# Hypothesis Testing for Proportions

Now, we want to test a hypothesis for population proportion $p$ → POP

&

$\hat{p}$ → SAMPLE

We still need the Central Limit Theorem *for proportions* to hold:

$$np_0 \geq 10 \quad \text{and} \quad n(1 - p_0) \geq 10$$

$\mu_0$, $p_0$ → VALUE WE ASSUME IN THE NULL.

Where $p_0$ is the population proportion specified by $H_0$

> **Step 1:** State the null and alternate hypotheses

The null hypothesis is of the form:

$$H_0 : p = p_0$$

The alternate hypothesis is in one of the three forms:

- **Left-tailed:** $H_1 : p < p_0$
- **Right-tailed:** $H_1 : p > p_0$
- **Two-tailed:** $H_1 : p \neq p_0$

> **Step 2:** Choose a significance level $\alpha$

> **Step 3:** Compute the test statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

> **Step 4:** Compute the P-value of the test statistic $z$

**Left-tailed:** P-value = area under the standard normal distribution to the left of $z$

- i.e., $P(Z < z)$

**Right-tailed:** P-value = area under the standard normal distribution to the right of $z$

- i.e., $P(Z > z)$

**Two-tailed:** P-value = sum of the areas under the standard normal distribution to the left of $-|z|$ and right of $|z|$

- i.e., $2 * P(Z < -|z|)$

$\alpha = 0.05$

> **Step 5:** Determine whether to reject $H_0$:

Reject $H_0$ if P-value $\leq \alpha$

$P_1$-VALUE = 0.049

Do not reject $H_0$ if P-value $> \alpha$

$P_2$-VALUE = 0.051

> **Step 6:** State a conclusion

Suppose that 67% of all auto damage insurance claims in the US are made by singles under 25 years old. Also suppose that in a random sample of 53 auto damage claims in Manhattan, KS, there were 42 made by singles under 25.

Test at the 5% significance level whether the proportion of auto damage claims made by singles under 25 in Manhattan is different than the proportion for the entire US.

$P = 0.67$

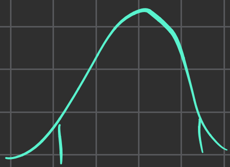
$\hat{P} = \frac{42}{53} = 0.792$

$\alpha = 0.05$

* STAGE YOUR HYPOTHESES

$H_0: \hat{P} = P_0$ $\qquad H_a: \hat{P} \neq P_0$

* TWO TAILED



* COMPUTE THE TEST STATISTIC

$$z^* = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \qquad \frac{0.792 - 0.67}{\sqrt{\frac{0.67(1-0.67)}{53}}} = 1.89$$

* DETERMINE REJECTION/DECISION $\alpha = 0.05$

P-VALUE → dF = 53 - 1 = 52 $\quad 2*(0.025 \cdot 0.05)$

CRIT VAL → $1.676 < z^*$

* CONCLUDE $\quad -1.67 > -z^*$

REJECT P-VALUE TEST