Self-learning Neural Network Based Traffic Signal Controller for an Isolated Intersection and Construction of a New Clustering Algorithm in Unsupervised Machine Learning

Ravil Mussabayev

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Problem statement

Build an adaptive traffic signal controller for an isolated intersection. In prospect, it will be extended to a transport network of arbitrary size.

Solution method

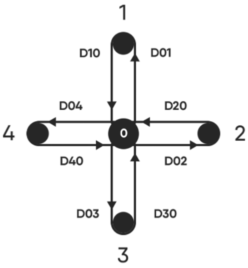
Reinforcement learning (RL) has been used to solve the problem.

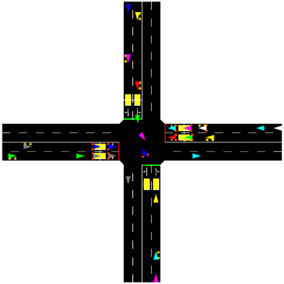
Basic elements of reinforcement learning:

1. The agent and the actions
2. The environment and its states
3. An action selection policy
4. A reward signal

Objective: through successive interaction with the environment train the agent to take actions that maximize the total reward it receives over the long run.

State space





$$NS \doteq D10 + D30$$

$$WE \doteq D40 + D20$$

$$S \doteq (NS, WE)$$

The state space: $\mathcal{S} \doteq \{(s_0, s_1) \mid s_0, s_1 \in \mathbb{Z}^+\}$

Action space

There are 3 actions:

1. $a_0 \doteq (0, 0)$ – do nothing
2. $a_1 \doteq (+dt, -dt)$ – extend the NS green phase and shorten the WE green phase
3. $a_2 \doteq (-dt, +dt)$ – shorten the NS greed phase and extend the WE green phase

The action space: $\mathcal{A} \doteq \{a_0, a_1, a_2\}$

Reward formula

Throughout the proposed algorithm the following exlusive <u>continuous</u> reward formula has been used:

$$S_t = (NS, WE) \to S_{t+1} = (NS', WE') \tag{1}$$

$$\begin{aligned} R(S_t, S_{t+1}) \quad & \doteq \mu \left(|NS - WE| - |NS' - WE'|\right) + \\ & +(1-\mu)\left(NS + WE - \left(NS' + WE'\right)\right) \end{aligned} \tag{2}$$

$\mu \in [0, 1]$ is the trade-off between queue reduction and equilibrium terms.

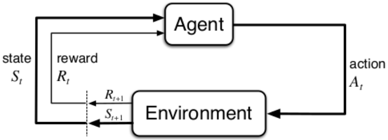
The reward space: $\mathcal{R} \doteq \{R(S_0, S_1) \mid S_0, S_1 \in \mathcal{S}\}$

Markov decision process (MDP)

Discrete time steps: $t = 0, 1, 2, 3, \ldots$
At each time step the agent-environment interaction is:

$$S_t \xrightarrow[A_t]{} (R_{t+1}, S_{t+1}),$$

where $S_t, S_{t+1} \in \mathcal{S},\ A_t \in \mathcal{A},\ R_{t+1} \in \mathcal{R}$.



$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$$

Return and policy

The return:

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots, \ 0 \leq \gamma \leq 1 \tag{3}$$

$$\begin{aligned} G_t &\doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots \\ &= R_{t+1} + \gamma (R_{t+2} + \gamma R_{t+3} + \dots) \\ &= R_{t+1} + \gamma G_{t+1} \end{aligned} \tag{4}$$

The policy $\pi : \mathcal{S} \times \mathcal{A} \to [0, 1]$:

$$\pi(a|s) \doteq \mathsf{Pr}\{A_t = a \mid S_t = s\} \tag{5}$$

State value function

Given π, the state value function:

$$
\begin{aligned}
v_\pi(s) \quad & \doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\
& = \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\
& = \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s] \\
& = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)[r + \gamma v_\pi(s')]
\end{aligned} \tag{6}
$$

State-action value function

Given π, the state-action value function:

$$
\begin{aligned}
q_\pi(s,a) \quad &\doteq \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s, A_t = a] \\
&= \sum_{s',r} p(s', r|s,a)[r + \gamma v_\pi(s')]
\end{aligned}
\tag{7}
$$

The goal

Let $\pi \geq \pi'$ if $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathcal{S}$

$$v_*(s) \doteq \max_\pi v_\pi(s), \ \forall s \in \mathcal{S} \tag{8}$$

$$q_*(s,a) \doteq \max_\pi q_\pi(s,a), \ \forall s \in \mathcal{S} \text{ and } \forall a \in \mathcal{A}(s) \tag{9}$$

Bellman optimality: $v_*(s) = \max_{a \in \mathcal{A}(s)} q_*(s,a)$.

