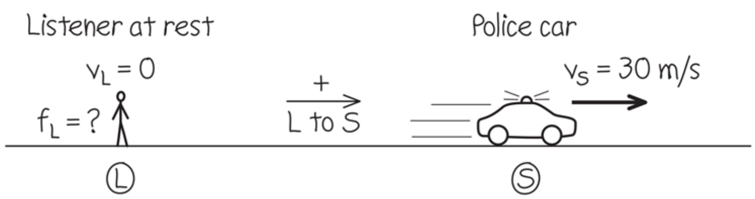
Example 16.15 Doppler effect II: Frequencies

WITH VARIATION PROBLEMS

If a listener L is at rest and the siren in Example 16.14 is moving away from L at 30 m/s, what frequency does the listener hear?

IDENTIFY and SET UP Our target variable is the frequency f_{L} heard by a listener behind the moving source. Figure 16.31 shows the situation. We have $v_{\mathrm{L}} = 0$ and $v_{\mathrm{S}} = +30\ \mathrm{m}/s$ (positive, since the velocity of the source is in the direction from listener to source).

Figure 16.31



Our sketch for this problem.

EXECUTE From Eq. (16.29),

$$f_L = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + 30 \text{ m/s}} (300 \text{ Hz}) = 276 \text{ Hz}$$

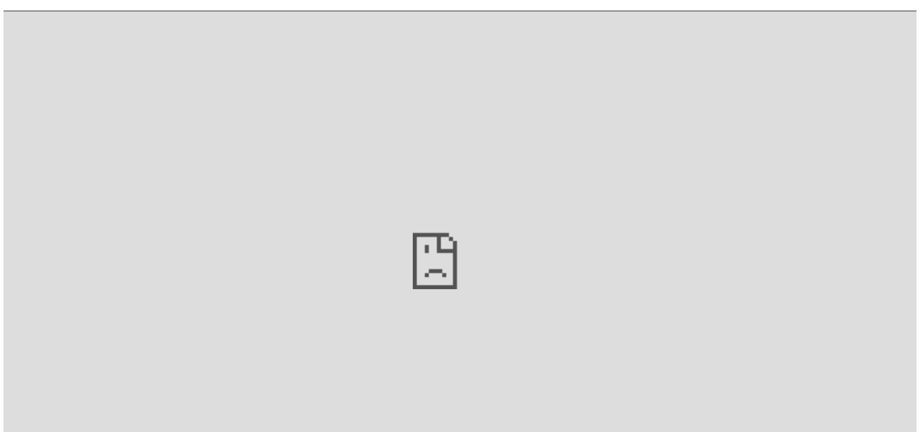
EVALUATE The source and listener are moving apart, so $f_L < f_S$. Here's a check on our numerical result. From Example 16.14, the wavelength behind the source (where the listener in Fig. 16.31 is located) is 1.23 m. The wave speed relative to the stationary listener is $v = 340 \text{ m}/s$ even though the source is moving, so

$$f_L = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.23 \text{ m}} = 276 \text{ Hz}$$

KEY CONCEPT

If a source of sound is moving through still air, a listener behind the source hears a sound of decreased frequency. A listener in front of the source hears a sound of increased frequency.

Video Tutor Solution: **Example 16.15**



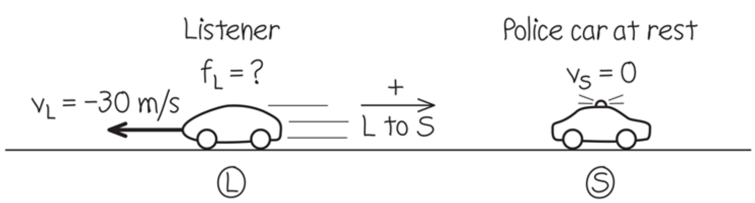
Example 16.16 Doppler effect III: A moving listener

WITH VARIATION PROBLEMS

If the siren is at rest and the listener is moving away from it at $30\ \mathrm{m}/s$, what frequency does the listener hear?

IDENTIFY and SET UP Again our target variable is f_L, but now L is in motion and S is at rest. Figure 16.32 shows the situation. The velocity of the listener is $v_L = -30\ \mathrm{m}/s$ (negative, since the motion is in the direction from source to listener).

Figure 16.32



Our sketch for this problem.

EXECUTE From Eq. (16.29),

$$f_L = \frac{v + v_L}{v} f_S = \frac{340\ \mathrm{m/s} + (-30\ \mathrm{m/s})}{340\ \mathrm{m/s}} (300\ \mathrm{Hz}) = 274\ \mathrm{Hz}$$

EVALUATE Again the source and listener are moving apart, so $f_L < f_S$. Note that the *relative velocity* of source and listener is the same as in Example 16.15, but the Doppler shift is different because v_S and v_L are different.

KEY CONCEPT

If a listener is moving away from a source of sound, the listener hears a sound of decreased frequency. If the listener is moving toward the source, the listener hears a sound of increased frequency.

Video Tutor Solution: **Example 16.16**

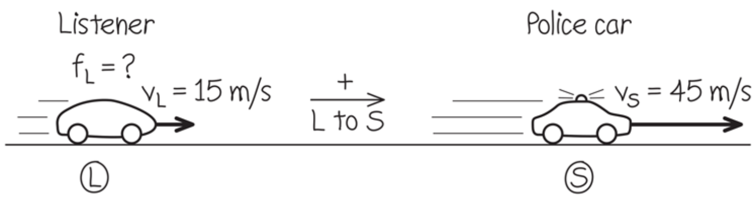
Example 16.17 Doppler effect IV: Moving source, moving listener

WITH VARIATION PROBLEMS

The siren is moving away from the listener with a speed of $45\ \mathrm{m/s}$ relative to the air, and the listener is moving toward the siren with a speed of $15\ \mathrm{m}/s$ relative to the air. What frequency does the listener hear?

IDENTIFY and SET UP Now *both* L and S are in motion (Fig. 16.33). Again our target variable is f_L. Both the source velocity $v_\mathrm{S} = +45\ \mathrm{m/s}$ and the listener's velocity $v_\mathrm{L} = +15\ \mathrm{m}/s$ are positive because both velocities are in the direction from listener to source.

Figure 16.33



Our sketch for this problem.

EXECUTE From Eq. (16.29),

$$f_{\mathrm{L}} = \frac{v + v_{\mathrm{L}}}{v + v_{\mathrm{S}}} f_{\mathrm{S}} = \frac{340\ \mathrm{m/s} + 15\ \mathrm{m/s}}{340\ \mathrm{m/s} + 45\ \mathrm{m/s}} (300\ \mathrm{Hz}) = 277\ \mathrm{Hz}$$

EVALUATE As in Examples 16.15 and 16.16, the source and listener again move away from each other at 30 m/s, so again $f_{\mathrm{L}} < f_S$. But f_{L} is different in all three cases because the Doppler effect for sound depends on how the source and listener are moving relative to the *air*, not simply on how they move relative to each other.

KEY CONCEPT

When a listener and source of sound are both moving relative to the air, the listener hears a sound of decreased frequency if the listener and source are moving apart. The listener hears a sound of increased frequency if the listener and source are moving closer together.

Video Tutor Solution: **Example 16.17**

Example 16.18 Doppler effect V: A double Doppler shift

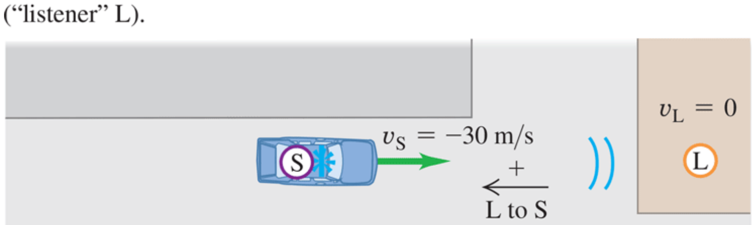
WITH VARIATION PROBLEMS

The police car is moving toward a warehouse at 30 m/s. What frequency does the driver hear reflected from the warehouse?

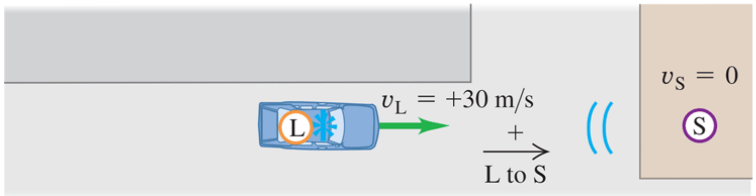
IDENTIFY This situation has *two* Doppler shifts (Fig. 16.34). In the first shift, the warehouse is the stationary "listener." The frequency of sound reaching the warehouse, which we call f_W, is greater than 300 Hz because the source is approaching. In the second shift, the warehouse acts as a source of sound with frequency f_W, and the listener is the driver of the police car; she hears a frequency greater than f_W because she is approaching the source.

Figure 16.34

(a) Sound travels from police car's siren (source S) to warehouse ("listener" L).



(b) Reflected sound travels from warehouse (source S) to police car (listener L).



Two stages of the sound wave's motion from the police car to the warehouse and back to the police car.

SET UP To determine f_W, we use Eq. (16.29) with f_L replaced by f_W. For this part of the problem, $v_L = v_W = 0$ (the warehouse is at rest) and $v_S = -30 \text{ m}/s$ (the siren is moving in the negative direction from source to listener).

To determine the frequency heard by the driver (our target variable), we again use Eq. (16.29) but now with f_S replaced by f_W. For this second part of the problem, $v_S = 0$ because the stationary warehouse is the source and the velocity of the listener (the driver) is $v_L = +30 \text{ m/s}$. (The listener's velocity is positive because it is in the direction from listener to source.)

EXECUTE The frequency reaching the warehouse is

$$f_W = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + (-30 \text{ m/s})} (300 \text{ Hz})$$

$$= 329 \text{ Hz}$$

Then the frequency heard by the driver is

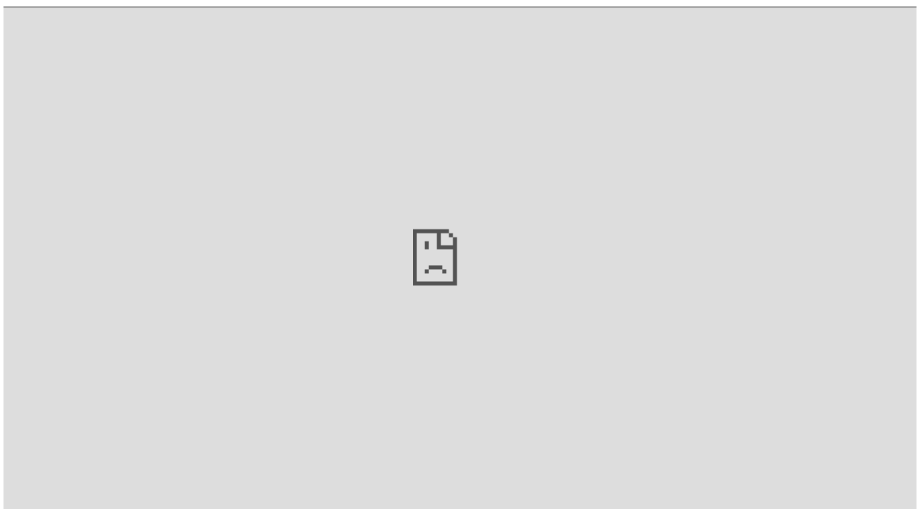
$$f_L = \frac{v + v_L}{v} f_W = \frac{340 \text{ m/s} + 30 \text{ m/s}}{340 \text{ m/s}} (329 \text{ Hz}) = 358 \text{ Hz}$$

EVALUATE Because there are two Doppler shifts, the reflected sound heard by the driver has an even higher frequency than the sound heard by a stationary listener in the warehouse.

KEY CONCEPT

If a source of sound moves relative to a wall (or other reflecting surface), there are two shifts in the frequency of the sound: The frequency received by and reflected from the wall is shifted compared to the sound emitted by the source, and the frequency received back at the source is shifted compared to the sound reflected from the wall. Both shifts increase the frequency if the source is approaching the wall, and both shifts decrease the frequency if the source is moving away from the wall.

Video Tutor Solution: **Example 16.18**



Doppler Effect for Electromagnetic Waves

In the Doppler effect for sound, the velocities v_L and v_S are always measured relative to the *air* or whatever medium we are considering. There is also a Doppler effect for *electromagnetic* waves in empty space, such as light waves or radio waves. In this case there is no medium that we can use as a reference to measure velocities, and all that matters is the *relative* velocity of source and receiver. (By contrast, the Doppler effect for sound does not depend simply on this relative velocity, as discussed in Example 16.17.)

To derive the expression for the Doppler frequency shift for light, we have to use the special theory of relativity. We'll discuss this in Chapter 37, but for now we quote the result without derivation. The wave speed is the speed of light, usually denoted by c, and it is the same for both source and receiver. In the frame of reference in which the receiver is at rest, the source is moving away from the receiver with velocity v. (If the source is *approaching* the receiver, v is negative.) The source frequency is again f_S. The frequency f_R measured by the receiver R (the frequency of arrival of the waves at the receiver) is then

(16.30)

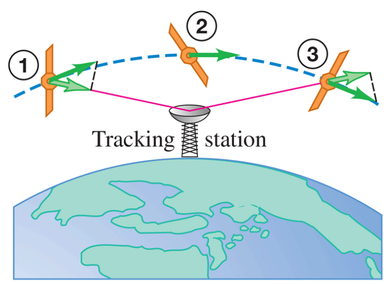
$$f_{\mathrm{R}} = \sqrt{\frac{c-v}{c+v}}\, f_{\mathrm{S}} \quad \text{(Doppler effect for light)}$$

When v is positive, the source is moving directly *away* from the receiver and f_R is always *less* than f_S; when v is negative, the source is moving directly *toward* the receiver and f_R is *greater* than f_S. The qualitative effect is the same as for sound, but the quantitative relationship is different.

A familiar application of the Doppler effect for radio waves is the radar device mounted on the side window of a police car to check other cars' speeds. The electromagnetic wave emitted by the device is reflected from a moving car, which acts as a moving source, and the wave reflected back to the device is Doppler-shifted in frequency. The transmitted and reflected signals are combined to produce beats, and the speed can be computed from the frequency of the beats. Similar techniques ("Doppler radar") are used to measure wind velocities in the atmosphere.

The Doppler effect is also used to track satellites and other space vehicles. In Fig. 16.35 a satellite emits a radio signal with constant frequency f_S. As the satellite orbits past, it first approaches and then moves away from the receiver; the frequency f_R of the signal received on earth changes from a value greater than f_S to a value less than f_S as the satellite passes overhead.

Figure 16.35



Change of velocity component along the line of sight of a satellite passing a tracking station. The frequency received at the tracking station changes from high to low as the satellite passes overhead.

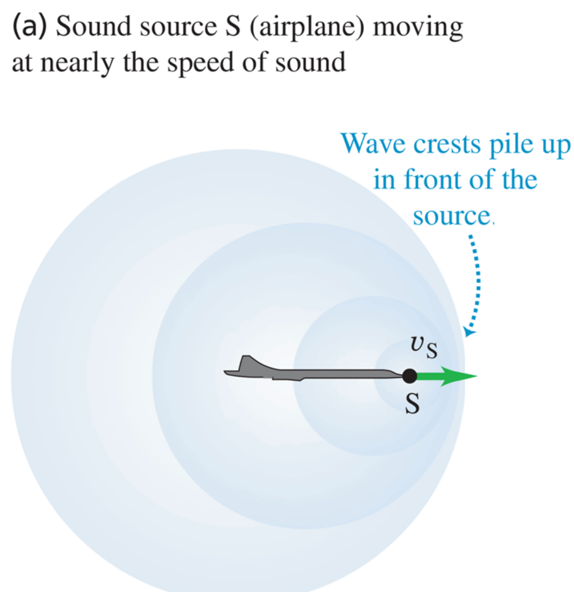
Test Your Understanding of Section 16.8

You are at an outdoor concert with a wind blowing at 10 m/s from the performers toward you. Is the sound you hear Doppler-shifted? If so, is it shifted to lower or higher frequencies?

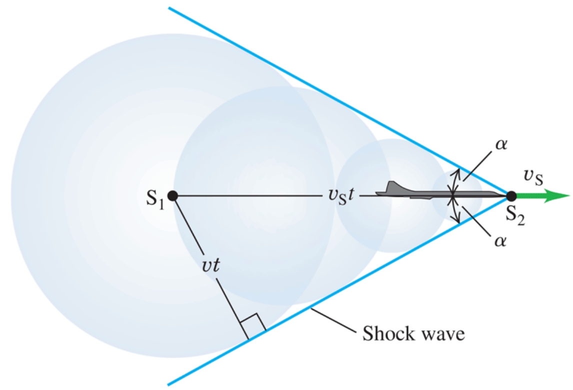
16.9 Shock Waves

You may have experienced "sonic booms" caused by an airplane flying overhead faster than the speed of sound. We can see qualitatively why this happens from Fig. 16.36. Let denote the *speed* of the airplane relative to the air, so that it is always positive. The motion of the airplane through the air produces sound; if is less than the speed of sound the waves in front of the airplane are crowded together with a wavelength given by Eq. (16.27):

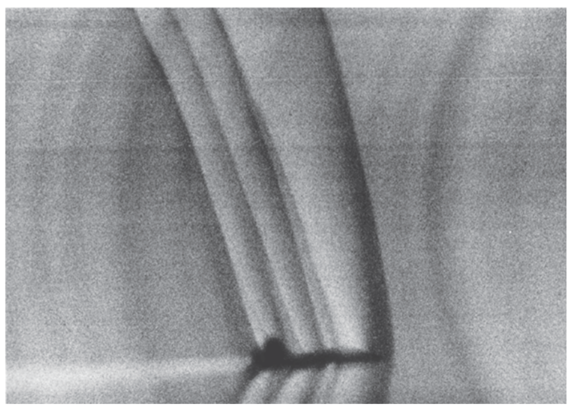
Figure 16.36



(b) Sound source moving faster than the speed of sound



(c) Shock waves around a supersonic airplane



Wave crests around a sound source S moving (a) slightly slower than the speed of sound and (b) faster than the sound speed (c) This photograph shows a T-38 jet airplane moving at 1.1 times the speed of sound. Separate shock waves are produced by the nose, wings, and tail. The angles of these waves vary because the air speeds up and slows down as it moves around the airplane, so the relative speed of the airplane and air is different for shock waves produced at different points.

As the speed of the airplane approaches the speed of sound the wavelength approaches zero and the wave crests pile up on each other (Fig. 16.36a). The airplane must exert a large force to compress the air in front of it; by Newton's third law, the air exerts an equally large force back on the airplane. Hence there is a large increase in aerodynamic drag (air resistance) as the airplane approaches the speed of sound, a phenomenon known as the "sound barrier."

When is greater in magnitude than the source of sound is **supersonic,** and Eqs. (16.27) and (16.29) for the Doppler effect no longer describe the sound wave in front of the source. Figure 16.36b shows a cross section of what happens. As the airplane moves, it displaces the surrounding air and produces sound. A series of wave crests is emitted from the nose of the airplane; each spreads out in a circle centered at the position of the airplane when it emitted the crest. After a time the crest emitted from point has spread to a circle with radius and the airplane has moved a greater distance to position You can see that the circular crests interfere constructively at points along the blue line that makes an angle with the direction of the airplane velocity, leading to a very-large-amplitude wave crest along this line. This large-amplitude crest is called a **shock wave** (Fig. 16.36c).

From the right triangle in Fig. 16.36b we can see that , or

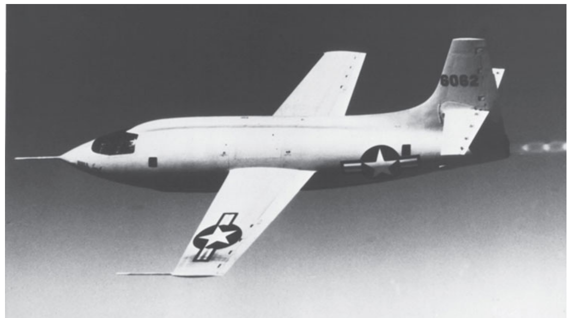
(16.31)

Shock wave produced by sound source moving faster than sound:

$$\sin \overset{\text{Angle of shock wave}}{\alpha} = \frac{v \leftarrow \text{Speed of sound}}{v_S \leftarrow \text{Speed of source}} \qquad (16.31)$$

The ratio is called the **Mach number.** It is greater than unity for all supersonic speeds, and in Eq. (16.31) is the reciprocal of the Mach number. The first person to break the sound barrier was Capt. Chuck Yeager of the U.S. Air Force, flying the Bell X-1 at Mach 1.06 on October 14, 1947 (Fig. 16.37).

Figure 16.37



The first supersonic airplane, the Bell X-1, was shaped much like a 50 caliber bullet—which was known to be able to travel faster than sound.

Shock waves are actually three-dimensional; a shock wave forms a *cone* around the direction of motion of the source. If the source (possibly a supersonic jet airplane or a rifle bullet) moves with constant velocity, the angle is constant, and the shock-wave cone moves along with the source. It's the arrival of this shock wave that causes the sonic boom you hear after a supersonic airplane has passed by. In front of the shock-wave cone, there is no sound. Inside the cone a stationary listener hears the Doppler-shifted sound of the airplane moving away.

CAUTION Shock waves A shock wave is produced *continuously* by any object that moves through the air at supersonic speed, not only at the instant that it "breaks the sound barrier." The sound waves that combine to form the shock wave, as in Fig. 16.36b, are created by the motion of the object itself, not by any sound source that the object may carry. The cracking noises of a bullet and of the tip of a circus whip are due to their supersonic motion. A supersonic jet airplane may have very loud engines, but these do not cause the shock wave. If the pilot were to shut the engines off, the airplane would continue to produce a shock wave as long as its speed remained supersonic.

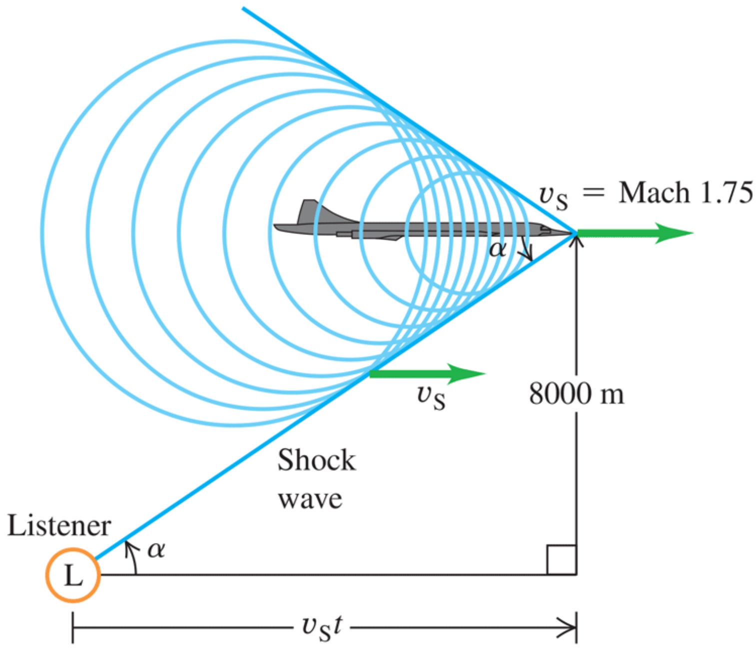
Shock waves have applications outside of aviation. They are used to break up kidney stones and gallstones without invasive surgery, using a technique with the impressive name *extracorporeal shock-wave lithotripsy.* A shock wave produced outside the body is focused by a reflector or acoustic lens so that as much of it as possible converges on the stone. When the resulting stresses in the stone exceed its tensile strength, it breaks into small pieces and can be eliminated. This technique requires accurate determination of the location of the stone, which may be done using ultrasonic imaging techniques (see Fig. 16.9).

Example 16.19 Sonic boom from a supersonic airplane

An airplane is flying at Mach 1.75 at an altitude of 8000 m, where the speed of sound is How long after the plane passes directly overhead will you hear the sonic boom?

IDENTIFY and SET UP The shock wave forms a cone trailing backward from the airplane, so the problem is really asking for how much time elapses from when the airplane flies overhead to when the shock wave reaches you at point L (Fig. 16.38). During the time (our target variable) since the airplane traveling at speed passed overhead, it has traveled a distance . Equation (16.31) gives the shock cone angle ; we use trigonometry to solve for .

Figure 16.38



You hear a sonic boom when the shock wave reaches you at L (*not* just when the plane breaks the sound barrier). A listener to the right of L has not yet heard the sonic boom but will shortly; a listener to the left of L has already heard the sonic boom.

EXECUTE From Eq. (16.31) the angle of the shock cone is

The speed of the plane is the speed of sound multiplied by the Mach number:

From Fig. 16.38 we have

EVALUATE You hear the boom 20.5 s after the airplane passes overhead, at which time it has traveled

since it passed overhead. We have assumed that the speed of sound is the same at all altitudes, so that is constant and the shock wave forms a perfect cone. In fact, the speed of sound decreases with increasing altitude. How would this affect the value of ?

KEY CONCEPT

An object moving through the air faster than the speed of sound continuously produces a cone-shaped shock wave. The angle of the cone depends on the object's Mach number (the ratio of its speed to the speed of sound).

Video Tutor Solution: **Example 16.19**

Test Your Understanding of Section 16.9

What would you hear if you were directly behind (to the left of) the supersonic airplane in Fig. 16.38? (i) A sonic boom; (ii) the sound of the airplane, Doppler-shifted to higher frequencies; (iii) the sound of the airplane, Doppler-shifted to lower frequencies; (iv) nothing.

Chapter 16 Summary

Sound waves: Sound consists of longitudinal waves in a medium. A sinusoidal sound wave is characterized by its frequency f and wavelength λ (or angular frequency ω and wave number k) and by its displacement amplitude A. The pressure amplitude $p_{\max}$ is directly proportional to the displacement amplitude, the wave number, and the bulk modulus B of the wave medium. (See Examples 16.1 and 16.2.)

The speed of a sound wave in a fluid depends on the bulk modulus B and density ρ. If the fluid is an ideal gas, the speed can be expressed in terms of the temperature T, molar mass M, and ratio of heat capacities γ of the gas. The speed of longitudinal waves in a solid rod depends on the density and Young's modulus Y. (See Examples 16.3 and 16.4.)

(16.5)

$$p_{\max} = BkA$$

(sinusoidal sound wave)

(16.7)

$$v = \sqrt{\frac{B}{\rho}}$$

(longitudinal wave in a fluid)

(16.10)

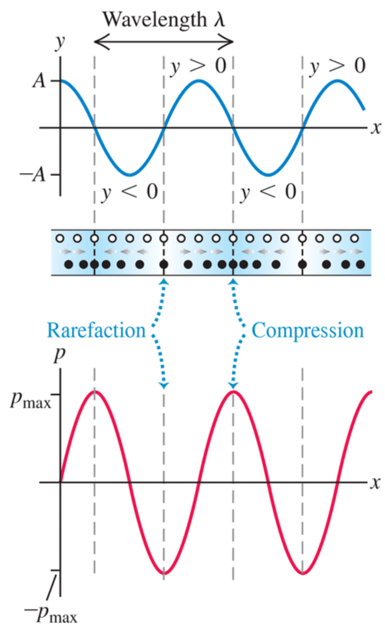
$$v = \sqrt{\frac{\gamma RT}{M}}$$

(sound wave in an ideal gas)

(16.8)

$$v = \sqrt{\frac{Y}{\rho}}$$

(longitudinal wave in a solid rod)



Intensity and sound intensity level: The intensity I of a sound wave is the time average rate at which energy is transported by the wave, per unit area. For a sinusoidal wave, the intensity can be expressed in terms of the displacement amplitude A or the pressure amplitude $p_{\max}$. (See Examples 16.5, 16.6 and 16.7.)

The sound intensity level β of a sound wave is a logarithmic measure of its intensity. It is measured relative to I_0 , an arbitrary intensity defined to be $10^{-12}\ \mathrm{W/m^2}$. Sound intensity levels are expressed in decibels (dB). (See Examples 16.8 and 16.9.)

(16.12), (16.14)

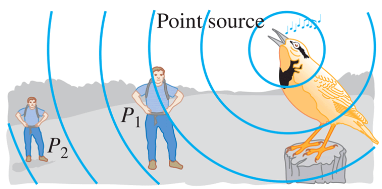
$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 = \frac{{p_{\max}}^2}{2\rho v} = \frac{{p_{\max}}^2}{2\sqrt{\rho B}}$$

(intensity of a sinusoidal sound wave in a fluid)

(16.15)

$$\beta = (10\text{ dB})\log\frac{I}{I_0}$$

(definition of sound intensity level)



Standing sound waves: Standing sound waves can be set up in a pipe or tube. A closed end is a displacement node and a pressure antinode; an open end is a displacement antinode and a pressure node. For a pipe of length L open at both ends, the normal-mode frequencies are integer multiples of the sound speed divided by $2L$. For a stopped pipe (one that is open at only one end), the normal-mode frequencies are the odd multiples of the sound speed divided by $4L$. (See Examples 16.10 and 16.11.)

A pipe or other system with normal-mode frequencies can be driven to oscillate at any frequency. A maximum response, or resonance, occurs if the driving frequency is close to one of the normal-mode frequencies of the system. (See Example 16.12.)

(16.18)

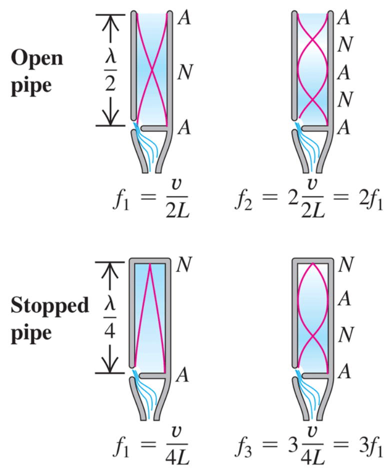
$$f_n = \frac{nv}{2L} \qquad (n = 1,\ 2,\ 3,\ \ldots)$$

(open pipe)

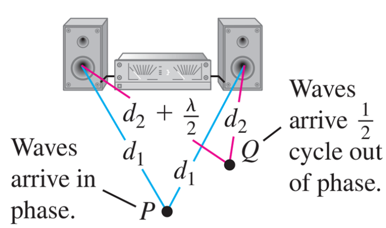
(16.22)

$$f_n = \frac{nv}{4L} \qquad (n = 1,\ 3,\ 5,\ \ldots)$$

(stopped pipe)



Interference: When two or more waves overlap in the same region of space, the resulting effects are called interference. The resulting amplitude can be either larger or smaller than the amplitude of each individual wave, depending on whether the waves are in phase (constructive interference) or out of phase (destructive interference). (See Example 16.13.)

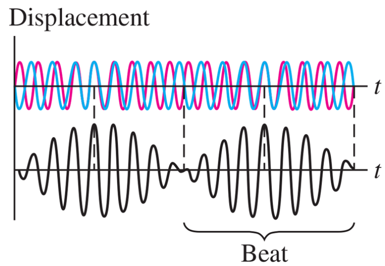


Beats: Beats are heard when two tones with slightly different frequencies f_a and f_b are sounded together. The beat frequency f_{beat} is the difference between f_a and f_b.

(16.24)

$$f_{\text{beat}} = f_a - f_b$$

(beat frequency)

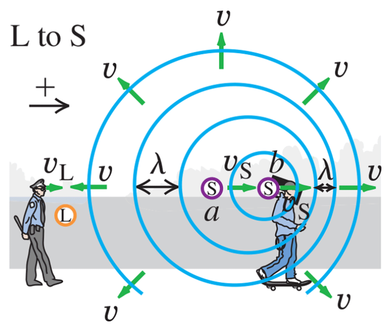


Doppler effect: The Doppler effect for sound is the frequency shift that occurs when there is motion of a source of sound, a listener, or both, relative to the medium. The source and listener frequencies f_S and f_L are related by the source and listener velocities v_S and v_L relative to the medium and to the speed of sound v. (See Examples 16.14, 16.15, 16.16, 16.17 and 16.18.)

(16.29)

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

(Doppler effect, moving source and moving listener)

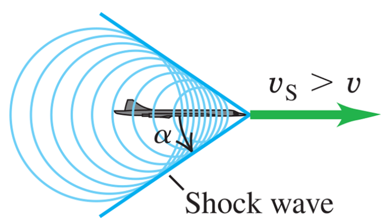


Shock waves: A sound source moving with a speed v_S greater than the speed of sound v creates a shock wave. The wave front is a cone with angle α. (See Example 16.19.)

(16.31)

$$\sin\alpha = \frac{v}{v_S}$$

(shock wave)



Guided Practice: Sound and Hearing

For assigned homework and other learning materials, go to **Mastering Physics**.

Key Example Variation Problems

Be sure to review Examples 16.5, 16.6, 16.7, 16.8, and 16.9 (Section 16.3) before attempting these problems.

VP16.9.1 A 256 Hz sound wave in air (density $1.20\,\mathrm{kg/m^3}$, speed of sound $344\,\mathrm{m/s}$) has intensity $5.50 \times 10^{-8}\,\mathrm{W/m^2}$. (a) What is the wave's pressure amplitude? (b) If the intensity remains the same but the frequency is doubled to 512 Hz, how does this affect the pressure amplitude?

VP16.9.2 At a certain distance from a fire alarm, the sound intensity level is 85.0 dB. (a) What is the intensity of this sound? (b) How many times greater is the intensity of this sound than that of a 67.0 dB sound?

VP16.9.3 A lion can produce a roar with a sound intensity level of 114 dB at a distance of 1.00 m. What is the sound intensity level at a distance of (a) 4.00 m and (b) 15.8 m from the lion? Assume that intensity obeys the inverse-square law.

VP16.9.4 The sound intensity level inside a typical modern airliner in flight is 66.0 dB. The air in the cabin has density

$0.920\,\mathrm{kg/m^3}$ (less than in the atmosphere at sea level) and speed of sound $344\,\mathrm{m/s}$. (a) What is the pressure amplitude of this sound? (b) If the pressure amplitude were increased by a factor of 10.0, what would the new sound intensity level be?

Be sure to review Examples 16.11 and 16.12 (Sections 16.4 and 16.5) before attempting these problems.

VP16.12.1 A particular open organ pipe has a fundamental frequency of 220 Hz (known to musicians as A_3 or "A below middle C") when the speed of sound waves in air is $344\,\mathrm{m/s}$. (a) What is the length of this pipe? (b) The third harmonic of this pipe has the same frequency as the fundamental frequency of a stopped pipe. What is the length of this stopped pipe?

VP16.12.2 You have two organ pipes, one open and one stopped. Which harmonic (if any) of the stopped pipe has the same frequency as the third harmonic of the open pipe if the stopped pipe length is (a) $\frac{1}{6}$, (b) $\frac{1}{2}$, or (c) $\frac{1}{3}$ that of the open pipe?

VP16.12.3 One of the strings of a bass viol is 0.680 m long and has a fundamental frequency of 165 Hz. (a) What is the speed of waves on this string? (b) When this string vibrates at its fundamental frequency, it causes the air in a nearby stopped organ pipe to vibrate at that pipe's fundamental frequency. The speed of sound in the pipe is $344\,\mathrm{m/s}$. What is the length of this pipe?

VP16.12.4 A stopped pipe 1.00 m in length is filled with helium at $20^\circ\mathrm{C}$ (speed of sound $999\,\mathrm{m/s}$). When the helium in this pipe vibrates at its third harmonic frequency, it causes the air at $20^\circ\mathrm{C}$ (speed of sound $344\,\mathrm{m/s}$) in a nearby open pipe to vibrate at its fifth harmonic frequency. What are the frequency and wavelength of the sound wave (a) in the helium in the stopped pipe and (b) in the air in the open pipe? (c) What is the length of the open pipe?

Be sure to review Examples 16.14, 16.15, 16.16, 16.17, and 16.18 (Section 16.8) before attempting these problems.

VP16.18.1 The siren on an ambulance emits a sound of frequency 2.80×10^3 Hz. If the ambulance is traveling at $26.0\,\mathrm{m/s}$ (93.6 km/h, or 58.2 mi/h), the speed of sound is $340\,\mathrm{m/s}$, and the air is still, what are the frequency and wavelength that you hear if you are standing (a) in front of the ambulance or (b) behind the ambulance?

VP16.18.2 A stationary bagpiper is playing a Highland bagpipe, in which one reed produces a continuous sound of frequency 440 Hz. The air is still and the speed of sound is $340\,\mathrm{m/s}$. (a) What is the wavelength of the sound wave produced by the bagpipe? What are the frequency and wavelength of the sound wave that a bicyclist hears if she is (b) approaching the bagpiper at $10.0\,\mathrm{m/s}$ or (c) moving away from the bagpiper at $10.0\,\mathrm{m/s}$?

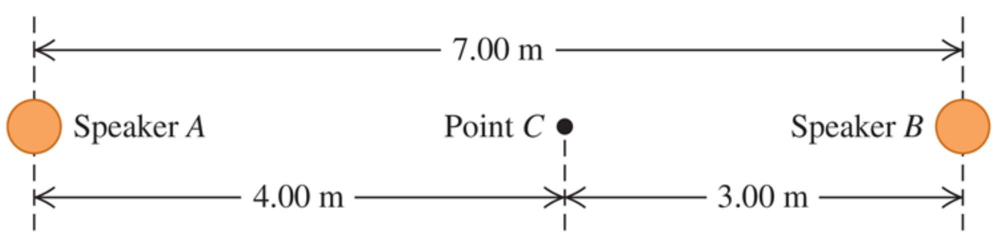
VP16.18.3 A police car moving east at $40.0\,\mathrm{m/s}$ is chasing a speeding sports car moving east at $35.0\,\mathrm{m/s}$. The police car's siren has frequency 1.20×10^3 Hz, the speed of sound is $340\,\mathrm{m/s}$, and the air is still. (a) What is the frequency of sound that the driver of the speeding sports car hears? (b) If the speeding sports car were to turn around and drive west at $35.0\,\mathrm{m/s}$ toward the approaching police car, what frequency would the driver of the sports car hear?

VP16.18.4 For a scene in an action movie, a car drives at $25.0\,\mathrm{m/s}$ directly toward a wall. The car's horn is on continuously and produces a sound of frequency 415 Hz. (a) If the speed of sound is $340\,\text{m/s}$ and the air is still, what is the frequency of the sound that the driver of the car hears reflected from the wall? (b) How fast would the car have to move for the reflected sound that the driver hears to have frequency 495 Hz?

Bridging Problem: Loudspeaker Interference

Loudspeakers A and B are 7.00 m apart and vibrate in phase at 172 Hz. They radiate sound uniformly in all directions. Their acoustic power outputs are 8.00×10^{-4} W and 6.00×10^{-5} W, respectively. The air temperature is 20°C. (a) Determine the difference in phase of the two signals at a point C along the line joining A and B, 3.00 m from B and 4.00 m from A (Fig. 16.39). (b) Determine the intensity and sound intensity level at C from speaker A alone (with B turned off) and from speaker B alone (with A turned off). (c) Determine the intensity and sound intensity level at C from both speakers together.

Figure 16.39



The situation for this problem.

Solution Guide

IDENTIFY and SET UP

1. Choose the equations that relate power, distance from the source, intensity, pressure amplitude, and sound intensity level.
2. Decide how you'll determine the phase difference in part (a). Once you have found the phase difference, how can you use it to find the amplitude of the combined wave at C due to both sources?
3. List the unknown quantities for each part of the problem and identify your target variables.

EXECUTE

4. Determine the phase difference at point C.
5. Find the intensity, sound intensity level, and pressure amplitude at C due to each speaker alone.
6. Use your results from steps 4 and 5 to find the pressure amplitude at C due to both loudspeakers together.
7. Use your result from step 6 to find the intensity and sound intensity level at C due to both loudspeakers together.

EVALUATE

8. How do your results from part (c) for intensity and sound intensity level at C compare to those from part (b)? Does this make sense?
9. What result would you have gotten in part (c) if you had (incorrectly) combined the *intensities* from A and B directly, rather than (correctly) combining the *pressure amplitudes* as you did in step 6?

Video Tutor Solution: **Chapter 16 Bridging Problem**

Questions/Exercises/Problems: Sound and Hearing

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

Discussion Questions

Q16.1 When sound travels from air into water, does the frequency of the wave change? The speed? The wavelength? Explain your reasoning.

Q16.2 The hero of a western movie listens for an oncoming train by putting his ear to the track. Why does this method give an earlier warning of the approach of a train than just listening in the usual way?

Q16.3 Would you expect the pitch (or frequency) of an organ pipe to increase or decrease with increasing temperature? Explain.

Q16.4 In most modern wind instruments the pitch is changed by using keys or valves to change the length of the vibrating air column. The bugle, however, has no valves or keys, yet it can play many notes. How might this be possible? Are there restrictions on what notes a bugle can play?

Q16.5 Symphonic musicians always “warm up” their wind instruments by blowing into them before a performance. What purpose does this serve?

Q16.6 In a popular and amusing science demonstration, a person inhales helium and then his voice becomes high and squeaky.

Why does this happen? (*Warning:* Inhaling too much helium can cause unconsciousness or death.)

Q16.7 Lane dividers on highways sometimes have regularly spaced ridges or ripples. When the tires of a moving car roll along such a divider, a musical note is produced. Why? Explain how this phenomenon could be used to measure the car's speed.

Q16.8 (a) Does a sound level of 0 dB mean that there is no sound? (b) Is there any physical meaning to a sound having a negative intensity level? If so, what is it? (c) Does a sound intensity of zero mean that there is no sound? (d) Is there any physical meaning to a sound having a negative intensity? Why?

Q16.9 Which has a more direct influence on the loudness of a sound wave: the *displacement* amplitude or the *pressure* amplitude? Explain.

Q16.10 If the pressure amplitude of a sound wave is halved, by what factor does the intensity of the wave decrease? By what factor must the pressure amplitude of a sound wave be increased in order to increase the intensity by a factor of 16? Explain.

Q16.11 Does the sound intensity level β obey the inverse-square law? Why?

Q16.12 A small fraction of the energy in a sound wave is absorbed by the air through which the sound passes. How does this modify the inverse-square relationship between intensity and distance from the source? Explain.

Q16.13 A small metal band is slipped onto one of the tines of a tuning fork. As this band is moved closer and closer to the end of the tine, what effect does this have on the wavelength and frequency of the sound the tine produces? Why?

Q16.14 An organist in a cathedral plays a loud chord and then releases the keys. The sound persists for a few seconds and gradually dies away. Why does it persist? What happens to the sound energy when the sound dies away?

Q16.15 Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 860 Hz. Point P is 12.0 m from A and 13.4 m from B. Is the interference at P constructive or destructive? Give the reasoning behind your answer.

Q16.16 Two vibrating tuning forks have identical frequencies, but one is stationary and the other is mounted at the rim of a rotating platform. What does a listener hear? Explain.

Q16.17 A large church has part of the organ in the front of the church and part in the back. A person walking rapidly down the aisle while both segments are playing at once reports that the two segments sound out of tune. Why?

Q16.18 A sound source and a listener are both at rest on the earth, but a strong wind is blowing from the source toward the listener. Is there a Doppler effect? Why or why not?

Q16.19 Can you think of circumstances in which a Doppler effect would be observed for surface waves in water? For elastic waves propagating in a body of water deep below the surface? If so, describe the circumstances and explain your reasoning. If not, explain why not.

Q16.20 Stars other than our sun normally appear featureless when viewed through telescopes. Yet astronomers can readily use the light from these stars to determine that they are rotating and even measure the speed of their surface. How do you think they can do this?

Q16.21 If you wait at a railroad crossing as a train approaches and passes, you hear a Doppler shift in its sound. But if you listen closely, you hear that the change in frequency is continuous; it does not suddenly go from one high frequency to another low frequency. Instead the frequency *smoothly* (but rather quickly) changes from high to low as the train passes. Why does this smooth change occur?

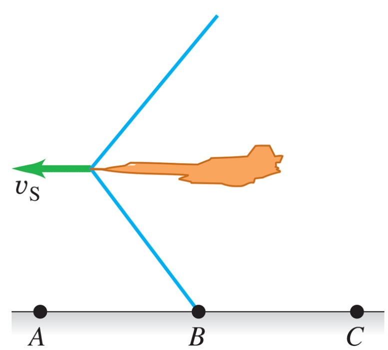
Q16.22 In case 1, a source of sound approaches a stationary observer at speed u. In case 2, the observer moves toward the stationary source at the same speed u. If the source is always producing the same frequency sound, will the observer hear the same frequency in both cases, since the relative speed is the same each time? Why or why not?

Q16.23 Does an aircraft make a sonic boom only at the instant its speed exceeds Mach 1? Explain.

Q16.24 If you are riding in a supersonic aircraft, what do you hear? Explain. In particular, do you hear a continuous sonic boom? Why or why not?

Q16.25 A jet airplane is flying at a constant altitude at a steady speed v_S greater than the speed of sound. Describe what observers at points A, B, and C hear at the instant shown in Fig. Q16.25, when the shock wave has just reached point B. Explain.

Figure Q16.25



Exercises

Unless indicated otherwise, assume the speed of sound in air to be $v = 344\ \mathrm{m/s}$.

Section 16.1 Sound Waves

16.1 • Example 16.1 (Section 16.1) showed that for sound waves in air with frequency 1000 Hz, a displacement amplitude of 1.2×10^{-8} m produces a pressure amplitude of 3.0×10^{-2} Pa. (a) What is the wavelength of these waves? (b) For 1000 Hz waves in air, what displacement amplitude would be needed for the pressure amplitude to be at the pain threshold, which is 30 Pa? (c) For what wavelength and frequency will waves with a displacement amplitude of 1.2×10^{-8} m produce a pressure amplitude of 1.5×10^{-3} Pa?

16.2 • A loud factory machine produces sound having a displacement amplitude of 1.00 μm, but the frequency of this sound can be adjusted. In order to prevent ear damage to the workers, the maximum pressure amplitude of the sound waves is limited to 10.0 Pa. Under the conditions of this factory, the bulk modulus of air is 1.42×10^{5} Pa. What is the highest-frequency sound to which this machine can be adjusted without exceeding the prescribed limit? Is this frequency audible to the workers?

16.3 • Consider a sound wave in air that has displacement amplitude 0.0200 mm. Calculate the pressure amplitude for frequencies of (a) 150 Hz; (b) 1500 Hz; (c) 15,000 Hz. In each case compare the result to the pain threshold, which is 30 Pa.

16.4 • BIO **Ultrasound and Infrasound.** (a) **Whale communication.** Blue whales apparently communicate with each other using sound of frequency 17 Hz, which can be heard nearly 1000 km away in the ocean. What is the wavelength of such a sound in seawater, where the speed of sound is 1531 m/s? (b) **Dolphin clicks.** One type of sound that dolphins emit is a sharp click of wavelength 1.5 cm in the ocean. What is the frequency of such clicks? (c) **Dog whistles.** One brand of dog whistles claims a frequency of 25 kHz for its product. What is the wavelength of this sound? (d) **Bats.** While bats emit a wide variety of sounds, one type emits pulses of sound having a frequency between 39 kHz and 78 kHz. What is the range of wavelengths of this sound? (e) **Sonograms.** Ultrasound is used to view the interior of the body, much as x rays are utilized. For sharp imagery, the wavelength of the sound should be around one-fourth (or less) the size of the objects to be viewed. Approximately what frequency of sound is needed to produce a clear image of a tumor that is 1.0 mm across if the speed of sound in the tissue is $1550\,\mathrm{m/s}$?

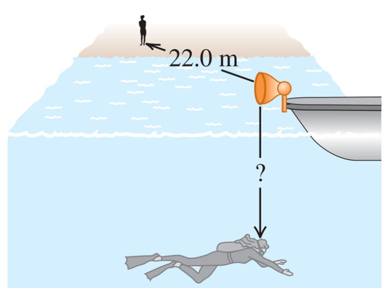
Section 16.2 Speed of Sound Waves

16.5 •• A 60.0-m-long brass rod is struck at one end. A person at the other end hears two sounds as a result of two longitudinal waves, one traveling in the metal rod and the other traveling in air. What is the time interval between the two sounds? (The speed of sound in air is 344 m/s; see Tables 11.1 and 12.1 for relevant information about brass.)

16.6 • (a) In a liquid with density 1300 kg/m^3, longitudinal waves with frequency 400 Hz are found to have wavelength 8.00 m. Calculate the bulk modulus of the liquid. (b) A metal bar with a length of 1.50 m has density 6400 kg/m^3. Longitudinal sound waves take 3.90×10^{-4} s to travel from one end of the bar to the other. What is Young's modulus for this metal?

16.7 • A submerged scuba diver hears the sound of a boat horn directly above her on the surface of the lake. At the same time, a friend on dry land 22.0 m from the boat also hears the horn (Fig. E16.7). The horn is 1.2 m above the surface of the water. What is the distance (labeled "?") from the horn to the diver? Both air and water are at 20°C.

Figure E16.7



16.8 • At a temperature of 27.0°C, what is the speed of longitudinal waves in (a) hydrogen (molar mass 2.02 g/mol); (b) helium (molar mass 4.00 g/mol); (c) argon (molar mass

39.9 g/mol)? See Table 19.1 for values of γ. (d) Compare your answers for parts (a), (b), and (c) with the speed in air at the same temperature.

16.9 • An oscillator vibrating at 1250 Hz produces a sound wave that travels through an ideal gas at 325 m/s when the gas temperature is 22.0°C. For a certain experiment, you need to have the same oscillator produce sound of wavelength 28.5 cm in this gas. What should the gas temperature be to achieve this wavelength?

16.10 •• **CALC** (a) Show that the fractional change in the speed of sound (dv/v) due to a very small temperature change dT is given by $dv/v = \frac{1}{2}\, dT/T$. [**HINT:** Start with Eq. (16.10).] (b) The speed of sound in air at 20°C is found to be 344 m/s. Use the result in part (a) to find the change in the speed of sound for a 1.0°C change in air temperature.

Section 16.3 Sound Intensity

16.11 •• BIO **Energy Delivered to the Ear.** Sound is detected when a sound wave causes the tympanic membrane (the eardrum) to vibrate. Typically, the diameter of this membrane is about 8.4 mm in humans. (a) How much energy is delivered to the eardrum each second when someone whispers (20 dB) a secret in your ear? (b) To comprehend how sensitive the ear is to very small amounts of energy, calculate how fast a typical 2.0 mg mosquito would have to fly (in mm/s) to have this amount of kinetic energy.

16.12 • (a) By what factor must the sound intensity be increased to raise the sound intensity level by 13.0 dB? (b) Explain why you don't need to know the original sound intensity.

16.13 •• **Eavesdropping!** You are trying to overhear a juicy conversation, but from your distance of 15.0 m, it sounds like only an average whisper of 20.0 dB. How close should you move to the chatterboxes for the sound level to be 60.0 dB?

16.14 • A small source of sound waves emits uniformly in all directions. The total power output of the source is P. By what factor must P increase if the sound intensity level at a distance of 20.0 m from the source is to increase 5.00 dB?

16.15 • A sound wave in air at 20°C has a frequency of 320 Hz and a displacement amplitude of 5.00×10^{-3} mm. For this sound wave calculate the (a) pressure amplitude (in Pa); (b) intensity (in $\mathrm{W/m^2}$); (c) sound intensity level (in decibels).

16.16 •• You live on a busy street, but as a music lover, you want to reduce the traffic noise. (a) If you install special sound-reflecting windows that reduce the sound intensity level (in dB) by 30 dB, by what fraction have you lowered the sound intensity (in W/m^2)? (b) If, instead, you reduce the intensity by half, what change (in dB) do you make in the sound intensity level?

16.17 • BIO For a person with normal hearing, the faintest sound that can be heard at a frequency of 400 Hz has a pressure amplitude of about 6.0×10^{-5} Pa. Calculate the (a) intensity; (b) sound intensity level; (c) displacement amplitude of this sound wave at 20°C.

16.18 •• The intensity due to a number of independent sound sources is the sum of the individual intensities. (a) When four quadruplets cry simultaneously, how many decibels greater is the sound intensity level than when a single one cries? (b) To increase the sound intensity level again by the same number of decibels as in part (a), how many more crying babies are required?

16.19 • CP A baby's mouth is 30 cm from her father's ear and 1.50 m from her mother's ear. What is the difference between the sound intensity levels heard by the father and by the mother?

16.20 •• (a) If two sounds differ by 5.00 dB, find the ratio of the intensity of the louder sound to that of the softer one. (b) If one sound is 100 times as intense as another, by how much do they differ in sound intensity level (in decibels)? (c) If you increase the volume of your stereo so that the intensity doubles, by how much does the sound intensity level increase?

16.21 •• CP At point A, 3.0 m from a small source of sound that is emitting uniformly in all directions, the sound intensity level is 53 dB. (a) What is the intensity of the sound at A? (b) How far from the source must you go so that the intensity is one-fourth of what it was at A? (c) How far must you go so that the sound intensity level is one-fourth of what it was at A? (d) Does intensity obey the inverse-square law? What about sound intensity level?

16.22 •• The pattern of displacement nodes N and antinodes A in a pipe is *ANANANANANA* when the standing-wave frequency is 1710 Hz. The pipe contains air at $20^\circ\mathrm{C}$ (See Table 16.1.) (a) Is it an open or a closed (stopped) pipe? (b) Which harmonic is this? (c) What is the length of the pipe? (d) What is the fundamental frequency? (e) What would be the fundamental frequency of the pipe if it contained helium at $20^\circ\mathrm{C}$?

Section 16.4 Standing Sound Waves and Normal Modes

16.23 • Standing sound waves are produced in a pipe that is 1.20 m long. For the fundamental and first two overtones, determine the locations along the pipe (measured from the left end) of the displacement nodes and the pressure nodes if (a) the pipe is open at both ends and (b) the pipe is closed at the left end and open at the right end.

16.24 • The fundamental frequency of a pipe that is open at both ends is 524 Hz. (a) How long is this pipe? If one end is now closed, find (b) the wavelength and (c) the frequency of the new fundamental.

16.25 • BIO **The Human Voice.** The human vocal tract is a pipe that extends about 17 cm from the lips to the vocal folds (also called "vocal cords") near the middle of your throat. The vocal folds behave rather like the reed of a clarinet, and the vocal tract acts like a stopped pipe. Estimate the first three standing-wave frequencies of the vocal tract. Use $v = 344 \text{ m/s}$. (The answers are only an estimate, since the position of lips and tongue affects the motion of air in the vocal tract.)

16.26 •• BIO **The Vocal Tract.** Many professional singers have a range of $2\frac{1}{2}$ octaves or even greater. Suppose a soprano's range extends from A below middle C (frequency 220 Hz) up to E-flat above high C (frequency 1244 Hz). Although the vocal tract is complicated, we can model it as a resonating air column, like an organ pipe, that is open at the top and closed at the bottom. The column extends from the mouth down to the diaphragm in the chest cavity. Assume that the lowest note is the fundamental. How long is this column of air if

$v = 354$ m/s? Does your result seem reasonable, on the basis of observations of your body?

16.27 • The longest pipe found in most medium-size pipe organs is 4.88 m (16 ft) long. What is the frequency of the note corresponding to the fundamental mode if the pipe is (a) open at both ends, (b) open at one end and closed at the other?

16.28 • **Singing in the Shower.** A pipe closed at both ends can have standing waves inside of it, but you normally don't hear them because little of the sound can get out. But you *can* hear them if you are *inside* the pipe, such as someone singing in the shower. (a) Show that the wavelengths of standing waves in a pipe of length L that is closed at both ends are $\lambda_n = 2L/n$ and the frequencies are given by $f_n = nv/2L = nf_1$, where $n = 1,\ 2,\ 3,\ \ldots$. (b) Modeling it as a pipe, find the frequency of the fundamental and the first two overtones for a shower 2.50 m tall. Are these frequencies audible?

16.29 •• The pattern of displacement nodes N and antinodes A in a pipe is *NANANANANA* when the standing-wave frequency is 1710 Hz. The pipe contains air at 20°C. (See Table 16.1.) (a) Is it an open or a closed (stopped) pipe? (b) Which harmonic is this? (c) What is the length of the pipe? (d) What is the fundamental frequency?

Section 16.5 Resonance and Sound

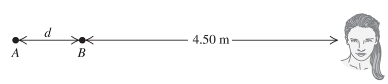
16.30 •• **CP** You have a stopped pipe of adjustable length close to a taut 62.0 cm, 7.25 g wire under a tension of 4110 N. You want to adjust the length of the pipe so that, when it produces sound at its fundamental frequency, this sound causes the wire to vibrate in its second *overtone* with very large amplitude. How long should the pipe be?

16.31 • You blow across the open mouth of an empty test tube and produce the fundamental standing wave in the 14.0-cm-long air column in the test tube, which acts as a stopped pipe. (a) What is the frequency of this standing wave? (b) What is the frequency of the fundamental standing wave in the air column if the test tube is half filled with water?

Section 16.6 Interference of Waves

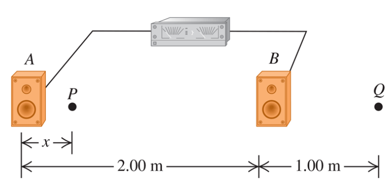
16.32 • Small speakers A and B are driven in phase at 725 Hz by the same audio oscillator. Both speakers start out 4.50 m from the listener, but speaker A is slowly moved away (Fig. E16.32). (a) At what distance d will the sound from the speakers first produce destructive interference at the listener's location? (b) If A is moved even farther away than in part (a), at what distance d will the speakers next produce destructive interference at the listener's location? (c) After A starts moving away from its original spot, at what distance d will the speakers first produce constructive interference at the listener's location?

Figure E16.32



16.33 • Two loudspeakers, A and B (Fig. E16.33), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.00 m to the right of speaker B. Both speakers emit sound waves that travel directly from the speaker to point Q. What is the lowest frequency for which (a) *constructive* interference occurs at point Q; (b) *destructive* interference occurs at point Q?

Figure E16.33



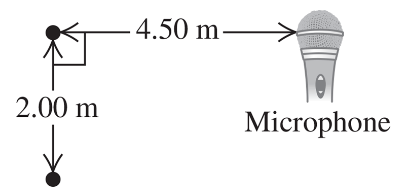
16.34 •• Two loudspeakers, A and B (see Fig. E16.33), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. The frequency of the sound waves produced by the loudspeakers is 206 Hz. Consider a point P between the speakers and along the line connecting them, a distance x to the right of A. Both speakers emit sound waves that travel directly from the speaker to point P. For what values of x will (a) *destructive* interference occur at P; (b) *constructive* interference occur at P? (c) Interference effects like those in parts (a) and (b) are almost never a factor in listening to home stereo equipment. Why not?

16.35 •• Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 12.0 m to the right of speaker A. The frequency of the waves emitted by each speaker is 688 Hz. You are standing between the speakers, along the line connecting them, and are at a point of constructive interference. How far must you walk toward speaker B to move to a point of destructive interference?

16.36 • Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 172 Hz. You are 8.00 m from A. What is the closest you can be to B and be at a point of destructive interference?

16.37 •• Two small stereo speakers are driven in step by the same variable- frequency oscillator. Their sound is picked up by a microphone arranged as shown in Fig. E16.37. For what frequencies does their sound at the speakers produce (a) constructive interference and (b) destructive interference?

Figure E16.37



4.50 m
2.00 m
Microphone

Section 16.7 Beats

16.38 •• Two guitarists attempt to play the same note of wavelength 64.8 cm at the same time, but one of the instruments is slightly out of tune and plays a note of wavelength 65.2 cm instead. What is the frequency of the beats these musicians hear when they play together?

16.39 •• **Tuning a Violin.** A violinist is tuning her instrument to concert A (440 Hz). She plays the note while listening to an electronically generated tone of exactly that frequency and hears a beat frequency of 3 Hz, which increases to 4 Hz when she tightens her violin string slightly. (a) What was the frequency of the note played by her violin when she heard the 3 Hz beats? (b) To get her violin perfectly tuned to concert A, should she tighten or loosen her string from what it was when she heard the 3 Hz beats?

16.40 •• Two organ pipes, open at one end but closed at the other, are each 1.14 m long. One is now lengthened by 2.00 cm. Find the beat frequency that they produce when playing together in their fundamentals.

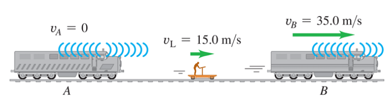
Section 16.8 The Doppler Effect

16.41 •• On the planet Arrakis a male ornithoid is flying toward his mate at 25.0 m/s while singing at a frequency of 1200 Hz. If the stationary female hears a tone of 1240 Hz, what is the speed of sound in the atmosphere of Arrakis?

16.42 • A railroad train is traveling at 25.0 m/s in still air. The frequency of the note emitted by the locomotive whistle is 400 Hz. What is the wavelength of the sound waves (a) in front of the locomotive and (b) behind the locomotive? What is the frequency of the sound heard by a stationary listener (c) in front of the locomotive and (d) behind the locomotive?

16.43 • Two train whistles, A and B, each have a frequency of 392 Hz. A is stationary and B is moving toward the right (away from A) at a speed of 35.0 m/s. A listener is between the two whistles and is moving toward the right with a speed of 15.0 m/s (Fig. E16.43). No wind is blowing. (a) What is the frequency from A as heard by the listener? (b) What is the frequency from B as heard by the listener? (c) What is the beat frequency detected by the listener?

Figure E16.43



16.44 • **Moving Source vs. Moving Listener.** (a) A sound source producing 1.00 kHz waves moves toward a stationary listener at one-half the speed of sound. What frequency will the listener hear? (b) Suppose instead that the source is stationary and the listener moves toward the source at one-half the speed of sound. What frequency does the listener hear? How does your answer compare to that in part (a)? Explain on physical grounds why the two answers differ.

16.45 • A swimming duck paddles the water with its feet once every 1.6 s, producing surface waves with this period. The duck is moving at constant speed in a pond where the speed of surface waves is 0.32 m/s, and the crests of the waves ahead of the duck are spaced 0.12 m apart. (a) What is the duck's speed? (b) How far apart are the crests behind the duck?

16.46 • A railroad train is traveling at 30.0 m/s in still air. The frequency of the note emitted by the train whistle is 352 Hz. What frequency is heard by a passenger on a train moving in the opposite direction to the first at 18.0 m/s and (a) approaching the first and (b) receding from the first?

16.47 • A car alarm is emitting sound waves of frequency 520 Hz. You are on a motorcycle, traveling directly away from the parked car. How fast must you be traveling if you detect a frequency of 490 Hz?

16.48 •• While sitting in your car by the side of a country road, you are approached by your friend, who happens to be in an identical car. You blow your car's horn, which has a frequency of 260 Hz. Your friend blows his car's horn, which is identical to yours, and you hear a beat frequency of 6.0 Hz. How fast is your friend approaching you?

16.49 • A police car is traveling due east at a speed of 15.0 m/s relative to the earth. You are in a convertible following behind the police car. Your car is also moving due east at 15.0 m/s relative to the earth, so the speed of the police car relative to you is zero. The siren of the police car is emitting sound of frequency 500 Hz. The speed of sound in the still air is 340 m/s. (a) What is the speed of the sound waves relative to you? (b) What is the wavelength of the sound waves at your location? (c) What frequency do you detect?

16.50 •• The siren of a fire engine that is driving northward at 30.0 m/s emits a sound of frequency 2000 Hz. A truck in front of this fire engine is moving northward at 20.0 m/s. (a) What is the frequency of the siren's sound that the fire engine's driver hears reflected from the back of the truck? (b) What wavelength would this driver measure for these reflected sound waves?

16.51 •• A stationary police car emits a sound of frequency 1200 Hz that bounces off a car on the highway and returns with a frequency of 1250 Hz. The police car is right next to the highway, so the moving car is traveling directly toward or away from it. (a) How fast was the moving car going? Was it moving toward or away from the police car? (b) What frequency would the police car have received if it had been traveling toward the other car at 20.0 m/s?

16.52 •• A stationary source emits sound waves of frequency f_s. There is no wind blowing. A device for detecting sound waves and measuring their observed frequency moves toward the source with speed v_L, and the observed frequency of the sound waves is f_L. The measurement is repeated for different values of v_L. You plot the results as f_L versus v_L and find that your data lie close to a straight line that has slope $1.75\ \text{m}^{-1}$ and y-intercept 600.0 Hz. What are your experimental results for the speed of sound in the still air and for the frequency f_s of the source?

Section 16.9 Shock Waves

16.53 •• A jet plane flies overhead at Mach 1.70 and at a constant altitude of 1250 m. (a) What is the angle α of the shock-wave cone? (b) How much time after the plane passes directly overhead do you hear the sonic boom? Neglect the variation of the speed of sound with altitude.

16.54 • The shock-wave cone created by a space shuttle at one instant during its reentry into the atmosphere makes an angle of 58.0° with its direction of motion. The speed of sound at this altitude is 331 m/s. (a) What is the Mach number of the shuttle at this instant, and (b) how fast (in m/s and in mi/h) is it traveling relative to the atmosphere? (c) What would be its Mach number and the angle of its shock-wave cone if it flew at the same speed but at low altitude where the speed of sound is 344 m/s?

Problems

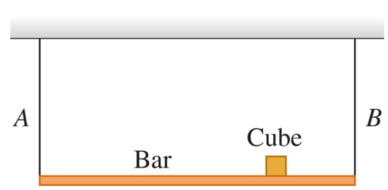
16.55 •• Use Eq. (16.10) and the information given in Example 16.4 to show that the speed of sound in air at 0°C is 332 m/s. (a) Using the first two terms of the power series expansion of $(1+x)^n$ (see Appendix B), show that the speed of sound in air at Celsius temperature T_{C} is given approximately by $v = (332\ \mathrm{m/s})(1 + T_{\mathrm{C}}/546)$. (b) Use the result in part (a) to calculate v at 20°C. Compare your result to the value given in Table 16.1. Is the expression in part (a) accurate at 20°C? (c) Do you expect the expression in part (a) to be accurate at 120°C? Explain. (**HINT:** Compare the second and third terms in the power series expansion.)

16.56 •• **CP** The sound from a trumpet radiates uniformly in all directions in 20°C air. At a distance of 5.00 m from the trumpet the sound intensity level is 52.0 dB. The frequency is 587 Hz. (a) What is the pressure amplitude at this distance? (b) What is the displacement amplitude? (c) At what distance is the sound intensity level 30.0 dB?

16.57 ••• CALC The air temperature over a lake decreases linearly with height after sunset, since air cools faster than water. (a) If the temperature at the surface is 25.00°C and the temperature at a height of 300.0 m is 5.000°C, how long does it take sound to rise 300.0 m directly upward? [**HINT:** Use Eq. (16.10) and integrate.] (b) At a height of 300.0 m, how far does sound travel horizontally in this same time interval? The change in wave speed with altitude due to the nocturnal temperature inversion over the lake gives rise to a change in direction of the sound waves. This phenomenon is called refraction and will be discussed in detail for electromagnetic waves in Chapter 33.

16.58 •• CP A uniform 165 N bar is supported horizontally by two identical wires A and B (Fig. P16.58). A small 185 N cube of lead is placed three-fourths of the way from A to B. The wires are each 75.0 cm long and have a mass of 5.50 g. If both of them are simultaneously plucked at the center, what is the frequency of the beats that they will produce when vibrating in their fundamental?

Figure P16.58

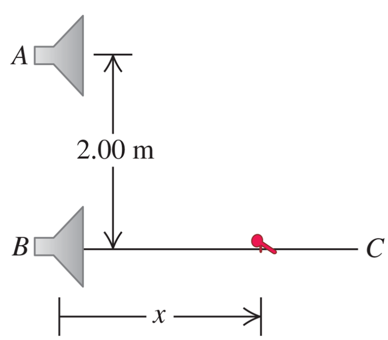


16.59 • An organ pipe has two successive harmonics with frequencies 1372 and 1764 Hz. (a) Is this an open or a stopped pipe? Explain. (b) What two harmonics are these? (c) What is the length of the pipe?

16.60 •• A Kundt's tube is filled with helium gas. The speed of sound for helium at $20^\circ C$ is given in Table 16.1. A tuning fork that produces sound waves with frequency 1200 Hz is used to set up standing waves inside the tube. You measure the node-to-node distance to be 47.0 cm. What is the temperature of the helium gas in the tube?

16.61 •• Two identical loudspeakers are located at points A and B, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B (line BC in Fig. P16.61). (a) At what distances from B will there be *destructive* interference? (b) At what distances from B will there be *constructive* interference? (c) If the frequency is made low enough, there will be no positions along the line BC at which destructive interference occurs. How low must the frequency be for this to be the case?

Figure P16.61



16.62 •• A bat flies toward a wall, emitting a steady sound of frequency 1.70 kHz. This bat hears its own sound plus the sound reflected by the wall. How fast should the bat fly in order to hear a beat frequency of 8.00 Hz?

16.63 •• The sound source of a ship's sonar system operates at a frequency of 18.0 kHz. The speed of sound in water (assumed to be at a uniform $20^\circ C$) is 1482 m/s. (a) What is the wavelength of the waves emitted by the source? (b) What is the difference in frequency between the directly radiated waves and the waves reflected from a whale traveling directly toward the ship at 4.95 m/s? The ship is at rest in the water.

16.64 •• Consider a thunderstorm with a flash of lightning followed by a crash of thunder. (a) Estimate the time delay between the lightning flash and the sound of the thunder. (b) Determine the distance sound travels in that time, which provides an estimate of the distance to the lightning flash. (c) By comparing to the sounds listed in Table 16.2, estimate for your location the average intensity level of the sound in decibels and calculate the intensity in W/m^2. (d) Assume the sound was generated at the site of the lightning flash and assume the sound was transmitted uniformly in all directions. Use the estimated distance to estimate the average sound power generated by the thunder. (e) Estimate the duration of the thunderclap. Multiply your estimate by the average power to determine the sound energy released by a lightning strike.

16.65 ••• **CP** Suppose that you are at a bowling alley. (a) By comparing to the sounds listed in Table 16.2, for your location estimate the sound intensity level in decibels and the intensity in W/m^2 of the crashing pins after a well-executed strike. (b) Using 18 m as the length of a bowling alley, determine the average power associated with that sound. (c) Estimate the duration of the crashing sound.

Multiply your estimate by the average power to obtain the sound energy released in the strike. (d) Estimate the time it took the bowling ball to travel down the alley prior to the strike. Use that time to estimate the speed of the ball. (e) Assume the ball has a mass of 6.4 kg. Account for both translational and rotational motions to estimate the kinetic energy of the bowling ball immediately prior to the strike. (f) What fraction of the ball's energy was converted to sound?

16.66 ••• BIO **Ultrasound in Medicine.** A 2.00 MHz sound wave travels through a pregnant woman's abdomen and is reflected from the fetal heart wall of her unborn baby. The heart wall is moving toward the sound receiver as the heart beats. The reflected sound is then mixed with the transmitted sound, and 72 beats per second are detected. The speed of sound in body tissue is 1500 m/s. Calculate the speed of the fetal heart wall at the instant this measurement is made.

