A New Demapper for BICM System with HARQ

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***Abstract*—In this paper, a new demapper generating log-likelihood ratio (LLR) is proposed by using a linear approximation for the nonlinear factor of an exact LLR. The exact LLR is sufficient for the probability based decoding, but its calculation requires a large number of computations in the case of high order modulations such as 16 quadrature amplitude modulation (QAM) and 64QAM. To alleviate the computational complexity a piecewise-linear demapper was proposed by neglecting the nonlinear factor of the exact LLR. However, it is observed that the throughput performance of the piecewise-linear demapper in hybrid automatic repeat request (HARQ) system is deficient in low signal-to-noise ratio region. Therefore, the new demapper is proposed which has an equivalent throughput performance to the exact LLR in HARQ system, and at the same time, only needs a few elementary operations for calculating the simplified LLRs. Experimental evidences which show the viability of the proposed demapper are also provided.**

***Index Terms*—bit interleaved coded modulation (BICM), demapper, hybrid automatic repeat request (HARQ), log-likelihood ratio (LLR), quadrature amplitude modulation (QAM), throughput**

I. Introduction

Hybrid automatic repeat request (HARQ) combining forward error control (FEC) and ARQ is a well-known error control technique with good error correction capability and high transmission efficiency [**?**]. Modern HARQ systems improve bit error rate (BER) performance by employing Shannon limit approaching low density parity check (LDPC) or turbo codes as FEC scheme [**?**][**?**]. Through near Shannon limit performance of those codes, HARQ systems can decrease the number of retransmissions, and consequently, have good throughput performance. Therefore HARQ technique is adopted by 3GPP LTE and IEEE 802.16e [**?**][**?**].

LDPC and turbo codes use log-likelihood ratio (LLR) for probability based decoding such as belief propagation and log maximum a posteriori, respectively. The LLR is generated by a demapper using posteriori probabilities, and the demapper is supposed to pass the exact LLR for each bit of codeword to the decoder of LDPC or turbo code. The exact LLR reflects full information of likelihood functions for all possible constellation symbols. Accordingly, the computational complexity of the exact LLR for 16 quadrature amplitude modulation (QAM) and 64QAM increases with modulation order. The calculation of the nonlinear factor of the exact LLR is especially complicated due to logarithmic functions. Thus, a simplified algorithm for generating the LLR, a piecewise-linear demapper, was proposed [**?**]. The piecewise-linear demapper disregards the nonlinear factor of the exact LLR by some steps of approximation. It is finally the first-order equation whose computation quantity is maintained reasonably small even in the case of the higher order modulations. Another advantage of the piecewise-linear demapper is that the decoding performance loss is insignificant. Hence, it has been widely utilized in industry.

In this paper, however, we have discovered that the piecewise-linear demapper has a deficient throughput performance at low signal-to-noise ratio (SNR) in HARQ system. The lower the operating SNR is due to HARQ operation, the larger the effect of the nonlinear factor of the exact LLR becomes. As a result, the difference between the two conventional demappers, negligible in high SNR region, increases in low SNR region. To compensate this degradation, a proposed demapper reflects the effect of the nonlinear factor by converting it into another piecewise-linear function. The proposed demapper can achieve the equivalent HARQ throughput performance to the exact LLR in low SNR region while requiring a small number of elementary operations.

II. System Model

Consider a bit interleaved coded modulating (BICM) system with HARQ that the LTE standard is based on [**?**][**?**]. By exploiting the bit interleaver between the channel encoder and the QAM mapper, the BICM system obtains code diversity to improve performance in a Rayleigh fading channel [**?**]. The system with HARQ detects errors in a frame and requests additional retransmissions to provide the receiver with extra information to successfully decode the frame.

