The Max-tree Image Representation

Roberto Souza
Assistant Professor
Electrical and Software Engineering

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UNIVERSITY OF CALGARY

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Outline

- Basic Image Processing Concepts

- Graph-based Image Representation: The Max-tree
  - Max-tree
  - Filters
  - Data structure and algorithms
  - Signature analysis
  - Extinction Values and Filters

Learning Objectives

- Develop knowledge regarding basic image processing concepts
- Introduce the concept of extrema persistence (i.e., extinction values)
- Learn about the max-tree data structure and how to use it for filtering and morphological image analysis

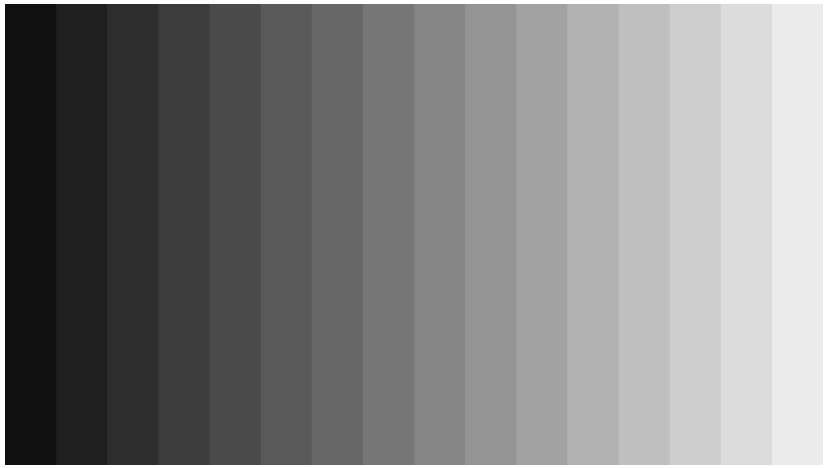
Basic Image Processing Concepts

Grayscale Image

Definition: *A grayscale image is* a function:

$$\boldsymbol{I(z): E \rightarrow k, E \in \aleph^2 \; and \; k \; \in \mathbf{Z}}$$

- Low values are black, dark gray;
- High values are bright gray, white;

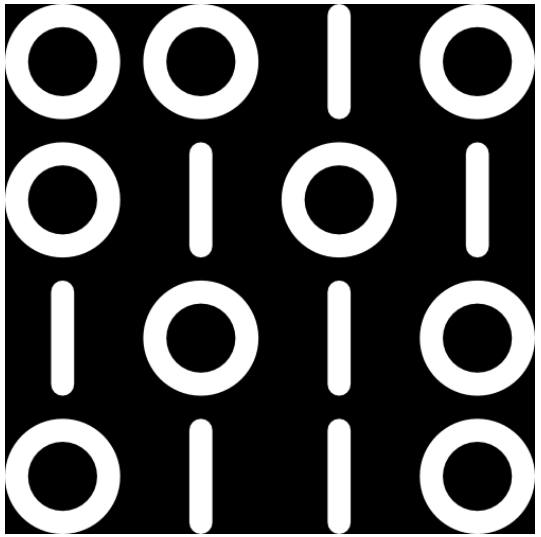


Grayscale ramp image.

Binary Image

Definition: A binary image is a function:

$$\boldsymbol{I(z): E \rightarrow k, E \in \aleph^2 \; and \; k \; \in [0,1]}$$

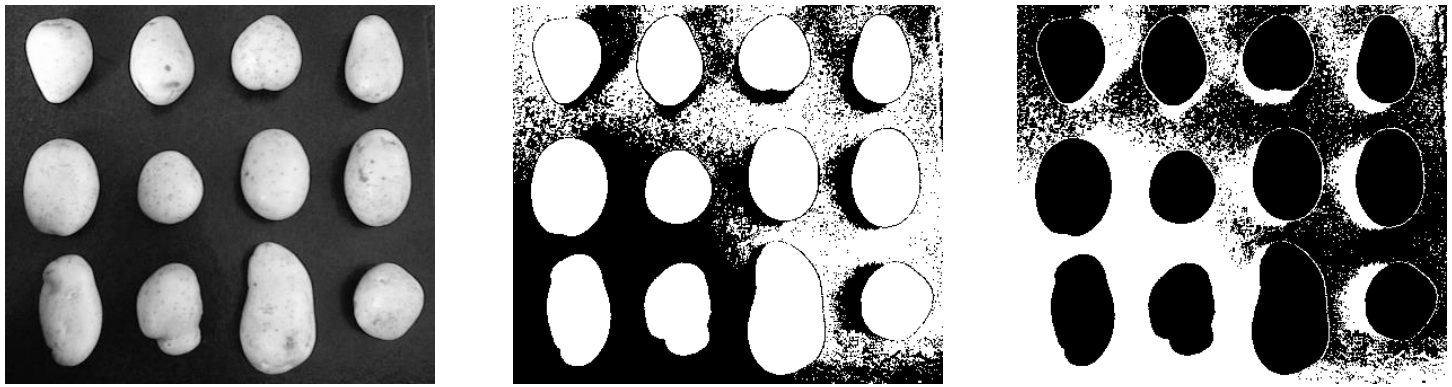


Binary image.

- 0 → black;
- 1 → White;

Image Threshold

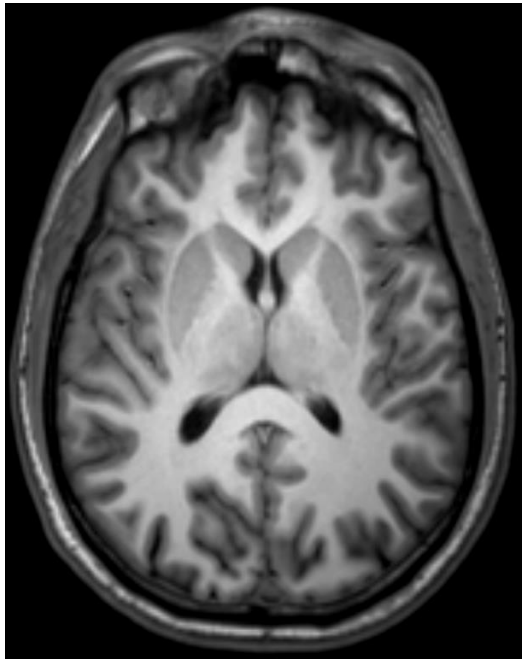
- Upper threshold: $\chi_h^{\geq} = \begin{cases} 1 \; if \; I(z) \geq h \\ 0 \; otherwise \end{cases}$
- Lower threshold: $\chi_h^{<} = \begin{cases} 1 \; if \; I(z) < h \\ 0 \; otherwise \end{cases}$



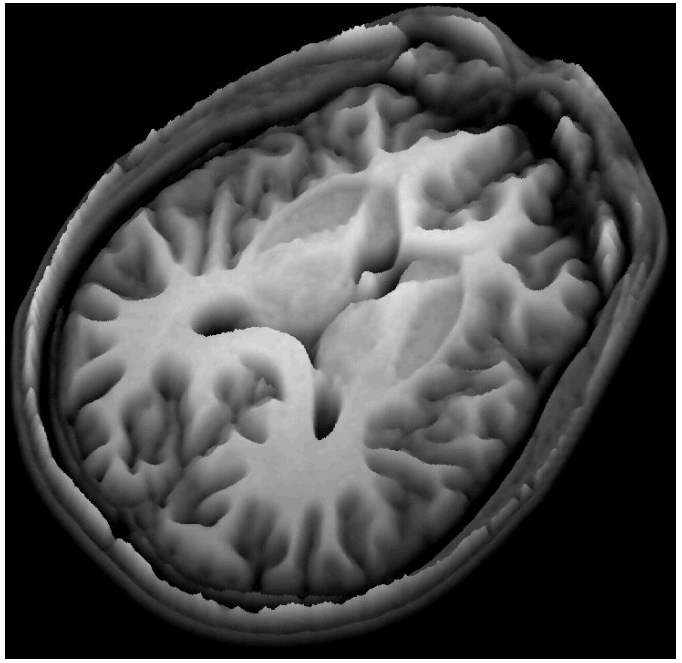
Left to right: original image, upper threshold, lower threshold.

Connected Components (CCs)

- “White islands in a binary image”

