ECE 504 Analysis of Deterministic Signals and Systems

Class Project

Dr. Bo Tang
Department of Electrical and Computer Engineering
Worcester Polytechnic Institute

This project will demonstrate the techniques we have learned in this class for modeling, analyzing, and controlling linear systems on a realistic case study. This project will study the problem of reference tracking for a helicopter from [1].

State	Meaning
θ	Pitch attitude
ϕ	Roll attitude
p	Roll rate
q	Pitch rate
ξ	Yaw rate
v_x	Forward velocity
v_y	Lateral velocity
v_z	Vertical velocity

Table 1: State variables for helicopter control example.

The helicopter has state variables shown in Table 1. This model represents a twin-engineered helicopter with a four-blade main rotor. You will develop a full-authority controller, which is designed to have complete control over the blade angles given a reference signal from the pilot. The model has four control variables, each representing a different blade angle. The outputs consist of the heave velocity, pitch attitude, roll attitude, heading rate, roll rate, and pitch rate. The system dynamics are given by the linear state-space equations:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx \end{aligned}$$

where the matrices *A*, *B*, *C* and *D* can be found in the Matlab file on the Canvas course website.

Your Assignment

Your assignment is to complete the following steps and prepare a **project report**, including plots and illustrations, to back up your results. In addition to the report, please **turn in the Matlab codes** that you used to complete the assignment.

Part One Download the *A*, *B*, *C*, and *D* matrices.

Part Two Compute the eigenvalues of matrix *A*. Is the system asymptotically stable, marginally stable, or unstable?

Part Three Is the system BIBO stable?

Part Four Compute the transfer function from input $u_1(t)$ to output $y_1(t)$. (Note: The system is a MIMO system, and $u_1(t)$ and $y_1(t)$ are the first input and the first output, respectively.)

Part Five Plot each of the output variables (in separate plots) when the input is a unit step (for all input variables) and the initial state is zero.

Part Six Is (*A, B*) controllable? Is (*A, C*) observable?

Part Seven Compute a stabilizing state feedback controller for this system.

Part Eight Compute an estimator for the system, and a stabilizing output feedback controller (i.e., state feedback controller + state estimator).

Part Nine Plot the closed-loop response (with the stabilizing output feedback controller that you designed) to a unit step input with zero initial state.

Part Ten Plot the zero-input response for the closed-loop system when the initial state is (1.2, 2.5, 3.1, -0.5, -1, 4.9. -2, -0.7).

Part Eleven Design a controller in unity feedback configuration to ensure that the output $y_1(t)$ can track any step reference input at $u_1(t)$. Provide plots to illustrate the effectiveness of your controller.

References

[1] S. Skogestad and I. Postlethwaite, *Multivariable feedback control: analysis and design*. Wiley New York, 2007, vol. 2.