3rd Assignment 776, computer vision

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0 CODE

Line1.m build the data set of Line1, x + y = 3

Line2.m build the data set of Line2, x - y = 3

NoisePoints.m, build the data set of Noise Points

leastSquare.m, fit the dataset using Least Square method

HoughTransform.m, fit the dataset using Hough Transformation method

RANSAC.m, fit the dataset using RANSAC method

RANSAC_demo.m RANSAC's demo

LeastSquare1.m LeastSquare's demo using Line1

LeastSquare2.m LeastSquare's demo using Line2

HoughTransform_demo.m HoughTransform's demo

main.m run all the demos above.

1 DATA PREPARATION

1. Build x set by sampling from the range [0,10]
   X = rand(L,1) * 10
2. Apply function to build the y
   y = 3 – x
   y = x – 3

3. Add Gaussian noise of (0, 0.1)
   Y = y + normrnd(0, 0.1, L, 1)

The data is shown by plot function as follows

2 LEAST SQUARES

The data is (x_1, y_1) to (x_n, y_n), the line equation is $y_i = mx_i + b$.

$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

Let

$$Y = \begin{bmatrix} y_1 \\ \dots \\ y_n \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 & 1 \\ \dots & \dots \\ x_n & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} m \\ b \end{bmatrix}$$

To minimize the least squares $E = \|Y - XB\|^2 = Y^T Y - 2(XB)^T Y + (XB)^T (XB)$

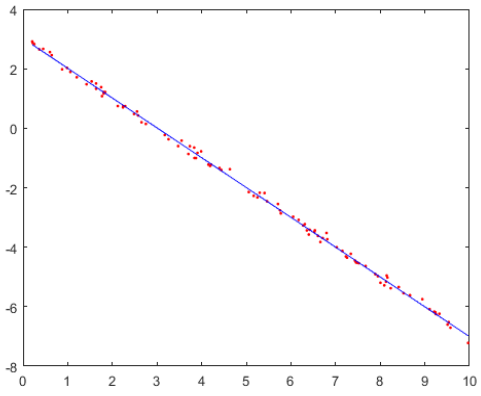
$$\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0$$

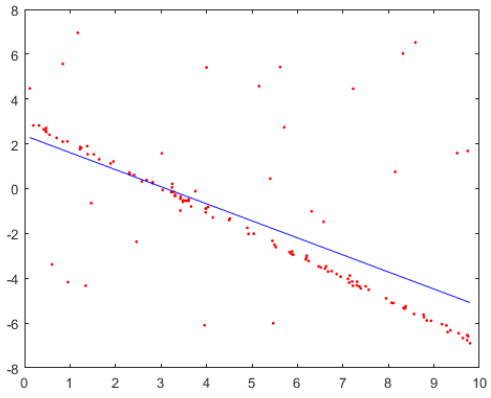
To solve this

$$\mathrm{B} = (X^T X)^{-1} X^T Y$$

And the residual error is the sum of the distance from the points to the line, which is

$$Error = \frac{|\, XB - Y \,|}{\sqrt{m^2 + 1}}$$





The result test on the Line1, Line1 + NoisePoints is shown above.

Using least squares method on Line1 is repeated 100 times following the instruction,

The mean of B is

[-0.9998, 2.9985]

The variance of B is

[1.0e-03*0.0113,1.0e-03*0.3464]

The residual error is 5.6270. Using least squares method on Line1 joins NoisePoints's residual error is 192.9047, which is much bigger than the fitted Line1.

3 RANSAC

The procedure of the algorithm RANSAC is shown as follows:

1. Randomly select minimal subset of points. The size of the subset is **S**.
2. Hypothesize a model. I use the least square method to fit the subset of points.
3. Compute error function.
4. Select points consistent with model. Using the distance threshold **t** to find the inliers that close to the fitted line.
5. Repeat for at least **N** times. If the sample ratio is larger than the thresh d, end the loop.

After that, using least square method to fit the inliers to get the final result.

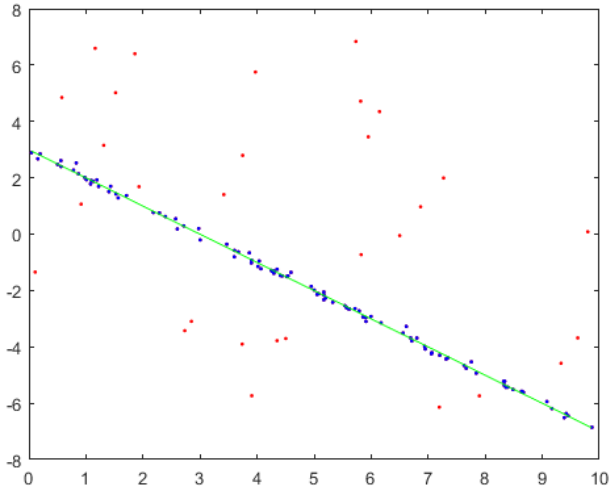
The N, S, t and the d are parameters of the RANSAC method. To improve the method by automatically determine the parameters of RANSAC, it can be computed by the probability that at least one random sample is free from outliers. For N,

$$N = \log(1-p)\,/\log(1-(1-e)^s)$$

In order to reduce N, set s as the minimum as 2.

For t, it can be computed by

$$t = \sqrt{3.84\sigma^2}$$



The result is shown as above. The blue points are inliers, and the red points are outliers. The fitted line is in green. For this case, the residual error = 4.4233, and the line is y = -1.0002x + 3.0035. N = 5.8428, which means it repeats 6 times.

In order to follow the instructions, I repeat the computation for 100 times for the same data set.