The greedy policy is one of the optimal ones:

$$\pi_*(s) \doteq \operatorname*{argmax}_a q_*(s,a) \tag{10}$$

Shallow Q-Network (SQN)

We approximate the true $q_*(s,a)$ by $\hat{q}(s,a,w)$.

At each time t the cost function is the squared error:

$$J_t(w_t) \doteq \frac{1}{2}\left(q_*(S_t,A_t) - \hat{q}(S_t,A_t,w_t)\right)^2 \tag{11}$$

We don't know $q_*(S_t,A_t)$, so we estimate it:

$$q_*(S_t,A_t) \approx U_t(S_t,A_t) \doteq R_{t+1} + \gamma \max_{a' \in \mathcal{A}} \hat{q}(S_{t+1},a',w_t) \tag{12}$$

Shallow Q-Network (SQN)

$$J_t(w_t) \doteq \frac{1}{2}\left(U_t(S_t, A_t) - \hat{q}(S_t, A_t, w_t)\right)^2 \tag{13}$$

$$\nabla J_t(w_t) = \left(U_t(S_t, A_t) - \hat{q}(S_t, A_t, w_t)\right) \nabla_w \hat{q}(S_t, A_t, w_t) \tag{14}$$

Update the weights by the gradient descent:

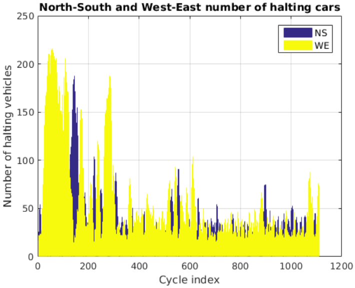
$$w_{t+1} \doteq w_t - \alpha \nabla J_t(w_t) \tag{15}$$

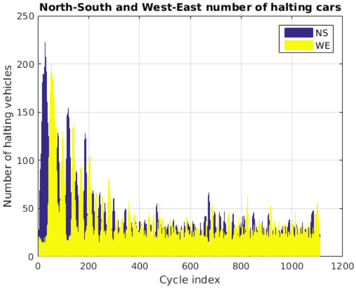
$\nabla J_t(w_t)$ is computed by the backpropagation algorithm.
α is the learning rate.

Shallow Q-Network (SQN). Implementation

- ε-greedy policy ($\varepsilon = 0.1$): choose action $\operatorname{argmax}_a \hat{q}(S_t, a, w_t)$ with probability 0.9, and a random action with probability 0.1
- Discounting factor $\gamma = 0.6$
- No-library Python 2-25-3 NN implementation. ReLU activation function in the hidden layer. Gradient descent with $\alpha = 0.001$, 5 iterations with regularization $\lambda = 0.03$
- Feature scaling and mean normalization ([-1,1])
- 1000 cycles of SUMO simulation

Experiments. SQN (right) vs. Q-learning (left)





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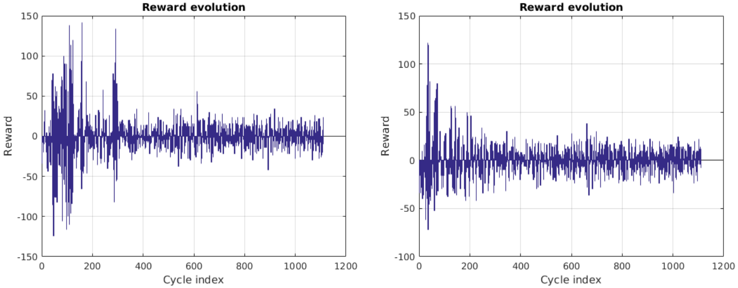

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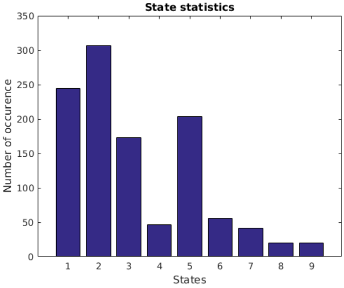