Fig. 1 describes the basic structure of the BICM-HARQ system. At the transmitter, the input bits are encoded by LDPC or turbo codes. Then the encoded bits are bit interleaved, and mapped onto QAM symbols according to Gray-coded constellation mapping as in the LTE standard [**?**]. Note that a minimum distance between two QAM constellation points is $2d$. To make the unit average power of each QAM symbol, d is determined to be $1/\sqrt{10}$ and $1/\sqrt{42}$ for 16QAM and 64QAM, respectively. The received symbols through the channel are first demodulated by the soft-output demapper. The outputs of the demapper, LLRs, are deinterleaved, and then passed to the channel decoder. If the received symbols are the retransmitted data, the current LLRs are combined with the past LLRs which are properly weighted.

Let the transmitted symbol x, then the received symbol is given as

$$y = hx + n, \tag{1}$$

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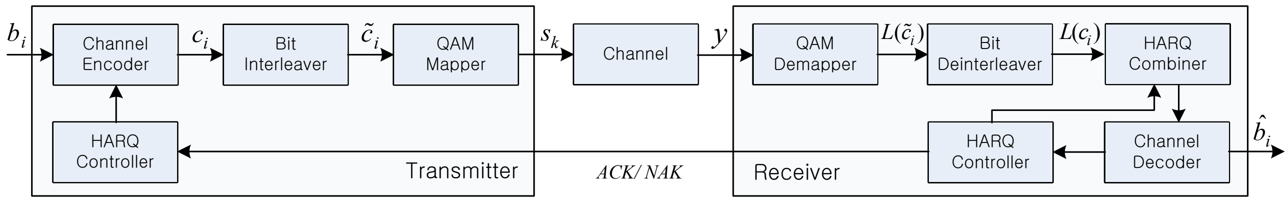

Fig. 1. Block diagram of BICM-HARQ system.

where h is the channel response and n is the complex additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2. With the assumption of perfect channel estimation at the receiver, the phase compensation is performed by multiplying the complex conjugate of the channel response with the received symbol.

$$y' = h^* y = |h|^2 x + n', \tag{2}$$

where $n' = h^* n$ which is still the complex AWGN with variance $|h|^2\sigma_n^2$. In (2), the minimum distance between two QAM constellation points also becomes $2|h|^2 d$.

III. Conventional Demappers

A. Exact LLR

The exact LLR is derived from posteriori probabilities and an assumption of uniformly distributed priori probability [**?**]. Let the interleaved bits $\left\{\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_M\right\}$ corresponding to the QAM symbol $s_k (k = 1, 2, ..., 2^M)$. The exact LLR of $\tilde{c}_i$ is given by

$$L(\tilde{c}_i) = \log \frac{\sum_{s_k \in S_i^{(0)}} p(y|x = s_k)}{\sum_{s_k \in S_i^{(1)}} p(y|x = s_k)}, \tag{3}$$

where $\log$ is a natural logarithmic function and $p(\cdot)$ is a probability density function (pdf). $S_i^{(0)}$ comprises the QAM symbol set where the i-th bits are equal to zero, and $S_i^{(1)}$ is its complementary. In (3), the exact LLR reflects full information of likelihood functions for all possible QAM symbols. Therefore, it is required a large number of computations to calculate the exact LLR in the case of higher order modulation such as 16QAM and 64QAM.

B. Piecewise-linear demapper

The piecewise-linear demapper was proposed to alleviate high computational complexity of the exact LLR for 16QAM and 64QAM [**?**]. For 4QAM, since the exact LLR has a simple linear form, it is not considered. The LLR of the piecewise-linear demapper is first obtained by the log-sum approximation taking the largest term of the summation in the logarithmic function.

$$L(\tilde{c}_i) \approx \log \frac{\max_{s_k \in S_i^{(0)}} p(y|x = s_k)}{\max_{s_k \in S_i^{(1)}} p(y|x = s_k)}. \tag{4}$$

Secondly, Gaussian pdfs with the phase compensated symbol in (2) are applied to (4).

$$L(\tilde{c}_i) \approx \frac{1}{\sigma_n^2}\Big\{ \min_{s_k \in S_i^{(1)}} (y' - h^* s_k)^2 - \min_{s_k \in S_i^{(0)}} (y' - h^* s_k)^2 \Big\}. \tag{5}$$