16.67 ••• BIO Horseshoe bats (genus *Rhinolophus*) emit sounds from their nostrils and then listen to the frequency of the sound reflected from their prey to determine the prey's speed. (The "horseshoe" that gives the bat its name is a depression around the nostrils that acts like a focusing mirror, so that the bat emits sound in a narrow beam like a flashlight.) A *Rhinolophus* flying at speed v_{bat} emits sound of frequency f_{bat}; the sound it hears reflected from an insect flying toward it has a higher frequency f_{refl}. (a) Show that the speed of the insect is

$$v_{\mathrm{insect}} = v\left[\frac{f_{\mathrm{refl}}(v - v_{bat}) - f_{\mathrm{bat}}(v + v_{bat})}{f_{\mathrm{refl}}(v - v_{bat}) + f_{\mathrm{bat}}(v + v_{bat})}\right]$$

where v is the speed of sound. (b) If $f_{\text{bat}} = 80.7\ \text{kHz}$, $f_{\text{refl}} = 83.5\ \text{kHz}$, and $v_{\text{bat}} = 3.9\ \text{m/s}$, calculate the speed of the insect.

16.68 • **CP** A police siren of frequency f_{siren} is attached to a vibrating platform. The platform and siren oscillate up and down in simple harmonic motion with amplitude A_{p} and frequency f_{p}. (a) Find the maximum and minimum sound frequencies that you would hear at a position directly above the siren. (b) At what point in the motion of the platform is the maximum frequency heard? The minimum frequency? Explain.

16.69 •• **CP** A turntable 1.50 m in diameter rotates at 75 rpm. Two speakers, each giving off sound of wavelength 31.3 cm, are attached to the rim of the table at opposite ends of a diameter. A listener stands in front of the turntable. (a) What is the greatest beat frequency the listener will receive from this system? (b) Will the listener be able to distinguish individual beats?

16.70 •• **DATA** A long, closed cylindrical tank contains a diatomic gas that is maintained at a uniform temperature that can be varied. When you measure the speed of sound v in the gas as a function of the temperature T of the gas, you obtain these results:

T (°C)	−20.0	0.0	20.0	40.0	60.0	80.0
v (m/s)	324	337	349	361	372	383

(a) Explain how you can plot these results so that the graph will be well fit by a straight line. Construct this graph and verify that the plotted points do lie close to a straight line. (b) Because the gas is diatomic, $\gamma = 1.40$. Use the slope of the line in part (a) to calculate M, the molar mass of the gas. Express M in grams/imole. What type of gas is in the tank?

16.71 •• **DATA** A long tube contains air at a pressure of 1.00 atm and a temperature of 77.0°C. The tube is open at one end and closed at the other by a movable piston. A tuning fork that vibrates with a frequency of 500 Hz is placed near the open end. Resonance is produced when the piston is at distances 18.0 cm, 55.5 cm, and 93.0 cm from the open end. (a) From these values, what is the speed of sound in air at 77.0°C? (b) From the result of part (a), what is the value of γ? (c) These results show that a displacement antinode is slightly outside the open end of the tube. How far outside is it?

16.72 ••• **DATA** **Supernova!** (a) Equation (16.30) can be written as

$$f_{\text{R}} = f_{\text{S}}\left(1 - \frac{v}{c}\right)^{1/2}\left(1 + \frac{v}{c}\right)^{-1/2}$$

where c is the speed of light in vacuum, 3.00×10^8 m/s. Most objects move much slower than this (v/c is very small), so calculations made with Eq. (16.30) must be done carefully to avoid rounding errors. Use the binomial theorem to show that if $v \ll c$, Eq. (16.30) approximately reduces to $f_{\text{R}} = f_{\text{S}}[1 - (v/c)]$. (b) The gas cloud known as the Crab Nebula can be seen with even a small telescope. It is the remnant of a *supernova*, a cataclysmic explosion of a star. (The explosion was seen on the earth on July 4, 1054 C.E.) Its streamers glow with the characteristic red color of heated hydrogen gas. In a laboratory on the earth, heated hydrogen produces red light with frequency 4.568×10^{14} Hz; the red light received from streamers in the Crab Nebula that are pointed toward the earth has frequency 4.586×10^{14} Hz. Estimate the speed with which the outer edges of the Crab Nebula are expanding. Assume that the speed of the center of the nebula relative to the earth is negligible. (c) Assuming that the expansion speed of the Crab Nebula has been constant since the supernova that produced it, estimate the diameter of the Crab Nebula. Give your answer in meters and in light-years. (d) The angular diameter of the Crab Nebula as seen from the earth is about 5 arc-minutes $\left(1\,\text{arc-minute} = \frac{1}{60}\,\text{degree}\right)$ Estimate the distance (in light-years) to the Crab Nebula, and estimate the year in which the supernova actually took place.

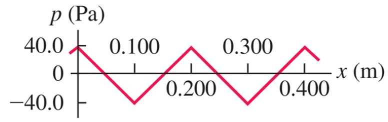
16.73 ••• **CP** A one-string Aeolian harp is constructed by attaching the lower end of a 1.56-m-long rigid and uniform pole with a mass of 8.00 kg to a tree at a pivot and then using a light rope to hang a 39.0-cm-long hollow steel tube with a mass of 4.00 kg from its upper end. A horizontal wire between the pole and the tree is attached to the pole at a height h above the pivot and holds the pole at a 45.0° angle with the vertical. The wire is uniform and has linear mass density $\mu = 1.40\ \text{g/m}$. When the wind blows, the wire resonates at its fundamental frequency. The height h is chosen such that the sound emitted by the wire stimulates the fundamental standing wave in the tube, which harmonizes with the resonating wire. (a) What is the frequency of the note produced by this instrument? (b) At what height h should the wire be placed? (**HINT:** The pole is in static equilibrium, so the net torque on it is zero and this determines the tension in the wire as a function of h.)

16.74 ••• Two powerful speakers, separated by 15.00 m, stand on the floor in front of the stage in a large amphitheater. An aisle perpendicular to the stage is directly in front of one of the speakers and extends 50.00 m to an exit door at the back of the amphitheater. (a) If the speakers produce in-phase, coherent 440 Hz tones, at how many points along the aisle is the sound minimal? (b) What is the distance between the farthest such point and the door at the back of the aisle? (c) Suppose the coherent sound emitted from both speakers is a linear superposition of a 440 Hz tone and another tone with frequency f. What is the smallest value of f so that minimal sound is heard at any point where the 440 Hz sound is minimal? (d) At how many additional points in the aisle is the 440 Hz tone present but the second tone is minimal? (e) What is the distance from the closest of these points to the speaker at the front of the aisle?

Challenge Problems

16.75 ••• **CALC** Figure P16.75 shows the pressure fluctuation p of a nonsinusoidal sound wave as a function of x for $t = 0$. The wave is traveling in the $+x$-direction. (a) Graph the pressure fluctuation p as a function of t for $x = 0$. Show at least two cycles of oscillation. (b) Graph the displacement y in this sound wave as a function of x at $t = 0$. At $x = 0$, the displacement at $t = 0$ is zero. Show at least two wavelengths of the wave. (c) Graph the displacement y as a function of t for $x = 0$. Show at least two cycles of oscillation. (d) Calculate the maximum velocity and the maximum acceleration of an element of the air through which this sound wave is traveling. (e) Describe how the cone of a loudspeaker must move as a function of time to produce the sound wave in this problem.

Figure P16.75



16.76 ••• **CP** **Longitudinal Waves on a Spring.** A long spring such as a Slinky™ is often used to demonstrate longitudinal waves. (a) Show that if a spring that obeys Hooke's law has mass m, length L, and force constant k', the speed of longitudinal waves on the spring is $v = L\sqrt{k'/m}$ (see Section 16.2). (b) Evaluate v for a spring with $m = 0.250\ \text{kg}$, $L = 2.00\ \text{m}$, and $k' = 1.50\ \text{N/m}$.

MCAT-Style Passage Problems

BIO **Ultrasound Imaging.** A typical ultrasound transducer used for medical diagnosis produces a beam of ultrasound with a frequency of 1.0 MHz. The beam travels from the transducer through tissue and partially reflects when it encounters different structures in the tissue. The same transducer that produces the ultrasound also detects the reflections. The transducer emits a short pulse of ultrasound and waits to receive the reflected echoes before emitting the next pulse. By measuring the time between the initial pulse and the arrival of the reflected signal, we can use the speed of ultrasound in tissue, $1540\,\text{m/s}$, to determine the distance from the transducer to the structure that produced the reflection.

As the ultrasound beam passes through tissue, the beam is attenuated through absorption. Thus deeper structures return weaker echoes. A typical attenuation in tissue is $-100\ \text{dB/m} \cdot \text{MHz}$; in bone it is $-500\ \text{dB/m} \cdot \text{MHz}$. In determining attenuation, we take the reference intensity to be the intensity produced by the transducer.

16.77 If the deepest structure you wish to image is 10.0 cm from the transducer, what is the maximum number of pulses per second that can be emitted? (a) 3850; (b) 7700; (c) 15,400; (d) 1,000,000.

16.78 After a beam passes through 10 cm of tissue, what is the beam's intensity as a fraction of its initial intensity from the transducer? (a) 1×10^{-11}; (b) 0.001; (c) 0.01; (d) 0.1.

16.79 Because the speed of ultrasound in bone is about twice the speed in soft tissue, the distance to a structure that lies beyond a bone can be measured incorrectly. If a beam passes through 4 cm of tissue, then 2 cm of bone, and then another 1 cm of tissue before echoing off a cyst and returning to the transducer, what is the difference between the true distance to the cyst and the distance that is measured by assuming the speed is always 1540 m/s? Compared with the measured distance, the structure is actually (a) 1 cm farther; (b) 2 cm farther; (c) 1 cm closer; (d) 2 cm closer.

16.80 In some applications of ultrasound, such as its use on cranial tissues, large reflections from the surrounding bones can produce standing waves. This is of concern because the large pressure amplitude in an antinode can damage tissues. For a frequency of 1.0 MHz, what is the distance between antinodes in tissue? (a) 0.38 mm; (b) 0.75 mm; (c) 1.5 mm; (d) 3.0 mm.

16.81 For cranial ultrasound, why is it advantageous to use frequencies in the kHZ range rather than the MHz range? (a) The antinodes of the standing waves will be closer together at the lower frequencies than at the higher frequencies; (b) there will be no standing waves at the lower frequencies; (c) cranial bones will attenuate the ultrasound more at the lower frequencies than at the higher frequencies; (d) cranial bones will attenuate the ultrasound less at the lower frequencies than at the higher frequencies.

Answers: Sound and Hearing

Chapter Opening Question ?

(iv) Equation (16.10) in Section 16.2 says that the speed of sound in a gas depends on the temperature and on the kind of gas (through the ratio of heat capacities and the molar mass). Winter air in the mountains has a lower temperature than summer air at sea level, but they have essentially the same composition. Hence the lower temperature alone explains the slower speed of sound in winter in the mountains.

Test Your Understanding

16.1 **(v)** From Eq. (16.5), the displacement amplitude is $A = p_{\text{max}}/Bk$. The pressure amplitude p_{max} and bulk modulus B remain the same, but the frequency f increases by a factor of 4. Hence the wave number $k = \omega/v = 2\pi f/v$ also increases by a factor of 4. Since A is inversely proportional to k, the displacement amplitude becomes $\frac{1}{4}$ as great. In other words, at higher frequency a smaller maximum displacement is required to produce the same maximum pressure fluctuation.

16.2 **(i)** From Eq. (16.7), the speed of longitudinal waves (sound) in a fluid is $v = \sqrt{B/\rho}$. We can rewrite this to give an expression for the bulk modulus B in terms of the fluid density ρ and the sound speed v: $B = \rho v^2$. At 20°C the speed of sound in mercury is slightly less than in water (1451 m/s versus 1482 m/s), but the density of mercury is greater than that of water by a large factor (13.6). Hence the bulk modulus of mercury is greater than that of water by a factor of $(13.6)(1451/1482)^2 = 13.0$.

16.3 **A and p_{max} increase by a factor of $\sqrt{2}$, B and v are unchanged, β increases by 3.0 dB** Equations (16.9) and (16.10) show that the bulk modulus B and sound speed v remain the same because the physical properties of the air are unchanged. From Eqs.

(16.12) and (16.14), the intensity is proportional to the square of the displacement amplitude or the square of the pressure amplitude. Hence doubling the intensity means that A and $p_{\max}$ both increase by a factor of $\sqrt{2}$. Example 16.9 shows that *multiplying* the intensity by a factor of $2\ (I_2/I_1 = 2)$ corresponds to *adding* to the sound intensity level by $(10\text{ dB})\log\ (I_2/I_1) = (10\text{ dB})\log 2 = 3.0\text{ dB}$.

16.4 **(ii)** Helium is less dense and has a lower molar mass than air, so sound travels faster in helium than in air. The normal-mode frequencies for a pipe are proportional to the sound speed v, so the frequency and hence the pitch increase when the air in the pipe is replaced with helium.

16.5 **(i) and (iv)** There will be a resonance if 660 Hz is one of the pipe's normal-mode frequencies. A stopped organ pipe has normal-mode frequencies that are odd multiples of its fundamental frequency [see Eq. (16.22) and Fig. 16.18]. Hence pipe (i),which has fundamental frequency 220 Hz, also has a normal-mode frequency of $3(220\text{ Hz}) = 660\text{ Hz}$. Pipe (ii) has twice the length of pipe (i); from Eq. (16.20), the fundamental frequency of a stopped pipe is inversely proportional to the length, so pipe (ii) has a fundamental frequency of $\left(\frac{1}{2}\right)(220\text{ Hz}) = 110\text{ Hz}$. Its other normal-mode frequencies are 330 Hz, 550 Hz, 770 Hz,. . ., so a 660 Hz tuning fork will not cause resonance. Pipe (iii) is an open pipe of the same length as pipe (i), so its fundamental frequency is twice as great as for pipe (i) [compare Eqs. (16.16) and (16.20)], or $2(220\text{ Hz}) = 440\text{ Hz}$. Its other normal-mode frequencies are integer multiples of the fundamental frequency [see Eq. (16.19)], or 880 Hz, 1320 Hz,. . . , none of which match the 660 Hz frequency of the tuning fork. Pipe (iv) is also an open pipe but with twice the length of pipe (iii) [see Eq. (16.18)], so its normal-mode frequencies are one-half those of pipe (iii): 220 Hz, 440 Hz, 660 Hz,. . ., so the third harmonic will resonate with the tuning fork.

16.6 **(iii)** Constructive and destructive interference between two waves can occur only if the two waves have the same frequency. In this case the frequencies are different, so there are no points where the two waves always reinforce each other (constructive interference) or always cancel each other (destructive interference).

16.7 **(vi)** The beat frequency is 3 Hz, so the difference between the two tuning fork frequencies is also 3 Hz. Hence the second tuning fork vibrates at a frequency of either 443 Hz or 437 Hz. You can distinguish between the two possibilities by comparing the pitches of the two tuning forks sounded one at a time: The frequency is 437 Hz if the second tuning fork has a lower pitch and 443 Hz if it has a higher pitch.

16.8 **no** The air (the medium for sound waves) is moving from the source toward the listener. Hence, relative to the air, both the source and the listener are moving in the direction from listener to source. So both velocities are positive and $v_S = v_L = +10 \text{ m/s}$. The equality of these two velocities means that the numerator and the denominator in Eq. (16.29) are the same, so $f_L = f_S$ and there is *no* Doppler shift.

16.9 **(iii)** Figure 16.38 shows that there are sound waves inside the cone of the shock wave. Behind the airplane the wave crests are spread apart, just as they are behind the moving source in Fig. 16.28. Hence the waves that reach you have an increased wavelength and a lower frequency.

Key Example Variation Problems

VP16.9.1 **a.** $p_{max} = 6.74 \times 10^{-3} \text{ Pa}$

b. p_{max} is unchanged

VP16.9.2 **a.** $3.16 \times 10^{-4} \mathrm{W/m^2}$

b. 63.1

VP16.9.3 **a.** 102 dB

b. 90 dB

VP16.9.4 **a.** 5.02×10^{-2} Pa

b. 86.0 dB

VP16.12.1 **a.** 0.782 m

b. 0.130 m

VP16.12.2 **a.** $n = 1$ (the fundamental frequency)

b. $n = 3$ (the third harmonic)

c. none

VP16.12.3 **a.** 224 m/s

b. 0.521 m

VP16.12.4 **a.** $f = 749$ Hz, $\lambda = 1.33$ m

b. $f = 749$ Hz, $\lambda = 0.459$ m

c. 1.15 m

VP16.18.1 **a.** $f = 3.03 \times 10^3$ Hz, $\lambda = 0.112$ m

b. $f = 2.60 \times 10^3$ Hz, $\lambda = 0.131$ m

VP16.18.2 **a.** $\lambda = 0.773$ m

b. $f = 453$ Hz, $\lambda = 0.773$ m

c. $f = 427$ Hz, $\lambda = 0.773$ m

VP16.18.3 **a.** $1.22 \times 10^3 \mathrm{Hz}$

b. 1.50×10^3 Hz

VP16.18.4 **a.** 481 Hz

b. 29.9 m/s

Bridging Problem

(a) $180^\circ = \pi$ rad

(b) A alone: $I = 3.98 \times 10^{-6}$ W/m^2, $\beta = 66.0$ dB; B alone: $I = 5.31 \times 10^{-7}$ W/m^2, $\beta = 57.2$ dB

(c) $I = 1.60 \times 10^{-6}$ W/m^2, $\beta = 62.1$ dB

Part III: Thermodynamics

Chapter 17
Temperature and Heat

?◻ At a steelworks, molten iron is heated to Celsius to remove impurities. It is most accurate to say that the molten iron contains a large amount of (i) temperature; (ii) heat; (iii) energy; (iv) two of these; (v) all three of these.

Learning Outcomes

In this chapter, you'll learn...

17.1 The meaning of thermal equilibrium, and what thermometers really measure.

17.2 How different types of thermometers function.

17.3 The physics behind the absolute, or Kelvin, temperature scale.

17.4 How the dimensions of an object change as a result of a temperature change.

17.5 The meaning of heat, and how it differs from temperature.

17.6 How to do calculations that involve heat flow, temperature changes, and changes of phase.

17.7 How heat is transferred by conduction, convection, and radiation.

You'll need to review...

11.4 Stress and strain.

12.2 Measuring pressure.

14.4 Spring forces and interatomic forces.

Whether it's a sweltering summer day or a frozen midwinter night, your body needs to be kept at a nearly constant temperature. It has effective temperature-control mechanisms, but sometimes it needs help. On a hot day you wear less clothing to improve heat transfer from your body to the air and for better cooling by evaporation of perspiration. On a cold day you may sit by a roaring fire to absorb the energy that it radiates. The concepts in this chapter will help you understand the basic physics of keeping warm or cool.

The terms “temperature” and “heat” are often used interchangeably in everyday language. In physics, however, these two terms have very different meanings. In this chapter we’ll define temperature in terms of how it’s measured and see how temperature changes affect the dimensions of objects. We’ll see that heat refers to energy transfer caused by temperature differences only and learn how to calculate and control such energy transfers.

Our emphasis in this chapter is on the concepts of temperature and heat as they relate to *macroscopic* objects such as cylinders of gas, ice cubes, and the human body. In Chapter 18 we’ll look at these same concepts from a *microscopic* viewpoint in terms of the behavior of individual atoms and molecules. These two chapters lay the groundwork for the subject of **thermodynamics,** the study of energy transformations involving heat, mechanical work, and other aspects of energy and how these transformations relate to the properties of matter. Thermodynamics forms an indispensable part of the foundation of physics, chemistry, and the life sciences, and its applications turn up in such places as car engines, refrigerators, biochemical processes, and the structure of stars. We’ll explore the key ideas of thermodynamics in Chapters 19 and 20.

17.1 Temperature and Thermal Equilibrium

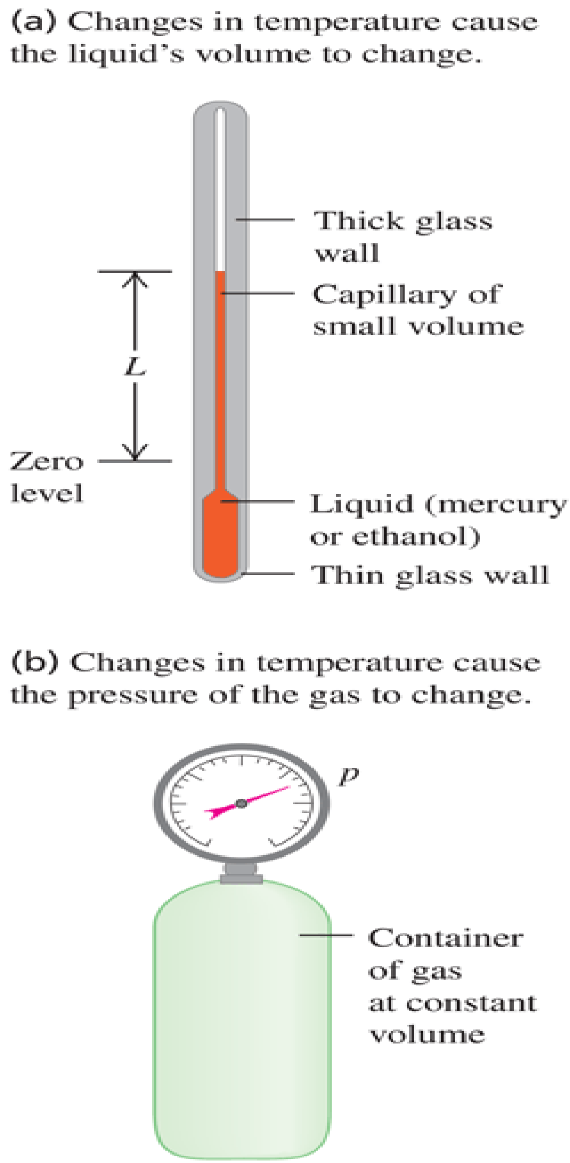
The concept of **temperature** is rooted in qualitative ideas based on our sense of touch. An object that feels “hot” usually has a higher temperature than a similar object that feels “cold.” That’s pretty vague, and the senses can be deceived. But many properties of matter that we can *measure*—including the length of a metal rod, steam pressure in a boiler, the ability of a wire to conduct an electric current, and the color of a very hot glowing object—depend on temperature.

Temperature is also related to the kinetic energies of the molecules of a material. In general this relationship is fairly complex, so it’s not a good place to start in *defining* temperature. In Chapter 18 we’ll look at the relationship between temperature and the energy of molecular motion for an ideal gas. However, we can define temperature and heat independently of any detailed molecular picture. In this section we’ll develop a *macroscopic* definition of temperature.

To use temperature as a measure of hotness or coldness, we need to construct a temperature scale. To do this, we can use any measurable property of a system that varies with its “hotness” or “coldness.” Figure 17.1a shows a familiar system that is used to measure temperature. When the system becomes hotter, the colored liquid (usually mercury or ethanol) expands and rises in the tube, and the value of L increases. Another simple system is a quantity of gas in a constant-volume container (Fig. 17.1b). The pressure p, measured by the gauge, increases or decreases as the gas becomes hotter or colder. A third example is the electrical resistance R of a conducting wire, which also varies when the wire becomes hotter or colder. Each of these properties gives us a number

(L, p, or R) that varies with hotness and coldness, so each property can be used to make a **thermometer.**

Figure 17.1



Temperature and Heat

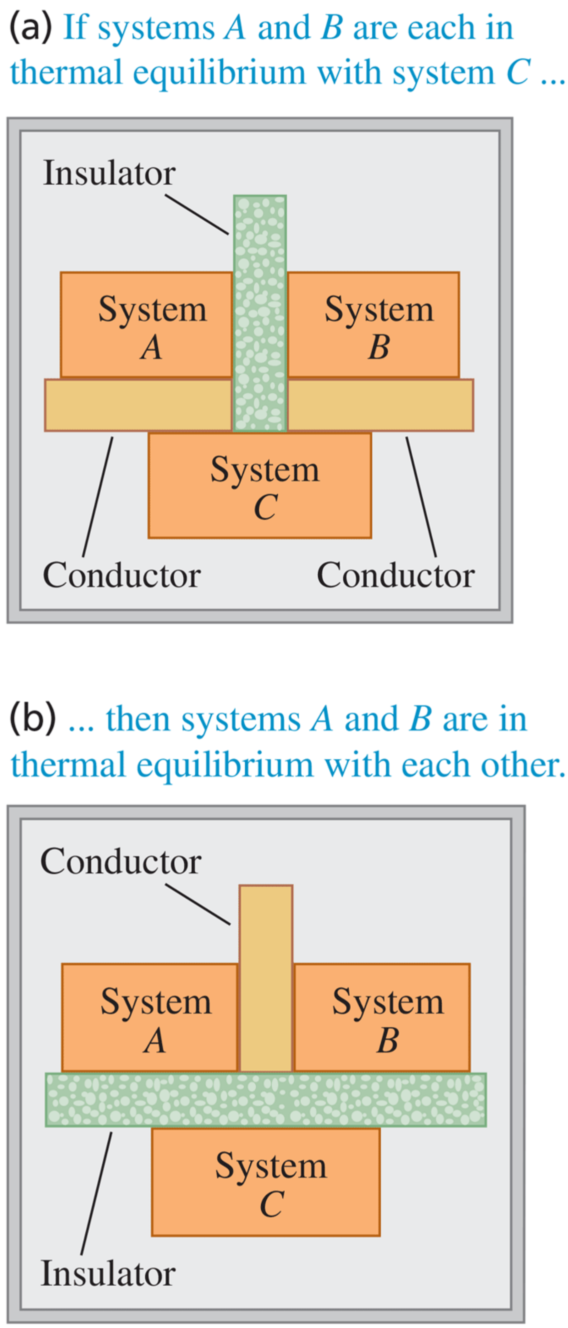
To measure the temperature of an object, you place the thermometer in contact with the object. If you want to know the temperature of a cup of hot coffee, you stick the thermometer in the coffee; as the two interact, the thermometer becomes hotter and the coffee cools off a little. After the thermometer settles down to a steady value, you read the temperature. The system has reached an *equilibrium* condition, in which the interaction between the thermometer and the coffee causes no further change in the system. We call this a state of **thermal equilibrium.**

If two systems are separated by an insulating material or **insulator** such as wood, plastic foam, or fiberglass, they influence each other more slowly. Camping coolers are made with insulating materials to delay the cold food inside from warming up and attaining thermal equilibrium with the hot summer air outside. An *ideal insulator* is an idealized material that permits no interaction at all between the two systems. It prevents the systems from attaining thermal equilibrium if they aren't in thermal equilibrium at the start. Real insulators, like those in camping coolers, aren't ideal, so the contents of the cooler will warm up eventually. But an ideal insulator is nonetheless a useful idealization, like a massless rope or a frictionless incline.

The Zeroth Law of Thermodynamics

We can discover an important property of thermal equilibrium by considering three systems, A, B, and C, that initially are not in thermal equilibrium (Fig. 17.2). We surround them with an ideal insulating box so that they cannot interact with anything except each other. We separate systems A and B with an ideal insulating wall (the green slab in Fig. 17.2a), but we let system C interact with both systems A and B. We show this interaction in the figure by a yellow slab representing a thermal **conductor,** a material that *permits* thermal interactions through it. We wait until thermal equilibrium is attained; then A and B are each in thermal equilibrium with C. But are they in thermal equilibrium *with each other?*

Figure 17.2



The zeroth law of thermodynamics.

To find out, we separate system C from systems A and B with an ideal insulating wall (Fig. 17.2b), then replace the insulating wall between A and B with a *conducting* wall that lets A and B interact. What happens?

Experiment shows that *nothing* happens; there are no additional changes to A or B. We can summarize this result as follows:

> ### Zeroth Law of Thermodynamics
> If C is initially in thermal equilibrium with both A and B, then A and B are also in thermal equilibrium with each other.

(The importance of the zeroth law was recognized only after the first, second, and third laws of thermodynamics had been named. Since it is fundamental to all of them, the name "zeroth" seemed appropriate. We'll learn about the other laws of thermodynamics in Chapters 19 and 20.)

Now suppose system C is a thermometer, such as the liquid-in-tube system of Fig. 17.1a. In Fig. 17.2a the thermometer C is in contact with both A and B. In thermal equilibrium, when the thermometer reading reaches a stable value, the thermometer measures the temperature of both A and B; hence both A and B have the *same* temperature. Experiment shows that thermal equilibrium isn't affected by adding or removing insulators, so the reading of thermometer C wouldn't change if it were in contact only with A or only with B. We conclude:

> ### Condition For Thermal Equilibrium
> Two systems are in thermal equilibrium if and only if they have the same temperature.

This is what makes a thermometer useful; a thermometer actually measures *its own* temperature, but when a thermometer is in thermal equilibrium with another object, the temperatures must be equal. When the temperatures of two systems are different, they *cannot* be in thermal equilibrium.

Test Your Understanding of Section 17.1

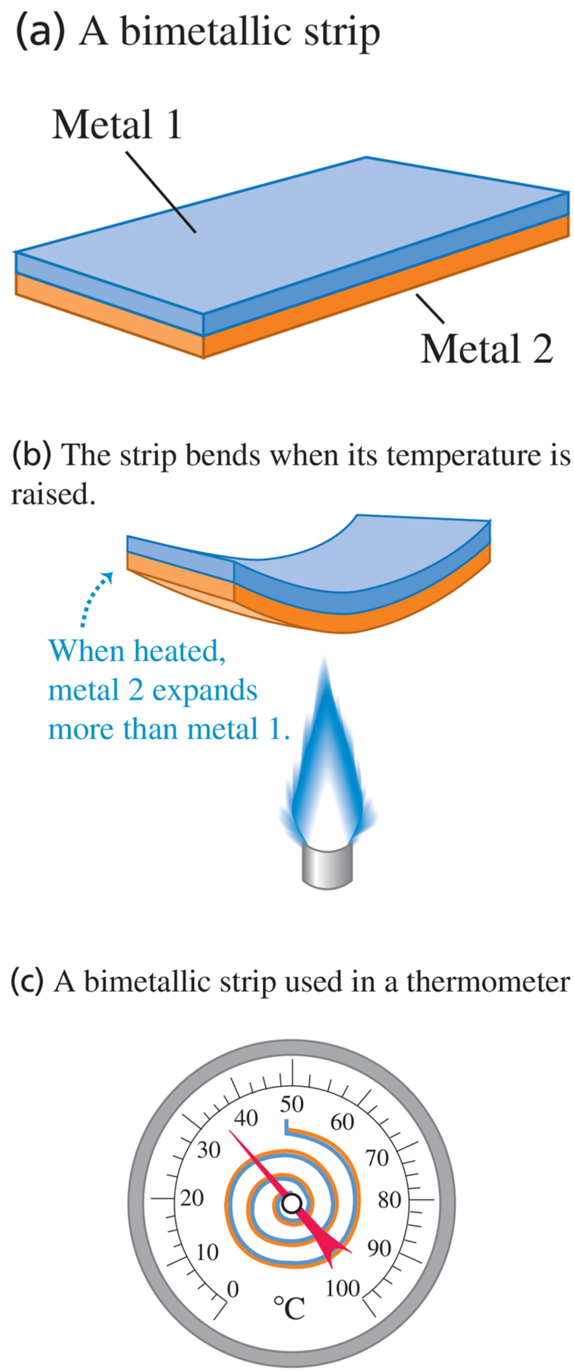
You put a thermometer in a pot of hot water and record the reading. What temperature have you recorded? (i) The temperature of the water; (ii) the temperature of the thermometer; (iii) an equal average of the temperatures of the water and thermometer; (iv) a weighted average of the temperatures of the water and thermometer, with more emphasis on the temperature of the water; (v) a weighted average of the water and thermometer, with more emphasis on the temperature of the thermometer.

17.2 Thermometers and Temperature Scales

To make the liquid-in-tube device shown in Fig. 17.1a into a useful thermometer, we need to mark a scale on the tube wall with numbers on it. Suppose we label the thermometer's liquid level at the freezing temperature of pure water "zero" and the level at the boiling temperature "100," and divide the distance between these two points into 100 equal intervals called *degrees*. The result is the **Celsius temperature scale** (formerly called the *centigrade* scale in English-speaking countries). The Celsius temperature for a state colder than freezing water is a negative number. The Celsius scale is used, both in everyday life and in science and industry, almost everywhere in the world.

Another common type of thermometer uses a *bimetallic strip*, made by bonding strips of two different metals together (Fig. 17.3a). When the temperature of the composite strip increases, one metal expands more than the other and the strip bends (Fig. 17.3b). This strip is usually formed into a spiral, with the outer end anchored to the thermometer case and the inner end attached to a pointer (Fig. 17.3c). The pointer rotates in response to temperature changes.

Figure 17.3



Use of a bimetallic strip as a thermometer.

In a *resistance thermometer* the changing electrical resistance of a coil of fine wire, a carbon cylinder, or a germanium crystal is measured.

Resistance thermometers are usually more precise than most other types.

Some thermometers detect the amount of infrared radiation emitted by an object. (We'll see in Section 17.7 that *all* objects emit electromagnetic radiation, including infrared, as a consequence of their temperature.) One example is a *temporal artery thermometer* (Fig. 17.4). A nurse runs this over a patient's forehead in the vicinity of the temporal artery, and an infrared sensor in the thermometer measures the radiation from the skin. This device gives more accurate values of body temperature than do oral or ear thermometers.

Figure 17.4

A temporal artery thermometer measures infrared radiation from the skin that overlies one of the important arteries in the head. Although the thermometer cover touches the skin, the infrared detector inside the cover does not.

In the **Fahrenheit temperature scale,** still used in the United States, the freezing temperature of water is $32^\circ\mathrm{F}$ and the boiling temperature is $212^\circ\mathrm{F}$, both at standard atmospheric pressure. There are 180 degrees between freezing and boiling, compared to 100 on the Celsius scale, so one Fahrenheit degree represents only $\frac{100}{180}$, or $\frac{5}{9}$, as great a temperature change as one Celsius degree.

To convert temperatures from Celsius to Fahrenheit, note that a Celsius temperature T_C is the number of Celsius degrees above freezing; the number of Fahrenheit degrees above freezing is $\frac{9}{5}$ of this. But freezing on the Fahrenheit scale is at $32^\circ F$, so to obtain the actual Fahrenheit temperature T_F, multiply the Celsius value by $\frac{9}{5}$ and then add 32°. Symbolically,

(17.1)

$$\underset{\text{Fahrenheit temperature}}{T_F} = \frac{9}{5}\underset{\text{Celsius temperature}}{T_C} + 32^\circ \tag{17.1}$$

To convert Fahrenheit to Celsius, solve this equation for T_C :

(17.2)

$$\underset{\text{Celsius temperature}}{T_C} = \frac{5}{9}\left(\underset{\text{Fahrenheit temperature}}{T_F} - 32^\circ\right) \tag{17.2}$$

In words, subtract 32° to get the number of Fahrenheit degrees above freezing, and then multiply by $\frac{5}{9}$ to obtain the number of Celsius degrees above freezing—that is, the Celsius temperature.

We don't recommend memorizing Eqs. (17.1) and (17.2). Instead, understand the reasoning that led to them so that you can derive them on the spot when you need them, checking your reasoning with the relationship $100^\circ C = 212^\circ F$.

It is useful to distinguish between an actual temperature and a temperature *interval* (a difference or change in temperature). An actual temperature of 20° is stated as $20^\circ C$ (twenty degrees Celsius), and a temperature *interval* of 15° is $15\ \mathrm{C}^{\circ}$ (fifteen Celsius degrees). A beaker of water heated from $20^{\circ}\mathrm{C}$ to $35^{\circ}\mathrm{C}$ undergoes a temperature change of $15\ \mathrm{C}^{\circ}$.

CAUTION Converting temperature differences Keep in mind that Eqs. (17.1) and (17.2) apply to *temperatures*, not *temperature differences*. To convert a temperature difference in Fahrenheit degrees (F°) to one in Celsius degrees (C°), simply multiply by $\frac{5}{9}$; to convert a temperature difference in C° to one in F°, multiply by $\frac{9}{5}$.

Test Your Understanding of Section 17.2

Which of the following types of thermometers have to be in thermal equilibrium with the object being measured in order to give accurate readings? (i) A bimetallic strip; (ii) a resistance thermometer; (iii) a temporal artery thermometer; (iv) both (i) and (ii); (v) all of (i), (ii), and (iii).

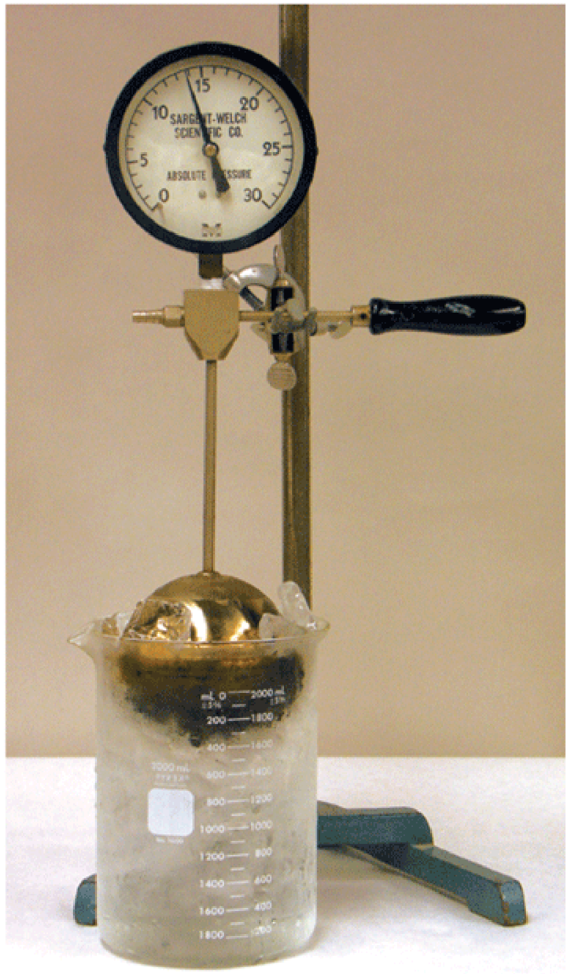
17.3 Gas Thermometers and the Kelvin Scale

When we calibrate two thermometers, such as a liquid-in-tube system and a resistance thermometer, so that they agree at $0^\circ C$ and $100^\circ C$, they may not agree exactly at intermediate temperatures. Any temperature scale defined in this way always depends somewhat on the specific properties of the material used. Ideally, we would like to define a temperature scale that *doesn't* depend on the properties of a particular material. To establish a truly material-independent scale, we first need to develop some principles of thermodynamics. We'll return to this fundamental problem in Chapter 20. Here we'll discuss a thermometer that comes close to the ideal, the *constant-volume gas thermometer.*

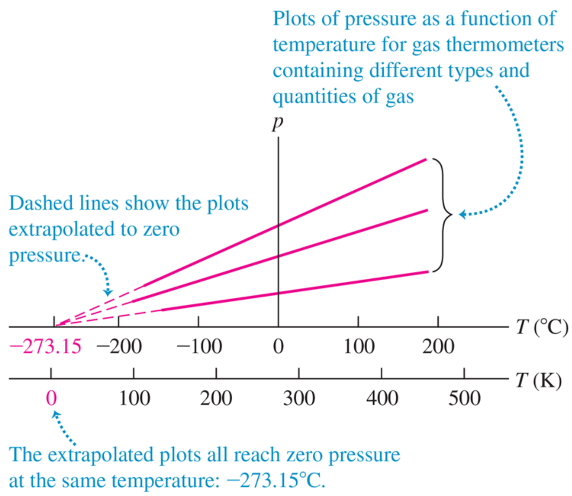
The principle of a constant-volume gas thermometer is that the pressure of a gas at constant volume increases with temperature. We place a quantity of gas in a constant-volume container (Fig. 17.5a) and measure its pressure by one of the devices described in Section 12.2. To calibrate this thermometer, we measure the pressure at two temperatures, say $0^\circ C$ and $100^\circ C$, plot these points on a graph, and draw a straight line between them. Then we can read from the graph the temperature corresponding to any other pressure. Figure 17.5b shows the results of three such experiments, each using a different type and quantity of gas.

Figure 17.5

(a) A constant-volume gas thermometer



(b) Graphs of pressure versus temperature at constant volume for three different types and quantities of gas



(a) Using a constant-volume gas thermometer to measure temperature. (b) The greater the amount of gas in the thermometer, the higher the graph of pressure p versus temperature T.

BIO Application

Mammalian Body Temperatures

Most mammals maintain body temperatures in the range from 36˚C to 40˚C (309 K to 313 K). A high metabolic rate warms the animal from within, and insulation (such as fur and body fat) slows heat loss.

By extrapolating this graph, we see that there is a hypothetical temperature, $-273.15^\circ\mathrm{C}$, at which the absolute pressure of the gas would become zero. This temperature turns out to be the *same* for many different gases (at least in the limit of very low gas density). We can't actually observe this zero-pressure condition. Gases liquefy and solidify at very low temperatures, and the proportionality of pressure to temperature no longer holds.

We use this extrapolated zero-pressure temperature as the basis for a temperature scale with its zero at this temperature. This is the **Kelvin temperature scale,** named for the British physicist Lord Kelvin (1824–

1907). The units are the same size as those on the Celsius scale, but the zero is shifted so that $0\ \mathrm{K} = -273.15^\circ\mathrm{C}$ and $273.15\ \mathrm{K} = 0^\circ\mathrm{C}$ (Fig. 17.5b); that is,

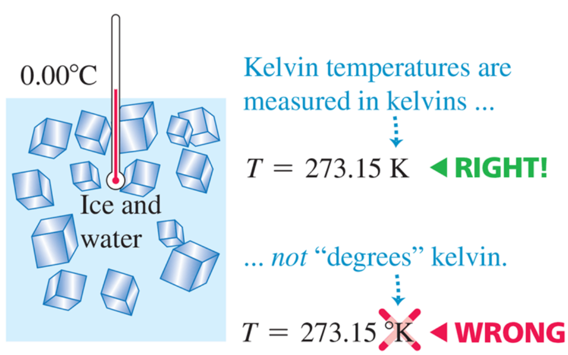
(17.3)

$$\underbrace{T_K}_{\text{Kelvin temperature}} = \underbrace{T_C}_{\text{Celsius temperature}} + 273.15 \qquad (17.3)$$

A common room temperature, $20^\circ\mathrm{C}$ ($= 68^\circ\mathrm{F}$), is $20 + 273.15$, or about 293 K.

CAUTION Never say "degrees kelvin" In SI nomenclature, the temperature mentioned above is read "293 kelvins," not "degrees kelvin" (Fig. 17.6). We capitalize Kelvin when it refers to the temperature scale; however, the *unit* of temperature is the *kelvin*, which is not capitalized (but is nonetheless abbreviated as a capital K).

Figure 17.6



Correct and incorrect uses of the Kelvin scale.

Example 17.1 Body temperature

You place a small piece of ice in your mouth. Eventually, the water all converts from ice at $T_1 = 32.00^\circ\text{F}$ to body temperature, $T_2 = 98.60^\circ\text{F}$. Express these temperatures in both Celsius degrees and kelvins, and find $\Delta T = T_2 - T_1$ in both cases.

IDENTIFY and SET UP Our target variables are stated above. We convert Fahrenheit temperatures to Celsius by using Eq. (17.2), and Celsius temperatures to Kelvin by using Eq. (17.3).

EXECUTE From Eq. (17.2), $T_1 = 0.00^\circ\text{C}$ and $T_2 = 37.00^\circ\text{C}$; then $\Delta T = T_2 - T_1 = 37.00\ \text{C}^\circ$. To get the Kelvin temperatures, just add 273.15 to each Celsius temperature: $T_1 = 273.15\ \text{K}$ and $T_2 = 310.15\ \text{K}$. The temperature difference is $\Delta T = T_2 - T_1 = 37.00\ \text{K}$.

EVALUATE The Celsius and Kelvin scales have different zero points but the same size degrees. Therefore *any* temperature difference ΔT is the *same* on the Celsius and Kelvin scales. However, ΔT is *not* the same on the Fahrenheit scale; here, for example, $\Delta T = 66.60\ \text{F}^\circ$.

KEY CONCEPT

The Celsius and Kelvin scales have different zero points, but differences in temperature are the same in both scales: Increasing the temperature by $37.00\ \text{C}^\circ$ is the same as increasing it by $37.00\ \text{K}$.

Video Tutor Solution: **Example 17.1**

The Kelvin Scale and Absolute Temperature

The Celsius scale has two fixed points: the normal freezing and boiling temperatures of water. But we can define the Kelvin scale by using a gas thermometer with only a single reference temperature. Figure 17.5b shows that the pressure p in a gas thermometer is directly proportional to the Kelvin temperature. So we can define the ratio of any two Kelvin temperatures T_1 and T_2 as the ratio of the corresponding gas-thermometer pressures p_1 and p_2 :

(17.4)

Definition of Kelvin scale: Ratio of two **temperatures** in **kelvins** ...

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \tag{17.4}$$

... equals ratio of corresponding **pressures** in **constant-volume gas thermometer.**

CAUTION Updating the Kelvin scale The definition of the Kelvin scale given here was accurate as of 2018. As of 2019, there is a new definition of this scale based on the definition of the joule and the value of the Boltzmann constant (which we'll introduce in Chapter 18). The change in definition has no effect, however, on the calculations in this textbook and calculations that you'll make.

To complete the definition of T, we need only specify the Kelvin temperature of a single state. For reasons of precision and reproducibility, the state chosen is the *triple point* of water, the unique combination of temperature and pressure at which solid water (ice), liquid water, and water vapor can all coexist. It occurs at a temperature of $0.01^\circ\mathrm{C}$ and a water-vapor pressure of 610 Pa (about 0.006 atm). (This is the pressure of the *water,* not the gas pressure in the *thermometer.*) The triple-point temperature of water is *defined* to have the value $T_{\text{triple}} = 273.16\ \mathrm{K}$, corresponding to $0.01^\circ\mathrm{C}$. From Eq. (17.4), if p_{triple} is the pressure in a gas thermometer at temperature T_{triple} and p is the pressure at some other temperature T, then T is given on the Kelvin scale by

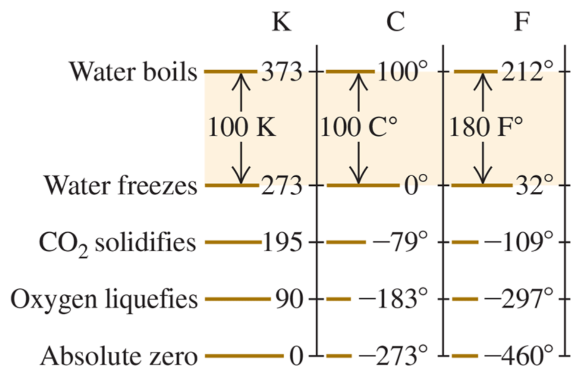
(17.5)

$$T = T_{\text{triple}} \frac{p}{p_{\text{triple}}} = (273.16\ \mathrm{K}) \frac{p}{p_{\text{triple}}}$$

Gas thermometers are impractical for everyday use. They are bulky and very slow to come to thermal equilibrium. They are used principally to establish high-precision standards and to calibrate other thermometers.

Figure 17.7 shows the relationships among the three temperature scales we have discussed. The Kelvin scale is called an **absolute temperature scale,** and its zero point [$T = 0\ \mathrm{K} = -273.15^\circ\mathrm{C}$, the temperature at which $p = 0$ in Eq. (17.5)] is called **absolute zero.** At absolute zero a system of molecules (such as a quantity of a gas, a liquid, or a solid) has its *minimum* possible total energy (kinetic plus potential); because of quantum effects, it is *not* correct to say that all molecular motion ceases at absolute zero. In Chapter 20 we'll define more completely what we mean by absolute zero through thermodynamic principles that we'll develop in the next several chapters.

Figure 17.7



Relationships among Kelvin (K), Celsius (c), and Fahrenheit (F) temperature scales. Temperatures have been rounded off to the nearest degree.

Test Your Understanding of Section 17.3

Rank the following temperatures from highest to lowest: (i) $0.00^\circ C$; (ii) $0.00^\circ F$; (iii) 260.00 K; (iv) 77.00 K; (v) $-180.00^\circ C$.

17.4 Thermal Expansion

Most materials expand when their temperatures increase. Rising temperatures make the liquid expand in a liquid-in-tube thermometer (Fig. 17.1a) and bend bimetallic strips (Fig. 17.3b). A completely filled and tightly capped bottle of water cracks when it is heated, but you can loosen a metal jar lid by running hot water over it. These are all examples of *thermal expansion*.

Linear Expansion

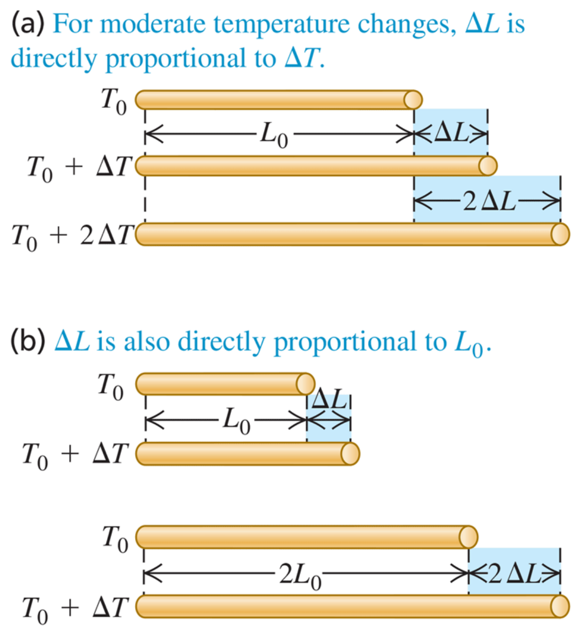
Suppose a solid rod has a length L_0 at some initial temperature T_0 . When the temperature changes by ΔT, the length changes by ΔL. Experiments show that if ΔT is not too large (say, less than 100 C° or so), ΔL is *directly proportional* to ΔT (Fig. 17.8a). If two rods made of the same material have the same temperature change, but one is twice as long as the other, then the *change* in its length is also twice as great. Therefore ΔL must also be proportional to L_0 (Fig. 17.8b). We may express these relationships in an equation:

(17.6)

Linear thermal expansion:

$$\underbrace{\Delta L}_{\text{Change in length}} = \underbrace{\alpha}_{\text{Coefficient of linear expansion}} \overbrace{L_0}^{\text{Original length}} \underbrace{\Delta T}_{\text{Temperature change}} \qquad (17.6)$$

Figure 17.8



How the length of a rod changes with a change in temperature. (Length changes are exaggerated for clarity.)

The constant α, which has different values for different materials, is called the **coefficient of linear expansion.** The units of α are K^{-1} or $(\mathrm{C}^{\circ})^{-1}$. (Remember that a temperature interval is the same on the Kelvin and Celsius scales.) If an object has length L_0 at temperature T_0, then its length L at a temperature $T = T_0 + \Delta T$ is

(17.7)

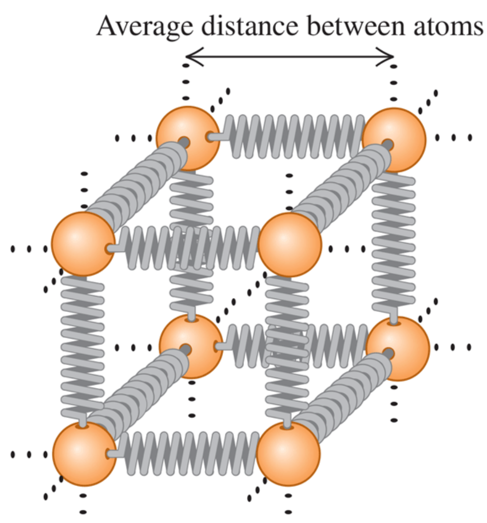
$$L = L_0 + \Delta L = L_0 + \alpha L_0\ \Delta T = L_0\ (1 + \alpha\ \Delta T)$$

For many materials, every linear dimension changes according to Eq. (17.6) or (17.7). Thus L could be the thickness of a rod, the side length of a square sheet, or the diameter of a hole. Some materials, such as wood or single crystals, expand differently in different directions. We won't consider this complication.

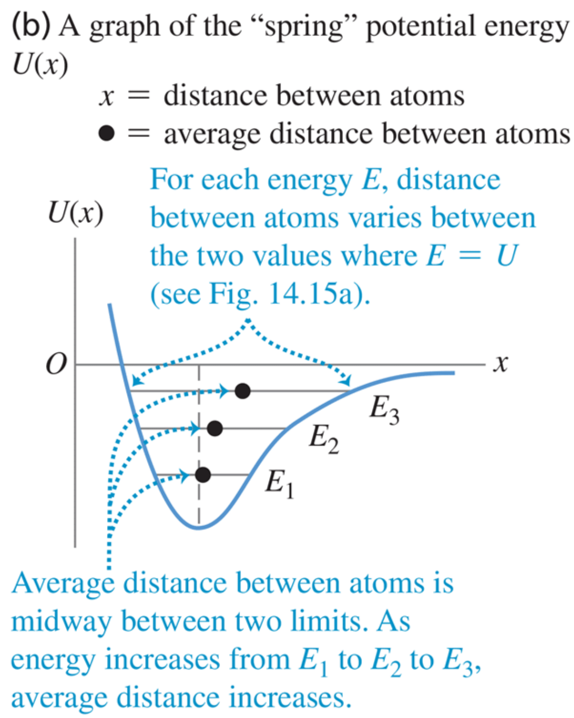
We can understand thermal expansion qualitatively on a molecular basis. Picture the interatomic forces in a solid as springs, as in Fig. 17.9a. (We explored the analogy between spring forces and interatomic forces in Section 14.4.) Each atom vibrates about its equilibrium position. When the temperature increases, the energy and amplitude of the vibration also increase. The interatomic spring forces are not symmetrical about the equilibrium position; they usually behave like a spring that is easier to stretch than to compress. As a result, when the amplitude of vibration increases, the *average* distance between atoms also increases (Fig. 17.9b). As the atoms get farther apart, every dimension increases.

Figure 17.9

(a) A model of the forces between neighboring atoms in a solid



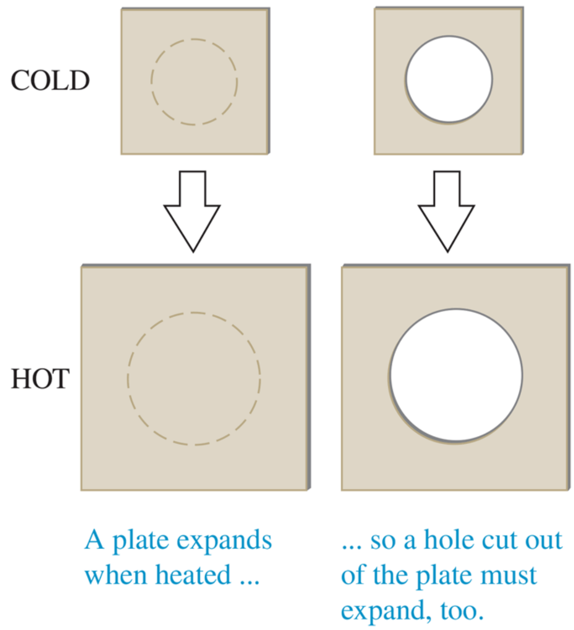
(b) A graph of the “spring” potential energy $U(x)$



(a) We can model atoms in a solid as being held together by “springs” that are easier to stretch than to compress. (b) A graph of the “spring” potential energy $U(x)$ versus distance x between neighboring atoms is *not* symmetrical (compare Fig. 14.20b). As the energy increases and the atoms oscillate with greater amplitude, the average distance increases.

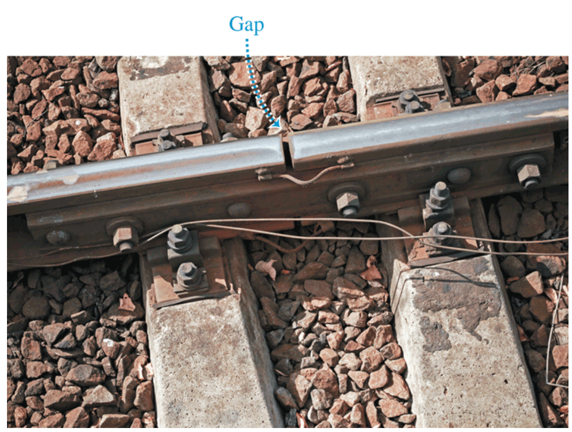
CAUTION Heating an object with a hole If a solid object has a hole in it, what happens to the size of the hole when the temperature of the object increases? A common misconception is that if the object expands, the hole will shrink because material expands into the hole. But, in fact, if the object expands, the hole will expand too (Fig. 17.10); *every* linear dimension of an object changes in the same way when the temperature changes. Think of the atoms in Fig. 17.9a as outlining a cubical hole. When the object expands, the atoms move apart and the hole increases in size. The only situation in which a “hole” will fill in due to thermal expansion is when two separate objects expand and close the gap between them (Fig. 17.11).

Figure 17.10



When an object undergoes thermal expansion, any holes in the object expand as well. (The expansion is exaggerated.)

Figure 17.11



This railroad track has a gap between segments to allow for thermal expansion. (The “clickety-clack” sound familiar to railroad passengers comes from the wheels passing over such gaps.) On hot days, the segments expand and fill in the gap. If there were no gaps, the track could buckle under very hot conditions.

The direct proportionality in Eq. (17.6) is not exact; it is *approximately* correct only for sufficiently small temperature changes. For a given material, α varies somewhat with the initial temperature T_0 and the size of the temperature interval. We'll ignore this complication here, however. Table 17.1 lists values of α for several materials. Within the precision of these values we don't need to worry whether T_0 is $0^\circ\mathrm{C}$ or $20^\circ\mathrm{C}$ or some other temperature. Typical values of α are very small; even for a temperature change of $100\ \mathrm{C}^\circ$, the fractional length change $\Delta L/L_0$ is only of the order of $\frac{1}{1000}$ for the metals in the table.

Table 17.1 Coefficients of Linear Expansion

Material	$\alpha\ [\mathrm{K}^{-1}$ or $(\mathrm{C}^\circ)^{-1}]$
Aluminum	2.4×10^{-5}
Brass	2.0×10^{-5}
Copper	1.7×10^{-5}
Glass	$0.4\text{–}0.9 \times 10^{-5}$
Invar (nickel–iron alloy)	0.09×10^{-5}
Quartz (fused)	0.04×10^{-5}
Steel	1.2×10^{-5}

Volume Expansion

Increasing temperature usually causes increases in *volume* for both solids and liquids. Just as with linear expansion, experiments show that if the temperature change ΔT is less than $100\ \mathrm{C}^\circ$ or so, the increase in volume ΔV is approximately proportional to both the temperature change ΔT and the initial volume V_0 :

(17.8)

$$\underbrace{\Delta V}_{\text{Change in volume}} = \underbrace{\beta}_{\text{Coefficient of volume expansion}} \overbrace{V_0}^{\text{Original volume}} \underbrace{\Delta T}_{\text{Temperature change}} \tag{17.8}$$

Volume thermal expansion:

The constant β characterizes the volume expansion properties of a particular material; it is called the **coefficient of volume expansion.** The units of β are K^{-1} or $(\mathrm{C}^\circ)^{-1}$. As with linear expansion, β varies somewhat with temperature, and Eq. (17.8) is an approximate relationship that is valid only for small temperature changes. For many substances, β decreases at low temperatures. Table 17.2 lists values of β for several materials near room temperature. Note that the values for liquids are generally much larger than those for solids.

Table 17.2 Coefficients of Volume Expansion

Solids	β [K^{-1} or $(\mathrm{C}^\circ)^{-1}$]	Liquids	β [K^{-1} or $(\mathrm{C}^\circ)^{-1}$]
Aluminum	7.2×10^{-5}	Ethanol	75×10^{-5}
Brass	6.0×10^{-5}	Carbon disulfide	115×10^{-5}
Copper	5.1×10^{-5}	Glycerin	49×10^{-5}
Glass	$1.2–2.7 \times 10^{-5}$	Mercury	18×10^{-5}
Invar	0.27×10^{-5}		
Quartz (fused)	0.12×10^{-5}		
Steel	3.6×10^{-5}		

For solid materials we can find a simple relationship between the volume expansion coefficient β and the linear expansion coefficient α. Consider a cube of material with side length L and volume $V = L^3$. At the initial temperature the values are L_0 and V_0. When the temperature increases by dT, the side length increases by dL and the volume increases by an amount dV :

$$dV = \frac{dV}{dL} dL = 3L^2 \, dL$$

Now we replace L and V by the initial values L_0 and V_0. From Eq. (17.6), dL is

$$dL = \alpha L_0 \, dT$$

Since $V_0 = {L_0}^3$, this means that dV can also be expressed as

$$dV = 3{L_0}^2 \alpha L_0 \, dT = 3\alpha V_0 \, dT$$

This is consistent with the infinitesimal form of Eq. (17.8), $dV = \beta V_0 \, dT$, only if

(17.9)

$$\beta = 3\alpha$$

(Check this relationship for some of the materials listed in Tables 17.1 and 17.2.)

Problem-Solving Strategy 17.1 Thermal Expansion

Identify *the relevant concepts:* Decide whether the problem involves changes in length (linear thermal expansion) or in volume (volume thermal expansion).

SET UP *the problem* using the following steps:

1. List the known and unknown quantities and identify the target variables.
2. Choose Eq. (17.6) for linear expansion and Eq. (17.8) for volume expansion.

EXECUTE *the solution* as follows:

1. Solve for the target variables. If you are given an initial temperature T_0 and must find a final temperature T corresponding to a given length or volume change, find ΔT and calculate $T = T_0 + \Delta T$. Remember that the size of a hole in a material varies with temperature just as any other linear dimension, and that the volume of a hole (such as the interior of a container) varies just as that of the corresponding solid shape.
2. Maintain unit consistency. Both L_0 and ΔL (or VV_0 and ΔV) must have the same units. If you use a value of α or β in K^{-1} or $(\mathrm{C}^{\circ})^{-1}$, then ΔT must be in either kelvins or Celsius degrees; from Example 17.1, the two scales are equivalent *for temperature differences*.

EVALUATE *your answer:* Check whether your results make sense.

Example 17.2 Length change due to temperature change

WITH VARIATION PROBLEMS

A surveyor uses a steel measuring tape that is exactly 50.000 m long at a temperature of $20^{\circ}\mathrm{C}$. The markings on the tape are calibrated for this temperature. (a) What is the length of the tape when the temperature is 35°C? (b) When it is 35°C, the surveyor uses the tape to measure a distance. The value that she reads off the tape is 35.794 m. What is the actual distance?

IDENTIFY and SET UP This problem concerns the linear expansion of a measuring tape. We are given the tape's initial length $L_0 = 50.000$ m at $T_0 = 20^\circ$C. In part (a) we use Eq. (17.6) to find the change ΔL in the tape's length at $T = 35^\circ$C, and use Eq. (17.7) to find L. (Table 17.1 gives the value of α for steel.) Since the tape expands, at 35°C the distance between two successive meter marks is greater than 1 m. Hence the actual distance in part (b) is *larger* than the distance read off the tape by a factor equal to the ratio of the tape's length L at 35°C to its length L_0 at 20°C.

EXECUTE (a) The temperature change is $\Delta T = T - T_0 = 15\ \mathrm{C}^\circ$=15 K; from Eqs. (17.6) and (17.7),

$$\begin{aligned} \Delta L &= \alpha L_0\, \Delta T = (1.2\times10^{-5}\mathrm{K}^{-1})(50\ \mathrm{m})(15\ \mathrm{K}) \\ &= 9.0\times10^{-3}\ \mathrm{m} = 9.0\ \mathrm{mm} \\ L &= L_0 + \Delta L = 50.000\ \mathrm{m} + 0.009\ \mathrm{m} = 50.009\ \mathrm{m} \end{aligned}$$

(b) Our result from part (a) shows that at 35°C, the slightly expanded tape reads a distance of 50.000 m when the true distance is 50.009 m. We can rewrite the algebra of part (a) as $L = L_0\,(1+\alpha\,\Delta T)$; at 35°C, *any* true distance will be greater than the reading by the factor $50.009/50.000 = 1 + \alpha\,\Delta T\ = 1 + 1.8\times10^{-4}$. The true distance is therefore

$$\left(1 + 1.8\times10^{-4}\right)(35.794\ \mathrm{m}) = 35.800\ \mathrm{m}$$

EVALUATE In part (a) we needed only two of the five significant figures of L_0 to compute ΔL to the same number of decimal places as L_0. Our result shows that metals expand very little under moderate temperature changes. However, even the small difference

$0.009\ \text{m} = 9\ \text{mm}$ found in part (b) between the scale reading and the true distance can be important in precision work.

KEY CONCEPT

A change in temperature causes the *length* of an object to change by an amount that is approximately proportional to the object's initial length and to the temperature change ΔT.

Video Tutor Solution: **Example 17.2**

Example 17.3 Volume change due to temperature change

WITH VARIATION PROBLEMS

A $200\ \text{cm}^3$ glass flask is filled to the brim with mercury at 20°C. How much mercury overflows when the temperature of the system is raised to 100°C? The coefficient of *linear* expansion of the glass is $0.40 \times 10^{-5}\text{K}^{-1}$.

IDENTIFY and SET UP This problem involves the volume expansion of the glass and of the mercury. The amount of overflow depends on the *difference* between the volume changes ΔV for these two materials, both given by Eq. (17.8). The mercury will overflow if its coefficient of volume expansion β (see Table 17.2) is greater than that of glass, which we find from Eq. (17.9) using the given value of α.

EXECUTE From Table 17.2, $\beta_{\text{Hg}} = 18 \times 10^{-5}\ \text{K}^{-1}$. That is indeed greater than $\beta_{\text{glass}} = 3\alpha_{\text{glass}} = 3(0.40 \times 10^{-5}\ \text{K}^{-1}) = 1.2 \times 10^{-5}\ \text{K}^{-1}$, from Eq. (17.9). The volume overflow is then

$$\begin{aligned} \Delta V_{\text{Hg}} - \Delta V_{\text{glass}} &= \beta_{\text{Hg}} V_0 \Delta T - \beta_{\text{glass}} V_0 \Delta T \\ &= V_0 \Delta T (\beta_{\text{Hg}} - \beta_{\text{glass}}) \\ &= (200\ \text{cm}^3)(80\ \text{C}^\circ)\left(18 \times 10^{-5} - 1.2 \times 10^{-5}\right) = 2.7\ \text{cm}^3 \end{aligned}$$

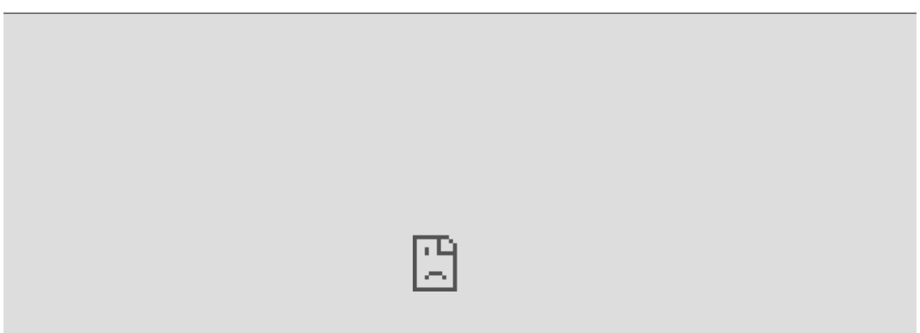
EVALUATE This is basically how a mercury-in-glass thermometer works; the column of mercury inside a sealed tube rises as T increases because mercury expands faster than glass.

As Tables 17.1 and 17.2 show, glass has smaller coefficients of expansion α and β than do most metals. This is why you can use hot water to loosen a metal lid on a glass jar; the metal expands more than the glass does.

KEY CONCEPT

A change in temperature causes the *volume* of an object to change by an amount that is approximately proportional to the object's initial volume and to the temperature change ΔT.

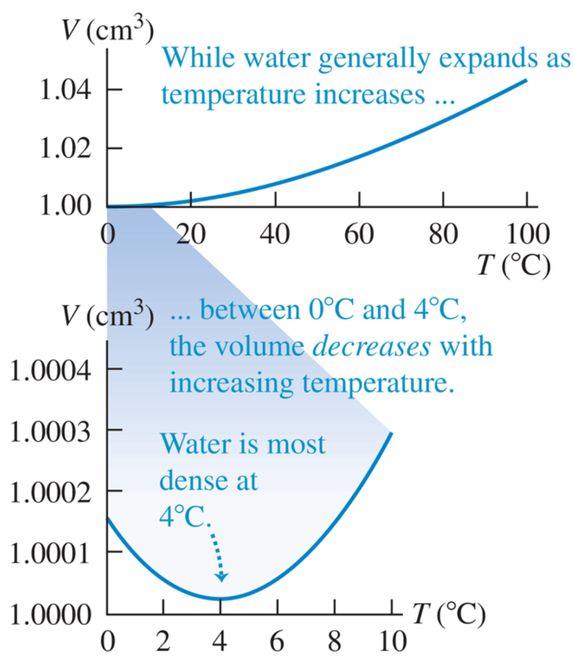
Video Tutor Solution: **Example 17.3**



Thermal Expansion of Water

Water, in the temperature range from 0˚C to 4˚C, *decreases* in volume with increasing temperature. In this range its coefficient of volume expansion is *negative*. Above 4˚C, water expands when heated (Fig. 17.12). Hence water has its greatest density at 4˚C. Water also expands when it freezes, which is why ice humps up in the middle of the compartments in an ice-cube tray. By contrast, most materials contract when they freeze.

Figure 17.12



The volume of 1 gram of water in the temperature range from 0˚C to 100˚C. By 100˚C the volume has increased to 1.043 cm^3. If the coefficient of volume expansion were constant, the curve would be a straight line.

This anomalous behavior of water has an important effect on plant and animal life in lakes. A lake cools from the surface down; above $4^\circ C$, the cooled water at the surface flows to the bottom because of its greater density. But when the surface temperature drops below $4^\circ C$, the water near the surface is less dense than the warmer water below. Hence the downward flow ceases, and the water near the surface remains colder than that at the bottom. As the surface freezes, the ice floats because it is less dense than water. The water at the bottom remains at $4^\circ C$ until nearly the entire lake is frozen. If water behaved like most substances, contracting continuously on cooling and freezing, lakes would freeze from the bottom up. Circulation due to density differences would continuously carry warmer water to the surface for efficient cooling, and lakes would freeze solid much more easily. This would destroy all plant and animal life that cannot withstand freezing. If water did not have its special properties, the evolution of life would have taken a very different course.

