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Simplified LLR algorithm for m-QAM demodulation

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Abstract: Here, a simplified algorithm to calculate log-likelihood ratio (LLR) for m-QAM demodulation is proposed. Traditional method calculating LLR for m-QAM demodulation is supposed to do distance computing between any two constellation points so that the high complexity of the hardware implementation is not acceptable. This simplified method is different from the traditional ways that the distance calculation between any two points in a plane can be omitted, indicating that there is an advantage in the amount of computation in this simplified method. Finally, the simplified method is compared with traditional methods from the perspective of complexity and system performance. The simulation results show that the simplified method can greatly reduce the amount of calculation and have little effect on the performance of the system.

1 Introduction

With the development of modern wireless communication system, 4G technology is widely used and 5G technology has also made considerable progress. These popular wireless communication technologies are based on high spectrum utilisation of communication system. The orthogonal frequency division multiplexing (OFDM) is a good communication system to improve the spectrum utilisation. It can greatly improve the utilisation rate of spectrum when m-QAM is employed in this system. Under certain conditions, however, the high-order modulation will lead to a high error rate of the system with transmission power limitation. In order to reduce the error rate of the system, the corresponding measures are usually taken at channel coding and channel decoding. Study [1] shows that in the additive white Gauss noise (AWGN) channel, there is a soft decision gain >2 dB, and this value is >3 dB in the fading channel. For high-order modulation such as m-QAM, however, the branches I and Q are mapped independently and each modulation mapping symbol corresponds to multiple bits, then the soft information of every bit must be calculated in the soft decision. Therefore, the computation of soft information is quite complex in high-order modulation. Literature [2] proposed a method widely used that calculates the Euclidean distance to achieve the purpose of simplifying the calculation for the grey mapping constellation. Literature [3] proposed a simplified calculation of non-square constellations by using the method of dividing the complex plane into blocks. It does not affect the performance of the system while simplifying the calculation. Literature [4] uses LLR computing or ML detection in the OFDM-IM system, and its system performance is better than that of the traditional OFDM system.

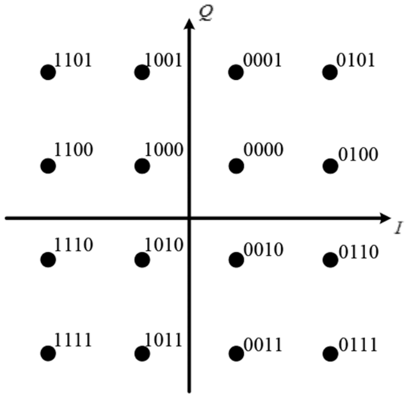

Fig. 1 *Constellations for grey-coded 16QAM*

This paper makes analysis on the traditional algorithm of soft decision method and the simplified scheme. The signal is supposed to be encoded by the low-density parity-check (LDPC) encoder and pass through the AWGN channel, then get decoded with the soft information calculated. The system is based on 16QAM, and it is simple to extend to m-QAM soft information calculation. Comparison of the simulation results of these schemes is operated and the simulation results show that the soft information decoding performance of the proposed simplified scheme is quite close to the traditional LLR calculation results.

2 Traditional method

The traditional soft decision algorithm is a log-likelihood ratio (LLR) calculation method based on the maximum a posteriori (MAP) criterion, and the soft demodulation output has the same positive and negative relationship with the hard demodulation output. Taking 16QAM as an example, in this paper, it is an amplitude and phase combined keying signal and its constellation uses orthogonal grey mapping, as shown in Fig. 1.

After mapping the 16QAM signal to the constellation point, each mapping symbol can be represented by a vector (b_1, b_2, b_3, b_4) of four bits, in which the values of b_1, b_2, b_3, b_4 are taken from 0 or 1.

Supposing the coordinates of any point in the constellation diagram are (A_i, A_q). The received signal passing the AWGN channel is $r = s + n = r_i + jr_q$, where $r_i = s_i + jn_i$, $r_q = s_q + jn_q$ and s is the signal received with information transmitted, and n is the Gauss white noise of variance δ^2. Its in-phase component r_i and quadrature component r_q can be regarded as two independent random variables. The conditional probability density function (PDF) of r is

$$p(r|b_k = A) = \frac{1}{\sqrt{2\pi}\delta}\exp\left\{-\frac{|r - A|^2}{2\delta^2}\right\} \qquad (1)$$

where $A = A_i + A_q$.