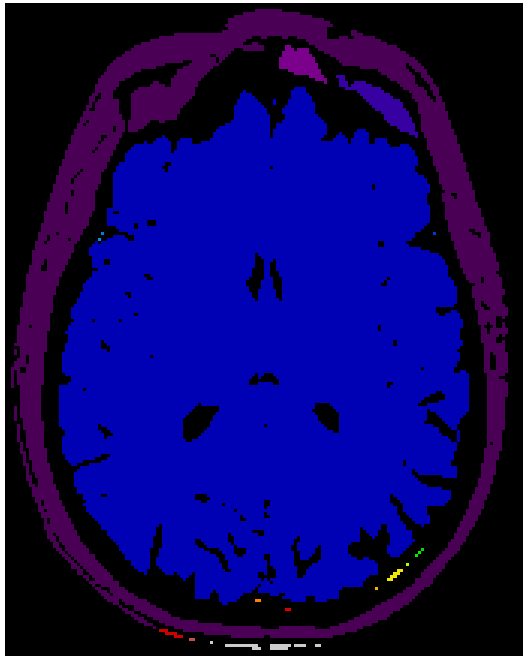


Axial brain image



Topographic view

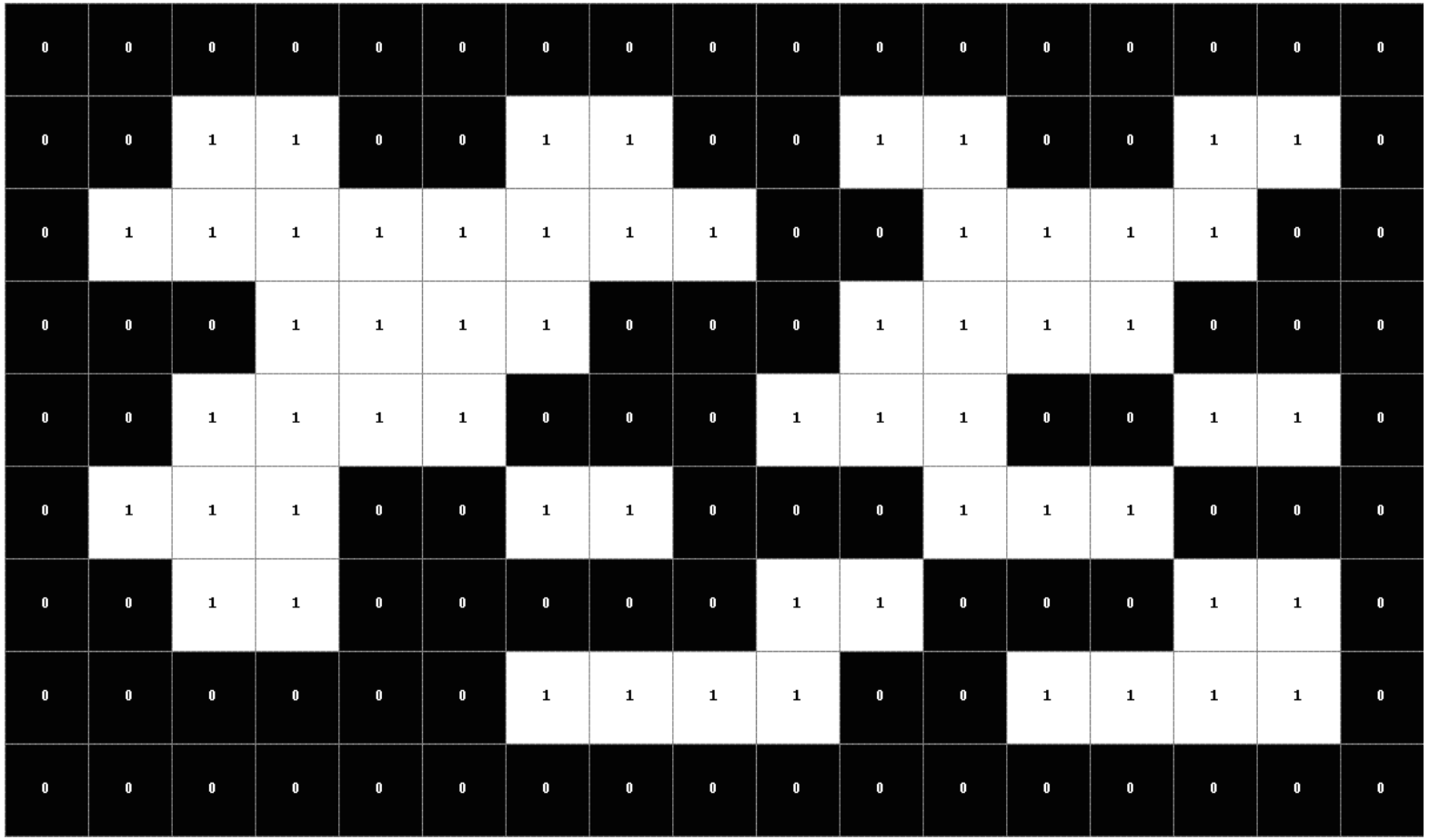
Upper threshold f≥60



Labeled image

9

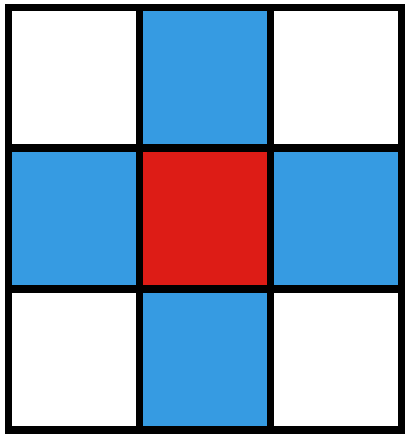
How many CCs are in this image?



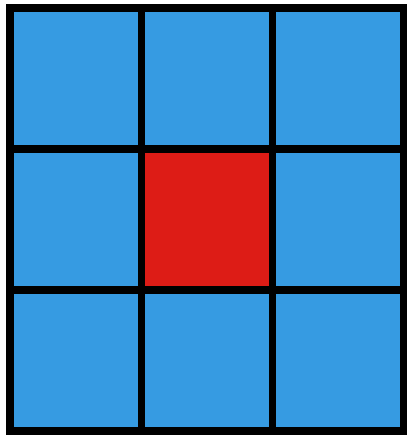
It depends on the connectivity rule you use!

Connectivity Rule

2D

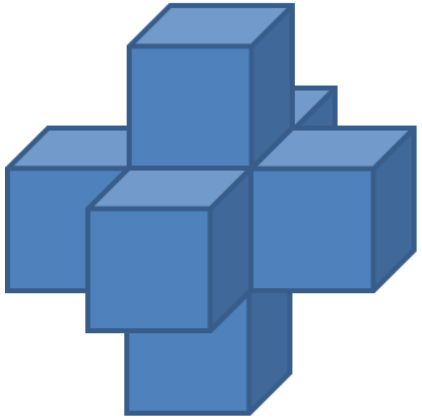


4-connectivity (C4)

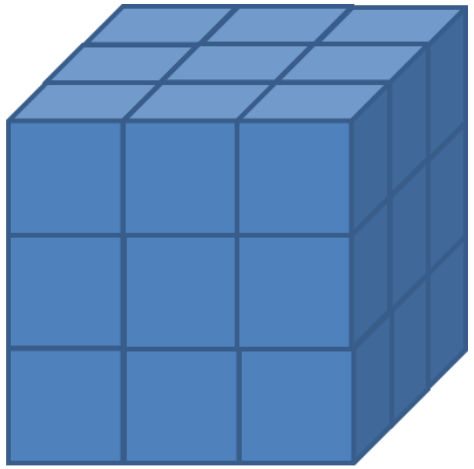


8-connectivity (C8)

3D

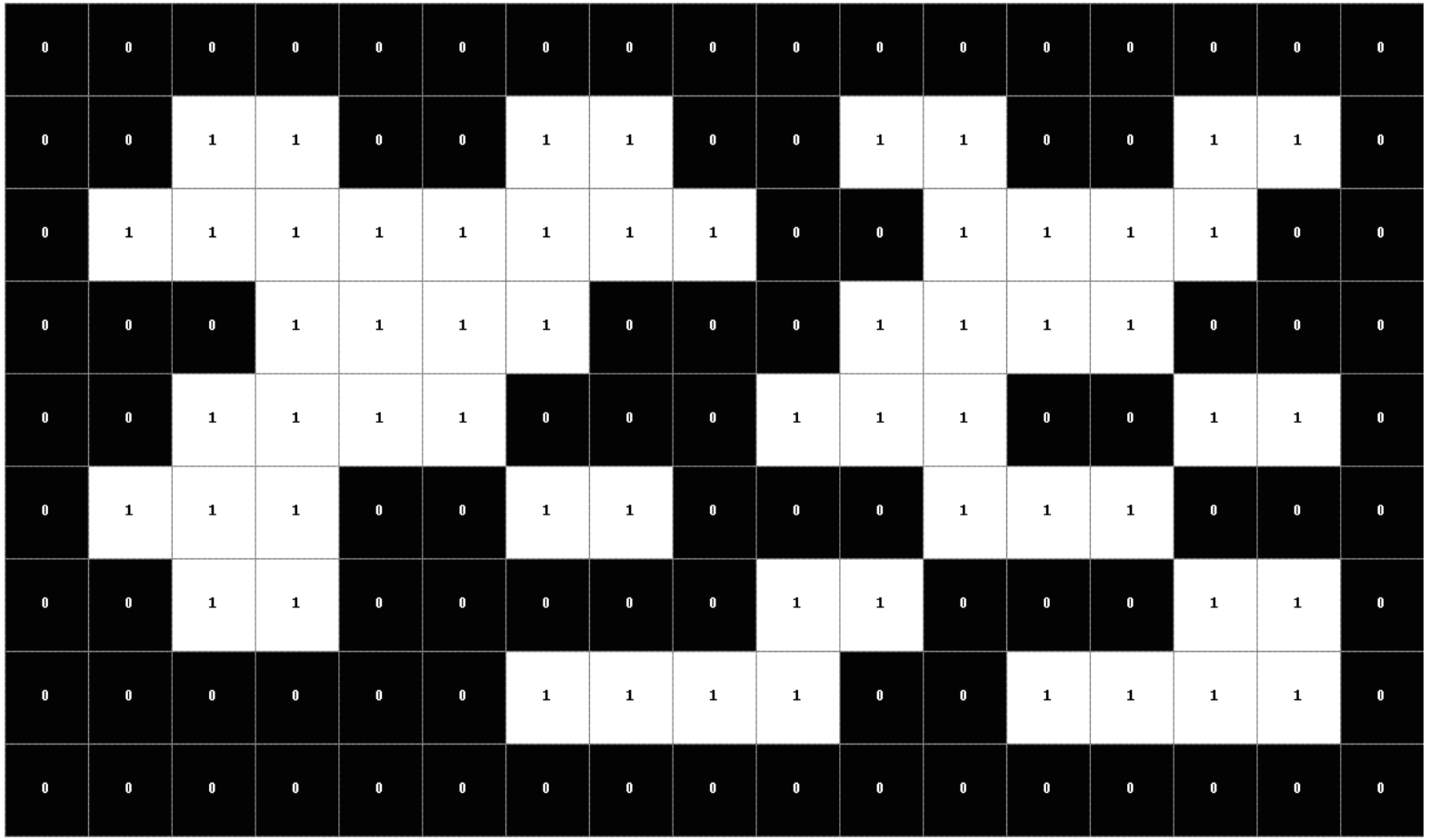


6-connectivity (C6)



26-connectivity (C26)

How many CCs are in this image?



- 2 if using connectivity C8
- 6 if using connectivity C4

Basic Definitions

- ***Flat zone-*** are connected component defined by pixels of the same gray-level.

- ***Threshold set -*** The threshold set is the set of connected componentes $\{C_{h,1}, C_{h,2}, \dots, C_{h,ncc}\}$.