Then we can separate the calculation of the LLR for in-phase and quadrature bits which are mapped onto real and imaginary values of the QAM symbols, respectively. For example, in LTE standard, the first and the third bits for 16QAM are in-phase bits. Finally, the piecewise-linear demapper for the in-phase bits (the same applies for the quadrature bits) can be expressed by

$$L(\tilde{c}_{I,i}) \approx \frac{4d}{\sigma_n^2}(m_0 y_I + n_0 |h|^2 d), \tag{6}$$

where y_I is the real value of the phase compensated symbol, $Re(h^* y)$, and m_0 and n_0 are constants determined by the QAM constellation and its range. The values of m_0 and n_0 for 16QAM and 64QAM are given in Tables I and II of Appendix, respectively. In (6), the piecewise-linear demapper is steadily expressed by the first-order equation even when higher order modulations are employed. Therefore it has low computational complexity, and also negligible BER performance loss compared with the exact LLR [**?**].

However, HARQ throughput degradation at low SNR is observed with the piecewise-linear demapper. HARQ keeps the system operating in lower SNR region. As a consequence, the effect of the nonlinear factor of the exact LLR which is disregarded in (6) by the log-sum approximation becomes significant relative to that in high SNR region. This situation results in increasing the gap between the conventional demappers.

IV. Proposed Demapper

A. Demapper for 16QAM

The proposed demapper linearly approximates the nonlinear factor of the exact LLR to take into account its increasing effects in low SNR region and reduce the computation quantities for calculating it. The exact LLR for 16QAM in (3) can be divided by a linear part which is the piecewise-linear demapper, and a nonlinear part as follows

$$L(\tilde{c}_{I,i}) = \frac{4d}{\sigma_n^2}(m_0 y_I + n_0 |h|^2 d) + A_1 - A_2. \tag{7}$$

Here $A_j (j = 1, 2)$ is the nonlinear term given by

$$A_j = \log\left[1 + \exp\Big\{\frac{4d}{\sigma_n^2}(m_j y_I + n_j |h|^2 d)\Big\}\right], \tag{8}$$

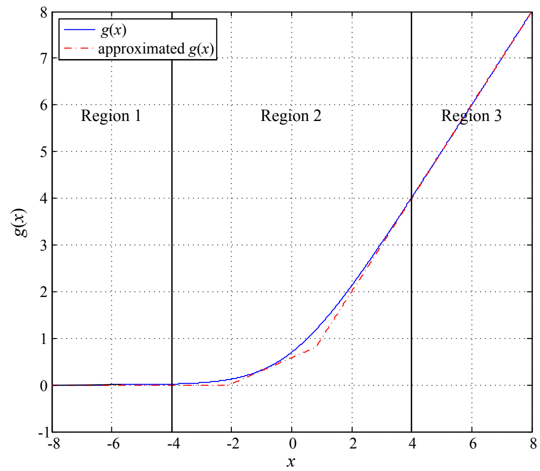

Fig. 2. Approximation of $g(x)$ with (a, b) = (0.28, 0.58).

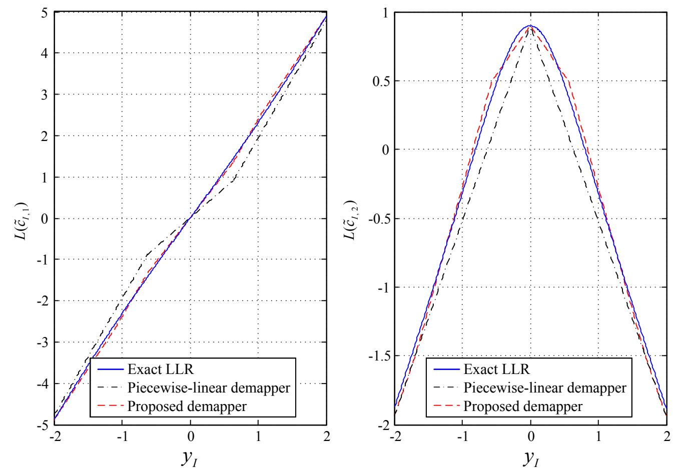

Fig. 3. Comparison of LLRs for the in-phase bits of 16QAM in the AWGN channel at SNR = 0.5 dB.