Thermal Stress

If we clamp the ends of a rod rigidly to prevent expansion or contraction and then change the temperature, **thermal stresses** develop. The rod would like to expand or contract, but the clamps won't let it. The resulting stresses may become large enough to strain the rod irreversibly or even break it. (Review the discussion of stress and strain in Section 11.4.)

Engineers must account for thermal stress when designing structures (see Fig. 17.11). Concrete highways and bridge decks usually have gaps between sections, filled with a flexible material or bridged by interlocking teeth (Fig. 17.13), to permit expansion and contraction of the concrete.

Long steam pipes have expansion joints or U-shaped sections to prevent buckling or stretching with temperature changes. If one end of a steel bridge is rigidly fastened to its abutment, the other end usually rests on rollers.

Figure 17.13

Expansion joints on bridges are needed to accommodate changes in length that result from thermal expansion.

To calculate the thermal stress in a clamped rod, we compute the amount the rod *would* expand (or contract) if not held and then find the stress needed to compress (or stretch) it back to its original length. Suppose that a rod with length L_0 and cross-sectional area A is held at constant length while the temperature is reduced, causing a tensile stress. From Eq. (17.6), the fractional change in length if the rod were free to contract would be

(17.10)

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} = \alpha\,\Delta T$$

Since the temperature decreases, both ΔL and ΔT are negative. The tension must increase by an amount F that is just enough to produce an equal and opposite fractional change in length $(\Delta L/L_0)_{\text{tension}}$. From the definition of Young's modulus, Eq. (11.10),

(17.11)

$$Y = \frac{F/A}{\Delta L/L_0} \qquad \text{so} \qquad \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \frac{F}{AY}$$

If the length is to be constant, the *total* fractional change in length must be zero. From Eqs. (17.10) and (17.11), this means that

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} + \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \alpha\,\Delta T + \frac{F}{AY} = 0$$

Solve for the tensile stress F/A required to keep the rod's length constant:

(17.12)

Thermal stress:

Force needed to keep length of rod constant

$$\frac{F}{A} = -Y\alpha\,\Delta T \qquad (17.12)$$

Young's modulus

Temperature change

Coefficient of linear expansion

Cross-sectional area of rod

For a decrease in temperature, ΔT is negative, so F and F/A are positive; this means that a *tensile* force and stress are needed to maintain the length. If ΔT is positive, F and F/A are negative, and the required force and stress are *compressive.*

If there are temperature differences within an object, nonuniform expansion or contraction will result and thermal stresses can be induced. You can break a glass bowl by pouring very hot water into it; the thermal stress between the hot and cold parts of the bowl exceeds the breaking stress of the glass, causing cracks. The same phenomenon makes ice cubes crack when dropped into warm water.

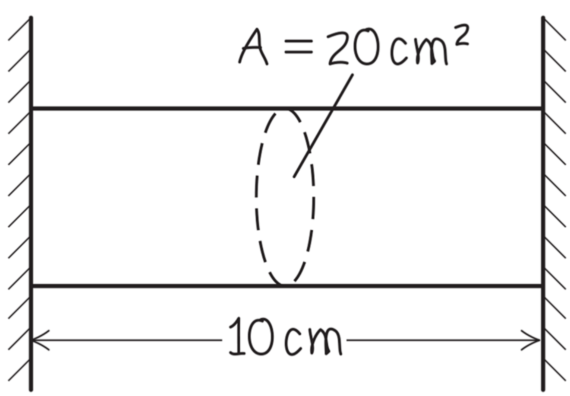
Example 17.4 Thermal stress

WITH VARIATION PROBLEMS

An aluminum cylinder 10 cm long, with a cross-sectional area of 20 cm^2, is used as a spacer between two steel walls. At 17.2°C it just slips between the walls. Calculate the stress in the cylinder and the total force it exerts on each wall when it warms to 22.3°C, assuming that the walls are perfectly rigid and a constant distance apart.

IDENTIFY and SET UP See Fig. 17.14. The cylinder of given cross-sectional area A exerts force F on each wall; our target variables are the stress F/A and F itself. We use Eq. (17.12) to relate F/A to the temperature change ΔT, and from that calculate F. (The length of the cylinder is irrelevant.) We find Young's modulus Y_{Al} and the coefficient of linear expansion α_{Al} from Tables 11.1 and 17.1, respectively.

Figure 17.14



Our sketch for this problem.

EXECUTE We have $Y_{\mathrm{Al}} = 7.0 \times 10^{10}$ Pa and $\alpha_{\mathrm{Al}} = 2.4 \times 10^{-5} \mathrm{K}^{-1}$, and $\Delta T = 22.3^{\circ}\mathrm{C} - 17.2^{\circ}\mathrm{C} = 5.1\ \mathrm{C}^{\circ} = 5.1$ K. From Eq. (17.12), the stress is

$$\begin{aligned} \frac{F}{A} &= -Y_{\mathrm{Al}}\alpha_{Al}\,\Delta T \\ &= -(7.0 \times 10^{10}\ \mathrm{Pa})(2.4 \times 10^{-5}\ \mathrm{K}^{-1})(5.1\ \mathrm{K}) \\ &= -8.6 \times 10^{6}\ \mathrm{Pa} = -1200\ \mathrm{lb/in.}^{2} \end{aligned}$$

The total force is the cross-sectional area times the stress:

$$\begin{aligned} F = A\left(\frac{F}{A}\right) &= (20 \times 10^{-4}\ \mathrm{m}^2)(-8.6 \times 10^{6}\ \mathrm{Pa}) \\ &= -1.7 \times 10^{4}\ \mathrm{N} = -1.9\ \mathrm{tons} \end{aligned}$$

EVALUATE The stress on the cylinder and the force it exerts on each wall are immense. Such thermal stresses must be accounted for in engineering.

KEY CONCEPT

To keep the length of an object constant when the temperature changes, forces must be applied to both of its ends. The required stress (force per unit area) is proportional to the temperature change.

Video Tutor Solution: **Example 17.4**

Test Your Understanding of Section 17.4

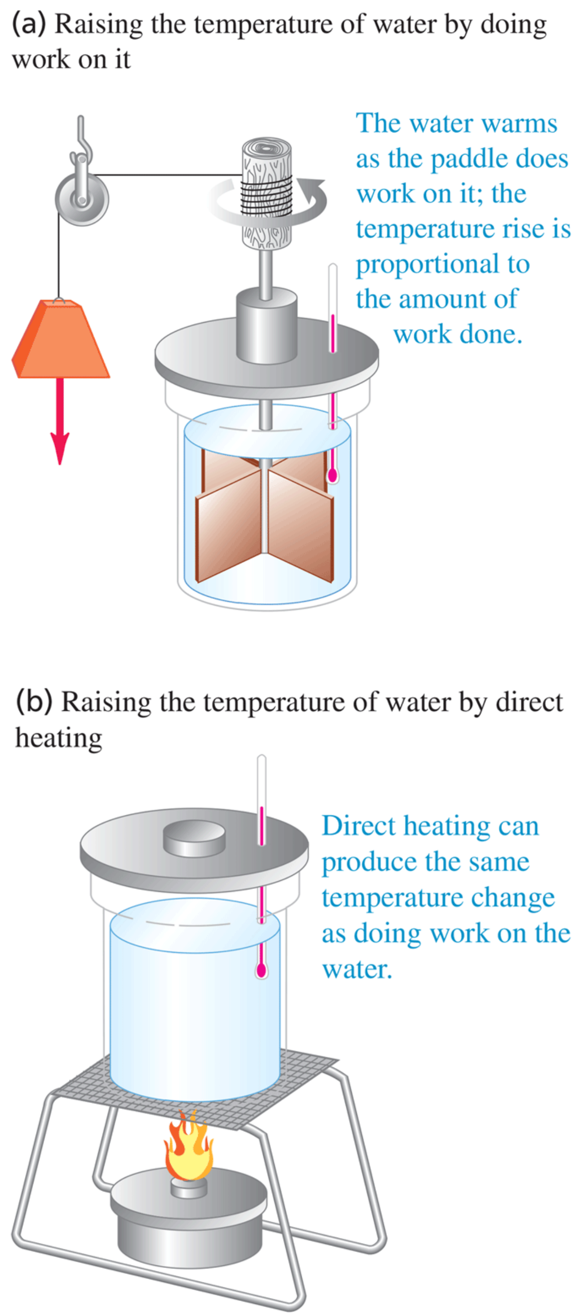
In the bimetallic strip shown in Fig. 17.3a, metal 1 is copper. Which of the following materials could be used for metal 2? (There may be more than one correct answer). (i) Steel; (ii) brass; (iii) aluminum.

17.5 Quantity of Heat

When you put a cold spoon into a cup of hot coffee, the spoon warms up and the coffee cools down as they approach thermal equilibrium. What causes these temperature changes is a transfer of *energy* from one substance to another. Energy transfer that takes place solely because of a temperature difference is called *heat flow* or *heat transfer,* and energy transferred in this way is called **heat.**

An understanding of the relationship between heat and other forms of energy emerged during the 18th and 19th centuries. Sir James Joule (1818–1889) studied how water can be warmed by vigorous stirring with a paddle wheel (Fig. 17.15a). The paddle wheel adds energy to the water by doing *work* on it, and Joule found that *the temperature rise is directly proportional to the amount of work done.* The same temperature change can also be caused by putting the water in contact with some hotter object (Fig. 17.15b); hence this interaction must also involve an energy exchange. We'll explore the relationship between heat and mechanical energy in Chapters 19 and 20.

Figure 17.15



The same temperature change of the same system may be accomplished by (a) doing work on it or (b) adding heat to it.

? CAUTION Temperature vs. heat It is absolutely essential for you to distinguish between *temperature* and *heat.* Temperature depends on the physical state of a material and is a quantitative description of its hotness or coldness. In physics the term “heat” always refers to energy in transit from one object or system to another because of a temperature difference, never to the amount of energy contained within a particular system. We can change the temperature of an object by adding heat to it or taking heat away, or by adding or subtracting energy in other ways, such as mechanical work (Fig. 17.15a). If we cut an object in half, each half has the same temperature as the whole; but to raise the temperature of each half by a given interval, we add *half* as much heat as for the whole.

We can define a *unit* of quantity of heat based on temperature changes of some specific material. The **calorie** (abbreviated cal) is *the amount of heat required to raise the temperature of 1 gram of water from* $14.5^\circ\mathrm{C}$ *to* $15.5^\circ\mathrm{C}$. A food-value calorie is actually a kilocalorie (kcal), equal to 1000 cal. A corresponding unit of heat that uses Fahrenheit degrees and British units is the **British thermal unit,** or Btu. One Btu is the quantity of heat required to raise the temperature of 1 pound (weight) of water $1\ \mathrm{F}^\circ$ from $63^\circ\mathrm{F}$ to $64^\circ\mathrm{F}$.

Because heat is energy in transit, there must be a definite relationship between these units and the familiar mechanical energy units such as the joule (Fig. 17.16). Experiments similar in concept to Joule’s have shown that

$$\begin{aligned} 1\ \mathrm{cal} &= 4.186\ \mathrm{J} \\ 1\ \mathrm{kcal} &= 1000\ \mathrm{cal} = 4186\ \mathrm{J} \\ 1\ \mathrm{Btu} &= 778\ \mathrm{ft}\cdot\mathrm{lb} = 252\ \mathrm{cal} = 1055\ \mathrm{J} \end{aligned}$$

Figure 17.16

The word "energy" is of Greek origin. This label on a can of Greek coffee shows that 100 milliliters of prepared coffee have an energy content $\left(\varepsilon\upsilon\acute{\varepsilon}\rho\gamma\varepsilon\iota\alpha\right)$ of 9.6 kilojoules or 2.3 kilocalories.

The calorie is not a fundamental SI unit. The International Committee on Weights and Measures recommends using the joule as the basic unit of energy in all forms, including heat. We'll follow that recommendation in this book.

Specific Heat

We use the symbol Q for quantity of heat. When it is associated with an infinitesimal temperature change dT, we call it dQ. The quantity of heat Q required to increase the temperature of a mass m of a certain material from T_1 to T_2 is found to be approximately proportional to the temperature change $\Delta T = T_2 - T_1$. It is also proportional to the mass m of material. When you're heating water to make tea, you need twice as much heat for two cups as for one if the temperature change is the same. The quantity of heat needed also depends on the nature of the material; raising the temperature of 1 kilogram of water by 1 C$^\circ$ requires 4190 J of heat, but only 910 J is needed to raise the temperature of 1 kilogram of aluminum by 1 C$^\circ$.

Putting all these relationships together, we have

(17.13)

$$\underbrace{Q}_{\text{Heat required to change temperature of a certain mass}} = \overbrace{m}^{\text{Mass of material}}\underbrace{c}_{\text{Specific heat of material}}\,\underbrace{\Delta T}_{\text{Temperature change}} \qquad (17.13)$$

The **specific heat** c has different values for different materials. For an infinitesimal temperature change dT and corresponding quantity of heat dQ,

(17.14)

$$dQ = mc\,dT$$

(17.15)

$$c = \frac{1}{m}\frac{dQ}{dT} \quad \text{(specific heat)}$$

In Eqs. (17.13), (17.14), and (17.15), when Q (or dQ) and ΔT (or dT) are positive, heat enters the object and its temperature increases. When they are negative, heat leaves the object and its temperature decreases.

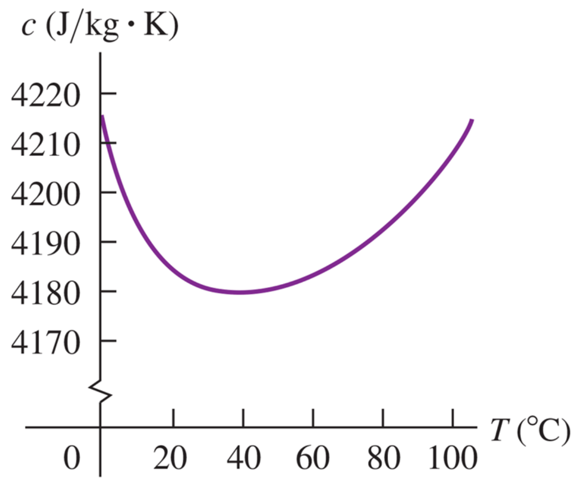
CAUTION The definition of heat Remember that dQ does not represent a change in the amount of heat *contained* in an object. Heat is always energy *in transit* as a result of a temperature difference. There is no such thing as "the amount of heat in an object."

The specific heat of water is approximately

$$4190\ \mathrm{J/kg\cdot K} \qquad 1\ \mathrm{cal/g\cdot C^\circ} \qquad \text{or} \qquad 1\ \mathrm{Btu/lb\cdot F^\circ}$$

The specific heat of a material always depends somewhat on the initial temperature and the temperature interval. Figure 17.17 shows this dependence for water. In this chapter we'll usually ignore this small variation.

Figure 17.17



Specific heat of water as a function of temperature. The value of c varies by less than 1% between $0^\circ\mathrm{C}$ and $100^\circ\mathrm{C}$.

Example 17.5 Feed a cold, starve a fever

During a bout with the flu an 80 kg man ran a fever of $39.0^\circ\mathrm{C}$ ($102.2^\circ\mathrm{F}$) instead of the normal body temperature of $37.0^\circ\mathrm{C}$ ($98.6^\circ\mathrm{F}$). Assuming that the human body is mostly water, how much heat is required to raise his temperature by that amount?

IDENTIFY and SET UP This problem uses the relationship among heat (the target variable), mass, specific heat, and temperature change. We use Eq. (17.13) to determine the required heat Q, with $m = 80\ \mathrm{kg}$, $c = 4190\ \mathrm{J/kg \cdot K}$ (for water), and $\Delta T = 39.0^\circ\mathrm{C} - 37.0^\circ\mathrm{C} = 2.0\ \mathrm{C}^\circ = 2.0\ \mathrm{K}$.

EXECUTE From Eq. (17.13),

$$Q = mc\,\Delta T = (80\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(2.0\ \text{K}) = 6.7 \times 10^5\ \text{J}$$

EVALUATE This corresponds to 160 kcal. In fact, the specific heat of the human body is about $3480\ \text{J/kg}\cdot\text{K}$, 83% that of water, because protein, fat, and minerals have lower specific heats. Hence a more accurate answer is $Q = 5.6 \times 10^5\ \text{J} = 133\ \text{kcal}$. Either result shows us that were it not for the body's temperature-regulating systems, taking in energy in the form of food would produce measurable changes in body temperature. (The elevated temperature of a person with the flu results from the body's extra activity in response to infection.)

KEY CONCEPT

To find the amount of heat required to change the temperature of a mass m of material by an amount ΔT, multiply m by ΔT and by the specific heat c of the material. Heat is positive when it flows into an object and negative when it flows out.

Video Tutor Solution: **Example 17.5**

Example 17.6 Overheating electronics

You are designing an electronic circuit element made of 23 mg of silicon. The electric current through it adds energy at the rate of $7.4\ \text{mW} = 7.4 \times 10^{-3}\ \text{J/s}$. If your design doesn't allow any heat transfer out of the element, at what rate does its temperature increase? The specific heat of silicon is $705\ \text{J/kg} \cdot \text{K}$.

IDENTIFY and SET UP The energy added to the circuit element gives rise to a temperature increase, just as if heat were flowing into the element at the rate $dQ/dt = 7.4 \times 10^{-3}\ \text{J/s}$. Our target variable is the rate of temperature change dT/dt. We can use Eq. (17.14),which relates infinitesimal temperature changes dT to the corresponding heat dQ, to obtain an expression for dQ/dt in terms of dT/dt.

EXECUTE We divide both sides of Eq. (17.14) by dT and rearrange:

$$\frac{dT}{dt} = \frac{dQ/dt}{mc} = \frac{7.4 \times 10^{-3}\ \text{J/s}}{(23 \times 10^{-6}\ \text{kg})(705\ \text{J/kg} \cdot \text{K})} = 0.46\ \text{K/s}$$

EVALUATE At this rate of temperature rise $(27\ \text{K/min})$, the circuit element would soon self-destruct. Heat transfer is an important design consideration in electronic circuit elements.

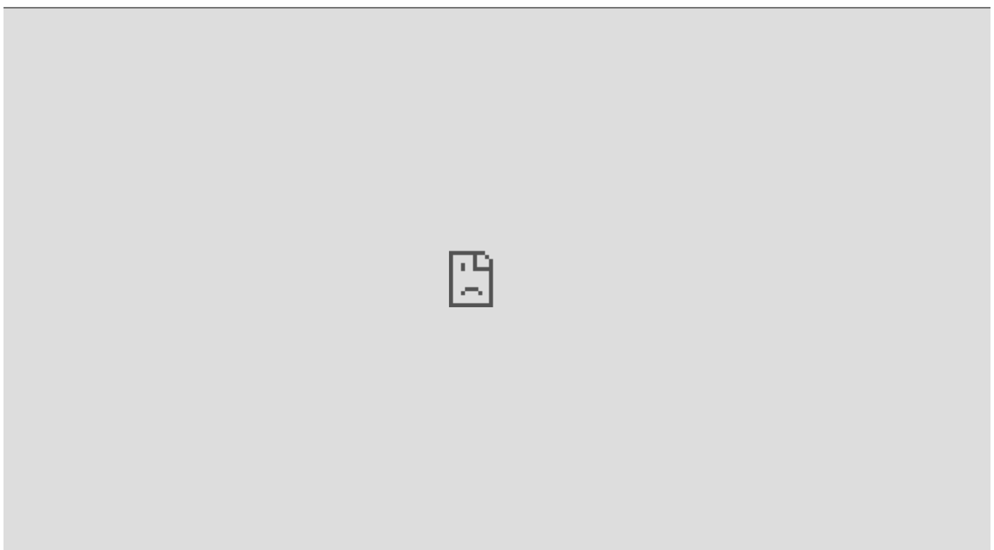
KEY CONCEPT

Any energy flow (not just heat) into or out of a quantity of material can cause the temperature of the material to change. The rate of energy flow is equal to the mass times the specific heat of the material times the rate of temperature change.

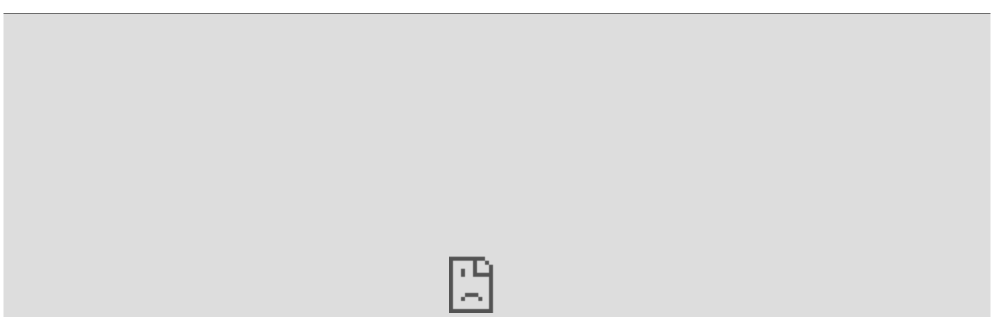
Video Tutor Solution: Example 17.6

Molar Heat Capacity

Video Tutor Demo: Water Balloon Held over Candle Flame



Video Tutor Demo: Heating Water and Aluminum



Sometimes it’s more convenient to describe a quantity of substance in terms of the number of *moles* n rather than the *mass* m of material. Recall from your study of chemistry that a mole of any pure substance always contains the same number of molecules. (We’ll discuss this point in more detail in Chapter 18.) The *molar mass* of any substance, denoted by M, is the mass per mole. (The quantity M is sometimes called *molecular weight*, but *molar mass* is preferable; the quantity depends on the mass of a molecule, not its weight.) For example, the molar mass of water is $18.0 \text{ g/mol} = 18.0 \times 10^{-3} \text{ kg/mol}$; 1 mole of water has a mass of $18.0 \text{ g} = 0.0180 \text{ kg}$. The total mass m of material is equal to the mass per mole M times the number of moles n :

(17.16)

$$m = nM$$

Replacing the mass m in Eq. (17.13) by the product nM, we find

(17.17)

$$Q = nMc\,\Delta T$$

The product Mc is called the **molar heat capacity** (or *molar specific heat*) and is denoted by C (capitalized). With this notation we rewrite Eq. (17.17) as

(17.18)

$$\underbrace{Q}_{\text{Heat required to change temperature of a certain number of moles}} = \overbrace{n}^{\text{Number of moles of material}}\underbrace{C}_{\text{Molar heat capacity of material}}\underbrace{\Delta T}_{\text{Temperature change}} \qquad (17.18)$$

Comparing to Eq. (17.15), we can express the molar heat capacity C (heat per mole per temperature change) in terms of the specific heat c (heat per mass per temperature change) and the molar mass M (mass per mole):

(17.19)

$$C = \frac{1}{n}\frac{dQ}{dT} = Mc \quad \text{(molar heat capacity)}$$

For example, the molar heat capacity of water is

$$\begin{aligned} C &= Mc = (0.0180\ \text{kg/mol})(4190\ \text{J/kg}\cdot\text{K}) \\ &= 75.4\ \text{J/mol}\cdot\text{K} \end{aligned}$$

Table 17.3 gives values of specific heat and molar heat capacity for several substances. Note the remarkably large specific heat for water (Fig. 17.18).

Table 17.3 **Approximate Specific Heats and Molar Heat Capacities (Constant Pressure)**

Substance	Specific Heat, *c* (J/kg · K)	Molar Mass, *M* (kg/mol)	Molar Heat Capacity, *C* (J/mol · K)
Aluminum	910	0.0270	24.6
Beryllium	1970	0.00901	17.7
Copper	390	0.0635	24.8
Ethanol	2428	0.0461	111.9
Ethylene glycol	2386	0.0620	148.0
Ice (near 0°C)	2100	0.0180	37.8
Iron	470	0.0559	26.3
Lead	130	0.207	26.9
Marble ($CaCO_3$)	879	0.100	87.9
Mercury	138	0.201	27.7
Salt (NaCl)	879	0.0585	51.4
Silver	234	0.108	25.3
Water (liquid)	4190	0.0180	75.4

Figure 17.18

Water has a much higher specific heat than the glass or metals used to make cookware. This helps explain why it takes several minutes to boil water on a stove, even though the pot or kettle reaches a high temperature very quickly.

CAUTION The meaning of "heat capacity" The term "heat capacity" is unfortunate because it gives the erroneous impression that an object *contains* a certain amount of heat. Remember, heat is energy in transit to or from an object, not the energy residing in the object.

Measurements of specific heats and molar heat capacities for solid materials are usually made at constant atmospheric pressure; the corresponding values are called the *specific heat* and *molar heat capacity at constant pressure,* denoted by c_p and C_p . For a gas it is usually easier to keep the substance in a container with constant *volume;* the corresponding values are called the *specific heat* and *molar heat capacity at constant volume,* denoted by c_V and C_V . For a given substance, C_V and C_p are different. If the system can expand while heat is added, there is additional energy exchange through the performance of *work* by the system on its surroundings. If the volume is constant, the system does no work. For gases the difference between C_p and C_V is substantial. We'll study heat capacities of gases in detail in Section 19.7.

The last column of Table 17.3 shows something interesting. The molar heat capacities for most elemental solids are about the same: about $25\ \mathrm{J/mol \cdot K}$. This correlation, named the *rule of Dulong and Petit* (for its discoverers), forms the basis for a very important idea. The number of atoms in 1 mole is the same for all elemental substances. This means that on a *per atom* basis, about the same amount of heat is required to raise the temperature of each of these elements by a given amount, even though the *masses* of the atoms are very different. The heat required for a given temperature increase depends only on *how many* atoms the sample contains, not on the mass of an individual atom. We'll see the reason the rule of Dulong and Petit works so well when we study the molecular basis of heat capacities in greater detail in Chapter 18.

Test Your Understanding of Section 17.5

You wish to raise the temperature of each of the following samples from $20^\circ C$ to $21^\circ C$. Rank these in order of the amount of heat needed to do this, from highest to lowest. (i) 1 kilogram of mercury; (ii) 1 kilogram of ethanol; (iii) 1 mole of mercury; (iv) 1 mole of ethanol.

17.6 Calorimetry and Phase Changes

Calorimetry means "measuring heat." We have discussed the energy transfer (heat) involved in temperature changes. Heat is also involved in *phase changes,* such as the melting of ice or boiling of water. Once we understand these additional heat relationships, we can analyze a variety of problems involving quantity of heat.

Phase Changes

We use the term **phase** to describe a specific state of matter, such as a solid, liquid, or gas. The compound H_2O exists in the *solid phase* as ice, in the *liquid phase* as water, and in the *gaseous phase* as steam. (These are also referred to as **states of matter:** the solid state, the liquid state, and the gaseous state.) A transition from one phase to another is called a **phase change** or *phase transition.* For any given pressure a phase change takes place at a definite temperature, usually accompanied by heat flowing in or out and a change of volume and density.

A familiar phase change is the melting of ice. When we add heat to ice at 0°C and normal atmospheric pressure, the temperature of the ice *does not* increase. Instead, some of it melts to form liquid water. If we add the heat slowly, to maintain the system very close to thermal equilibrium, the temperature remains at 0°C until all the ice is melted (Fig. 17.19). The effect of adding heat to this system is not to raise its temperature but to change its *phase* from solid to liquid.

Figure 17.19

The surrounding air is at room temperature, but this ice–water mixture remains at $0^\circ C$ until all of the ice has melted and the phase change is complete.

To change 1 kg of ice at $0^\circ C$ to 1 kg of liquid water at $0^\circ C$ and normal atmospheric pressure requires 3.34×10^5 J of heat. The heat required per unit mass is called the **heat of fusion** (or sometimes *latent heat of fusion*), denoted by L_f. For water at normal atmospheric pressure the heat of fusion is

$$L_f = 3.34 \times 10^5 \text{ J/kg} = 79.6 \text{ cal/g} = 143 \text{ Btu/lb}$$

More generally, to melt a mass m of material that has a heat of fusion L_f requires a quantity of heat Q given by

$$Q = mL_f$$

This process is *reversible.* To freeze liquid water to ice at $0^\circ C$, we have to *remove* heat; the magnitude is the same, but in this case, Q is negative because heat is removed rather than added. To cover both possibilities and to include other kinds of phase changes, we write

(17.20)

Mass of material that changes phase

Heat transfer in a phase change $Q = \pm mL$ Latent heat for this phase change (17.20)

+ if heat enters material, − if heat leaves

The plus sign (heat entering) is used when the material melts; the minus sign (heat leaving) is used when it freezes. The heat of fusion is different for different materials, and it also varies somewhat with pressure.

For any given material at any given pressure, the freezing temperature is the same as the melting temperature. At this unique temperature the liquid and solid phases can coexist in a condition called **phase equilibrium.**

We can go through this whole story again for *boiling* or *evaporation,* a phase transition between liquid and gaseous phases. The corresponding heat (per unit mass) is called the **heat of vaporization** L_{v} . At normal atmospheric pressure the heat of vaporization L_{v} for water is

$$L_{\mathrm{v}} = 2.256 \times 10^6\ \mathrm{J/kg} = 539\ \mathrm{cal/g} = 970\ \mathrm{Btu/lb}$$

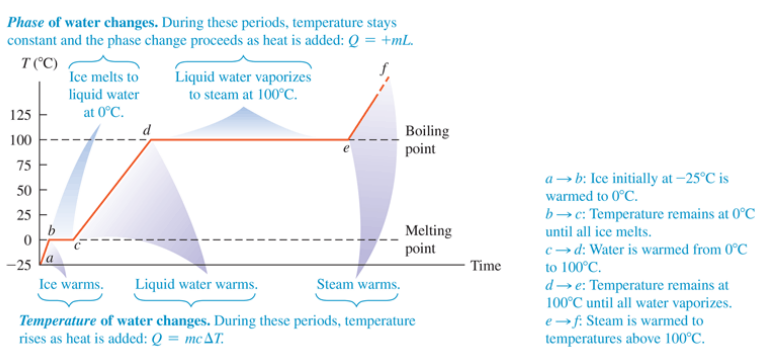
That is, it takes 2.256×10^6 J to change 1 kg of liquid water at $100^\circ\mathrm{C}$ to 1 kg of water vapor at $100^\circ\mathrm{C}$. By comparison, to raise the temperature of 1 kg of water from $0^\circ\mathrm{C}$ to $100^\circ\mathrm{C}$ requires $Q = mc\,\Delta T = (1.00\ \mathrm{kg})(4190\ \mathrm{J/kg \cdot C^\circ})(100\ \mathrm{C^\circ}) = 4.19 \times 10^5\ \mathrm{J}$, less than one-fifth as much heat as is required for vaporization at $100^\circ\mathrm{C}$. This agrees with everyday kitchen experience; a pot of water may reach boiling temperature in a few minutes, but it takes a much longer time to completely evaporate all the water away.

Like melting, boiling is a reversible transition. When heat is removed from a gas at the boiling temperature, the gas returns to the liquid phase, or *condenses,* giving up to its surroundings the same quantity of heat (heat of vaporization) that was needed to vaporize it. At a given pressure the boiling and condensation temperatures are always the same; at this temperature the liquid and gaseous phases can coexist in phase equilibrium.

Both L_v and the boiling temperature of a material depend on pressure. Water boils at a lower temperature (about $95^\circ C$) in Denver than in Pittsburgh because Denver is at higher elevation and the average atmospheric pressure is lower. The heat of vaporization is somewhat greater at this lower pressure, about 2.27×10^6 J/kg.

Figure 17.20 summarizes these ideas about phase changes. Table 17.4 lists heats of fusion and vaporization for some materials and their melting and boiling temperatures at normal atmospheric pressure. Very few *elements* have melting temperatures in the vicinity of ordinary room temperatures; one of the few is the metal gallium, shown in Fig. 17.21.

Figure 17.20



Graph of temperature versus time for a specimen of water initially in the solid phase (ice). Heat is added to the specimen at a constant rate. The temperature remains constant during each change of phase, provided that the pressure remains constant.

Table 17.4 Heats of Fusion and Vaporization

Substance	Normal Melting Point K	Normal Melting Point °C	Heat of Fusion, L_f (J/kg)	Normal Boiling Point K	Normal Boiling Point °C	Heat of Vaporization, L_v (J/kg)
Helium	*	*	*	4.216	−268.93	20.9×10^3
Hydrogen	13.84	−259.31	58.6×10^3	20.26	−252.89	452×10^3
Nitrogen	63.18	−209.97	25.5×10^3	77.34	−195.8	201×10^3
Oxygen	54.36	−218.79	13.8×10^3	90.18	−183.0	213×10^3
Ethanol	159	−114	104.2×10^3	351	78	854×10^3
Mercury	234	−39	11.8×10^3	630	357	272×10^3
Water	273.15	0.00	334×10^3	373.15	100.00	2256×10^3
Sulfur	392	119	38.1×10^3	717.75	444.60	326×10^3
Lead	600.5	327.3	24.5×10^3	2023	1750	871×10^3
Antimony	903.65	630.50	165×10^3	1713	1440	561×10^3
Silver	1233.95	960.80	88.3×10^3	2466	2193	2336×10^3
Gold	1336.15	1063.00	64.5×10^3	2933	2660	1578×10^3
Copper	1356	1083	134×10^3	1460	1187	5069×10^3

*A pressure in excess of 25 atmospheres is required to make helium solidify. At 1 atmosphere pressure, helium remains a liquid down to absolute zero.

Figure 17.21

The metal gallium, shown here melting in a person's hand, is one of the few elements that melt in the vicinity of room temperature. Its melting temperature is 29.8 °C, and its heat of fusion is 8.04×10^4 J/kg.

A substance can sometimes change directly from the solid to the gaseous phase. This process is called *sublimation*, and the solid is said to *sublime.* The corresponding heat is called the *heat of sublimation,* L_s. Liquid carbon dioxide cannot exist at a pressure lower than about 5×10^5 Pa (about 5 atm), and "dry ice" (solid carbon dioxide) sublimes at atmospheric pressure. Sublimation of water from frozen food causes freezer burn. The reverse process, a phase change from gas to solid, occurs when frost forms on cold objects such as refrigerator cooling coils.

Very pure water can be cooled several degrees below the freezing temperature without freezing; the resulting unstable state is described as *supercooled.* When a small ice crystal is dropped in or the water is agitated, it crystallizes within a second or less (Fig. 17.22). Supercooled water *vapor* condenses quickly into fog droplets when a disturbance, such as dust particles or ionizing radiation, is introduced. This principle is used in "seeding" clouds, which often contain supercooled water vapor, to cause condensation and rain.

Figure 17.22

When this airplane flew into a cloud at a temperature just below freezing, the plane struck supercooled water droplets in the cloud that rapidly crystallized and formed ice on the plane's nose (shown here) and wings. Such inflight icing can be extremely hazardous, which is why commercial airliners are equipped with devices to remove ice.

A liquid can sometimes be *superheated* above its normal boiling temperature. Any small disturbance such as agitation causes local boiling with bubble formation.

Steam heating systems for buildings use a boiling–condensing process to transfer heat from the furnace to the radiators. Each kilogram of water that is turned to steam in the boiler absorbs over 2×10^6 J (the heat of vaporization L_v of water) from the boiler and gives it up when it condenses in the radiators. Boiling–condensing processes are also used in refrigerators, air conditioners, and heat pumps. We'll discuss these systems in Chapter 20.

The temperature-control mechanisms of many warm-blooded animals make use of heat of vaporization, removing heat from the body by using it to evaporate water from the tongue (panting) or from the skin (sweating). Such *evaporative cooling* enables humans to maintain normal body temperature in hot, dry desert climates where the air temperature may reach $55^\circ C$ (about $130^\circ F$). The skin temperature may be as much as $30^\circ C$ cooler than the surrounding air. Under these conditions a normal person may perspire several liters per day, and this lost water must be replaced. Evaporative cooling also explains why you feel cold when you first step out of a swimming pool (Fig. 17.23).

Figure 17.23

The water may be warm and it may be a hot day, but these children will feel cold when they first step out of the swimming pool. That's because as water evaporates from their skin, it removes the heat of vaporization from their bodies. To stay warm, they will need to dry off immediately.

Evaporative cooling is also used to condense and recirculate "used" steam in coal-fired or nuclear-powered electric-generating plants. That's what goes on in the large, tapered concrete towers that you see at such plants.

Chemical reactions such as combustion are analogous to phase changes in that they involve definite quantities of heat. Complete combustion of 1 gram of gasoline produces about 46,000 J or about 11,000 cal, so the **heat of combustion** L_c of gasoline is

$$L_c = 46{,}000 \ \mathrm{J/g} = 4.6 \times 10^7 \ \mathrm{J/kg}$$

Energy values of foods are defined similarly. When we say that a gram of peanut butter "contains 6 calories," we mean that 6 kcal of heat (6000 cal or 25,000 J) is released when the carbon and hydrogen atoms in the peanut butter react with oxygen (with the help of enzymes) and are completely converted to CO_2 and H_2O. Not all of this energy is directly useful for mechanical work. We'll study the *efficiency* of energy utilization in Chapter 20.

Heat Calculations

Let's look at some examples of calorimetry calculations (calculations with heat). The basic principle is very simple: When heat flow occurs between two objects that are isolated from their surroundings, the amount of heat lost by one object must equal the amount gained by the other. Heat is energy in transit, so this principle is really just conservation of energy. Calorimetry, dealing entirely with one conserved quantity, is in many ways the simplest of all physical theories!

Problem-Solving Strategy 17.2 Calorimetry Problems

IDENTIFY *the relevant concepts:* When heat flow occurs between two or more objects that are isolated from their surroundings, the *algebraic sum* of the quantities of heat transferred to all the objects is zero. We take a quantity of heat *added* to an object as *positive* and a quantity *leaving* an object as *negative.*

SET UP *the problem* using the following steps:

1. Identify the objects that exchange heat.
2. Each object may undergo a temperature change only, a phase change at constant temperature, or both. Use Eq. (17.13) for the heat transferred in a temperature change and Eq. (17.20) for the heat transferred in a phase change.
3. Consult Table 17.3 for values of specific heat or molar heat capacity and Table 17.4 for heats of fusion or vaporization.
4. List the known and unknown quantities and identify the target variables.

EXECUTE *the solution* as follows:

1. Use Eq. (17.13) and/or Eq. (17.20) and the energy-conservation relationship $\sum Q = 0$ to solve for the target variables. Ensure that you use the correct algebraic signs for Q and ΔT terms, and that you correctly write $\Delta T = T_{\text{final}} - T_{\text{initial}}$ and not the reverse.
2. If a phase change occurs, you may not know in advance whether all, or only part, of the material undergoes a phase change. Make a reasonable guess; if that leads to an unreasonable result (such as a final temperature higher or lower than any initial temperature), the guess was wrong. Try again!

EVALUATE *your answer:* Double-check your calculations, and ensure that the results are physically sensible.

Example 17.7 A temperature change with no phase change

WITH VARIATION PROBLEMS

A camper pours 0.300 kg of coffee, initially in a pot at $70.0^\circ\mathrm{C}$, into a 0.120 kg aluminum cup initially at $20.0^\circ\mathrm{C}$. What is the equilibrium temperature? Assume that coffee has the same specific heat as water and that no heat is exchanged with the surroundings.

IDENTIFY and SET UP The target variable is the common final temperature T of the cup and coffee. No phase changes occur, so we need only Eq. (17.13). With subscripts C for coffee, W for water, and Al for aluminum, we have $T_{0\mathrm{C}} = 70.0^\circ\mathrm{C}$ and $T_{0\mathrm{Al}} = 20.0^\circ\mathrm{C}$; Table 17.3 gives $c_\mathrm{W} = 4190\ \mathrm{J/kg \cdot K}$ and $c_\mathrm{Al} = 910\ \mathrm{J/kg \cdot K}$.

EXECUTE The (negative) heat gained by the coffee is $Q_\mathrm{C} = m_\mathrm{C} c_\mathrm{W} \Delta T_\mathrm{C}$. The (positive) heat gained by the cup is $Q_\mathrm{Al} = m_\mathrm{Al} c_\mathrm{Al} \Delta T_\mathrm{Al}$. We set $Q_\mathrm{C} + Q_\mathrm{Al} = 0$ (see Problem-Solving Strategy 17.2) and substitute $\Delta T_\mathrm{C} = T - T_{0\mathrm{C}}$ and $\Delta T_\mathrm{Al} = T - T_{0\mathrm{Al}}$:

$$
\begin{aligned}
Q_\mathrm{C} + Q_\mathrm{Al} = m_\mathrm{C} c_\mathrm{W} \Delta T_\mathrm{C} + m_\mathrm{Al} c_\mathrm{Al} \Delta T_\mathrm{Al} &= 0 \\
m_\mathrm{C} c_\mathrm{W} (T - T_{0\mathrm{C}}) + m_\mathrm{Al} c_\mathrm{Al} (T - T_{0\mathrm{Al}}) &= 0
\end{aligned}
$$

Then we solve this expression for the final temperature T. A little algebra gives

$$
T = \frac{m_\mathrm{C} c_\mathrm{W} T_{0\mathrm{C}} + m_\mathrm{Al} c_\mathrm{Al} T_{0\mathrm{Al}}}{m_\mathrm{C} c_\mathrm{W} + m_\mathrm{Al} c_\mathrm{Al}} = 66.0^\circ\mathrm{C}
$$

EVALUATE The final temperature is much closer to the initial temperature of the coffee than to that of the cup; water has a much higher specific heat than aluminum, and we have more than twice as much mass of water. We can also find the quantities of heat by substituting the value $T = 66.0^\circ\text{C}$ back into the original equations. We find $Q_\text{C} = -5.0 \times 10^3$ J and $Q_\text{Al} = +5.0 \times 10^3$ J. As expected, Q_C is negative: The coffee loses heat to the cup.

KEY CONCEPT

In a calorimetry problem in which two objects at different temperatures interact by exchanging heat, energy is conserved: The sum of the heat flows (one positive, one negative) into the two objects is zero. The heat flow stops when the two objects reach the same temperature.

Video Tutor Solution: **Example 17.7**

Example 17.8 Changes in both temperature and phase

WITH VARIATION PROBLEMS

A glass contains 0.25 kg of Omni-Cola (mostly water) initially at 25°C. How much ice, initially at -20°C, must you add to obtain a final temperature of $0^\circ\mathrm{C}$ with all the ice melted? Ignore the heat capacity of the glass.

IDENTIFY and SET UP The Omni-Cola and ice exchange heat. The cola undergoes a temperature change; the ice undergoes both a temperature change and a phase change from solid to liquid. We use subscripts C for cola, I for ice, and W for water. The target variable is the mass of ice, m_I. We use Eq. (17.13) to obtain an expression for the amount of heat involved in cooling the drink to $T = 0^\circ\mathrm{C}$ and warming the ice to $T = 0^\circ\mathrm{C}$, and Eq. (17.20) to obtain an expression for the heat required to melt the ice at $0^\circ\mathrm{C}$. We have $T_{0\mathrm{C}} = 25^\circ\mathrm{C}$ and $T_{0\mathrm{I}} = -20^\circ\mathrm{C}$, Table 17.3 gives $c_\mathrm{W} = 4190\ \mathrm{J/kg\cdot K}$ and $c_\mathrm{I} = 2100\ \mathrm{J/kg\cdot K}$, and Table 17.4 gives $L_\mathrm{f} = 3.34 \times 10^5\ \mathrm{J/kg}$.

EXECUTE From Eq. (17.13), the (negative) heat gained by the Omni-Cola is $Q_\mathrm{C} = m_\mathrm{C} c_\mathrm{W} \Delta T_\mathrm{C}$. The (positive) heat gained by the ice in warming is $Q_\mathrm{I} = m_\mathrm{I} c_\mathrm{I} \Delta T_\mathrm{I}$. The (positive) heat required to melt the ice is $Q_2 = m_\mathrm{I} L_\mathrm{f}$. We set $Q_\mathrm{C} + Q_\mathrm{I} + Q_2 = 0$, insert $\Delta T_\mathrm{C} = T - T_{0\mathrm{C}}$ and $\Delta T_\mathrm{I} = T - T_{0\mathrm{I}}$, and solve for m_I :

$$\begin{aligned}
m_\mathrm{C} c_\mathrm{W} \Delta T_\mathrm{C} + m_\mathrm{I} c_\mathrm{I} \Delta T_\mathrm{I} + m_\mathrm{I} L_\mathrm{f} &= 0 \\
m_\mathrm{C} c_\mathrm{W} (T - T_{0\mathrm{C}}) + m_\mathrm{I} c_\mathrm{I} (T - T_{0\mathrm{I}}) + m_\mathrm{I} L_\mathrm{f} &= 0 \\
m_\mathrm{I} [c_\mathrm{I} (T - T_{0\mathrm{I}}) + L_\mathrm{f}] &= -m_\mathrm{C} c_\mathrm{W} (T - T_{0\mathrm{C}}) \\
m_\mathrm{I} &= m_\mathrm{C} \frac{c_\mathrm{W} (T_{0\mathrm{C}} - T)}{c_\mathrm{I} (T - T_{0\mathrm{I}}) + L_\mathrm{f}}
\end{aligned}$$

Substituting numerical values, we find that $m_\mathrm{I} = 0.070\ \mathrm{kg} = 70\ \mathrm{g}$.

EVALUATE Three or four medium-size ice cubes would make about 70 g, which seems reasonable given the 250 g of Omni-Cola to be cooled.

KEY CONCEPT

When heat flows between two objects and one or both of them change phase, your calculations must include the heat required to cause the phase change. This depends on the object's mass and material and on which phase change occurs.

Video Tutor Solution: **Example 17.8**

Example 17.9 What's cooking?

WITH VARIATION PROBLEMS

A hot copper pot of mass 2.0 kg (including its copper lid) is at a temperature of 150°C. You pour 0.10 kg of cool water at 25°C into the pot, then quickly replace the lid so no steam can escape. Find the final temperature of the pot and its contents, and determine the phase of the water (liquid, gas, or a mixture). Assume that no heat is lost to the surroundings.

IDENTIFY and SET UP The water and the pot exchange heat. Three outcomes are possible: (1) No water boils, and the final temperature T is less than 100°C; (2) some water boils, giving a mixture of water and steam at 100°C; or (3) all the water boils, giving 0.10 kg of steam at 100°C or greater. We use Eq. (17.13) for the heat transferred in a temperature change and Eq. (17.20) for the heat transferred in a phase change.

EXECUTE First consider case (1), which parallels Example 17.8 exactly. The equation that states that the heat flow into the water equals the heat flow out of the pot is

$$Q_{\mathrm{W}} + Q_{\mathrm{Cu}} = m_{\mathrm{W}} c_{\mathrm{W}} (T - T_{0\mathrm{W}}) + m_{\mathrm{Cu}} c_{\mathrm{Cu}} (T - T_{0\mathrm{Cu}}) = 0$$

Here we use subscripts W for water and Cu for copper, with $m_{\mathrm{W}} = 0.10$ kg, $m_{\mathrm{Cu}} = 2.0$ kg, $T_{0\mathrm{W}} = 25^{\circ}\mathrm{C}$, and $T_{0\mathrm{Cu}} = 150^{\circ}\mathrm{C}$. From Table 17.3, $c_{\mathrm{W}} = 4190\ \mathrm{J/kg \cdot K}$ and $c_{\mathrm{Cu}} = 390\ \mathrm{J/kg \cdot K}$. Solving for the final temperature T and substituting these values, we get

$$T = \frac{m_{\mathrm{W}} c_{\mathrm{W}} T_{0\mathrm{W}} + m_{\mathrm{Cu}} c_{\mathrm{Cu}} T_{0\mathrm{Cu}}}{m_{\mathrm{W}} c_{\mathrm{W}} + m_{\mathrm{Cu}} c_{\mathrm{Cu}}} = 106^{\circ}\mathrm{C}$$

But this is above the boiling point of water, which contradicts our assumption that no water boils! So at least some of the water boils.

So consider case (2), in which the final temperature is $T = 100^{\circ}\mathrm{C}$ and some unknown fraction x of the water boils, where (if this case is correct) x is greater than zero and less than or equal to 1. The (positive) amount of heat needed to vaporize this water is $x m_{\mathrm{W}} L_{\mathrm{v}}$. The energy-conservation condition $Q_{\mathrm{W}} + Q_{\mathrm{Cu}} = 0$ is then

$$m_{\mathrm{W}} c_{\mathrm{W}} (100^{\circ}\mathrm{C} - T_{0\mathrm{W}}) + x m_{\mathrm{W}} L_{\mathrm{v}} + m_{\mathrm{Cu}} c_{\mathrm{Cu}} (100^{\circ}\mathrm{C} - T_{0\mathrm{Cu}}) = 0$$

We solve for the target variable x :

$$x = \frac{-m_{\mathrm{Cu}} c_{\mathrm{Cu}} (100^{\circ}\mathrm{C} - T_{0\mathrm{Cu}}) - m_{\mathrm{W}} c_{\mathrm{W}} (100^{\circ}\mathrm{C} - T_{0\mathrm{W}})}{m_{\mathrm{W}} L_{\mathrm{v}}}$$

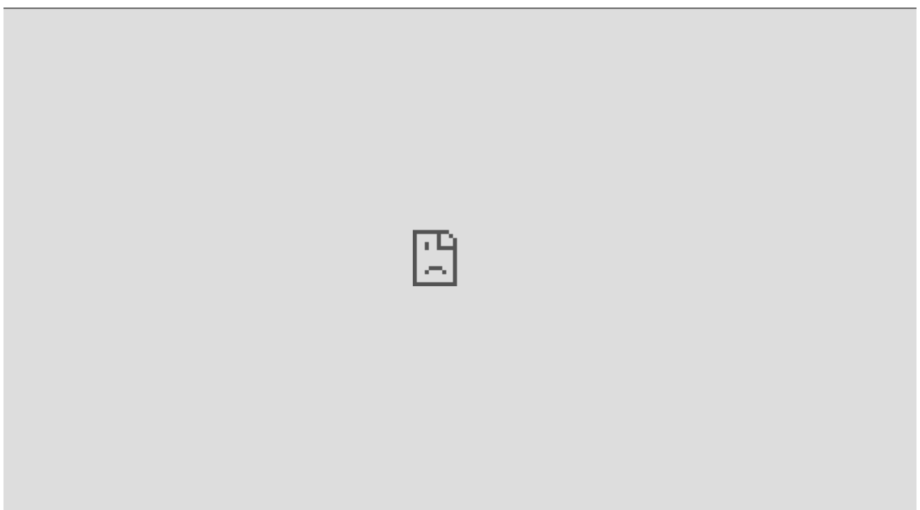
With $L_{\mathrm{v}} = 2.256 \times 10^{6}$ J from Table 17.4, this yields $x = 0.034$. We conclude that the final temperature of the water and copper is $100^{\circ}\mathrm{C}$ and that $0.034(0.10\ \mathrm{kg}) = 0.0034\ \mathrm{kg} = 3.4$ g of the water is converted to steam at $100^{\circ}\mathrm{C}$.

EVALUATE Had x turned out to be greater than 1, case (3) would have held; all the water would have vaporized, and the final temperature would have been greater than $100^\circ C$. Can you show that this would have been the case if we had originally poured less than 15 g of $25^\circ C$ water into the pot?

KEY CONCEPT

In many calorimetry problems you won't know whether or not a phase change occurs. To find out, try working the problem three ways (assuming no phase change, assuming part of the object changes phase, and assuming all of it changes phase). The way that leads to a sensible result is the correct one.

Video Tutor Solution: **Example 17.9**



Example 17.10 Combustion, temperature change, and phase change

In a particular camp stove, only 30% of the energy released in burning gasoline goes to heating the water in a pot on the stove.

How much gasoline must we burn to heat 1.00 L (1.00 kg) of water from $20^\circ C$ to $100^\circ C$ and boil away 0.25 kg of it?

IDENTIFY and SET UP All of the water undergoes a temperature change and part of it undergoes a phase change, from liquid to gas. We determine the heat required to cause both of these changes, and then use the 30% combustion efficiency to determine the amount of gasoline that must be burned (the target variable). We use Eqs. (17.13) and (17.20) and the idea of heat of combustion.

EXECUTE To raise the temperature of the water from $20^\circ C$ to $100^\circ C$ requires

$$Q_1 = mc\,\Delta T = (1.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})(80\text{ K}) = 3.35\times 10^5\text{ J}$$

To boil 0.25 kg of water at $100^\circ C$ requires

$$Q_2 = mL_\text{v} = (0.25\text{ kg})(2.256\times 10^6\text{ J/kg}) = 5.64\times 10^5\text{ J}$$

The total energy needed is $Q_1 + Q_2 = 8.99\times 10^5$ J. This is $30\% = 0.30$ of the total heat of combustion, which is therefore $(8.99\times 10^5\text{ J})/0.30 = 3.00\times 10^6$ J. As we mentioned earlier, the combustion of 1 g of gasoline releases 46,000 J, so the mass of gasoline required is $(3.00\times 10^6\text{ J})/(46{,}000\text{ J/g}) = 65$ g, or a volume of about 0.09 L of gasoline.

EVALUATE This result suggests the tremendous amount of energy released in burning even a small quantity of gasoline. Another 123 g of gasoline would be required to boil away the remaining water; can you prove this?

KEY CONCEPT

To find the amount of heat released when a quantity of substance undergoes combustion, multiply the mass that combusts by the heat of combustion of the substance.

Video Tutor Solution: Example 17.10

Test Your Understanding of Section 17.6

You take a block of ice at 0°C and add heat to it at a steady rate. It takes a time t to completely convert the block of ice to steam at 100°C. What do you have at time $t/2$? (i) All ice at 0°C; (ii) a mixture of ice and water at 0°C; (iii) water at a temperature between 0°C and 100°C; (iv) a mixture of water and steam at 100°C.

17.7 Mechanisms of Heat Transfer

We have talked about *conductors* and *insulators*, materials that permit or prevent heat transfer between objects. Now let's look in more detail at *rates* of energy transfer. In the kitchen you use a metal or glass pot for good heat transfer from the stove to whatever you're cooking, but your refrigerator is insulated with a material that *prevents* heat from flowing into the food inside the refrigerator. How do we describe the difference between these two materials?

The three mechanisms of heat transfer are conduction, convection, and radiation. *Conduction* occurs within an object or between two objects in contact. *Convection* depends on motion of mass from one region of space to another. *Radiation* is heat transfer by electromagnetic radiation, such as sunshine, with no need for matter to be present in the space between objects.

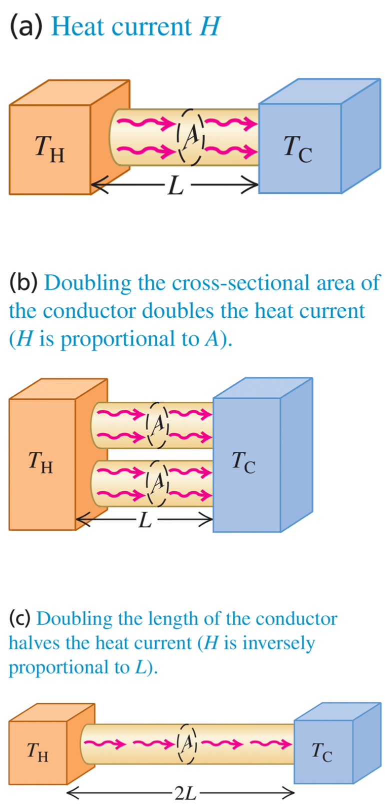
Conduction

If you hold one end of a copper rod and place the other end in a flame, the end you are holding gets hotter and hotter, even though it is not in direct contact with the flame. Heat reaches the cooler end by **conduction** through the material. The atoms in the hotter regions have more kinetic energy, on the average, than their cooler neighbors. They jostle their neighbors, giving them some of their energy. The neighbors jostle *their* neighbors, and so on through the material. The atoms don't move from one region of material to another, but their energy does.

Most metals also conduct heat by another, more effective mechanism. Within the metal, some electrons can leave their parent atoms and wander through the metal. These “free” electrons can rapidly carry energy from hotter to cooler regions of the metal, so metals are generally good conductors of heat. A metal rod at 20°C feels colder than a piece of wood at 20°C because heat can flow more easily from your hand into the metal. The presence of “free” electrons also causes most metals to be good electrical conductors.

In conduction, the direction of heat flow is always from higher to lower temperature. Figure 17.24a shows a rod of conducting material with cross-sectional area A and length L. The left end of the rod is kept at a temperature T_H and the right end at a lower temperature T_C, so heat flows from left to right. The sides of the rod are covered by an ideal insulator, so no heat transfer occurs at the sides.

Figure 17.24



Steady-state heat flow due to conduction in a uniform rod.

When a quantity of heat dQ is transferred through the rod in a time dT, the rate of heat flow is dQ/dt. We call this rate the **heat current,** denoted by H. That is, $H = dQ/dt$. Experiments show that the heat current is proportional to the cross-sectional area A of the rod (Fig. 17.24b) and to the temperature difference $(T_{\text{H}} - T_{\text{C}})$ and is inversely proportional to the rod length L (Fig. 17.24c):

(17.21)

$$\underbrace{H}_{\text{Heat current in conduction}} = \overbrace{\frac{dQ}{dt}}^{\text{Rate of heat flow}} = \underbrace{k}_{\text{Thermal conductivity of rod material}}\,\underbrace{A}_{\text{Cross-sectional area of rod}}\,\frac{\overbrace{T_{\text{H}} - T_{\text{C}}}^{\text{Temperatures of hot and cold ends of rod}}}{\underbrace{L}_{\text{Length of rod}}} \qquad (17.21)$$

The quantity $(T_{\text{H}} - T_{\text{C}})/L$ is the temperature difference *per unit length;* it is called the magnitude of the **temperature gradient.** The numerical value of the **thermal conductivity** k depends on the material of the rod. Materials with large k are good conductors of heat; materials with small k are poor conductors, or insulators. Equation (17.21) also gives the heat current through a slab or through *any* homogeneous object with uniform cross section A perpendicular to the direction of flow; L is the length of the heat-flow path.

The units of heat current H are units of energy per time, or power; the SI unit of heat current is the watt $(1 \text{ W} = 1 \text{ J/s})$. We can find the units of k by solving Eq. (17.21) for k; you can show that the SI units are $\text{W/m} \cdot \text{K}$. Table 17.5 gives some numerical values of k.

Table 17.5 Thermal Conductivities

Substance	k (W/m · K)
Metals	
Aluminum	205.0
Brass	109.0
Copper	385.0
Lead	34.7
Mercury	8.3
Silver	406.0
Steel	50.2
Solids (representative values)	
Brick, insulating	0.15
Brick, red	0.6
Concrete	0.8
Cork	0.04
Felt	0.04
Fiberglass	0.04
Glass	0.8
Ice	1.6
Rock wool	0.04
Styrofoam	0.027
Wood	0.12–0.04
Gases	
Air	0.024
Argon	0.016
Helium	0.14
Hydrogen	0.14
Oxygen	0.023

The thermal conductivity of "dead" (nonmoving) air is very small. A wool sweater keeps you warm because it traps air between the fibers. Many insulating materials such as Styrofoam and fiberglass are mostly dead air.

BIO Application

Fur Versus Blubber

The fur of an arctic fox is a good thermal insulator because it traps air, which has a low thermal conductivity k. (The value $k = 0.04\ \mathrm{W/m \cdot K}$ for fur is higher than for air, $k = 0.024\ \mathrm{W/m \cdot K}$, because fur also includes solid hairs.) The layer of fat beneath a bowhead whale's skin, called blubber, has six times the thermal conductivity of fur ($k = 0.024 \text{ W/m} \cdot \text{K}$). So a 6 cm thickness of blubber ($L = 6 \text{ cm}$) is required to give the same insulation as 1 cm of fur.

If the temperature varies in a nonuniform way along the length of the conducting rod, we introduce a coordinate x along the length and generalize the temperature gradient to be dT/dx. The corresponding generalization of Eq. (17.21) is

(17.22)

$$H = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

The negative sign indicates that heat flows in the direction of *decreasing* temperature. If temperature increases with increasing x, then $dT/dx > 0$ and $H < 0$; the negative value of H in this case means that heat flows in the negative x-direction, from high to low temperature.

For thermal insulation in buildings, engineers use the concept of **thermal resistance,** denoted by R. The thermal resistance R of a slab of material with area A is defined so that the heat current H through the slab is

(17.23)

$$H = \frac{A(T_{\mathrm{H}} - T_{\mathrm{C}})}{R}$$

where T_{H} and T_{C} are the temperatures on the two sides of the slab. Comparing this with Eq. (17.21), we see that R is given by

(17.24)

$$R = \frac{L}{k}$$

where L is the thickness of the slab. The SI unit of R is $1\ \mathrm{m}^2 \cdot \mathrm{K/W}$. In the units used for commercial insulating materials in the United States, H is expressed in Btu/h, A is in ft^2, and $T_{\mathrm{H}} - T_{\mathrm{C}}$ in F°. $(1\ \mathrm{Btu/h} = 0.293\ \mathrm{W.})$ The units of R are then $\mathrm{ft}^2 \cdot \mathrm{F}^\circ \cdot \mathrm{h/Btu}$ though values of R are usually quoted without units; a 6-inch-thick layer of fiberglass has an R value of 19 (that is, $R = 19\ \mathrm{ft}^2 \cdot \mathrm{F}^\circ \cdot \mathrm{h/Btu}$), a 2-inch-thick slab of polyurethane foam has an R value of 12, and so on. Doubling the thickness doubles the R value. Common practice in new construction in severe northern climates is to specify R values of around 30 for exterior walls and ceilings. When the insulating material is in layers, such as a plastered wall, fiberglass insulation, and wood exterior siding, the R values are additive. Do you see why?

Problem-Solving Strategy 17.3 Heat Conduction

IDENTIFY *the relevant concepts:* Heat conduction occurs whenever two objects at different temperatures are placed in contact.

SET UP *the problem* using the following steps:

1. Identify the direction of heat flow (from hot to cold). In Eq. (17.21), L is measured along this direction, and A is an

area perpendicular to this direction. You can often approximate an irregular-shaped container with uniform wall thickness as a flat slab with the same thickness and total wall area.

2. List the known and unknown quantities and identify the target variable.

EXECUTE *the solution* as follows:

1. If heat flows through a single object, use Eq. (17.21) to solve for the target variable.
2. If the heat flows through two different materials in succession (in *series*), the temperature T at the interface between them is intermediate between T_{H} and T_{C}, so that the temperature differences across the two materials are $(T_{\mathrm{H}} - T)$ and $(T - T_{\mathrm{C}})$. In steady-state heat flow, the same heat must pass through both materials, so the heat current H must be the *same* in both materials.
3. If heat flows through two or more *parallel* paths, then the total heat current H is the sum of the currents H_1, H_2, $\ldots$ for the separate paths. An example is heat flow from inside a room to outside, both through the glass in a window and through the surrounding wall. In parallel heat flow the temperature difference is the same for each path, but L, A, and k may be different for each path.
4. Be consistent with units. If k is expressed in $\mathrm{W/m \cdot K}$, for example, use distances in meters, heat in joules, and T in kelvins.

EVALUATE *your answer:* Are the results physically reasonable?

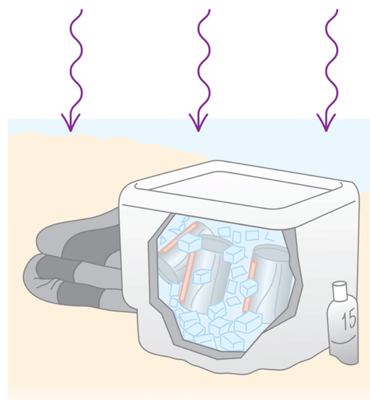
Example 17.11 Conduction into a picnic cooler

WITH VARIATION PROBLEMS

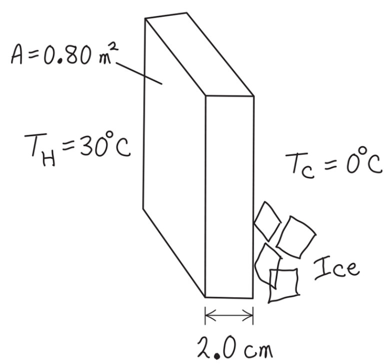
A Styrofoam cooler (Fig. 17.25a) has total wall area (including the lid) of 0.80 m^2 and wall thickness 2.0 cm. It is filled with ice, water, and cans of Omni-Cola, all at 0°C. What is the rate of heat flow into the cooler if the temperature of the outside wall is 30°C? How much ice melts in 3 hours?

Figure 17.25

(a) A cooler at the beach



(b) Our sketch for this problem



Conduction of heat across the walls of a Styrofoam cooler.

IDENTIFY and SET UP The target variables are the heat current H and the mass m of ice melted. We use Eq. (17.21) to determine H and Eq. (17.20) to determine m.

EXECUTE We assume that the total heat flow is the same as it would be through a flat Styrofoam slab of area $0.80\ \text{m}^2$ and thickness $2.0\ \text{cm} = 0.020\ \text{m}$ (Fig. 17.25b). We find k from Table 17.5. From Eq. (17.21),

$$\begin{aligned} H &= kA\frac{T_{\text{H}} - T_{\text{C}}}{L} = (0.027\ \text{W/m}\cdot\text{K})(0.80\ \text{m}^2)\frac{30^\circ\text{C} - 0^\circ\text{C}}{0.020\ \text{m}} \\ &= 32.4\ \text{W} = 32.4\ \ \text{J/s} \end{aligned}$$

The total heat flow is $Q = Ht$, with $t = 3\ \text{h} = 10,800\ \text{s}$. From Table 17.4, the heat of fusion of ice is $L_{\text{f}} = 3.34 \times 10^5\ \text{J/kg}$, so from Eq. (17.20) the mass of ice that melts is

$$m = \frac{Q}{L_{\text{f}}} = \frac{(32.4\ \ \text{J/s})(10,800\ \ \text{s})}{3.34 \times 10^5 \text{J/kg}} = 1.0\ \text{kg}$$

EVALUATE The low heat current is a result of the low thermal conductivity of Styrofoam.

KEY CONCEPT

If a temperature difference is maintained between the two sides of an object of thickness L and cross-sectional area A, there will be a steady heat current due to conduction from the high-temperature side to the low-temperature side. This conduction heat current is proportional to the temperature difference and to the ratio A/L.

Video Tutor Solution: **Example 17.11**

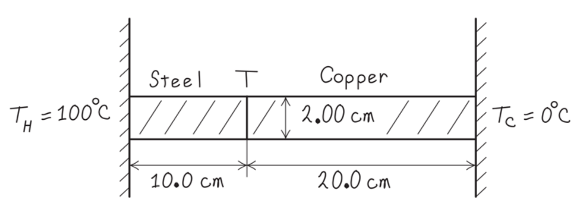
Example 17.12 Conduction through two bars I

WITH VARIATION PROBLEMS

A steel bar 10.0 cm long is welded end to end to a copper bar 20.0 cm long. Each bar has a square cross section, 2.00 cm on a side. The free end of the steel bar is kept at $100^\circ\mathrm{C}$ by placing it in contact with steam, and the free end of the copper bar is kept at $0^\circ\mathrm{C}$ by placing it in contact with ice. Both bars are perfectly insulated on their sides. Find the steady-state temperature at the junction of the two bars and the total rate of heat flow through the bars.

IDENTIFY and SET UP Figure 17.26 shows the situation. The heat currents in these end-to-end bars must be the same (see Problem-Solving Strategy 17.3). We are given "hot" and "cold" temperatures $T_\mathrm{H} = 100^\circ\mathrm{C}$ and $T_\mathrm{C} = 0^\circ\mathrm{C}$. With subscripts S for steel and Cu for copper, we write Eq. (17.21) separately for the heat currents H_S and H_Cu and set the resulting expressions equal to each other.

Figure 17.26



Our sketch for this problem.

EXECUTE Setting $H_{\mathrm{S}} = H_{\mathrm{Cu}}$, we have from Eq. (17.21)

$$H_{\mathrm{S}} = k_{\mathrm{S}} A \frac{T_{\mathrm{H}} - T}{L_{\mathrm{S}}} = H_{\mathrm{Cu}} = k_{\mathrm{Cu}} A \frac{T - T_{\mathrm{C}}}{L_{\mathrm{Cu}}}$$

We divide out the equal cross-sectional areas A and solve for T :

$$T = \frac{\dfrac{k_{\mathrm{S}}}{L_{\mathrm{S}}} T_{\mathrm{H}} + \dfrac{k_{\mathrm{Cu}}}{L_{\mathrm{Cu}}} T_{\mathrm{C}}}{\left(\dfrac{k_{\mathrm{S}}}{L_{\mathrm{S}}} + \dfrac{k_{\mathrm{Cu}}}{L_{\mathrm{Cu}}} \right)}$$

Substituting $L_{\mathrm{S}} = 10.0$ cm and $L_{\mathrm{Cu}} = 20.0$ cm, the given values of T_{H} and T_{C}, and the values of k_{S} and k_{Cu} from Table 17.5, we find $T = 20.7^{\circ}\mathrm{C}$.

We can find the total heat current by substituting this value of T into either the expression for H_{S} or the one for H_{Cu} :

$$\begin{aligned} H_{\mathrm{S}} &= (50.2\ \mathrm{W/m \cdot K})(0.0200\ \mathrm{m})^2 \frac{100^{\circ}\mathrm{C} - 20.7^{\circ}\mathrm{C}}{0.100\ \mathrm{m}} \\ &= 15.9\ \mathrm{W} \\ H_{\mathrm{Cu}} &= (385\ \mathrm{W/m \cdot K})(0.0200\ \mathrm{m})^2 \frac{20.7^{\circ}\mathrm{C}}{0.200\ \mathrm{m}} = 15.9\ \mathrm{W} \end{aligned}$$

EVALUATE Even though the steel bar is shorter, the temperature drop across it is much greater (from $100^{\circ}\mathrm{C}$ to $20.7^{\circ}\mathrm{C}$) than across the copper bar (from $20.7^{\circ}\mathrm{C}$ to $0^{\circ}\mathrm{C}$). That's because steel is a much poorer conductor than copper.

KEY CONCEPT

When there is a steady heat flow by conduction through two materials in succession, the heat current is the same in both materials: No energy is lost in going from one material to the next.

Video Tutor Solution: **Example 17.12**

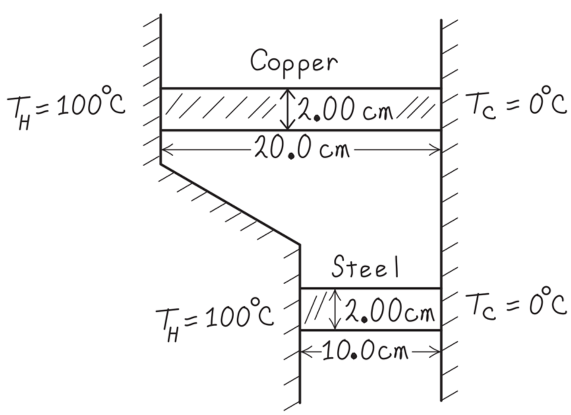
Example 17.13 Conduction through two bars II

WITH VARIATION PROBLEMS

Suppose the two bars of Example 17.12 are separated. One end of each bar is kept at $100^\circ \mathrm{C}$ and the other end of each bar is kept at $0^\circ \mathrm{C}$. What is the *total* heat current in the two bars?