- ***Partition –*** A partition P of a set X is a set of nonempty subsets of X, such that every element x in X is in exactly one of these subsets.
  - $P(X) = \{X_0, X_1, X_{n-1}\}, \;\; X_i \cap X_j = \emptyset, \;\; i \neq j$
  - $X = X_0 \cup X_1 \dots X_{n-1}$

Basic Definitions

- ***Image ordering:*** $\mathrm{f}[\mathrm{z}] \leq \mathrm{I}[\mathrm{z}], \quad \forall \mathrm{z}$

- ***Anti-extensive filter:*** $\boldsymbol{\psi(I) \leq I, \forall I}$

- ***Connected filter –*** is a filter in which the partition of the input image flat zones is always finer than the partition of the filtered image.
  - $P_I \subseteq P_{\psi(I)}, \forall I$

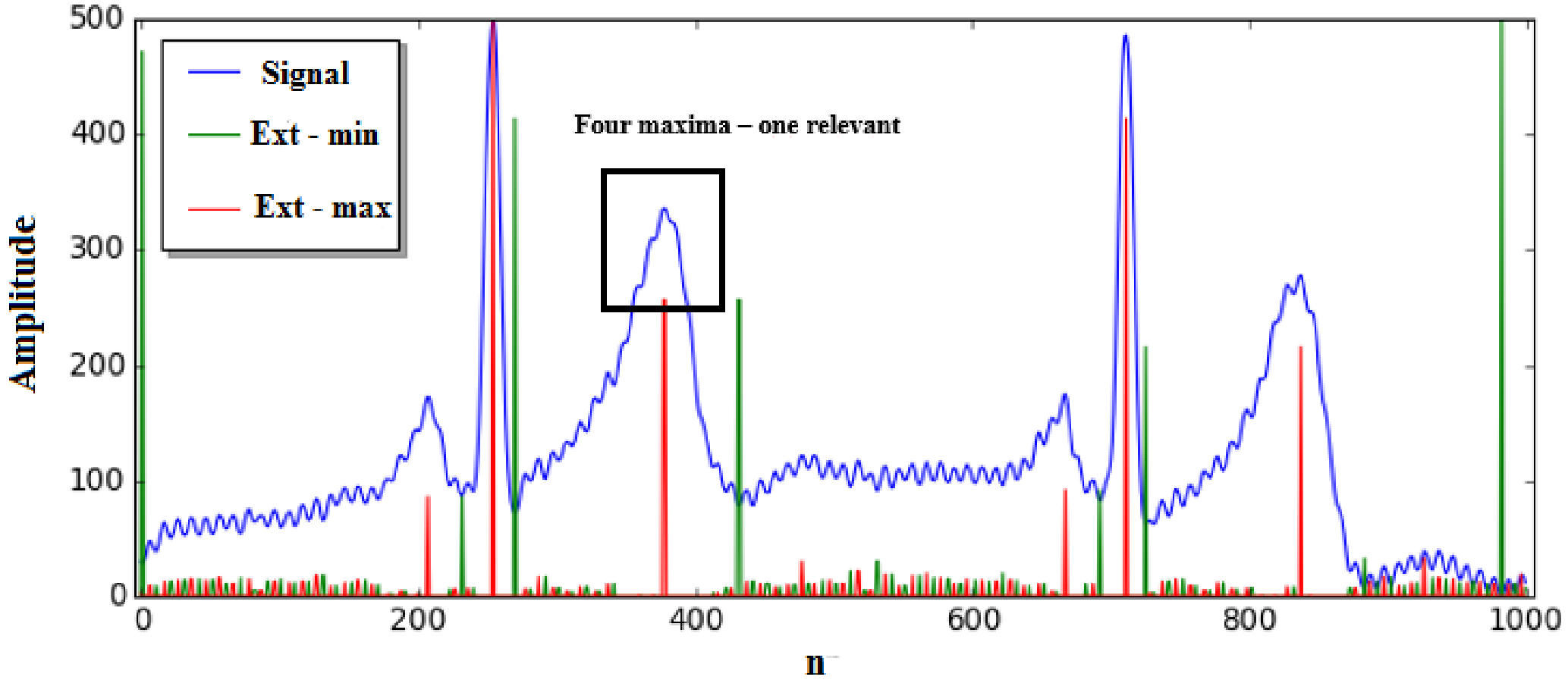
Extinction Values

- ***Extinction value***[+]: consider M a regional maximum of na image I, and $\Psi(\psi_\lambda)_\lambda$ is a family of decreasing connected anti-extensive transformations. The extinction value $\epsilon_\Psi(\mathrm{M})$ is given by:
  - $\epsilon_\Psi(\mathrm{M}) = \sup\{\lambda \geq 0 | \forall \mu \leq \lambda, \mathrm{M} \subset \mathrm{Max}(\psi_\mu(\mathrm{i}))\}$

- Extinction values are attributes of extrema (i.e., minima or maxima)
- Extinction values are associated to an increasing attribute
- Measure of extrema persistence when submitted to filters of different intensities

[+]Vachier, "Extinction value: a new measurement of persistence", *IEEE Workshop on Non- linear Signal and Image Processing.*

Extinction Values



Which extrema seem relevant to you?

Graph-based Image Representation

The Max-tree Data Structure

Component Tree

- The component tree[+] is a structure for image representation that represents every connected component of every possible threshold of the image
  - It stores intensity, shape and size information about its CCs
- It is an efficient structure for implementing connected anti-extensive filters and by duality extensive filters
- It provides an attribute signature as means of discriminating features in the image
- Little known outside the morphology community

[+]Jones, R. , “Component trees for image filtering and segmentation.” *IEEE Workshop on Nonlinear Signal and Image Processing*,1997.

Max-tree

- The max-tree* is an efficient representation in terms of memory of the component tree

- There are efficient algorithms to build and process it

- It has been used in many applications such as segmentation, feature extraction, filtering, remote sensing…;
  - Connected filtering
  - Selecting image markers

*Salembier et al. (1998), “Antiextensive connected operators for image and sequence processing”. IEEE Tran on Image Processing

Component Tree

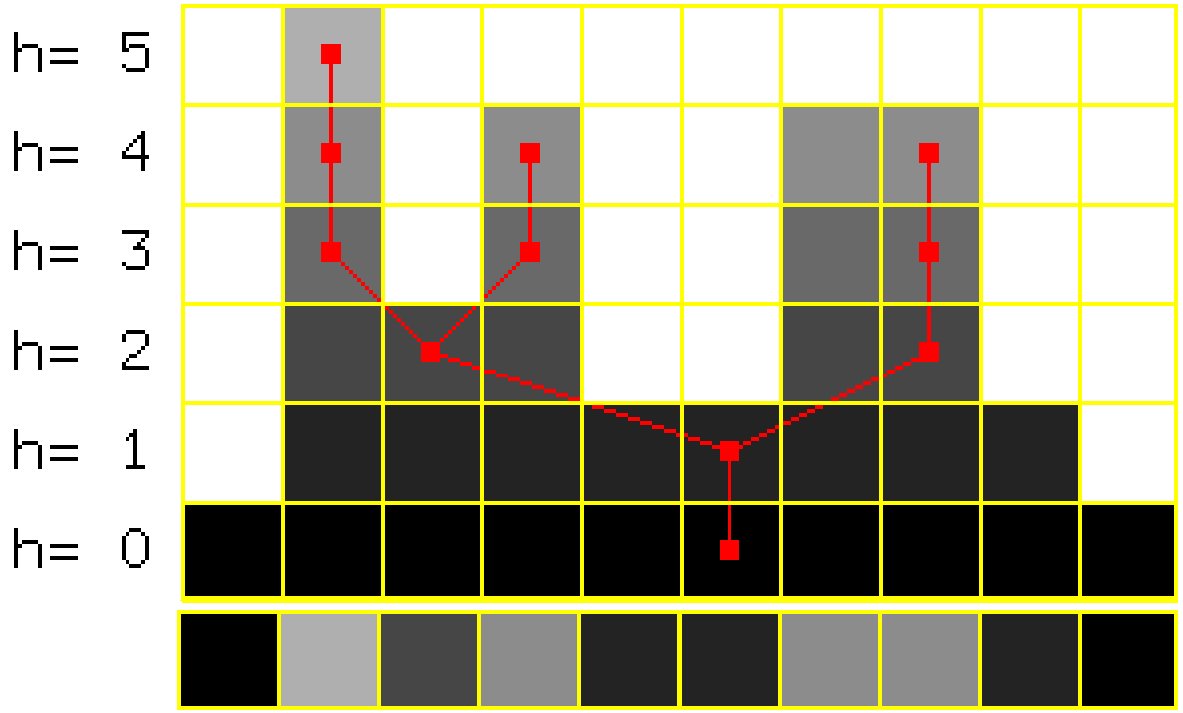


Image I = [0,5,2,4,1,1,4,4,1,0] (bottom) and corresponding component tree (top).

Max-tree

- Compact structure for the component tree representation.

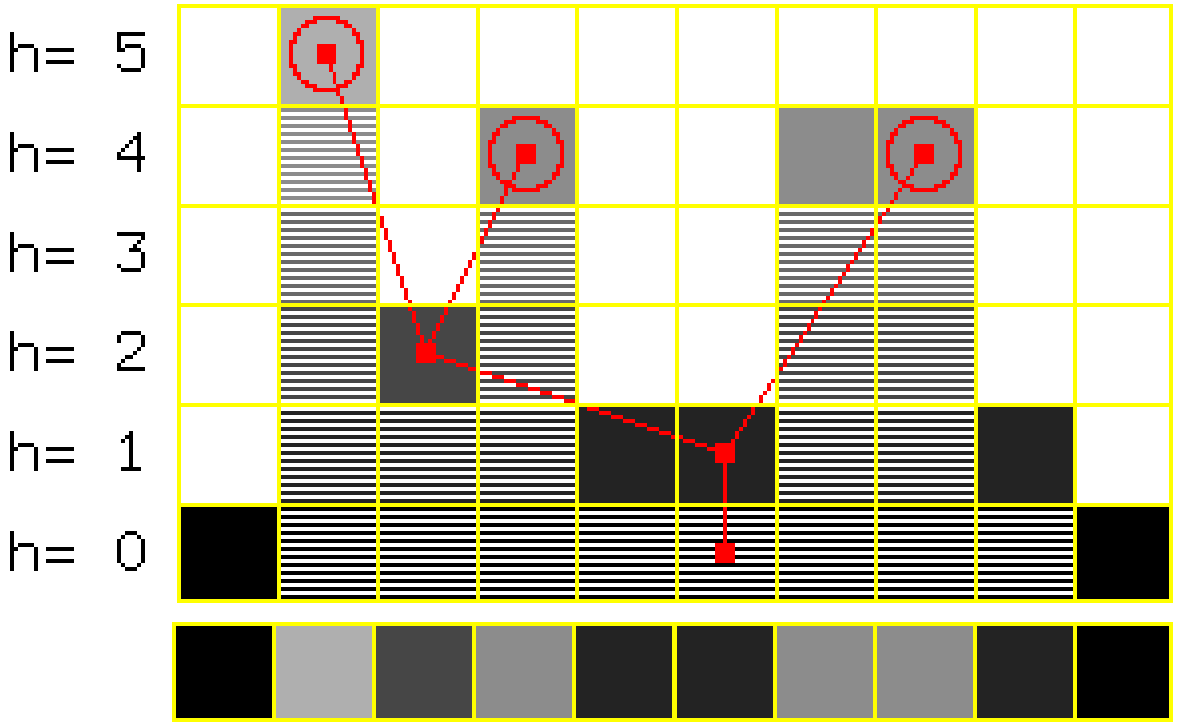
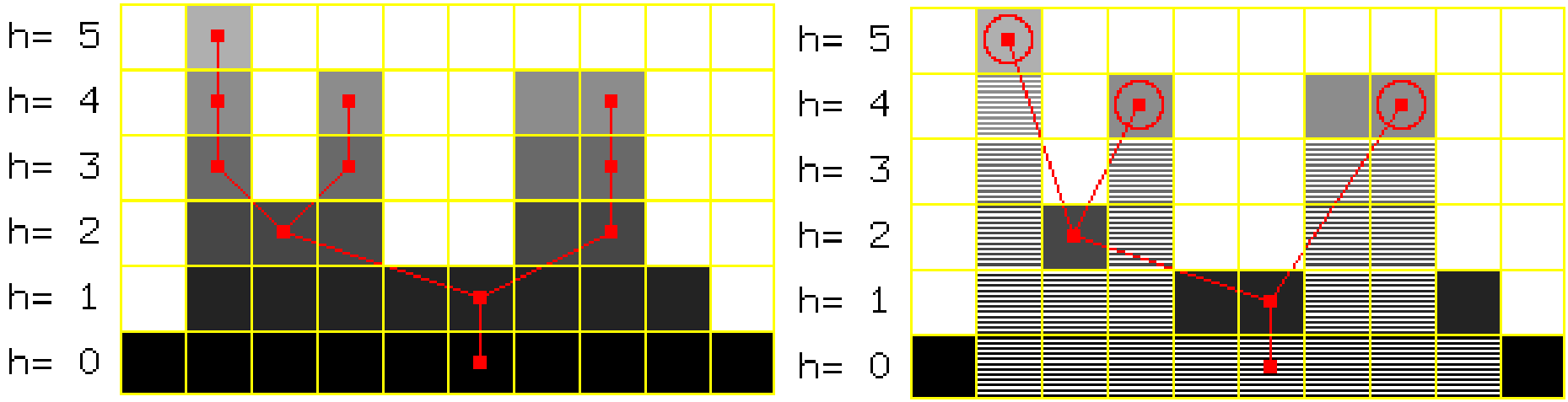


