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1 Reading 9: Correlation and Regressions

1.1 Sample covar and sample correlation coefficients

Sample covariance: $cov_{x,y} = \sum_i \frac{(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$

Sample correlation coeff: $r_{x,y} = \frac{cov_{x,y}}{s_x s_y}$, where s_x is the sample dev of X.

1.2 Limitations to correlations analysis

Outliers: The results will be affected by extreme data points.(outliers)

Spurious correlation: There might be some non-zero correlation coeff, but actually they have no correlation at all.

Nonlinear relationships: Correlation only describe the linear relations.

1.3 Hypothesis: determine if the population correlation coefficient is zero

Two-tailed hypothesis test:

$$H_0 : \rho = 0, H_a : \rho \neq 0$$

Assume that the two populations are **normally** distributed, then we can use t-test:

$$t = \frac{r\sqrt{n-2}}{1-r^2}$$

: Reject H_0 if $t > +t_{critical}$ or $t < -t_{critical}$. Here, r is the sample correlation. Remember, you need to check t-table to find the t-value.

1.4 Determine dependent/independent variables in a linear regression

Simple linear regression: Explain the variation in a dependent variable in terms of the variation in a single independent variable. **Independent variables** are called explanatory variable, the exogenous variable, or the predicting variable. **Dependent variable** is also called the explained variable, the endogenous variable, or the predicted variable.

1.5 Assumptions in linear regression and interpret regression coeff.

1. Assumptions of linear regression:

- (a) Linear relationship must exist.
- (b) The independent variable is uncorrelated with residuals.
- (c) Expected Residual term is value. $E(\epsilon) = 0$
- (d) variance of the residual term is const. $E(\epsilon_i^2) = \sigma_\epsilon^2$. Otherwise, it will be "heteroskedastic"
- (e) The residual term is independently distributed. otherwise - "auto correlation" $E(\epsilon_i \epsilon_j) = 0$
- (f) The residual term is normally distributed.

2. Simple Linear Regression Model

- (a) Model: $Y_i = b_0 + b_1 X_i + \epsilon_i$, where $i = 1 \dots n$, and Y_i is the actual observed data.
- (b) The fitted line, the line of best fit : $\hat{Y} = \hat{b}_0 + \hat{b}_1 X_i$. Where \hat{b}_0 is the estimated parameter of the model.

- (c) How to choose the best fitted line? **Sum of squared errors** is minimum.

$$\hat{b}_1 = \frac{cov_{x,y}}{sigma_x^2}$$

where X is the independent variable. \hat{b}_1 is "regression coefficient".

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

where \bar{X}, \bar{Y} are the mean.

3. Interpreting a regression coefficient: Similar to basic ideas of "slope". Keep in mind: any conclusion regarding this parameter needs the statistical significance of the slope coefficient.

1.6 Standard error of estimate, the coeff. of determination and a confidence interval for a regression coefficient.

1. Standard error of estimate (SEE): Standard deviation between $Y_{estimate}$ and Y_{actual} . - Smaller: better
2. Coefficient of Determination (R^2) The percentage of the total variance in the dependent variable that is predictable from the independent variable. - One independent variable: $R^2 = r^2$, where r^2 is the square of correlation coefficient.
3. Regression Coefficient confidence interval

(a) Hypothesis: $H_0 : b_1 = 0 \Leftrightarrow H_a : b_1 \neq 0$

(b) Confidence interval: $\hat{b}_1 - (t_c s_{\hat{b}_1}) < b_1 < \hat{b}_1 + (t_c s_{\hat{b}_1})$ $s_{\hat{b}_1}$ is the standard error of the regression coeffi.

1.7 Hypothesis: Determine if $\hat{b}_1 = b_1$

1. t-test statistic: $t_{b_1} = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}}$
2. Reject: if $t > +t_{critical}$ or $t < -t_{critical}$

1.8 Calculate the predicted value for the dependent variable

If an estimated regression model is known, $\hat{Y} = \hat{b}_0 + \hat{b}_1 X_p$

1.9 Calculate and interpret a confidence interval for the predicted value of the dependent variable

1. Eq: $\hat{Y} \pm (t_c s_f)$, where s_f is the **std error of the forecast**.
2. $s_f^2 = SEE^2 \left[1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_x^2} \right]$
 - (a) SEE^2 = variance of the residuals
 - (b) s_x^2 = variance of the independent variable
 - (c) X = value of the independent variable where the forecast was made.