IDENTIFY and SET UP Figure 17.27 shows the situation. For each bar, $T_\mathrm{H} - T_\mathrm{C} = 100^\circ \mathrm{C} - 0^\circ \mathrm{C} = 100\ \mathrm{K}$. The total heat current is the sum of the currents in the two bars, $H_\mathrm{S} + H_\mathrm{Cu}$.

Figure 17.27



Our sketch for this problem.

EXECUTE We write the heat currents for the two rods individually, and then add them to get the total heat current:

$$\begin{aligned} H &= H_{\mathrm{S}} + H_{\mathrm{Cu}} = k_{\mathrm{S}} A \frac{T_{\mathrm{H}} - T_{\mathrm{C}}}{L_{\mathrm{S}}} + k_{\mathrm{Cu}} A \frac{T_{\mathrm{H}} - T_{\mathrm{C}}}{L_{\mathrm{Cu}}} \\ &= (50.2\ \mathrm{W/m \cdot K})(0.0200\ \mathrm{m})^2 \frac{100\ \mathrm{K}}{0.100\ \mathrm{m}} \\ &\quad + (385\ \mathrm{W/m \cdot K})(0.0200\ \mathrm{m})^2 \frac{100\ \mathrm{K}}{0.200\ \mathrm{m}} \\ &= 20.1\ \mathrm{W} + 77.0\ \mathrm{W} = 97.1\ \mathrm{W} \end{aligned}$$

EVALUATE The heat flow in the copper bar is much greater than that in the steel bar, even though it is longer, because the thermal conductivity of copper is much larger. The total heat flow is greater than in Example 17.12 because the total cross section for heat flow is greater and because the full 100 K temperature difference appears across each bar.

KEY CONCEPT

Even if two different objects have the same constant temperature difference between their ends, the heat current H through the two objects can be different. The value of H depends on the dimensions of the object and the thermal conductivity of the material of the object.

Video Tutor Solution: **Example 17.13**

Convection

Video Tutor Demo: Candle Chimneys

Convection is the transfer of heat by mass motion of a fluid from one region of space to another. Familiar examples include hot-air and hot-water home heating systems, the cooling system of an automobile engine, and the flow of blood in the body. If the fluid is circulated by a blower or pump, the process is called *forced convection;* if the flow is caused by differences in density due to thermal expansion, such as hot air rising, the process is called *free convection* (Fig. 17.28).

Figure 17.28

A heating element in the tip of this submerged tube warms the surrounding water, producing a complex pattern of free convection.

Free convection in the atmosphere plays a dominant role in determining the daily weather, and convection in the oceans is an important global heat-transfer mechanism. On a smaller scale, soaring hawks and glider pilots make use of thermal updrafts from the warm earth. The most important mechanism for heat transfer within the human body (needed to maintain nearly constant temperature in various environments) is *forced* convection of blood, with the heart as the pump.

Convective heat transfer is a very complex process, and there is no simple equation to describe it. Here are a few experimental facts:

1. The heat current due to convection is directly proportional to the surface area. That's why radiators and cooling fins, which use convection to transfer heat, have large surface areas.

2. The viscosity of fluids slows natural convection near a stationary surface. For air, this gives rise to a surface film that on a vertical surface typically has about the same insulating value as 1.3 cm of plywood (R value $= 0.7$). Forced convection decreases the thickness of this film, increasing the rate of heat transfer. This is the reason for the "wind-chill factor"; you get cold faster in a cold wind than in still air with the same temperature.
3. The heat current due to free convection is found to be approximately proportional to the $\frac{5}{4}$ power of the temperature difference between the surface and the main body of fluid.

Radiation

Radiation is the transfer of heat by electromagnetic waves such as visible light, infrared, and ultraviolet radiation. Everyone has felt the warmth of the sun's radiation and the intense heat from a charcoal grill or the glowing coals in a fireplace. Most of the heat from these very hot objects reaches you not by conduction or convection in the intervening air but by *radiation.* This heat transfer would occur even if there were nothing but vacuum between you and the source of heat.

Every object, even at ordinary temperatures, emits energy in the form of electromagnetic radiation. Around 20°C, nearly all the energy is carried by infrared waves with wavelengths much longer than those of visible light (see Fig. 17.4 and Fig. 17.29). As the temperature rises, the wavelengths shift to shorter values. At 800°C, an object emits enough visible radiation to appear "red-hot," although even at this temperature most of the energy is carried by infrared waves. At 3000°C, the temperature of an incandescent lamp filament, the radiation contains enough visible light that the object appears "white-hot."

Figure 17.29

This false-color infrared photograph reveals radiation emitted by various parts of the man's body. The strongest emission (colored red) comes from the warmest areas, while there is very little emission from the bottle of cold beverage.

The rate of energy radiation from a surface is proportional to the surface area A and to the fourth power of the absolute (Kelvin) temperature T. The rate also depends on the nature of the surface; we describe this dependence by a quantity e called **emissivity.** A dimensionless number between 0 and 1, e is the ratio of the rate of radiation from a particular surface to the rate of radiation from an equal area of an ideal radiating surface at the same temperature. Emissivity also depends somewhat on temperature. Thus we can to nine significant figures, its express the heat current $H = dQ/dt$ due to radiation from a surface as

(17.25)

Area of emitting surface

Emissivity of surface

Heat current in radiation

$$H = Ae\sigma T^4 \qquad (17.25)$$

Absolute temperature of surface

Stefan–Boltzmann constant

This relationship is called the **Stefan–Boltzmann law** in honor of its late-19th-century discoverers. The **Stefan–Boltzmann constant** σ (Greek sigma) is a fundamental constant; to nine significant figures, its numerical value is

$$\sigma = 5.67037442 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$$

You should check unit consistency in Eq. (17.25). Emissivity (e) is often larger for dark surfaces than for light ones. The emissivity of a smooth copper surface is about 0.3, but e for a dull black surface can be close to unity.

Example 17.14 Heat transfer by radiation

WITH VARIATION PROBLEMS

A thin, square steel plate, 10 cm on a side, is heated in a blacksmith's forge to $800^\circ\mathrm{C}$. If the emissivity is 0.60, what is the total rate of radiation of energy from the plate?

IDENTIFY and SET UP The target variable is H, the rate of emission of energy from the plate's two surfaces. We use Eq. (17.25) to calculate H.

EXECUTE The total surface area is $2(0.10\ \mathrm{m})^2 = 0.020\ \mathrm{m}^2$, and $T = 800^\circ\mathrm{C} = 1073\ \mathrm{K}$. Then Eq. (17.25) gives

$$\begin{aligned} H &= Ae\sigma T^4 \\ &= (0.020\ \mathrm{m}^2)(0.60)(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(1073\ \mathrm{K})^4 = 900\ \mathrm{W} \end{aligned}$$

EVALUATE The nearby blacksmith will easily feel the heat radiated from this plate.

KEY CONCEPT

All objects emit energy in the form of electromagnetic radiation due to their temperature. The heat current of this radiation is proportional to the object's surface area, to the emissivity of its surface, and to the fourth power of the object's Kelvin temperature.

Video Tutor Solution: **Example 17.14**

Radiation and Absorption

While an object at absolute temperature T is radiating, its surroundings at temperature T_{s} are also radiating, and the object *absorbs* some of this radiation. If it is in thermal equilibrium with its surroundings, $T = T_{\mathrm{s}}$ and the rates of radiation and absorption must be equal. For this to be true, the rate of absorption must be given in general by $H = Ae\sigma T_{\mathrm{s}}{}^4$. Then the *net* rate of radiation from an object at temperature T with surroundings at temperature T_{s} is $Ae\sigma T^4 - Ae\sigma T_{\mathrm{s}}{}^4$, or

(17.26)

Area of emitting surface　Emissivity of surface

Net heat current in radiation

$$H_{\mathrm{net}} = Ae\sigma(T^4 - T_{\mathrm{s}}^4) \tag{17.26}$$

Stefan–Boltzmann constant　Absolute temperatures of surface (T) and surroundings (T_{s})

In Eq. (17.26) a positive value of H means a net heat flow *out of* the object. This will be the case if $T > T_{\mathrm{s}}$.

Example 17.15 Radiation from the human body

WITH VARIATION PROBLEMS

What is the total rate of radiation of energy from a human body with surface area $1.20\ \mathrm{m}^2$ and surface temperature $30^\circ\mathrm{C} = 303\ \mathrm{K}$? If the surroundings are at a temperature of $20^\circ\mathrm{C}$, what is the *net* rate of radiative heat loss from the body? The emissivity of the human body is very close to unity, irrespective of skin pigmentation.

IDENTIFY and SET UP We must consider both the radiation that the body emits and the radiation that it absorbs from its surroundings. Equation (17.25) gives the rate of radiation of energy from the body, and Eq. (17.26) gives the net rate of heat loss.

EXECUTE Taking $e = 1$ in Eq. (17.25), we find that the body radiates at a rate

$$\begin{aligned} H &= Ae\sigma T^4 \\ &= (1.20\ \mathrm{m}^2)(1)(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(303\ \mathrm{K})^4 = 574\ \mathrm{W} \end{aligned}$$

This loss is partly offset by absorption of radiation, which depends on the temperature of the surroundings. From Eq. (17.26), the *net* rate of radiative energy transfer is

$$
\begin{aligned}
H_{\text{net}} &= Ae\sigma\left(T^4 - T_{\text{s}}^{\,4}\right) \\
&= (1.20\ \text{m}^2)(1)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)\left[(303\ \text{K})^4 - (293\ \text{K})^4\right] \\
&= 72\ \text{W}
\end{aligned}
$$

EVALUATE The value of H_{net} is positive because the body is losing heat to its colder surroundings.

KEY CONCEPT

An object at Kelvin temperature T emits electromagnetic radiation but also absorbs radiation from its surroundings at Kelvin temperature T_{s}. The *net* heat current is proportional to the object's surface area, to the emissivity of its surface, and to the difference between T^4 and $T_{\text{s}}^{\,4}$.

Video Tutor Solution: Example 17.15

Applications of Radiation

Heat transfer by radiation is important in some surprising places. A premature baby in an incubator can be cooled dangerously by radiation if the walls of the incubator happen to be cold, even when the *air* in the incubator is warm. Some incubators regulate the air temperature by measuring the baby's skin temperature.

An object that is a good absorber must also be a good emitter. An ideal radiator, with emissivity $e = 1$, is also an ideal absorber, absorbing *all* of the radiation that strikes it. Such an ideal surface is called an ideal black body or simply a **blackbody.** Conversely, an ideal *reflector,* which absorbs *no* radiation at all, is also a very ineffective radiator.

This is the reason for the silver coatings on vacuum ("Thermos") bottles, invented by Sir James Dewar (1842–1923). A vacuum bottle has double glass walls. The air is pumped out of the spaces between the walls; this eliminates nearly all heat transfer by conduction and convection. The silver coating on the walls reflects most of the radiation from the contents back into the container, and the wall itself is a very poor emitter. Thus a vacuum bottle can keep coffee or soup hot for several hours. The Dewar flask, used to store very cold liquefied gases, is exactly the same in principle.

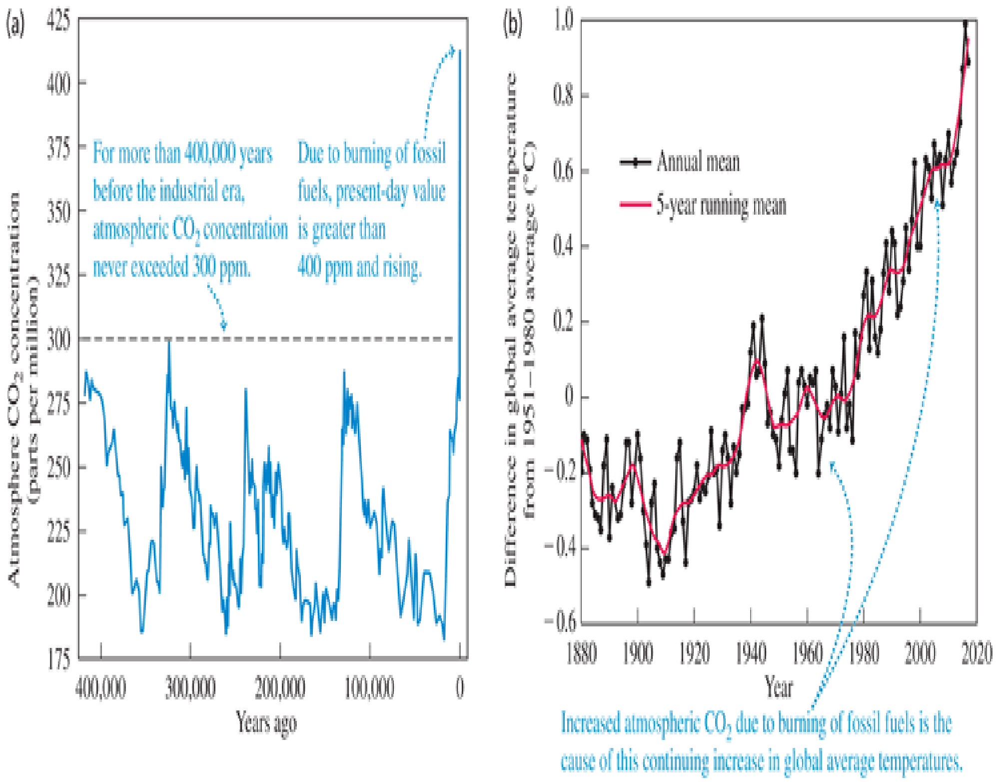
Radiation, Climate, and Climate Change

Our planet constantly absorbs radiation coming from the sun. In thermal equilibrium, the rate at which our planet absorbs solar radiation must equal the rate at which it emits radiation into space. The presence of an atmosphere on our planet has a significant effect on this equilibrium.

Most of the radiation emitted by the sun (which has a surface temperature of 5800 K) is in the visible part of the spectrum, to which our atmosphere is transparent. But the average surface temperature of the earth is only 287 K ($14^\circ C$). Hence most of the radiation that our planet emits into space is infrared radiation, just like the radiation from the person shown in Fig. 17.29. However, our atmosphere is *not* completely transparent to infrared radiation. This is because our atmosphere contains carbon dioxide (CO_2), which is its fourth most abundant constituent (after nitrogen, oxygen, and argon). Molecules of CO_2 in the atmosphere *absorb* some of the infrared radiation coming upward from the surface. They then re-radiate the absorbed energy, but some of the re-radiated energy is directed back down toward the surface instead of escaping into space. In order to maintain thermal equilibrium, the earth's surface must compensate for this by increasing its temperature T and hence its total rate of radiating energy (which is proportional to T^4). This phenomenon, called the **greenhouse effect,** makes our planet's surface temperature about $33^\circ C$ higher than it would be if there were no atmospheric CO_2. If CO_2 were absent, the earth's average surface temperature would be below the freezing point of water, and life as we know it would be impossible.

While atmospheric CO_2 has benefits, too much of it can have extremely negative consequences. Measurements of air trapped in ancient Antarctic ice show that over the past 650,000 years CO_2 has constituted less than 300 parts per million of our atmosphere. Since the beginning of the industrial age, however, the burning of fossil fuels such as coal and petroleum has elevated the atmospheric CO_2 concentration to unprecedented levels (Fig. 17.30a). As a consequence, since the 1950s the global average surface temperature has increased by $0.9^\circ C$ and the earth has experienced the hottest years ever recorded (Fig. 17.30b). If we continue to consume fossil fuels at the same rate, by 2050 the atmospheric CO_2 concentration will reach 600 parts per million, well off the scale of Fig. 17.30a. The resulting temperature increase will have dramatic effects on global climate. In polar regions massive quantities of ice will melt and run from solid land to the sea, thus raising ocean levels worldwide and threatening the homes and lives of hundreds of millions of people who live near the coast. Coping with these threats is one of the greatest challenges facing 21st-century civilization.

Figure 17.30



(a) Due to humans burning fossil fuels, the concentration of carbon dioxide in the atmosphere is now more than 33% greater than in the pre-industrial era. (b) Due to the increased CO_2 concentration, during the past 50 years the global average temperature has increased at an average rate of approximately 0.18 C$^\circ$ per decade.

Test Your Understanding of Section 17.7

A room has one wall made of concrete, one wall made of copper, and one wall made of steel. All of the walls are the same size and at the same temperature of $20^\circ C$. Which wall feels coldest to the touch? (i) The concrete wall; (ii) the copper wall; (iii) the steel wall; (iv) all three walls feel equally cold.

Chapter 17 Summary

Temperature and temperature scales: Two objects in thermal equilibrium must have the same temperature. A conducting material between two objects permits them to interact and come to thermal equilibrium; an insulating material impedes this interaction.

The Celsius and Fahrenheit temperature scales are based on the freezing ($0^\circ\mathrm{C} = 32^\circ\mathrm{F}$) and boiling ($100^\circ\mathrm{C} = 212^\circ\mathrm{F}$) temperatures of water. One Celsius degree equals $\frac{9}{5}$ Fahrenheit degrees. (See Example 17.1.)

The Kelvin scale has its zero at the extrapolated zero-pressure temperature for a gas thermometer, $-273.15^\circ\mathrm{C} = 0\ \mathrm{K}$. In the gas-thermometer scale, the ratio of two temperatures T_1 and T_2 is defined to be equal to the ratio of the two corresponding gas-thermometer pressures p_1 and p_2.

(17.1)

$$T_{\mathrm{F}} = \frac{9}{5}T_{\mathrm{C}} + 32^\circ$$

(17.2)

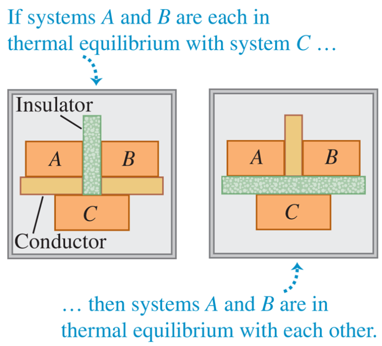
$$T_{\mathrm{C}} = \frac{5}{9}(T_{\mathrm{F}} - 32^\circ)$$

(17.3)

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15$$

(17.4)

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}$$



Thermal expansion and thermal stress: A temperature change ΔT causes a change in any linear dimension L_0 of a solid object. The change ΔL is approximately proportional to L_0 and ΔT. Similarly, a temperature change causes a change ΔV in the volume V_0 of any solid or liquid; ΔV is approximately proportional to V_0 and ΔT. The quantities α and β are the coefficients of linear expansion and volume expansion, respectively. For solids, $\beta = 3\alpha$. (See Examples 17.2 and 17.3.)

When a material is cooled or heated and held so it cannot contract or expand, it is under a tensile stress F/A. (See Example 17.4.)

(17.6)

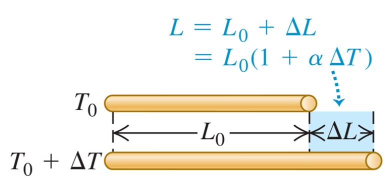
$$\Delta L = \alpha L_0 \,\Delta T$$

(17.8)

$$\Delta V = \beta V_0 \,\Delta T$$

(17.12)

$$\frac{F}{A} = -Y\alpha \,\Delta T$$



Heat, phase changes, and calorimetry: Heat is energy in transit from one object to another as a result of a temperature difference. Equations (17.13) and (17.18) give the quantity of heat Q required to cause a temperature change ΔT in a quantity of material with mass m and specific heat c (alternatively, with number of moles n and molar heat capacity $C = Mc$, where M is the molar mass and $m = nM$). When heat is added to an object, Q is positive; when it is removed, Q is negative. (See Examples 17.5 and 17.6.)

To change a mass m of a material to a different phase at the same temperature (such as liquid to vapor), a quantity of heat given by Eq. (17.20) must be added or subtracted. Here L is the heat of fusion, vaporization, or sublimation.

In an isolated system whose parts interact by heat exchange, the algebraic sum of the Q's for all parts of the system must be zero. (See Examples 17.7, 17.8, 17.9 and 17.10.)

(17.13)

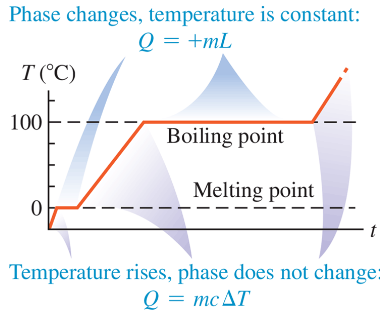
$$Q = mc\,\Delta T$$

(17.18)

$$Q = nC\,\Delta T$$

(17.20)

$$Q = \pm mL$$



Conduction, convection, and radiation: Conduction is the transfer of heat within materials without bulk motion of the materials. The heat current H depends on the area A through which the heat flows, the length L of the heat-flow path, the temperature difference $(T_H - T_C)$, and the thermal conductivity k of the material. (See Examples 17.11, 17.12 and 17.13.)

Convection is a complex heat-transfer process that involves mass motion from one region to another.

Radiation is energy transfer through electromagnetic radiation. The radiation heat current H depends on the surface area A, the emissivity e of the surface (a pure number between 0 and 1), and the Kelvin temperature T. Here σ is the Stefan–Boltzmann constant. The *net* radiation heat current H_{net} from an object at temperature T to its surroundings at temperature T_s depends on both T and T_s. (See Examples 17.14 and 17.15.)

(17.21)

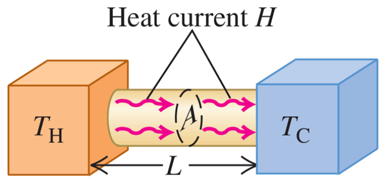
$$H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$$

(17.25)

$$H = Ae\sigma T^4$$

(17.26)

$$H_{\text{net}} = Ae\sigma\left(T^4 - T_s^4\right)$$



Heat current $H = kA\dfrac{T_H - T_C}{L}$

Guided Practice: Temperature and Heat

For assigned homework and other learning materials, go to
Mastering Physics.

Key Example Variation Problems

Be sure to review Examples 17.2, 17.3, and 17.4 (Section 17.4) before attempting these problems.

VP17.4.1 A metal rod is 0.500 m in length at a temperature of 15.0°C. When you raise its temperature to 37.0°C, its length increases by 0.220 mm. (a) What is the coefficient of linear expansion of the metal? (b) If a second rod of the same metal has length 0.300 m at 25.0°C, how will its length change if the temperature drops to −20.0°C?

VP17.4.2 A copper mug that can hold 250 cm^3 of liquid is filled to the brim with ethanol at 20.0°C. If you lower the temperature of the mug and ethanol to -50.0°C, what is the maximum additional volume of ethanol you can add to the mug without spilling any? (See Table 17.2. Ethanol remains a liquid at temperatures down to −114°C.)

VP17.4.3 A cylindrical brass rod is 10.0 cm in length and 0.500 cm in radius at 25.0°C. How much force do you have to apply to each end of the rod to maintain its length when the temperature is decreased to 13.0 $^\circ$C? Are the required forces tensile or compressive? (See Table 17.1. Brass has Young's modulus 9.0×10^{10} Pa)

VP17.4.4 A rod made of metal A is attached end to end to another rod made of metal B, making a combined rod of overall length L. The coefficients of linear expansion of metals A and B are α_A and α_B, respectively, and $\alpha_B > \alpha_A$. When the temperature of the combined rod is increased by ΔT, the overall length increases by ΔL. What was the initial length of the rod of metal A?

Be sure to review Examples 17.7, 17.8, and 17.9 (Section 17.6) before attempting these problems.

VP17.9.1 You place a piece of aluminum at 250.0 $^\circ$C in 5.00 kg of liquid water at 20.0 $^\circ$C. None of the water boils, and the final temperature of the water and aluminum is 22.0 $^\circ$C. What is the mass of the piece of aluminum? Assume no heat is exchanged with the container that holds the water. (See Table 17.3.)

VP17.9.2 You place an ice cube of mass 7.50×10^{-3} kg and temperature 0.00 $^\circ$C on top of a copper cube of mass 0.460 kg. All of the ice melts, and the final equilibrium temperature of the two substances is 0.00 $^\circ$C. What was the initial temperature of the copper cube? Assume no heat is exchanged with the surroundings. (See Tables17.3 and 17.4.)

VP17.9.3 You have 1.60 kg of liquid ethanol at 28.0 $^\circ$C that you wish to cool. What mass of ice at initial temperature -5.00 $^\circ$C should you add to the ethanol so that all of the ice melts and the resulting ethanol–water mixture has temperature

10.0 $^\circ$C? Assume no heat is exchanged with the container that holds the ethanol. (See Tables17.3 and 17.4.)

VP17.19.4 You put a silver ingot of mass 1.25 kg and initial temperature 315 $^\circ$C in contact with 0.250 kg of ice at initial temperature $-8.00\,^\circ$C. Assume no heat is exchanged with the surroundings. (a) What is the final equilibrium temperature? (b) What fraction of the ice melts? (See Tables17.3 and 17.4.)

Be sure to review Examples 17.11, 17.12, 17.13, 17.14, and 17.15 (Section 17.7) before attempting these problems.

VP17.15.1 A square pane of glass 0.500 m on a side is 6.00 mm thick. When the temperatures on the two sides of the glass are 25.0 $^\circ$C and $-10.0\,^\circ$C, the heat current due to conduction through the glass is 1.10×10^3 W. (a) What is the thermal conductivity of the glass? (b) If the thickness of the glass is increased to 9.00 mm, what will be the heat current?

VP17.15.2 A brass rod and a lead rod, each 0.250 m long and each with cross-sectional area $2.00 \times 10^{-4}\,\mathrm{m}^2$, are joined end to end to make a composite rod of overall length 0.500 m. The free end of the brass rod is maintained at a high temperature, and the free end of the lead rod is maintained at a low temperature. The temperature at the junction of the two rods is 185 $^\circ$C, and the heat current due to conduction through the composite rod is 6.00 W. What are the temperatures of (a) the free end of the brass rod and (b) the free end of the lead rod? (See Table 17.5.)

VP17.15.3 The emissivity of the surface of a star is approximately 1. The star Sirius A emits electromagnetic radiation at a rate of 9.7×10^{27} W and has a surface temperature of 9940 K. What is the radius of Sirius in meters and as a multiple of the sun's radius (6.96×10^8 m)?

VP17.15.4 A building in the desert is made of concrete blocks (emissivity 0.91) and has an exposed surface area of 525 m^2. If the building is maintained at 20.0 °C but the temperature on a hot desert night is 35.0 °C, what is the net rate at which the building absorbs energy by radiation?

Bridging Problem: Steady-State Heat Flow: Radiation and Conduction

One end of a solid cylindrical copper rod 0.200 m long and 0.0250 m in radius is inserted into a large block of solid hydrogen at its melting temperature, 13.84 K. The other end is blackened and exposed to thermal radiation from surrounding walls at 500.0 K. (Some telescopes in space employ a similar setup. A solid refrigerant keeps the telescope very cold—required for proper operation—even though it is exposed to direct sunlight.) The sides of the rod are insulated, so no energy is lost or gained except at the ends of the rod. (a) When equilibrium is reached, what is the temperature of the blackened end? The thermal conductivity of copper at temperatures near 20 K is 1670 W/m · K. (b) At what rate (in kg/h) does the solid hydrogen melt?

Solution Guide

IDENTIFY and SET UP

1. Draw a sketch of the situation, showing all relevant dimensions.
2. List the known and unknown quantities, and identify the target variables.

3. In order for the rod to be in equilibrium, how must the radiation heat current from the walls into the blackened end of the rod compare to the conduction heat current from this end to the other end and into the solid hydrogen? Use your answers to select the appropriate equations for part (a).
4. How does the heat current from the rod into the hydrogen determine the rate at which the hydrogen melts? (*Hint:* See Table 17.4.) Use your answer to select the appropriate equations for part (b).

EXECUTE

5. Solve for the temperature of the blackened end of the rod. (*Hint:* Since copper is an excellent conductor of heat at low temperature, you can assume that the temperature of the blackened end is only slightly higher than 13.84 K.)
6. Use your result from step 5 to find the rate at which the hydrogen melts.

EVALUATE

7. Is your result from step 5 consistent with the hint in that step?
8. How would your results from steps 5 and 6 be affected if the rod had twice the radius?

Video Tutor Solution: **Chapter 17 Bridging Problem**

Questions/Exercises/Problems: Temperature and Heat

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

Discussion Questions

Q17.1 Explain why it would not make sense to use a full-size glass thermometer to measure the temperature of a thimbleful of hot water.

Q17.2 If you heat the air inside a rigid, sealed container until its Kelvin temperature doubles, the air pressure in the container will also double. Is the same thing true if you double the Celsius temperature of the air in the container? Explain.

Q17.3 Many automobile engines have cast-iron cylinders and aluminum pistons. What kinds of problems could occur if the engine gets too hot? (The coefficient of volume expansion of cast iron is approximately the same as that of steel.)

Q17.4 Why do frozen water pipes burst? Would a mercury thermometer break if the temperature went below the freezing temperature of mercury? Why or why not?

Q17.5 Two objects made of the same material have the same external dimensions and appearance, but one is solid and the other is hollow. When their temperature is increased, is the overall volume expansion the same or different? Why?

Q17.6 Why is it sometimes possible to loosen caps on screw-top bottles by dipping the capped bottle briefly into hot water?

Q17.7 The inside of an oven is at a temperature of $200^\circ\mathrm{C}$ ($392^\circ\mathrm{F}$). You can put your hand in the oven without injury as long as you don't touch anything. But since the air inside the oven is also at $200^\circ\mathrm{C}$, why isn't your hand burned just the same?

Q17.8 A newspaper article about the weather states that "the temperature of an object measures how much heat the object contains." Is this description correct? Why or why not?

Q17.9 A student asserts that a suitable unit for specific heat is $1\ \mathrm{m}^2/\mathrm{s}^2 \cdot \mathrm{C}^\circ$. Is she correct? Why or why not?

Q17.10 In some household air conditioners used in dry climates, air is cooled by blowing it through a water-soaked filter, evaporating some of the water. How does this cool the air? Would such a system work well in a high-humidity climate? Why or why not?

Q17.11 The units of specific heat c are $\mathrm{J/kg \cdot K}$, but the units of heat of fusion L_f or heat of vaporization L_v are simply $\mathrm{J/kg}$. Why do the units of L_f and L_v not include a factor of $(\mathrm{K})^{-1}$ to account for a temperature change?

Q17.12 Why is a hot, humid day in the tropics generally more uncomfortable for human beings than a hot, dry day in the desert?

Q17.13 A piece of aluminum foil used to wrap a potato for baking in a hot oven can usually be handled safely within a few seconds after the potato is removed from the oven. The same is not true of the potato, however! Give two reasons for this difference.

Q17.14 Desert travelers sometimes keep water in a canvas bag. Some water seeps through the bag and evaporates. How does this cool the water inside the bag?

Q17.15 When you first step out of the shower, you feel cold. But as soon as you are dry you feel warmer, even though the room temperature does not change. Why?

Q17.16 The climate of regions adjacent to large bodies of water (like the Pacific and Atlantic coasts) usually features a narrower range of temperature than the climate of regions far from large bodies of water (like the prairies). Why?

Q17.17 When water is placed in ice-cube trays in a freezer, why doesn't the water freeze all at once when the temperature has reached $0^{\circ}C$? In fact, the water freezes first in a layer adjacent to the sides of the tray. Why?

Q17.18 Before giving you an injection, a physician swabs your arm with isopropyl alcohol at room temperature. Why does this make your arm feel cold? (*Hint:* The reason is *not* the fear of the injection! The boiling point of isopropyl alcohol is $82.4^{\circ}C$.)

Q17.19 A cold block of metal feels colder than a block of wood at the same temperature. Why? A *hot* block of metal feels hotter than a block of wood at the same temperature. Again, why? Is there any temperature at which the two blocks feel equally hot or cold? What temperature is this?

Q17.20 A person pours a cup of hot coffee, intending to drink it five minutes later. To keep the coffee as hot as possible, should she put cream in it now or wait until just before she drinks it? Explain.

Q17.21 When a freshly baked apple pie has just been removed from the oven, the crust and filling are both at the same temperature. Yet if you sample the pie, the filling will burn your tongue but the crust will not. Why is there a difference? (*Hint:* The filling is moist while the crust is dry.)

Q17.22 Old-time kitchen lore suggests that things cook better (evenly and without burning) in heavy cast-iron pots. What desirable characteristics do such pots have?

Q17.23 In coastal regions in the winter, the temperature over the land is generally colder than the temperature over the nearby ocean; in the summer, the reverse is usually true. Explain. (*Hint:* The specific heat of soil is only 0.2–0.8 times as great as that of water.)

Q17.24 It is well known that a potato bakes faster if a large nail is stuck through it. Why? Does an aluminum nail work better than a steel one? Why or why not? (*Note:* Don't try this in a microwave oven!) There is also a gadget on the market to hasten the roasting of meat; it consists of a hollow metal tube containing a wick and some water. This is claimed to work much better than a solid metal rod. How does it work?

Q17.25 Glider pilots in the Midwest know that thermal updrafts are likely to occur in the vicinity of freshly plowed fields. Why?

Q17.26 Some folks claim that ice cubes freeze faster if the trays are filled with hot water, because hot water cools off faster than cold water. What do you think?

Q17.27 We're lucky that the earth isn't in thermal equilibrium with the sun (which has a surface temperature of 5800 K). But why aren't the two objects in thermal equilibrium?

Q17.28 When energy shortages occur, magazine articles sometimes urge us to keep our homes at a constant temperature day and night to conserve fuel. They argue that when we turn down the heat at night, the walls, ceilings, and other areas cool off and must be reheated in the morning. So if we keep the temperature constant, these parts of the house will not cool off and will not have to be reheated. Does this argument make sense? Would we really save energy by following this advice?

Exercises

Section 17.2 Thermometers and Temperature Scales

17.1 • Convert the following Celsius temperatures to Fahrenheit: (a) $-62.8^\circ C$, the lowest temperature ever recorded in North America (February 3, 1947, Snag, Yukon); (b) $56.7^\circ C$, the highest temperature ever recorded in the United States (July 10, 1913, Death Valley, California); (c) $31.1^\circ C$, the world's highest average annual temperature (Lugh Ferrandi, Somalia).

17.2 • BIO **Temperatures in Biomedicine.** (a) **Normal body temperature.** The average normal body temperature measured in the mouth is 310 K. What would Celsius and Fahrenheit thermometers read for this temperature? (b) **Elevated body temperature.** During very vigorous exercise, the body's temperature can go as high as $40^\circ C$. What would Kelvin and Fahrenheit thermometers read for this temperature? (c) **Temperature difference in the body.** The surface temperature of the body is normally about $7\ C^\circ$ lower than the internal temperature. Express this temperature difference in kelvins and in Fahrenheit degrees. (d) **Blood storage.** Blood stored at $4.0^\circ C$ lasts safely for about 3 weeks, whereas blood stored at $-160^\circ C$ lasts for 5 years. Express both temperatures on the Fahrenheit and Kelvin scales. (e) **Heat stroke.** If the body's temperature is above $105^\circ F$ for a prolonged period, heat stroke can result. Express this temperature on the Celsius and Kelvin scales.

17.3 • (a) On January 22, 1943, the temperature in Spearfish, South Dakota, rose from $-4.0^\circ F$ to $45.0^\circ F$ in just 2 minutes. What was the temperature change in Celsius degrees? (b) The temperature in Browning, Montana, was $44.0^\circ F$ on January 23, 1916. The next day the temperature plummeted to $-56^\circ F$. What was the temperature change in Celsius degrees?

Section 17.3 Gas Thermometers and the Kelvin Scale

17.4 • Derive an equation that gives T_K as a function of T_F to the nearest hundredth of a degree. Solve the equation and thereby obtain an equation for T_F as a function of T_K.

17.5 •• You put a bottle of soft drink in a refrigerator and leave it until its temperature has dropped 10.0 K. What is its temperature change in (a) F° and (b) C°?

17.6 • (a) Calculate the one temperature at which Fahrenheit and Celsius thermometers agree with each other. (b) Calculate the one temperature at which Fahrenheit and Kelvin thermometers agree with each other.

17.7 • The pressure of a gas at the triple point of water is 1.35 atm. If its volume remains unchanged, what will its pressure be at the temperature at which $\mathrm{CO_2}$ solidifies?

17.8 • Convert the following Kelvin temperatures to the Celsius and Fahrenheit scales: (a) the midday temperature at the surface of the moon (400 K); (b) the temperature at the tops of the clouds in the atmosphere of Saturn (95 K); (c) the temperature at the center of the sun (1.55×10^7 K).

17.9 •• **A Constant-Volume Gas Thermometer.** An experimenter using a gas thermometer found the pressure at the triple point of water ($0.01^{\circ}\mathrm{C}$) to be 4.80×10^4 Pa and the pressure at the normal boiling point ($100^{\circ}\mathrm{C}$) to be 6.50×10^4 Pa. (a) Assuming that the pressure varies linearly with temperature, use these two data points to find the Celsius temperature at which the gas pressure would be zero (that is, find the Celsius temperature of absolute zero). (b) Does the gas in this thermometer obey Eq. (17.4) precisely? If that equation were precisely obeyed and the pressure at $100^{\circ}\mathrm{C}$ were 6.50×10^4 Pa, what pressure would the experimenter have measured at 0.01 °C? (As we'll learn in Section 18.1, Eq. (17.4) is accurate only for gases at very low density.)

17.10 •• A constant-volume gas thermometer registers an absolute pressure corresponding to 325 mm of mercury when in contact with water at the triple point. What pressure does it read when in contact with water at the normal boiling point?

Section 17.4 Thermal Expansion

17.11 • The Humber Bridge in England has the world's longest single span, 1410 m. Calculate the change in length of the steel deck of the span when the temperature increases from $-5.0^\circ\mathrm{C}$ to $18.0^\circ\mathrm{C}$.

17.12 • One of the tallest buildings in the world is the Taipei 101 in Taiwan, at a height of 1671 feet. Assume that this height was measured on a cool spring day when the temperature was $15.5^\circ\mathrm{C}$. You could use the building as a sort of giant thermometer on a hot summer day by carefully measuring its height. Suppose you do this and discover that the Taipei 101 is 0.471 foot taller than its official height. What is the temperature, assuming that the building is in thermal equilibrium with the air and that its entire frame is made of steel?

17.13 • A U.S. penny has a diameter of 1.9000 cm at $20.0^\circ\mathrm{C}$. The coin is made of a metal alloy (mostly zinc) for which the coefficient of linear expansion is $2.6 \times 10^{-5}\ \mathrm{K}^{-1}$. What would its diameter be on a hot day in Death Valley ($48.0^\circ\mathrm{C}$)? On a cold night in the mountains of Greenland ($-53^\circ\mathrm{C}$)?

17.14 • **Ensuring a Tight Fit.** Aluminum rivets used in airplane construction are made slightly larger than the rivet holes and cooled by "dry ice" (solid CO_2) before being driven. If the diameter of a hole is 4.500 mm, what should be the diameter of a rivet at $23.0^\circ\mathrm{C}$ if its diameter is to equal that of the hole when the rivet is cooled to $-78.0^\circ\mathrm{C}$, the temperature of dry ice? Assume that the expansion coefficient remains constant at the value given in Table 17.1.

17.15 •• A copper cylinder is initially at $20.0^\circ\mathrm{C}$. At what temperature will its volume be 0.150% larger than it is at $20.0^\circ\mathrm{C}$?

17.16 •• A geodesic dome constructed with an aluminum framework is a nearly perfect hemisphere; its diameter measures 55.0 m on a winter day at a temperature of $-15^\circ\mathrm{C}$. How much more interior space does the dome have in the summer, when the temperature is $35^\circ\mathrm{C}$?

17.17 •• A glass flask whose volume is $1000.00\ \mathrm{cm}^3$ at $0.0^\circ\mathrm{C}$ is completely filled with mercury at this temperature. When flask and mercury are warmed to $55.0^\circ\mathrm{C}$, $8.95\ \mathrm{cm}^3$ of mercury overflow. If the coefficient of volume expansion of mercury is $18.0 \times 10^{-5}\ \mathrm{K}^{-1}$, compute the coefficient of volume expansion of the glass.

17.18 •• A steel tank is completely filled with $1.90\ \ \mathrm{m}^3$ of ethanol when both the tank and the ethanol are at $32.0^\circ\mathrm{C}$. When the tank and its contents have cooled to $18.0^\circ\mathrm{C}$, what additional volume of ethanol can be put into the tank?

17.19 •• A machinist bores a hole of diameter 1.35 cm in a steel plate that is at $25.0^\circ\mathrm{C}$. What is the cross-sectional area of the hole (a) at $25.0^\circ\mathrm{C}$ and (b) when the temperature of the plate is increased to $175^\circ\mathrm{C}$? Assume that the coefficient of linear expansion remains constant over this temperature range.

17.20 • Consider a flat metal plate with width w and length l, so its area is $A = lw$. The metal has coefficient of linear expansion α. Derive an expression, in terms of α, that gives the change ΔA in area for a change ΔT in temperature.

17.21 •• Steel train rails are laid in 12.0-m-long segments placed end to end. The rails are laid on a winter day when their temperature is $-9.0^\circ\mathrm{C}$. (a) How much space must be left between adjacent rails if they are just to touch on a summer day when their temperature is $33.0^\circ\mathrm{C}$? (b) If the rails are originally laid in contact, what is the stress in them on a summer day when their temperature is $33.0^\circ\mathrm{C}$?

17.22 •• A brass rod is 185 cm long and 1.60 cm in diameter. What force must be applied to each end of the rod to prevent it from contracting when it is cooled from 120.0˚C to 10.0˚C?

17.23 • The increase in length of an aluminum rod is twice the increase in length of an Invar rod with only a third of the temperature increase. Find the ratio of the lengths of the two rods.

Section 17.5 Quantity of Heat

17.24 • In an effort to stay awake for an all-night study session, a student makes a cup of coffee by first placing a 200 W electric immersion heater in 0.320 kg of water. (a) How much heat must be added to the water to raise its temperature from $20.0^\circ C$ to $80.0^\circ C$? (b) How much time is required? Assume that all of the heater's power goes into heating the water.

17.25 •• An aluminum tea kettle with mass 1.10 kg and containing 1.80 kg of water is placed on a stove. If no heat is lost to the surroundings, how much heat must be added to raise the temperature from $20.0^\circ C$ to $85.0^\circ C$?

17.26 • BIO **Heat Loss During Breathing.** In very cold weather a significant mechanism for heat loss by the human body is energy expended in warming the air taken into the lungs with each breath. (a) On a cold winter day when the temperature is $-20^\circ C$, what amount of heat is needed to warm to body temperature ($37^\circ C$) the 0.50 L of air exchanged with each breath? Assume that the specific heat of air is $1020 \text{ J} / \text{kg} \cdot \text{K}$ and that 1.0 L of air has mass 1.3×10^{-3} kg. (b) How much heat is lost per hour if the respiration rate is 20 breaths per minute?

17.27 • BIO While running, a 70 kg student generates thermal energy at a rate of 1200 W. For the runner to maintain a constant body temperature of $37^\circ C$, this energy must be removed by perspiration or other mechanisms. If these mechanisms failed and the energy could not flow out of the student's body, for what amount of time could a student run before irreversible body damage occurred? (*Note:* Protein structures in the body are irreversibly damaged if body temperature rises to $44^\circ C$ or higher. The specific heat of a typical human body is $3480 \text{ J/kg} \cdot \text{K}$, slightly less than that of water. The difference is due to the presence of protein, fat, and minerals, which have lower specific heats.)

17.28 •• **On-Demand Water Heaters.** Conventional hot-water heaters consist of a tank of water maintained at a fixed temperature. The hot water is to be used when needed. The drawbacks are that energy is wasted because the tank loses heat when it is not in use and that you can run out of hot water if you use too much. Some utility companies are encouraging the use of *on-demand* water heaters (also known as *flash heaters*), which consist of heating units to heat the water as you use it. No water tank is involved, so no heat is wasted. A typical household shower flow rate is 2.5 gal/min (9.46 L/min) with the tap water being heated from 50°F (10°C) to 120°F (49°C) by the on-demand heater. What rate of heat input (either electrical or from gas) is required to operate such a unit, assuming that all the heat goes into the water?

17.29 • You are given a sample of metal and asked to determine its specific heat. You weigh the sample and find that its weight is 28.4 N. You carefully add 1.25×10^4 J of heat energy to the sample and find that its temperature rises 18.0 C°. What is the sample's specific heat?

17.30 • **CP** A 25,000 kg subway train initially traveling at 15.5 m/s slows to a stop in a station and then stays there long enough for its brakes to cool. The station's dimensions are 65.0 m long by 20.0 m wide by 12.0 m high. Assuming all the work done by the brakes in stopping the train is transferred as heat uniformly to all the air in the station, by how much does the air temperature in the station rise? Take the density of the air to be 1.20 $\mathrm{kg/m^3}$ and its specific heat to be 1020 $\mathrm{J/kg \cdot K}$.

17.31 • **CP** While painting the top of an antenna 225 m in height, a worker accidentally lets a 1.00 L water bottle fall from his lunchbox. The bottle lands in some bushes at ground level and does not break. If a quantity of heat equal to the magnitude of the change in mechanical energy of the water goes into the water, what is its increase in temperature?

17.32 • **CP** A nail driven into a board increases in temperature. If we assume that 60% of the kinetic energy delivered by a 1.80 kg hammer with a speed of 7.80 m/s is transformed into heat that flows into the nail and does not flow out, what is the temperature increase of an 8.00 g aluminum nail after it is struck ten times?

17.33 •• **CP** A 15.0 g bullet traveling horizontally at 865 m/s passes through a tank containing 13.5 kg of water and emerges with a speed of 534 m/s. What is the maximum temperature increase that the water could have as a result of this event?

Section 17.6 Calorimetry and Phase Changes

17.34 • You have 750 g of water at 10.0˚C in a large insulated beaker. How much boiling water at 100.0˚C must you add to this beaker so that the final temperature of the mixture will be 75˚C?

17.35 •• A 500.0 g chunk of an unknown metal, which has been in boiling water for several minutes, is quickly dropped into an insulating Styrofoam beaker containing 1.00 kg of water at room temperature (20.0˚C). After waiting and gently stirring for 5.00 minutes, you observe that the water's temperature has reached a constant value of 22.0˚C. (a) Assuming that the Styrofoam absorbs a negligibly small amount of heat and that no heat was lost to the surroundings, what is the specific heat of the metal? (b) Which is more useful for storing thermal energy: this metal or an equal weight of water? Explain. (c) If the heat absorbed by the Styrofoam actually is not negligible, how would the specific heat you calculated in part (a) be in error? Would it be too large, too small, or still correct? Explain.

17.36 • BIO **Treatment for a Stroke.** One suggested treatment for a person who has suffered a stroke is immersion in an ice-water bath at 0˚C to lower the body temperature, which prevents damage to the brain. In one set of tests, patients were cooled until their internal temperature reached 32.0˚C. To treat a 70.0 kg patient, what is the minimum amount of ice (at 0˚C) you need in the bath so that its temperature remains at 0˚C? The specific heat of the human body is 3480 J/kg · C˚, and recall that normal body temperature is 37.0˚C.

17.37 •• A blacksmith cools a 1.20 kg chunk of iron, initially at 650.0°C, by trickling 15.0°C water over it. All of the water boils away, and the iron ends up at 120.0°C. How much water did the blacksmith trickle over the iron?

17.38 •• A copper calorimeter can with mass 0.100 kg contains 0.160 kg of water and 0.0180 kg of ice in thermal equilibrium at atmospheric pressure. If 0.750 kg of lead at 255°C is dropped into the calorimeter can, what is the final temperature? Assume that no heat is lost to the surroundings.

17.39 •• A copper pot with a mass of 0.500 kg contains 0.170 kg of water, and both are at 20.0°C. A 0.250 kg block of iron at 85.0°C is dropped into the pot. Find the final temperature of the system, assuming no heat loss to the surroundings.

17.40 • In a container of negligible mass, 0.200 kg of ice at an initial temperature of -40.0°C is mixed with a mass m of water that has an initial temperature of 80.0°C. No heat is lost to the surroundings. If the final temperature of the system is 28.0°C, what is the mass m of the water that was initially at 80.0°C?

17.41 • A 6.00 kg piece of solid copper metal at an initial temperature T is placed with 2.00 kg of ice that is initially at -20.0°C. The ice is in an insulated container of negligible mass and no heat is exchanged with the surroundings. After thermal equilibrium is reached, there is 1.20 kg of ice and 0.80 kg of liquid water. What was the initial temperature of the piece of copper?

17.42 •• An ice-cube tray of negligible mass contains 0.290 kg of water at 18.0°C. How much heat must be removed to cool the water to 0.00°C and freeze it? Express your answer in joules, calories, and Btu.

17.43 • How much heat is required to convert 18.0 g of ice at -10.0°C to steam at 100.0°C? Express your answer in joules, calories, and Btu.

17.44 •• An open container holds 0.550 kg of ice at -15.0°C. The mass of the container can be ignored. Heat is supplied to the container at the constant rate of $800.0\ \text{J/min}$ for 500.0 min. (a) After how many minutes does the ice *start* to melt? (b) After how many minutes, from the time when the heating is first started, does the temperature begin to rise above 0.0°C? (c) Plot a curve showing the temperature as a function of the elapsed time.

17.45 • **CP** What must the initial speed of a lead bullet be at 25.0°C so that the heat developed when it is brought to rest will be just sufficient to melt it? Assume that all the initial mechanical energy of the bullet is converted to heat and that no heat flows from the bullet to its surroundings. (Typical rifles have muzzle speeds that exceed the speed of sound in air, which is 347 m/s at 25.0°C.)

17.46 •• **BIO** **Steam Burns Versus Water Burns.** What is the amount of heat input to your skin when it receives the heat released (a) by 25.0 g of steam initially at 100.0°C, when it is cooled to skin temperature (34.0°C)? (b) By 25.0 g of water initially at 100.0°C, when it is cooled to 34.0°C? (c) What does this tell you about the relative severity of burns from steam versus burns from hot water?

17.47 • **BIO** **"The Ship of the Desert."** Camels require very little water because they are able to tolerate relatively large changes in their body temperature. While humans keep their body temperatures constant to within one or two Celsius degrees, a dehydrated camel permits its body temperature to drop to 34.0°C overnight and rise to 40.0°C during the day. To see how effective this mechanism is for saving water, calculate how many liters of water a 400 kg camel would have to drink if it attempted to keep its body temperature at a constant $34.0^\circ C$ by evaporation of sweat during the day (12 hours) instead of letting it rise to $40.0^\circ C$. (*Note:* The specific heat of a camel or other mammal is about the same as that of a typical human, $3480\ J/kg \cdot K$. The heat of vaporization of water at $34^\circ C$ is $2.42 \times 10^6\ J/kg$.)

17.48 • **BIO** Evaporation of sweat is an important mechanism for temperature control in some warm-blooded animals. (a) What mass of water must evaporate from the skin of a 70.0 kg man to cool his body $1.00\ C^\circ$? The heat of vaporization of water at body temperature ($37^\circ C$) is $2.42 \times 10^6\ J/kg$. The specific heat of a typical human body is $3480\ J/kg \cdot K$ (see Exercise 17.27). (b) What volume of water must the man drink to replenish the evaporated water? Compare to the volume of a soft-drink can ($355\ cm^3$).

17.49 •• **CP** An asteroid with a diameter of 10 km and a mass of 2.60×10^{15} kg impacts the earth at a speed of $32.0\ km/s$, landing in the Pacific Ocean. If 1.00% of the asteroid's kinetic energy goes to boiling the ocean water (assume an initial water temperature of $10.0^\circ C$), what mass of water will be boiled away by the collision? (For comparison, the mass of water contained in Lake Superior is about 2×10^{15} kg.)

17.50 • A laboratory technician drops a 0.0850 kg sample of unknown solid material, at $100.0^\circ C$, into a calorimeter. The calorimeter can, initially at $19.0^\circ C$, is made of 0.150 kg of copper and contains 0.200 kg of water. The final temperature of the calorimeter can and contents is $26.1^\circ C$. Compute the specific heat of the sample.

17.51 •• An insulated beaker with negligible mass contains 0.250 kg of water at 75.0˚C. How many kilograms of ice at -20.0˚C must be dropped into the water to make the final temperature of the system 40.0˚C?

17.52 • A 4.00 kg silver ingot is taken from a furnace at 750.0˚C and placed on a large block of ice at 0.0˚C. Assuming that all the heat given up by the silver is used to melt the ice, how much ice is melted?

17.53 •• A plastic cup of negligible mass contains 0.280 kg of an unknown liquid at a temperature of 30.0˚C. A 0.0270 kg mass of ice at a temperature of 0.0˚C is added to the liquid, and when thermal equilibrium is reached the temperature of the combined substances is 14.0˚C. Assuming no heat is exchanged with the surroundings, what is the specific heat capacity of the unknown liquid?

Section 17.7 Mechanisms of Heat Transfer

17.54 •• Two rods, one made of brass and the other made of copper, are joined end to end. The length of the brass section is 0.300 m and the length of the copper section is 0.800 m. Each segment has cross-sectional area 0.00500 m^2. The free end of the brass segment is in boiling water and the free end of the copper segment is in an ice–water mixture, in both cases under normal atmospheric pressure. The sides of the rods are insulated so there is no heat loss to the surroundings. (a) What is the temperature of the point where the brass and copper segments are joined? (b) What mass of ice is melted in 5.00 min by the heat conducted by the composite rod?

17.55 •• A copper bar is welded end to end to a bar of an unknown metal. The two bars have the same lengths and cross-sectional areas. The free end of the copper bar is maintained at a temperature T_H that can be varied. The free end of the unknown metal is kept at $0.0^\circ C$. To measure the thermal conductivity of the unknown metal, you measure the temperature T at the junction between the two bars for several values of T_H. You plot your data as T versus T_H, both in kelvins, and find that your data are well fit by a straight line that has slope 0.710. What do your measurements give for the value of the thermal conductivity of the unknown metal?

17.56 •• One end of an insulated metal rod is maintained at $100.0^\circ C$, and the other end is maintained at $0.00^\circ C$ by an ice–water mixture. The rod is 60.0 cm long and has a cross-sectional area of 1.25 cm^2. The heat conducted by the rod melts 8.50 g of ice in 10.0 min. Find the thermal conductivity k of the metal.

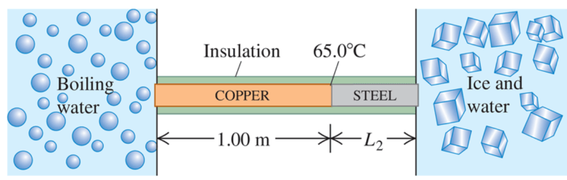
17.57 •• A carpenter builds an exterior house wall with a layer of wood 3.0 cm thick on the outside and a layer of Styrofoam insulation 2.2 cm thick on the inside wall surface. The wood has $k = 0.080\ \mathrm{W/m \cdot K}$, and the Styrofoam has $k = 0.027\ \mathrm{W/m \cdot K}$. The interior surface temperature is $19.0^{\circ}\mathrm{C}$, and the exterior surface temperature is $-10.0^{\circ}\mathrm{C}$. (a) What is the temperature at the plane where the wood meets the Styrofoam? (b) What is the rate of heat flow per square meter through this wall?

17.58 • An electric kitchen range has a total wall area of $1.40\ \mathrm{m}^2$ and is insulated with a layer of fiberglass 4.00 cm thick. The inside surface of the fiberglass has a temperature of $175^{\circ}\mathrm{C}$, and its outside surface is at $35.0^{\circ}\mathrm{C}$. The fiberglass has a thermal conductivity of $0.040\ \mathrm{W/m \cdot K}$. (a) What is the heat current through the insulation, assuming it may be treated as a flat slab with an area of $1.40\ \mathrm{m}^2$? (b) What electric-power input to the heating element is required to maintain this temperature?

17.59 • Air has a very low thermal conductivity. This explains why we feel comfortable wearing short sleeves in a $20^{\circ}\mathrm{C}$ environment even though our body temperature is $37^{\circ}\mathrm{C}$. Material objects feel cooler to our immediate touch than the air, owing to relatively high thermal conductivities. (a) Touch a few surfaces in a room-temperature environment and rank them in order of which feel the coolest to which feel the warmest. Objects that feel cooler have larger thermal conductivities. Consider a wood surface, a metallic surface, and a glass surface, and rank these in order from coolest to warmest. (b) How does your ranking compare to the thermal conductivities listed in Table 17.5?

17.60 • A long rod, insulated to prevent heat loss along its sides, is in perfect thermal contact with boiling water (at atmospheric pressure) at one end and with an ice–water mixture at the other (Fig. E17.60). The rod consists of a 1.00 m section of copper (one end in boiling water) joined end to end to a length L_2 of steel (one end in the ice–water mixture). Both sections of the rod have cross-sectional areas of 4.00 cm^2. The temperature of the copper–steel junction is 65.0°C after a steady state has been set up. (a) How much heat per second flows from the boiling water to the ice–water mixture? (b) What is the length L_2 of the steel section?

Figure E17.60



17.61 • A pot with a steel bottom 8.50 mm thick rests on a hot stove. The area of the bottom of the pot is 0.150 m^2. The water inside the pot is at 100.0°C, and 0.390 kg are evaporated every 3.00 min. Find the temperature of the lower surface of the pot, which is in contact with the stove.

17.62 •• You are asked to design a cylindrical steel rod 50.0 cm long, with a circular cross section, that will conduct 190.0 J/s from a furnace at 400.0°C to a container of boiling water under 1 atmosphere. What must the rod's diameter be?

17.63 •• A picture window has dimensions of 1.40 m × 2.50 m and is made of glass 5.20 mm thick. On a winter day, the temperature of the outside surface of the glass is −20.0°C, while the temperature of the inside surface is a comfortable 19.5°C. (a) At what rate is heat being lost through the window by conduction? (b) At what rate would heat be lost through the window if you covered it with a 0.750-mm-thick layer of paper (thermal conductivity 0.0500 $W/m \cdot K$)?

17.64 • What is the rate of energy radiation per unit area of a blackbody at (a) 273 K and (b) 2730 K?

17.65 • **Size of a Light-Bulb Filament.** The operating temperature of a tungsten filament in an incandescent light bulb is 2450 K, and its emissivity is 0.350. Find the surface area of the filament of a 150 W bulb if all the electrical energy consumed by the bulb is radiated by the filament as electromagnetic waves. (Only a fraction of the radiation appears as visible light.)

17.66 •• The emissivity of tungsten is 0.350. A tungsten sphere with radius 1.50 cm is suspended within a large evacuated enclosure whose walls are at 290.0 K. What power input is required to maintain the sphere at 3000.0 K if heat conduction along the supports is ignored?

17.67 • **The Sizes of Stars.** The hot glowing surfaces of stars emit energy in the form of electromagnetic radiation. It is a good approximation to assume $e = 1$ for these surfaces. Find the radii of the following stars (assumed to be spherical): (a) Rigel, the bright blue star in the constellation Orion, which radiates energy at a rate of 2.7×10^{32} W and has surface temperature 11,000 K; (b) Procyon B (visible only using a telescope), which radiates energy at a rate of 2.1×10^{23} W and has surface temperature 10,000 K. (c) Compare your answers to the radius of the earth, the radius of the sun, and the distance between the earth and the sun. (Rigel is an example of a *supergiant* star, and Procyon B is an example of a *white dwarf* star.)

Problems

17.68 •• Figure 17.12 shows that the graph of the volume of 1 gram of liquid water can be closely approximated by a parabola in the temperature range between $0^\circ\mathrm{C}$ and $10^\circ\mathrm{C}$. (a) Show that the equation of this parabola has the form $V = A + B(T_\mathrm{C} - 4.0^\circ\mathrm{C})^2$ and find the values of the constants A and B. (b) Define the temperature-dependent quantity $\beta(T_\mathrm{C})$ in terms of the equation $dV = \beta(T_\mathrm{C})\, V\, dT$. Use the result of part (a) to find the value of $\beta(T_\mathrm{C})$ for $T_\mathrm{C} = 1.0^\circ\mathrm{C}$, $4.0^\circ\mathrm{C}$, $7.0^\circ\mathrm{C}$, and $10.0^\circ\mathrm{C}$. Your results show that β is not constant in this temperature range but is approximately constant above $7.0^\circ\mathrm{C}$.

17.69 •• **CP** A Foucault pendulum consists of a brass sphere with a diameter of 35.0 cm suspended from a steel cable 10.5 m long (both measurements made at $20.0^\circ\mathrm{C}$). Due to a design oversight, the swinging sphere clears the floor by a distance of only 2.00 mm when the temperature is $20.0^\circ\mathrm{C}$. At what temperature will the sphere begin to brush the floor?

17.70 •• A steel wire has density 7800 kg/m^3 and mass 2.50 g. It is stretched between two rigid supports separated by 0.400 m. (a) When the temperature of the wire is $20.0^\circ\mathrm{C}$, the frequency of the fundamental standing wave for the wire is 440 Hz. What is the tension in the wire? (b) What is the temperature of the wire if its fundamental standing wave has frequency 460 Hz? For steel the coefficient of linear expansion is $1.2 \times 10^{-5}\ \mathrm{K}^{-1}$ and Young's modulus is 20×10^{10} Pa.

17.71 •• An unknown liquid has density ρ and coefficient of volume expansion β. A quantity of heat Q is added to a volume V of the liquid, and the volume of the liquid increases by an amount ΔV. There is no phase change. In terms of these quantities, what is the specific heat capacity c of the liquid?

17.72 •• **CP** A small fused quartz sphere swings back and forth as a simple pendulum on the lower end of a long copper wire that is attached to the ceiling at its upper end. The amplitude of swing is small. When the wire has a temperature of 20.0°C, its length is 3.00 m. What is the percentage change in the period of the motion if the temperature of the wire is increased to 220°C? (*Hint:* Use the power series expansion for $(1 + x)^n$ in Appendix B.)

17.73 ••• You propose a new temperature scale with temperatures given in $^\circ\text{M}$. You define 0.0°M to be the normal melting point of mercury and 100.0°M to be the normal boiling point of mercury. (a) What is the normal boiling point of water in $^\circ\text{M}$? (b) A temperature change of $10.0\ \text{M}^\circ$ corresponds to how many C°?

17.74 • **CP CALC** A 250 kg weight is hanging from the ceiling by a thin copper wire. In its fundamental mode, this wire vibrates at the frequency of concert A (440 Hz). You then increase the temperature of the wire by $40\ \text{C}^\circ$. (a) By how much will the fundamental frequency change? Will it increase or decrease? (b) By what percentage will the speed of a wave on the wire change? (c) By what percentage will the wavelength of the fundamental standing wave change? Will it increase or decrease?

17.75 ••• You are making pesto for your pasta and have a cylindrical measuring cup 10.0 cm high made of ordinary glass $\left[\beta = 2.7 \times 10^{-5}\ (\text{C}^\circ)^{-1}\right]$ that is filled with olive oil $\left[\beta = 6.8\ \times 10^{-4}\ (\text{C}^\circ)^{-1}\right]$ to a height of 3.00 mm below the top of the cup. Initially, the cup and oil are at room temperature (22.0°C). You get a phone call and forget about the olive oil, which you inadvertently leave on the hot stove. The cup and oil heat up slowly and have a common temperature. At what temperature will the olive oil start to spill out of the cup?

17.76 •• A surveyor's 30.0 m steel tape is correct at 20.0°C. The distance between two points, as measured by this tape on a day when its temperature is 5.00°C, is 25.970 m. What is the true distance between the points?

17.77 •• A metal rod that is 30.0 cm long expands by 0.0650 cm when its temperature is raised from 0.0°C to 100.0°C. A rod of a different metal and of the same length expands by 0.0350 cm for the same rise in temperature. A third rod, also 30.0 cm long, is made up of pieces of each of the above metals placed end to end and expands 0.0580 cm between 0.0°C and 100.0°C. Find the length of each portion of the composite rod.

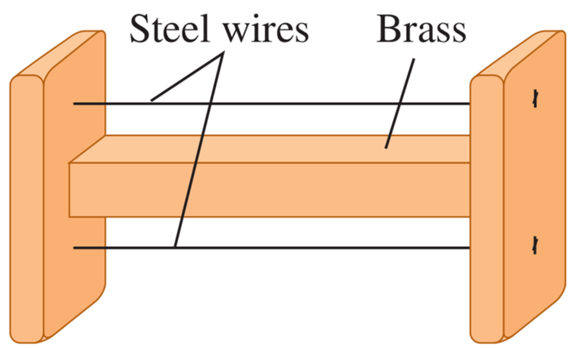
17.78 •• A copper sphere with density 8900 kg/m^3, radius 5.00 cm, and emissivity $e = 1.00$ sits on an insulated stand. The initial temperature of the sphere is 300 K. The surroundings are very cold, so the rate of absorption of heat by the sphere can be neglected. (a) How long does it take the sphere to cool by 1.00 K due to its radiation of heat energy? Neglect the change in heat current as the temperature decreases. (b) To assess the accuracy of the approximation used in part (a), what is the fractional change in the heat current H when the temperature changes from 300 K to 299 K?

17.79 ••• (a) Equation (17.12) gives the stress required to keep the length L of a rod constant as its temperature T changes. Show that if L is permitted to change by an amount ΔL when T changes by ΔT, the stress is

$$\frac{F}{A} = Y\left(\frac{\Delta L}{L_0} - \alpha\,\Delta T\right)$$

where F is the tension on the rod, L_0 is the original length of the rod, A its cross-sectional area, α its coefficient of linear expansion, and Y its Young's modulus. (b) A heavy brass bar has projections at its ends (Fig. P17.79). Two fine steel wires, fastened between the projections, are just taut (zero tension) when the whole system is at $20^\circ\mathrm{C}$. What is the tensile stress in the steel wires when the temperature of the system is raised to $140^\circ\mathrm{C}$? Make any simplifying assumptions you think are justified, but state them.

Figure P17.79



17.80 •• CP A metal wire, with density ρ and Young's modulus Y, is stretched between rigid supports. At temperature T, the speed of a transverse wave is found to be v_1. When the temperature is increased to $T + \Delta T$, the speed decreases to $v_2 < v_1$. Determine the coefficient of linear expansion of the wire.

17.81 •• A steel ring with a 2.5000 in. inside diameter at $20.0^\circ\mathrm{C}$ is to be warmed and slipped over a brass shaft with a 2.5020 in. outside diameter at $20.0^\circ\mathrm{C}$. (a) To what temperature should the ring be warmed? (b) If the ring and the shaft together are cooled by some means such as liquid air, at what temperature will the ring just slip off the shaft?

17.82 • BIO **Doughnuts: Breakfast of Champions!** A typical doughnut contains 2.0 g of protein, 17.0 g of carbohydrates, and 7.0 g of fat. Average food energy values are 4.0 kcal/g for protein and carbohydrates and 9.0 kcal/g for fat. (a) During heavy exercise, an average person uses energy at a rate of 510 kcal/h. How long would you have to exercise to "work off" one doughnut? (b) If the energy in the doughnut could somehow be converted into the kinetic energy of your body as a whole, how fast could you move after eating the doughnut? Take your mass to be 60 kg, and express your answer in m/s and in km/h.

17.83 •• There is 0.050 kg of an unknown liquid in a plastic container of negligible mass. The liquid has a temperature of 90.0°C. To measure the specific heat capacity of the unknown liquid, you add a mass m_{w} of water that has a temperature of 0.0°C to the liquid and measure the final temperature T after the system has reached thermal equilibrium. You repeat this measurement for several values of m_{w}, with the initial temperature of the unknown liquid always equal to 90.0°C. The plastic container is insulated, so no heat is exchanged with the surroundings. You plot your data as m_{w} versus T^{-1}, the inverse of the final temperature T. Your data points lie close to a straight line that has slope 2.15 kg $\cdot$ C$^\circ$. What does this result give for the value of the specific heat capacity of the unknown liquid?

17.84 •• You cool a 100.0 g slug of red-hot iron (temperature 745°C) by dropping it into an insulated cup of negligible mass containing 85.0 g of water at 20.0°C. Assuming no heat exchange with the surroundings, (a) what is the final temperature of the water and (b) what is the final mass of the iron and the remaining water?

17.85 •• CALC **Debye's T^3 Law.** At very low temperatures the molar heat capacity of rock salt varies with temperature according to Debye's T^3 law:

$$C = k\frac{T^3}{\theta^3}$$

where $k = 1940\ \mathrm{J/mol \cdot K}$ and $\theta = 281\ \mathrm{K}$. (a) How much heat is required to raise the temperature of 1.50 mol of rock salt from 10.0 K to 40.0 K? (*Hint:* Use Eq. (17.18) in the form $dQ = nC\,dT$ and integrate.) (b) What is the average molar heat capacity in this range? (c) What is the true molar heat capacity at 40.0 K?

17.86 •• The heat one feels when sitting near the fire in a fireplace or at a campfire is due almost entirely to thermal radiation. (a) Estimate the diameter and length of an average campfire log. (b) Compute the surface area of such a log. (c) Use the Stefan–Boltzmann law to determine the power emitted by thermal radiation by such a log when it burns at a typical temperature of $700^\circ\mathrm{C}$ in a surrounding air temperature of $20.0^\circ\mathrm{C}$. The emissivity of a burning log is close to unity.

17.87 • **Hot Air in a Physics Lecture.** (a) A typical student listening attentively to a physics lecture has a heat output of 100 W. How much heat energy does a class of 140 physics students release into a lecture hall over the course of a 50 min lecture? (b) Assume that all the heat energy in part (a) is transferred to the $3200\ \mathrm{m}^3$ of air in the room. The air has specific heat $1020\ \mathrm{J/kg \cdot K}$ and density $1.20\ \mathrm{kg/m}^3$. If none of the heat escapes and the air conditioning system is off, how much will the temperature of the air in the room rise during the 50 min lecture? (c) If the class is taking an exam, the heat output per student rises to 280 W. What is the temperature rise during 50 min in this case?

17.88 ••• CALC The molar heat capacity of a certain substance varies with temperature according to the empirical equation

$$C = 29.5\ \mathrm{J/mol \cdot K} + (8.20 \times 10^{-3}\ \mathrm{J/mol \cdot K^2})T$$

How much heat is necessary to change the temperature of 3.00 mol of this substance from 27°C to 227°C? (*Hint:* Use Eq. (17.18) in the form $dQ = nC\,dT$ and integrate.)

17.89 •• BIO **Bicycling on a Warm Day.** If the air temperature is the same as the temperature of your skin (about 30°C), your body cannot get rid of heat by transferring it to the air. In that case, it gets rid of the heat by evaporating water (sweat). During bicycling, a typical 70 kg person's body produces energy at a rate of about 500 W due to metabolism, 80% of which is converted to heat. (a) How many kilograms of water must the person's body evaporate in an hour to get rid of this heat? The heat of vaporization of water at body temperature is $2.42 \times 10^6\ \mathrm{J/kg}$. (b) The evaporated water must, of course, be replenished, or the person will dehydrate. How many 750 mL bottles of water must the bicyclist drink per hour to replenish the lost water? (Recall that the mass of a liter of water is 1.0 kg.)