Image I = [0,5,2,4,1,1,4,4,1,0] (bottom) and corresponding max-tree (top).

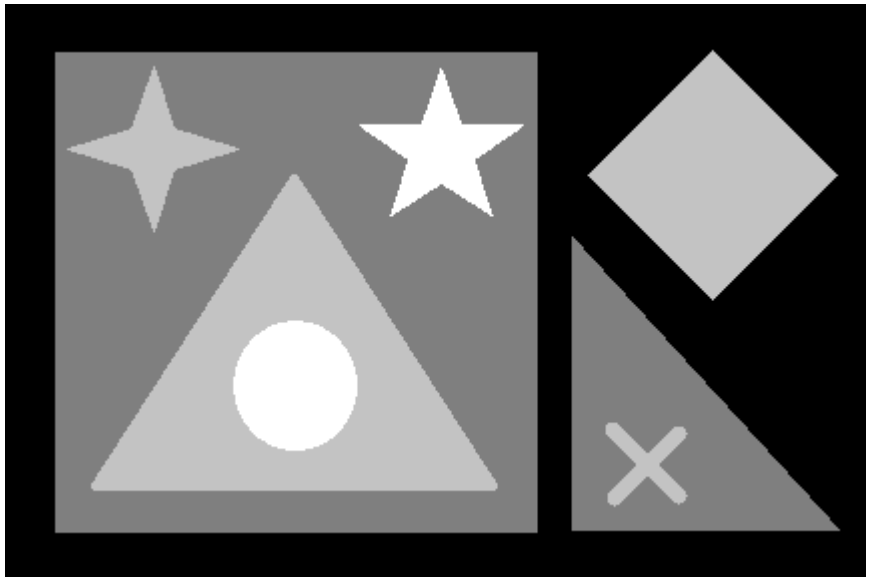
Component Tree and Max-tree



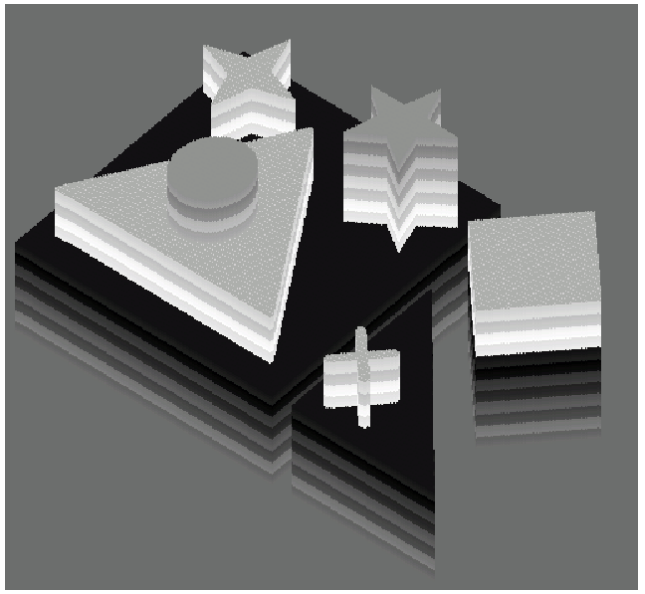
Component tree (left) and max-tree (right) of the image I = [0,5,2,4,1,1,4,4,1,0].

We will work with the max-tree but we will often think in terms of the component tree!

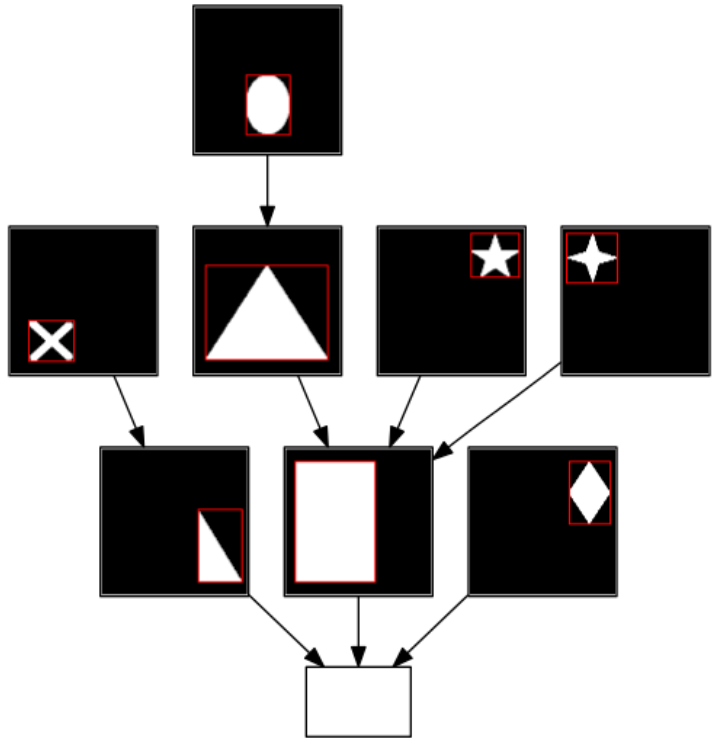
Max-tree Representation of a Synthetic Image



