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Array signal processing - Summer 2012

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1. Introduction

- What is array signal processing?
  - Using multi-antenna systems for receiving (and/or transmitting) and extracting useful information from the aquired signals
- Why use arrays?
  - For example: signal diversity, improved SNR, **source localization**...

1. Introduction

- Some applications for antenna-arrays:
  - Radar, sonar, astronomy and small-scale positioning applications

Figure: From left to right: Raymarine Super HD Color radar [2], the Very Large Array (VLA) in New Mexico [3] and a Uniform rectangular patch array [4].

2. Framework

- Here the focus will be on source Direction-Of-Arrival (DOA) estimation
- In 3D-space the DOA can be described by two angles: azimuth (ϕ) *and elevation* (θ)

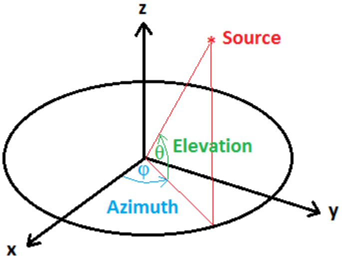

Figure: Azimuth and elevation.

2. Framework

- The standard signal model used in array processing is:

$$\boldsymbol{X} = \boldsymbol{A}(\phi, \theta) \cdot \boldsymbol{S} + \boldsymbol{N}$$

- Each column of **A** corresponds to a DOA
- **A** is determined by the structure of the array and the DOA angles
- **A** cannot be exactly determined in real-world arrays due to calibration errors, mutual coupling, nonuniform spacing, varying beam patterns, etc.

2. Framework

- How does source localization work?
  - Phase difference between the array elements
  - Usual assumption: far-field point sources

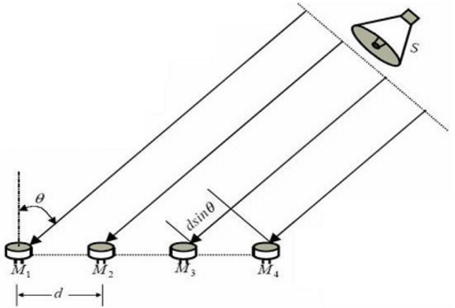

Figure: Impinging wavefront on a uniform linear array. [1]

2. Framework

- This summer's main topics have been:
  - **Beamforming techniques** (CBF, MW, IAA)
  - Subspace methods (MUSIC, ES root-MUSIC)
  - Model order selection (AIC, MDL, EEF)
  - Cramér-Rao bound
  - Parametric MIMO-systems in multipath environments
  - **Manifold Separation Technique / Effective Aperture Distribution Function**
  - Polynomial rooting
  - **Autocalibration**

  \+ a lot of estimation theory and matrix calculus...

3. Beamforming

- Beamforming is a central technique used for estimating for example source DOAs, powers or signal waveforms

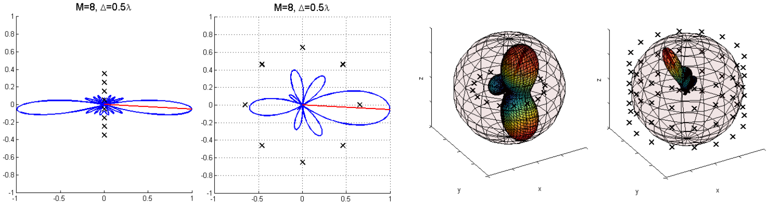

Figure: Beampatterns of different array configurations. From left to right: ULA, UCA (fixed elevation), UCA (variable elevation) and SUCA.

3. Beamforming

- Beamforming is essentially spatial filtering, i.e.:

$$\hat{\boldsymbol{s}}(\phi) = \boldsymbol{w}^{\boldsymbol{H}}(\phi) \cdot \boldsymbol{x} \quad \textit{and} \quad \hat{\boldsymbol{P}}(\phi) = \boldsymbol{w}^{\boldsymbol{H}}(\phi) \cdot \hat{\boldsymbol{R}}_{\boldsymbol{x}} \cdot \boldsymbol{w}(\phi)$$

- The weight vector **w** is determined by the selected beamforming technique

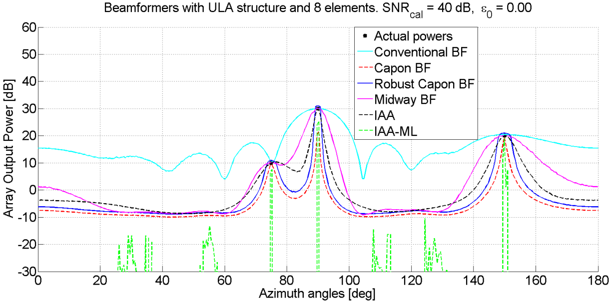

Figure: Power spectra of some beamformers.

