Math 645 - Homework 6 - Due Friday, March 1, 2013

1. Prove the remaining Bernstein inequality statements that were not proven in class. In particular prove statements (1), (2) and (4) from your class notes.

2. Prove that the end point Sobolev embedding theorem is implied by the fractional integration version. In particular, suppose that $0 < s < n$, $1 < p < q < \infty$ and $\frac{n}{p} = \frac{n}{q} + s$. Then show that $\|\Lambda^{-s} f\|_{L^q(\mathbb{R}^n)} \leq C\|f\|_{L^p(\mathbb{R}^n)}$ implies

$$\|f\|_{L^q(\mathbb{R}^n)} \leq C\|\Lambda^s f\|_{L^p(\mathbb{R}^n)},$$

for all $f \in \mathcal{S}(\mathbb{R}^n)$. One needs to justify the inversion rigorously.

3. Prove the Gagliardo-Nirenburg inequality using LP theory: If $1 < p < q < \infty$ satisfies

$$\frac{1}{p} = \frac{1}{q} + \frac{\theta}{n}$$

and $0 < \theta < 1$ then we have

$$\|f\|_{L^q(\mathbb{R}^n)} \leq C\|f\|^{1-\theta}_{L^p(\mathbb{R}^n)} \|\nabla f\|^{\theta}_{L^p(\mathbb{R}^n)}$$

for all $f \in \mathcal{S}(\mathbb{R}^n)$. (Notice that the previous problem is not enough.)