Synthetic image



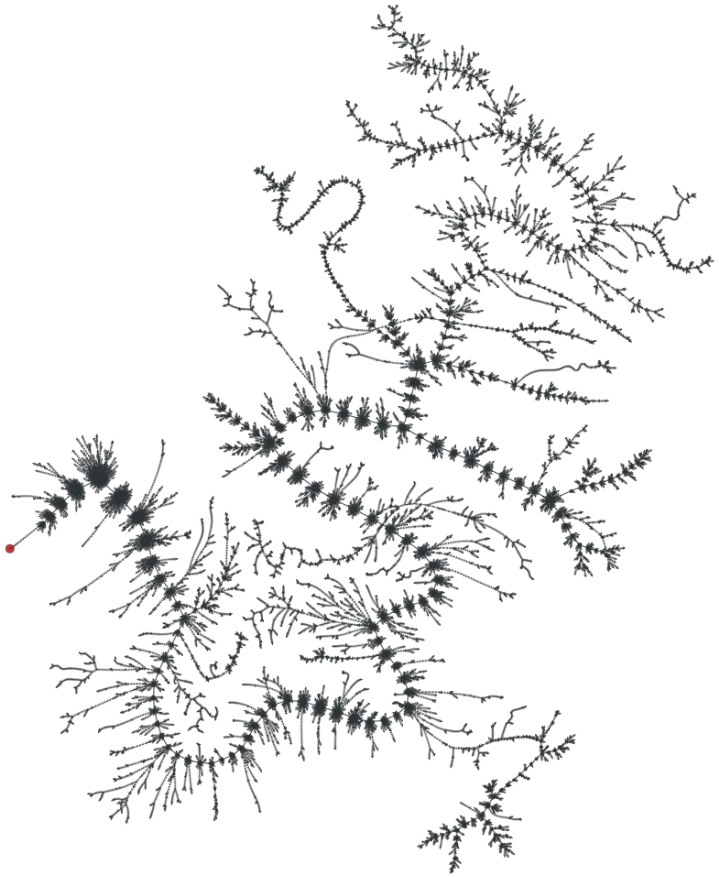
Topographic view



Max-tree

Max-tree Representation of a Natural Image

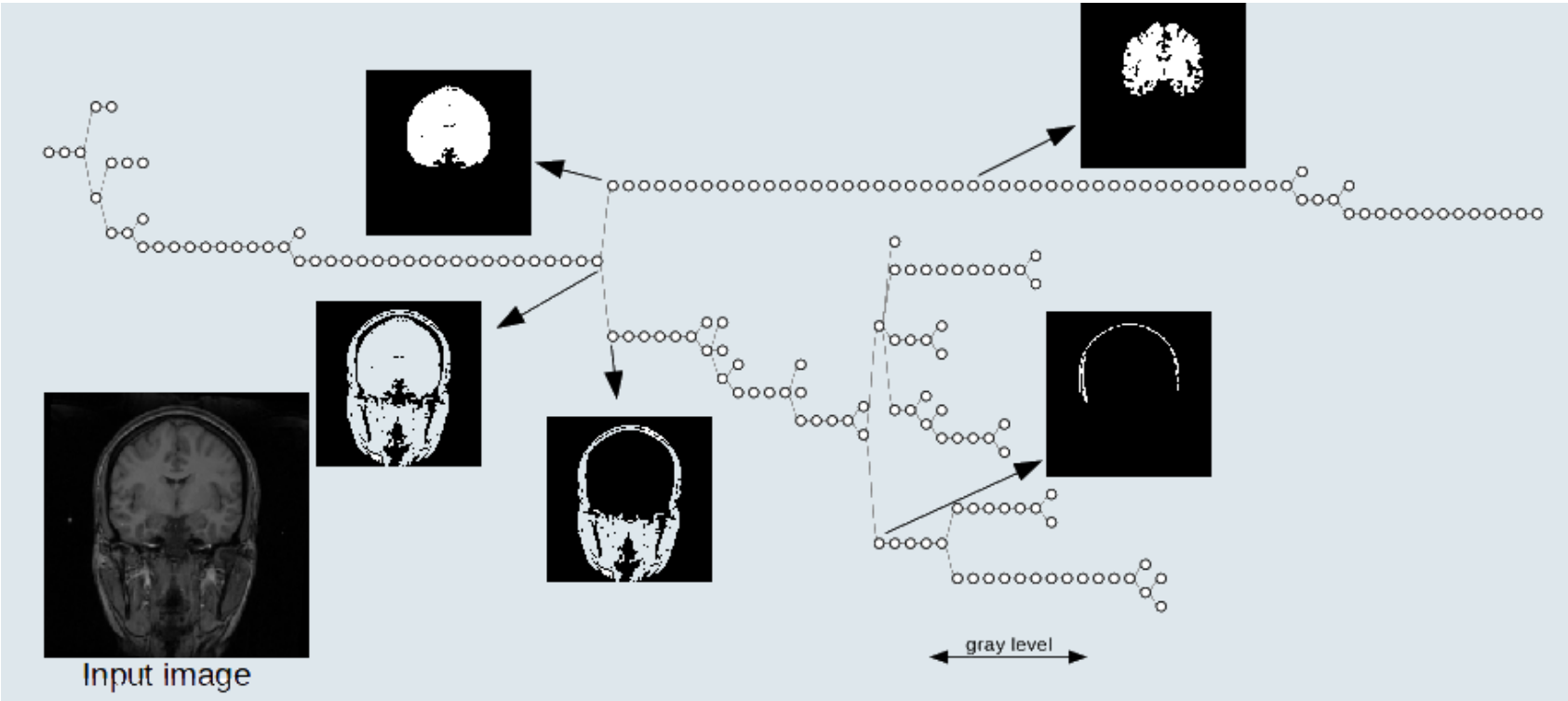
Lena image (512x512 pixels)



Corresponding max-tree with connectivity C8 (~41,00 nodes)

Max-tree

- Hierarchical representation of an image based on threshold decomposition

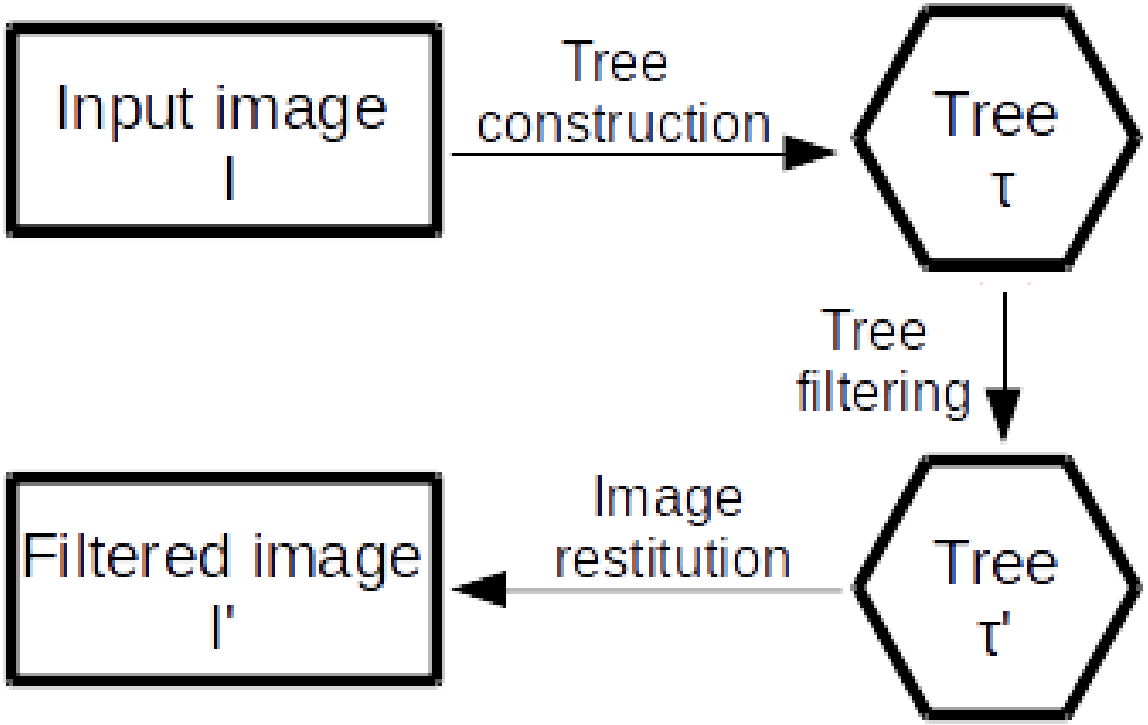


Max-tree illustration

25

Max-tree Filtering

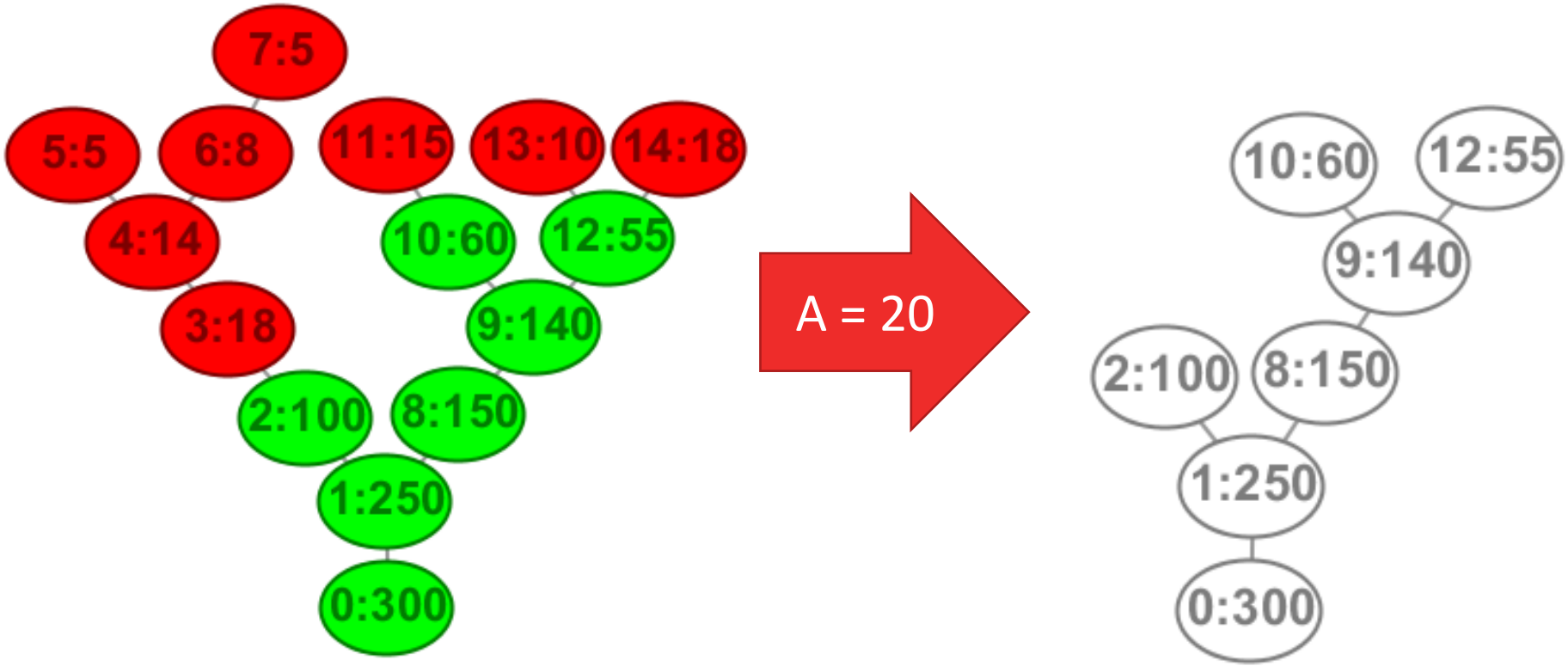
- Max-tree filters are connected filters, i.e. do not blur the image



