CS 162 Programming languages

Lecture 10: Type Checking II

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Outline

- We will talk about types in λ^+

Motivation

- When writing programs, everything is great as long as the program works.
- Unfortunately, this is usually not the case
- Programs crash, don’t compute what we want them to compute, etc.
- This is arguably the biggest problem software faces today

Software correctness

- Problem: Rice’s theorem. Any non-trivial property about a Turing machine is undecidable

- This means that we can never give an algorithm, that for all programs can decide if this program has an error on some inputs.

- What can we do?

Big idea

- Big Idea: Just because we cannot prove something about the original program does not mean we cannot prove something about an *abstraction* of the program.

- Strategy: In addition to the operational semantics, we will also define *abstract semantics* that will overapproximate the states a program is in.

- Example: In λ^{+}, the operational semantics compute a concrete integer or list, while our abstract semantics only compute the if the result is of kind integer or list.

Abstraction

- Of course, any abstraction will be less precise than the program
- One popular abstraction: types
- Let’s assume we have types **Int** and **List**
- Example: let x = 10 in x
- Operational semantics yield concrete value 10
- Abstract semantics that only differentiate the kind (or type) of the expression yield: Integer

Abstraction

- But we don’t just want any abstraction, we need abstractions that *overapproximate* the result of the concrete program

- Recall the example: let x = 10 in x

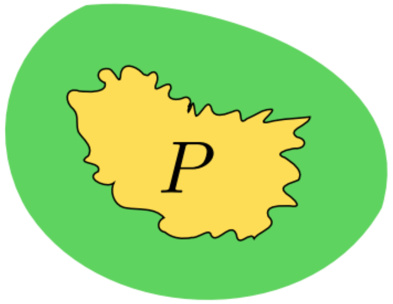
- Abstract value *Integer* overapproximates 10 since 10 is a kind of integer

- On the other hand, abstract value *List* does not overapproximate 10.

Soundness

- The reason we only care about sound abstract semantics is the following:

- Theorem: If some abstract semantics are sound and an expression is of abstract value x, then its concrete value y is always part of the abstract value x.



- Why is this useful?

- This means that if a program has no error in the abstract semantics, it is guaranteed not to have an error in the concrete semantics.

- ASTREE tools: http://www.astree.ens.fr/

Types

- In this class, we will focus on one kind of abstraction: types
- This means abstract values are the types in the language
- What is a type? An abstract value representing an (usually) infinite set of concrete values
- Question: For proving what kind of properties are types as abstract values useful?
- Answer: To avoid run-time type errors!

Adding types to λ$^+$

- Adding types to λ$^+$

$$\mathsf{T} \;::=\; \mathsf{T} \to \mathsf{T} \mid \mathsf{Int} \mid \mathsf{List}[\mathsf{T}]$$

- Adding type annotation to λ$^+$ expressions:

$$\begin{array}{rcll}
e & ::= & \ldots & \text{(as before)} \\
 & \mid & \mathsf{Nil}[\mathsf{T}] & \text{empty list} \\
 & \mid & \mathsf{lambda}\, x : \mathsf{T}.\ e & \text{lambda abstraction} \\
 & \mid & \mathsf{fix}\, f : \mathsf{T}\ \mathsf{is}\ e & \text{fixed-point operator} \\
 & \mid & (e \,@\, \mathsf{T}) & \text{type annotation}
\end{array}$$

Typing rules for integers

$$\frac{}{\Gamma \vdash i : \mathsf{Int}}\ \textsc{T-Int}$$

Any integer constant i is of type **integer**

$$\frac{\Gamma \vdash e_1 : \mathsf{Int} \qquad \Gamma \vdash e_2 : \mathsf{Int} \qquad \Box \in \{+, -, *\}}{\Gamma \vdash e_1 \Box e_2 : \mathsf{Int}}\ \textsc{T-Arith}$$

if e_1 and e_2 are both integers, then e_1 □ e_2 will also be integer

$$\frac{\Gamma \vdash e_1 : \mathsf{Int} \qquad \Gamma \vdash e_2 : \mathsf{Int} \qquad \Box \in \{<, >, =\}}{\Gamma \vdash e_1 \Box e_2 : \mathsf{Bool}}\ \textsc{T-Rel}$$

if e_1 and e_2 are both integers, then e_1 □ e_2 will a boolean

Typing rules for booleans

$$\frac{}{\Gamma \vdash \mathsf{true} : \mathsf{Bool}}\ \textsc{T-True} \qquad\qquad \frac{}{\Gamma \vdash \mathsf{false} : \mathsf{Bool}}\ \textsc{T-False}$$

