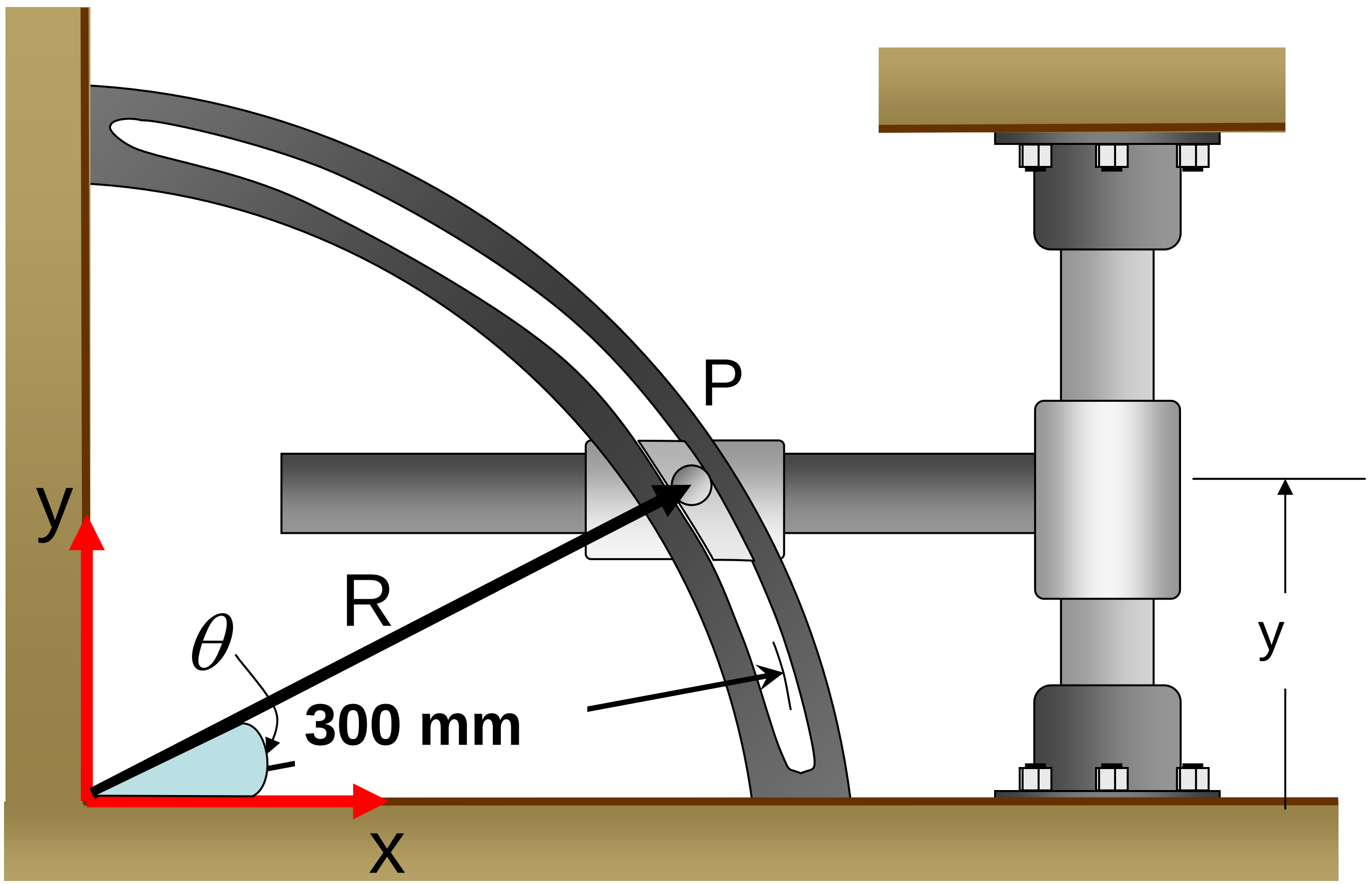
EXERCICE S7.2

Sachant que y = 100 mm, dy/dt = 200 mm/s et $d^2y/dt^2 = 0$, évaluez la vitesse et l'accélération du point P en terme de ses composantes normale et tangentielle.