Assuming that $A \in \{0, 1\}$ and $p(b_k = 0) = p(b_k = 1) = 1/2$, we have

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$$
\begin{aligned}
p(r|b_k = A) &= \sum_{b:b_k = A} p(r|b) \\
&= \sum_{b:b_k = A} p(r - c(b)) \\
&= \sum_{b:b_k = A} \frac{\exp(-\parallel r - c(b) \parallel^2 / 2\delta^2)}{2\pi\delta^2}
\end{aligned} \tag{2}
$$

where the first line of the equation holds because it is a sum of two disjoint components, and the third line is the PDF of complex Gaussian random variable and both of its real and imaginary components have a variance of δ^2. The relation of norm is written as

$$
\parallel r - c \parallel^2 = \parallel r \parallel^2 - 2\langle r, c\rangle + \parallel c \parallel^2 \tag{3}
$$

where the inner product is given by

$$
\langle r, c\rangle \triangleq \text{real}(r)\text{real}(c) + \text{imag}(r)\text{imag}(c) \tag{4}
$$

Soft decision decoders take as input the LLR for each code bit [5].

$$
\begin{aligned}
\text{LLR}(k) &\triangleq \ln\left[\frac{P(b_k = 0|r)}{P(b_k = 1|r)}\right] \\
&= \ln\left[\frac{p(r|b_k = 0)P(b_k = 0)/p(r)}{p(r|b_k = 1)P(b_k = 1)/p(r)}\right] \\
&= \ln\left[\frac{p(r|b_k = 0)}{p(r|b_k = 1)}\right]
\end{aligned} \tag{5}
$$

where we use P to indicate the probability and the equation follows because we assume $p(b_k = 0) = p(b_k = 1)$. Substituting (2) into (5), we have

$$
\text{LLR}(k) = \ln\left[\frac{\sum_{b:b_k = 0} \exp(-\parallel r - c(b) \parallel^2 / 2\delta^2)}{\sum_{b:b_k = 1} \exp(-\parallel r - c(b) \parallel^2 / 2\delta^2)}\right] \tag{6}
$$

Therefore, the LLR of the received signal can be calculated, according to formula (6), by computing the squared distance of the constellation points referring to bit 1 and the ones referring to bit 0.

When the modulation symbols are all of the same energy, such as m-PSK modulations, we can reach a simpler form (see (7)) . Equation (6) and (7) are computing the exact LLR, while they are too difficult for real-time implementation. It is obvious that the one which has the lowest $\parallel r - c(b) \parallel^2$ value that counts most in (6). Therefore, a common approximation method to the LLR is to compute the minimum $\parallel r - c(b) \parallel^2$ value only, that is, to use only the nearest constellation point referring to bit 1 and the ones referring to bit 0 to calculate the distance. Supposing the nearest neighbour constellation points as

$$
c^*(k, A) \triangleq c(\underset{b:b_k = A}{\arg\min} \parallel r - c(b) \parallel^2) \tag{8}
$$

We might rewrite (6) as

$$
\begin{aligned}
\text{LLR}(k) &\simeq \ln\left[\frac{\exp(-(\parallel r - c^*(k, 0) \parallel^2 / 2\delta^2))}{\exp(-(\parallel r - c^*(k, 1) \parallel^2 / 2\delta^2))}\right] \\
&= \frac{1}{2\delta^2}(\parallel r - c^*(k, 1) \parallel^2 - \parallel r - c^*(k, 0) \parallel^2)
\end{aligned} \tag{9}
$$

Thus, the LLR of the received signal can be calculated, according to (9), by computing the distance between the received points and the constellation points referring to bit 1 and the ones referring to bit 0. This common approximation method might lead to an SNR loss by decreasing the complexity of calculation.

3 Improved method

However, (9) leads to the LLR with the calculation of the distance from the received points to the specific nearest constellation points one referring to bit 1 and the other referring to bit 0. The distance of one uncertain point to a certain nearest constellation point is needed, that is, five subtractions and four multiplications are required.

As far as we are concerned, it is not an optimal method to do the computing with four multiplications, thus, the simplification continues. We can split (9) into two fractions M and N (see (10)) . Thus, we have

$$
M(k) = 2\langle r, c^*(k, 0) - c^*(k, 1)\rangle \tag{11}
$$

$$
N(k) = \parallel c^*(k, 1) \parallel^2 - \parallel c^*(k, 0) \parallel^2 \tag{12}
$$

Two subtractions and two multiplications are required to calculate M and more steps are required to calculate N. If we consider this problem from the other side, however, things might go even more smoothly. The comparison of the flow chart of the two methods is as shown in Fig. 2.