The mean and variance of the line parameters is

[-1.0004 3.0159]

[0.0000 0.0005]

And the mean and variance of the residual error is

[4.8476 0.1910]

And the mean and variance of the number of samples (required trials) is

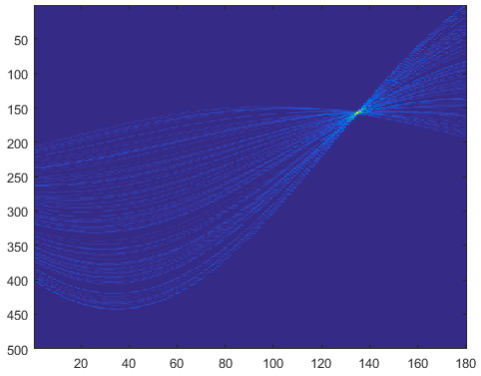
[5.2540 1.4996]

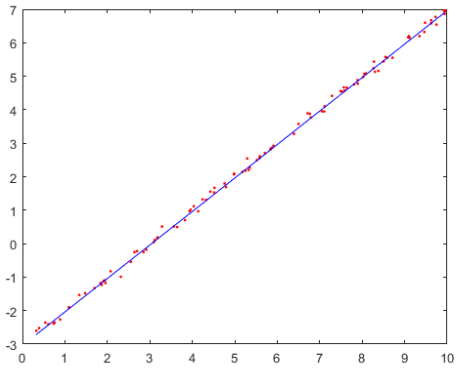
4 HOUGH TRANSFORMATION

The core idea of Hough transformation is to using parameter space representation. I used the polar representation as follows:

$$x\cos\theta + y sin\theta = \rho$$

For each point in the dataset, choose the different θ to get the different ρ. For each point (θ, ρ) in polar coordinates, accumulate the value to accumulator matrix H. Apply this method to the line2 data, we can get the accumulator matrix H.





The visualized accumulator matrix H is shown above left.

After that, find the maxima of the matrix H, which is (θ_m, ρ_m).

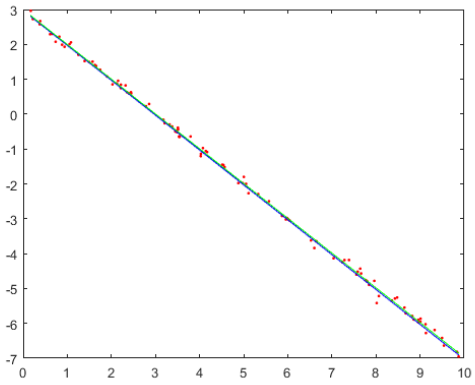
$$m = -\cot\theta, \quad b = \rho/\sin\theta$$

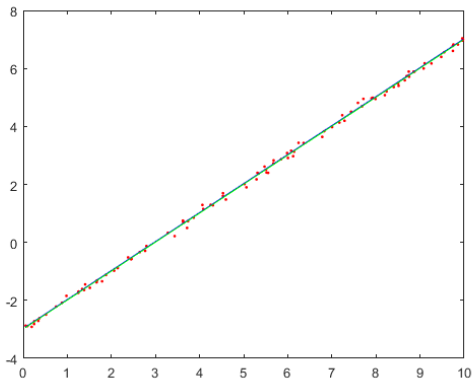
And the fitted line using Hough Transformation is shown above right.

In order to implement the approximate accumulator matrix, sample θ from 1 to 180. And using a transform to make it as a positive integer.

$$\rho' = round(\, k\rho + b\,)$$

And using Hough Transformation and Least Squares to fit the Line1 and Line2, the result is shown as follows, the blue line is the Hough Transformation's fit result and the green line is the Least Square's fit result.



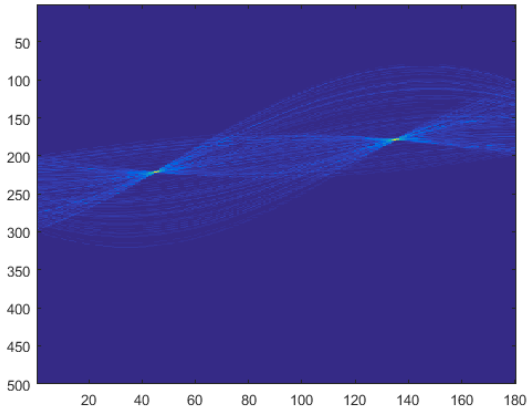


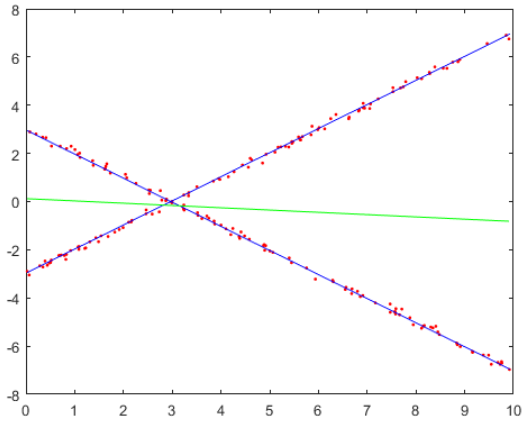
Their parameters are:

Hough transform: [1.0000, 2.9698] [-1.0000, 2.9698]

Least Squares: [0.9996, -2.9909] [-0.9987 2.9904]

For the data set Line1+Line2, the result is shown as follows:





To find all the lines by Hough Transformation, I set the threshold of the least points to 40.

In this case, Least Squares and RANSAC cannot handle it for they are design for fit one line. The Hough Transformation can handle it correctly. The result of Hough Transformation is [1.0000, -2.9698] and [-1.0000, 2.9698]. It is the same as the result on Line1 or Line1.