where the m_j and n_j are the constants given in Table I of Appendix. In (8), the value in the exponential function can be acquired from the channel and the noise power estimations, and the phase compensation in the receiver. Therefore the value can be viewed as a variable whose nonlinear function can be expressed by $g(x)$.

$$A_j = g(x) = \log\Big\{1 + \exp(x)\Big\}. \tag{9}$$

Fig. 2 represents the function $g(x)$ divided by three regions. Now the problem is how close to approximate $g(x)$ using linear functions. In the first and the third regions, the curve of $g(x)$ is closely approximated by two asymptotic curves: $g(x) = 0$ and $g(x) = x$. For approximation in the second region, another linear equation is additionally introduced: $g(x) = ax + b$. By utilizing these linear functions the approximated $g(x)$ is derived as

$$g(x) \approx \hat{g}(x) = \max(0, ax + b, x), \tag{10}$$

where $\max$ function holds the maximum value according to x. In Fig. 2, the approximated curve according to (10) is also presented when the values of (a, b) are $(0.28, 0.58)$ which is determined heuristically based on the simulation results. The accuracy of this approximation can vary with the values of (a, b). However, it is observed by the simulation that the performance is not so sensitive to the value of the parameters.

The proposed demapper adopts the approximation of (10) to convert the nonlinear part of the exact LLR into a simple linear form. For 16QAM, the proposed demapper is expressed as

$$L(\tilde{c}_{I,i}) \approx \frac{4d}{\sigma_n^2}(m_0 y_I + n_0 |h|^2 d) + \hat{A}_1 - \hat{A}_2, \tag{11}$$

where $\hat{A}_0$ and $\hat{A}_1$ are the outputs from the approximation of (8). Although the proposed demapper has additional computational complexity compared with the piecewise-linear demapper in (6), it only requires a few elementary operations. With little additional computation, the proposed demapper can provide improvement of throughput performance.

Fig. 3 compares the LLRs for 16QAM according to the received symbol in the AWGN channel when SNR is equal to 0.5 dB. The LLR curve of the proposed demapper is closer to the exact LLR than that of the piecewise-linear LLR. This confirms that the proposed demapper successfully approximates the nonlinear part of the exact LLR.

B. Demapper for 64QAM

The proposed demapper for 64QAM requires an additional step of approximation since the nonlinear part of the exact LLR for 64QAM has three exponential terms in the logarithmic function. The nonlinear term for 64QAM can be written as

$$A_j = \log(1 + \sum_{l=3(j-1)+1}^{3(j-1)+3} E_l), \tag{12}$$

where $j = 1, 2$ and $E_l = \exp\Big\{\frac{4d}{\sigma_n^2}(m_l y_I + n_l |h|^2 d)\Big\}$. The values of m_j and n_j are given in Table II of Appendix. Due to the complicated logarithmic function, its simplification is first applied to employ the approximation of (10). The new approximation is exploited in which the summation of exponential functions is dominated by the largest exponential term.

$$A_j \approx \log\Big\{1 + \max_l(E_l)\Big\}. \tag{13}$$

The form of the above equation is identical to the nonlinear term of the exact LLR for 16QAM so that the approximation of (10) can be used to piecewise-linearize (13). Consequently, the proposed demapper for 64QAM is derived in the same form as (11).