In the case of 16QAM, for instance, there are 16 possibilities for the positions of any received signals on the axis of the coordinate, and the distributions of each bit 0 or 1 are like the following diagram, from the MSB to the LSB Fig. 3.

The value of part N in (12) is a constant by calculating the distance of two certain constellation points. How to deal with (11), in other word, by what means to obtain the value of M easily, has become the top priority. According to the perpendicular bisector theorem, it can be obtained that the nearest constellation points one referring to bit 1 and the other referring to bit 0 in the 16 separated areas and we can name these specific areas from area 0000 to area 1111, where the specific area means that all received points falling in this specific area have a shorter distance to the specific constellation point in this area than any other constellation points. That is, for instance, when the received signal located in the place where we called area 0000, the constellation point which has the shortest distance to the received signal is the one represents grey-coded 0000 and the constellation points which have the second shortest distance to the received signal are the ones represent grey-coded 0001, 0010, 0100, and 1000 and so on. Under this circumstance, the values of M and N are limited to a few certain values, and more briefly the value of vector N is easily accessible and the distributions of vector M are available now. The distributions of each bit are listed as follows Fig. 4.

$$
\begin{aligned}
\text{LLR}(k) &= \ln\left[\frac{\sum_{b:b_k = 0} \exp(-(\parallel r \parallel^2 - 2\langle r, c(b)\rangle + \parallel c(b) \parallel^2 / 2\delta^2))}{\sum_{b:b_k = 1} \exp(-(\parallel r \parallel^2 - 2\langle r, c(b)\rangle + \parallel c(b) \parallel^2 / 2\delta^2))}\right] \\
&= \ln\left[\frac{\sum_{b:b_k = 0} \exp(\langle r, c(b)\rangle / \delta^2) \sum_{b:b_k = 0} \exp(-(\parallel r \parallel^2 / 2\delta^2)) \sum_{b:b_k = 0} \exp(-(\parallel c(b) \parallel^2 / 2\delta^2))}{\sum_{b:b_k = 1} \exp((\langle r, c(b)\rangle / \delta^2)) \sum_{b:b_k = 1} \exp(-(\parallel r \parallel^2 / 2\delta^2)) \sum_{b:b_k = 1} \exp(-(\parallel c(b) \parallel^2 / 2\delta^2))}\right] \\
&= \ln\left[\frac{\sum_{b:b_k = 0} \exp(\langle r, c(b)\rangle / \delta^2)}{\sum_{b:b_k = 1} \exp(\langle r, c(b)\rangle / \delta^2)}\right]
\end{aligned} \tag{7}
$$

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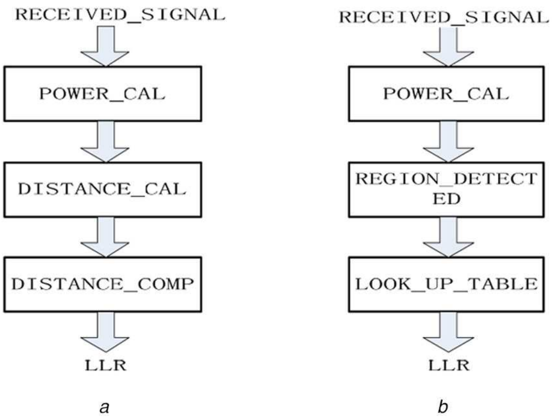

Fig. 2 *Flow chart of two methods*
(a) Traditional method, ***(b)*** Simplified method