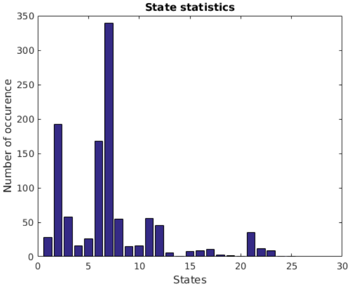

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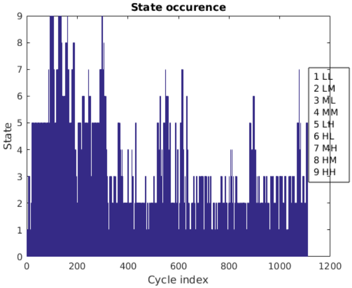
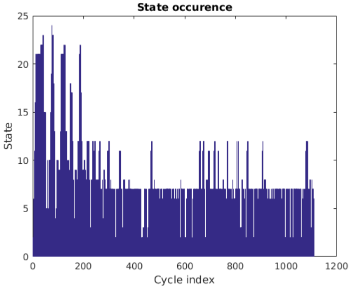




Why clustering?

Apply an efficient clustering algorithm to a history of occured states to obtain a good discretization of the state space.

Clustering. Problem statement

Let the data $D \doteq \{(x_i, y_i)\}_{i=1}^{N}$ be given.

Objective: identify groups of points that are in some sense similar to each other.

The new clustering algorithm should:

- address the problem of clusters of various densities
- be able to identify clusters of complex shapes

Local density function

$$f(x,y) \doteq \sum_{(x_i,y_i)\in D} e^{-\lambda((x-x_i)^2+(y-y_i)^2)}$$

$$I = \begin{pmatrix} E & F \\ F & G \end{pmatrix} = \begin{pmatrix} 1+f_x^2 & f_x f_y \\ f_x f_y & 1+f_y^2 \end{pmatrix}$$

$$II = \begin{pmatrix} L & M \\ M & N \end{pmatrix} = \begin{pmatrix} -f_{xx} & -f_{xy} \\ -f_{xy} & -f_{yy} \end{pmatrix}$$

Mean curvature:

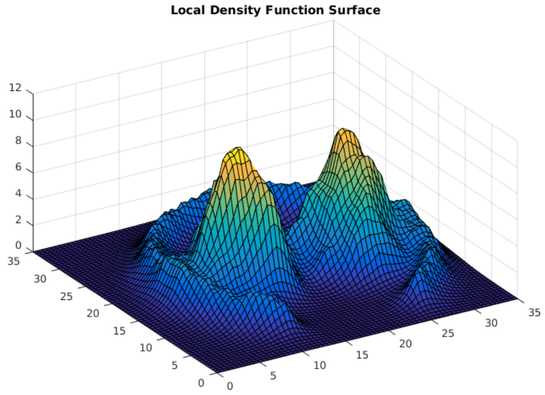
$$H = \frac{EN + GL - 2FM}{2(EG - F^2)}$$

Hypothesis

My conjecture: shift the points along the gradient of the mean curvature in order to obtain a skeletonization of each cluster.

Classification of a new point: classify to the cluster the skeleton of which is nearest to the point.

Experiments



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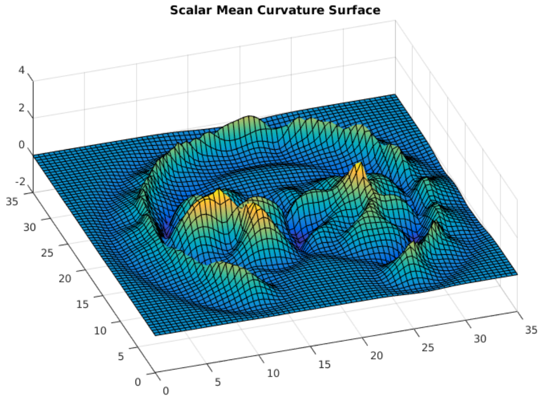

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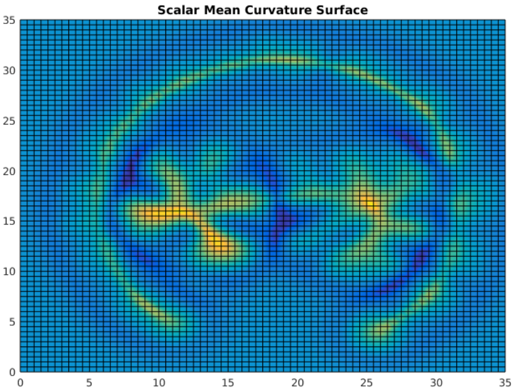