V. Simulation Results

We evaluate HARQ throughput performance, defined as the rate of acknowledgment (ACK) per transmission, of the proposed demapper compared with the two conventional demap-

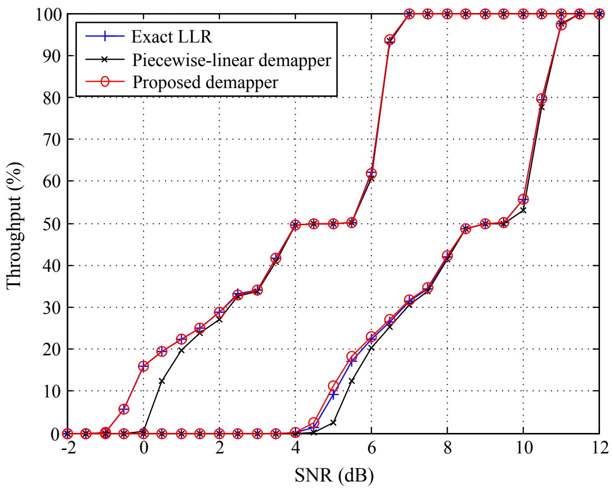

Fig. 4. HARQ throughput performances in the AWGN channel.

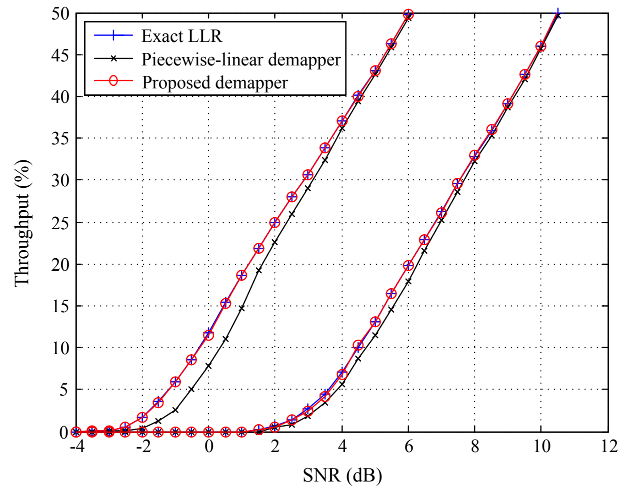

Fig. 5. HARQ throughput performances in the 3-path fading channel.

pers. In the simulations, the BICM-HARQ system employs the LDPC code whose length is 2304 and code rate is 0.5, given in IEEE 802.16e [**?**]. Note that when employing the LDPC code in the BICM system, the bit interleaving is omitted since LDPC codes have the nature of bit-interleaving. The maximum number of retransmissions for HARQ operation is determined as six, and Type I Chase combining [**?**] is used. Considered channel environments are the AWGN channel and a 3-path fading channel in which each path has an equal average power.

Fig. 4 illustrates HARQ throughput performances in the AWGN channel. For 16QAM and 64QAM, the performance of the piecewise-linear demapper is degraded in low SNR region. In Fig. 4, the proposed demapper achieves about 16% throughput for 16QAM at 0 dB, which is equivalent to the performance by the exact LLR, though the piecewise-linear demapper fails. The performance gap between the proposed and the piecewise-linear demappers decreases with 64QAM, but the proposed demapper still outperforms the piecewise-linear demapper.

The proper scheduling of modulation and code rate may prevent this degradation. In Fig. 4, for example, 16QAM can be employed at 5 dB, instead of 64QAM. However, in a real system, the scheduler does not operate in the best way due to feedback delay, channel measurement error, or other environment [**?**]. On the other hand, the proposed demapper can stably guarantee HARQ throughput performance close to the exact LLR over a wide range of SNR.

Fig 5 describes HARQ throughput performances in a 3-path fading channel where each path has an equal average power. In Fig. 5, the similar analysis in the performance can be addressed. The proposed demapper achieves equivalent throughput to the exact LLR, and outperforms about 0.5 dB for 16QAM and 0.2 dB for 64QAM at the 6th retransmission compared with the piecewise-linear demapper.