17.90 •• BIO **Overheating.** (a) By how much would the body temperature of the bicyclist in Problem 17.89 increase in an hour if he were unable to get rid of the excess heat? (b) Is this temperature increase large enough to be serious? To find out, how high a fever would it be equivalent to, in °F? (Recall that the normal internal body temperature is 98.6°F and the specific heat of the body is $3480\ \mathrm{J/kg \cdot C^\circ}$.)

17.91 • **BIO A Thermodynamic Process in an Insect.** The African bombardier beetle (*Stenaptinus insignis*) can emit a jet of defensive spray from the movable tip of its abdomen (Fig. P17.91). The beetle's body has reservoirs containing two chemicals; when the beetle is disturbed, these chemicals combine in a reaction chamber, producing a compound that is warmed from $20^\circ\mathrm{C}$ to $100^\circ\mathrm{C}$ by the heat of reaction. The high pressure produced allows the compound to be sprayed out at speeds up to $19\ \mathrm{m/s}$ ($68\ \mathrm{km}/h$), scaring away predators of all kinds. (The beetle shown in Fig. P17.91 is 2 cm long.) Calculate the heat of reaction of the two chemicals (in J/kg). Assume that the specific heat of the chemicals and of the spray is the same as that of water, $4.19 \times 10^3\ \mathrm{J/kg \cdot K}$, and that the initial temperature of the chemicals is $20^\circ\mathrm{C}$.

Figure P17.91

17.92 ••• **CP** A industrious explorer of the polar regions has devised a contraption for melting ice. It consists of a sealed 10 L cylindrical tank with a porous grate separating the top half from the bottom half. The bottom half includes a paddle wheel attached to an axle that passes outside the cylinder, where it is attached by a gearbox and pulley system to a stationary bicycle. Pedaling the bicycle rotates the paddle wheel inside the cylinder. The tank includes 6.00 L of water and 3.00 kg of ice at $0.0^{\circ}\mathrm{C}$. The water fills the bottom chamber, where it may be agitated by the paddle wheel, and partially fills the upper chamber, which also includes the ice. The bicycle is pedaled with an average torque of $25.0\ \mathrm{N} \cdot m$ at a rate of 30.0 revolutions per minute. The system is 70% efficient. (a) For what length of time must the explorer pedal the bicycle to melt all the ice? (b) How much longer must he pedal to raise the temperature of the water to $10.5^{\circ}\mathrm{C}$?

17.93 •• You have 1.50 kg of water at $28.0^{\circ}\mathrm{C}$ in an insulated container of negligible mass. You add 0.600 kg of ice that is initially at $-22.0^{\circ}\mathrm{C}$. Assume that no heat exchanges with the surroundings. (a) After thermal equilibrium has been reached, has all of the ice melted? (b) If all of the ice has melted, what is the final temperature of the water in the container? If some ice remains, what is the final temperature of the water in the container, and how much ice remains?

17.94 •• A thirsty nurse cools a 2.00 L bottle of a soft drink (mostly water) by pouring it into a large aluminum mug of mass 0.257 kg and adding 0.120 kg of ice initially at $-15.0^{\circ}\mathrm{C}$. If the soft drink and mug are initially at $20.0^{\circ}\mathrm{C}$, what is the final temperature of the system, assuming that no heat is lost?

17.95 ••• A copper calorimeter can with mass 0.446 kg contains 0.0950 kg of ice. The system is initially at $0.0^{\circ}\mathrm{C}$. (a) If 0.0350 kg of steam at $100.0^{\circ}\mathrm{C}$ and 1.00 atm pressure is added to the can, what is the final temperature of the calorimeter can and its contents? (b) At the final temperature, how many kilograms are there of ice, how many of liquid water, and how many of steam?

17.96 • A Styrofoam bucket of negligible mass contains 1.75 kg of water and 0.450 kg of ice. More ice, from a refrigerator at -15.0°C, is added to the mixture in the bucket, and when thermal equilibrium has been reached, the total mass of ice in the bucket is 0.884 kg. Assuming no heat exchange with the surroundings, what mass of ice was added?

17.97 ••• In a container of negligible mass, 0.0400 kg of steam at 100°C and atmospheric pressure is added to 0.200 kg of water at 50.0°C. (a) If no heat is lost to the surroundings, what is the final temperature of the system? (b) At the final temperature, how many kilograms are there of steam and how many of liquid water?

17.98 •• BIO **Mammal Insulation.** Animals in cold climates often depend on *two* layers of insulation: a layer of body fat (of thermal conductivity $0.20\ \text{W/m}\cdot\text{K}$) surrounded by a layer of air trapped inside fur or down. We can model a black bear (*Ursus americanus*) as a sphere 1.5 m in diameter having a layer of fat 4.0 cm thick. (Actually, the thickness varies with the season, but we are interested in hibernation, when the fat layer is thickest.) In studies of bear hibernation, it was found that the outer surface layer of the fur is at 2.7°C during hibernation, while the inner surface of the fat layer is at 31.0°C. (a) What is the temperature at the fat–inner fur boundary so that the bear loses heat at a rate of 50.0 W? (b) How thick should the air layer (contained within the fur) be?

17.99 •• **Effect of a Window in a Door.** A carpenter builds a solid wood door with dimensions $2.00\ \text{m} \times 0.95\ \text{m} \times 5.0\ \text{cm}$. Its thermal conductivity is $k = 0.120\ \text{W/m}\cdot\text{K}$. The air films on the inner and outer surfaces of the door have the same combined thermal resistance as an additional 1.8 cm thickness of solid wood. The inside air temperature is

$20.0^\circ C$, and the outside air temperature is $-8.0^\circ C$. (a) What is the rate of heat flow through the door? (b) By what factor is the heat flow increased if a window 0.500 m on a side is inserted in the door? The glass is 0.450 cm thick, and the glass has a thermal conductivity of $0.80 \ W/m \cdot K$. The air films on the two sides of the glass have a total thermal resistance that is the same as an additional 12.0 cm of glass.

17.100 ••• **CP** At $0^\circ C$, a cylindrical metal bar with radius r and mass M is slid snugly into a circular hole in a large, horizontal, rigid slab of thickness d. For this metal, Young's modulus is Y and the coefficient of linear expansion is α. A light but strong hook is attached to the underside of the metal bar; this apparatus is used as part of a hoist in a shipping yard. The coefficient of static friction between the bar and the slab is μ_s. At a temperature T above $0^\circ C$, the hook is attached to a large container and the slab is raised. What is the largest mass the container can have without the metal bar slipping out of the slab as the container is slowly lifted? The slab undergoes negligible thermal expansion.

17.101 •• Compute the ratio of the rate of heat loss through a single-pane window with area $0.15 \ m^2$ to that for a double-pane window with the same area. The glass of a single pane is 4.2 mm thick, and the air space between the two panes of the double-pane window is 7.0 mm thick. The glass has thermal conductivity $0.80 \ W/m \cdot K$. The air films on the room and outdoor surfaces of either window have a combined thermal resistance of $0.15 \ m^2 \cdot K/W$.

17.102 • Rods of copper, brass, and steel—each with cross-sectional area of $2.00 \ cm^2$—are welded together to form a Y-shaped figure. The free end of the copper rod is maintained at $100.0^\circ C$, and the free ends of the brass and steel rods at $0.0^\circ C$. Assume that there is no heat loss from the surfaces of the rods. The lengths of the rods are: copper, 13.0 cm; brass, 18.0 cm; steel, 24.0 cm. What is (a) the temperature of the junction point; (b) the heat current in each of the three rods?

17.103 •• BIO **Jogging in the Heat of the Day.** You have probably seen people jogging in extremely hot weather. There are good reasons not to do this! When jogging strenuously, an average runner of mass 68 kg and surface area 1.85 m^2 produces energy at a rate of up to 1300 W, 80% of which is converted to heat. The jogger radiates heat but actually absorbs more from the hot air than he radiates away. At such high levels of activity, the skin's temperature can be elevated to around 33°C instead of the usual 30°C. (Ignore conduction, which would bring even more heat into his body.) The only way for the body to get rid of this extra heat is by evaporating water (sweating). (a) How much heat per second is produced just by the act of jogging? (b) How much *net* heat per second does the runner gain just from radiation if the air temperature is 40.0°C (104°F)? (Remember: He radiates out, but the environment radiates back in.) (c) What is the *total* amount of excess heat this runner's body must get rid of per second? (d) How much water must his body evaporate every minute due to his activity? The heat of vaporization of water at body temperature is 2.42×10^6 J/kg. (e) How many 750 mL bottles of water must he drink after (or preferably before!) jogging for a half hour? Recall that a liter of water has a mass of 1.0 kg.

17.104 ••• BIO **Basal Metabolic Rate.** The *basal metabolic rate* is the rate at which energy is produced in the body when a person is at rest. A 75 kg (165 lb) person of height 1.83 m (6 ft) has a body surface area of approximately 2.0 m^2. (a)

What is the net amount of heat this person could radiate per second into a room at 18°C (about 65°F) if his skin's surface temperature is 30°C? (At such temperatures, nearly all the heat is infrared radiation, for which the body's emissivity is 1.0, regardless of the amount of pigment.)
(b) Normally, 80% of the energy produced by metabolism goes into heat, while the rest goes into things like pumping blood and repairing cells. Also normally, a person at rest can get rid of this excess heat just through radiation. Use your answer to part (a) to find this person's basal metabolic rate.

17.105 ••• CALC **Time Needed for a Lake to Freeze Over.** (a) When the air temperature is below 0°C, the water at the surface of a lake freezes to form an ice sheet. Why doesn't freezing occur throughout the entire volume of the lake? (b) Show that the thickness of the ice sheet formed on the surface of a lake is proportional to the square root of the time if the heat of fusion of the water freezing on the underside of the ice sheet is conducted through the sheet. (c) Assuming that the upper surface of the ice sheet is at −10°C and the bottom surface is at 0°C, calculate the time it will take to form an ice sheet 25 cm thick. (d) If the lake in part (c) is uniformly 40 m deep, how long would it take to freeze all the water in the lake? Is this likely to occur?

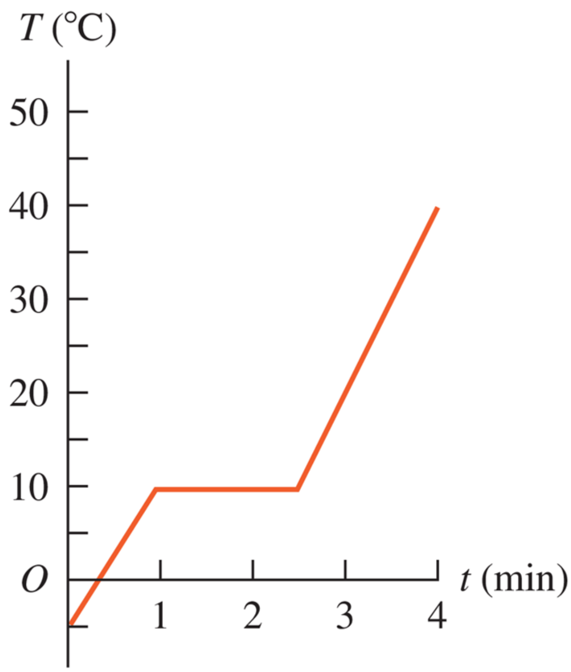
17.106 • The rate at which radiant energy from the sun reaches the earth's upper atmosphere is about 1.50 kW/m^2. The distance from the earth to the sun is 1.50×10^{11} m, and the radius of the sun is 6.96×10^8 m. (a) What is the rate of radiation of energy per unit area from the sun's surface? (b) If the sun radiates as an ideal blackbody, what is the temperature of its surface?

17.107 ••• **A Thermos for Liquid Helium.** A physicist uses a cylindrical metal can 0.250 m high and 0.090 m in diameter to store liquid helium at 4.22 K; at that temperature the heat of vaporization of helium is 2.09×10^4 J/kg. Completely surrounding the metal can are walls maintained at the temperature of liquid nitrogen, 77.3 K, with vacuum between the can and walls. How much liquid helium boils away per hour? The emissivity of the metal can is 0.200. The only heat transfer between the metal can and the surrounding walls is by radiation.

17.108 •• A metal sphere with radius 3.20 cm is suspended in a large metal box with interior walls that are maintained at 30.0°C. A small electric heater is embedded in the sphere. Heat energy must be supplied to the sphere at the rate of 0.660 J/s to maintain the sphere at a constant temperature of 41.0°C. (a) What is the emissivity of the metal sphere? (b) What power input to the sphere is required to maintain it at 82.0°C? What is the ratio of the power required for 82.0°C to the power required for 41.0°C? How does this ratio compare with 2^4? Explain.

17.109 •• DATA As a physicist, you put heat into a 500.0 g solid sample at the rate of 10.0 kJ/min while recording its temperature as a function of time. You plot your data as shown in Fig. P17.109. (a) What is the latent heat of fusion for this solid? (b) What are the specific heats of the liquid and solid states of this material?

Figure P17.109



17.110 ••• **DATA** At a chemical plant where you are an engineer, a tank contains an unknown liquid. You must determine the liquid's specific heat capacity. You put 0.500 kg of the liquid into an insulated metal cup of mass 0.200 kg. Initially the liquid and cup are at 20.0°C. You add 0.500 kg of water that has a temperature of 80.0°C. After thermal equilibrium has been reached, the final temperature of the two liquids and the cup is 58.1°C. You then empty the cup and repeat the experiment with the same initial temperatures, but this time with 1.00 kg of the unknown liquid. The final temperature is 49.3°C. Assume that the specific heat capacities are constant over the temperature range of the experiment and that no heat is lost to the surroundings. Calculate the specific heat capacity of the liquid and of the metal from which the cup is made.

17.111 •• **DATA** As a mechanical engineer, you are given two uniform metal bars A and B, made from different metals, to determine their thermal conductivities. You measure both bars to have length 40.0 cm and uniform cross-sectional area 2.50 cm^2. You place one end of bar A in thermal contact with a very large vat of boiling water at 100.0 $^\circ\mathrm{C}$ and the other end in thermal contact with an ice–water mixture at 0.0 $^\circ\mathrm{C}$. To prevent heat loss along the bar's sides, you wrap insulation around the bar. You weigh the amount of ice initially as 300 g. After 45.0 min, you weigh the ice again; 191 g of ice remains. The ice–water mixture is in an insulated container, so the only heat entering or leaving it is the heat conducted by the metal bar.

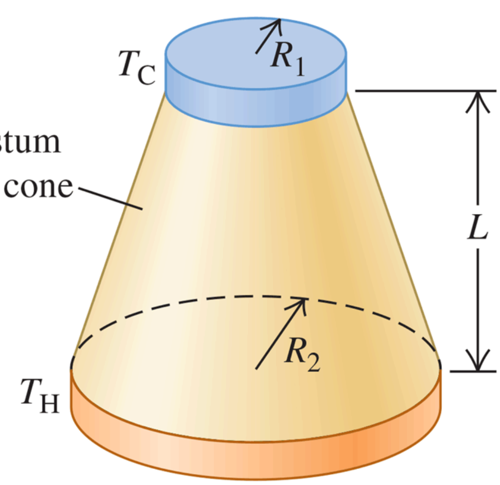
You are confident that your data will allow you to calculate the thermal conductivity k_A of bar A. But this measurement was tedious—you don't want to repeat it for bar B. Instead, you glue the bars together end to end, with adhesive that has very large thermal conductivity, to make a composite bar 80.0 m long. You place the free end of A in thermal contact with the boiling water and the free end of B in thermal contact with the ice–water mixture. The composite bar is thermally insulated. Hours later, you notice that ice remains in the ice–water mixture. Measuring the temperature at the junction of the two bars, you find that it is 62.4 $^\circ\mathrm{C}$. After 10 min you repeat that measurement and get the same temperature, with ice remaining in the ice–water mixture. From your data, calculate the thermal conductivities of bar A and of bar B.

Challenge Problems

17.112 • At a remote arctic research base, liquid water is obtained by melting ice in a propane-fueled conversion tank. Propane has a heat of combustion of 25.6 MJ/L, and 30% of the released energy supplies heat to the tank. Liquid water at $0^\circ\mathrm{C}$ is drawn off the tank at a rate of 500 mL/min, while a corresponding amount of ice at $0^\circ\mathrm{C}$ is continually inserted into the tank from a hopper. How long will an 18 L tank of propane fuel this operation?

17.113 ••• **CALC** A frustum of a cone (Fig. P17.113) has smaller radius R_1, larger radius R_2, and length L and is made from a material with thermal conductivity k. Derive an expression for the conductive heat current through the frustum when the side with radius R_1 is kept at temperature T_H and the side with radius R_2 is kept at temperature T_C. [*Hint:* Parameterize the axis of the frustum using coordinate x. Use Eq. (17.21) for the heat current H through a differential slice of the frustum with length dx, area $A = \pi r^2$ (where r is a function of x), and temperature difference dT. Separate variables and integrate on dT and dx.]

Figure P17.113

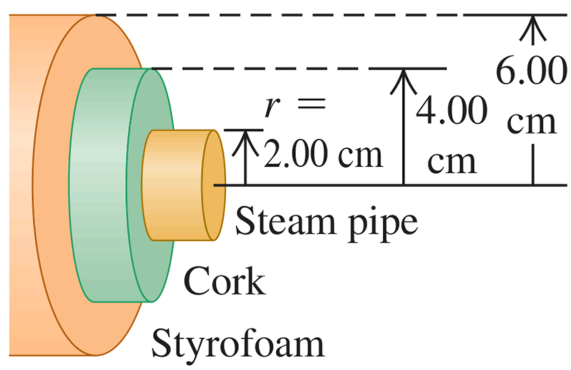


17.114 ••• **BIO** **A Walk in the Sun.** Consider a poor lost soul walking at 5 km/h on a hot day in the desert, wearing only a bathing suit. This person's skin temperature tends to rise due to four mechanisms: (i) energy is generated by metabolic reactions in the body at a rate of 280 W, and almost all of this energy is converted to heat that flows to the skin; (ii) heat is delivered to the skin by convection from the outside air at a rate equal to $k\prime A_{\text{skin}}(T_{\text{air}} - T_{\text{skin}})$, where $k\prime$ is $54\ \text{J/h}\cdot\text{C}^{\circ}\cdot\text{m}^2$, the exposed skin area A_{skin} is $1.5\ \text{m}^2$, the air temperature T_{air} is 47°C, and the skin temperature T_{skin} is 36°C; (iii) the skin absorbs radiant energy from the sun at a rate of $1400\ \text{W/m}^2$; (iv) the skin absorbs radiant energy from the environment, which has temperature 47°C. (a) Calculate the net rate (in watts) at which the person's skin is heated by all four of these mechanisms. Assume that the emissivity of the skin is $e = 1$ and that the skin temperature is initially 36°C. Which mechanism is the most important? (b) At what rate (in L/h) must perspiration evaporate from this person's skin to maintain a constant skin temperature? (The heat of vaporization of water at 36°C is $2.42 \times 10^6\ \text{J/kg}$.) (c) Suppose the person is protected by light-colored clothing ($e \approx 0$) and only $0.45\ \text{m}^2$ of skin is exposed. What rate of perspiration is required now? Discuss the usefulness of the traditional clothing worn by desert peoples.

17.115 ••• A hollow cylinder has length L, inner radius a, and outer radius b, and the temperatures at the inner and outer surfaces are T_2 and T_1. (The cylinder could represent an insulated hot-water pipe.) The thermal conductivity of the material of which the cylinder is made is k. Derive an equation for (a) the total heat current through the walls of the cylinder; (b) the temperature variation inside the cylinder walls. (c) Show that the equation for the total heat current reduces to Eq. (17.21) for linear heat flow when the cylinder wall is very thin. (d) A steam pipe with a radius of 2.00 cm, carrying steam at 140°C, is surrounded by a cylindrical jacket with inner and outer radii 2.00 cm and

4.00 cm and made of a type of cork with thermal conductivity 4.00×10^{-2} W/m · K. This in turn is surrounded by a cylindrical jacket made of a brand of Styrofoam with thermal conductivity 2.70×10^{-2} W/m · K and having inner and outer radii 4.00 cm and 6.00 cm (Fig. P17.115). The outer surface of the Styrofoam has a temperature of 15°C. What is the temperature at a radius of 4.00 cm, where the two insulating layers meet? (e) What is the total rate of transfer of heat out of a 2.00 m length of pipe?

Figure P17.115



MCAT-Style Passage Problems

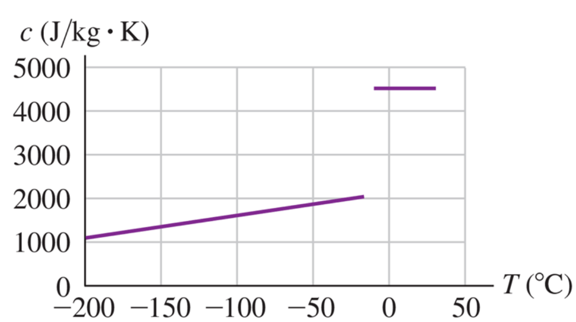
BIO Preserving Cells at Cold Temperatures. In cryopreservation, biological materials are cooled to a very low temperature to slow down chemical reactions that might damage the cells or tissues. It is important to prevent the materials from forming ice crystals during freezing. One method for preventing ice formation is to place the material in a protective solution called a *cryoprotectant*. Stated values of the thermal properties of one cryoprotectant are listed here:

Melting point	$-20^\circ C$
Latent heat of fusion	2.80×10^5 J/kg
Specific heat (liquid)	4.5×10^3 J/kg · K
Specific heat (solid)	2.0×10^3 J/kg · K
Thermal conductivity (liquid)	1.2 W/m · K
Thermal conductivity (solid)	2.5 W/m · K

17.116 You place 35 g of this cryoprotectant at $22^\circ C$ in contact with a cold plate that is maintained at the boiling temperature of liquid nitrogen (77 K). The cryoprotectant is thermally insulated from everything but the cold plate. Use the values in the table to determine how much heat will be transferred from the cryoprotectant as it reaches thermal equilibrium with the cold plate. (a) 1.5×10^4 J; (b) 2.9×10^4 J; (c) 3.4×10^4 J; (d) 4.4×10^4 J.

17.117 Careful measurements show that the specific heat of the solid phase depends on temperature (Fig. P17.117). How will the actual time needed for this cryoprotectant to come to equilibrium with the cold plate compare with the time predicted by using the values in the table? Assume that all values other than the specific heat (solid) are correct. The actual time (a) will be shorter; (b) will be longer; (c) will be the same; (d) depends on the density of the cryoprotectant.

Figure P17.117



17.118 In another experiment, you place a layer of this cryoprotectant between one $10\ \mathrm{cm} \times 10\ \mathrm{cm}$ cold plate maintained at $-40^{\circ}\mathrm{C}$ and a second cold plate of the same size maintained at liquid nitrogen's boiling temperature (77 K). Then you measure the rate of heat transfer. Another lab wants to repeat the experiment but uses cold plates that are $20\ \mathrm{cm} \times 20\ \mathrm{cm}$, with one at $-40^{\circ}\mathrm{C}$ and the other at 77 K. How thick does the layer of cryoprotectant have to be so that the rate of heat transfer by conduction is the same as that when you use the smaller plates? (a) One-quarter the thickness; (b) half the thickness; (c) twice the thickness; (d) four times the thickness.

17.119 To measure the specific heat in the liquid phase of a newly developed cryoprotectant, you place a sample of the new cryoprotectant in contact with a cold plate until the solution's temperature drops from room temperature to its freezing point. Then you measure the heat transferred to the cold plate. If the system isn't sufficiently isolated from its room-temperature surroundings, what will be the effect on the measurement of the specific heat? (a) The measured specific heat will be greater than the actual specific heat; (b) the measured specific heat will be less than the actual specific heat; (c) there will be no effect because the thermal conductivity of the cryoprotectant is so low; (d) there will be no effect on the specific heat, but the temperature of the freezing point will change.

Answers: Temperature and Heat

Chapter Opening Question ?

(iii) The molten iron contains a large amount of energy. An object *has* a temperature but does not *contain* temperature. By "heat" we mean energy that is in transit from one object to another as a result of temperature difference between the objects. Objects do not *contain* heat.

Test Your Understanding

17.1 **(ii)** A liquid-in-tube thermometer actually measures its own temperature. If the thermometer stays in the hot water long enough, it will come to thermal equilibrium with the water and its temperature will be the same as that of the water.

17.2 **(iv)** Both a bimetallic strip and a resistance thermometer measure their own temperature. For this to be equal to the temperature of the object being measured, the thermometer and object must be in thermal equilibrium. A temporal artery thermometer detects the infrared radiation from a person's skin; the detector and skin need not be at the same temperature.

17.3 **(i), (iii), (ii), (v), (iv)** To compare these temperatures, convert them all to the Kelvin scale. For (i), the Kelvin temperature is

for (ii),

— — and

for (iii),

for (iv), and for (v),

17.4 **(ii) and (iii)** Metal 2 must expand more than metal 1 when heated and so must have a larger coefficient of linear expansion From Table 17.1, brass and aluminum have larger values of than copper, but steel does not.

17.5 **(ii), (i), (iv), (iii)** For (i) and (ii), the relevant quantity is the specific heat of the substance, which is the amount of heat required to raise the temperature of 1 *kilogram* of that substance by 1 K From Table 17.3, these values are (i) 138 J for mercury and (ii) 2428 J for ethanol. For (iii) and (iv) we need the molar heat capacity which is the amount of heat required to raise the temperature of 1 *mole* of that substance by Again from Table 17.3, these values are (iii) 27.7 J for mercury and (iv) 111.9 J for ethanol. (The ratio of molar heat capacities is different from the ratio of the specific heats because a mole of mercury and a mole of ethanol have different masses.)

17.6 **(iv)** In time the system goes from point to point in Fig. 17.20. According to this figure, at time (halfway along the horizontal axis from to), the system is at and is still boiling; that is, it is a mixture of liquid and gas. This says that most of the heat added goes into boiling the water.

17.7 **(ii)** When you touch one of the walls, heat flows from your hand to the lower-temperature wall. The more rapidly heat flows from your hand, the colder you'll feel. Equation (17.21) shows that the rate of heat flow is proportional to the thermal conductivity From Table 17.5, copper has a much higher thermal conductivity than steel or concrete and so the copper wall feels the coldest.

Key Example Variation Problems

VP17.4.1 **a.**

b.

VP17.4.2

VP17.4.3 tensile

VP17.4.4 ________ ____

VP17.9.1

VP17.9.2

VP17.9.3 0.181 kg

VP17.9.4 **a.**

b. all of it

VP17.15.1 **a.**

b. 733 W

VP17.15.2 **a.**

b.

VP17.15.3 1.7 times the sun's radius

VP17.15.4

Bridging Problem

(a) 14.26 K

(b)

Chapter 18
Thermal Properties of Matter

?◻ The higher the temperature of a gas, the greater the average kinetic energy of its molecules. How much faster are molecules moving in the air above a frying pan (100˚C) than in the surrounding kitchen air (25˚C) (i) 4 times faster; (ii) twice as fast; (iii) 1.25 times as fast; (iv) 1.12 times as fast; (v) 1.06 times as fast.

Learning Outcomes

In this chapter, you'll learn...

18.1 How to relate the pressure, volume, and temperature of a gas.

18.2 How the interactions between the molecules of a substance determine the properties of the substance.

18.3 How the pressure and temperature of a gas are related to the kinetic energy of its molecules.

18.4 How the heat capacities of a gas reveal whether its molecules are rotating or vibrating.

18.5 How the speeds of molecules are distributed in a gas.

18.6 What determines whether a substance is a gas, a liquid, or a solid.

You'll need to review...

7.4 Potential energy and force.

11.4 Bulk stress.

12.2 Fluids in equilibrium.

13.3 Escape speed.

14.4 Interatomic forces and oscillations.

17.1, 17.2, 17.3, 17.4, 17.5, 17.6 Temperature, heat, thermal expansion, specific heat, molar heat capacity, phase changes.

The kitchen is a great place to learn about how the properties of matter depend on temperature. When you boil water in a tea kettle, the increase in temperature produces steam that whistles out of the spout at high pressure. If you forget to poke holes in a potato before baking it, the high-pressure steam produced inside the potato can cause it to explode messily. Water vapor in the air can condense into liquid on the sides of a glass of ice water; if the glass is just out of the freezer, water vapor will solidify and form frost on its sides.

These examples show the relationships among the large-scale or *macroscopic* properties of a substance, such as pressure, volume, temperature, and mass. But we can also describe a substance by using a *microscopic* perspective. This means investigating small-scale quantities such as the masses, speeds, kinetic energies, and momenta of the individual molecules that make up a substance.

The macroscopic and microscopic descriptions are intimately related. For example, the (microscopic) forces that occur when air molecules strike a solid surface (such as your skin) cause (macroscopic) atmospheric pressure. To produce standard atmospheric pressure of 1.01×10^5 Pa, 10^{32} molecules strike your skin every day with an average speed of over $1700\ \text{km/h}\ (1000\ \text{mi/h})$*!*

In this chapter we'll begin by looking at some macroscopic aspects of matter in general. We'll pay special attention to the *ideal gas,* one of the simplest types of matter to understand. We'll relate the macroscopic properties of an ideal gas to the microscopic behavior of its molecules. We'll also use microscopic ideas to understand the heat capacities of gases and solids. Finally, we'll look at the various phases of matter—gas, liquid, and solid—and the conditions under which each occurs.

18.1 Equations of State

Quantities such as pressure, volume, temperature, and amount of substance describe the conditions, or *state*, in which a particular material exists. (For example, a tank of medical oxygen has a pressure gauge and a label stating the volume within the tank. We can add a thermometer and put the tank on a scale to measure the mass of oxygen.) These quantities are called **state variables.**

The volume V of a substance is usually determined by its pressure p, temperature T, and amount of substance, described by the mass m_{total} or number of moles n. (We are calling the total mass of a substance m_{total} because later in the chapter we'll use m for the mass of one molecule.) Ordinarily, we can't change one of these variables without causing a change in another. When the tank of oxygen gets hotter, the pressure increases. If the tank gets too hot, it explodes.

In a few cases the relationship among p, V, T, and m_{total} (or n) is simple enough that we can express it as an equation called the **equation of state.** When it's too complicated for that, we can use graphs or numerical tables. Even then, the relationship among the variables still exists; we call it an equation of state even when we don't know the actual equation.

Here's a simple (though approximate) equation of state for a solid material. The temperature coefficient of volume expansion β (see Section 17.4) is the fractional volume change $\Delta V/V_0$ per unit temperature change, and the compressibility k (see Section 11.4) is the negative of the fractional volume change $\Delta V/V_0$ per unit pressure change. If a certain amount of material has volume V_0 when the pressure is p_0 and the temperature is T_0, the volume V at slightly differing pressure p and temperature T is approximately

(18.1)

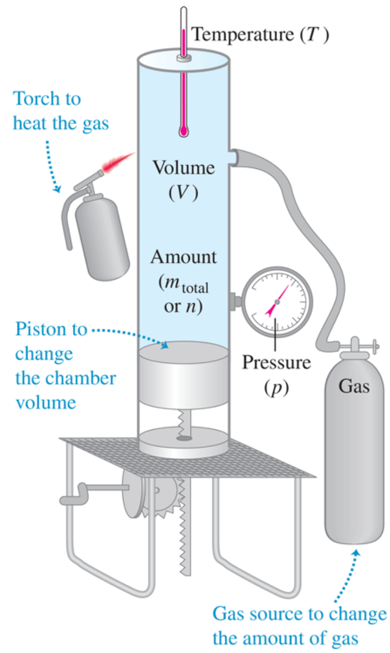
$$V = V_0[1 + \beta (T - T_0) - k (p - p_0)]$$

(There is a negative sign in front of the term $k (p - p_0)$ because an *increase* in pressure causes a *decrease* in volume.)

The Ideal-Gas Equation

Another simple equation of state is the one for an *ideal gas*. Figure 18.1 shows an experimental setup to study the behavior of a gas. The cylinder has a movable piston to vary the volume, the temperature can be varied by heating, and we can pump in any desired amount of gas. We then measure the pressure, volume, temperature, and amount of gas. Note that *pressure* refers both to the force per unit area exerted by the cylinder on the gas and to that exerted by the gas on the cylinder; by Newton's third law, these must be equal.

Figure 18.1



A hypothetical setup for studying the behavior of gases. By heating the gas, varying the volume with a movable piston, and adding more gas, we can control the gas pressure p, volume V, temperature T, and number of moles n.

It is usually easiest to describe the amount of gas in terms of the number of moles n, rather than the mass. (We did this when we defined molar heat capacity in Section 17.5.) The **molar mass** M of a compound (sometimes confusingly called *molecular weight*) is the mass per mole:

(18.2)

Total mass of substance

Number of moles of substance

Molar mass of substance

$$m_{\text{total}} = nM \tag{18.2}$$

Hence if we know the number of moles of gas in the cylinder, we can determine the mass of gas from Eq. (18.2).

Measurements of the behavior of various gases lead to three conclusions:

1. The volume V is proportional to the number of moles n. If we double n, keeping pressure and temperature constant, the volume doubles.
2. The volume varies *inversely* with the absolute pressure p. If we double p while holding the temperature T and number of moles n constant, the gas compresses to one-half of its initial volume. In other words, $pV =$ constant when n and T are constant.
3. The pressure is proportional to the *absolute* temperature T. If we double T, keeping the volume and number of moles constant, the pressure doubles. In other words, $p = (\text{constant}) \times T$ when n and V are constant.

We can combine these three relationships into a single **ideal-gas equation:**

(18.3)

Ideal-gas equation:

$$\underbrace{p}_{\text{Gas pressure}}\underbrace{V}_{\text{Gas volume}} = \overbrace{n}^{\text{Number of moles of gas}}\underbrace{R}_{\text{Gas constant}}\overbrace{T}^{\text{Absolute temperature of gas}} \qquad (18.3)$$

An **ideal gas** is one for which Eq. (18.3) holds precisely for *all* pressures and temperatures. This is an idealized model; it works best at very low pressures and high temperatures, when the gas molecules are far apart and in rapid motion. It is valid within a few percent at moderate pressures (such as a few atmospheres) and at temperatures well above those at which the gas liquefies (Fig. 18.2).

CAUTION **Use absolute pressure and absolute temperature in the ideal-gas equation** The pressure p in Eq. (18.3), the ideal-gas equation, is the *absolute* pressure, not the gauge pressure (see Section 12.2). Furthermore, the temperature T in Eq. (18.3) is the *absolute* (Kelvin) temperature, not the Celsius temperature. If you use gauge pressure or Celsius temperature in Eq. (18.3), you'll get nonsensical results, such as the pressure of a gas being negative below $0^\circ\mathrm{C} = 273.15\,\mathrm{K}$!

Figure 18.2

The ideal-gas equation $pV = nRT$ gives a good description of the air inside an inflated vehicle tire, where the pressure is about 3 atmospheres and the temperature is much too high for nitrogen or oxygen to liquefy. As the tire warms (T increases), the volume V changes only slightly but the pressure p increases.

We might expect that the proportionality constant R in the ideal-gas equation would have different values for different gases, but it turns out to have the same value for *all* gases, at least at sufficiently high temperature and low pressure. It is called the **gas constant** (or *ideal-gas constant*). In SI units, in which the unit of p is Pa ($1\ \mathrm{Pa} = 1\ \mathrm{N/m^2}$) and the unit of V is m^3, the numerical value of R (to nine significant figures) is

$$R = 8.31446262\ \mathrm{J/mol \cdot K}$$

or $R = 8.314\ \mathrm{J/mol \cdot K}$ to four significant figures. Note that the units of pressure times volume are the same as the units of work or energy (for example, $\mathrm{N/m^2}$ times $\mathrm{m^3}$); that's why R has units of energy per mole per unit of absolute temperature. In chemical calculations, volumes are often expressed in liters (L) and pressures in atmospheres (atm). In this system, to four significant figures,

$$R = 0.08206\,\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$$

We can express the ideal-gas equation, Eq. (18.3), in terms of the mass m_{total} of gas, using $m_{\text{total}} = nM$ from Eq. (18.2):

(18.4)

$$pV = \frac{m_{\text{total}}}{M}\,RT$$

From this we can get an expression for the density $\rho = m_{\text{total}}/V$ of the gas:

(18.5)

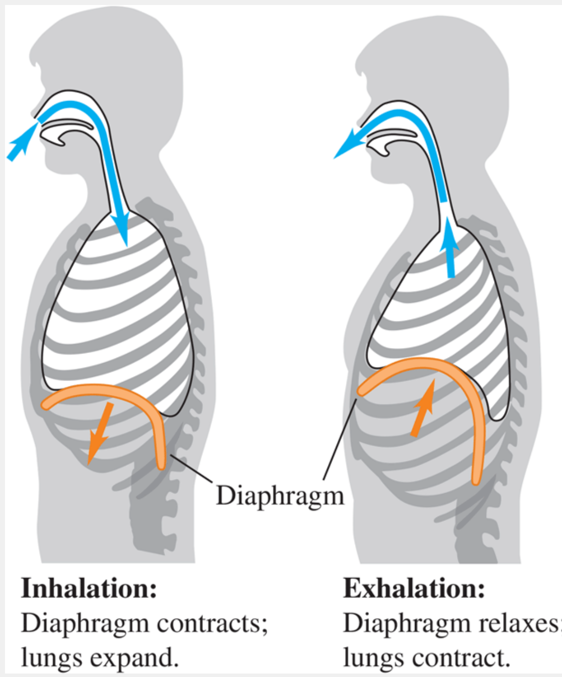
$$\rho = \frac{pM}{RT}$$

CAUTION Density vs. pressure When using Eq. (18.5), be certain that you distinguish between the Greek letter ρ (rho) for density and the letter p for pressure.

BIO Application

Respiration and the Ideal-Gas Equation

To breathe, you rely on the ideal-gas equation $pV = nRT$. Contraction of the dome-shaped diaphragm muscle increases the volume V of the thoracic cavity (which encloses the lungs), decreasing its pressure p. The lowered pressure causes the lungs to expand and fill with air. (The temperature T is kept constant.) When you exhale, the diaphragm relaxes, allowing the lungs to contract and expel the air.



For a *constant mass* (or constant number of moles) of an ideal gas the product nR is constant, so the quantity pV/T is also constant. If the subscripts 1 and 2 refer to any two states of the same mass of a gas, then

(18.6)

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} = \text{constant} \qquad \text{(ideal gas, constant mass)}$$

Notice that you don't need the value of R to use this equation.

We used the proportionality of pressure to absolute temperature in Chapter 17 to define a temperature scale in terms of pressure in a constant-volume gas thermometer. That may make it seem that the pressure–temperature relationship in the ideal-gas equation, Eq. (18.3), is just a result of the way we define temperature. But the ideal-gas equation also tells us what happens when we change the volume or the amount of substance. Also, we'll see in Chapter 20 that the gas-thermometer scale corresponds closely to a temperature scale that does *not* depend on the properties of any particular material. For now, consider Eq. (18.6) as being based on this genuinely material-independent temperature scale.

Problem-Solving Strategy 18.1 Ideal Gases

IDENTIFY *the relevant concepts:* Unless the problem states otherwise, you can use the ideal-gas equation to find quantities related to the state of a gas, such as pressure p, volume V, temperature T, and>or number of moles n.

SET UP *the problem* using the following steps:

1. List the known and unknown quantities. Identify the target variables.
2. If the problem concerns only one state of the system, use Eq. (18.3), $pV = nRT$ (or Eq. (18.5), $\rho = pM/RT$ if the problem involves the density ρ rather than n and V).
3. In problems that concern two states (call them 1 and 2) of the same amount of gas, if all but one of the six quantities p_1, p_2, V_1, V_2, T_1, and T_2 are known, use Eq. (18.6), $p_1V_1/T_1 = p_2V_2/T_2 =$ constant. Otherwise, use Eq. (18.3) or Eq. (18.5).

EXECUTE *the solution* as follows:

1. Use consistent units. (SI units are entirely consistent.) The problem statement may make one system of units more

convenient than others. Make appropriate unit conversions, such as from atmospheres to pascals or from liters to cubic meters.

2. You may have to convert between mass m_{total} and number of moles n, using $m_{\text{total}} = Mn$, where M is the molar mass. If you use Eq. (18.4), you *must* use the same mass units for m_{total} and M. So if M is in grams per mole (the usual units for molar mass), then m_{total} must also be in grams. To use m_{total} in kilograms, you must convert M to kg/mol. For example, the molar mass of oxygen is 32 g/mol or 32×10^{-3} kg/mol.
3. Remember that in the ideal-gas equations, T is always an *absolute* (Kelvin) temperature and p is always an absolute (not gauge) pressure.
4. Solve for the target variables.

EVALUATE *your answer:* Do your results make physical sense? Use benchmarks, such as the result of Example 18.1 below that a mole of an ideal gas at 1 atmosphere pressure occupies a volume of 22.4 liters.

Example 18.1 Volume of an ideal gas at STP

WITH VARIATION PROBLEMS

What is the volume of a container that holds exactly 1 mole of an ideal gas at *standard temperature and pressure* (STP), defined as $T = 0^\circ\text{C} = 273.15\text{ K}$ and $p = 1\text{ atm} = 1.013 \times 10^5\text{ Pa}$?

IDENTIFY and SET UP This problem involves the properties of a single state of an ideal gas, so we use Eq. (18.3). We are given the pressure p, temperature T, and number of moles n; our target variable is the corresponding volume V.

EXECUTE From Eq. (18.3), using R in $\mathrm{J/mol \cdot K}$, we get

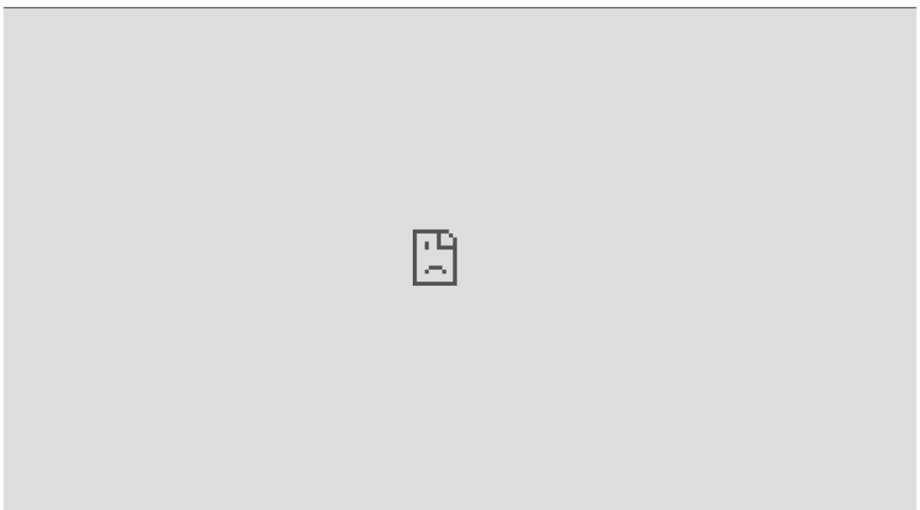
$$V = \frac{nRT}{p} = \frac{(1\ \mathrm{mol})(8.314\ \mathrm{J/mol{\cdot}K})(273.15\ \mathrm{K})}{1.013 \times 10^5\ \mathrm{Pa}} = 0.0224\ \mathrm{m}^3 = 22.4\ \mathrm{L}$$

EVALUATE At STP, 1 mole of an ideal gas occupies 22.4 L. This is the volume of a cube 0.282 m (11.1 in.) on a side, or of a sphere 0.350 m (13.8 in.) in diameter.

KEY CONCEPT

The ideal-gas equation, Eq. (18.3), relates the pressure, volume, absolute temperature, and number of moles for a quantity of an ideal gas.

Video Tutor Solution: **Example 18.1**

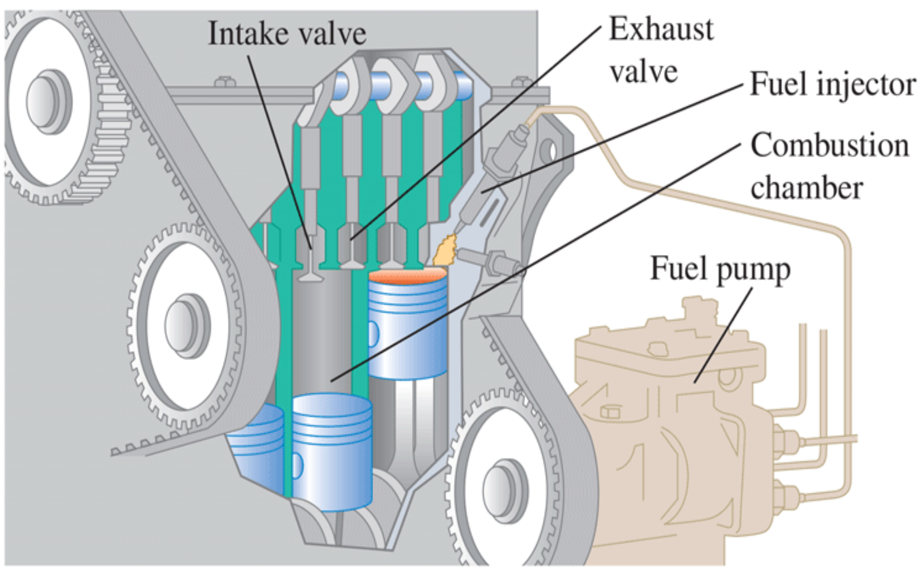


Example 18.2 Compressing gas in an automobile engine

WITH VARIATION PROBLEMS

In an automobile engine, a mixture of air and vaporized gasoline is compressed in the cylinders before being ignited. A typical engine has a compression ratio of 9.00 to 1; that is, the gas in the cylinders is compressed to $\frac{1}{9.00}$ of its original volume (Fig. 18.3). The intake and exhaust valves are closed during the compression, so the quantity of gas is constant. What is the final temperature of the compressed gas if its initial temperature is $27^\circ\mathrm{C}$ and the initial and final pressures are 1.00 atm and 21.7 atm, respectively?

Figure 18.3



Cutaway of an automobile engine. While the air–gasoline mixture is being compressed prior to ignition, both the intake and exhaust valves are in the closed (up) position.

IDENTIFY and SET UP We must compare two states of the same quantity of ideal gas, so we use Eq. (18.6). In the uncompressed state 1, $p_1 = 1.00$ atm and $T_1 = 27^\circ\mathrm{C} = 300$ K. In the compressed state 2, $p_2 = 21.7$ atm. The cylinder volumes are not given, but we have $V_1 = 9.00V_2$. The temperature T_2 of the compressed gas is the target variable.

Execute We solve Eq. (18.6) for T_2 :

$$T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} = (300\ \text{K}) \frac{(21.7\ \text{atm}) V_2}{(1.00\ \text{atm})\,(9.00 V_2)} = 723\ \text{K} = 450^{\circ}\text{C}$$

EVALUATE This is the temperature of the air–gasoline mixture *before* the mixture is ignited; when burning starts, the temperature becomes higher still.

KEY CONCEPT

For a fixed quantity of an ideal gas, the pressure p, volume V, and absolute temperature T may all change, but the quantity pV/T remains constant.

Video Tutor Solution: Example 18.2

Example 18.3 Mass of air in a scuba tank

WITH VARIATION PROBLEMS

An "empty" aluminum scuba tank contains 11.0 L of air at 21°C and 1 atm. When the tank is filled rapidly from a compressor, the air temperature is 42°C and the gauge pressure is 2.10×10^7 Pa. What mass of air was added? (Air is about 78% nitrogen, 21% oxygen, and 1% miscellaneous; its average molar mass is $28.8\ \text{g/mol} = 28.8 \times 10^{-3}\text{kg/mol}$.)

IDENTIFY and SET UP Our target variable is the difference $m_2 - m_1$ between the masses present at the end (state 2) and at the beginning (state 1). We are given the molar mass M of air, so we can use Eq. (18.2) to find the target variable if we know the number of moles present in states 1 and 2. We determine n_1 and n_2 by applying Eq. (18.3) to each state individually.

EXECUTE We convert temperatures to the Kelvin scale by adding 273 and convert the pressure to absolute by adding 1.013×10^5 Pa. The tank's volume is hardly affected by the increased temperature and pressure, so $V_2 = V_1$. From Eq. (18.3), the numbers of moles in the empty tank (n_1) and the full tank (n_2) are as follows:

$$\begin{aligned} n_1 &= \frac{p_1 V_1}{RT_1} = \frac{(1.013 \times 10^5\ \text{Pa})(11.0 \times 10^{-3}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(294\ \text{K})} = 0.46\ \text{mol} \\ n_2 &= \frac{p_2 V_2}{RT_2} = \frac{(2.11 \times 10^7\ \text{Pa})(11.0 \times 10^{-3}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(315\ \text{K})} = 88.6\ \text{mol} \end{aligned}$$

We added $n_2 - n_1 = 88.6\ \text{mol} - 0.46\ \text{mol} = 88.1\ \text{mol}$ to the tank. From Eq. (18.2), the added mass is $M(n_2 - n_1) = (28.8 \times 10^{-3}\ \text{kg/mol})(88.1\ \text{mol}) = 2.54\ \text{kg}$.

EVALUATE The added mass is not insubstantial: You could certainly use a scale to determine whether the tank was empty or full.

KEY CONCEPT

You can determine the mass of a quantity of an ideal gas from its pressure, volume, and absolute temperature, and the molar mass of the gas.

Video Tutor Solution: **Example 18.3**

Example 18.4 Variation of atmospheric pressure with elevation

WITH VARIATION PROBLEMS

Find the variation of atmospheric pressure with elevation in the earth's atmosphere. Assume that at all elevations, $T = 0^\circ\mathrm{C}$ and $g = 9.80\ \mathrm{m/s^2}$.

IDENTIFY and SET UP As the elevation y increases, both the atmospheric pressure p and the density ρ decrease. Hence we have *two* unknown functions of y; to solve for them, we need two independent equations. One is the ideal-gas equation, Eq. (18.5), which is expressed in terms of p and ρ. The other is Eq. (12.4), the relationship that we found in Section 12.2 among p, ρ, and y in a fluid in equilibrium: $dp/dy = -\rho g$. We are told to assume that g and T are the same at all elevations; we also assume that the atmosphere has the same chemical composition, and hence the same molar mass M, at all heights. We combine the two equations and solve for $p(y)$.

EXECUTE We substitute $\rho = pM/RT$ into $dp/dy = -\rho g$, separate variables, and integrate, letting p_1 be the pressure at elevation y_1 and p_2 be the pressure at y_2 :

$$\begin{aligned}
\frac{dp}{dy} &= -\frac{pM}{RT}\,g \\
\int_{p_1}^{p_2} \frac{dp}{p} &= -\frac{Mg}{RT}\int_{y_1}^{y_2} dy \\
\ln\frac{p_2}{p_1} &= -\frac{Mg}{RT}\,(y_2 - y_1) \\
\frac{p_2}{p_1} &= e^{-Mg(y_2-y_1)/RT}
\end{aligned}$$

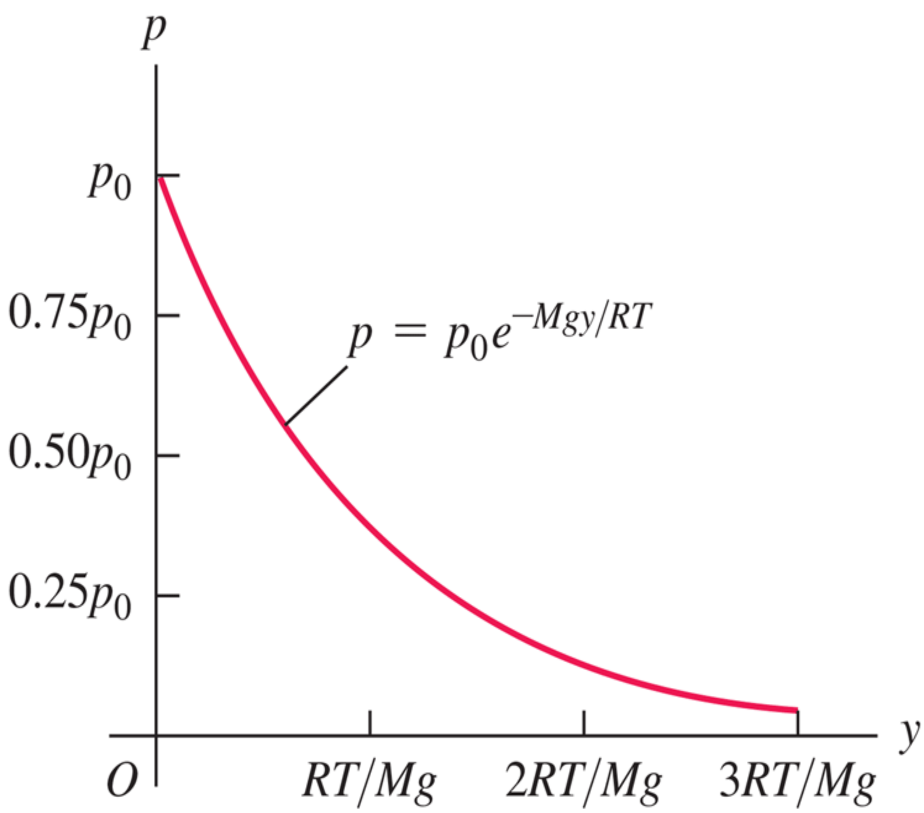
Now let $y_1 = 0$ be at sea level and let the pressure at that point be $p_0 = 1.013 \times 10^5$ Pa. Then the pressure p at any height y is

$$p = p_0\, e^{-Mgy/RT}$$

EVALUATE According to our calculation, the pressure decreases exponentially with elevation. The graph in Fig. 18.4 shows that the slope dp/dy becomes less negative with greater elevation. That result makes sense, since $dp/dy = -\rho g$ and the density also decreases with elevation. At the summit of Mount Everest, where $y = 8848$ m,

$$\begin{aligned}
\frac{Mgy}{RT} &= \frac{(28.8\times10^{-3}\ \mathrm{kg/mol})(9.80\ \mathrm{m/s^2})(8848\ \mathrm{m})}{(8.314\ \mathrm{J/mol\cdot K})(273\ \mathrm{K})} = 1.10 \\
p &= (1.013\times10^5\ \mathrm{Pa})e^{-1.10} = 0.337\times10^5\ \mathrm{Pa} = 0.33\ \mathrm{atm}
\end{aligned}$$

Figure 18.4



The variation of atmospheric pressure p with elevation y, assuming a constant temperature T.

The assumption of constant temperature isn't realistic, and g decreases a little with increasing elevation (see Challenge Problem 18.84). Even so, this example shows why most mountaineers carry oxygen on Mount Everest. It also shows why jet airliners, which typically fly at altitudes of 8000 to 12,000 m, *must* have pressurized cabins for passenger comfort and health.

KEY CONCEPT

The ideal-gas equation can also be expressed as a relationship among the pressure, density, molar mass, and absolute temperature of an ideal gas [Eq. (12.4)].

Video Tutor Solution: **Example 18.4**

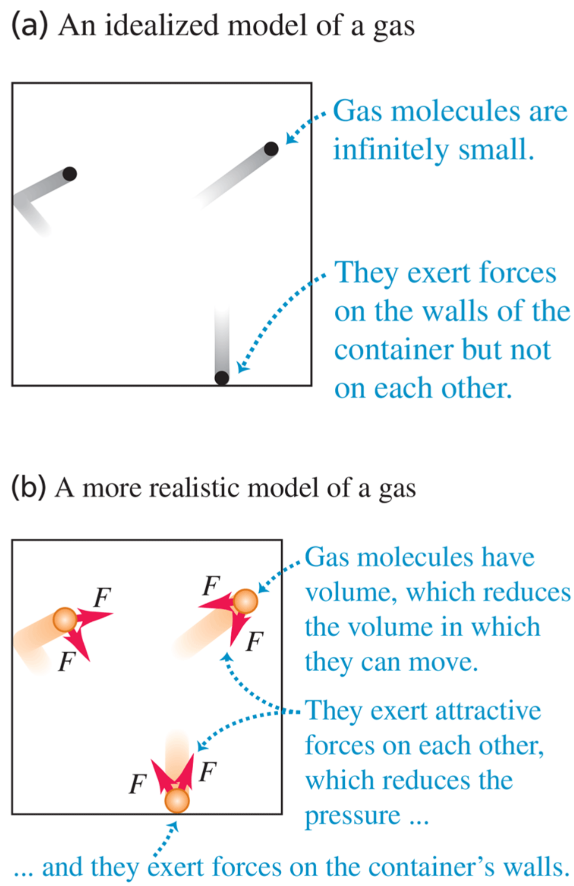
The van der Waals Equation

In Section 18.3 we'll obtain the ideal-gas equation, Eq. (18.3), from a simple molecular model that ignores the volumes of the molecules themselves and the attractive forces between them (Fig. 18.5a). Another equation of state, the **van der Waals equation,** makes approximate corrections for these two omissions (Fig. 18.5b). This equation was developed by the 19th-century Dutch physicist J. D. van der Waals; the interaction between atoms that we discussed in Section 14.4 is named the *van der Waals interaction*. The van der Waals equation is

(18.7)

$$\left(p + \frac{an^2}{V^{1}}\right)(V - nb) = nRT$$

Figure 18.5



A gas as modeled by (a) the ideal-gas equation and (b) the van der Waals equation.

The constants a and b are different for different gases. Roughly speaking, b represents the volume of a mole of molecules; the total volume of the molecules is nb, and the volume remaining in which the molecules can move is $V - nb$. The constant a depends on the attractive intermolecular forces, which reduce the pressure of the gas by *pulling* the molecules together as they *push* on the walls of the container. The decrease in pressure is proportional to the number of molecules per unit volume in a layer near the wall (which are exerting the pressure on the wall) and is also proportional to the number per unit volume in the next layer beyond the wall (which are doing the attracting). Hence the decrease in pressure due to intermolecular forces is proportional to $n^2/V^{\,1}$.

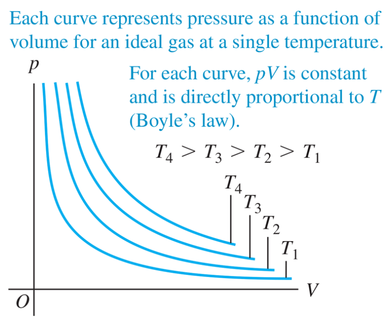
When n/V is small (that is, when the gas is *dilute*), the average distance between molecules is large, the corrections in the van der Waals equation become insignificant, and Eq. (18.7) reduces to the ideal-gas equation. As an example, for carbon dioxide gas (CO_2) the constants in the van der Waals equation are $a = 0.364\ \mathrm{J{\cdot}m^3/mol^2}$ and $b = 4.27 \times 10^{-5}\ \mathrm{m^3/mol}$. We saw in Example 18.1 that 1 mole of an ideal gas at $T = 0^\circ\mathrm{C} = 273.15\ \mathrm{K}$ and $p = 1\ \mathrm{atm}\ =\ 1.013 \times 10^5\ Pa$ occupies a volume $V = 0.0224\ \mathrm{m}^3$; according to Eq. (18.7), 1 mole of CO_2 occupying this volume at this temperature would be at a pressure 532 Pa less than 1 atm, a difference of only 0.5% from the ideal-gas value.

pV-Diagrams

We could in principle represent the p-V-T relationship graphically as a *surface* in a three-dimensional space with coordinates p, V, and T. This representation is useful (see Section 18.6), but ordinary two-dimensional graphs are usually more convenient. One of the most useful of these is a set of graphs of pressure as a function of volume, each for a particular constant temperature. Such a diagram is called a ***pV*-diagram.** Each curve, representing behavior at a specific temperature, is called an **isotherm,** or a pV*-isotherm*.

Figure 18.6 shows pV-isotherms for a constant amount of an ideal gas. Since $p = nRT/V$ from Eq. (18.3), along an isotherm (constant T) the pressure p is inversely proportional to the volume V and the isotherms are hyperbolic curves.

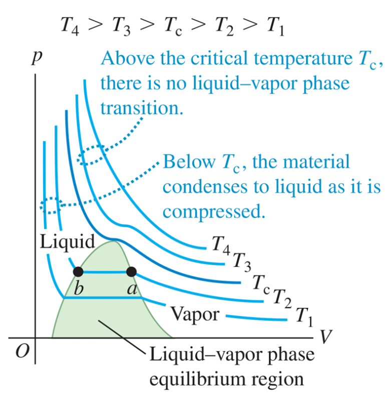
Figure 18.6



Isotherms, or constant- temperature curves, for a constant amount of an ideal gas. The highest temperature is T_4; the lowest is T_1. This is a graphical representation of the ideal-gas equation of state.

Figure 18.7 shows a pV-diagram for a material that *does not* obey the ideal-gas equation. At temperatures below T_c the isotherms develop flat regions in which we can compress the material (that is, reduce the volume V) without increasing the pressure p. Observation shows that the gas is *condensing* from the vapor (gas) to the liquid phase. The flat parts of the isotherms in the shaded area of Fig. 18.7 represent conditions of liquid–vapor *phase equilibrium*. As the volume decreases, more and more material goes from vapor to liquid, but the pressure does not change. (To keep the temperature constant during condensation, we have to remove the heat of vaporization, discussed in Section 17.6.)

Figure 18.7



A pV-diagram for a nonideal gas, showing isotherms for temperatures above and below the critical temperature T_c. The liquid–vapor equilibrium region is shown as a green shaded area. At still lower temperatures the material might undergo phase transitions from liquid to solid or from gas to solid; these are not shown here.

When we compress such a gas at a constant temperature T_2 in Fig. 18.7, it is vapor until point a is reached. Then it begins to liquefy; as the volume decreases further, more material liquefies, and *both* the pressure and the temperature remain constant. At point b, all the material is in the liquid state. After this, any further compression requires a very rapid rise of pressure, because liquids are in general much less compressible than gases. At a lower constant temperature T_1, similar behavior occurs, but the condensation begins at lower pressure and greater volume than at the constant temperature T_2. At temperatures greater than T_c, *no* phase transition occurs as the material is compressed; at the highest temperatures, such as T_4, the curves resemble the ideal-gas curves of Fig. 18.6. We call T_c the *critical temperature* for this material. In Section 18.6 we'll discuss what happens to the phase of the gas above the critical temperature.

We'll use pV-diagrams often in the next two chapters. We'll show that the *area* under a pV-curve (whether or not it is an isotherm) represents the *work* done by the system during a volume change. This work, in turn, is directly related to heat transfer and changes in the *internal energy* of the system.

Test Your Understanding of Section 18.1

Rank the following ideal gases in order from highest to lowest number of moles: (i) Pressure $p = 1$ atm, volume $V = 1$ L, temperature $T = 300$ K; (ii) $p = 2$ atm, $V = 1$ L, T = 300 K; (iii) $p = 1$ atm, $V = 2$ L, $T = 300$ K; (iv) $p = 1$ atm, $V = 1$ L, T = 600 K; (v) $p = 2$ atm, $V = 1$ L, T = 600 K.

18.2 Molecular Properties of Matter

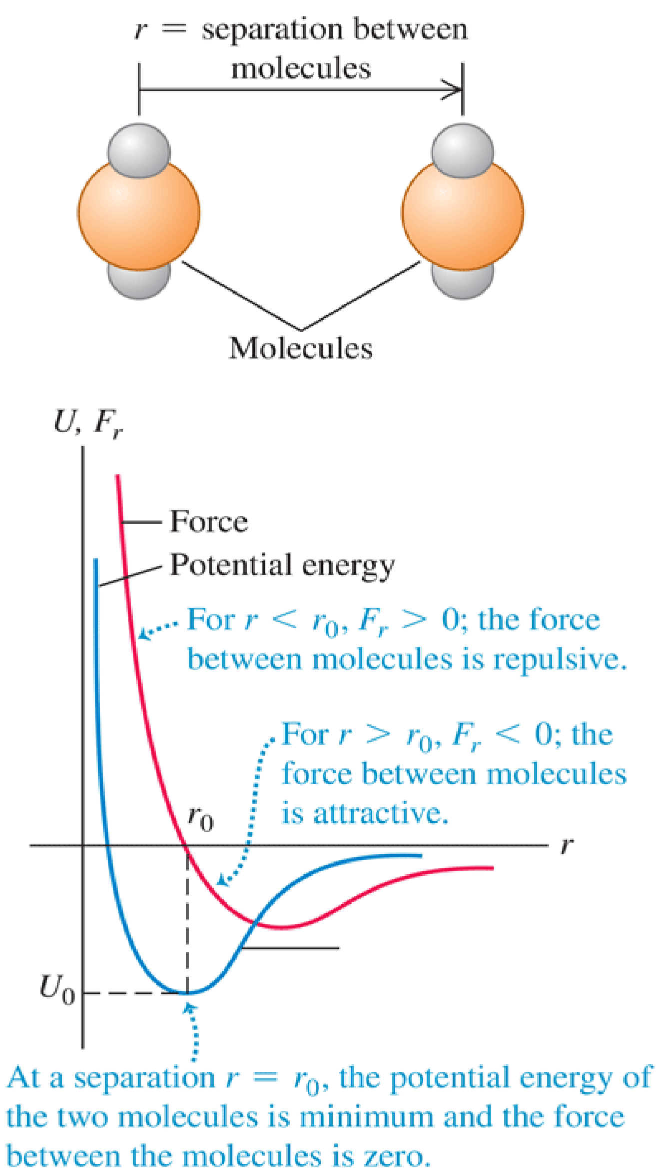
We have studied several properties of matter in bulk, including elasticity, density, surface tension, heat capacities, and equations of state. Now we want to look in more detail at the relationship of bulk behavior to *molecular* structure. We begin with a general discussion of the molecular structure of matter. Then in the next two sections we develop the kinetic-molecular model of an ideal gas, obtaining from this molecular model the equation of state and an expression for heat capacity.

Molecules and Intermolecular Forces

Any specific chemical compound is made up of identical **molecules.** The smallest molecules contain one atom each and are of the order of 10^{-10} m in size; the largest contain many atoms and are at least 10,000 times larger. In gases the molecules move nearly independently; in liquids and solids they are held together by intermolecular forces. These forces arise from interactions among the electrically charged particles that make up the molecules. Gravitational forces between molecules are negligible in comparison with electric forces.

The interaction of two *point* electric charges is described by a force (repulsive for like charges, attractive for unlike charges) with a magnitude proportional to $1/r^2$, where r is the distance between the points. We'll study this relationship, called *Coulomb's law,* in Chapter 21. Molecules are *not* point charges but complex structures containing both positive and negative charge, and their interactions are more complex. The force between molecules in a gas varies with the distance r between molecules somewhat as shown in Fig. 18.8, where a positive F_r corresponds to a repulsive force and a negative F_r to an attractive force. When molecules are far apart, the intermolecular forces are very small and usually attractive. As a gas is compressed and its molecules are brought closer together, the attractive forces increase. The intermolecular force becomes zero at an equilibrium spacing r_0, corresponding roughly to the spacing between molecules in the liquid and solid states. In liquids and solids, relatively large pressures are needed to compress the substance appreciably. This shows that at molecular distances slightly *less* than r_0, the forces become *repulsive* and relatively large.

Figure 18.8



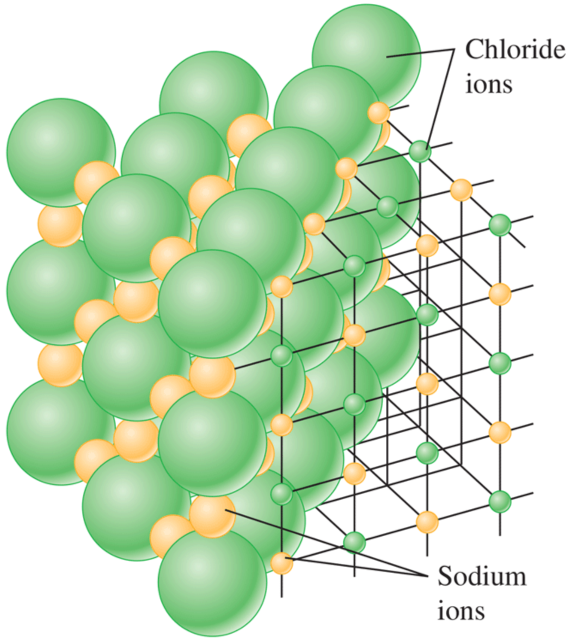
How the force between molecules and their potential energy of interaction depend on their separation r.

Figure 18.8 also shows the potential energy as a function of r. This function has a *minimum* at r_0, where the force is zero. The two curves are related by $F_r(r) = -dU/dr$, as we showed in Section 7.4. Such a potential-energy function is often called a **potential well.** A molecule at rest at a distance r_0 from a second molecule would need an additional energy $|U_0|$, the "depth" of the potential well, to "escape" to an indefinitely large value of r.

Molecules are always in motion; their kinetic energies usually increase with temperature. At very low temperatures the average kinetic energy of a molecule may be much *less* than the depth of the potential well. The molecules then condense into the liquid or solid phase with average intermolecular spacings of about r_0. But at higher temperatures the average kinetic energy becomes larger than the depth $|U_0|$ of the potential well. Molecules can then escape the intermolecular force and become free to move independently, as in the gaseous phase of matter.

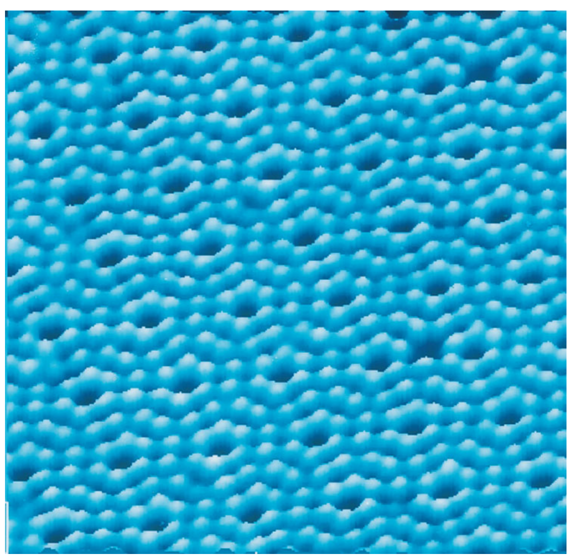
In *solids*, molecules vibrate about more or less fixed points. (See Section 17.4.) In a crystalline solid these points are arranged in a *crystal lattice*. Figure 18.9 shows the cubic crystal structure of sodium chloride, and Fig. 18.10 shows a scanning tunneling microscope image of individual silicon atoms on the surface of a crystal.

Figure 18.9



Schematic representation of the cubic crystal structure of sodium chloride (ordinary salt).