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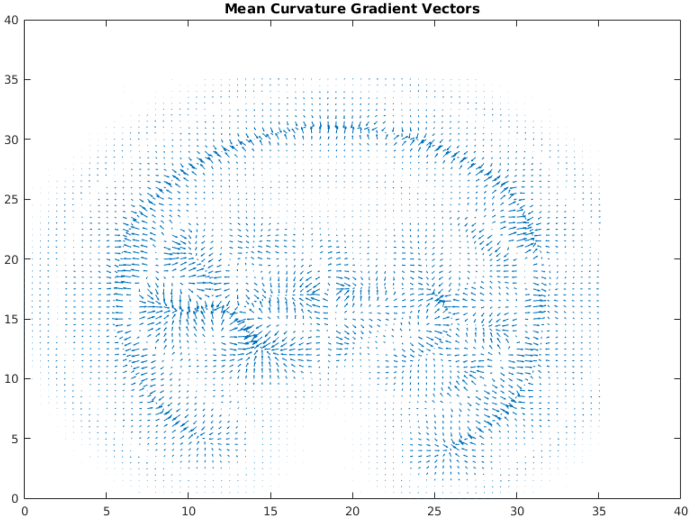

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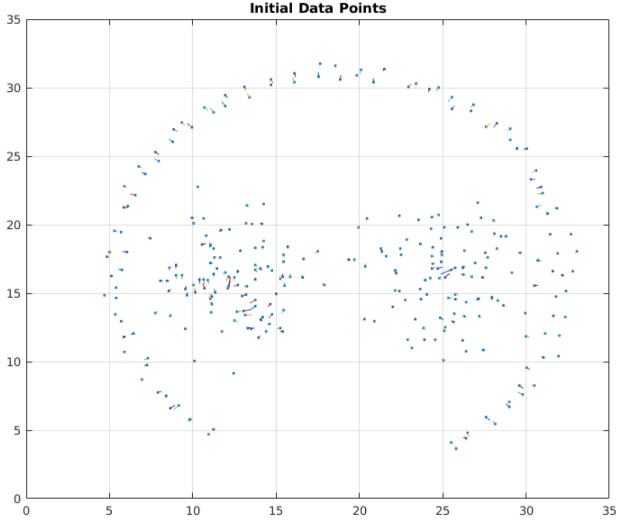

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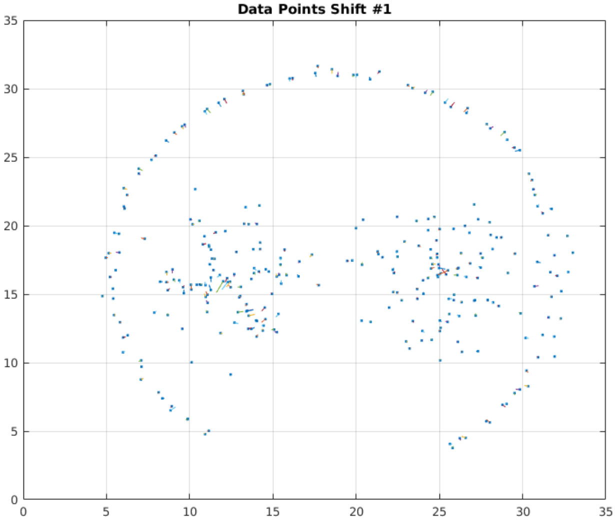

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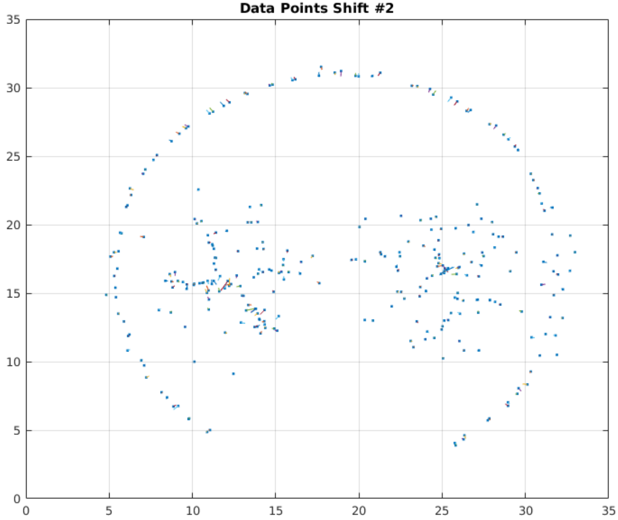