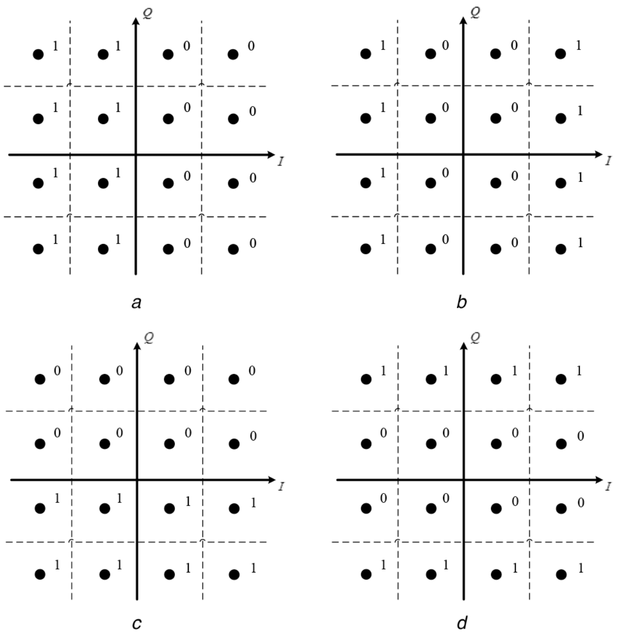

Fig. 3 *Polarity distributions of each bit for grey-coded 16QAM*
(a) The MSB, ***(b)*** The second bit, ***(c)*** The third bit, ***(d)*** The LSB

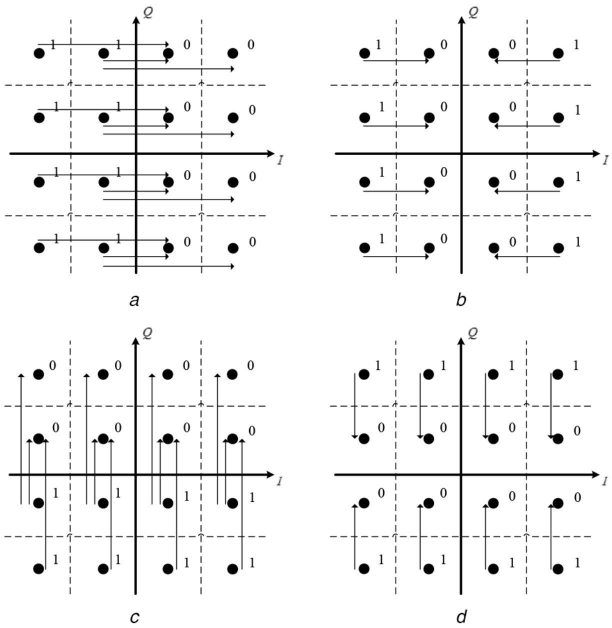

Fig. 4 *Distributions of vector M of each bit for grey-coded 16QAM*
(a) The MSB, ***(b)*** The second bit, ***(c)*** The third bit, ***(d)*** The LSB

As we can see, for each case, for each bit, the values of vector *M* and vector *N* are determined and it is simple to calculate the values (see the Appendix) by (11) and (12). then, the LLRs of each bit are available at the same time.

Assuming that the coordinate of the receiving signal is (I, Q), where the real component is *I* and imaginary component is *Q*, and the values here are the results of the normalisation, where the *P* is the average energy of the received symbols. The marks 3–0 represent MSB-LSB. The left item in the Table is vector *M* and the right one is vector *N* (no right item represents vector *N* equals 0), and it is worth mentioning that vector *N* is a constant. The LLRs of each bit are as follows (Table 1).

In the case of m-QAM, it is obvious that we can follow the previous method to deduce the values of LLRs of each separated area. In this condition, we have *m* possibilities for the positions of any received signals on the axis of the coordinate and the distributions of each bit 0 or 1 from the MSB to the LSB is readily available, then we obtain the value of vector *M* and vector *N* and the addition refers smoothly to the LLRs.

4 Simulation results

Here, taking 16QAM for example, the performance of the traditional method without simplification, the traditional method with simplification and the simplification method proposed by this paper is simulated on the MATLAB for AWGN channel, and the performance comparison diagram is obtained as follows Fig. 5.

It is shown that the traditional method without simplification has about a 0.1 dB performance gain compared with the simplified ones while the performance of the traditional method with simplification and the simplification method proposed by this paper are almost the same. Even a 6-bit-quantisation of the received signal will lead to a performance loss <0.1 dB, the difference between the performance of the traditional method and

$$\begin{aligned}\mathrm{LLR}(k) &\simeq \frac{1}{2\delta^2}\left(\left|\left|r - c^*(k,1)\right|\right|^2 - \left|\left|r - c^*(k,0)\right|\right|^2\right)\\ &= \frac{1}{2\delta^2}\left(2\langle r, c^*(k,0) - c^*(k,1)\rangle + \left|\left|c^*(k,1)\right|\right|^2 - \left|\left|c^*(k,0)\right|\right|^2\right)\\ &= \frac{1}{2\delta^2}(M(k) + N(k))\end{aligned} \tag{10}$$