Figure 18.10



A scanning tunneling microscope image of the surface of a silicon crystal. The area shown is only 9.0 nm (9.0×10^{-9} m) across. Each blue “bead” is one silicon atom; these atoms are arranged in a (nearly) perfect array of hexagons.

In a *liquid,* the intermolecular distances are usually only slightly greater than in the solid phase of the same substance, but the molecules have much greater freedom of movement. Liquids show regularity of structure only in the immediate neighborhood of a few molecules.

The molecules of a *gas* are usually widely separated and so have only very small attractive forces. A gas molecule moves in a straight line until it collides with another molecule or with a wall of the container. In molecular terms, an *ideal gas* is a gas whose molecules exert *no* attractive forces on each other (see Fig. 18.5a) and therefore have no *potential* energy.

At low temperatures, most common substances are in the solid phase. As the temperature rises, a substance melts and then vaporizes. From a molecular point of view, these transitions are in the direction of increasing molecular kinetic energy. Thus temperature and molecular kinetic energy are closely related.

Moles and Avogadro's Number

We have used the mole as a measure of quantity of substance. One **mole** of any pure chemical element or compound contains a definite number of molecules, the same number for all elements and compounds. The official SI definition is:

> One mole is the amount of substance that contains $6.02214076 \times 10^{23}$ elementary entities.

In our discussion, the "elementary entities" are molecules. (In a monatomic substance such as carbon or helium, each molecule is a single atom.) Until 2018 the number of elementary entities per mole was defined in terms of the number of atoms in 0.012 kilogram of carbon-12.

The new definition is based on the redefinition of the kilogram (see Section 1.3) and on careful measurements of nearly perfect spheres of pure silicon-28.

The number of molecules in a mole is called **Avogadro's number,** denoted by N_A. The numerical value of N_A is *defined* to be

$$N_A = 6.02214076 \times 10^{23} \text{ molecules/mol} \quad \text{(Avogadro's number)}$$

The *molar mass* M of a compound is the mass of 1 mole. It is equal to the mass m of a single molecule multiplied by Avogadro's number:

(18.8)

$$\underset{\text{Molar mass of a substance}}{M} = \underset{\text{Avogadro's number}}{N_A}\,\underset{\text{Mass of a molecule of substance}}{m} \tag{18.8}$$

When the molecule consists of a single atom, the term *atomic mass* is often used instead of molar mass.

Example 18.5 Atomic and molecular mass

Find the mass of a single hydrogen atom and of a single oxygen molecule.

IDENTIFY and SET UP This problem involves the relationship between the mass of a molecule or atom (our target variable) and the corresponding molar mass M. We use Eq. (18.8) in the form $m = M/N_A$ and the values of the atomic masses from the periodic table of the elements (see Appendix D).

EXECUTE For atomic hydrogen the atomic mass (molar mass) is $M_H = 1.008$ g/mol so the mass m_H of a single hydrogen atom is

$$m_{\mathrm{H}} = \frac{1.008 \text{ g/mol}}{6.022 \times 10^{23} \text{ atoms/mol}} = 1.674 \times 10^{-24} \text{ g}/atom$$

For oxygen the atomic mass is 16.0 g/mol, so for the diatomic (two-atom) oxygen molecule the molar mass is 32.0 g/mol. Then the mass of a single oxygen molecule is

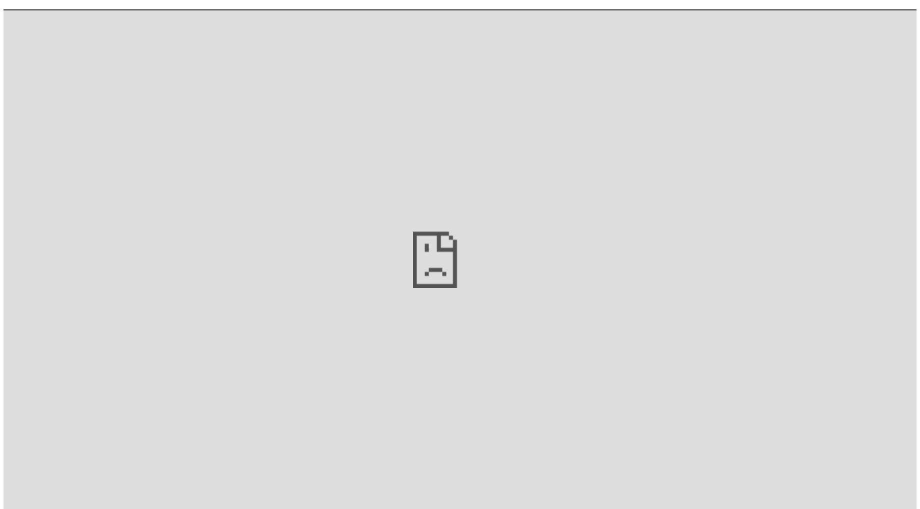
$$m_{\mathrm{O_2}} = \frac{32.0 \text{ g/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} = 53.1 \times 10^{-24} \text{ g/molecule}$$

EVALUATE We note that the values in Appendix D are for the *average* atomic masses of a natural sample of each element. Such a sample may contain several *isotopes* of the element, each with a different atomic mass. Natural samples of hydrogen and oxygen are almost entirely made up of just one isotope.

KEY CONCEPT

To find the mass of a single molecule of a substance, divide the molar mass of that substance by Avogadro’s number (the number of molecules in a mole).

Video Tutor Solution: **Example 18.5**



Test Your Understanding of Section 18.2

Suppose you could adjust the value of r_0 for the molecules of a certain chemical compound (Fig. 18.8) by turning a dial. If you doubled the value of r_0, the density of the solid form of this compound would become (i) twice as great; (ii) four times as great; (iii) eight times as great; (iv) $\frac{1}{2}$ as great; (v) $\frac{1}{4}$ as great; (vi) $\frac{1}{8}$ as great.

18.3 Kinetic-Molecular Model of an Ideal Gas

The goal of any molecular theory of matter is to understand the *macroscopic* properties of matter in terms of its atomic or molecular structure and behavior. Once we have this understanding, we can design materials to have specific desired properties. Theories have led to the development of high-strength steels, semiconductor materials for electronic devices, and countless other materials essential to contemporary technology.

Let's consider a simple molecular model of an ideal gas. This *kinetic-molecular model* represents the gas as a large number of particles bouncing around in a closed container. In this section we use the kinetic-molecular model to understand how the ideal-gas equation of state, Eq. (18.3), is related to Newton's laws. In the following section we use the kinetic-molecular model to predict the molar heat capacity of an ideal gas. We'll go on to elaborate the model to include "particles" that are not points but have a finite size.

Our discussion of the kinetic-molecular model has several steps, and you may need to go over them several times. Don't get discouraged!

Here are the assumptions of our model:

1. A container with volume V contains a very large number N of identical molecules, each with mass m.
2. The molecules behave as point particles that are small compared to the size of the container and to the average distance between molecules.

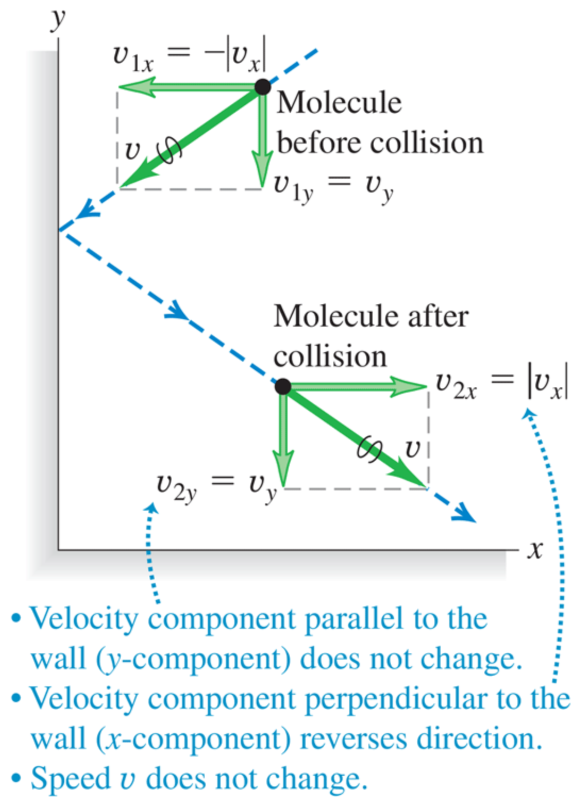
3. The molecules are in constant motion. Each molecule collides occasionally with a wall of the container. These collisions are perfectly elastic.
4. The container walls are rigid and infinitely massive and do not move.

CAUTION Molecules vs. moles Don't confuse N, the number of *molecules* in the gas, with n, the number of *moles*. The number of molecules is equal to the number of moles multiplied by Avogadro's number: $N = nN_{\text{A}}$.

Collisions and Gas Pressure

During collisions the molecules exert *forces* on the walls of the container; this is the origin of the *pressure* that the gas exerts. In a typical collision (Fig. 18.11) the velocity component parallel to the wall is unchanged, and the component perpendicular to the wall reverses direction but does not change in magnitude.

Figure 18.11



Elastic collision of a molecule with an idealized container wall.

We'll first determine the *number* of collisions that occur per unit time for a certain area A of wall. Then we find the total momentum change associated with these collisions and the force needed to cause this momentum change. From this we can determine the pressure (force per unit area) and compare to the ideal-gas equation. We'll find a direct connection between the temperature of the gas and the kinetic energy of its molecules.

To begin, we'll assume that all molecules in the gas have the same *magnitude* of x-velocity, $|v_x|$. Later we'll see that our results don't depend on making this overly simplistic assumption.

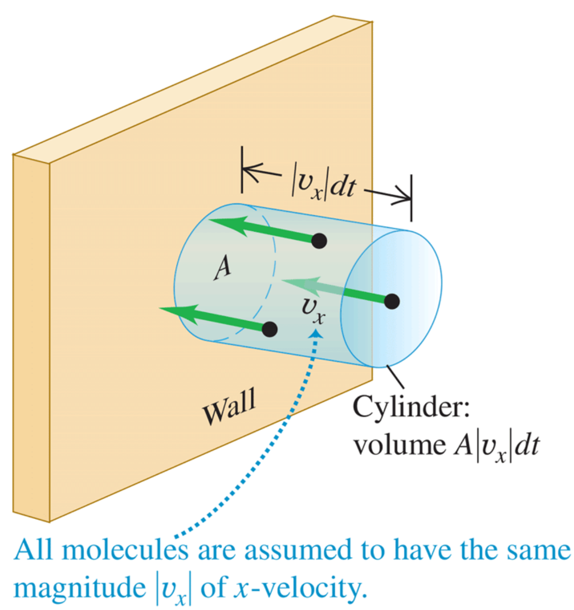
As Fig. 18.11 shows, for each collision the x-component of velocity changes from $-|v_x|$ to $+|v_x|$. So the x-component of momentum p_x

changes from $-m|v_x|$ to $+m|v_x|$, and the *change* in p_x is $m|v_x| - (-m|v_x|) = 2m|v_x|$.

If a molecule is going to collide with a given wall area A during a small time interval dt, then at the beginning of dt it must be within a distance $|v_x|\,dt$ from the wall (Fig. 18.12) and it must be headed toward the wall. So the number of molecules that collide with A during dt is equal to the number of molecules within a cylinder with base area A and length $|v_x|\,dt$ that have their x-velocity aimed toward the wall. The volume of such a cylinder is $A|v_x|\,dt$. Assuming that the number of molecules per unit volume (N/V) is uniform, the *number* of molecules in this cylinder is $(N/V)(A|v_x|\,dt)$. On the average, half of these molecules are moving toward the wall and half are moving away from it. So the number of collisions with A during dt is

$$\frac{1}{2}\left(\frac{N}{V}\right)(A|v_x|\,dt)$$

Figure 18.12



For a molecule to strike the wall in area A during a time interval dt, the molecule must be headed for the wall and be within the shaded cylinder of length $|v_x|\ dt$ at the beginning of the interval.

For the system of all molecules in the gas, the total momentum change dP_x during dt is the *number* of collisions multiplied by $2m\,|v_x|$:

(18.9)

$$dP_x = \frac{1}{2}\left(\frac{N}{V}\right)(A\,|v_x|\ dt)(2m\,|v_x|) = \frac{NAmv_x^2\ dt}{V}$$

(We are using capital P for total momentum and lowercase p for pressure. Be careful!) We wrote v_x^2 rather than $|v_x|^2$ in the final expression because the square of the absolute value of a number is equal to the square of that number. The *rate* of change of momentum component P_x is

(18.10)

$$\frac{dP_x}{dt} = \frac{NAmv_x^2}{V}$$

According to Newton's second law, this rate of change of momentum equals the force exerted by the wall area A on the gas molecules. From Newton's *third* law this is equal and opposite to the force exerted *on* the wall *by* the molecules. Pressure p is the magnitude of the force exerted on the wall per unit area:

(18.11)

$$p = \frac{F}{A} = \frac{Nmv_x^2}{V}$$

The pressure exerted by the gas depends on the number of molecules per volume (N/V), the mass m per molecule, and the speed of the molecules.

Pressure and Molecular Kinetic Energies

We mentioned that $|v_x|$ is really *not* the same for all the molecules. But we could have sorted the molecules into groups having the same $|v_x|$ within each group, then added up the resulting contributions to the pressure. The net effect of all this is just to replace v_x^2 in Eq. (18.11) by the *average* value of v_x^2, which we denote by $\left(v_x^2\right)_{\text{av}}$. We can relate $\left(v_x^2\right)_{\text{av}}$. to the *speeds* of the molecules. The speed v of a molecule is related to the velocity components v_x, v_y, and v_z by

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

We can average this relationship over all molecules:

$$\left(v^2\right)_{\text{av}} = (v_x^2)_{\text{av}} + (v_y^2)_{\text{av}} + (v_z^2)_{\text{av}}$$

But there is no real difference in our model between the x-, y-, and z-directions. (Molecular speeds are very fast in a typical gas, so the effects of gravity are negligibly small.) It follows that $\left(v_x^2\right)_{\text{av}}$, $\left(v_y^2\right)_{\text{av}}$, and $\left(v_z^2\right)_{\text{av}}$ must all be *equal*. Hence $\left(v^2\right)_{\text{av}}$ is equal to $3\left(v_x^2\right)_{\text{av}}$ and

$$\left(v_x^2\right)_{\text{av}} = \frac{1}{3}\left(v^2\right)_{\text{av}}$$

so Eq. (18.11) becomes

(18.12)

$$pV = \frac{1}{3}Nm\left(v^2\right)_{\text{av}} = \frac{2}{3}N\left[\frac{1}{2}m\left(v^2\right)_{\text{av}}\right]$$

We notice that $\frac{1}{2}m\left(v^2\right)_{\text{av}}$ is the average translational kinetic energy of a single molecule. The product of this and the total number of molecules N equals the total random kinetic energy K_{tr} of translational motion of all the molecules. (The notation K_{tr} reminds us that this is the energy of *translational* motion. There may also be energies associated with molecular rotation and vibration.) The product pV equals two-thirds of the total translational kinetic energy:

(18.13)

$$pV = \frac{2}{3}\,K_{tr}$$

Now compare Eq. (18.13) to the ideal-gas equation $pV = nRT$, Eq. (18.3), which is based on experimental studies of gas behavior. For the two equations to agree, we must have

(18.14)

Number of moles of gas

Average translational kinetic energy of an ideal gas

$$K_{\text{tr}} = \frac{3}{2}nRT \qquad (18.14)$$

Absolute temperature of gas

Gas constant

So K_{tr} is *directly proportional* to the absolute temperature T (Fig. 18.13).

Figure 18.13

Summer air (top) is warmer than winter air (bottom); that is, the average translational kinetic energy of air molecules is greater in summer.

The average translational kinetic energy of a single molecule is the total translational kinetic energy K_{tr} of all molecules divided by the number of molecules, N:

$$\frac{K_{\mathrm{tr}}}{N} = \frac{1}{2}\, m\left(v^2\right)_{\mathrm{av}} = \frac{3nRT}{2N}$$

Also, the total number of molecules N is the number of moles n multiplied by Avogadro's number N_A, so $N = nN_A$ and $n/N = 1/N_A$. Thus the above equation becomes

(18.15)

$$\frac{K_{tr}}{N} = \frac{1}{2}\, m\left(v^2\right)_{av} = \frac{3}{2}\left(\frac{R}{N_A}\right)T$$

The ratio R/N_A is called the **Boltzmann constant,** k:

$$k = \frac{R}{N_A} = \frac{8.314\ \mathrm{J/mol{\cdot}K}}{6.022 \times 10^{23}\ \mathrm{molecules/mol}} = 1.381 \times 10^{-23}\ \mathrm{J/molecule \cdot K}$$

(The exact numerical value of k is *defined* to be $1.380649 \times 10^{-23}\ \mathrm{J/molecule \cdot K}$.) In terms of k we can rewrite Eq. (18.15) as

(18.16)

Average translational kinetic energy of a gas molecule:

$$\frac{1}{2}\overset{\text{Mass of a molecule}}{m}\underbrace{\left(v^2\right)_{av}}_{\text{Average value of the square of molecular speeds}} = \frac{3}{2}\underset{\text{Boltzmann constant}}{k}\overset{\text{Absolute temperature of gas}}{T} \qquad (18.16)$$

CAUTION **The Boltzmann constant, Avogadro's number, the gas constant, and the Kelvin scale** In our discussion we began with the gas constant R and Avogadro's number N_A and used them to define the Boltzmann constant k. In fact, in the International System (SI; see Section 1.3), both N_A and k are fundamental quantities with defined values. The value of the gas constant is the product of these two defined values:

$R = N_A k$. Furthermore, the value of k provides the official definition of the Kelvin scale: If the temperature increases by one kelvin, the quantity kT increases by exactly 1.380649×10^{-23} J.

This shows that the average translational kinetic energy *per molecule* depends only on the temperature, not on the pressure, volume, or kind of molecule. We can obtain the average translational kinetic energy *per mole* by multiplying Eq. (18.16) by Avogadro's number and using the relationship $M = N_A m$:

(18.17)

$$N_A \tfrac{1}{2} m\left(v^2\right)_{av} = \tfrac{1}{2} M\left(v^2\right)_{av} = \tfrac{3}{2} RT \quad \text{(average translational kinetic energy per mole of gas)}$$

The translational kinetic energy of a mole of an ideal gas depends only on T.

Finally, it can be helpful to rewrite the ideal-gas equation on a "per-molecule" basis. We use $N = N_A n$ and $R = N_A k$ to obtain this alternative form:

(18.18)

$$pV = NkT$$

This shows that we can think of the Boltzmann constant k as a gas constant on a "per-molecule" basis instead of the usual "per-mole" basis for R.

Molecular Speeds

? From Eqs. (18.16) and (18.17) we can obtain expressions for the square root of $\left(v^2\right)_{av}$, called the **root-mean-square speed** (or **rms speed**)

v_{rms}:

(18.19)

$$\underbrace{v_{rms}}_{\text{Root-mean-square speed of a gas molecule}} = \sqrt{\underbrace{(v^2)_{av}}_{\text{Average value of the square of molecular speeds}}} = \sqrt{\frac{3\overbrace{k}^{\text{Boltzmann constant}}T}{\underbrace{m}_{\text{Mass of a molecule}}}} = \sqrt{\frac{3\underbrace{R}_{\text{Gas constant}}\overbrace{T}^{\text{Absolute temperature of gas}}}{\underbrace{M}_{\text{Molar mass}}}} \qquad (18.19)$$

It might seem more natural to give the *average* speed rather than v_{rms}, but v_{rms} follows more directly from Eqs. (18.16) and (18.17). To compute the rms speed, we square each molecular speed, add, divide by the number of molecules, and take the square root; v_{rms} is the *root* of the *mean* of the *squares*.

Equations (18.16) and (18.19) show that at a given temperature T, gas molecules of different mass m have the same average kinetic energy but different root-mean-square speeds. On average, the nitrogen molecules ($M = 28$ g/mol) in the air around you are moving faster than are the oxygen molecules ($M = 32$ g/mol). Hydrogen molecules ($M = 2$ g/mol) are fastest of all; this is why there is hardly any hydrogen in the earth's atmosphere, despite its being the most common element in the universe (Fig. 18.14). A sizable fraction of any H_2 molecules in the atmosphere would have speeds greater than the earth's escape speed of 1.12×10^4 m/s (calculated in Example 13.5 in Section 13.3) and would escape into space. The heavier, slower-moving gases cannot escape so easily, which is why they predominate in our atmosphere.

Figure 18.14

While hydrogen is a desirable fuel for vehicles, it is only a trace constituent of our atmosphere (0.00005% by volume). Hence hydrogen fuel has to be generated by electrolysis of water, which is itself an energy-intensive process.

The assumption that individual molecules undergo perfectly elastic collisions with the container wall is a little too simple. In most cases, molecules actually adhere to the wall for a short time and then leave again with speeds that are characteristic of the temperature *of the wall.* However, the gas and the wall are ordinarily in thermal equilibrium and have the same temperature. So there is no net energy transfer between gas and wall, and our conclusions remain valid.

Problem-Solving Strategy 18.2 Kinetic-Molecular Theory

IDENTIFY *the relevant concepts:* Use the results of the kinetic-molecular model to relate the macroscopic properties of a gas, such as temperature and pressure, to microscopic properties, such as molecular speeds.

SET UP *the problem* using the following steps:

1. List knowns and unknowns; identify the target variables.

2. Choose appropriate equation(s) from among Eqs. (18.14), (18.16), and (18.19).

EXECUTE *the solution* as follows: Maintain consistency in units.

1. The usual units for molar mass M are grams per mole; these units are often omitted in tables. In equations such as Eq. (18.19), when you use SI units you must express M in kilograms per mole. For example, for oxygen

$$M_{O_2} = 32\,\mathrm{g/mol} = 32 \times 10^{-3}\,\mathrm{kg/mol}.$$

2. Are you working on a "per-molecule" basis (with m, N, and k) or a "per-mole" basis (with M, n, and R)? To check units, think of N as having units of "molecules"; then m has units of mass per molecule, and k has units of joules per molecule per kelvin. Similarly, n has units of moles; then M has units of mass per mole and R has units of joules per mole per kelvin.
3. Remember that T is always *absolute* (Kelvin) temperature.

EVALUATE *your answer:* Are your answers reasonable? Here's a benchmark: Typical molecular speeds at room temperature are several hundred meters per second.

Example 18.6 Molecular kinetic energy and *urms*

WITH VARIATION PROBLEMS

(a) What is the average translational kinetic energy of an ideal-gas molecule at 27°C? (b) What is the total random translational kinetic energy of the molecules in 1 mole of this gas? (c) What is the rms speed of oxygen molecules at this temperature?

IDENTIFY and SET UP This problem involves the translational kinetic energy of an ideal gas on a per-molecule and per-mole basis, as well as the root-mean-square molecular speed v_{rms}. We are given $T = 27^{\circ}\text{C} = 300\text{ K}$ and $n = 1\text{ mol}$; we use the molecular mass m for oxygen. We use Eq. (18.16) to determine the average kinetic energy of a molecule, Eq. (18.14) to find the total molecular kinetic energy K_{tr} of 1 mole, and Eq. (18.19) to find v_{rms}.

EXECUTE (a) From Eq. (18.16),

$$\begin{aligned}\tfrac{1}{2}\,m\left(v^2\right)_{\text{av}} = \tfrac{3}{2}\,kT &= \tfrac{3}{2}\,(1.38\times10^{-23}\text{ J/K})(300\text{ K})\\ &= 6.21\times10^{-21}\text{ J}\end{aligned}$$

(b) From Eq. (18.14), the kinetic energy of one mole is

$$K_{\text{tr}} = \tfrac{3}{2}\,nRT = \tfrac{3}{2}\,(1\text{ mol})(8.314\text{ J/mol·K})(300\text{ K}) = 3740\text{ J}$$

(c) We found the mass per molecule m and molar mass M of molecular oxygen in Example 18.5. Using Eq. (18.19), we can calculate v_{rms} in two ways:

$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(300\text{ K})}{5.31\times10^{-26}\text{ kg}}}\\ &= 484\text{ m/s} = 1740\text{ km/h} = 1080\text{ mi/h}\\ v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{32.0\times10^{-3}\text{ kg/mol}}} = 484\text{ m/s}\end{aligned}$$

EVALUATE The answer in part (a) does not depend on the mass of the molecule. We can check our result in part (b) by noting that the translational kinetic energy per mole must be equal to the product of the average translational kinetic energy per molecule from part (a) and Avogadro's number

$N_{\text{A}}:\ K_{\text{tr}} = \left(6.21\times10^{-21}\text{ J/molecule}\right)\times(6.022\times10^{23}\text{ molecules}) = 3740\text{ J}.$

KEY CONCEPT

The average translational kinetic energy of a molecule in an ideal gas at absolute temperature T is $\frac{3}{2}\,kT$, where k is the Boltzmann constant, no matter what the mass of the molecules. However, the root-mean-square speed v_{rms} of molecules (that is, the square root of the average value of v^2) in an ideal gas does depend on the mass per molecule m: $v_{\mathrm{rms}} = \sqrt{\dfrac{3kT}{m}}$.

Video Tutor Solution: **Example 18.6**

Example 18.7 Calculating rms and average speeds

WITH VARIATION PROBLEMS

Five gas molecules have speeds of 500, 600, 700, 800, and 900 m/s. What is the rms speed? What is the *average* speed?

IDENTIFY and SET UP We use the definitions of the root mean square and the average of a collection of numbers. To find v_{rms}, we square each speed, find the average (mean) of the squares, and take the square root of the result. We find v_{av} as usual.

EXECUTE The average value of v^2 and the resulting v_{rms} for the five molecules are

$$\left(v^2\right)_{\mathrm{av}} = \frac{500^2 + 600^2 + 700^2 + 800^2 + 900^2}{5}\ \mathrm{m}^2/s^2$$

$$= 5.10 \times 10^5\ \mathrm{m}^2/\mathrm{s}^2$$

$$v_{\mathrm{rms}} = \sqrt{\left(v^2\right)_{\mathrm{av}}} = 714\ \mathrm{m/s}$$

The average speed v_{av} is

$$v_{\mathrm{av}} = \frac{500 + 600 + 700 + 800 + 900}{5}\ \mathrm{m/s} = 700\ \mathrm{m/s}$$

EVALUATE In general v_{rms} and v_{av} are *not* the same. Roughly speaking, v_{rms} gives greater weight to the higher speeds than does v_{av}.

KEY CONCEPT

In general, for any collection of numbers, the root-mean-square value is *not* equal to the average value. For molecules in an ideal gas, the root-mean-square speed is always greater than the average speed.

Video Tutor Solution: **Example 18.7**

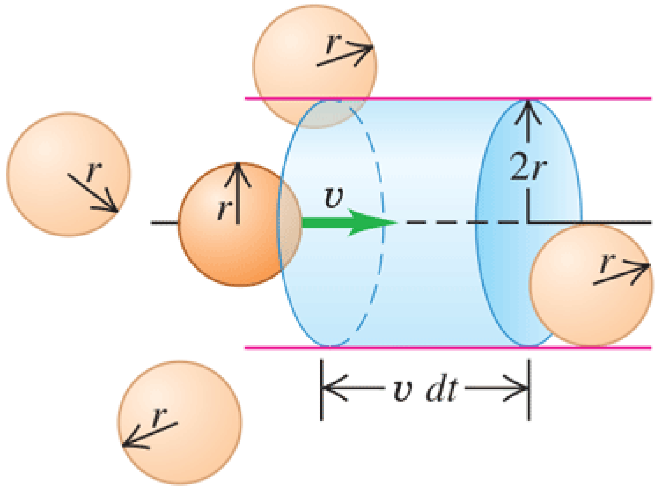
Collisions Between Molecules

We have ignored the possibility that two gas molecules might collide. If they are really points, they *never* collide. But consider a more realistic model in which the molecules are rigid spheres with radius r. How often do they collide with other molecules? How far do they travel, on average, between collisions? We can get approximate answers from the following rather primitive model.

Consider N spherical molecules with radius r in a volume V. Suppose only one molecule is moving. When it collides with another molecule, the distance between centers is $2r$. Suppose we draw a cylinder with radius 2 r, with its axis parallel to the velocity of the molecule (Fig. 18.15). The moving molecule collides with any other molecule whose center is inside this cylinder. In a short time dt a molecule with speed v travels a distance $v\ dt$; during this time it collides with any molecule that is in the cylindrical volume of radius $2r$ and length $v$dt. The volume of the cylinder is $4\pi r^2 v\ dt$. There are N/V molecules per unit volume, so the number dN with centers in this cylinder is

$$dN = 4\pi r^2 v\ dtN/V$$

Figure 18.15



In a time dt a molecule with radius r will collide with any other molecule within a cylindrical volume of radius $2r$ and length $v$dt.

Thus the number of collisions *per unit time* is

$$\frac{dN}{dt} = \frac{4\pi r^2 v \mathrm{N}}{V}$$

This result assumes that only one molecule is moving. It turns out that collisions are more frequent when all the molecules move at once, and the above equation has to be multiplied by a factor of $\sqrt{2}$:

$$\frac{dN}{dt} = \frac{4\pi\sqrt{2}\,r^2 v \mathrm{N}}{V}$$

The average time t_{mean} between collisions, called the *mean free time*, is the reciprocal of this expression:

(18.20)

$$t_{\text{mean}} = \frac{V}{4\pi\sqrt{2}\,r^2 v \mathrm{N}}$$

The average distance traveled between collisions is called the **mean free path.** In our model, this is just the molecule's speed v multiplied by t_{mean} :

(18.21)

$$\underset{\text{a gas molecule}}{\overset{\text{Mean free path of}}{\lambda}} = \overset{\text{Speed of molecule}}{v}\,\underset{\text{between collisions}}{\underset{\text{Mean free time}}{t_{\text{mean}}}} = \frac{\overset{\text{Volume of gas}}{V}}{4\pi\sqrt{2}\,\underset{\text{molecule}}{\underset{\text{Radius of a}}{r^2}}\,\underset{\text{molecules in gas}}{\underset{\text{Number of}}{N}}} \qquad (18.21)$$

The mean free path λ (the Greek letter lambda) is inversely proportional to the number of molecules per unit volume (N/V) and inversely proportional to the cross-sectional area πr^1 of a molecule; the more molecules there are and the larger the molecule, the shorter the mean distance between collisions (Fig. 18.16). Note that the mean free path *does not* depend on the speed of the molecule.

Figure 18.16

If you try to walk through a crowd, your mean free path—the distance you can travel on average without running into another person—depends on how large the people are and how closely they are spaced.

We can express Eq. (18.21) in terms of macroscopic properties of the gas, using the ideal-gas equation in the form of Eq. (18.18), $pV = NkT$. We find

(18.22)

$$\lambda = \frac{kT}{4\pi\sqrt{2}\,r^2 p}$$

If the temperature is increased at constant pressure, the gas expands, the average distance between molecules increases, and λ increases. If the pressure is increased at constant temperature, the gas compresses and λ decreases.

Example 18.8 Calculating mean free path

WITH VARIATION PROBLEMS

(a) Estimate the mean free path of a molecule of air at $27^\circ\mathrm{C}$ and 1 atm. Model the molecules as spheres with radius $r = 2.0 \times 10^{-10}$ m. (b) Estimate the mean free time of an oxygen molecule with $v = v_{\mathrm{rms}}$ at $27^\circ\mathrm{C}$ and 1 atm.

IDENTIFY and SET UP This problem uses the concepts of mean free path and mean free time (our target variables). We use Eq. (18.22) to determine the mean free path λ. We then use $\lambda = vt_{\mathrm{mean}}$ in Eq. (18.21), with $v = v_{\mathrm{rms}}$, to find the mean free time t_{mean}.

EXECUTE (a) From Eq. (18.22),

$$\begin{aligned}\lambda &= \frac{kT}{4\pi\sqrt{2}r^2p} = \frac{(1.38\times10^{-23}\ \mathrm{J/K})(300\ \mathrm{K})}{4\pi\sqrt{2}(2.0\times10^{-10}\ \mathrm{m})^2(1.01\times10^5\ \mathrm{Pa})}\\ &= 5.8\times10^{-8}\ \mathrm{m}\end{aligned}$$

(b) From Example 18.6, for oxygen at 27°C the root-mean-square speed is $v_{\text{rms}} = 484\ \text{m/s}$, so the mean free time for a molecule with this speed is

$$t_{\text{mean}} = \frac{\lambda}{v} = \frac{5.8 \times 10^{-8}\ \text{m}}{484\ \text{m/s}} = 1.2 \times 10^{-10}\ \text{s}$$

This molecule undergoes about 10^{10} collisions per second!

EVALUATE Note that from Eqs. (18.21) and (18.22) the mean free *path* doesn't depend on the molecule's speed, but the mean free *time* does. Slower molecules have a longer average time interval t_{mean} between collisions than do fast ones, but the average *distance* λ between collisions is the same no matter what the molecule's speed. Our answer to part (a) says that the molecule doesn't go far between collisions, but the mean free path is still several hundred times the molecular radius r.

KEY CONCEPT

The mean free path and mean free time are, respectively, the average distance and average time that a molecule in a gas travels between collisions with other molecules. Both depend on the temperature and pressure of the gas and the radius of a molecule. The mean free time also depends on the speed of the molecule; the mean free path does not.

Video Tutor Solution: **Example 18.8**

Test Your Understanding of Section 18.3

Rank the following gases in order from (a) highest to lowest rms speed of molecules and (b) highest to lowest average translational kinetic energy of a molecule: (i) oxygen ($M = 32.0\ \mathrm{g/mol}$) at 300 K; (ii) nitrogen ($M = 28.0\ \mathrm{g/mol}$) at 300 K; (iii) oxygen at 330 K; (iv) nitrogen at 330 K.

18.4 Heat Capacities

When we introduced the concept of heat capacity in Section 17.5, we talked about ways to *measure* the specific heat or molar heat capacity of a particular material. Now we'll see how to *predict* these on theoretical grounds.

Heat Capacities of Gases

The basis of our analysis is that heat is *energy* in transit. When we add heat to a substance, we are increasing its molecular energy. We'll assume that the volume of the gas remains constant; if we were to let the gas expand, it would do work by pushing on the moving walls of its container, and this additional energy transfer would have to be included in our calculations. We'll return to this more general case in Chapter 19. For now we are concerned with C_V, the molar heat capacity *at constant volume.*

In the simple kinetic-molecular model of Section 18.3 the molecular energy consists of only the translational kinetic energy K_{tr} of the pointlike molecules. This energy is directly proportional to the absolute temperature T, as shown by Eq. (18.14), $K_{\mathrm{tr}} = \frac{3}{2}\,nRT$. When the temperature changes by a small amount dT, the corresponding change in kinetic energy is

(18.23)

$$dK_{\mathrm{tr}} = \frac{3}{2}\,nR\,dT$$

From the definition of molar heat capacity at constant volume, C_V (see Section 17.5), we also have

(18.24)

$$dQ = nC_V\, dT$$

where dQ is the heat input needed for a temperature change dT. Now if K_{tr} represents the total molecular energy, as we have assumed, then dQ and dK_{tr} must be *equal* (Fig. 18.17). From Eqs. (18.23) and (18.24), this says

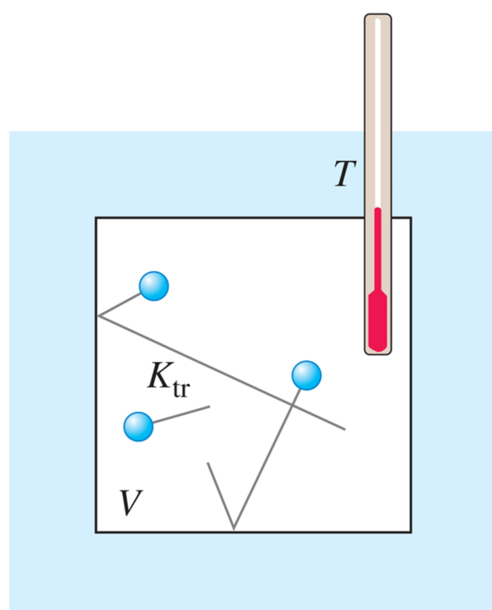
$$nC_V\, dT = \tfrac{3}{2}\, nR\, dT$$

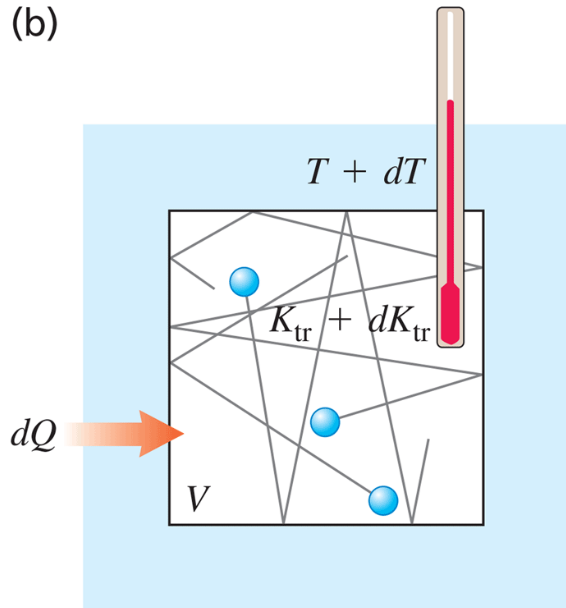
(18.25)

$$\underset{\text{Molar heat capacity at constant volume, ideal gas of point particles}}{C_V} = \frac{3}{2}\underset{\text{Gas constant}}{R} \qquad (18.25)$$

Figure 18.17

(a)





(a) A fixed volume V of a monatomic ideal gas. (b) When an amount of heat dQ is added to the gas, the total translational kinetic energy increases by $dK_{tr} = dQ$ and the temperature increases by $dT = dQ/nC_V$.

This surprisingly simple result says that the molar heat capacity at constant volume is $\frac{3}{2}R$ for *any* gas whose molecules can be represented as points.

Does Eq. (18.25) agree with experiment? In SI units, Eq. (18.25) gives

$$C_V = \frac{3}{2}\,(8.314\ \mathrm{J/mol{\cdot}K}) = 12.47\ \mathrm{J/mol} \cdot K$$

For comparison, Table 18.1 gives measured values of C_V for several gases. We see that for *monatomic* gases our prediction is right on the money, but that it is way off for diatomic and polyatomic gases.

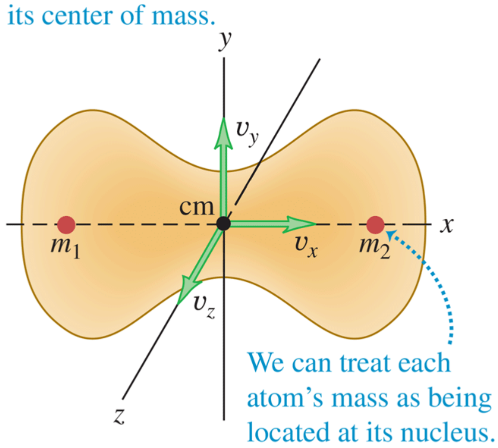
Table 18.1 Molar Heat Capacities of Gases

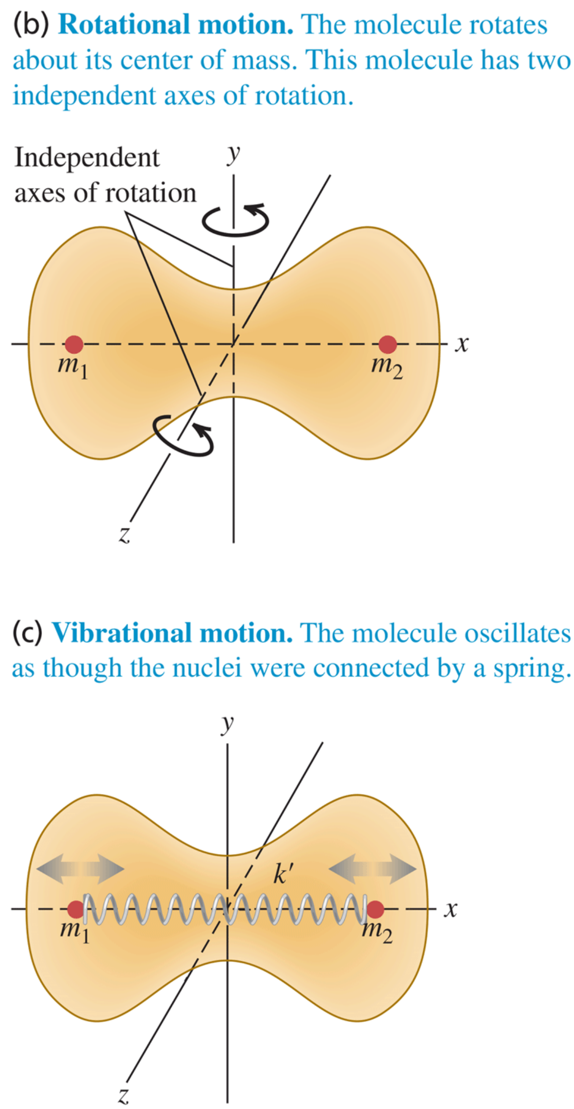
Type of Gas	Gas	C_V (J/mol · K)
Monatomic	He	12.47
	Ar	12.47
Diatomic	H_2	20.42
	N_2	20.76
	O_2	20.85
	CO	20.85
Polyatomic	CO_2	28.46
	SO_2	31.39
	H_2S	25.95

This comparison tells us that our point-molecule model is good enough for monatomic gases but that for diatomic and polyatomic molecules we need something more sophisticated. For example, we can picture a diatomic molecule as *two* point masses, like a little elastic dumbbell (see Fig. 18.18), with an interaction force between the atoms of the kind shown in Fig. 18.8. Such a molecule can have additional kinetic energy associated with *rotation* about axes through its center of mass. The atoms may also vibrate along the line joining them, with additional kinetic and potential energies.

Figure 18.18

(a) **Translational motion.** The molecule moves as a whole; its velocity may be described as the x-, y-, and z-velocity components of its center of mass.





Motions of a diatomic molecule.

When heat flows into a *monatomic* gas at constant volume, *all* of the added energy goes into an increase in random *translational* molecular kinetic energy. Equation (18.23) shows that this gives rise to an increase in temperature. But when the temperature is increased by the same amount in a *diatomic* or *polyatomic* gas, additional heat is needed to supply the increased rotational and vibrational energies. Thus polyatomic gases have *larger* molar heat capacities than monatomic gases, as Table 18.1 shows.

But how do we know how much energy is associated with each additional kind of motion of a complex molecule, compared to the translational kinetic energy? The new principle that we need is called the principle of **equipartition of energy.** It can be derived from sophisticated statistical-mechanics considerations; that derivation is beyond our scope, and we'll treat the principle as an axiom.

The principle of equipartition of energy states that each velocity component (either linear or angular) has, on average, an associated kinetic energy per molecule of $\frac{1}{2}\,kT$, or one-half the product of the Boltzmann constant and the absolute temperature. The number of velocity components needed to describe the motion of a molecule completely is called the number of **degrees of freedom.** For a monatomic gas, there are three degrees of freedom (for the velocity components v_x, v_y, and v_z); this gives a total average kinetic energy per molecule of $3\left(\frac{1}{2}\,kT\right)$, consistent with Eq. (18.16).

For a *diatomic* molecule there are two possible axes of rotation, perpendicular to each other and to the molecule's axis. (We don't include rotation about the molecule's own axis because in ordinary collisions there is no way for this rotational motion to change.) If we add two rotational degrees of freedom for a diatomic molecule, the average total kinetic energy per molecule is $\frac{5}{2}\,kT$ instead of $\frac{3}{2}\,kT$. The total kinetic energy of n moles is $K_{\text{total}} = nN_{\text{A}}\left(\frac{5}{2}\,kT\right) = \frac{5}{2}\,n(kN_{\text{A}})T = \frac{5}{2}\,nRT$, and the molar heat capacity (at constant volume) is

(18.26)

Molar heat capacity at constant volume, ideal diatomic gas

$$C_V = \frac{5}{2}R \quad \text{(18.26)}$$

Gas constant

In SI units,

$$C_V = \frac{5}{2}\,(8.314\ \mathrm{J/mol{\cdot}K}) = 20.79\ \mathrm{J/mol} \cdot K$$

This value is close to the measured values for diatomic gases in Table 18.1.

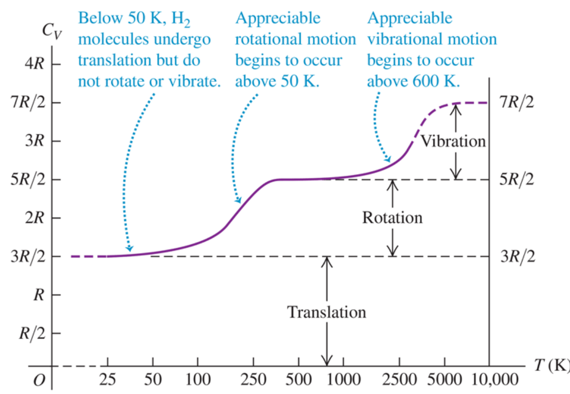
Vibrational motion can also contribute to the heat capacities of gases. Molecular bonds can stretch and bend, and the resulting vibrations lead to additional degrees of freedom and additional energies. For most diatomic gases, however, vibration does *not* contribute appreciably to heat capacity. The reason for this involves some concepts of quantum mechanics. Briefly, vibrational energy can change only in finite steps. If the energy change of the first step is much larger than the energy possessed by most molecules, then nearly all the molecules remain in the minimum-energy state of motion. Changing the temperature does not change their average vibrational energy appreciably, and the vibrational degrees of freedom are said to be “frozen out.” In more complex molecules the gaps between permitted energy levels can be much smaller, and then vibration *does* contribute to heat capacity. The rotational energy of a molecule also changes by finite steps, but they are usually much smaller; the “freezing out” of rotational degrees of freedom occurs only in rare instances.

In Table 18.1 the large values of C_V for polyatomic molecules show the effects of vibrational energy. In addition, a molecule with three or more atoms that are not in a straight line has *three* rotational degrees of freedom.

From this discussion we expect heat capacities to be temperature-dependent, generally increasing with increasing temperature. Figure 18.19 is a graph of the temperature dependence of C_V for hydrogen gas

(H_2), showing the temperatures at which the rotational and vibrational energies begin to contribute.

Figure 18.19

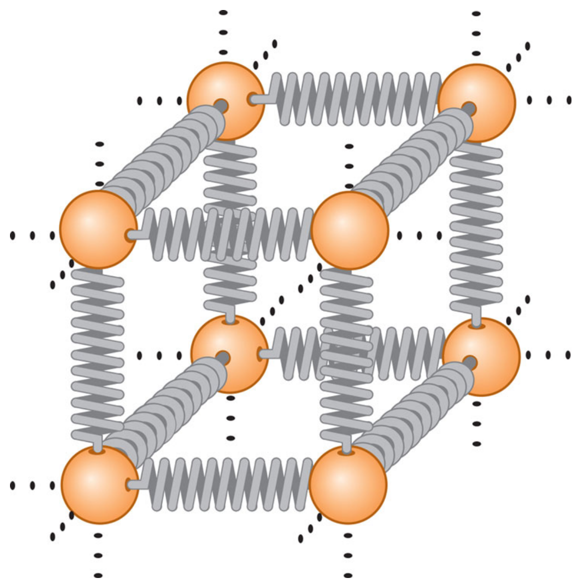


Experimental values of C_V, the molar heat capacity at constant volume, for hydrogen gas (H_2). The temperature is plotted on a logarithmic scale.

Heat Capacities of Solids

We can carry out a similar heat-capacity analysis for a crystalline solid. Consider a crystal consisting of N identical atoms (a *monatomic solid*). Each atom is bound to an equilibrium position by interatomic forces. Solid materials are elastic, so forces must permit stretching and bending of the bonds. We can think of a crystal as an array of atoms connected by little springs (Fig. 18.20).

Figure 18.20



To visualize the forces between neighboring atoms in a crystal, envision every atom as being attached to its neighbors by springs.

Each atom can *vibrate* around its equilibrium position and has three degrees of freedom, corresponding to its three components of velocity. According to the equipartition principle, each atom has an average kinetic energy of $\frac{1}{2}\,kT$ for each degree of freedom. In addition, there is *potential* energy associated with the elastic deformation. For a simple harmonic oscillator (discussed in Chapter 14) it is not hard to show that the average kinetic energy is *equal* to the average potential energy. In our model of a crystal, each atom is a three-dimensional harmonic oscillator; it can be shown that the equality of average kinetic and potential energies also holds here, provided that the “spring” forces obey Hooke’s law.

Thus we expect each atom to have an average kinetic energy $\frac{3}{2}\,kT$ and an average potential energy $\frac{3}{2}\,kT$, or an average total energy $3kT$ per atom. If the crystal contains N atoms or n moles, its total energy is

(18.27)

$$E_{\text{total}} = 3NkT = 3nRT$$

From this we conclude that the molar heat capacity of a crystal should be

(18.28)

Molar heat capacity of an ideal monatomic solid (rule of Dulong and Petit)

$$C_V = 3R \qquad (18.28)$$

Gas constant

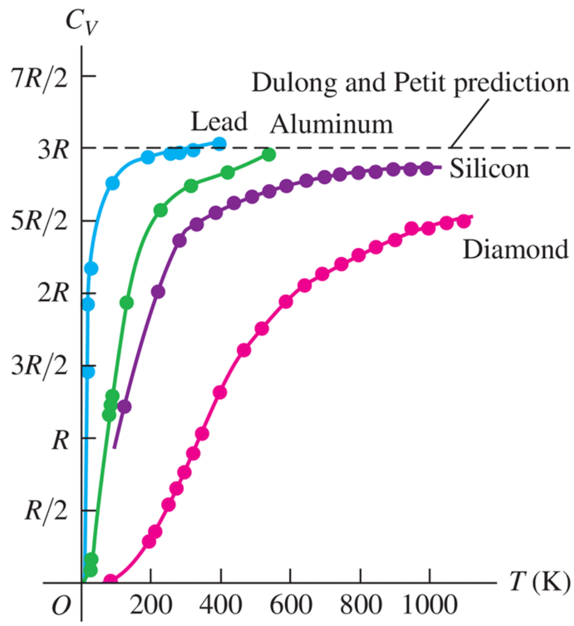
In SI units,

$$C_V = (3)(8.314\ \mathrm{J/mol{\cdot}K}) = 24.9\ \mathrm{J/mol \cdot K}$$

We have *derived* the **rule of Dulong and Petit,** which we encountered as an *empirical* finding in Section 17.5: Monatomic solids all have molar heat capacities of about 25 J/mol·K. The agreement is only approximate, but given the very simple nature of our model, it is quite significant.

At low temperatures, the heat capacities of most solids *decrease* with decreasing temperature (Fig. 18.21) for the same reason that vibrational degrees of freedom of molecules are frozen out at low temperatures. At very low temperatures the quantity kT is much *smaller* than the smallest energy step the vibrating atoms can take. Hence most of the atoms remain in their lowest energy states because the next higher energy level is out of reach. The average vibrational energy per atom is then *less* than $3kT$, and the heat capacity per molecule is *less* than $3k$. At higher temperatures when kT is *large* in comparison to the minimum energy step, the equipartition principle holds, and the total heat capacity is $3k$ per molecule or $3R$ per mole as the rule of Dulong and Petit predicts. Quantitative understanding of the temperature variation of heat capacities was one of the triumphs of quantum mechanics during its initial development in the 1920s.

Figure 18.21



Experimental values of C_V for lead, aluminum, silicon, and diamond. At high temperatures, C_V for each solid approaches about $3R$, in agreement with the rule of Dulong and Petit. At low temperatures, C_V is much less than $3R$.

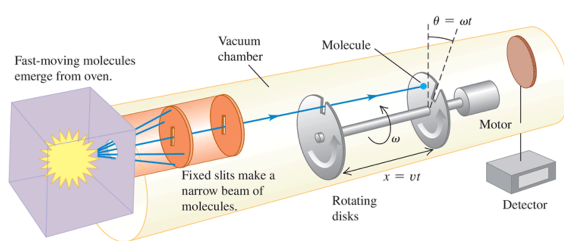
Test Your Understanding of Section 18.4

A cylinder with a fixed volume contains hydrogen gas (H_2) at 25 K. You then add heat to the gas at a constant rate until its temperature reaches 500 K. Does the temperature of the gas increase at a constant rate? Why or why not? If not, does the temperature increase most rapidly near the beginning or near the end of this process?

18.5 Molecular Speeds

As we mentioned in Section 18.3, the molecules in a gas don't all have the same speed. Figure 18.22 shows one experimental scheme for measuring the distribution of molecular speeds. A substance is vaporized in a hot oven; molecules of the vapor escape through an aperture in the oven wall and into a vacuum chamber. A series of slits blocks all molecules except those in a narrow beam, which is aimed at a pair of rotating disks. A molecule passing through the slit in the first disk is blocked by the second disk unless it arrives just as the slit in the second disk is lined up with the beam. The disks function as a speed selector that passes only molecules within a certain narrow speed range. This range can be varied by changing the disk rotation speed, and we can measure how many molecules lie within each of various speed ranges.

Figure 18.22



A molecule with a speed v passes through the slit in the first rotating disk. When the molecule reaches the second rotating disk, the disks have rotated through the offset angle θ. If $v = \omega x/\theta$, the molecule passes through the slit in the second rotating disk and reaches the detector.

To describe the results of such measurements, we define a function $f(v)$ called a *distribution function*. If we observe a total of N molecules, the number dN having speeds in the range between v and $v + dv$ is given by

(18.29)

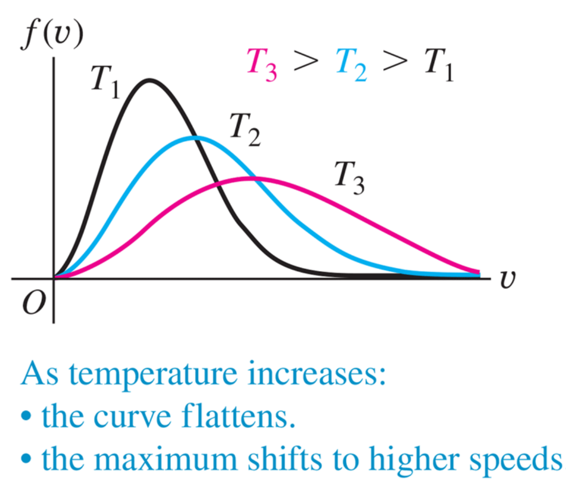
$$dN = Nf(v)\,dv$$

The *probability* that a randomly chosen molecule will have a speed in the interval v to $v + dv$ is $f(v)dv$. Hence $f(v)$ is the probability per unit speed *interval;* it is *not* the probability that a molecule has speed exactly equal to v. Since a probability is a pure number, $f(v)$ has units of reciprocal speed $(\mathrm{s/m})$.

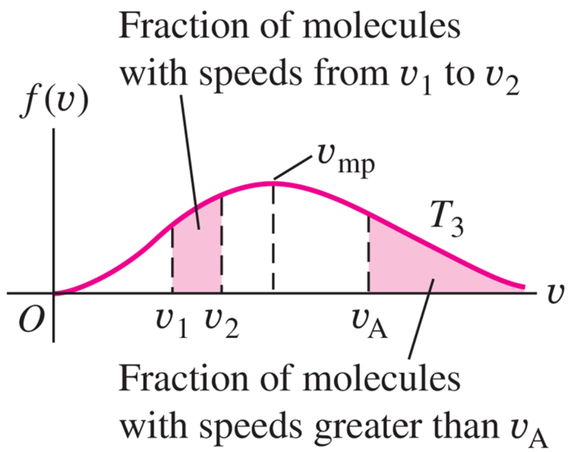
Figure 18.23a shows distribution functions for three different temperatures. At each temperature the height of the curve for any value of v is proportional to the number of molecules with speeds near v. The peak of the curve represents the *most probable speed* v_{mp} for the corresponding temperature. As the temperature increases, the average molecular kinetic energy increases, and so the peak of $f(v)$ shifts to higher and higher speeds.

Figure 18.23

(a)



(b)



(a) Curves of the Maxwell–Boltzmann distribution function $f(v)$ for three temperatures. (b) The shaded areas under the curve represent the fractions of molecules within certain speed ranges. The most probable speed v_{mp} for a given temperature is at the peak of the curve.

Figure 18.23b shows that the area under a curve between any two values of v represents the fraction of all the molecules having speeds in that range. Every molecule must have *some* value of v, so the integral of $f(v)$ over all v must be unity for any T.

If we know $f(v)$, we can calculate the most probable speed v_{mp}, the average speed v_{av}, and the rms speed v_{rms}. To find v_{mp}, we simply find the point where $df/dv = 0$; this gives the value of the speed where the curve has its peak. To find v_{av}, we take the number $Nf(v)\,dv$ having speeds in each interval dv, multiply each number by the corresponding speed v, add all these products (by integrating over all v from zero to infinity), and finally divide by N. That is,

(18.30)

$$v_{av} = \int_0^\infty v f(v)\,dv$$

We can find the rms speed in a similar way; the average of v^2 is

(18.31)

$$\left(v^2\right)_{\text{av}} = \int_0^\infty v^2 f(v)\, dv$$

and v_{rms} is the square root of this.

The Maxwell–Boltzmann Distribution

The function $f(v)$ describing the actual distribution of molecular speeds is called the **Maxwell–Boltzmann distribution.** It can be derived from statistical-mechanics considerations, but that derivation is beyond our scope. Here is the result:

(18.32)

Mass of a gas molecule
Molecular speed
Mass of a gas molecule
Molecular speed
Maxwell-Boltzmann distribution function

$$f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} \qquad (18.32)$$

Boltzmann constant
Boltzmann constant
Absolute temperature of gas

We can also express this function in terms of the translational kinetic energy of a molecule, which we denote by ϵ; that is, $\epsilon = \frac{1}{2} mv^2$. We invite you to verify that when this is substituted into Eq. (18.32), the result is

(18.33)

$$f\ (\epsilon) = \frac{8\pi}{m}\left(\frac{m}{2\pi kT}\right)^{3/2} \epsilon e^{-\epsilon/kT}$$

This form shows that the exponent in the Maxwell–Boltzmann distribution function is $-\epsilon/kT$; the shape of the curve is determined by the relative magnitude of ϵ and kT at any point. You can prove that the *peak* of each curve occurs where $\epsilon = kT$, corresponding to a most probable speed v_{mp}:

(18.34)

$$v_{\mathrm{mp}} = \sqrt{\frac{2kT}{m}}$$

To find the average speed, we substitute Eq. (18.32) into Eq. (18.30), make a change of variable $v^2 = x$, and integrate by parts. The result is

(18.35)

$$v_{\mathrm{av}} = \sqrt{\frac{8kT}{\pi m}}$$

Finally, to find the rms speed, we substitute Eq. (18.32) into Eq. (18.31). Evaluating the resulting integral takes some mathematical acrobatics, but we can find it in a table of integrals. The result is

(18.36)

$$v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}}$$

This result agrees with Eq. (18.19); it *must* agree if the Maxwell–Boltzmann distribution is to be consistent with our kinetic-theory calculations.

Table 18.2 shows the fraction of all the molecules in an ideal gas that have speeds *less than* various multiples of v_{rms}. These numbers were obtained by numerical integration; they are the same for all ideal gases.

Table 18.2 Fractions of Molecules in an Ideal Gas with Speeds Less Than Various Multiples of v_{rms}

v/v_{rms}	Fraction
0.20	0.011
0.40	0.077
0.60	0.218
0.80	0.411
1.00	0.608
1.20	0.771
1.40	0.882
1.60	0.947
1.80	0.979
2.00	0.993

The distribution of molecular speeds in liquids is similar, although not identical, to that for gases. We can understand evaporation and the vapor pressure of a liquid on this basis. Suppose a molecule must have a speed at least as great as v_A in Fig. 18.23b to escape from the surface of a liquid into the adjacent vapor. The number of such molecules, represented by the area under the "tail" of each curve (to the right of v_A), increases rapidly with temperature. Thus the rate at which molecules can escape is strongly temperature-dependent. This process is balanced by one in which molecules in the vapor phase collide inelastically with the surface and are trapped into the liquid phase. The number of molecules suffering this fate per unit time is proportional to the pressure in the vapor phase. Phase equilibrium between liquid and vapor occurs when these two competing processes proceed at the same rate. So if the molecular speed distributions are known for various temperatures, we can make a theoretical prediction of vapor pressure as a function of temperature. When liquid evaporates, it's the high-speed molecules that escape from the surface. The ones that are left have less energy on average; this gives us a molecular view of evaporative cooling.

BIO Application

Activation Energy and Moth Activity

This hawkmoth of genus *Manduca* cannot fly if the temperature of its muscles is below 29°C. The reason is that the enzyme-catalyzed reactions that power aerobic metabolism and enable muscle action require a minimum molecular energy (activation energy). Just like the molecules in an ideal gas, at low temperatures very few of the molecules involved in these reactions have high energy. As the temperature increases, more molecules have the required minimum energy and the reactions take place at a greater rate. Above 29°C, enough power is generated to allow the hawkmoth to fly.

Rates of chemical reactions are often strongly temperature-dependent, and the reason is contained in the Maxwell–Boltzmann distribution.

When two reacting molecules collide, the reaction can occur only when the molecules are close enough for their electrons to interact strongly. This requires a minimum energy, called the *activation energy,* and thus a minimum molecular speed. Figure 18.23a shows that the number of molecules in the high-speed tail of the curve increases rapidly with temperature. Thus we expect the rate of any reaction with an activation energy to increase rapidly with temperature.

Test Your Understanding of Section 18.5

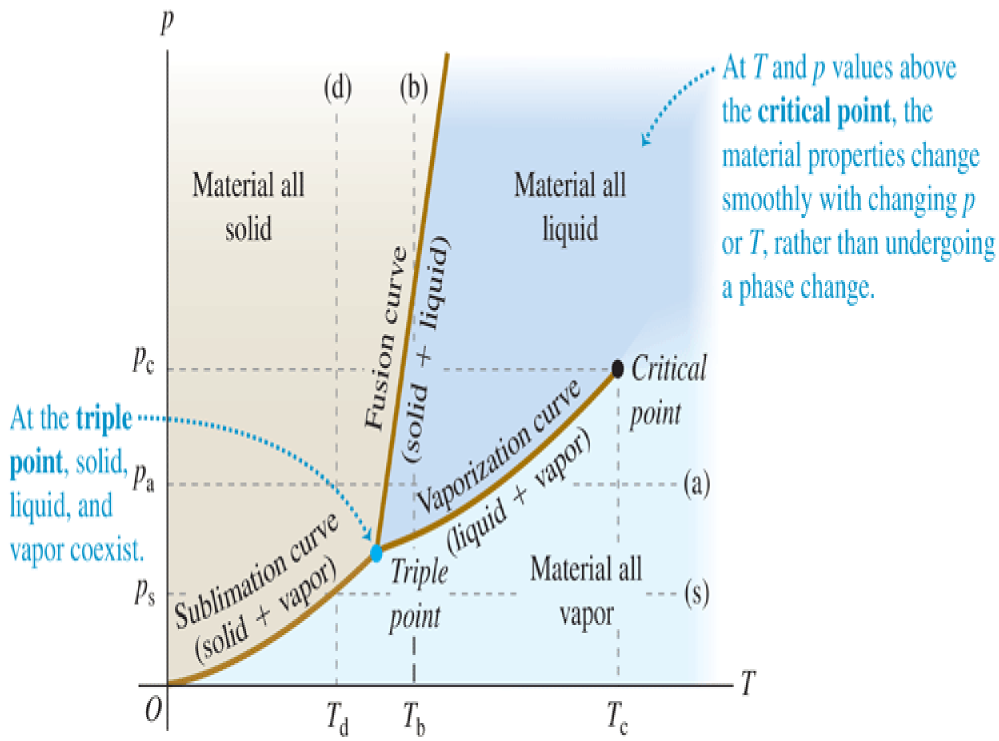
A quantity of gas containing N molecules has a speed distribution function $f(v)$. How many molecules have speeds between v_1 and $v_2 > v_1$? (i) $\int_0^{v_2} f(v)dv - \int_0^{v_1} f(v)dv$ (ii) $N\left[\int_0^{v_2} f(v)dv - \int_0^{v_1} f(v)dv\right]$ (iii) $\int_0^{v_1} f(v)dv - \int_0^{v_2} f(v)dv$ (iv) $N\left[\int_0^{v_1} f(v)dv - \int_0^{v_2} f(v)dv\right]$ (v) none of these.

18.6 Phases of Matter

An ideal gas is the simplest system to analyze from a molecular viewpoint because we ignore the interactions between molecules. But those interactions are the very thing that makes matter condense into the liquid and solid phases under some conditions. So it's not surprising that theoretical analysis of liquid and solid structure and behavior is a lot more complicated than that for gases. We won't try to go far here with a microscopic picture, but we can talk in general about phases of matter, phase equilibrium, and phase transitions.

In Section 17.6 we learned that each phase is stable in only certain ranges of temperature and pressure. A transition from one phase to another ordinarily requires **phase equilibrium** between the two phases, and for a given pressure this occurs at only one specific temperature. We can represent these conditions on a graph with axes p and T, called a **phase diagram;** Fig. 18.24 shows an example. Each point on the diagram represents a pair of values of p and T.

Figure 18.24



A typical pT phase diagram, showing regions of temperature and pressure at which the various phases exist and where phase changes occur.

Only a single phase can exist at each point in Fig. 18.24, except for points on the solid lines, where two phases can coexist in phase equilibrium. The fusion curve separates the solid and liquid areas and represents possible conditions of solid-liquid phase equilibrium. The vaporization curve separates the liquid and vapor areas, and the sublimation curve separates the solid and vapor areas. All three curves meet at the **triple point,** the only condition under which all three phases can coexist (Fig. 18.25). In Section 17.3 we used the triple-point temperature of water to define the Kelvin temperature scale. Table 18.3 gives triple-point data for several substances.

Figure 18.25

Atmospheric pressure on earth is higher than the triple-point pressure of water (see line (a) in Fig. 18.24). Depending on the temperature, water can exist as a vapor (in the atmosphere), as a liquid (in the ocean), or as a solid (like the iceberg shown here).

Table 18.3 Triple-Point Data

Substance	Temperature (K)	Pressure (Pa)
Hydrogen	13.80	0.0704×10^5
Deuterium	18.63	0.171×10^5
Neon	24.56	0.432×10^5
Nitrogen	63.18	0.125×10^5
Oxygen	54.36	0.00152×10^5
Ammonia	195.40	0.0607×10^5
Carbon dioxide	216.55	5.17×10^5
Sulfur dioxide	197.68	0.00167×10^5
Water	273.16	0.00610×10^5

If we heat a substance at a constant pressure p_a, it goes through a series of states represented by the horizontal line (a) in Fig. 18.24. The melting and boiling temperatures at this pressure are the temperatures at which the line intersects the fusion and vaporization curves, respectively. When the pressure is p_s, constant-pressure heating transforms a substance from solid directly to vapor. This process is called *sublimation;* the intersection of line (s) with the sublimation curve gives the temperature T_s at which it occurs for a pressure p_s. At any pressure less than the triple-point pressure, no liquid phase is possible. The triple-point pressure for carbon dioxide (CO_2) is 5.1 atm. At normal atmospheric pressure, solid CO_2 ("dry ice") undergoes sublimation; there is no liquid phase.

Line (b) in Fig. 18.24 represents compression at a constant temperature T_b. The material passes from vapor to liquid and then to solid at the points where line (b) crosses the vaporization curve and fusion curve, respectively. Line (d) shows constant-temperature compression at a lower temperature T_d; the material passes from vapor to solid at the point where line (d) crosses the sublimation curve.

We saw in the pV-diagram of Fig. 18.7 that a liquid-vapor phase transition occurs only when the temperature and pressure are less than those at the point at the top of the green shaded area labeled "Liquid-vapor phase equilibrium region." This point corresponds to the endpoint at the top of the vaporization curve in Fig. 18.24. It is called the **critical point,** and the corresponding values of p and T are called the critical pressure and temperature, p_c and T_c. A gas at a pressure *above* the critical pressure does not separate into two phases when it is cooled at constant pressure (along a horizontal line above the critical point in Fig. 18.24). Instead, its properties change gradually and continuously from those we ordinarily associate with a gas (low density, large compressibility) to those of a liquid (high density, small compressibility) *without a phase transition.*

You can understand this by thinking about liquid-phase transitions at successively higher points on the vaporization curve. As we approach the critical point, the *differences* in physical properties (such as density and compressibility) between the liquid and vapor phases become smaller. Exactly *at* the critical point they all become zero, and at this point the distinction between liquid and vapor disappears. The heat of vaporization also grows smaller as we approach the critical point, and it too becomes zero at the critical point.

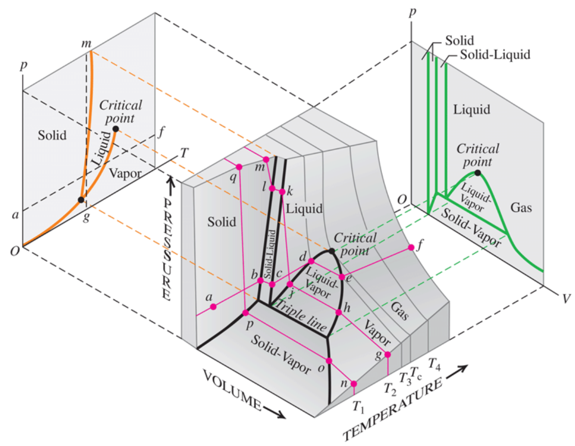
For nearly all familiar materials the critical pressures are much greater than atmospheric pressure, so we don't observe this behavior in everyday life. For example, the critical point for water is at 647.4 K and 221.2×10^5 Pa (about 218 atm or 3210 psi). But high-pressure steam boilers in electric generating plants regularly run at pressures and temperatures well above the critical point.

Many substances can exist in more than one solid phase. A familiar example is carbon, which exists as noncrystalline soot and crystalline graphite and diamond. Water is another example; more than a dozen types of ice, differing in crystal structure and physical properties, have been observed at very high pressures.

pVT-Surfaces

We remarked in Section 18.1 that for any material, it can be useful to represent the equation of state as a surface in a three-dimensional space with coordinates p, V, and T. Figure 18.26 shows a typical pVT-surface. The light lines represent pV-isotherms; projecting them onto the pV-plane gives a diagram similar to Fig. 18.7. The pV-isotherms represent contour lines on the pVT-surface, just as contour lines on a topographic map represent the elevation (the third dimension) at each point. The projections of the edges of the surface onto the pT-plane give the pT phase diagram of Fig. 18.24.