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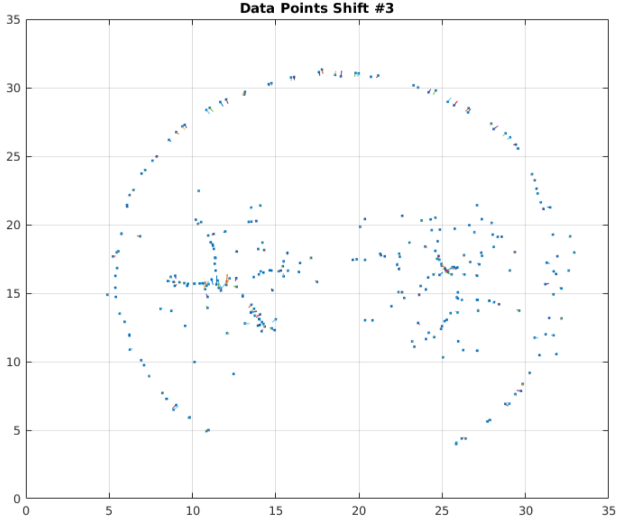

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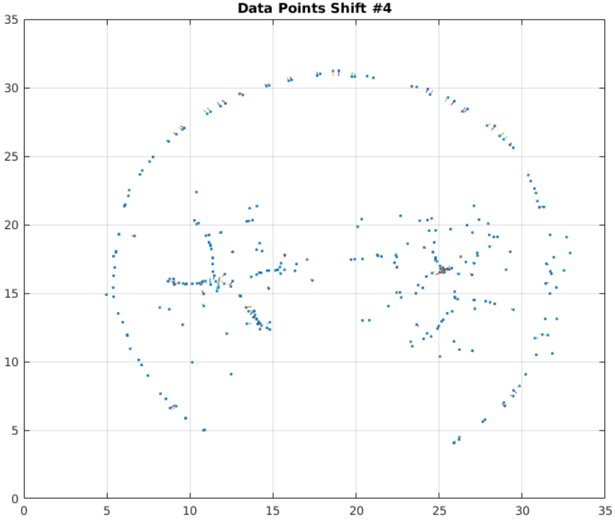

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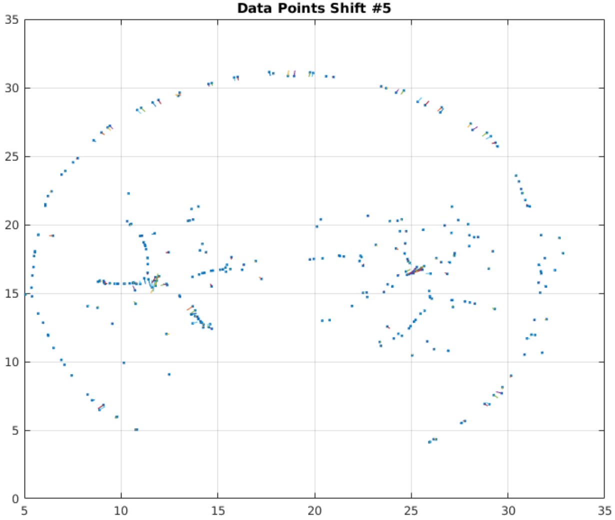

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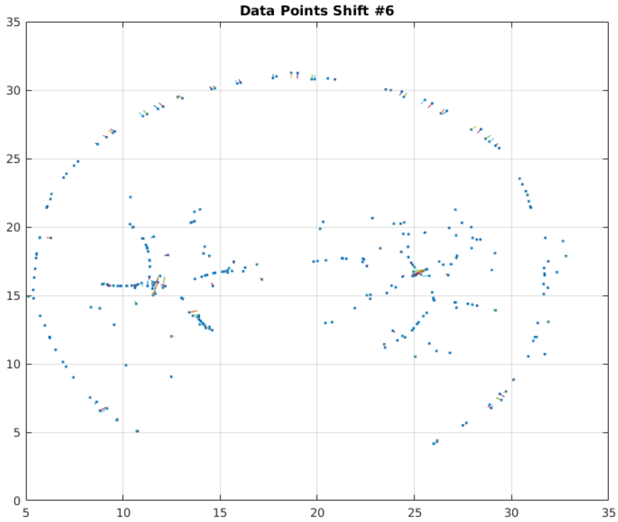