26

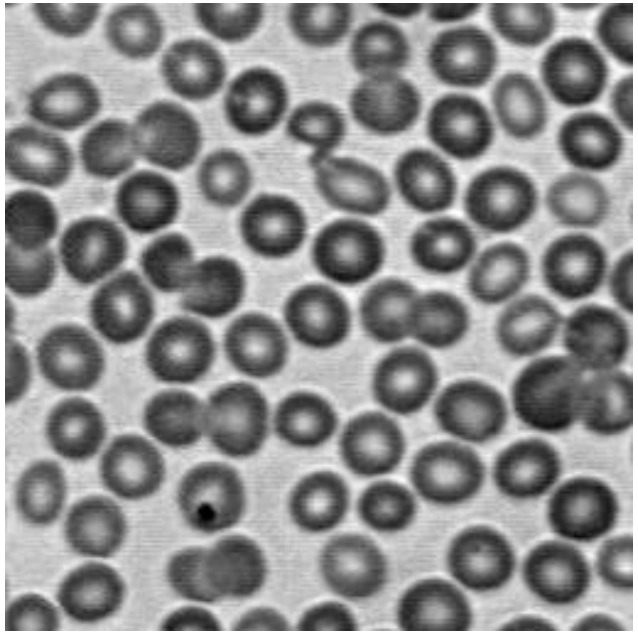
Area-open Filter

$$\tau_1 = \psi_A(\tau_0) = \{ i \in \tau_0 \,|\, Area(i) \geq A \}$$



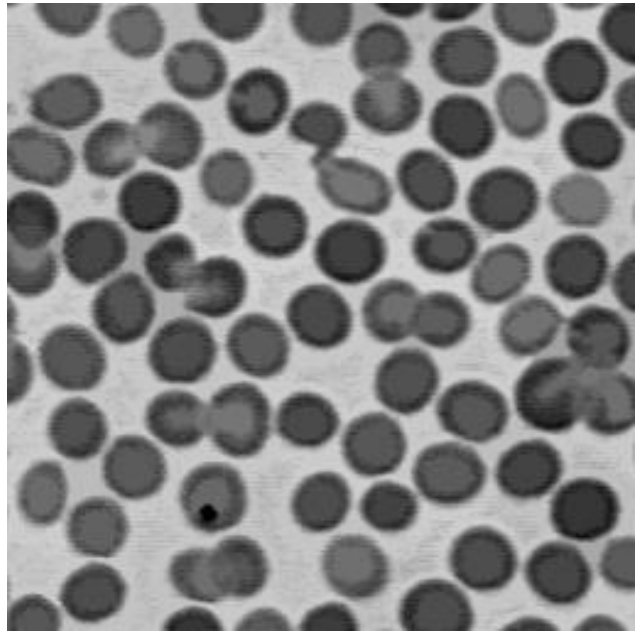
Area-open Filter

$$\tau_1 = \psi_A(\tau_0) = \{ i \in \tau_0 \,|\, Area(i) \geq A \}$$



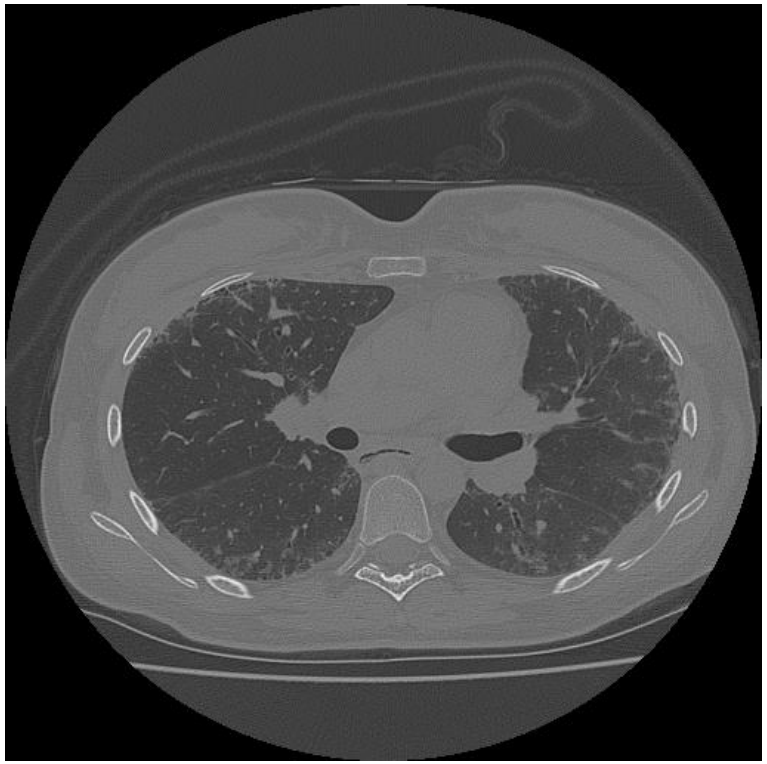
Input

A = 200

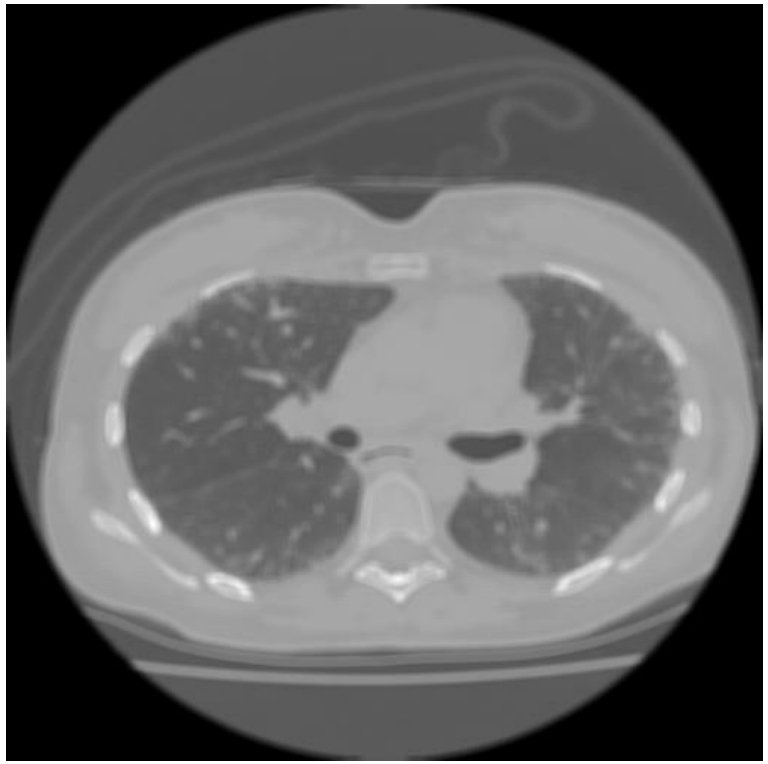


Max-tree, area-open filter

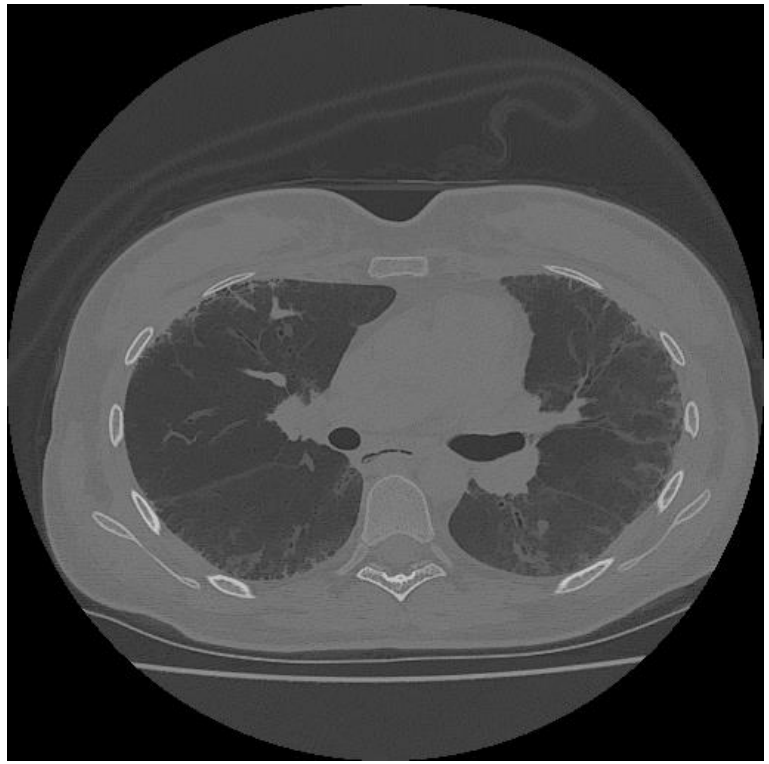
Area-open Filter



Original



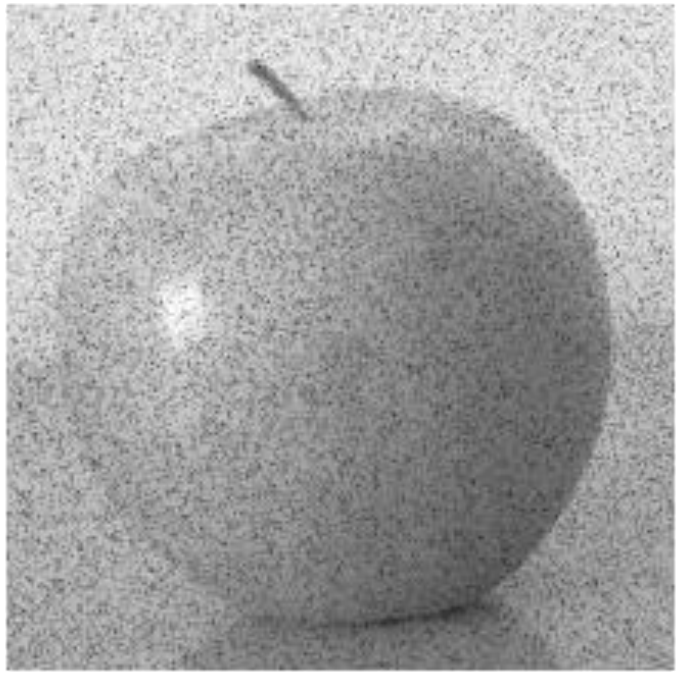
Mean filter (3x3 kernel)