VI. Conclusion

In this paper, we have proposed a new simplified demapper for BICM system with HARQ. From the observation of deficient HARQ throughput performance of the piecewise-linear demapper in low SNR region, we investigated the difference of the conventional demappers. As a result, the proposed demapper tightly approximates the nonlinear factor of the exact LLR by compounding simple linear functions, and achieves the equivalent HARQ throughput performance to the exact LLR. Furthermore, the proposed LLR can be obtained by a small number of elementary operations.

Appendix

The values of m_j and n_j of the exact LLR are determined by the QAM constellation and its range. The calculation is based on the minimum distance of two QAM constellation points, d, which becomes $d'(=|h|^2 d)$ after the phase compensation of the received symbol. By practically substituting Gaussian pdfs into (3) and arranging the equation in the order of the pdf having the largest value, m_j and n_j can be obtained and given in Table I for 16QAM and Table II for 64QAM.

TABLE I
m_k AND n_k FOR THE IN-PHASE BITS OF 16QAM.

		m_0	n_0	m_1	n_1	m_2	n_2
$\tilde{c}_{I,0}$	$y_I < -2d'$	2	2	1	-2	1	2
	$-2d' \le y_I < 2d'$	1	0	1	-2	-1	-2
	$2d' \le y_I$	2	-2	-1	2	-1	-2
$\tilde{c}_{I,1}$	$y_I < 0$	1	2	1	0	3	0
	$0 \le y_I$	-1	2	-1	0	-3	0

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TABLE II
$m_{k,l}$ AND $n_{k,l}$ FOR THE IN-PHASE BITS OF 64QAM.

		m_0	n_0	m_1	n_1	m_2	n_2	m_3	n_3	m_4	n_4	m_5	n_5	m_6	n_6
$\tilde{c}_{I,0}$	$y_I < -6d'$	4	12	1	-2	2	-6	3	-12	1	6	2	10	3	12
	$-6d' \le y_I < -4d'$	3	6	1	-2	2	-6	3	-12	1	4	2	6	-1	-6
	$-4d' \le y_I < -2d'$	2	2	1	-2	2	-6	3	-12	1	2	-2	-10	-1	-4
	$-2d' \le y_I < 2d'$	1	0	1	-2	2	-6	3	-12	-1	-2	-2	-6	-3	-12
	$2d' \le y_I < 4d'$	2	-2	-1	2	2	-10	1	-4	-1	-2	-2	-6	-3	-12
	$4d' \le y_I < 6d'$	3	-6	-1	4	-2	6	1	-6	-1	-2	-2	-6	-3	-12
	$6d' \le y_I$	4	-12	-1	6	-2	10	-3	12	-1	-2	-2	-6	-3	-12
$\tilde{c}_{I,1}$	$y_I < -6d'$	2	10	1	2	2	2	3	0	1	6	6	6	7	0
	$-6d' \le y_I < -2d'$	1	4	1	2	2	2	3	0	-1	-6	6	-6	5	0
	$-2d' \le y_I < 0$	2	6	-1	-2	2	-2	1	0	-1	-6	6	-6	5	0
	$0 \le y_I < 2d'$	-2	6	1	-2	-2	-2	-1	0	1	-6	-6	-6	-5	0
	$2d' \le y_I < 6d'$	-1	4	-1	2	-2	2	-3	0	1	-6	-6	-6	-5	0
	$6d' \le y_I$	-2	10	-1	2	-2	2	-3	0	-1	6	-6	6	-7	0
$\tilde{c}_{I,2}$	$y_I < -4d'$	1	6	1	4	4	4	5	0	3	12	4	12	1 7	0
	$-4d' \le y_I < 0$	-1	-2	-1	-4	4	-4	3	0	-3	-12	4	-12	1	0
	$0 \le y_I < 4d'$	1	-2	1	-4	-4	-4	-3	0	3	-12	-4	-12	-1	0
	$4d' \le y_I$	-1	6	-1	4	-4	4	-5	0	-3	12	-4	12	-7	0

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