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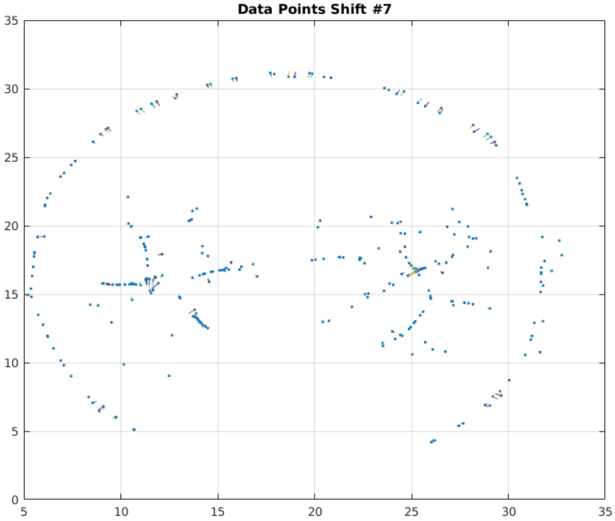

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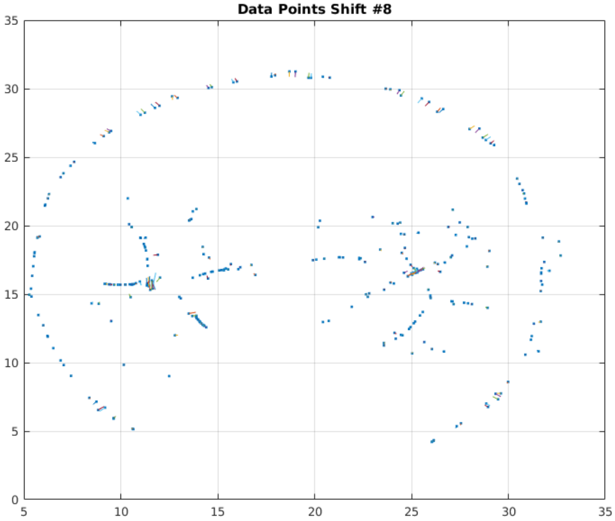

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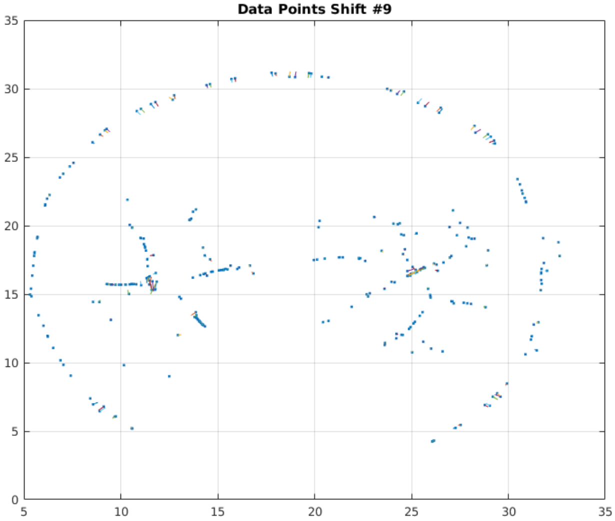

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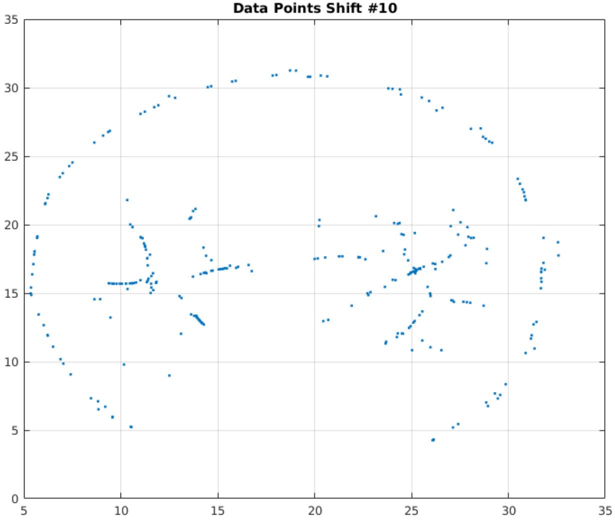

Conclusion

Yet I have not devised the way to distinguish among clusters.