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Table 1 Values of LLRs of each separated area of each bit for grey-coded 16QAM

AREA	LLR3 = M3 + N3	LLR2 = M2 + N2	LLR1 = M1 + N1	LLR0 = M0 + N0
0000	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
0001	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
0010	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$
0011	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$
0100	$\frac{8I}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
0101	$\frac{8I}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
0110	$\frac{8I}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$
0111	$\frac{8I}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$
1000	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
1001	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
1010	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$
1011	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$
1100	$\frac{8I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
1101	$\frac{8I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}-\frac{8}{10}$	$\frac{-4Q}{\sqrt{10P}}+\frac{8}{10}$
1110	$\frac{8I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$
1111	$\frac{8I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4I}{\sqrt{10P}}+\frac{8}{10}$	$\frac{8Q}{\sqrt{10P}}+\frac{8}{10}$	$\frac{4Q}{\sqrt{10P}}+\frac{8}{10}$

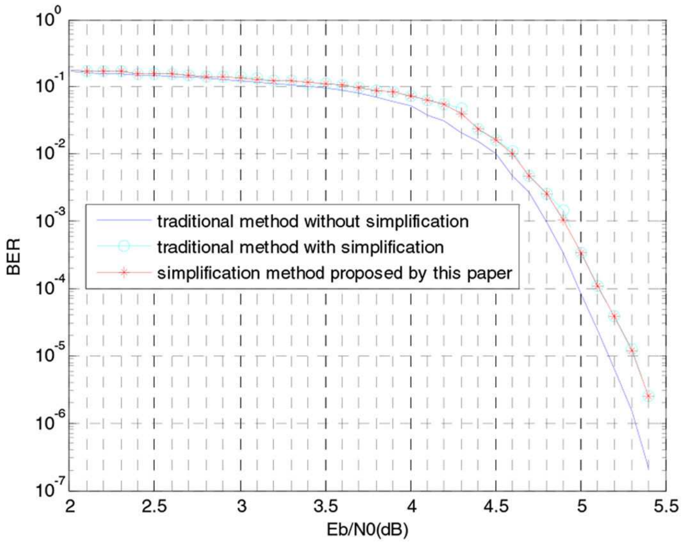

Fig. 5 *Comparison of the performance of the three methods for (4096, 2048) LDPC decoder, grey-coded 16QAM*

the proposed simplification method is acceptable considering the resources factors, as shown in Fig. 6.

For further verifying the performance of this simplified method computing the LLR proposed in this paper, the Verilog code is written and the hardware implementation is done in the FPGA. The polarities of the test data of the MATLAB results and the FPGA results are compared and the comparison result is demonstrated as follows in Fig. 7.

Comparing and analysing the data, we can see that the polarities of the hardware simulation results of the proposed scheme are exactly the same with the ones of software simulation results, indicating that this proposed algorithm is practically feasible.

On the other hand, this method proposed by this paper can spare less resources to achieve an almost-same performance compared with the traditional method with simplification. The following table lists the comparison results of the resource occupancy for FPGA implementation of these two simplified algorithms, As shown in Table 2.

It is worth mentioning that using these two DSPs is to calculate the average power of the received signals and is necessary for almost all m-QAM demodulation. If this usage can be ignored, this

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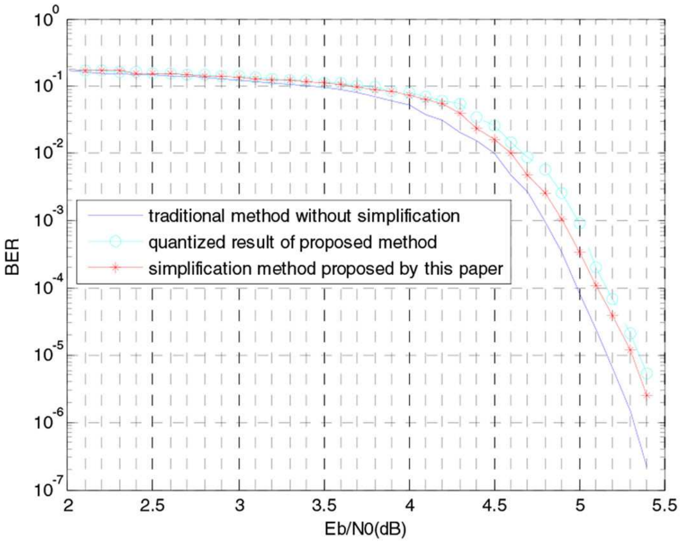

