Math 644 - Short Homework 1 - Due Friday, Sept. 14, 2012

1. Consider the following differential operator:

$$L = \sum_{\alpha} c_\alpha D_x^\alpha.$$

Above the $c_\alpha \in \mathbb{R}$ are constants. Let $u = u(x,t)$ be the solution to the following problem

$$\begin{aligned} \partial_t u + Lu &= 0, \text{ on } \mathbb{R}_x^n \times (0,\infty), \\ u|_{t=0} &= \Phi, \text{ on } \mathbb{R}_x^n. \end{aligned}$$

We denote this with the solution operator $S(t)$ as $u(x,t) = S(t)\Phi(x)$.

Show that the solution to

$$\begin{aligned} \partial_t u + Lu &= f(x,t), \text{ on } \mathbb{R}_x^n \times (0,\infty), \\ u|_{t=0} &= g(x), \text{ on } \mathbb{R}_x^n, \end{aligned}$$

is given by

$$u(x,t) = S(t)g(x) + \int_0^t S(t-\tau)f(x,\tau)\, d\tau.$$

This is the general version of Duhamels principle mentioned in class. For the purposes of this exercise, let us assume that Φ, f and g are smooth and that we can differentiate under the integral sign without additional justification. (HINT: Consider first the case when $g = 0$. Be careful when using the Chain rule.)