4. Manifold separation

- The Manifold Separation Technique (MST) [8] expands the array steering matrix **A** into two independent components:

$$\boldsymbol{A}(\phi) = \boldsymbol{G} \cdot \boldsymbol{D}(\phi)$$

- **G** depends on the array configuration only
- $\boldsymbol{D}(\phi)$ has the same structure for all arrays and it depends only on the DOAs
- The MST conveniently extends different array processing algorithms to arrays of arbitrary configuration and it automatically takes care of DOA interpolation
- The MST exploits the fact that **A** is periodic in ϕ, allowing for a Fourier series representation
- **G** is essentially a matrix containing the Fourier coefficients obtained from the DFT of **A**
- This infinite series can be truncated without major information loss in order to obtain a practical $\hat{\mathbf{G}}$

4. Manifold separation

- Real world arrays usually need to be calibrated in an anechoic chamber to get **A**
- The finite angular resolution makes interpolation necessary for any DOAs in-between the calibration points
- The MST automatically solves this problem since **D** is a continous function of ϕ

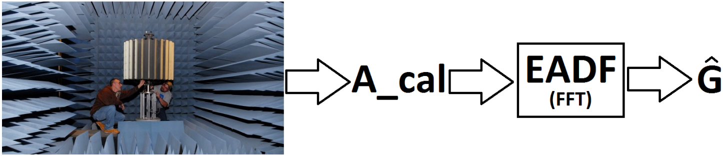

Figure: The MST in practice. [9]

5. Autocalibration - introduction

- Consider the following signal model:

$$\boldsymbol{X} = \boldsymbol{A}(\phi) \cdot \boldsymbol{S} + \boldsymbol{N} = \boldsymbol{G} \cdot \boldsymbol{D}(\phi) \cdot \boldsymbol{S} + \boldsymbol{N} = \boldsymbol{G} \cdot \boldsymbol{\Psi}(\phi) + \boldsymbol{N}$$

- Autocalibration is the simultaneous estimation of both the array response and the wavefield
- With a "good enough" initial guess of either one, the algorithm should improve on these
- For this study, the autocalibration problem was:
  - The wavefield $\mathbf{\Psi}$ is to be sequentially/ recursively estimated starting from a pertubed sampling matrix $\tilde{\mathbf{G}}$
  - The estimated wavefield is then used to estimate $\mathbf{G}$ and improve on the initial guess ($\tilde{\mathbf{G}}$)

5. Autocalibration - introduction

- The estimation is done in an alternating manner using an algorithm of choice for estimating $\mathbf{\Psi}$ and Sequential Least Squares (SLS) for updating $\hat{\mathbf{G}}$

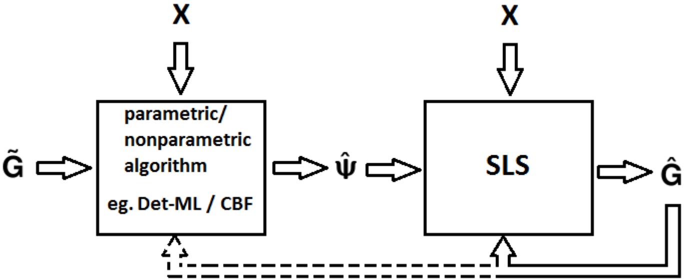

Figure: Block diagram of the autocalibration algorithm

5. Autocalibration - choice of wavefield estimator

- Both parametric and nonparametric algorithms for estimating $\boldsymbol{\Psi}$ were implemented
- Parametric techniques rely on the point-source assumption and knowledge of the number of sources, whereas these assumptions can be relaxed for nonparametric techniques
- The implemented nonparametric algorithms [5] included polynomial rooting versions of:
  - Standard- (SCBF) and Robust Capon Beamformers (RCBF)
  - Midway method (MW)
  - Iterative Adaptive Approach (IAA) and Maximum Likelihood Iterative Adaptive Approach (IAA-ML)
- However, all of the implemented nonparametric algorithms only converged to the initial guess $\tilde{\mathbf{G}}$, which made them useless

5. Autocalibration - choice of wavefield estimator

- On the other hand, the (parametric) Deterministic Maximum Likelihood -algorithm [6] with Exponentially Embedded Families (EEF) [7] model order selection was found to beat the initial guess $\tilde{\mathbf{G}}$
- Consequently, it was the only algorithm used in the simulations