Fig. 6 *Influence to the performance of 6-bit-quantisation for (4096, 2048) LDPC decoder, grey-coded 16QAM*

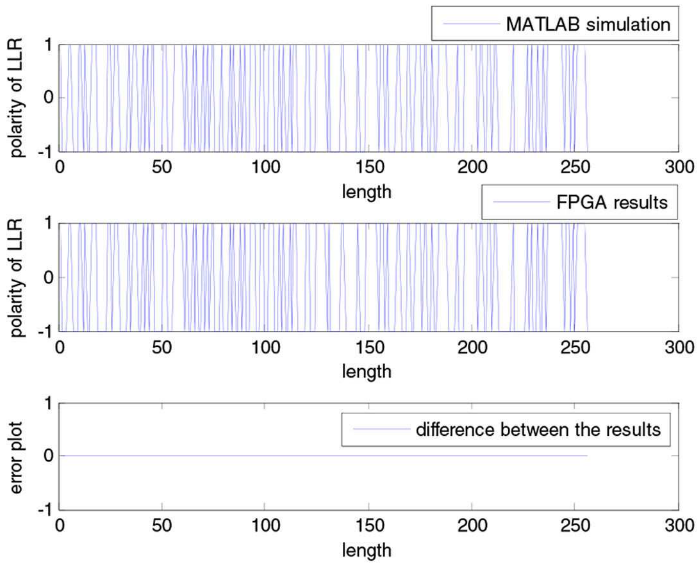

Fig. 7 *Comparison of the polarities of LLR between the simulation and the implementation for grey-coded 16QAM*

Table 2 Comparison of the resource occupancy of these two simplified algorithms for grey-coded 16QAM

Method	FPGA resources		
	Slices	LUTs	DSP
traditional method	462	632	4
proposed method	205	369	2

algorithm proposed in this paper has no requirement for DSP. Instead, we look up the value of the table calculated in advance.

5 Conclusion

In this paper, analysis is made for the soft information calculation of the two traditional methods with or without simplification. Based on this, we proposed a simplified method calculating the soft information for grey-coded m-QAM. The performance of these three methods is simulated on the MATLAB, and it is shown that the performance of these two simplified methods is almost the same and there is only a little performance loss compared with the traditional method without simplification. Finally, the feasibility of the scheme proposed by this paper is verified by FPGA implementation, and the FPGA resources occupied by the two simplified schemes are compared. In general, the LLR calculation scheme proposed in this paper can reduce the complexity of the LLR algorithm with a little performance loss, and it is simpler and easier to implement so that this scheme has certain reference and application value for LLR calculation.

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7 Appendix

See Table 3.

Table 3 Values of vector M and N of each separated area of each bit for grey-coded 16QAM

Area	M3	M2	M1	M0	N3	N2	N1	N0
0000	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	0	8/10	0	8/10
0001	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	0	8/10	−8/10	8/10
0010	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	0	8/10	0	8/10
0011	$\frac{4I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	0	8/10	8/10	8/10
0100	$\frac{8I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	−8/10	8/10	0	8/10
0101	$\frac{8I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	−8/10	8/10	−8/10	8/10
0110	$\frac{8I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	−8/10	8/10	0	8/10
0111	$\frac{8I}{\sqrt{10P}}$	$\frac{-4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	−8/10	8/10	8/10	8/10
1000	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	0	8/10	0	8/10
1001	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	0	8/10	−8/10	8/10
1010	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	0	8/10	0	8/10
1011	$\frac{4I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	0	8/10	8/10	8/10
1100	$\frac{8I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	8/10	8/10	0	8/10
1101	$\frac{8I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{-4Q}{\sqrt{10P}}$	8/10	8/10	−8/10	8/10
1110	$\frac{8I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	8/10	8/10	0	8/10
1111	$\frac{8I}{\sqrt{10P}}$	$\frac{4I}{\sqrt{10P}}$	$\frac{8Q}{\sqrt{10P}}$	$\frac{4Q}{\sqrt{10P}}$	8/10	8/10	8/10	8/10

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