Figure 18.26

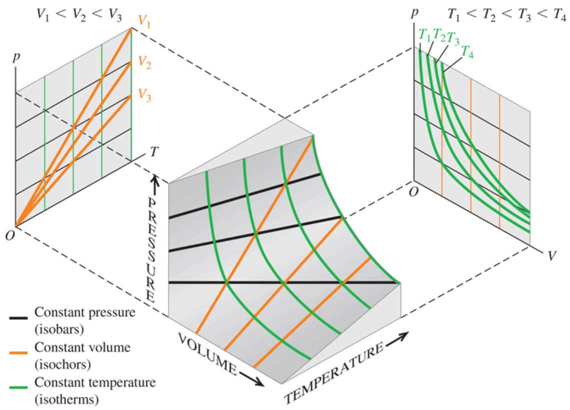


A pVT-surface for a substance that expands on melting. Projections of the boundaries on the surface onto the pT- and pV-planes are also shown.

Line *abcdef* in Fig. 18.26 represents constant-pressure heating, with melting along *bc* and vaporization along *de*. Note the volume changes that occur as T increases along this line. Line *ghjklm* corresponds to an isothermal (constant temperature) compression, with liquefaction along *hj* and solidification along *kl*. Between these, segments *gh* and *jk* represent isothermal compression with increase in pressure; the pressure increases are much greater in the liquid region *jk* and the solid region *lm* than in the vapor region *gh*. Finally, line *nopq* represents isothermal solidification directly from vapor, as in the formation of snowflakes or frost.

Figure 18.27 shows the much simpler pVT-surface for a substance that obeys the ideal-gas equation of state under all conditions. The projections of the constant-temperature curves onto the pV-plane correspond to the curves of Fig. 18.6, and the projections of the constant-volume curves onto the pT-plane show that pressure is directly proportional to absolute temperature. Figure 18.27 also shows the *isobars* (curves of constant pressure) and *isochors* (curves of constant volume) for an ideal gas.

Figure 18.27



A pVT-surface for an ideal gas. At the left, each orange line corresponds to a certain constant volume; at the right, each green line corresponds to a certain constant temperature.

Test Your Understanding of Section 18.6

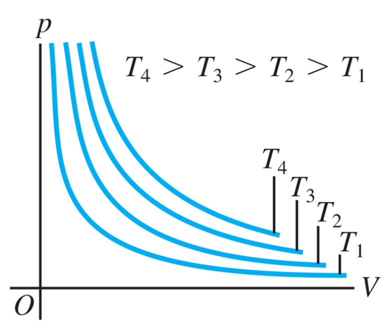
The average atmospheric pressure on Mars is 6.0×10^2 Pa. Could there be lakes of liquid water on the surface of Mars today? What about in the past, when the atmospheric pressure is thought to have been substantially greater?

Chapter 18 Summary

Equations of state: The pressure p, volume V, and absolute temperature T of a given quantity of a substance are related by an equation of state. This relationship applies only for equilibrium states, in which p and T are uniform throughout the system. The ideal-gas equation of state, Eq. (18.3), involves the number of moles n and a constant R that is the same for all gases. (See Examples 18.1, 18.2, 18.3 and 18.4.)

(18.3)

$$pV = nRT$$



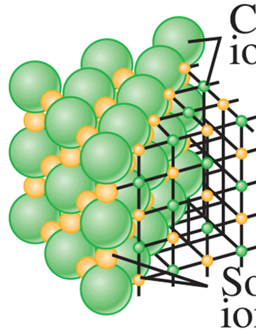
Molecular properties of matter: The molar mass M of a pure substance is the mass per mole. The mass m_{total} of a quantity of substance equals M multiplied by the number of moles n. Avogadro's number N_A is the number of molecules in a mole. The mass m of an individual molecule is M divided by N_A. (See Example 18.5.)

(18.2)

$$m_{\text{total}} = nM$$

(18.8)

$$M = N_{\mathrm{A}} m$$



Kinetic-molecular model of an ideal gas: In an ideal gas, the total translational kinetic energy of the gas as a whole (K_{tr}) and the average translational kinetic energy per molecule $\left[\frac{1}{2} m\left(v^2\right)_{\mathrm{av}}\right]$ are proportional to the absolute temperature T, and the root-mean-square speed of molecules is proportional to the square root of T. These expressions involve the Boltzmann constant $k = R/N_{\mathrm{A}}$. (See Examples 18.6 and 18.7.) The mean free path λ of molecules in an ideal gas depends on the number of molecules per volume (N/V) and the molecular radius r. (See Example 18.8.)

(18.14)

$$K_{\mathrm{tr}} = \frac{3}{2} nRT$$

(18.16)

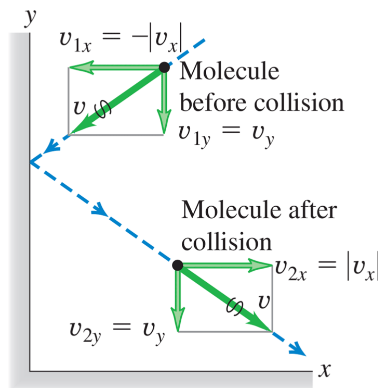
$$\frac{1}{2} m\left(v^2\right)_{\mathrm{av}} = \frac{3}{2} kT$$

(18.19)

$$v_{\mathrm{rms}} = \sqrt{\left(v^2\right)_{\mathrm{av}}} = \sqrt{\frac{3kT}{m}}$$

(18.21)

$$\lambda = vt_{\text{mean}} = \frac{V}{4\pi\sqrt{2}\, r^2 N}$$



Heat capacities: The molar heat capacity at constant volume C_V is a simple multiple of the gas constant R for certain idealized cases: an ideal monatomic gas [Eq. (18.25)]; an ideal diatomic gas including rotational energy [Eq. (18.26)]; and an ideal monatomic solid [Eq. (18.28)]. Many real systems are approximated well by these idealizations.

(18.25)

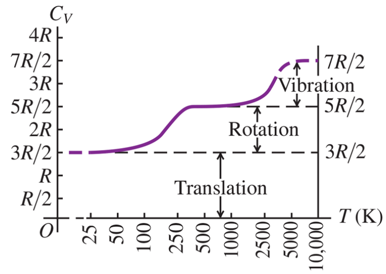
$$C_V = \frac{3}{2} R \,(\text{monatomic gas})$$

(18.26)

$$C_V = \frac{5}{2} R \,(\text{diatomic gas})$$

(18.28)

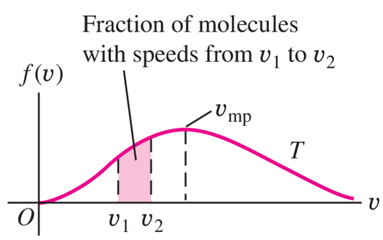
$$C_V = 3R \,(\text{monatomic solid})$$



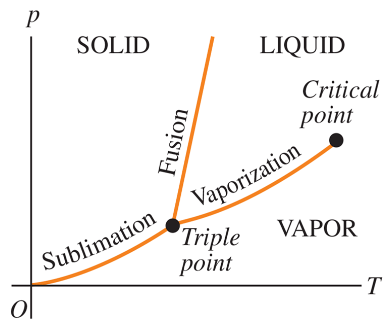
Molecular speeds: The speeds of molecules in an ideal gas are distributed according to the Maxwell–Boltzmann distribution $f(v)$. The quantity $f(v)\,dv$ describes what fraction of the molecules have speeds between v and $v + dv$.

(18.32)

$$f\ (v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$



Phases of matter: Ordinary matter exists in the solid, liquid, and gas phases. A phase diagram shows conditions under which two phases can coexist in phase equilibrium. All three phases can coexist at the triple point. The vaporization curve ends at the critical point, above which the distinction between the liquid and gas phases disappears.



p
SOLID
LIQUID
Critical point
Fusion
Vaporization
Sublimation
Triple point
VAPOR
O
T

Guided Practice: Thermal Properties of Matter

For assigned homework and other learning materials, go to

Mastering Physics.

Key Example Variation Problems

Be sure to review Examples 18.1, 18.2, 18.3, and 18.4 (Section 18.1) before attempting these problems.

VP18.4.1 When the temperature is 30.0°C, the pressure of the air inside a bicycle tire of fixed volume $1.40 \times 10^{-3}\,\text{m}^3$ is $5.00 \times 10^5\,\text{Pa}$. (a) What will be the pressure inside the tire when the temperature drops to 10.0°C? (b) How many moles of air are inside the tire?

VP18.4.2 When a research balloon is released at sea level, where the temperature is 15.0°C and the atmospheric pressure is $1.01 \times 10^5\,\text{Pa}$, the helium in it has volume $13.0\,\text{m}^3$. (a) When the balloon reaches an altitude of 32.0 km, where the temperature is -44.5°C and the pressure is 868 Pa, what is the volume of the helium? (b) If this balloon is spherical, how many times larger is its radius at 32.0 km than at sea level?

VP18.4.3 The dwarf planet Pluto has a very thin atmosphere made up almost entirely of nitrogen (N_2, molar mass $2.8 \times 10^{-2}\,\mathrm{kg/mol}$). At Pluto's surface the temperature is 42 K and the atmospheric pressure is 1.0 Pa. At the surface, (a) how many moles of gas are there per cubic meter of atmosphere, and (b) what is the density of the atmosphere in $\mathrm{kg/m^3}$? (For comparison, the values are $42\,\mathrm{mol/m^3}$ and $1.2\,\mathrm{kg/m^3}$ at the earth's surface.)

VP18.4.4 When the pressure on n moles of helium gas is suddenly changed from an initial value of p_1 to a final value of p_2, the density of the gas changes from its initial value of ρ_1 to a final value of $\rho_2 = \rho_1(p_2 / p_1)^{3/5}$. (a) If the initial absolute temperature of the gas is T_1, what is its final absolute temperature T_2 in terms of T_1, p_1, and p_2? (b) If the final pressure is 0.500 times the initial pressure, what are the ratio of the final density to the initial density and the ratio of the final temperature to the initial temperature? (c) Repeat part (b) if the final pressure is 2.00 times the initial pressure.

Be sure to review Examples 18.6 and 18.7 (Section 18.3) before attempting these problems.

VP18.7.1 At what temperature (in $^\circ\mathrm{C}$) is the rms speed of helium atoms (molar mass $4.00\,\mathrm{g/mol}$) the same as the rms speed of nitrogen molecules (molar mass $28.0\,\mathrm{g/mol}$) at $20.0^\circ\mathrm{C}$? (Note that helium remains a gas at temperatures above $-269^\circ\mathrm{C}$.)

VP18.7.2 For the first 380,000 years after the Big Bang, the temperature of the matter in the universe was too high for nuclei and electrons to form atoms. The first hydrogen atoms (mass per atom $1.67 \times 10^{-27}\,\mathrm{kg}$) did not form until the temperature had dropped to about 3000 K. (a) What was the average translational kinetic energy of hydrogen atoms when the temperature was $3.0 \times 10^3\,\mathrm{K}$? (b) What was the rms speed of these atoms?

VP18.7.3 The air in a room with dimensions $5.0\,\mathrm{m} \times 5.0\,\mathrm{m} \times 2.4\,\mathrm{m}$ is at temperature $20.0^\circ\mathrm{C}$ and pressure $1.00\,\mathrm{atm} = 1.01 \times 10^5\,\mathrm{Pa}$. (a) How many air molecules are in this room? (b) What is the total translational kinetic energy of these molecules? (c) How fast would a car of mass $1.5 \times 10^3\,\mathrm{kg}$ have to move to have the same translational kinetic energy?

VP18.7.4 (a) What is the average value of the first 10 integers (1 through 10)? (b) What is the average value of the squares of the first 10 integers? (c) What is the rms value of the first 10 integers?

Be sure to review Example 18.8 (Section 18.3) before attempting these problems.

VP18.8.1 At the surface of Mars the atmosphere has average pressure $6.0 \times 10^2\,\mathrm{Pa}$ and average temperature $-63^\circ\mathrm{C}$. (a) What is the mean free path of atmospheric molecules (assumed to be spheres of radius $2.0 \times 10^{-10}\,\mathrm{m}$) at the surface of Mars? (b) How many times greater is your answer in part (a) than the mean free path of atmospheric molecules at the earth's surface, where the average pressure is $1.01 \times 10^5\,\mathrm{Pa}$ and the average temperature is $15^\circ\mathrm{C}$?

VP18.8.2 A cubical box 1.00 m on a side contains air at $20.0^\circ\mathrm{C}$. (a) What would the pressure inside the box have to be in order for the mean free path of air molecules (assumed to be spheres of radius $2.0 \times 10^{-10}\,\mathrm{m}$) to be 1.00 m, so that a typical molecule suffers no collisions with other molecules as it travels the width of the box? (b) How many moles of air would be inside the box at the pressure calculated in part (a)? (At 1 atm pressure, this box would contain 41.4 mol of air.)

VP18.8.3 A cylinder for storing helium (molar mass $4.00\,\mathrm{g/mol}$) has an interior volume of $50.0\,\mathrm{L} = 5.00 \times 10^{-2}\,\mathrm{m}^3$. The cylinder holds $4.00 \times 10^2\,\mathrm{mol}$ of helium under pressure at $27.0^\circ\mathrm{C}$. If you assume the helium atoms are spheres of radius $3.1 \times 10^{-11}\,\mathrm{m}$, what are (a) the mean free path of a helium atom in the tank and (b) the mean free time for a helium atom moving at the rms speed?

VP18.8.4 A gas with molecules of radius r and mass per molecule m is at temperature T and pressure p. (a) Write an expression for the mean free time for a molecule moving at the rms speed for this gas. (b) Which single change would have the greatest effect on the mean free time: doubling the radius r, doubling the pressure p, or doubling the temperature T?

Bridging Problem: Gas on Jupiter's Moon Europa

An astronaut visiting Jupiter's satellite Europa leaves a canister of 1.20 mol of nitrogen gas (28.0 g/mol) at 25.0°C on the satellite's surface. Europa has no significant atmosphere, and the acceleration due to gravity at its surface is $1.30\ \mathrm{m/s^2}$. The canister springs a leak, allowing molecules to escape from a small hole. (a) What is the maximum height (in km) above Europa's surface that is reached by a nitrogen molecule whose speed equals the rms speed? Assume that the molecule is shot straight up out of the hole in the canister, and ignore the variation in g with altitude. (b) The escape speed from Europa is $2025\,\mathrm{m/s}$. Can any of the nitrogen molecules escape from Europa and into space?

Solution Guide

IDENTIFY and SET UP

1. Sketch the situation, showing all relevant dimensions.
2. List the unknown quantities, and decide which are the target variables.
3. How will you find the rms speed of the nitrogen molecules? What principle will you use to find the maximum height that a molecule with this speed can reach?
4. Does the rms molecular speed in the gas represent the maximum molecular speed? If not, what is the maximum speed?

EXECUTE

5. Solve for the rms speed. Use this to calculate the maximum height that a molecule with this speed can reach.
6. Use your result from step 5 to answer the question in part (b).

EVALUATE

7. Do your results depend on the amount of gas in the container? Why or why not?
8. How would your results from steps 5 and 6 be affected if the gas cylinder were instead left on Jupiter's satellite Ganymede, which has higher surface gravity than Europa and a higher escape speed? Like Europa, Ganymede has no significant atmosphere.

Video Tutor Solution: **Chapter 18 Bridging Problem**

Questions/Exercises/Problems: Thermal Properties of Matter

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

Discussion Questions

Q18.1 Section 18.1 states that ordinarily, pressure, volume, and temperature cannot change individually without one affecting the others. Yet when a liquid evaporates, its volume changes, even though its pressure and temperature are constant. Is this inconsistent? Why or why not?

Q18.2 In the ideal-gas equation, could an equivalent Celsius temperature be used instead of the Kelvin one if an appropriate numerical value of the constant is used? Why or why not?

Q18.3 When a car is driven some distance, the air pressure in the tires increases. Why? Should you let out some air to reduce the pressure? Why or why not?

Q18.4 The coolant in an automobile radiator is kept at a pressure higher than atmospheric pressure. Why is this desirable? The radiator cap will release coolant when the gauge pressure of the coolant reaches a certain value, typically or so. Why not just seal the system completely?

Q18.5 Unwrapped food placed in a freezer experiences dehydration, known as “freezer burn.” Why?

Q18.6 A group of students drove from their university (near sea level) up into the mountains for a skiing weekend. Upon arriving at the slopes, they discovered that the bags of potato chips they had brought for snacks had all burst open. What caused this to happen?

Q18.7 The derivation of the ideal-gas equation included the assumption that the number of molecules is very large, so that we could compute the average force due to many collisions. However, the ideal-gas equation holds accurately only at low pressures, where the molecules are few and far between. Is this inconsistent? Why or why not?

Q18.8 A rigid, perfectly insulated container has a membrane dividing its volume in half. One side contains a gas at an absolute temperature and pressure while the other half is completely empty. Suddenly a small hole develops in the membrane, allowing the gas to leak out into the other half until it eventually occupies twice its original volume. In terms of and what will be the new temperature and pressure of the gas when it is distributed equally in both halves of the container? Explain your reasoning.

Q18.9 (a) Which has more atoms: a kilogram of hydrogen or a kilogram of lead? Which has more mass? (b) Which has more atoms: a mole of hydrogen or a mole of lead? Which has more mass? Explain your reasoning.

Q18.10 Use the concepts of the kinetic-molecular model to explain: (a) why the pressure of a gas in a rigid container increases as heat is added to the gas and (b) why the pressure of a gas increases as we compress it, even if we do not change its temperature.

Q18.11 The proportions of various gases in the earth's atmosphere change somewhat with altitude. Would you expect the proportion of oxygen at high altitude to be greater or less than at sea level compared to the proportion of nitrogen? Why?

Q18.12 Comment on the following statement: *When two gases are mixed, if they are to be in thermal equilibrium, they must have the same average molecular speed.* Is the statement correct? Why or why not?

Q18.13 The kinetic-molecular model contains a hidden assumption about the temperature of the container walls. What is this assumption? What would happen if this assumption were not valid?

Q18.14 The temperature of an ideal gas is directly proportional to the average kinetic energy of its molecules. If a container of ideal gas is moving past you at is the temperature of the gas higher than if the container was at rest? Explain your reasoning.

Q18.15 If the pressure of an ideal monatomic gas is increased while the number of moles is kept constant, what happens to the average translational kinetic energy of one atom of the gas? Is it possible to change *both* the volume and the pressure of an ideal gas and keep the average translational kinetic energy of the atoms constant? Explain.

Q18.16 In deriving the ideal-gas equation from the kinetic-molecular model, we ignored potential energy due to the earth's gravity. Is this omission justified? Why or why not?

Q18.17 Imagine a special air filter placed in a window of a house. The tiny holes in the filter allow only air molecules moving faster than a certain speed to exit the house, and allow only air molecules moving slower than that speed to enter the house from outside. What effect would this filter have on the temperature inside the house? (It turns out that the second law of thermodynamics—which we'll discuss in

Chapter 20—tells us that such a wonderful air filter would be impossible to make.)

Q18.18 A gas storage tank has a small leak. The pressure in the tank drops more quickly if the gas is hydrogen or helium than if it is oxygen. Why?

Q18.19 Consider two specimens of ideal gas at the same temperature. Specimen A has the same total mass as specimen B, but the molecules in specimen A have greater molar mass than they do in specimen B. In which specimen is the total kinetic energy of the gas greater? Does your answer depend on the molecular structure of the gases? Why or why not?

Q18.20 The temperature of an ideal monatomic gas is increased from to . Does the average translational kinetic energy of each gas atom double? Explain. If your answer is no, what would the final temperature be if the average translational kinetic energy was doubled?

Q18.21 If the root-mean-square speed of the atoms of an ideal gas is to be doubled, by what factor must the Kelvin temperature of the gas be increased? Explain.

Q18.22 (a) If you apply the same amount of heat to 1.00 mol of an ideal monatomic gas and 1.00 mol of an ideal diatomic gas, which one (if any) will increase more in temperature? (b) Physically, *why* do diatomic gases have a greater molar heat capacity than monatomic gases?

Q18.23 The discussion in Section 18.4 concluded that all ideal monatomic gases have the same heat capacity Does this mean that it takes the same amount of heat to raise the temperature of 1.0 g of each one by 1.0 K? Explain your reasoning.

Q18.24 In a gas that contains molecules, is it accurate to say that the number of molecules with speed is equal to ? Is it accurate to say that this number is given by Explain your answers.

Q18.25 The atmosphere of the planet Mars is 95.3% carbon dioxide and about 0.03% water vapor. The atmospheric pressure is only about 600 Pa, and the surface temperature varies from to The polar ice caps contain both ice and water ice. Could there be *liquid* on the surface of Mars? Could there be liquid water? Why or why not?

Q18.26 A beaker of water at room temperature is placed in an enclosure, and the air pressure in the enclosure is slowly reduced. When the air pressure is reduced sufficiently, the water begins to boil. The temperature of the water does not rise when it boils; in fact, the temperature *drops* slightly. Explain these phenomena.

Q18.27 When you stand on ice with skates, the blades on the bottom have a low surface area, and so they pressurize the ice beneath the blades. When ice is pressurized, is the melting temperature greater than or less than ? Explain.

Q18.28 Hydrothermal vents are openings in the ocean floor that discharge very hot water. The water emerging from one such vent off the Oregon coast, 2400 m below the surface, is at Despite its high temperature, the water doesn't boil. Why not?

Q18.29 The dark areas on the moon's surface are called *maria*, Latin for "seas," and were once thought to be bodies of water. In fact, the maria are not "seas" at all, but plains of solidified lava. Given that there is no atmosphere on the moon, how can you explain the absence of liquid water on the moon's surface?

Q18.30 In addition to the normal cooking directions printed on the back of a box of rice, there are also "high-altitude directions." The only difference is that the "high-altitude directions" suggest increasing the cooking time and using a greater volume of boiling water in which to cook the rice. Why should the directions depend on the altitude in this way?

Exercises

Section 18.1 Equations of State

18.1 • A 20.0 L tank contains of helium at The molar mass of helium is (a) How many moles of helium are in the tank? (b) What is the pressure in the tank, in pascals and in atmospheres?

18.2 ••• Helium gas with a volume of 3.20 L, under a pressure of 0.180 atm and at is warmed until both pressure and volume are doubled. (a) What is the final temperature? (b) How many grams of helium are there? The molar mass of helium is

18.3 • A cylindrical tank has a tight-fitting piston that allows the volume of the tank to be changed. The tank originally contains of air at a pressure of 0.355 atm. The piston is slowly pulled out until the volume of the gas is increased to If the temperature remains constant, what is the final value of the pressure?

18.4 • A 3.00 L tank contains air at 3.00 atm and The tank is sealed and cooled until the pressure is 1.00 atm. (a) What is the temperature then in degrees Celsius? Assume that the volume of the tank is constant. (b) If the temperature is kept at the value found in part (a) and the gas is compressed, what is the volume when the pressure again becomes 3.00 atm?

18.5 • The discussion following Eq. (18.7) gives the constants in the van der Waals equation for gas. It also says that at STP the van der Waals equation gives only a small (0.5%) correction in the ideal-gas equation. Consider 1 mole of gas at and a volume of . (a) What is the pressure of the gas calculated by the ideal-gas equation? (b) What does the van der Waals equation give for the pressure? What is the percentage difference from the ideal-gas result?

18.6 •• You have several identical balloons. You experimentally determine that a balloon will break if its volume exceeds 0.900 L.The pressure of the gas inside the balloon equals air pressure (1.00 atm). (a) If the air inside the balloon is at a constant and behaves as an ideal gas, what mass of air can you blow into one of the balloons before it bursts? (b) Repeat part (a) if the gas is helium rather than air.

18.7 •• A Jaguar XK8 convertible has an eight-cylinder engine. At the beginning of its compression stroke, one of the cylinders contains of air at atmospheric pressure and a temperature of At the end of the stroke, the air has been compressed to a volume of and the gauge pressure has increased to Compute the final temperature.

18.8 •• A welder using a tank of volume fills it with oxygen at a gauge pressure of and temperature of The tank has a small leak, and in time some of the oxygen leaks out. On a day when the temperature is the gauge pressure of the oxygen in the tank is Find (a) the initial mass of oxygen and (b) the mass of oxygen that has leaked out.

18.9 •• A large cylindrical tank contains of nitrogen gas at and (absolute pressure). The tank has a tight-fitting piston that allows the volume to be changed. What will be the pressure if the volume is decreased to and the temperature is increased to ?

18.10 • An empty cylindrical canister 1.50 m long and 90.0 cm in diameter is to be filled with pure oxygen at to store in a space station. To hold as much gas as possible, the absolute pressure of the oxygen will be 21.0 atm. The molar mass of oxygen is (a) How many moles of oxygen does this canister hold? (b) For someone lifting this canister, by how many kilograms does this gas increase the mass to be lifted?

18.11 • The gas inside a balloon will always have a pressure nearly equal to atmospheric pressure, since that is the pressure applied to the outside of the balloon. You fill a balloon with helium (a nearly ideal gas) to a volume of 0.600 L at What is the volume of the balloon if you cool it to the boiling point of liquid nitrogen (77.3 K)?

18.12 • An ideal gas has a density of at and . Identify the gas.

18.13 •• If a certain amount of ideal gas occupies a volume at STP on earth, what would be its volume (in terms of) on Venus, where the temperature is and the pressure is 92 atm?

18.14 • A diver observes a bubble of air rising from the bottom of a lake (where the absolute pressure is 3.50 atm) to the surface (where the pressure is 1.00 atm). The temperature at the bottom is and the temperature at the surface is (a) What is the ratio of the volume of the bubble as it reaches the surface to its volume at the bottom? (b)

Would it be safe for the diver to hold his breath while ascending from the bottom of the lake to the surface? Why or why not?

18.15 • How many moles of an ideal gas exert a gauge pressure of 0.876 atm in a volume of 5.43 L at a temperature of ?

18.16 • Three moles of an ideal gas are in a rigid cubical box with sides of length 0.300 m. (a) What is the force that the gas exerts on each of the six sides of the box when the gas temperature is (b) What is the force when the temperature of the gas is increased to

18.17 •• (a) Calculate the mass of nitrogen present in a volume of if the gas is at and the absolute pressure of is a partial vacuum easily obtained in laboratories. (b) What is the density (in) of the

18.18 • At an altitude of 11,000 m (a typical cruising altitude for a jet airliner), the air temperature is and the air density is What is the pressure of the atmosphere at that altitude? (*Note:* The temperature at this altitude is not the same as at the surface of the earth, so the calculation of Example 18.4 in Section 18.1 doesn't apply.)

Section 18.2 Molecular Properties of Matter

18.19 • How many moles are in a 1.00 kg bottle of water? How many molecules? The molar mass of water is

18.20 • A large organic molecule has a mass of

What is the molar mass of this compound?

18.21 •• Modern vacuum pumps make it easy to attain pressures of the order of in the laboratory. Consider a volume of air and treat the air as an ideal gas. (a) At a pressure of and an ordinary temperature of 300.0 K, how many molecules are present in a volume of (b) How many molecules would be present at the same temperature but at 1.00 atm instead?

18.22 •• The Lagoon Nebula (Fig. E18.22) is a cloud of hydrogen gas located 3900 light-years from the earth. The cloud is about 45 light-years in diameter and glows because of its high temperature of 7500 K. (The gas is raised to this temperature by the stars that lie within the nebula.) The cloud is also very thin; there are only 80 molecules per cubic centimeter. (a) Find the gas pressure (in atmospheres) in the Lagoon Nebula. Compare it to the laboratory pressure referred to in Exercise 18.21. (b) Science-fiction films sometimes show starships being buffeted by turbulence as they fly through gas clouds such as the Lagoon Nebula. Does this seem realistic? Why or why not?

Figure E18.22

18.23 •• **How Close Together Are Gas Molecules?** Consider an ideal gas at and 1.00 atm. To get some idea how close these molecules are to each other, on the average, imagine them to be uniformly spaced, with each molecule at the center of a small cube. (a) What is the length of an edge of each cube if adjacent cubes touch but do not overlap? (b) How does this distance compare with the diameter of a typical molecule? (c) How does their separation compare with the spacing of atoms in solids, which typically are about 0.3 nm apart?

Section 18.3 Kinetic-Molecular Model of an Ideal Gas

18.24 •• A container with rigid walls holds moles of a monatomic ideal gas. In terms of , how many moles of the gas must be removed from the container to double the pressure while also doubling the rms speed of the gas atoms?

18.25 • (a) What is the total translational kinetic energy of the air in an empty room that has dimensions if the air is treated as an ideal gas at 1.00 atm? (b) What is the speed of a 2000 kg automobile if its kinetic energy equals the translational kinetic energy calculated in part (a)?

18.26 • A flask contains a mixture of neon (Ne), krypton (Kr), and radon (Rn) gases. Compare (a) the average kinetic energies of the three types of atoms and (b) the root-mean-square speeds. (*Hint:* Appendix D shows the molar mass (in) of each element under the chemical symbol for that element.)

18.27 • A container with volume and rigid walls holds a monatomic ideal gas. To determine the number of gas atoms in the container, you measure the pressure of the gas in atmospheres as a function of the Celsius temperature of the gas. You plot versus and find that your data lie close to a straight line that has slope . What is your experimental result for the number of gas atoms?

18.28 • A container with volume 1.64 L is initially evacuated. Then it is filled with 0.226 g of . Assume that the pressure of the gas is low enough for the gas to obey the ideal-gas law to a high degree of accuracy. If the root-mean-square speed of the gas molecules is , what is the pressure of the gas?

18.29 •• (a) A deuteron, , is the nucleus of a hydrogen isotope and consists of one proton and one neutron. The plasma of deuterons in a nuclear fusion reactor must be heated to about 300 million K. What is the rms speed of the deuterons? Is this a significant fraction of the speed of light in vacuum (b) What would the temperature of the plasma be if the deuterons had an rms speed equal to ?

18.30 • **Martian Climate.** The atmosphere of Mars is mostly (molar mass) under a pressure of 650 Pa, which we shall assume remains constant. In many places the temperature varies from in summer to in winter. Over the course of a Martian year, what are the ranges of (a) the rms speeds of the molecules and (b) the density (in) of the atmosphere?

18.31 •• Oxygen has a molar mass of What is (a) the average translational kinetic energy of an oxygen molecule at a temperature of 300 K; (b) the average value of the square of its speed; (c) the root-mean-square speed; (d) the momentum of an oxygen molecule traveling at this speed? (e) Suppose an oxygen molecule traveling at this speed bounces back and forth between opposite sides of a cubical vessel 0.10 m on a side. What is the average force the molecule exerts on one of the walls of the container? (Assume that the molecule's velocity is perpendicular to the two sides that it strikes.) (f) What is the average force per unit area? (g) How many oxygen molecules traveling at this speed are necessary to produce an average pressure of 1 atm? (h) Compute the number of oxygen molecules that are contained in a vessel of this size at 300 K and atmospheric pressure. (i) Your answer for part (h) should be three times as large as the answer for part (g). Where does this discrepancy arise?

18.32 •• Calculate the mean free path of air molecules at and 300 K. (This pressure is readily attainable in the laboratory; see Exercise 18.21.) As in Example 18.8, model the air molecules as spheres of radius

18.33 •• At what temperature is the root-mean-square speed of nitrogen molecules equal to the root-mean-square speed of hydrogen molecules at (*Hint:* Appendix D shows the molar mass (in) of each element under the chemical symbol for that element. The molar mass of is twice the molar mass of hydrogen atoms, and similarly for)

18.34 • Smoke particles in the air typically have masses on the order of The Brownian motion (rapid, irregular movement) of these particles, resulting from collisions with air molecules, can be observed with a microscope. (a) Find the root-mean-square speed of Brownian motion for a particle with a mass of in air at 300 K. (b) Would the root-mean-square speed be different if the particle were in hydrogen gas at the same temperature? Explain.

Section 18.4 Heat Capacities

18.35 •• Three moles of helium gas (molar mass are in a rigid container that keeps the volume of the gas constant. Initially the rms speed of the gas atoms is . What is the rms speed of the gas atoms after 2400 J of heat energy is added to the gas?

18.36 •• A rigid container holds 4.00 mol of a monatomic ideal gas that has temperature 300 K. The initial pressure of the gas is . What is the pressure after 6000 J of heat energy is added to the gas?

18.37 • How much heat does it take to increase the temperature of 1.80 mol of an ideal gas by 50.0 K near room temperature if the gas is held at constant volume and is (a) diatomic; (b) monatomic?

18.38 •• Perfectly rigid containers each hold moles of ideal gas, one being hydrogen and the other being neon If it takes 300 J of heat to increase the temperature of the hydrogen by by how many degrees will the same amount of heat raise the temperature of the neon?

18.39 •• (a) Compute the specific heat at constant volume of nitrogen gas, and compare it with the specific heat of liquid water. The molar mass of is (b) You warm 1.00 kg of water at a constant volume of 1.00 L from to in a kettle. For the same amount of heat, how many kilograms of air would you be able to warm to What volume (in liters) would this air occupy at and a pressure of 1.00 atm? Make the simplifying assumption that air is 100%

18.40 •• (a) Calculate the specific heat at constant volume of water vapor, assuming the nonlinear triatomic molecule has three translational and three rotational degrees of freedom and that vibrational motion does not contribute. The molar mass of water is (b) The actual specific heat of water vapor at low pressures is about Compare this with your calculation and comment on the actual role of vibrational motion.

Section 18.5 Molecular Speeds

18.41 • For diatomic carbon dioxide gas (molar mass) at calculate (a) the most probable speed (b) the average speed (c) the root-mean-square speed

18.42 • For a gas of nitrogen molecules what must the temperature be if 94.7% of all the molecules have speeds less than (a) (b) (c) Use Table 18.2. The molar mass of is

18.43 • The speed of sound for an ideal gas is given by [Eq. (16.10)]. We'll see in Chapter 19 that for a monatomic ideal gas, . (a) What is the ratio ? (b) The average speed of the gas atoms is given by Eq. (18.35). What is the ratio ?

18.44 • In 0.0345 mol of a monatomic ideal gas, how many of the atoms have speeds that are within 20% of the rms speed? (Use Table 18.2.)

Section 18.6 Phases of Matter

18.45 • Solid water (ice) is slowly warmed from a very low temperature. (a) What minimum external pressure must be applied to the solid if a melting phase transition is to be observed? Describe the sequence of phase transitions that occur if the applied pressure is such that (b) Above a certain maximum pressure no boiling transition is observed. What is this pressure? Describe the sequence of phase transitions that occur if

18.46 • **Meteorology.** The *vapor pressure* is the pressure of the vapor phase of a substance when it is in equilibrium with the solid or liquid phase of the substance. The *relative humidity* is the partial pressure of water vapor in the air divided by the vapor pressure of water at that same temperature, expressed as a percentage. The air is saturated when the humidity is 100%. (a) The vapor pressure of water at is Pa. If the air temperature is and the relative humidity is 60%, what is the partial pressure of water vapor in the atmosphere (that is, the pressure due to water vapor alone)? (b) Under the conditions of part (a), what is the mass of water in of air? (The molar mass of water is Assume that water vapor can be treated as an ideal gas.)

Problems

18.47 •• **CP** Imagine the sound made when a latex balloon with a diameter of 30 cm pops 2 m from your ear. Estimate the sound intensity level in decibels, using Table 16.2 as a guide. (a) Assuming the duration of the popping event was 100 ms, use the intensity of the popping sound to determine the average power of the pop and the energy released in the pop. This provides an estimate of the energy stored in the balloon prior to the pop. (b) Let be the ratio of the energy stored in the stretched balloon to its surface area. Use your estimate of the stored energy and the size of the balloon to estimate the value of . (c) The gauge pressure of a latex balloon depends on and radius according to . Use this information to estimate the gauge pressure in the balloon.

18.48 • A physics lecture room at 1.00 atm and has a volume of (a) Use the ideal-gas law to estimate the number of air molecules in the room. Assume that all of the air is Calculate (b) the particle density—that is, the number of molecules per cubic centimeter—and (c) the mass of the air in the room.

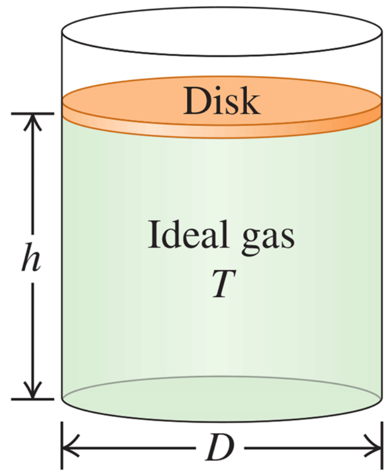
18.49 •• CP BIO **The Effect of Altitude on the Lungs.** (a) Calculate the *change* in air pressure you'll experience if you climb a 1000 m mountain, assuming for simplicity that the temperature and air density do not change over this distance and that they were and respectively, at the bottom of the mountain. (*Note:* The result of Example 18.4 doesn't apply, since the expression derived in that example accounts for the variation of air density with altitude and we are told to ignore that here.) (b) If you took a 0.50 L breath at the foot of the mountain and managed to hold it until you reached the top, what would be the volume of this breath when you exhaled it there?

18.50 •• CP BIO **The Bends.** If deep-sea divers rise to the surface too quickly, nitrogen bubbles in their blood can expand and prove fatal. This phenomenon is known as the *bends.* If a scuba diver rises quickly from a depth of 25 m in Lake Michigan (which is fresh water), what will be the volume at the surface of an bubble that occupied in his blood at the lower depth? Does it seem that this difference is large enough to be a problem? (Assume that the pressure difference is due to only the changing water pressure, not to any temperature difference. This assumption is reasonable, since we are warm-blooded creatures.)

18.51 ••• **CP** A hot-air balloon stays aloft because hot air at atmospheric pressure is less dense than cooler air at the same pressure. If the volume of the balloon is and the surrounding air is at 15.0°C, what must the temperature of the air in the balloon be for it to lift a total load of 290 kg (in addition to the mass of the hot air)? The density of air at and atmospheric pressure is

18.52 •• In an evacuated enclosure, a vertical cylindrical tank of diameter is sealed by a 3.00 kg circular disk that can move up and down without friction. Beneath the disk is a quantity of ideal gas at temperature in the cylinder (Fig. P18.52). Initially the disk is at rest at a distance of above the bottom of the tank. When a lead brick of mass 9.00 kg is gently placed on the disk, the disk moves downward. If the temperature of the gas is kept constant and no gas escapes from the tank, what distance above the bottom of the tank is the disk when it again comes to rest?

Figure P18.52



18.53 ••• A cylinder 1.00 m tall with inside diameter 0.120 m is used to hold propane gas (molar mass) for use in a barbecue. It is initially filled with gas until the gauge pressure is at The temperature of the gas remains constant as it is partially emptied out of the tank, until the gauge pressure is Pa. Calculate the mass of propane that has been used.

18.54 • CP During a test dive in 1939, prior to being accepted by the U.S. Navy, the submarine *Squalus* sank at a point where the depth of water was The temperature was at the surface and at the bottom. The density of seawater is (a) A diving bell was used to rescue 33 trapped crewmen from the *Squalus*. The diving bell was in the form of a circular cylinder 2.30 m high, open at the bottom and closed at the top. When the diving bell was lowered to the bottom of the sea, to what height did water rise within the diving bell? (*Hint:* Ignore the relatively small variation in water pressure between the bottom of the bell and the surface of the water within the bell.) (b) At what gauge pressure must compressed air have been supplied to the bell while on the bottom to expel all the water from it?

18.55 •• A parcel of air over a campfire feels an upward buoyant force because the heated air is less dense than the surrounding air. By estimating the acceleration of the air immediately above a fire, one can estimate the fire's temperature. The mass of a volume of air is , where is the number of moles of air molecules in the volume and is the molar mass of air. The net upward force on a parcel of air above a fire is roughly given by , where is the mass of a volume of ambient air and is the mass of a similar volume of air in the hot zone. (a) Use the ideal-gas law, along with the knowledge that the pressure of the air above the fire is the same as that of the ambient air, to derive an expression for the acceleration of an air parcel as a function of , where is the absolute temperature of the air above the fire and is the absolute temperature of the ambient air. (b) Rearrange your formula from part (a) to obtain an expression for as a function of and . (c) Based on your experience with campfires, estimate the acceleration of the air above the fire by comparing in your mind the upward trajectory of sparks with the acceleration of falling objects. Thus you can estimate as a multiple of . (d) Assuming an ambient temperature of , use your formula and your estimate of to estimate the temperature of the fire.

18.56 • **Pressure on Venus.** At the surface of Venus the average temperature is a balmy due to the greenhouse effect (global warming!), the pressure is 92 earth-atmospheres, and the acceleration due to gravity is The atmosphere is nearly all (molar mass), and the temperature remains remarkably constant. Assume that the temperature does not change with altitude. (a) What is the atmospheric pressure 1.00 km above the surface of Venus? Express your answer in Venus-atmospheres and earth-atmospheres. (b) What is the root-mean-square speed of the molecules at the surface of Venus and at an altitude of 1.00 km?

18.57 **••• CP CALC** A cylindrical diving bell has a radius of 750 cm and a height of 2.50 m. The bell includes a top compartment that holds an undersea adventurer. A bottom compartment separated from the top by a sturdy grating holds a tank of compressed air with a valve to release air into the bell, a second valve that can release air from the bell into the sea, a third valve that regulates the entry of seawater for ballast, a pump that removes the ballast to increase buoyancy, and an electric heater that maintains a constant temperature of . The total mass of the bell and all of its apparatuses is 4350 kg. The density of seawater is . (a) An 80.0 kg adventurer enters the bell. How many liters of seawater should be moved into the bell so that it is neutrally buoyant? (b) By carefully regulating ballast, the bell is made to descend into the sea at a rate of . Compressed air is released from the tank to raise the pressure in the bell to match the pressure of the seawater outside the bell. As the bell descends, at what rate should air be released through the first valve? (*Hint:* Derive an expression for the number of moles of air in the bell as a function of depth ; then differentiate this to obtain as a function of .) (c) If the compressed air tank is a fully loaded, specially designed, 600 ft^3 tank, which means it contains that volume of air at standard temperature and pressure (and 1 atm), how deep can the bell descend?

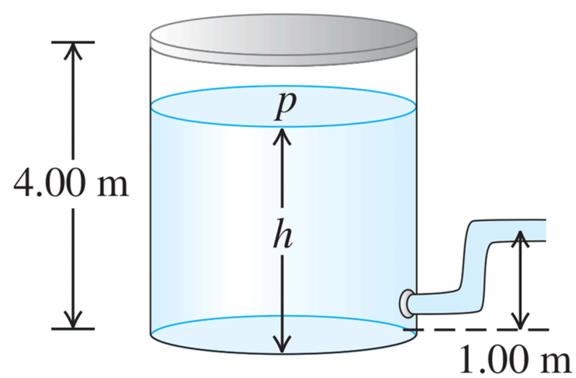
18.58 •• A flask with a volume of 1.50 L, provided with a stopcock, contains ethane gas at 300 K and atmospheric pressure The molar mass of ethane is The system is warmed to a temperature of 550 K, with the stopcock open to the atmosphere. The stopcock is then closed, and the flask is cooled to its original temperature. (a) What is the final pressure of the ethane in the flask? (b) How many grams of ethane remain in the flask?

18.59 •• CP A balloon of volume is to be filled with hydrogen at atmospheric pressure (a) If the hydrogen is stored in cylinders with volumes of at a gauge pressure of how many cylinders are required? Assume that the temperature of the hydrogen remains constant. (b) What is the total weight (in addition to the weight of the gas) that can be supported by the balloon if both the gas in the balloon and the surrounding air are at The molar mass of hydrogen is The density of air at and atmospheric pressure is See Chapter 12 for a discussion of buoyancy. (c) What weight could be supported if the balloon were filled with helium (molar mass) instead of hydrogen, again at

18.60 •• A vertical cylindrical tank contains 1.80 mol of an ideal gas under a pressure of 0.300 atm at The round part of the tank has a radius of 10.0 cm, and the gas is supporting a piston that can move up and down in the cylinder without friction. There is a vacuum above the piston. (a) What is the mass of this piston? (b) How tall is the column of gas that is supporting the piston?

18.61 •• CP A large tank of water has a hose connected to it (Fig. P18.61). The tank is sealed at the top and has compressed air between the water surface and the top. When the water height has the value 3.50 m, the absolute pressure of the compressed air is Assume that the air above the water expands at constant temperature, and take the atmospheric pressure to be (a) What is the speed with which water flows out of the hose when (b) As water flows out of the tank, decreases. Calculate the speed of flow for and for (c) At what value of does the flow stop?

Figure P18.61



18.62 •• CP A light, plastic sphere with mass and density is suspended in air by thread of negligible mass. (a) What is the tension in the thread if the air is at and ? The molar mass of air is . (b) How much does the tension in the thread change if the temperature of the gas is increased to ? Ignore the change in volume of the plastic sphere when the temperature is changed.

18.63 •• BIO **How Many Atoms Are You?** Estimate the number of atoms in the body of a physics student. Note that the human body is mostly water, which has molar mass and that each water molecule contains three atoms.

18.64 •• BIO A person at rest inhales 0.50 L of air with each breath at a pressure of 1.00 atm and a temperature of The inhaled air is 21.0% oxygen. (a) How many oxygen molecules does this person inhale with each breath? (b) Suppose this person is now resting at an elevation of 2000 m but the temperature is still Assuming that the oxygen percentage and volume per inhalation are the same as stated above, how many oxygen molecules does this person now inhale with each breath? (c) Given that the body still requires the same number of oxygen molecules per second as at sea level to maintain its functions, explain why some people report “shortness of breath” at high elevations.

18.65 •• You blow up a spherical balloon to a diameter of 50.0 cm until the absolute pressure inside is 1.25 atm and the temperature is Assume that all the gas is of molar mass (a) Find the mass of a single molecule. (b) How much translational kinetic energy does an average molecule have? (c) How many molecules are in this balloon? (d) What is the *total* translational kinetic energy of all the molecules in the balloon?

18.66 • The size of an oxygen molecule is about Make a rough estimate of the pressure at which the finite volume of the molecules should cause noticeable deviations from ideal-gas behavior at ordinary temperatures

18.67 •• CP CALC **The Lennard-Jones Potential.** A commonly used potential-energy function for the interaction of two molecules (see Fig. 18.8) is the Lennard-Jones 6-12 potential:

where is the distance between the centers of the molecules and and are positive constants. The corresponding force is given in Eq. (14.26). (a) Graph and versus . (b) Let be the value of at which and let be the value of at which Show the locations of and on your graphs of and Which of these values represents the equilibrium separation between the molecules? (c) Find the values of and in terms of and find the ratio (d) If the molecules are located a distance apart [as calculated in part (c)], how much work must be done to pull them apart so that ?

18.68 • (a) What is the total random translational kinetic energy of 5.00 L of hydrogen gas (molar mass) with pressure Pa and temperature 300 K? (*Hint:* Use the procedure of Problem 18.65 as a guide.) (b) If the tank containing the gas is placed on a swift jet moving at by what percentage is the *total* kinetic energy of the gas increased? (c) Since the kinetic energy of the gas molecules is greater when it is on the jet, does this mean that its temperature has gone up? Explain.

18.69 •• It is possible to make crystalline solids that are only one layer of atoms thick. Such "two-dimensional" crystals can be created by depositing atoms on a very flat surface. (a) If the atoms in such a two-dimensional crystal can move only within the plane of the crystal, what will be its molar heat capacity near room temperature? Give your answer as a multiple of and in (b) At very low temperatures, will the molar heat capacity of a two-dimensional crystal be greater than, less than, or equal to the result you found in part (a)? Explain why.

18.70 • **Hydrogen on the Sun.** The surface of the sun has a temperature of about 5800 K and consists largely of hydrogen atoms. (a) Find the rms speed of a hydrogen atom at this temperature. (The mass of a single hydrogen atom is) (b) The escape speed for a particle to leave the gravitational influence of the sun is given by where is the sun's mass, its radius, and the gravitational constant (see Example 13.5 of Section 13.3). Use Appendix F to calculate this escape speed. (c) Can appreciable quantities of hydrogen escape from the sun? Can *any* hydrogen escape? Explain.

18.71 •• CP (a) Show that a projectile with mass can "escape" from the surface of a planet if it is launched vertically upward with a kinetic energy greater than where is the acceleration due to gravity at the planet's surface and is the planet's radius. Ignore air resistance. (See Problem 18.70.) (b) If the planet in question is the earth, at what temperature does the average translational kinetic energy of a nitrogen molecule (molar mass) equal that required to escape? What about a hydrogen molecule (molar mass)? (c) Repeat part (b) for the moon, for which and (d) While the earth and the moon have similar average surface temperatures, the moon has essentially no atmosphere. Use your results from parts (b) and (c) to explain why.

18.72 •• Helium gas is in a cylinder that has rigid walls. If the pressure of the gas is 2.00 atm, then the root-mean-square speed of the helium atoms is . By how much (in atmospheres) must the pressure be increased to increase the of the He atoms by ? Ignore any change in the volume of the cylinder.

18.73 •• **CALC** Calculate the integral in Eq. (18.31), and compare this result to as given by Eq. (18.16). (*Hint:* You may use the tabulated integral

where is a positive integer and is a positive constant.)

18.74 •• (a) Calculate the total *rotational* kinetic energy of the molecules in 1.00 mol of a diatomic gas at 300 K. (b) Calculate the moment of inertia of an oxygen molecule for rotation about either the or shown in Fig. 18.18b. Treat the molecule as two massive points (representing the oxygen atoms) separated by a distance of The molar mass of oxygen *atoms* is (c) Find the rms angular velocity of rotation of an oxygen molecule about either the or shown in Fig. 18.18b. How does your answer compare to the angular velocity of a typical piece of rapidly rotating machinery

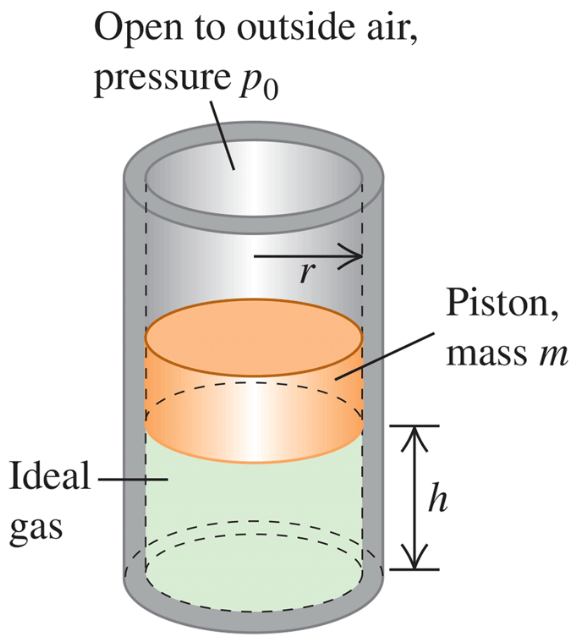
18.75 •• **CALC** (a) Explain why in a gas of molecules, the number of molecules having speeds in the *finite* interval to is (b) If is small, then is approximately constant over the interval and For oxygen gas at use this approximation to calculate the number of molecules with speeds within of Express your answer as a multiple of . (c) Repeat part (b) for speeds within of (d) Repeat parts (b) and (c) for a temperature of 600 K. (e) Repeat parts (b) and (c) for a temperature of 150 K. (f) What do your results tell you about the shape of the distribution as a function of temperature? Do your conclusions agree with what is shown in Fig. 18.23?

18.76 •• **CALC** Calculate the integral in Eq. (18.30), and compare this result to as given by Eq. (18.35). (*Hint:* Make the change of variable and use the tabulated integral

where is a positive integer and is a positive constant.)

18.77 ••• **CP** **Oscillations of a Piston.** A vertical cylinder of radius contains an ideal gas and is fitted with a piston of mass that is free to move (Fig. P18.77). The piston and the walls of the cylinder are frictionless, and the entire cylinder is placed in a constant-temperature bath. The outside air pressure is In equilibrium, the piston sits at a height above the bottom of the cylinder. (a) Find the absolute pressure of the gas trapped below the piston when in equilibrium. (b) The piston is pulled up by a small distance and released. Find the net force acting on the piston when its base is a distance above the bottom of the cylinder, where (c) After the piston is displaced from equilibrium and released, it oscillates up and down. Find the frequency of these small oscillations. If the displacement is not small, are the oscillations simple harmonic? How can you tell?

Figure P18.77



18.78 •• **DATA** A steel cylinder with rigid walls is evacuated to a high degree of vacuum; you then put a small amount of helium into the cylinder. The cylinder has a pressure gauge that measures the pressure of the gas inside the cylinder. You place the cylinder in various temperature environments, wait for thermal equilibrium to be established, and then measure the pressure of the gas. You obtain these results:

	T (°C)	p (Pa)
Normal boiling point of nitrogen	−195.8	254
Ice–water mixture	0.0	890
Outdoors on a warm day	33.3	999
Normal boiling point of water	100.0	1214
Hot oven	232	1635

(a) Recall (Chapter 17) that absolute zero is the temperature at which the pressure of an ideal gas becomes zero. Use the data in the table to calculate the value of absolute zero in . Assume that the pressure of the gas is low enough for it to be treated as an ideal gas, and ignore the change in volume of the cylinder as its temperature is changed. (b) Use the coefficient of volume expansion for steel in Table 17.2 to calculate the percentage change in the volume of the cylinder between the lowest and highest temperatures in the table. Is it accurate to ignore the volume change of the cylinder as the temperature changes? Justify your answer.

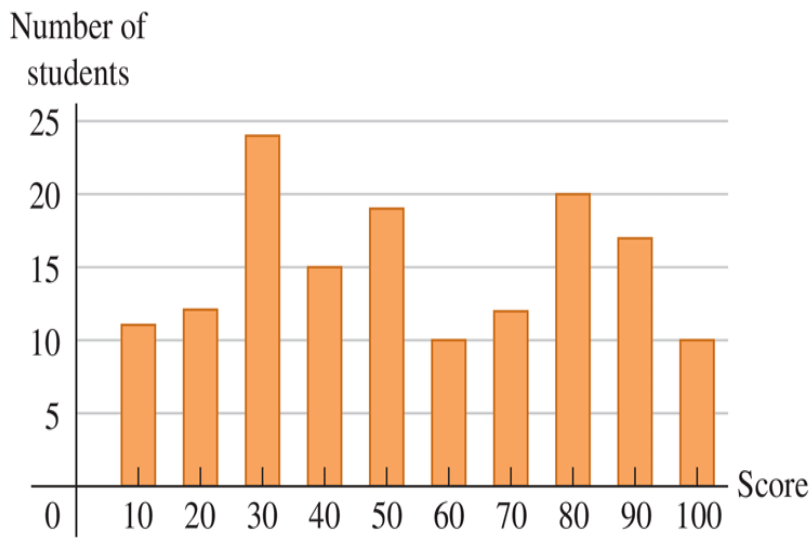
18.79 ••• DATA **The Dew Point and Clouds.** The vapor pressure of water (see Exercise 18.46) decreases as the temperature decreases. The table lists the vapor pressure of water at various temperatures:

Temperature (°C)	Vapor Pressure (Pa)
10.0	1.23×10^3
12.0	1.40×10^3
14.0	1.60×10^3
16.0	1.81×10^3
18.0	2.06×10^3
20.0	2.34×10^3
22.0	2.65×10^3
24.0	2.99×10^3
26.0	3.36×10^3
28.0	3.78×10^3
30.0	4.25×10^3

If the amount of water vapor in the air is kept constant as the air is cooled, the *dew point* temperature is reached, at which the partial pressure and vapor pressure coincide and the vapor is saturated. If the air is cooled further, the vapor condenses to liquid until the partial pressure again equals the vapor pressure at that temperature. The temperature in a room is (a) A meteorologist cools a metal can by gradually adding cold water. When the can's temperature reaches water droplets form on its outside surface. What is the relative humidity of the air in the room? On a spring day in the midwestern United States, the air temperature at the surface is Puffy cumulus clouds form at an altitude where the air temperature equals the dew point. If the air temperature decreases with altitude at a rate of at approximately what height above the ground will clouds form if the relative humidity at the surface is (b) 35%; (c) 80%?

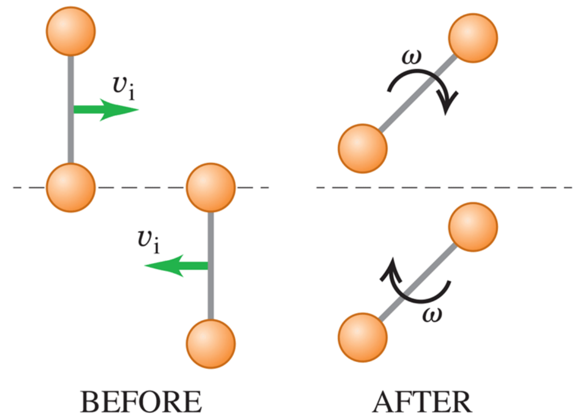
18.80 •• **DATA** The statistical quantities "average value" and "root-mean-square value" can be applied to any distribution. Figure P18.80 shows the scores of a class of 150 students on a 100 point quiz. (a) Find the average score for the class. (b) Find the rms score for the class. (c) Which is higher: the average score or the rms score? Why?

Figure P18.80



18.81 ••• CP We can crudely model a nitrogen molecule as a pair of small balls, each with the mass of a nitrogen atom, , attached by a rigid massless rod with length . (a) What is the moment of inertia of this molecule with respect to an axis passing through the midpoint and perpendicular to the molecular axis? (b) Consider air at 1 atm pressure and temperature. Suppose two nitrogen molecules with rms speeds have an elastic collision such that only one nitrogen atom in one molecule collides with one nitrogen atom in the other molecule in the manner depicted in Fig. P18.81. Write equations for the conservation of energy and the conservation of angular momentum, in terms of , , , and , where and are, respectively, the center-of-mass speed and the angular speed of either molecule after the collision. (c) Solve these equations for and in terms of and . Note that after the collision. (d) Using for the conditions specified above, what is the frequency of rotation?

Figure P18.81



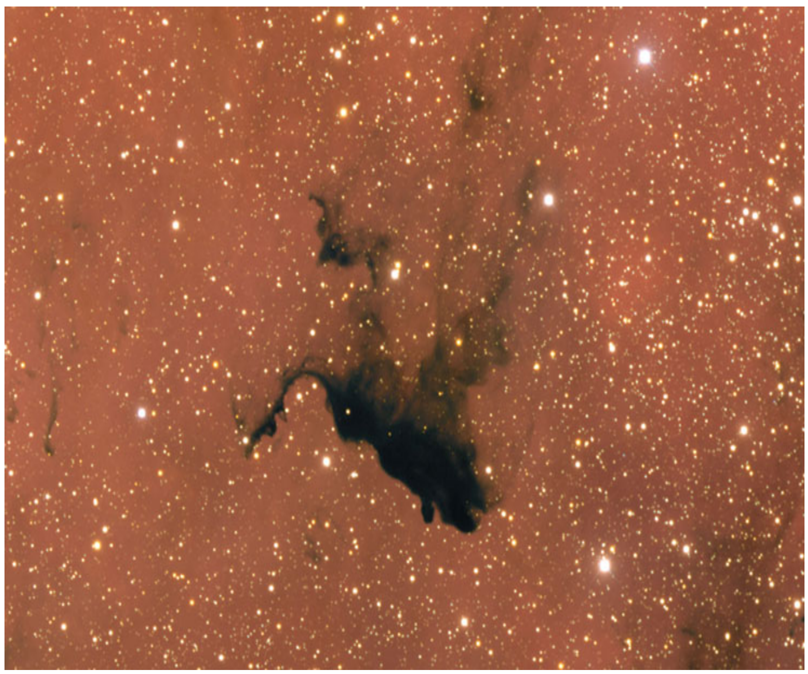
Challenge Problems

18.82 ••• A pneumatic lift consists of a vertical cylinder with a radius of 10.0 cm. A movable piston slides within the cylinder at its upper end and supports a platform on which loads are placed. An intake valve allows compressed air from a tank to enter the cylinder, and an exhaust valve allows air to be removed from the cylinder. In either case the rate of air transfer is sufficiently low that the temperature in the cylinder remains constant. When neither valve is activated, the cylinder is airtight. The piston and platform together have a mass of 50.0 kg, the temperature is and the pressure outside the cylinder is 1.00 atm. (a) There is 1.00 mol of air in the cylinder and no load on the platform. What is the height between the bottom of the piston and the bottom of the cylinder? (b) A 200 kg load is placed on the platform. By what distance does the platform drop? (c) The intake valve is activated and compressed air enters the cylinder so that the platform moves back to its original height. How many moles of air were introduced? (d) How many more moles of air should be introduced so that the platform rises 2.00 m above its original height? (e) At what rate should this air be introduced, in , so that the platform rises at a rate of ?

18.83 ••• **CP** **Dark Nebulae and the Interstellar Medium.** The dark area in Fig. P18.83 that appears devoid of stars is a *dark nebula,* a cold gas cloud in interstellar space that contains enough material to block out light from the stars behind it. A typical dark nebula is about 20 light-years in diameter and contains about 50 hydrogen atoms per cubic centimeter (monatomic hydrogen, *not*) at about 20 K. (A light-year is the distance light travels in vacuum in one year and is equal to) (a) Estimate the mean free path for a hydrogen atom in a dark nebula. The radius of a hydrogen atom is (b) Estimate the rms speed of a hydrogen atom and the mean free time (the average time between collisions for a given atom). Based on this result, do you think that atomic collisions, such as those leading to molecule formation, are very important in determining the composition of the nebula? (c) Estimate the pressure inside a dark nebula. (d) Compare the rms speed of a hydrogen atom to the escape speed at the surface of the nebula (assumed spherical). If the space around the nebula were a vacuum, would such a cloud be stable or would it tend to evaporate? (e) The stability of dark nebulae is explained by the presence of the *interstellar medium* (ISM), an even thinner gas that permeates space and in which the dark nebulae are embedded. Show that for dark nebulae to be in equilibrium with the ISM, the numbers of atoms per volume and the temperatures of dark nebulae and the ISM must be related by

(f) In the vicinity of the sun, the ISM contains about 1 hydrogen atom per Estimate the temperature of the ISM in the vicinity of the sun. Compare to the temperature of the sun's surface, about 5800 K. Would a spacecraft coasting through interstellar space burn up? Why or why not?

Figure P18.83



18.84 ••• CALC **Earth's Atmosphere.** In the *troposphere,* the part of the atmosphere that extends from earth's surface to an altitude of about the temperature is not uniform but decreases with increasing elevation. (a) Show that if the temperature variation is approximated by the linear relationship where is the temperature at the earth's surface and is the temperature at height , the pressure at height is

where is the pressure at the earth's surface and is the molar mass for air. The coefficient is called the lapse rate of temperature. It varies with atmospheric conditions, but an average value is about (b) Show that the above result reduces to the result of Example 18.4 (Section 18.1) in the limit that (c) With calculate for and compare your answer to the result of Example 18.4. Take and

MCAT-Style Passage Problems

Insulating Windows. One way to improve insulation in windows is to fill a sealed space between two glass panes with a gas that has a lower thermal conductivity than that of air. The thermal conductivity of a gas depends on its molar heat capacity , molar mass , and molecular radius *r*. The dependence on those quantities at a given temperature is approximated by . The noble gases have properties that make them particularly good choices as insulating gases. Noble gases range from helium (molar mass , molecular radius 0.13 nm) to xenon (molar mass , molecular radius 0.22 nm). (The noble gas radon is heavier than xenon, but radon is radioactive and so is not suitable for this purpose.)

18.85 What is one reason the noble gases are *preferable* to air (which is mostly nitrogen and oxygen) as an insulating material? (a) Noble gases are monatomic, so no rotational modes contribute to their molar heat capacity; (b) noble gases are monatomic, so they have lower molecular masses than do nitrogen and oxygen; (c) molecular radii in noble gases are much larger than those of gases that consist of diatomic molecules; (d) because noble gases are monatomic, they have many more degrees of freedom than do diatomic molecules, and their molar heat capacity is reduced by the number of degrees of freedom.

18.86 Estimate the ratio of the thermal conductivity of Xe to that of He. (a) 0.015; (b) 0.061; (c) 0.10; (d) 0.17.

18.87 The rate of *effusion*—that is, leakage of a gas through tiny cracks—is proportional to If tiny cracks exist in the material that's used to seal the space between two glass panes, how many times greater is the rate of He leakage out of the space between the panes than the rate of Xe leakage at the same temperature? (a) 370 times; (b) 19 times; (c) 6 times; (d) no greater—the He leakage rate is the same as for Xe.

Answers: Thermal Properties of Matter

Chapter Opening Question ?

(iv) From Eq. (18.19), the root-mean-square speed of a gas molecule is proportional to the square root of the absolute temperature T. The temperature range we're considering is $(25 + 273.15)\ \mathrm{K}\ = 298\ \mathrm{K}$ to $(100 + 273.15)\ \mathrm{K} = 373\ \mathrm{K}$. Hence the speeds increase by a factor of $\sqrt{(373\ \mathrm{K})/(298\ \mathrm{K})} = 1.12$; that is, there is a 12% increase. While $100^\circ\mathrm{C}$ feels far warmer than $25^\circ\mathrm{C}$, the difference in molecular speeds is relatively small.

Test Your Understanding

18.1 **(ii) and (iii) (tie), (i) and (v) (tie), (iv)** We can rewrite the ideal-gas equation, Eq. (18.3), as $n = pV/RT$. This tells us that the number of moles n is proportional to the pressure and volume and inversely proportional to the absolute temperature. Hence, compared to (i), the number of moles in each case is (ii) $(2)(1)/(1) = 2$ times as much, (iii) $(1)(2)/(1) = 2$ times as much, (iv) $(1)(1)/(2) = \frac{1}{2}$ as much, and (v) $(2)(1)/(2)\ = 1$ time as much (that is, equal).

18.2 **(vi)** The value of r_0 determines the equilibrium separation of the molecules in the solid phase, so doubling r_0 means that the separation doubles as well. Hence a solid cube of this compound might grow from 1 cm on a side to 2 cm on a side. The volume would then be $2^3 = 8$ times larger, and the density (mass divided by volume) would be $\frac{1}{8}$ as great.

18.3 (a) **(iv), (ii), (iii), (i);** (b) **(iii) and (iv) (tie), (i) and (ii) (tie)** (a) Equation (18.19) tells us that $v_{\mathrm{rms}} = \sqrt{3RT/M}$, so the rms speed is proportional to the square root of the ratio of absolute temperature T to molar mass M. Compared to (i) oxygen at 300 K, v_{rms} in the other cases is (ii) $\sqrt{(32.0\ \mathrm{g/mol})/(28.0\ \mathrm{g/mol})}\ = 1.07$ times faster, (iii)

$\sqrt{(330 \text{ K})/(300 \text{ K})} = 1.05$ times faster, and (iv) $\sqrt{(330 \text{ K})(32.0 \text{ g/mol})/(300 \text{ K})(28.0 \text{ g/mol})} = 1.12$ times faster. (b) From Eq. (18.16), the average translational kinetic energy per molecule is $\frac{1}{2} m(v^2)_{\text{av}} = \frac{3}{2} kT$, which is directly proportional to T and independent of M. We have $T = 300$ K for cases (i) and (ii) and $T = 330$ K for cases (iii) and (iv), so $\frac{1}{2} m(v^2)_{\text{av}}$ has equal values for cases (iii) and (iv) and equal (but smaller) values for cases (i) and (ii).

18.4 **no; near the beginning** Adding a small amount of heat dQ to the gas changes the temperature by dT, where $dQ = nC_V\, dT$ from Eq. (18.24). Figure 18.19 shows that C_V for H_2 varies with temperature between 25 K and 500 K, so a given amount of heat gives rise to different amounts of temperature change during the process. Hence the temperature will *not* increase at a constant rate. The temperature change $dT = dQ/nC_V$ is inversely proportional to C_V, so the temperature increases most rapidly at the beginning of the process when the temperature is lowest and C_V is smallest (see Fig. 18.19).

18.5 **(ii)** Figure 18.23b shows that the *fraction* of molecules with speeds between v_1 and v_2 equals the area under the curve of $f(v)$ versus v from $v = v_1$ to $v = v_2$. This is equal to the integral $\int_{v_1}^{v_2} f(v)\, dv$ which in turn is equal to the difference between the integrals $\int_0^{v_2} f(v)\, dv$ (the fraction of molecules with speeds from 0 to v_2) and $\int_0^{v_2} f(v)\, dv$ (the fraction of molecules with speeds from 0 to the slower speed v_1). The *number* of molecules with speeds from v_1 to v_2 equals the fraction of molecules in this speed range multiplied by N, the total number of molecules.

18.6 **no, yes** The triple-point pressure of water from Table 18.3 is 6.10×10^2 Pa. The present-day pressure on Mars is just less than this value, corresponding to the line labeled p_s in Fig. 18.24. Hence liquid water cannot exist on the present-day

Martian surface, and there are no rivers or lakes. Planetary scientists conclude that liquid water could have existed and almost certainly did exist on Mars in the past, when the atmosphere was thicker.

Key Example Variation Problems

VP18.4.1

a. $4.67 \times 10^5 \text{Pa}$

b. 0.278 mol

VP18.4.2

a. $1.20 \times 10^3 \text{ m}^3$

b. 4.52 times

VP18.4.3

a. $2.9 \times 10^{-3} \text{ mol/m}^3$

b. $8.0 \times 10^{-5} \text{ kg/m}^3$

VP18.4.4

a. $T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{2/5}$

b. $\frac{\rho_2}{\rho_1} = 0.660, \ \frac{T_2}{T_1} = 0.758$

c. $\frac{\rho_2}{\rho_1} - 1.52, \ \frac{T_2}{T_1} = 1.32$

VP18.7.1 -231°C

VP18.7.2

a. $6.2 \times 10^{-20} \text{ J}$

b. $8.6 \times 10^3 \text{ m/s}$

VP18.7.3

a. 1.5×10^{27}

b. $9.1 \times 10^6 \text{J}$

c. $1.1 \times 10^2 \text{ m/s}$, or $4.0 \times 10^2 \ km/\text{h}$

VP18.7.4

a. 5.50

b. 38.5

c. 6.20

VP18.8.1

a. $6.8 \times 10^{-6} \text{ m}$

b. 1.2×10^2 times greater

VP18.8.2

a. $5.7 \times 10^{-3} \text{ Pa}$

b. $2.3 \times 10^{-6} \text{ mol}$

VP18.8.3

a. $1.2 \times 10^{-8} \text{ m}$

b. 8.9×10^{-12}

VP18.8.4 **a.** $t = \frac{1}{4\pi r^2 p}\sqrt{\frac{mkT}{6}}$

b. doubling r

Bridging Problem

(a) 102 km

(b) yes

Chapter 19
The First Law of Thermodynamics

?⧉ A steam locomotive uses the first law of thermodynamics: Water is heated and boils, and the expanding steam does work to propel the locomotive. Would it be possible for the steam to propel the locomotive by doing work as it *condenses?* (i) Yes; (ii) no; (iii) answer depends on the details of how the steam condenses.

Learning Outcomes

In this chapter, you'll learn...