5. Autocalibration - simulations

- Some simulation results will be presented next
- The plots compare the MSEs of the starting value $\tilde{\mathbf{G}}$ to estimates $\hat{\mathbf{G}}$ produced using the exact wavefield and the deterministic-ML wavefield estimate
- The central parameters are:
  - Niter = Number of iteration (update) rounds
  - Nbatch = Number of samples available at each update instant (snapshots)
  - N = Total number of samples ($= \mathit{Niter} \cdot \mathit{Nbatch}$)
  - SNR_{cal} = The SNR of $\tilde{\mathbf{G}} = \mathbf{G}+\mathbf{N}$
- The simulations considered a UCA with M = 8 elements spaced half a wavelength apart
- The sequential estimation was done in batches of Nbatch samples $\geq$ M in order for $\hat{\boldsymbol{R}}_{\boldsymbol{x}}$ to be full rank
- For each batch of samples processed, the DOAs and signal powers were fixed (to random values)

5. Autocalibration - simulations

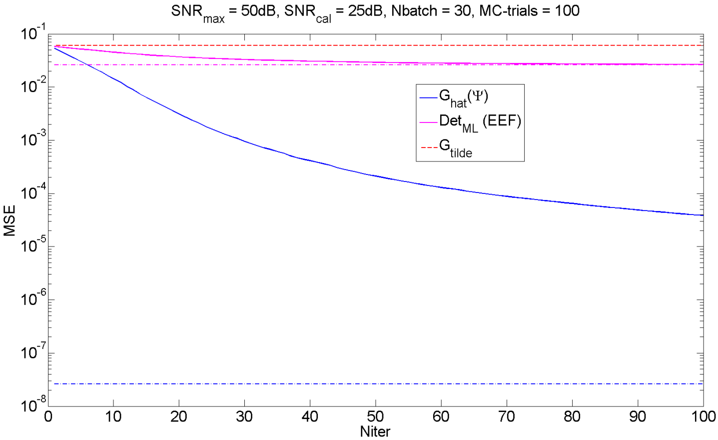

Figure: Number of batches (of size 30) vs MSE of $\hat{\mathbf{G}}$

5. Autocalibration - simulations

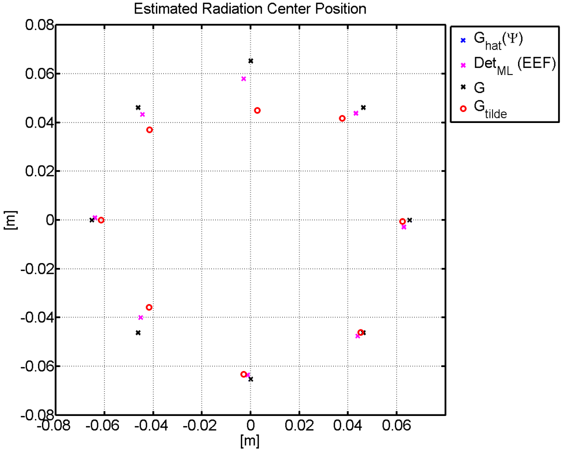

Figure: The array configuration for one realization

5. Autocalibration - simulations

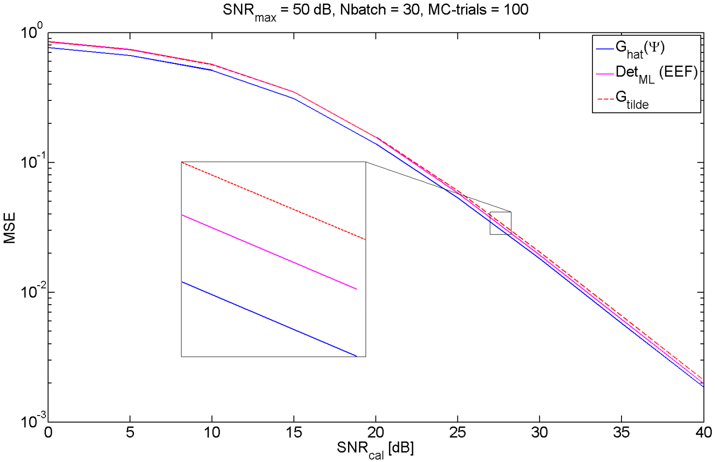

Figure: The pertubation SNR of $\tilde{\mathbf{G}}$ vs MSE of $\hat{\mathbf{G}}$

5. Autocalibration - results

- The main results from this autocalibration experiment were:
  - Autocalibration can achieve a significant gain ($\sim$ 10dB), provided that it is given a "good enough" starting point ($\tilde{\mathbf{G}}$ in this case)
  - Practically everything (DOAs, signals, noise power, number of sources...) is estimated in the simulated method, which makes it applicable in practice

5. Autocalibration - future work

- Some remaining topics for future work include:
  - The nonparametric algorithms - can they beat $\tilde{\mathbf{G}}$?
  - Can this be done in the single snapshot case (Nbatch = 1)?
  - What if $\mathbf{G}$ is unknown and we have an initial $\tilde{\boldsymbol{\Psi}}$?
  - Total Least Squares vs. SLS and least squares on the whole data

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