Max-tree, area-open

29

Area-open Filter



Original

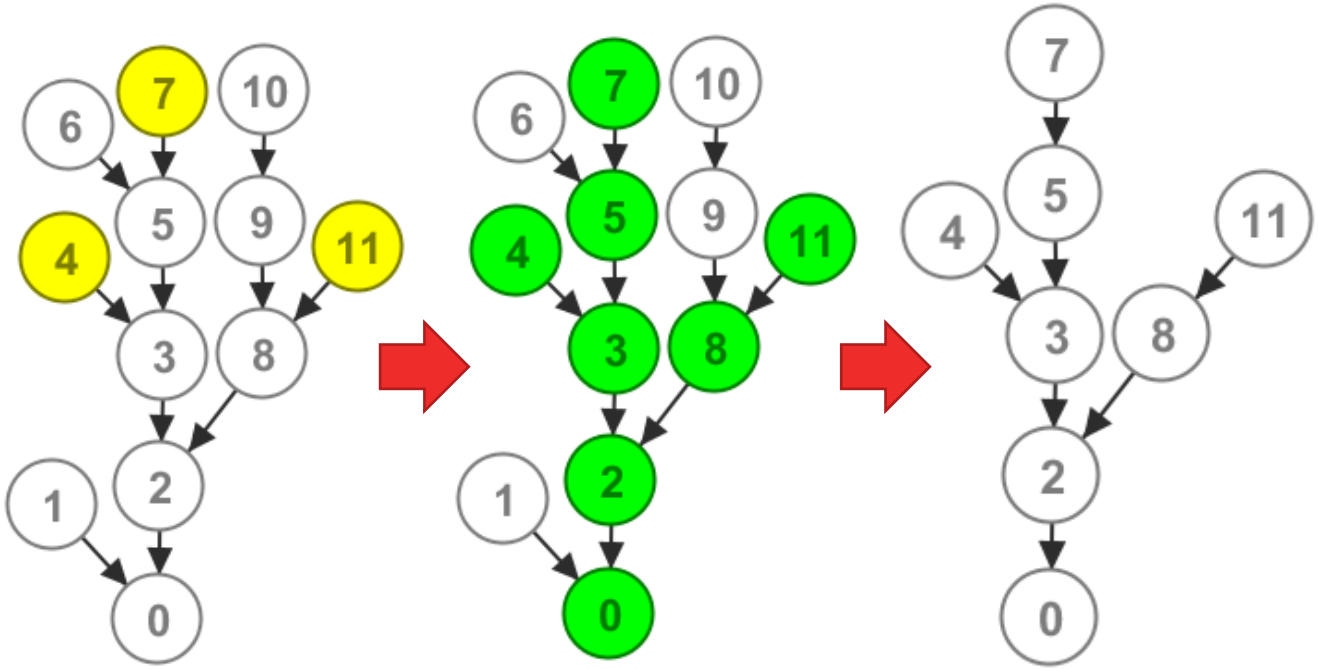
Max-tree, area-open

Extinction filter

$$\tau_1 = \psi_{NL,EX}(\tau_0)$$

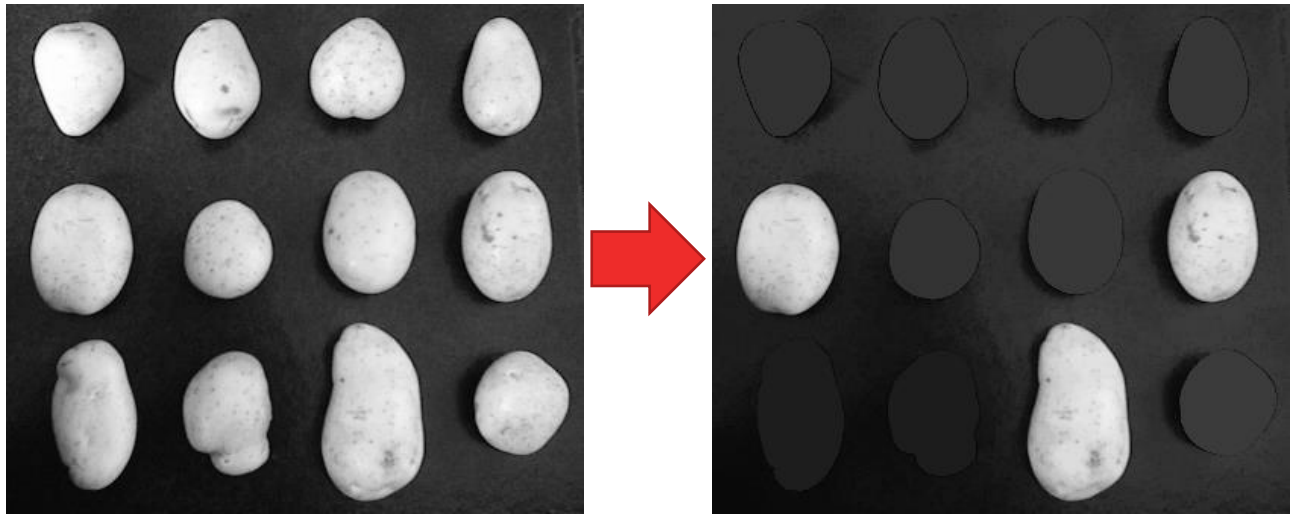
Preserves relevant extrema.

- **NL**: number of leaves to be preserved.
- **EX**: extinction values.



Extinction filter

- **NL**: 3 leaves.
- **EX**: extinction values of the volume attribute.

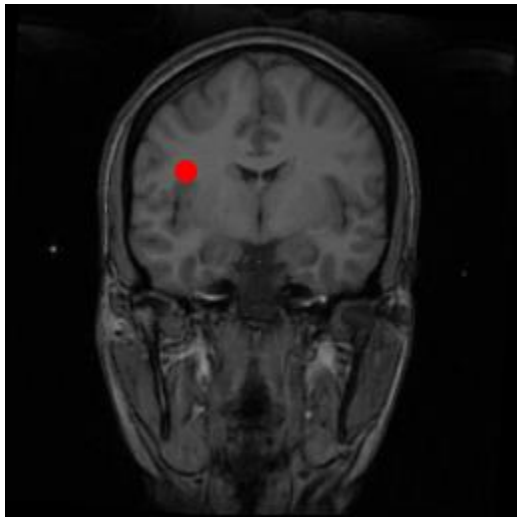


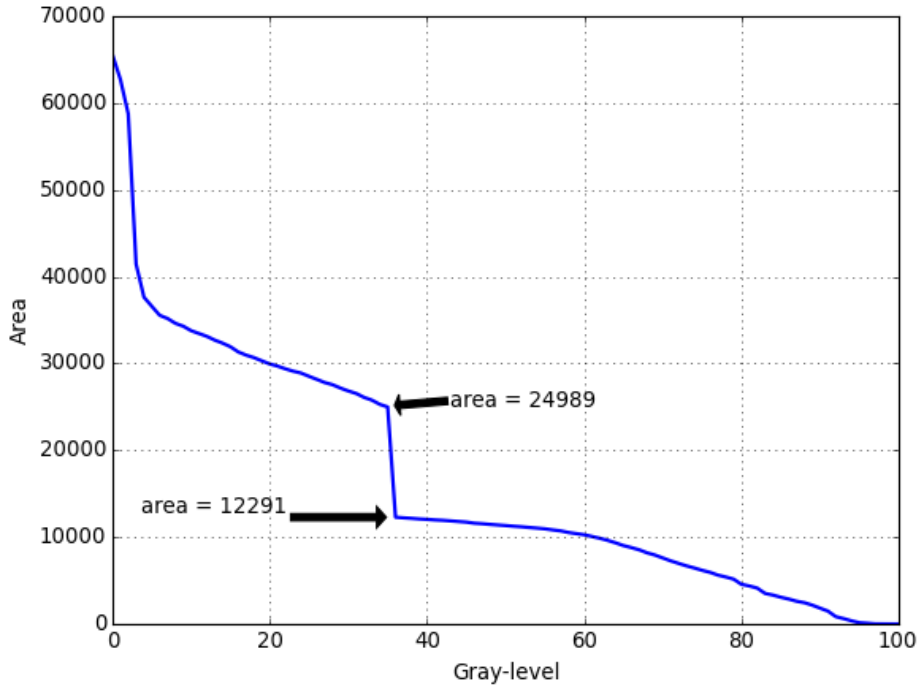
Input

Volume Extinction Filter

Signature Analysis

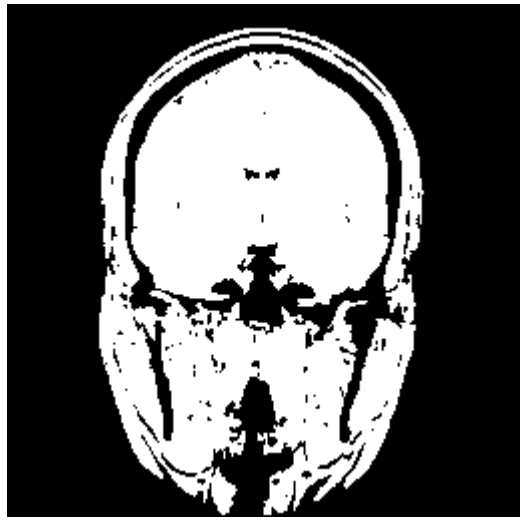
- The max-tree signature consists in analyzing an attribute variation starting at a leaf node and going towards the root.

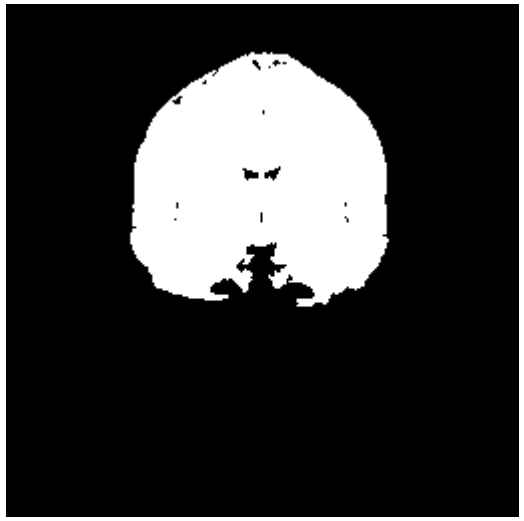




Brain MR image (left) and its area signature (right).

Signature Analysis





Node reconstruction before the sudden drop in the area signature value (right) and node reconstruction after (left).

Array-based Representation

- The max-tree representation we are going to use is array-based and node-oriented;
- It consists of two arrays: node array (NA) and node index (NI);
- NA is ordered in a way it is easier to perform tree traversals;
- NA minimally stores the parent relationship of the max-tree nodes and the gray level of each node. It can also store other attributes like area, bounding-box coordinates,...

Array-based Representation

Image

0:2	1:0	2:3
3:2	4:1	5:3
6:7	7:0	8:3

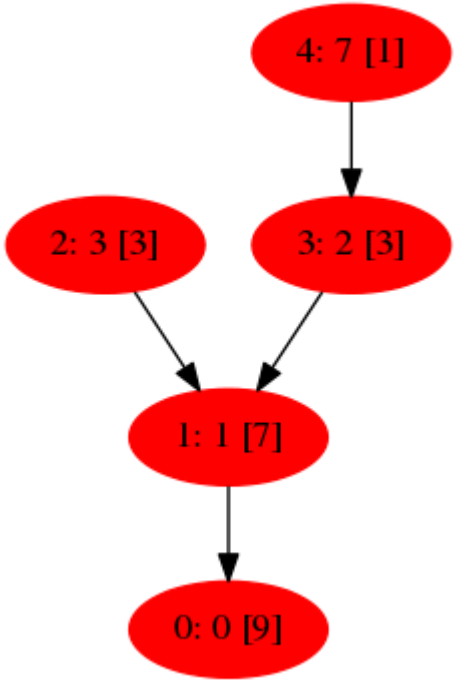
Root to leaves direction

Node Array

par	h	a
0	0	9
0	1	7
1	3	3
1	2	3
3	7	1

Node Index

3	0	2
3	1	2
4	0	2



Left to right: sample image. NA/NI max-tree representation of the sample image , and the node oriented max-tree illustration (node ID: h [area]).

Max-tree Filtering Algorithm

Algorithm for filtering the max-tree

```
1: function DIRECT-FILTER(NA, NI, to_keep)
2:     to_keep[1] ← True                          ▷ cannot remove the root
3:     M ← NA.lines
4:     N ← NI.size
5:     for i ← 1, M do                                        ▷ parallel loop
6:         lut[i] ← i
7:         nearest_ancestor_kept ← 0
8:     for i ← 1, M do
9:         if (not to_keep[i]) then
10:            temp ← nearest_ancestor_kept[par[i]]
11:            nearest_ancestor_kept[i] ← temp
12:            lut[i] ←= lut[temp]
13:        else
14:            nearest_ancestor_kept[i] ← i
15:            par[i] ← nearest_ancestor_kept[par[i]]
16:    index_fix[1] ← (1 − to_keep[1])
17:    for i ← 2, M do
18:        index_fix[i] ← index_fix[i − 1] + (1 − to_keep[i])
19:        lut[i] ← lut[i] + index_fix[i]
20:    for i ← 1, M do
21:        par[i] ← lut[par[i]]
22:    for i ← 1, N do                                        ▷ parallel loop
23:        NI[i] ← lut[NI[i]]
24:    Remove NA lines corresponding to filtered nodes ▷
   parallel loop
25:    return NA, NI
```

Number of nodes often much smaller than the number of image pixels (M << N)

Image Restitution from the Max-tree

- Algorithm for recovering the image from the max-tree

```
1: function GET-IMAGE(h, NI)
2:     N ← NI.size
3:     for i ← 1, N do                    ▷ parallel loop
4:         f[i] ← h[NI[i]]
       return f
```

siamxt Toolbox

- Developed by our research group with state-of-the-art algorithms;
- Accepts 2D and 3D images of integer (uint8 or uint16) type;
- Developed in Python/NumPy, loops optimized in C++ and wraped with SWIG.