19.1 The significance of thermodynamic systems and processes.

19.2 How to calculate work done by a system when its volume changes.

19.3 What is meant by a path between thermodynamic states.

19.4 How to interpret and use the first law of thermodynamics.

19.5 Four important kinds of thermodynamic processes.

19.6 Why the internal energy of an ideal gas depends on temperature only.

19.7 The difference between molar heat capacities at constant volume and at constant pressure.

19.8 How to analyze adiabatic processes in an ideal gas.

You'll need to review...

6.3 Work done by a force.

7.3 Internal energy.

17.5, 18.4 Specific heat and molar heat capacity.

18.1 pV-diagrams.

Every time you drive a gasoline-powered car, turn on an air conditioner, or cook a meal, you reap the benefits of *thermodynamics*, the study of relationships involving heat, mechanical work, and other aspects of energy and energy transfer. For example, in a car engine heat is generated by the chemical reaction of oxygen and vaporized gasoline in the engine's cylinders. The heated gas pushes on the pistons within the cylinders, doing mechanical work that is used to propel the car. This is an example of a *thermodynamic process*.

The first law of thermodynamics, central to the understanding of such processes, is an extension of the principle of conservation of energy. It broadens this principle to include energy exchange by both heat transfer and mechanical work and introduces the concept of the *internal energy* of a system. Conservation of energy plays a vital role in every area of physical science, and the first law has extremely broad usefulness. To state energy relationships precisely, we need the concept of a *thermodynamic system*. We'll discuss *heat* and *work* as two means of transferring energy into or out of such a system.

19.1 Thermodynamic Systems

We have studied energy transfer through mechanical work (Chapter 6) and through heat transfer (Chapters 17 and 18). Now we are ready to combine and generalize these principles.

We always talk about energy transfer to or from some specific *system*. The system might be a mechanical device, a biological organism, or a specified quantity of material, such as the refrigerant in an air conditioner or steam expanding in a turbine. In general, a **thermodynamic system** is any collection of objects that is convenient to regard as a unit, and that may have the potential to exchange energy with its surroundings. A familiar example is a quantity of popcorn kernels in a pot with a lid. When the pot is placed on a stove, energy is added to the popcorn by conduction of heat. As the popcorn pops and expands, it does work as it exerts an upward force on the lid and moves it through a displacement (Fig. 19.1). The *state* of the popcorn—its volume, temperature, and pressure—changes as it pops. A process such as this one, in which there are changes in the state of a thermodynamic system, is called a **thermodynamic process**.

Figure 19.1

The popcorn in the pot is a thermodynamic system. In the thermodynamic process shown here, heat is added to the system, and the system does work on its surroundings to lift the lid of the pot.

In mechanics we used the concept of *system* with free-body diagrams and with conservation of energy and momentum. For *thermodynamic* systems, as for all others, it is essential to define clearly at the start exactly what is and is not included in the system. Only then can we describe unambiguously the energy transfers into and out of that system. For instance, in our popcorn example we defined the system to include the popcorn but not the pot, lid, or stove.

Thermodynamics has its roots in many practical problems other than popping popcorn (Fig. 19.2). The gasoline engine in an automobile, the jet engines in an airplane, and the rocket engines in a launch vehicle use the heat of combustion of their fuel to perform mechanical work in propelling the vehicle. Muscle tissue in living organisms metabolizes chemical energy in food and performs mechanical work on the organism's surroundings. A steam engine or steam turbine uses the heat of combustion of coal or other fuel to perform mechanical work such as driving an electric generator or pulling a train.

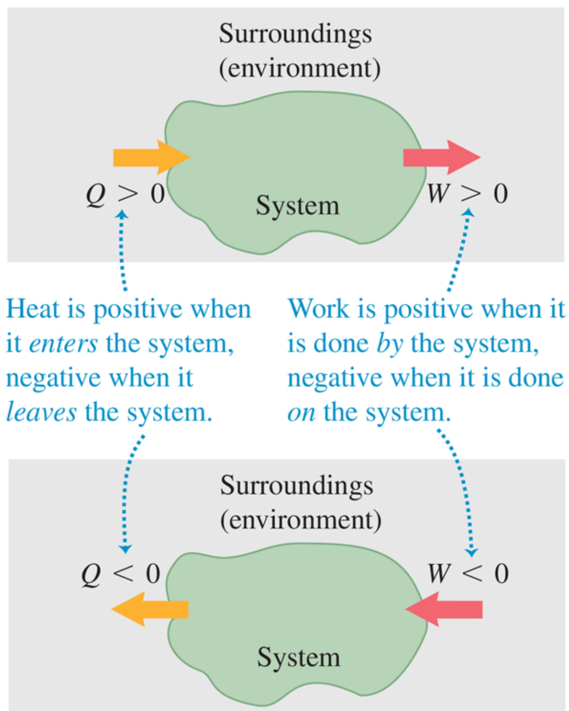
Figure 19.2

(a) A rocket engine uses the heat of combustion of its fuel to do work propelling the launch vehicle. (b) Humans and other biological organisms are more complicated systems than we can analyze fully in this book, but the same basic principles of thermodynamics apply to them.

Signs for Heat and Work in Thermodynamics

We describe the energy relationships in any thermodynamic process in terms of the quantity of heat Q added *to* the system and the work W done *by* the system. Both Q and W may be positive, negative, or zero (Fig. 19.3). A positive value of Q represents heat flow *into* the system, with a corresponding input of energy to it; negative Q represents heat flow *out of* the system. A positive value of W represents work done *by* the system against its surroundings, such as work done by an expanding gas, and hence corresponds to energy *leaving* the system. Negative W, such as work done during compression of a gas in which work is done *on the gas* by its surroundings, represents energy *entering* the system. We'll use these conventions consistently in the examples in this chapter and the next.

Figure 19.3



A thermodynamic system may exchange energy with its surroundings (environment) by means of heat, work, or both. Note the sign conventions for Q and W.

CAUTION Be careful with the sign of work W Note that our sign rule for work is *opposite* to the one we used in mechanics, in which we always spoke of the work done by the forces acting *on* an object. In thermodynamics it is usually more convenient to call W the work done *by* the system so that when a system expands, the pressure, volume change, and work are all positive. Use the sign rules for work and heat consistently!

Test Your Understanding of Section 19.1

In Example 17.7 (Section 17.6), what is the sign of Q for the coffee? For the aluminum cup? If a block slides along a horizontal surface with friction, what is the sign of W for the block?

19.2 Work Done During Volume Changes

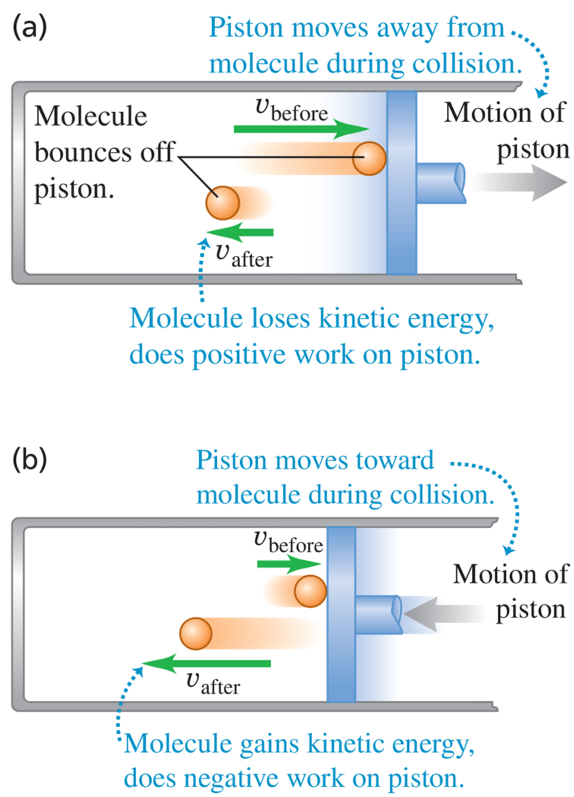
A simple example of a thermodynamic system is a quantity of gas enclosed in a cylinder with a movable piston. Internal-combustion engines, steam engines, and compressors in refrigerators and air conditioners all use some version of such a system. In the next several sections we'll use the gas-in-cylinder system to explore several kinds of thermodynamic processes.

We'll use a microscopic viewpoint, based on the kinetic and potential energies of individual molecules in a material, to develop intuition about thermodynamic quantities. But it is important to understand that the central principles of thermodynamics can be treated in a completely *macroscopic* way, without reference to microscopic models. Indeed, part of the great power and generality of thermodynamics is that it does *not* depend on details of the structure of matter.

First we consider the *work* done by the system during a volume change. When a gas expands, it pushes outward on its boundary surfaces as they move outward. Hence an expanding gas always does positive work. The same thing is true of any material that expands under pressure, such as the popcorn in Fig. 19.1.

? We can understand the work done by a gas in a volume change by considering the molecules that make up the gas. When one such molecule collides with a stationary surface, it exerts a momentary force on the wall but does no work because the wall does not move. But if the surface is moving, like a piston in a gasoline engine, the molecule *does* do work on the surface during the collision. If the piston in Fig. 19.4a moves to the right, so the volume of the gas increases, the molecules that strike the piston exert a force through a distance and do *positive* work on the piston. If the piston moves toward the left as in Fig. 19.4b, so the volume of the gas decreases, positive work is done *on* the molecule during the collision. Hence the gas molecules do *negative* work on the piston.

Figure 19.4



A molecule striking a piston (a) does positive work if the piston is moving away from the molecule and (b) does negative work if the piston is moving toward the molecule. Hence a gas does positive work when it expands as in (a) but does negative work when it compresses as in (b).

Figure 19.5 shows a system whose volume can change (a gas, liquid, or solid) in a cylinder with a movable piston. Suppose that the cylinder has cross-sectional area A and that the pressure exerted by the system at the piston face is p. The total force F exerted by the system on the piston is $F = pA$. When the piston moves out an infinitesimal distance dx, the work dW done by this force is

$$dW = F\,dx = pA\,dx$$

But

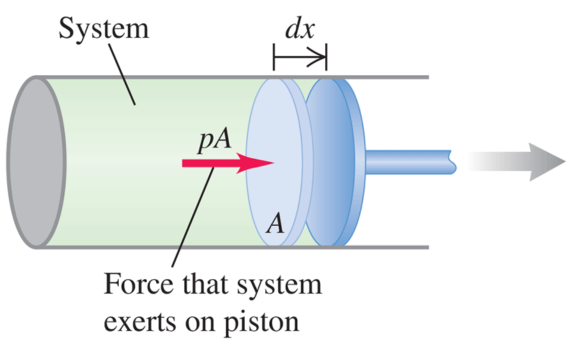
$$A\,dx = dV$$

where dV is the infinitesimal change of volume of the system. Thus we can express the work done by the system in this infinitesimal volume change as

(19.1)

$$dW = p\,dV$$

Figure 19.5



The infinitesimal work done by the system during the small expansion dx is $dW = pA\,dx$.

In a finite change of volume from V_1 to V_2,

(19.2)

Work done in a volume change

Upper limit = final volume

Integral of the pressure with respect to volume

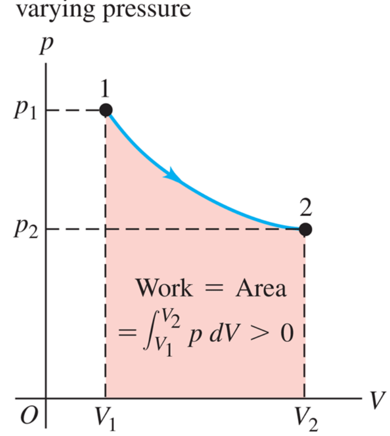
Lower limit = initial volume

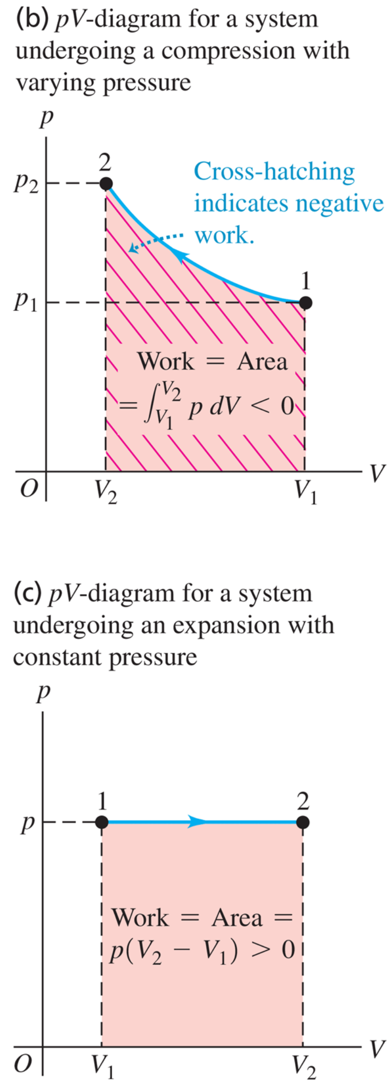
$$W = \int_{V_1}^{V_2} p\,dV \tag{19.2}$$

In general, the pressure of the system may vary during the volume change. For example, this is the case in the cylinders of an automobile engine as the pistons move back and forth. To evaluate the integral in Eq. (19.2), we have to know how the pressure varies as a function of volume. We can represent this relationship as a graph of p as a function of V (a pV-diagram, described at the end of Section 18.1). Figure 19.6 (next page) shows a simple example. In this figure, Eq. (19.2) is represented graphically as the *area* under the curve of p versus V between the limits V_1 and V_2. (In Section 6.3 we used a similar interpretation of the work done by a force F as the area under the curve of F versus x between the limits x_1 and x_2.)

Figure 19.6

(a) pV-diagram for a system undergoing an expansion with varying pressure





The work done equals the area under the curve on a pV-diagram.

According to the rule we stated in Section 19.1, work is *positive* when a system *expands*. In an expansion from state 1 to state 2 in Fig. 19.6a, the area under the curve and the work are positive. A *compression* from 1 to 2 in Fig. 19.6b gives a *negative* area; when a system is compressed, its volume decreases and it does *negative* work on its surroundings (see also Fig. 19.4b).

If the pressure p remains constant while the volume changes from V_1 to V_2 (Fig. 19.6c), the work done by the system is

(19.3)

Work done in a volume change at constant pressure

$$W = \overset{\text{Pressure}}{p}(\underset{\text{Final volume}}{V_2} - \underset{\text{Initial volume}}{V_1}) \tag{19.3}$$

If the volume is *constant*, there is no displacement and the system does no work.

Example 19.1 Isothermal expansion of an ideal gas

As an ideal gas undergoes an *isothermal* (constant-temperature) expansion at temperature T, its volume changes from V_1 to V_2. How much work does the gas do?

IDENTIFY and SET UP The ideal-gas equation, Eq. (18.3), tells us that if the temperature T of n moles of an ideal gas is constant, the quantity $pV = nRT$ is also constant. If V changes, p changes as well, so we *cannot* use Eq. (19.3) to calculate the work done. Instead we must evaluate the integral in Eq. (19.2), so we must know p as a function of V; for this we use Eq. (18.3).

EXECUTE From Eq. (18.3),

$$p = \frac{nRT}{V}$$

We substitute this into the integral of Eq. (19.2), take the constant factor nRT outside, and evaluate the integral:

$$\begin{aligned} W &= \int_{V_1}^{V_2} p\,dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} \\ &= nRT \ln \frac{V_2}{V_1} \qquad \text{(ideal gas, isothermal process)} \end{aligned}$$

We can rewrite this expression for W in terms of p_1 and p_2. Because $pV = nRT$ is constant,

$$p_1V_1 = p_2V_2 \qquad \text{or} \qquad \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

so

$$W = nRT \ln \frac{p_1}{p_2} \qquad \text{(ideal gas, isothermal process)}$$

EVALUATE We check our result by noting that in an expansion $V_2 > V_1$ and the ratio V_2/V_1 is greater than 1. The logarithm of a number greater than 1 is positive, so $W > 0$, as it should be. As an additional check, look at our second expression for W: In an isothermal expansion the volume increases and the pressure drops, so p_2p_1, the ratio $p_1/p_2 > 1$, and $W = nRT \ln(p_1/p_2)$ is again positive.

Our result for W also applies to an isothermal *compression* of a gas, for which V_2V_1 and $p_2 > p_1$.

KEY CONCEPT

To find the amount of work a system does on its surroundings during a volume change, you can either calculate the integral of pressure p with respect to volume V, or find the area under the curve of p versus V. The work is positive when the volume increases, and negative when the volume decreases.

Video Tutor Solution: **Example 19.1**

CAUTION **Be careful with subscripts 1 and 2** In Eq. (19.2), V_1 is the *initial* volume and V_2 is the *final* volume. That's why labels 1 and 2 are reversed in Fig. 19.6b compared to Fig. 19.6a, even though both processes move between the same two thermodynamic states.

Test Your Understanding of Section 19.2

A quantity of ideal gas undergoes an expansion that doubles its volume. Does the gas do more work on its surroundings if the expansion is at constant *pressure* or at constant *temperature*? (i) Constant pressure; (ii) constant temperature; (iii) the same amount in both cases; (iv) not enough information is given to decide.

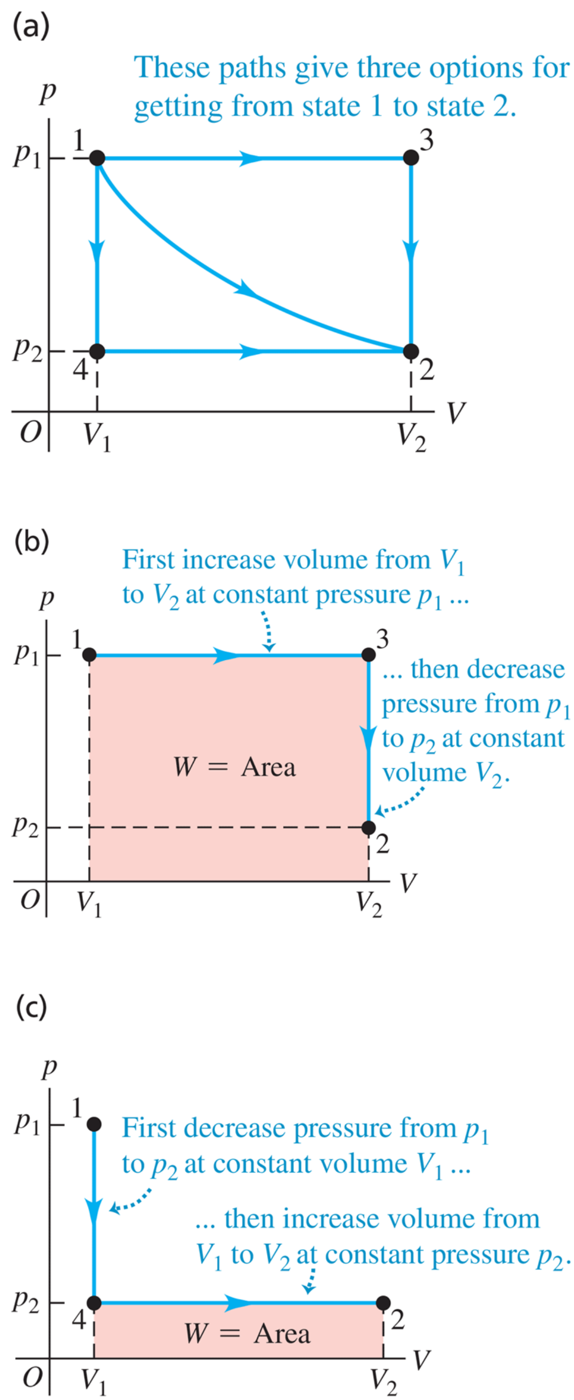
19.3 Paths Between Thermodynamic States

We've seen that if a thermodynamic process involves a change in volume, the system undergoing the process does work (either positive or negative) on its surroundings. Heat also flows into or out of the system during the process if there is a temperature difference between the system and its surroundings. Let's now examine how the work done by and the heat added to the system during a thermodynamic process depend on the details of how the process takes place.

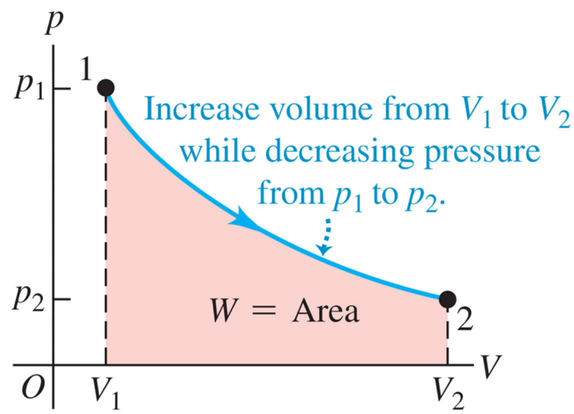
Work Done in a Thermodynamic Process

When a thermodynamic system changes from an initial state to a final state, it passes through a series of intermediate states. We call this series of states a **path**. There are always infinitely many possibilities for these intermediate states. When all are equilibrium states, the path can be plotted on a pV-diagram (**Fig. 19.7a**). Point 1 represents an initial state with pressure p_1 and volume V_1, and point 2 represents a final state with pressure p_2 and volume V_2. To pass from state 1 to state 2, we could keep the pressure constant at p_1 while the system expands to volume V_2 (point 3 in Fig. 19.7b), then reduce the pressure to p_2 (probably by decreasing the temperature) while keeping the volume constant at V_2 (to point 2). The work done by the system during this process is the area under the line $1 \rightarrow 3$; no work is done during the constant-volume process $3 \rightarrow 2$. Or the system might traverse the path $1 \rightarrow 4 \rightarrow 2$ (Fig. 19.7c); then the work is the area under the line $4 \rightarrow 2$, since no work is done during the constant-volume process $1 \rightarrow 4$. The smooth curve from 1 to 2 is another possibility (Fig. 19.7d), and the work for this path is different from that for either of the other paths.

Figure 19.7



(d)



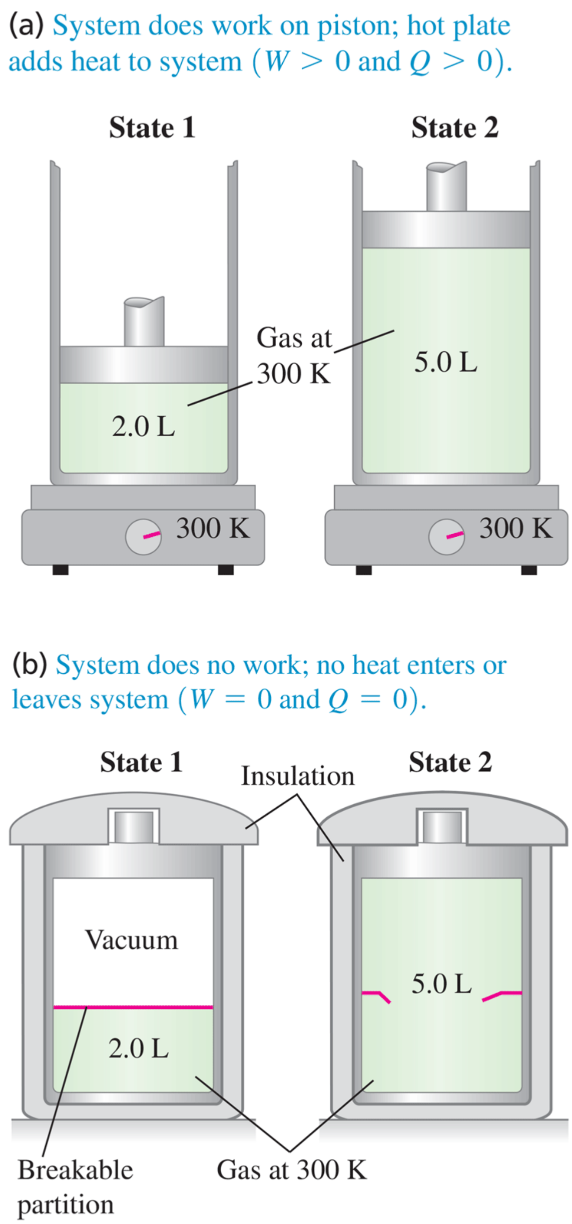
The work done by a system during a transition between two states depends on the path chosen.

We conclude that *the work done by the system depends not only on the initial and final states, but also on the intermediate states—that is, on the path*. Furthermore, we can take the system through a series of states forming a closed loop, such as $1 \to 3 \to 2 \to 4 \to 1$. In this case the final state is the same as the initial state, but the total work done by the system is *not* zero. (In fact, it is represented on the graph by the area enclosed by the loop; see Exercise 19.7.) So we can't talk about the amount of work *contained in* a system. In a particular state, a system may have definite values of the state coordinates p, V, and T, but it wouldn't make sense to say that it has a definite value of W.

Heat Added in a Thermodynamic Process

Like work, the *heat* added to a thermodynamic system when it undergoes a change of state depends on the path from the initial state to the final state. Here's an example. Suppose we want to change the volume of a certain quantity of an ideal gas from 2.0 L to 5.0 L while keeping the temperature constant at $T = 300$ K. Figure 19.8 (next page) shows two different ways to do this. In Fig. 19.8a the gas is contained in a cylinder with a piston, with an initial volume of 2.0 L. We let the gas expand slowly, supplying heat from the electric heater to keep the temperature at 300 K until the gas reaches its final volume of 5.0 L. The gas absorbs a definite amount of heat in this isothermal process.

Figure 19.8



(a) Slow, controlled isothermal expansion of a gas from an initial state 1 to a final state 2 with the same temperature but lower pressure. (b) Rapid, uncontrolled expansion of the same gas starting at the same state 1 and ending at the same state 2.

Figure 19.8b shows a different process leading to the same final state. The container is surrounded by insulating walls and is divided by a thin, breakable partition into two compartments. The lower part has volume 2.0 L and the upper part has volume 3.0 L. In the lower compartment we place the same amount of the same gas as in Fig. 19.8a, again at $T = 300\text{ K}$. The initial state is the same as before. Now we break the partition; the gas expands rapidly, with no heat passing through the insulating walls. The final volume is 5.0 L, the same as in Fig. 19.8a. The expanding gas does no work because it doesn't push against anything that moves. This uncontrolled expansion of a gas into vacuum is called a **free expansion;** we'll discuss it further in Section 19.6.

Experiments show that when an ideal gas undergoes a free expansion, there is no temperature change. So the final state of the gas is the same as in Fig. 19.8a. The intermediate pressures and volumes during the transition from state 1 to state 2 are entirely different in the two cases; Figs. 19.8a and 19.8b represent *two different paths* connecting the *same states* 1 and 2. For the path in Fig. 19.8b, *no* heat is transferred into the system, and the system does no work. Like work, *heat depends not only on the initial and final states but also on the path*.

Because of this path dependence, it would not make sense to say that a system "contains" a certain quantity of heat. To see this, suppose we assign an arbitrary value to the "heat in an object" in some reference state. Then presumably the "heat in the object" in some other state would equal the heat added when the object goes to the second state. But that's ambiguous, as we have just seen; the heat added depends on the *path* we take from the reference state to the second state. We are forced to conclude that there is *no* consistent way to define “heat in an object”; it is not a useful concept.

While it doesn’t make sense to talk about “work in an object” or “heat in an object,” it *does* make sense to speak of the amount of *internal energy* in an object. This important concept is our next topic.

Test Your Understanding of Section 19.3

The system described in Fig. 19.7a undergoes four different thermodynamic processes. Each process is represented in a pV-diagram as a straight line from the initial state to the final state. (These processes are different from those shown in the pV-diagrams of Fig. 19.7.) Rank the processes in order of the amount of work done by the system, from the most positive to the most negative. (i) $1 \rightarrow 2$; (ii) $2 \rightarrow 1$; (iii) $3 \rightarrow 4$; (iv) $4 \rightarrow 3$.

19.4 Internal Energy and the First Law of Thermodynamics

Internal energy is one of the most important concepts in thermodynamics. In Section 7.3, when we discussed energy changes for an object sliding with friction, we stated that warming an object increased its internal energy and that cooling the object decreased its internal energy. But what *is* internal energy? We can look at it in various ways; let's start with one based on the ideas of mechanics. Matter consists of atoms and molecules, and these are made up of particles having kinetic and potential energies. We *tentatively* define the **internal energy** of a system as the sum of the kinetic energies of all of its constituent particles, plus the sum of all the potential energies of interaction among these particles.

CAUTION Is it internal? Internal energy does *not* include potential energy arising from the interaction between the system and its surroundings. If the system is a glass of water, placing it on a high shelf increases the gravitational potential energy arising from the interaction between the glass and the earth. But this has no effect on the interactions among the water molecules, and so the internal energy of the water does not change.

We use the symbol U for internal energy. (We used this symbol in our study of mechanics to represent potential energy. However, U has a different meaning in thermodynamics.) During a change of state of the system, the internal energy may change from an initial value U_1 to a final value U_2. We denote the change in internal energy as $\Delta U = U_2 - U_1$.

When we add a quantity of heat Q to a system and the system does no work during the process (so $W = 0$), the internal energy increases by an amount equal to Q; that is, $\Delta U = Q$. When a system does work W by expanding against its surroundings and no heat is added during the process, energy leaves the system and the internal energy decreases: W is positive, Q is zero, and $\Delta U = -W$. When *both* heat transfer and work occur, the *total* change in internal energy is

(19.4)

First law of thermodynamics:

$$\overset{\text{Internal energy change of thermodynamic system}}{\Delta U} = \underset{\text{Heat added to system}}{Q} - \underset{\text{Work done by system}}{W} \qquad (19.4)$$

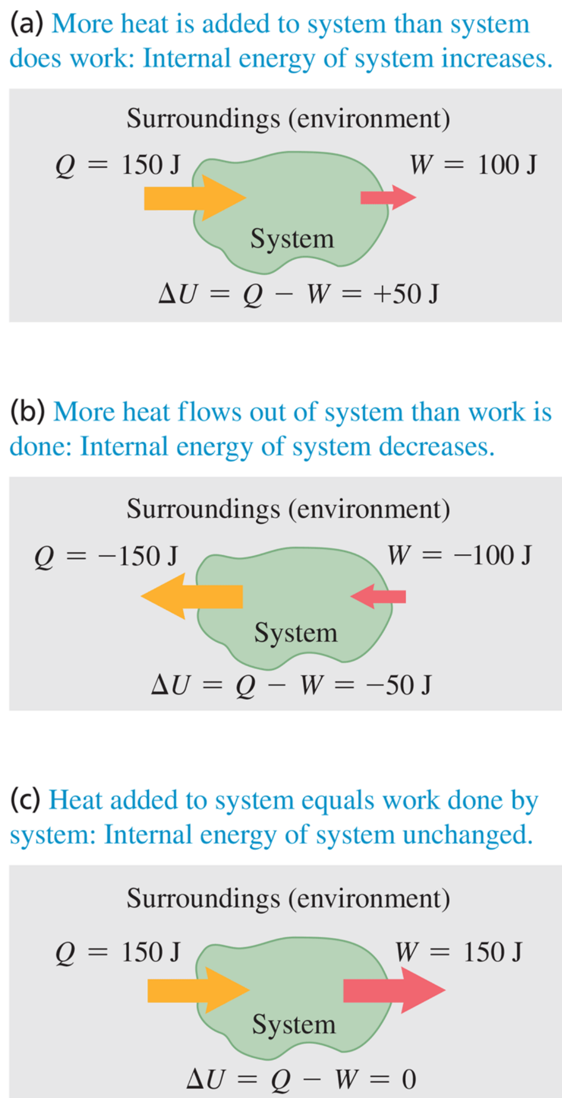
We can rearrange this to the form

(19.5)

$$Q = \Delta U + W$$

The message of Eq. (19.5) is that when heat Q is added to a system, some of this added energy remains within the system, changing its internal energy by ΔU; the remainder leaves the system as the system does work W on its surroundings. Because W and Q may be positive, negative, or zero, ΔU can be positive, negative, or zero for different processes (Fig. 19.9).

Figure 19.9



In a thermodynamic process, the internal energy U of a system may (a) increase ($\Delta U > 0$), (b) decrease ($\Delta U < 0$), or (c) remain the same ($\Delta U = 0$).

Equation (19.4) or (19.5) is the **first law of thermodynamics**. It is a generalization of the principle of conservation of energy to include energy transfer through heat as well as mechanical work. As you'll see in later chapters, this principle can be extended to ever-broader classes of phenomena by identifying additional forms of energy and energy transfer. In every situation in which it seems that the total energy in all known forms is not conserved, it has been possible to identify a new form of energy such that the total energy, including the new form, *is* conserved.

Understanding the First Law of Thermodynamics

At the beginning of this discussion we tentatively defined internal energy in terms of microscopic kinetic and potential energies. But actually *calculating* internal energy in this way for any real system would be hopelessly complicated. Furthermore, this definition isn't an *operational* one: It doesn't describe how to determine internal energy from physical quantities that we can measure.

BIO Application

The First Law of Exercise Thermodynamics

Your body is a thermodynamic system. When you exercise, your body does work (such as the work done to lift your body as a whole in a push-up). Hence $W > 0$. Your body also warms up during exercise; by perspiration and other means the body rids itself of this heat, so $Q < 0$. Since Q is negative and W is positive, $\Delta U = Q - W < 0$ and the body's internal energy decreases. That's why exercise helps you lose weight: It uses up some of the internal energy stored in your body in the form of fat.

So let’s look at internal energy in another way. Starting over, we define the *change* in internal energy ΔU during any change of a system as the quantity given by Eq. (19.4), $\Delta U = Q - W$. This *is* an operational definition because we can measure Q and W. It does not define U itself, only ΔU. This is not a shortcoming because we can *define* the internal energy of a system to have a specified value in some reference state, and then use Eq. (19.4) to define the internal energy in any other state. This is analogous to our treatment of potential energy in Chapter 7, in which we arbitrarily defined the potential energy of a mechanical system to be zero at a certain position.

This new definition trades one difficulty for another. If we define ΔU by Eq. (19.4), then when the system goes from state 1 to state 2 by two different paths, how do we know that ΔU is the same for the two paths? We have already seen that Q and W are, in general, *not* the same for different paths. If ΔU, which equals $Q - W$, is also path dependent, then ΔU is ambiguous. If so, the concept of internal energy of a system is subject to the same criticism as the erroneous concept of quantity of heat in a system, as we discussed at the end of Section 19.3.

The only way to answer this question is through *experiment*. For various materials we measure Q and W for various changes of state and various paths to learn whether ΔU is or is not path dependent. The results of many such investigations are clear and unambiguous: While Q and W depend on the path, $\Delta U = Q - W$ *is independent of path. The change in internal energy of a system during any thermodynamic process depends only on the initial and final states, not on the path leading from one to the other*.

Experiment, then, is the ultimate justification for believing that a thermodynamic system in a specific state has a unique internal energy that depends only on that state. An equivalent statement is that the internal energy U of a system is a function of the state coordinates p, V, and T (actually, any two of these, since the three variables are related by the equation of state).

To say that the first law of thermodynamics, given by Eq. (19.4) or (19.5), represents conservation of energy for thermodynamic processes is correct, as far as it goes. But an important *additional* aspect of the first law is the fact that internal energy depends only on the state of a system (Fig. 19.10). In changes of state, the change in internal energy is independent of the path.

Figure 19.10

The internal energy of a cup of coffee depends on just its thermo-dynamic state—how much water and ground coffee it contains, and what its temperature is. It does not depend on the history of how the coffee was prepared—that is, the thermodynamic path that led to its current state.

All this may seem a little abstract if you are satisfied to think of internal energy as microscopic mechanical energy. There's nothing wrong with that view, and we'll make use of it at various times during our discussion. But as for heat, a precise *operational* definition of internal energy must be independent of the detailed microscopic structure of the material.

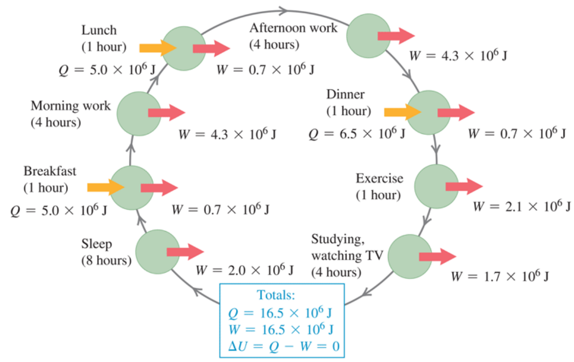
Cyclic Processes and Isolated Systems

Two special cases of the first law of thermodynamics are worth mentioning. A process that eventually returns a system to its initial state is called a *cyclic process*. For such a process, the final state is the same as the initial state, and so the *total* internal energy change must be zero. Then

$$U_2 = U_1 \qquad \text{and} \qquad Q = W$$

If a net quantity of work W is done by the system during this process, an equal amount of energy must have flowed into the system as heat Q. But there is no reason either Q or W individually has to be zero (Fig. 19.11).

Figure 19.11



Every day, your body (a thermodynamic system) goes through a cyclic thermodynamic process like this one. Heat Q is added by metabolizing food, and your body does work W in breathing, walking, and other activities. If you return to the same state at the end of the day, $Q = W$ and the net change in your internal energy is zero.

Another special case occurs in an *isolated system*, one that does no work on its surroundings and has no heat flow to or from its surroundings. For any process taking place in an isolated system,

$$W = Q = 0$$

and therefore

$$U_2 = U_1 = \Delta U = 0$$

In other words, *the internal energy of an isolated system is constant*.

Problem-Solving Strategy 19.1 The First Law of Thermodynamics

IDENTIFY *the relevant concepts*: The first law of thermodynamics is the statement of the law of conservation of energy in its most general form. You can apply it to *any* thermodynamic process in which the internal energy of a system changes, heat flows into or out of the system, and/or work is done by or on the system.

SET UP *the problem* using the following steps:

1. Define the thermodynamic system to be considered.
2. If the thermodynamic process has more than one step, identify the initial and final states for each step.
3. List the known and unknown quantities and identify the target variables.
4. Confirm that you have enough equations. You can apply the first law, $\Delta U = Q - W$, just once to each step in a thermodynamic process, so you'll often need additional equations. These may include Eq. (19.2), $w = \int_{V_1}^{V_2} p dV$, which gives the work W done in a volume change, and the equation of state of the material that makes up the thermodynamic system (for an ideal gas, $pV = nRT$).

EXECUTE *the solution* as follows:

1. Be sure to use consistent units. If p is in Pa and V in m^3, then W is in joules. If a heat capacity is given in terms of calories, convert it to joules. When you use $n = m_{\mathrm{total}}/M$ to relate total mass m_{total} to number of moles n, remember that if m_{total} is in kilograms, M must be in *kilograms* per mole; M is usually tabulated in *grams* per mole.
2. The internal energy change ΔU in any thermodynamic process or series of processes is independent of the path, whether the substance is an ideal gas or not. If you can calculate ΔU for *any* path between given initial and final states, you know ΔU for *every possible path* between those states; you can then relate the various energy quantities for any of those other paths.
3. In a process comprising several steps, tabulate Q, W, and ΔU for each step, with one line per step and with the Q's, W's, and ΔU's forming columns (see Example 19.4). You

can apply the first law to each line, and you can add each column and apply the first law to the sums. Do you see why?

4. Using steps 1–3, solve for the target variables.

EVALUATE *your answer*: Check your results for reasonableness. Ensure that each of your answers has the correct algebraic sign. A positive Q means that heat flows *into* the system; a negative Q means that heat flows *out of* the system. A positive W means that work is done *by* the system on its environment; a negative W means that work is done *on* the system by its environment.

Example 19.2 Working off your dessert

WITH VARIATION PROBLEMS

You propose to climb several flights of stairs to work off the energy you took in by eating a 900 calorie hot fudge sundae. How high must you climb? Assume that your mass is 60.0 kg.

IDENTIFY and SET UP The thermodynamic system is your body. You climb the stairs to make the final state of the system the same as the initial state (no fatter, no leaner). There is therefore no net change in internal energy: $\Delta U = 0$. Eating the hot fudge sundae corresponds to a heat flow into your body, and you do work climbing the stairs. We can relate these quantities by using the first law of thermodynamics. We are given that $Q = 900$ food calories (900 kcal) of heat flow into your body. The work you must do to raise your mass m a height h is $W = mgh$; our target variable is h.

EXECUTE From the first law of thermodynamics, $\Delta U = 0 = Q - W$, so $W = mgh = Q$. Hence you must climb to height $h = Q/mg$. First convert units: $Q = (900 \text{ kcal})(4186 \text{ J}/1 \text{ kcal}) = 3.77 \times 10^6 \text{ J}$. Then

$$h = \frac{Q}{mg} = \frac{3.77 \times 10^6 \text{ J}}{(60.0 \text{ kg})(9.80 \text{ m/s}^2)} = 6410 \text{ m}$$

EVALUATE We have unrealistically assumed 100% efficiency in the conversion of food energy into mechanical work. The actual efficiency is roughly 25%, so the work W you do as you "burn off" the sundae is only about $(0.25)\,(900\ kcal) = 225\ kcal$. (The remaining 75%, or 675 kcal, is transferred to your surroundings as heat.) Hence you actually must climb about $(0.25)\,(6410\ m) = 1600\ m$, or one *mile*! Do you really want that sundae?

KEY CONCEPT

The first law of thermodynamics states that the change in internal energy ΔU of a system equals the heat flow Q into the system minus the work W that the system does on its surroundings. Q is positive if heat flows into the system, negative if heat flows out; W is positive if the system does work on its surroundings, negative if the surroundings do work on the system.

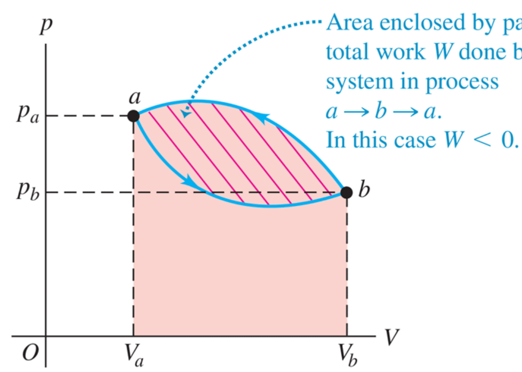
Video Tutor Solution: **Example 19.2**

Example 19.3 A cyclic process

WITH VARIATION PROBLEMS

Figure 19.12 shows a pV-diagram for a *cyclic* process in which the initial and final states of some thermodynamic system are the same. The state of the system starts at point a and proceeds counterclockwise in the pV-diagram to point b, then back to a; the total work is $W = -500$ J. (a) Why is the work negative? (b) Find the change in internal energy and the heat added during this process.

Figure 19.12



The net work done by the system in process $a \rightarrow b \rightarrow a$ is -500 J. What would it have been if the process had proceeded clockwise in this pV-diagram?

IDENTIFY and SET UP We must relate the change in internal energy, the heat added, and the work done in a thermodynamic process. Hence we can apply the first law of thermodynamics. The process is cyclic, and it has two steps: $a \rightarrow b$ via the lower curve in Fig. 19.12 and $b \rightarrow a$ via the upper curve. We are asked only about the *entire* cyclic process $a \rightarrow b \rightarrow a$.

EXECUTE

(a) The work done in any step equals the area under the curve in the pV-diagram, with the area taken as positive if $V_2 > V_1$ and negative if $V_2 V_1$; this rule yields the signs that result from the actual integrations in Eq. (19.2), $W = \int_{V_1}^{V_2} p\, dV$. The area under the lower curve $a \rightarrow b$ is therefore positive, but it is smaller than the absolute value of the (negative) area under the upper curve $b \rightarrow a$. Therefore the net area (the area enclosed by the path, shown with red stripes) and the net work W are negative. In other words, 500 J more work is done *on* the system than *by* the system in the complete process.

(b) In any cyclic process, $\Delta U = 0$, so $Q = W$. Here, that means $Q = -500$ J; that is, 500 J of heat flows *out of* the system.

EVALUATE In cyclic processes, the total work is positive if the process goes clockwise around the pV-diagram representing the cycle, and negative if the process goes counterclockwise (as here).

KEY CONCEPT

In a cyclic thermodynamic process a system returns to the same state it was in initially, so the net change ΔU in internal energy is zero. The net work W done by the system in a cyclic process equals the area enclosed by the path that the system follows on a pV-diagram. Since $\Delta U = 0$, the net heat flow Q into the system in a cyclic process is equal to W.

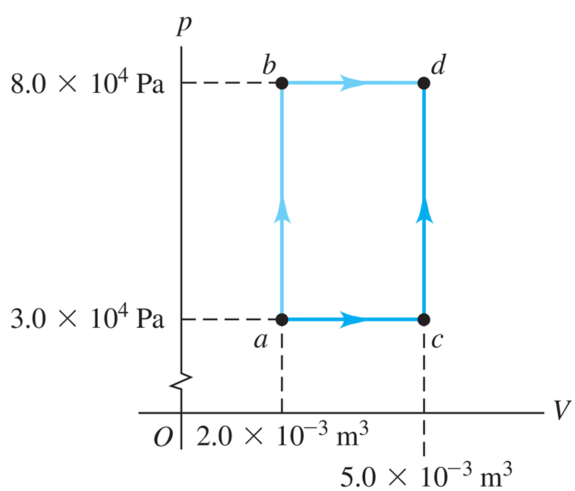
Video Tutor Solution: **Example 19.3**

Example 19.4 Comparing thermodynamic processes

WITH VARIATION PROBLEMS

The pV-diagram of **Fig. 19.13** shows a series of thermodynamic processes. In process ab, 150 J of heat is added to the system; in process bd, 600 J of heat is added. Find (a) the internal energy change in process ab; (b) the internal energy change in process abd (shown in light blue); and (c) the total heat added in process acd (shown in dark blue).

Figure 19.13



A pV-diagram showing the various thermodynamic processes.

IDENTIFY and SET UP In each process we use $\Delta U = Q - W$ to determine the desired quantity. We are given $Q_{ab} = +150\ \mathrm{J}$ and $Q_{bd} = +600\ \mathrm{J}$ (both values are positive because heat is *added* to the system). Our target variables are (a) ΔU_{ab}, (b) ΔU_{abd}, and (c) Q_{acd}.

EXECUTE (a) No volume change occurs during process *ab*, so the system does no work: $W_{ab} = 0$ and so $\Delta U_{ab} = Q_{ab} = 150\ \mathrm{J}$.

(b)Process *bd* is an expansion at constant pressure, so from Eq. (19.3),

$$\begin{aligned} W_{bd} &= p\,(V_2 - V_1) \\ &= (8.0 \times 10^4\ \mathrm{Pa})(5.0 \times 10^{-3}\ \mathrm{m}^3 - 2.0 \times 10^{-3}\ \mathrm{m}^3) = 240\ \mathrm{J} \end{aligned}$$

The total work for the two-step process *abd* is then

$$W_{abd} = W_{ab} + W_{bd} = 0 + 240\ \mathrm{J} = 240\ \mathrm{J}$$

and the total heat is

$$Q_{abd} = Q_{ab} + Q_{bd} = 150\ \mathrm{J} + 600\ \mathrm{J} = 750\ \mathrm{J}$$

Applying Eq. (19.4) to *abd*, we then have

$$\Delta U_{abd} = Q_{abd} - W_{abd} = 750\ \mathrm{J} - 240\ \mathrm{J} = 510\ \mathrm{J}$$

(c) Because ΔU is *independent of the path* from a to d, the internal energy change is the same for path *acd* as for path *abd*:

$$\Delta U_{acd} = \Delta U_{abd} = 510\ \mathrm{J}$$

The total work for path *acd* is

$$\begin{aligned} W_{acd} &= W_{ac} + W_{cd} = p\,(V_2 - V_1) + 0 \\ &= (3.0 \times 10^4\ \mathrm{Pa})(5.0 \times 10^{-3}\ \mathrm{m}^3 - 2.0 \times 10^{-3}\ \mathrm{m}^3) \\ &= 90\ \mathrm{J} \end{aligned}$$

Now we apply Eq. (19.5) to process *acd*:

$$Q_{acd} = \Delta U_{acd} + W_{acd} = 510\ \mathrm{J} + 90\ \mathrm{J} = 600\ \mathrm{J}$$

We tabulate the quantities above:

Step	Q	W	$\Delta U = Q - W$	Step	Q	W	$\Delta U = Q - W$
ab	150 J	0 J	150 J	ac	?	90 J	?
bd	600 J	240 J	360 J	cd	?	0 J	?
abd	750 J	240 J	510 J	acd	600 J	90 J	510 J

EVALUATE Be sure that you understand how each entry in the table above was determined. Although ΔU is the same (510 J) for *abd* and *acd*, W (240 J versus 90 J) and Q (750 J versus 600 J) are quite different. Although we couldn't find Q or ΔU for processes *ac* and *cd*, we could analyze the composite process *acd* by comparing it with process *abd*, which has the same initial and final states and for which we have more information.

KEY CONCEPT

When a system starts in one state and ends in a different state, the internal energy change ΔU of the system is the same no matter what path the system takes between the two states. However, the heat flow Q into the system and the work W done by the system *do* depend on the path taken.

Video Tutor Solution: **Example 19.4**

Example 19.5 Thermodynamics of boiling water

WITH VARIATION PROBLEMS

One gram of water (1 cm^3) becomes 1671 cm^3 of steam when boiled at a constant pressure of 1 atm (1.013×10^5 Pa). The heat of vaporization at this pressure is $L_v = 2.256 \times 10^6$ J/kg. Compute (a) the work done by the water when it vaporizes and (b) its increase in internal energy.

IDENTIFY and SET UP The heat added causes the system (water) to change phase from liquid to vapor. We can analyze this process by using the first law of thermodynamics. The water is boiled at constant pressure, so we can use Eq. (19.3) to calculate the work W done by the vaporizing water as it expands. We are given the mass of water and the heat of vaporization, so we can use Eq. (17.20), $Q = mL_v$, to calculate the heat Q added to the water. We can then find the internal energy change from Eq. (19.4), $\Delta U = Q - W$.

EXECUTE (a) From Eq. (19.3), the water does work

$$\begin{aligned} W &= p(V_2 - V_1) \\ &= (1.013 \times 10^5 \text{ Pa})(1671 \times 10^{-6} \text{ m}^3 - 1 \times 10^{-6} \text{ m}^3) = 169 \text{ J} \end{aligned}$$

(b) From Eq. (17.20), the heat added to the water is

$$Q = mL_v = (10^{-3} \text{ kg})(2.256 \times 10^6 \text{ J/kg}) = 2256 \text{ J}$$

Then from Eq. (19.4),

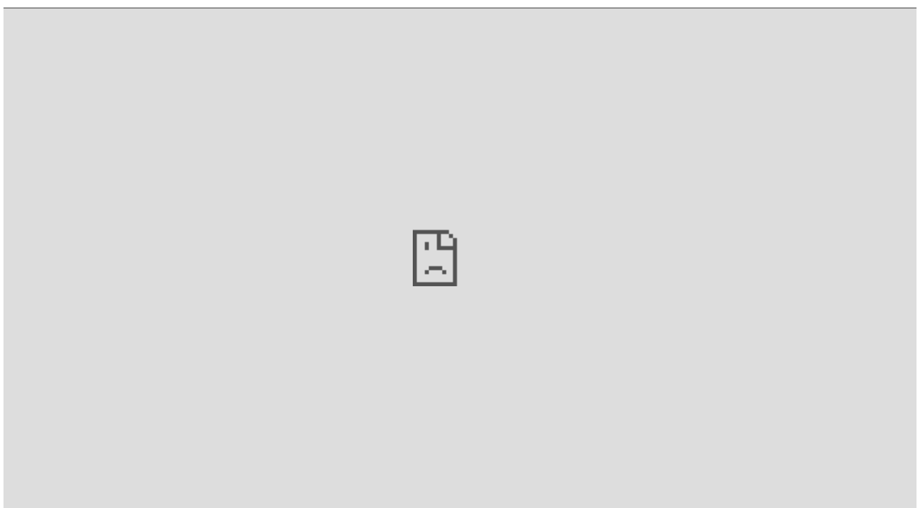
$$\Delta U = Q - W = 2256 \text{ J} - 169 \text{ J} = 2087 \text{ J}$$

EVALUATE To vaporize 1 g of water, we must add 2256 J of heat, most of which (2087 J) remains in the system as an increase in internal energy. The remaining 169 J leaves the system as the system expands from liquid to vapor and does work against the surroundings. (The increase in internal energy is associated mostly with the attractive intermolecular forces. The associated potential energies are greater after work has been done to pull apart the molecules in the liquid, forming the vapor state. It's like increasing gravitational potential energy by pulling an elevator farther from the center of the earth.)

KEY CONCEPT

For a system that undergoes a thermodynamic process, if you can calculate any two of the quantities ΔU (internal energy change), Q (heat flow into the system), and W (work done by the system), you can calculate the third using the first law of thermodynamics, $\Delta U = Q - W$.

Video Tutor Solution: Example 19.5



Infinitesimal Changes of State

In the preceding examples the initial and final states differ by a finite amount. Later we'll consider *infinitesimal* changes of state in which a small amount of heat dQ is added to the system, the system does a small amount of work dW, and its internal energy changes by an amount dU. For such a process,

(19.6)

First law of thermodynamics, infinitesimal process:

$$\overset{\text{Infinitesimal internal energy change}}{dU} = \underset{\text{Infinitesimal heat added}}{dQ} - \underset{\text{Infinitesimal work done}}{dW} \qquad (19.6)$$

For the systems we'll discuss, the work dW is given by $dW = p\,dV$, so we can also state the first law as

(19.7)

$$dU = dQ - p\,dV$$

Test Your Understanding of Section 19.4

Rank the following thermodynamic processes according to the change in internal energy in each process, from most positive to most negative. (i) As you do 250 J of work on a system, it transfers 250 J of heat to its surroundings; (ii) as you do 250 J of work on a system, it absorbs 250 J of heat from its surroundings; (iii) as a system does 250 J of work on you, it transfers 250 J of heat to its surroundings; (iv) as a system does 250 J of work on you, it absorbs 250 J of heat from its surroundings.

19.5 Kinds of Thermodynamic Processes

In this section we describe four specific kinds of thermodynamic processes that occur often in practical situations. We can summarize these briefly as "no heat transfer" or *adiabatic*, "constant volume" or *isochoric*, "constant pressure" or *isobaric*, and "constant temperature" or *isothermal*. For some of these processes we can use a simplified form of the first law of thermodynamics.

Adiabatic Process

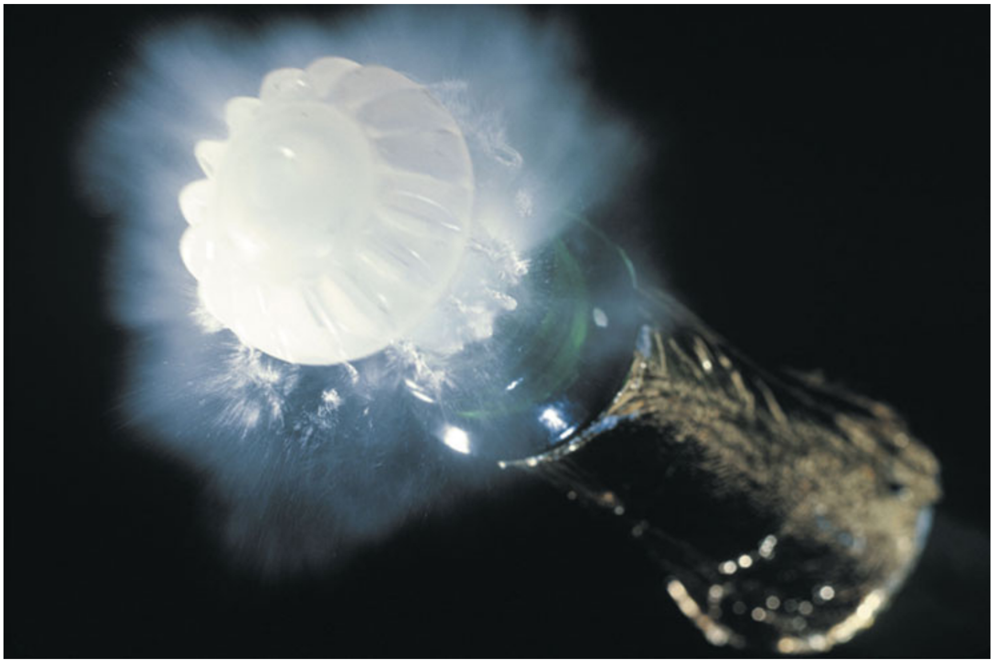
An **adiabatic process** (pronounced "ay-dee-ah-*bat*-ic") is defined as one with no heat transfer into or out of a system; $Q = 0$. We can prevent heat flow either by surrounding the system with thermally insulating material or by carrying out the process so quickly that there is not enough time for appreciable heat flow. From the first law we find that for every adiabatic process,

(19.8)

$$U_2 - U_1 = \Delta U = -W \qquad \text{(adiabatic process)}$$

When a system expands adiabatically, W is positive (the system does work on its surroundings), so ΔU is negative and the internal energy decreases. When a system is *compressed* adiabatically, W is negative (work is done on the system by its surroundings) and U increases. In many (but not all) systems an increase of internal energy is accompanied by a rise in temperature, and a decrease in internal energy by a drop in temperature (Fig. 19.14).

Figure 19.14



When the cork is popped on a bottle of champagne, the pressurized gases inside the bottle expand rapidly and do work on the outside air ($W > 0$). There is little time for the gases to exchange heat with their surroundings, so the expansion is nearly adiabatic ($Q = 0$). Hence the internal energy of the expanding gases decreases ($\Delta U = -W < 0$) and their temperature drops. This makes water vapor condense and form a miniature cloud.

The compression stroke in an internal-combustion engine is an approximately adiabatic process. The temperature rises as the air–fuel mixture in the cylinder is compressed. The expansion of the burned fuel during the power stroke is also an approximately adiabatic expansion with a drop in temperature. In Section 19.8 we'll consider adiabatic processes in an ideal gas.

Isochoric Process

An **isochoric process** (pronounced "eye-so-*kor*-ic") is a *constant-volume* process. When the volume of a thermodynamic system is constant, it does no work on its surroundings. Then $W = 0$ and

(19.9)

$$U_2 - U_1 = \Delta U = Q \quad \text{(isochoric process)}$$

In an isochoric process, all the energy added as heat remains in the system as an increase in internal energy. Heating a gas in a closed constant-volume container is an example of an isochoric process. The processes *ab* and *cd* in Example 19.4 are also examples of isochoric processes. (Note that there are types of work that do not involve a volume change. For example, we can do work on a fluid by stirring it. In some literature, "isochoric" is used to mean that no work of any kind is done.)

Isobaric Process

An **isobaric process** (pronounced "eye-so-*bear*-ic") is a *constant-pressure* process. In general, none of the three quantities ΔU, Q, and W is zero in an isobaric process, but calculating W is easy nonetheless. From Eq. (19.3),

(19.10)

$$W = p(V_2 - V_1) \quad \text{(isobaric process)}$$

Boiling water at constant pressure is an isobaric process (Fig. 19.15).

Figure 19.15

Most cooking involves isobaric processes. That’s because the air pressure above a saucepan or frying pan, or inside a microwave oven, remains essentially constant while the food is being heated.

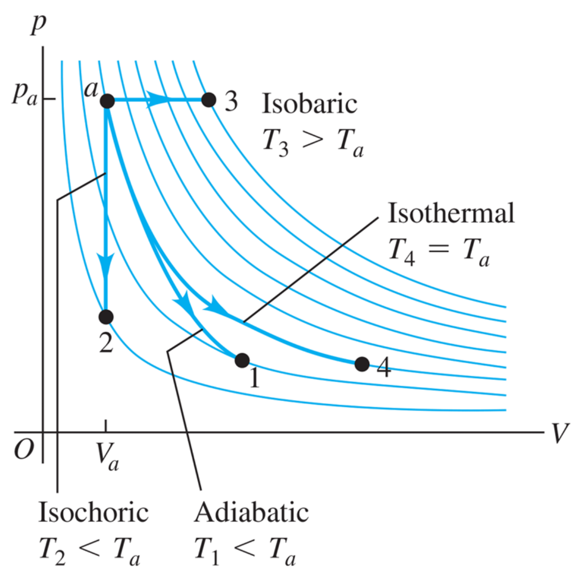
Isothermal Process

An **isothermal process** is a *constant-temperature* process. For a process to be isothermal, any heat flow into or out of the system must occur slowly enough that thermal equilibrium is maintained. In general, none of the quantities ΔU, Q, or W is zero in an isothermal process.

In some special cases the internal energy of a system depends *only* on its temperature, not on its pressure or volume. The most familiar system having this special property is an ideal gas, as we’ll discuss in the next section. For such systems, if the temperature is constant, the internal energy is also constant; $\Delta U = 0$ and $Q = W$. That is, any energy entering the system as heat Q must leave it again as work W done by the system. Example 19.1, involving an ideal gas, is an example of an isothermal process in which U is also constant. For most systems other than ideal gases, the internal energy depends on pressure as well as temperature, so U may vary even when T is constant.

Figure 19.16 shows a pV-diagram for these four processes for a constant amount of an ideal gas. The path followed in an adiabatic process (a to 1) is called an **adiabat.** A vertical line (constant volume) is an **isochor**, a horizontal line (constant pressure) is an **isobar**, and a curve of constant temperature (shown as light blue lines in Fig. 19.16) is an **isotherm.**

Figure 19.16



Four different processes for a constant amount of an ideal gas, all starting at state a. For the adiabatic process, $Q = 0$; for the isochoric process, $W = 0$; and for the isothermal process, $\Delta U = 0$. The temperature increases only during the isobaric expansion.

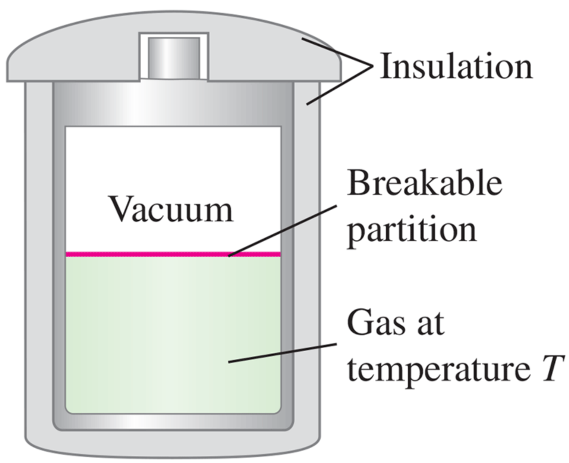
Test Your Understanding of Section 19.5

Which of the processes in Fig. 19.7 are isochoric? Which are isobaric? Is it possible to tell if any of the processes are isothermal or adiabatic?

19.6 Internal Energy of an Ideal Gas

We now show that for an ideal gas, the internal energy U depends only on temperature, not on pressure or volume. Let's think again about the free-expansion experiment described in Section 19.3. A thermally insulated container with rigid walls is divided into two compartments by a partition (Fig. 19.17). One compartment has a quantity of an ideal gas and the other is evacuated.

Figure 19.17



The partition is broken (or removed) to start the free expansion of gas into the vacuum region.

When the partition is removed or broken, the gas expands to fill both parts of the container. There is no heat flow through the insulation, and the gas does no work on its surroundings because the walls of the container don't move. So both Q and W are zero and the internal energy U is constant.

Does the *temperature* T change during a free expansion? Suppose it *does* change, while the internal energy stays the same. In that case we have to conclude that the internal energy depends on both T and the volume V or on both T and the pressure p, but certainly not on T alone. But if T is constant during a free expansion, for which we know that U is constant even though both p and V change, then we have to conclude that U depends only on T, not on p or V.

Many experiments have shown that when a low-density gas (essentially an ideal gas) undergoes a free expansion, its temperature *does not* change. The conclusion is:

> The internal energy U of an ideal gas depends only on its temperature T, not on its pressure or volume.

This property, in addition to the ideal-gas equation of state, is part of the ideal-gas model. We'll make frequent use of this property.

For nonideal gases, some temperature change occurs during free expansions, even though the internal energy is constant. This shows that the internal energy cannot depend *only* on temperature; it must depend on pressure as well. From the microscopic viewpoint, in which internal energy U is the sum of the kinetic and potential energies for all the particles that make up the system, this is not surprising. Nonideal gases usually have attractive intermolecular forces, and when molecules move farther apart, the associated potential energies increase. If the total internal energy is constant, the kinetic energies must decrease. Temperature is directly related to molecular *kinetic* energy, and for such a gas a free expansion is usually accompanied by a *drop* in temperature.

Test Your Understanding of Section 19.6

Is the internal energy of a solid likely to be independent of its volume, as is the case for an ideal gas? Explain your reasoning. (*Hint:* See Fig. 18.20.)

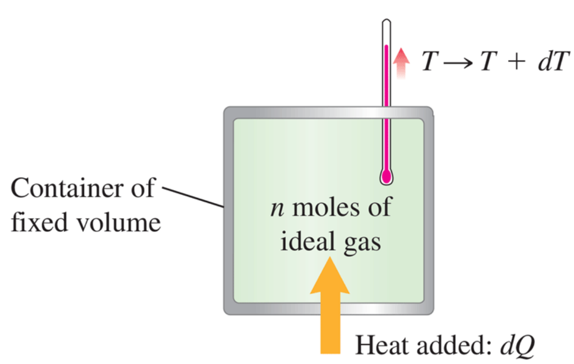
19.7 Heat Capacities of an Ideal Gas

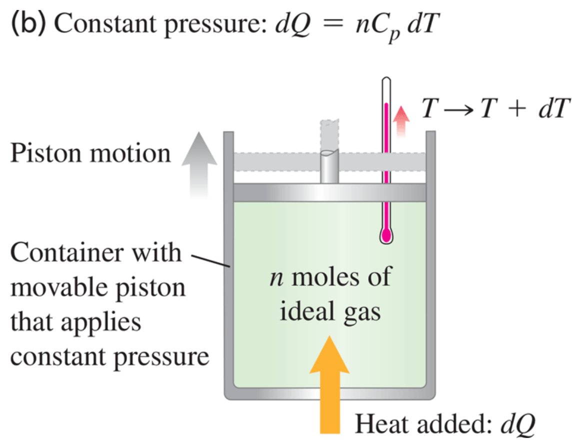
We defined specific heat and molar heat capacity in Section 17.5. We also remarked at the end of that section that the specific heat or molar heat capacity of a substance depends on the conditions under which the heat is added. The heat capacity of a gas is usually measured in a closed container under constant-volume conditions. The corresponding heat capacity is the **molar heat capacity at constant volume**, denoted by C_V. Heat capacity measurements for solids and liquids are usually carried out under constant atmospheric pressure, and we call the corresponding heat capacity the **molar heat capacity at constant pressure**, C_p.

Let's consider C_V and C_p for an ideal gas. To measure C_V, we raise the temperature of an ideal gas in a rigid container with constant volume, ignoring its thermal expansion (Fig. 19.18a). To measure C_p, we let the gas expand just enough to keep the pressure constant as the temperature rises (Fig. 19.18b).

Figure 19.18

(a) Constant volume: $dQ = nC_V\,dT$





Measuring the molar heat capacity of an ideal gas (a) at constant volume and (b) at constant pressure.

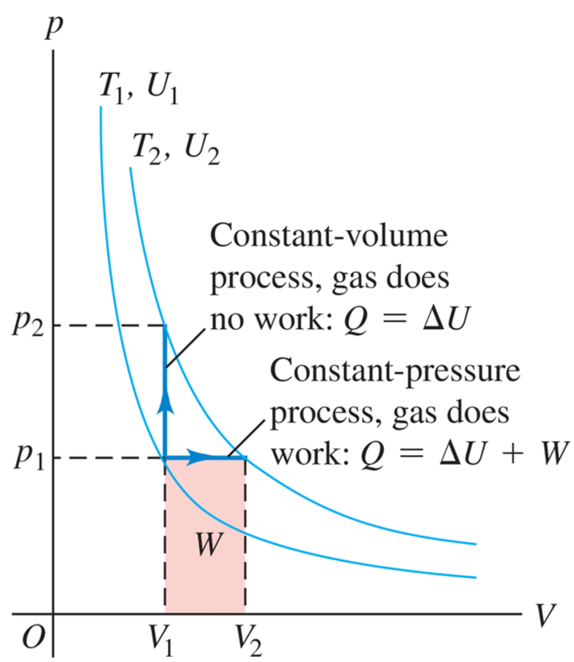
Why should these two molar heat capacities be different? The answer lies in the first law of thermodynamics. In a constant-volume temperature increase, the system does no work, and the change in internal energy ΔU equals the heat added Q. In a constant-pressure temperature increase, on the other hand, the volume *must* increase; otherwise, the pressure (given by the ideal-gas equation of state, $p = nRT/V$) could not remain constant. As the material expands, it does an amount of work W. According to the first law,

(19.11)

$$Q = \Delta U + W$$

For a given temperature increase, the internal energy change ΔU of an ideal gas has the same value no matter what the process (remember that the internal energy of an ideal gas depends only on temperature, not on pressure or volume). Equation (19.11) then shows that the heat input for a constant-pressure process must be *greater* than that for a constant-volume process because additional energy must be supplied to account for the work W done during the expansion. So C_p is greater than C_V for an ideal gas. The pV-diagram in Fig. 19.19 shows this relationship. For air, C_p is 40% greater than C_V .

Figure 19.19



Raising the temperature of an ideal gas from T_1 to T_2 by a constant-volume or a constant-pressure process. For an ideal gas, U depends only on T, so ΔU is the same for both processes. But for the constant-pressure process, more heat Q must be added to both increase U and do work W. Hence $C_p > C_V$.

For a very few substances (one of which is water between 0°C and 4°C) the volume *decreases* during heating. In this case, W is negative and the internal energy change ΔU is greater than the heat input Q.

Relating C_p and C_V for an Ideal Gas

We can derive a simple relationship between C_p and C_V for an ideal gas. First consider the constant-*volume* process. We place n moles of an ideal gas at temperature T in a constant-volume container. We place it in thermal contact with a hotter object; an infinitesimal quantity of heat dQ flows into the gas, and its temperature increases by an infinitesimal amount dT. By the definition of C_V, the molar heat capacity at constant volume,

(19.12)

$$dQ = nC_V\, dT$$

The pressure increases during this process, but the gas does no work $(dW = 0)$ because the volume is constant. The first law in differential form, Eq. (19.6), is $dQ = dU + dW$. Since $dW = 0$, $dQ = dU$ and Eq. (19.12) can also be written as

(19.13)

$$dU = nC_V\, dT$$

Now consider a constant-*pressure* process with the same temperature change dT. We place the same gas in a cylinder with a piston that we allow to move just enough to maintain constant pressure (Fig. 19.18b). Again we bring the system into contact with a hotter object. As heat flows into the gas, it expands at constant pressure and does work. By the definition of C_p, the molar heat capacity at constant pressure, the amount of heat entering the gas is

(19.14)

$$dQ = nC_p\, dT$$

The work dW done by the gas in this constant-pressure process is

$$dW = p\, dV$$