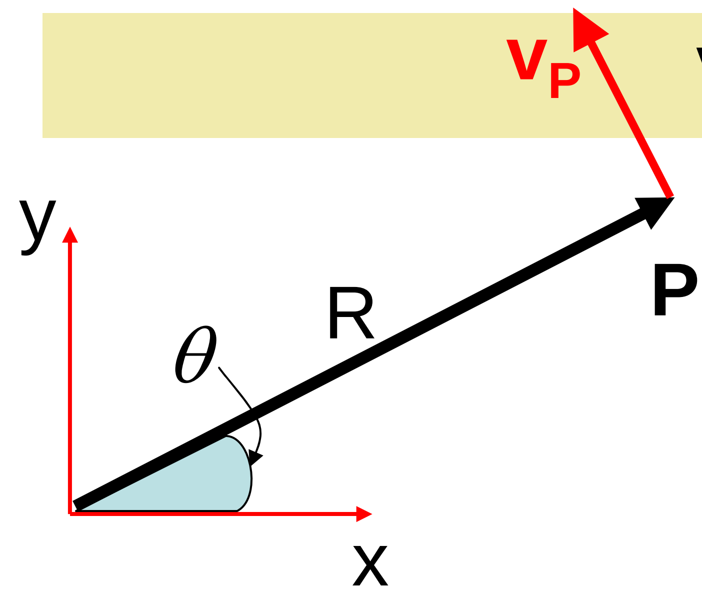


Données : y = 100 mm, dy/dt = 200 mm/s, $d^2y/dt^2 = 0$

R = 300 mm

On cherche: v_t, v_n, a_t, a_n

VITESSE



$V_n = 0$

Trajectoire de P :

$$x^2 + y^2 = R^2$$

En dérivant par rapport au temps :

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \Rightarrow \quad \frac{dx}{dt} = -\left(\frac{y}{x}\right)\frac{dy}{dt}$$

Données : $y = 0{,}1$ m, $\dfrac{dy}{dt} = 0{,}2$ m/s,

$$V_n = 0$$

$$x = \sqrt{R^2 - y^2} = 0{,}283\ \text{m}$$

Donc :

$$\frac{dx}{dt} = -\left(\frac{y}{x}\right)\frac{dy}{dt} = -\left(\frac{0{,}1}{0{,}283}\right)0{,}2 = -0{,}0707\ m/s$$

On trouve la norme avec les valeurs trouvées et l'on obtient:

$$v_P = v_t = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 0{,}212\ m/s$$

ACCÉLÉRATION

$$a_x = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\left(\frac{y}{x}\right)\frac{\mathrm{d}y}{\mathrm{d}t}$$

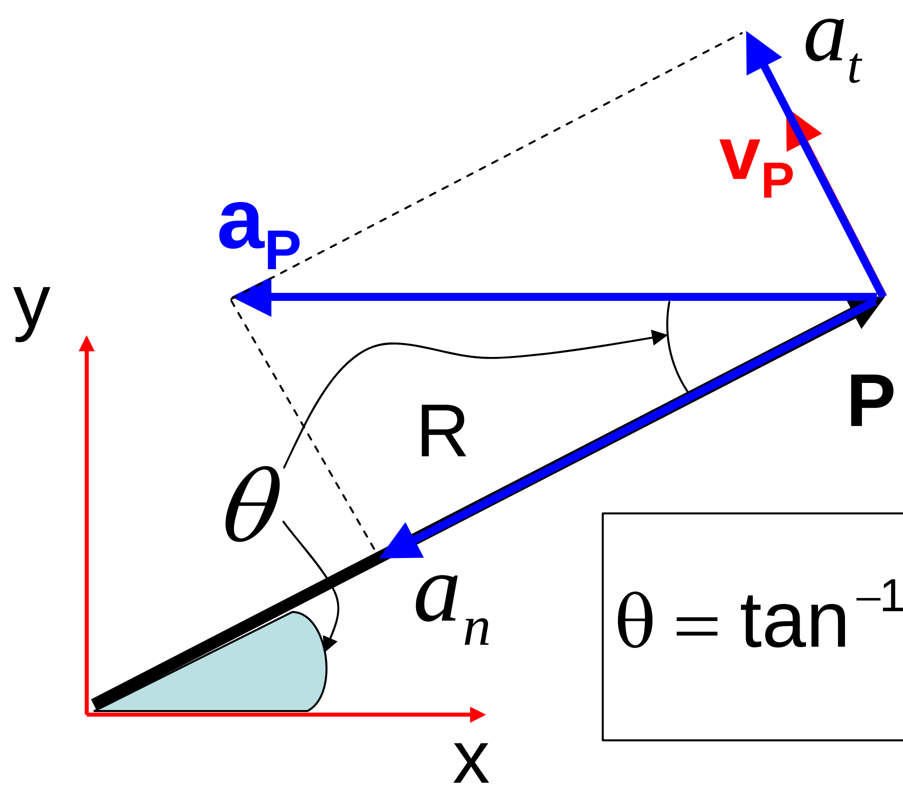
$$a_x = -\frac{1}{x}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \frac{y}{x^2}\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}t} - \frac{y}{x}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$$

x = 0,283 ; y = 0,1

dx/dt = 0,0707 ; dy/dt = 0,2

$$a_x = -\frac{1}{0{,}283}(0{,}2)^2 + \frac{0{,}1}{0{,}283^2}(0{,}2)(-{,}0707) - 0$$

$$\boxed{\left.\begin{array}{l} a_x = -0{,}1590\ \mathrm{m/s}^2 \\ a_y = 0 \end{array}\right\} \quad a_P = a_x}$$



$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{100}{283}\right) = 19{,}46^{o}$$

$$a_n = a\cos\theta = 0{,}159\cos 19{,}46^{o} = 0{,}15\ \mathrm{m/s}^2$$

$$a_t = a\sin\theta = 0{,}159\sin 19{,}46^{o} = 0{,}053\ \mathrm{m/s}^2$$