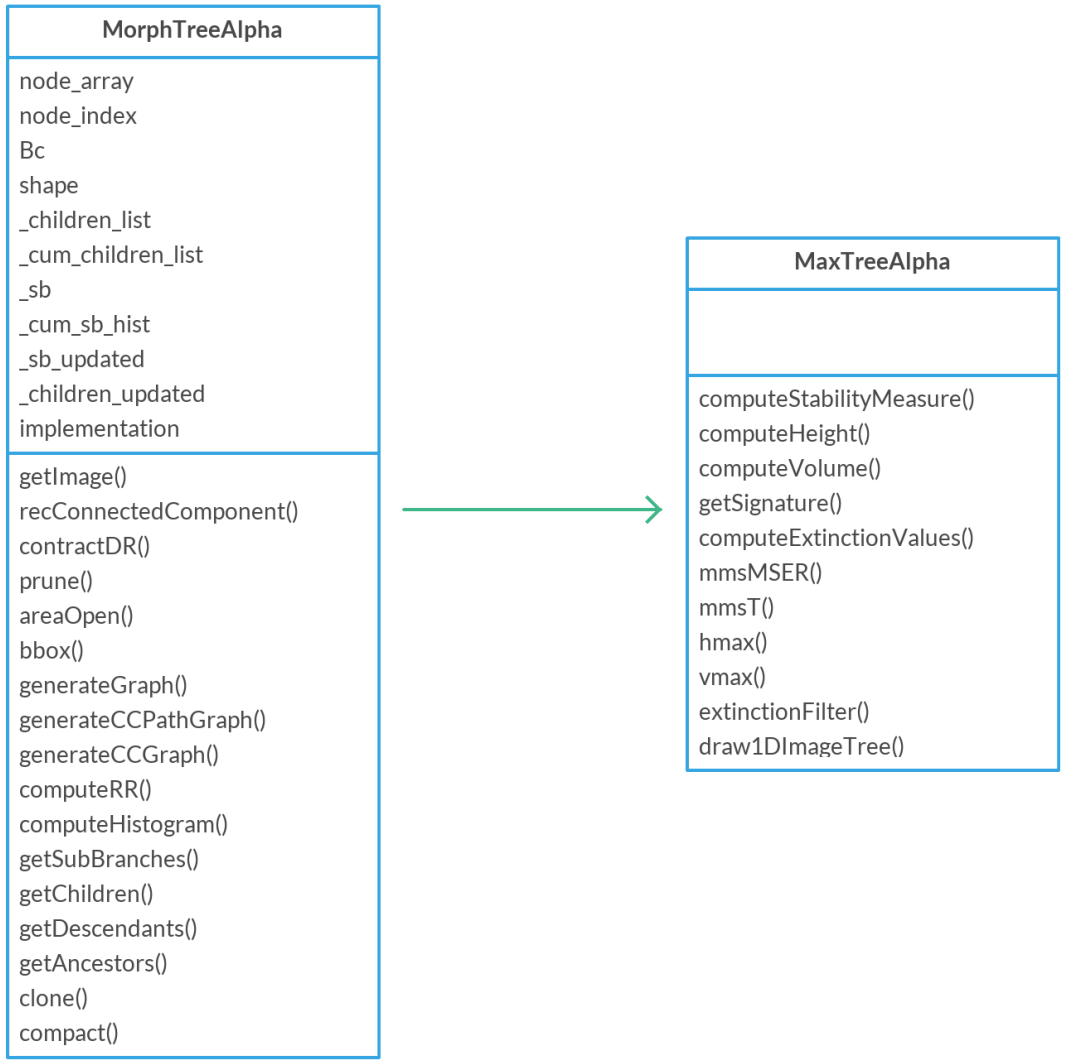
siamxt Toolbox

Attributes stored in node array of the s***iamxt*** toolbox.

Line index	Attribute
0	par
1	nchild
2	Level
3	area
4	seed
5	sumx
6	xmin

Line index	Attribute
7	xmax
8	sumy
9	ymin
10	ymax
11	sumz
12	zmin
13	zmax

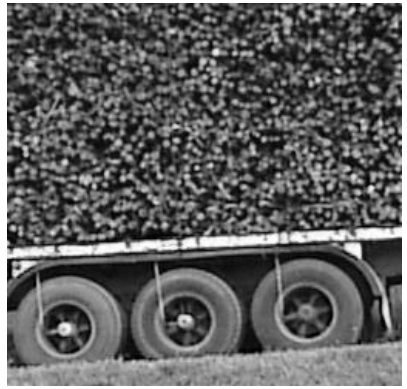
siamxt Toolbox



Class diagram of the ***siamxt*** toolbox.

siamxt Toolbox





Sample images.

Dimensions	Construction	Filtering	Restitution	Total
256x256	17.7	1.7	0.08	19.5
512x512	72.7	2.6	0.3	75.6
1024x1024	216.7	4.3	1.3	222.3

Average siamxt processing times in miliseconds. Time measured using old hardware (2016).

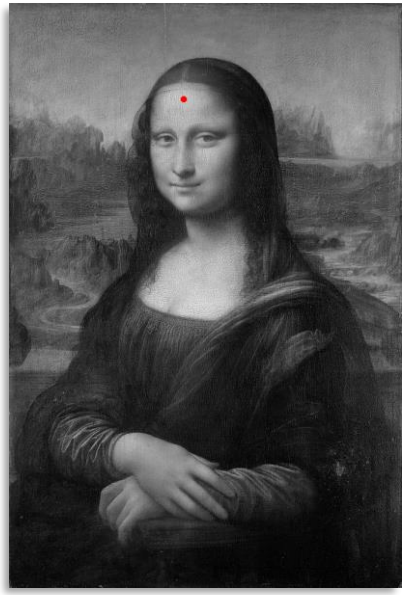
Summary

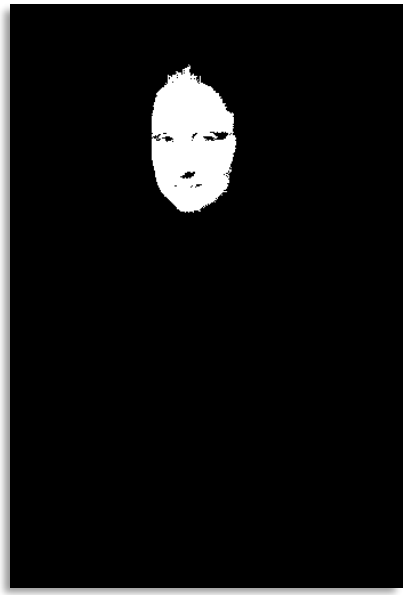
- The max-tree is a versatile structure for imgae processing
- It allows for size and shape analysis of the CCs present in the image
- Efficient algorithms for processing it
- Many applications:
  - Connected filters
  - Signature analysis
  - Selection of image markers

Thank you!

Assignment #01

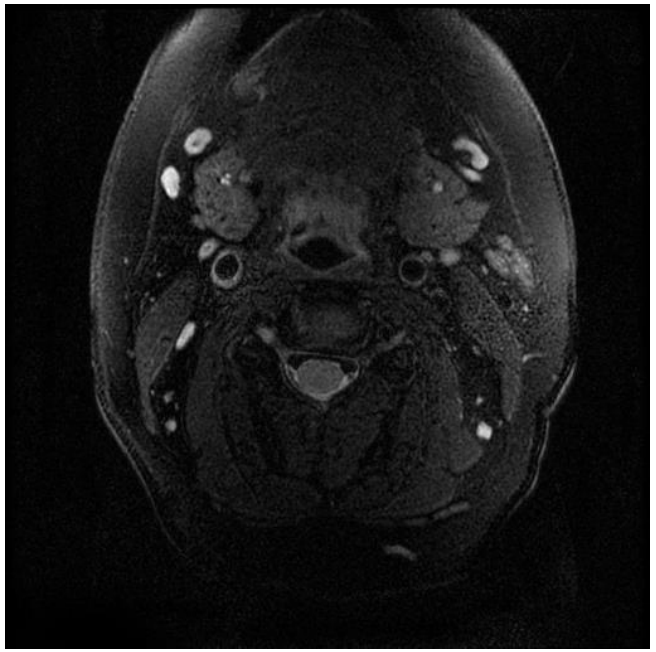
- Using the max-tree area signature analysis, determined CCs in the max-tree that separate Mona Lisa's face from the background



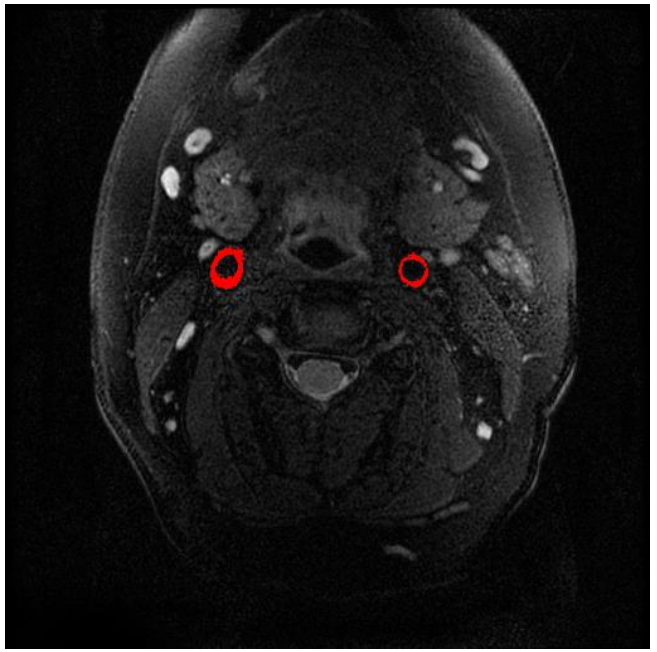


Assignment #01

- Apply a series of filters to this image to segment the carotid arteries wall



Input Image



Segmentation

Assignment #01

- Apply a series of filters to remove the white artifacts in the image

Input Image

Filtered Image

Assignment #01

- Apply a series of filters that will remove all objects in the image except for the two pens