Boolean constants have type **Bool**

$$\frac{\Gamma \vdash e_1 : \mathsf{Bool} \qquad \Gamma \vdash e_2 : \mathsf{T}_1 \qquad \Gamma \vdash e_3 : \mathsf{T}_2 \qquad \mathsf{T}_1 = \mathsf{T}_2}{\Gamma \vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 : \mathsf{T}_1}\ \textsc{T-If}$$

if e_1 is of type **Bool**, both e_2 and e_3 have the same type $T_1 = T_2$, then the whole if-else expression is of type T_1

Typing rules for lambda

$$\frac{x : \mathsf{T}_1, \Gamma \vdash e : \mathsf{T}_2}{\Gamma \vdash (\textsf{lambda}\, x : \mathsf{T}_1.\ e) : \mathsf{T}_1 \to \mathsf{T}_2}\ \textrm{T-L{\scriptsize AMBDA}}$$

If x is of type T_1 and e is of type T_2,
then the lambda expression has type $T_1 \rightarrow T_2$

$$\frac{\Gamma \vdash e_1 : \mathsf{T}_1 \to \mathsf{T}_2 \qquad \Gamma \vdash e_2 : \mathsf{T}_3 \qquad \mathsf{T}_1 = \mathsf{T}_3}{\Gamma \vdash (e_1\ e_2) : \mathsf{T}_2}\ \textrm{T-A{\scriptsize PP}}$$

if e_1 has type $T_1 \rightarrow T_2$, and e_2 has type T_3 such that $T_1 = T_3$
then lambda app return a value of type T_2

Typing rules for let-binding

$$\frac{\Gamma(x) = \mathsf{T}}{\Gamma \vdash x : \mathsf{T}}\ \textsc{T-Var}$$

If the type of x is T in the current type environment, then x is of type T

$$\frac{\Gamma \vdash e_1 : \mathsf{T}_1 \qquad x : \mathsf{T}_1, \Gamma \vdash e_2 : \mathsf{T}_2}{\Gamma \vdash \mathsf{let}\, x = e_1\, \mathsf{in}\, e_2 : \mathsf{T}_2}\ \textsc{T-Let}$$

if x and e_1 are of type T_1, and under the extended environment, e_2 is of type T_2, then the whole expression has type T_2

Typing rules for list

$$\frac{}{\Gamma \vdash \mathsf{Nil}[\mathsf{T}] : \mathsf{List}[\mathsf{T}]}\ \textsc{T-Nil}$$

An empty list is a list

$$\frac{\Gamma \vdash e_1 : \mathsf{T} \qquad \Gamma \vdash e_2 : \mathsf{List}[\mathsf{T}]}{\Gamma \vdash e_1 :: e_2 : \mathsf{List}[\mathsf{T}]}\ \textsc{T-Cons}$$

if e_1 is of type T and e_2 is of type List[T],
then e_1::e_2 is of type List[T]

$$\frac{\Gamma \vdash e_1 : \mathsf{List}[\mathsf{T}_1] \qquad \Gamma \vdash e_2 : \mathsf{T}_2 \qquad x : \mathsf{T}_1, y : \mathsf{List}[\mathsf{T}_1], \Gamma \vdash e_3 : \mathsf{T}_3 \qquad \mathsf{T}_2 = \mathsf{T}_3}{\Gamma \vdash \mathsf{match}\ e_1\ \mathsf{with}\ \mathsf{Nil} \to e_2 \mid x :: y \to e_3\ \mathsf{end} : \mathsf{T}_2}\ \textsc{T-Match}$$

If e is a list, and both the Nil branch and the cons branch have the same type,
then the overall pattern-match has that type.

Typing checking by example

wrong!

$$\dfrac{[1,2] : List[int] \qquad \dfrac{\dfrac{x : int \qquad x+2 : int}{\lambda x : int . x+2 : \boxed{int} \to int} \qquad y : \boxed{List[int]}}{\lambda x : int . x+2 \;\; y}}{let \;\; y = \underbrace{[1,2]}_{e_1} \;\; in \;\; \underbrace{\lambda x : int . x+2}_{e_2} \;\; y))}$$

TODOs by next lecture

- TBD