1.10 ANOVA in regression. Interpret results, and calculate F-statistic

- Analysis of variance (ANOVA) is used to analyze the total variability of the dependent variable.
 - Total sum of squares(SST): $SST = \sum_{i=1}^n (Y_i - \bar{Y})^2$
SST is the total variation in the dependent variable. $Variance = SST/(n - 1)$
 - Regression sum of squares(RSS): $RSS = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2$
RSS is the explained variation.
 - Sum of squared errors(SSE): $SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$
SSE is the unexplained variation.
 - $SST = RSS + SSE$ I cannot get this equation yet You need to know how to use these squares.
 - Degree of freedom: i) Regression(Explained): $k = 1$, since we only estimate one parameters. ii) Error(Unexplained) $df = n - k - 1 = n - 2$ iii) Total variation $df = n - 1$
- Calculating R^2 and **SEE**
 - $R^2 = explainedvariation/totalvarn = RSS/SST$
 - $SEE = \sqrt{\frac{SSE}{n-2}}$ SEE is the std deviation of the regression error terms.
- The F-Statistic: used to explain whether *at least one* independent parameter can significantly explain the dependent parameter.
 - F-statistic eq: $F = \frac{MSR}{MSE} = \frac{RSS/k}{SSE/n-k-1}$ where MSR = mean regression sum of squares. MSE = mean squared errors. Note: **One tailed test!**
- F-statistic with one independent variable.
 - Hypothesis: $H_0 : b_1 = 0 \Leftrightarrow H_a : b_1 \neq 0$
 - degree of freedom: $df_{rss} = k = 1, df_{sse} = n - k - 1$
 - Decision rule: reject H_0 if $F > F_c$

1.11 Limitations of regression analysis

- Parameter instability: the estimation eq may not be useful for other times.
- Limited usefulness: other participants may also use the same eq.
- Assumptions does not hold: i) Heteroskedastic, i.e., non-const variance of the error terms. ii) autocorrelation, i.e., error terms are not independent.

2 Reading 10: Multiple Regression and Issues in Regression Analysis

Some basic ides

- Model: $Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \dots + b_kX_{ki} + \epsilon_i$
- Multiple regression methodology estimates the intercept and slope coefficients so that $\sum_i^n \epsilon_i^2$ is minimized.

2.1 Interpret estimated regression coefficients and their p-values.

They are just simple linear functions with multiple parameters. Ignore.

2.2 Formulate a null/alternative hypothesis, do corresponding calculations

1. Hypothesis Testing of Regression coefficient. (Multi-parameters).
Use t-statistics to determine if one parameter significantly contribute to the model.

$$t = \frac{\hat{b}_j - b_j}{s_{\hat{b}_j}}, df = n - k - 1$$

where k is the number of regression coefficients, and 1 corresponds to the intercept term, and $s_{\hat{b}_j}$ is the coefficient standard error of b_j

2. Determining statistical significance.
“testing statistical significance” $\Rightarrow H_0 : b_j = 0, H_a : b_j \neq 0$
3. Interpreting p-values.
(a) Def: p-value is **the smallest level of significance for which the null hypothesis can be rejected**. If the p-value is less than significance level, the null
4. Other Tests of the Regression Coefficients: $H_0 : a = \text{some value}$

2.3 Calculate and Interpret a confidence interval for the population value of a regression coefficient or a predicted value for the dependent variable if an estimated regression model.

1. Confidence intervals for a regress. coeff.: $\hat{b}_j \pm (t_c \times s_{\hat{b}_j})$
2. predicting the dependent variable: $\hat{Y}_i = \hat{b}_0 + \hat{b}_1 \hat{X}_{1i} + \dots + \hat{b}_k \hat{X}_{ki}$
Even if you may conclude that some b_i are not statistically significantly, you cannot treat them as 0 and keep other parameters unchanged. You should use the original model, or you can throw \hat{b}_k away and make a new regression model.

2.4 Assumptions of a multiple regression model

1. Linear relationships exist.
2. The independent variables are not random, and there is no exact linear relation between independent variables.
3. $E[\epsilon | X_1, \dots, X_k] = 0$
4. Variance of $\epsilon = 0$, i.e. $E[\epsilon_i] = 0$
5. $E(\epsilon_i \epsilon_j) = 0$
6. ϵ is normally distributed.

2.5 Calculate and interpret F-statistic

F-test: whether at least **one** of the independent variables explains a significant portion of the variation of the dependent variable. F test is a one-tail test.

1. $H_0 : b_1 = b_2 = b_3 = 0$ vs $H_a : \text{at least one } b_j \neq 0$
2. $F = \frac{MSR}{MSE} = \frac{RSS/k}{SSE/(n-k-1)}$
3. Degree of freedom: $df_{\text{numerator}} = k, df_{\text{denominator}} = n - k - 1$
4. Rules: reject H_0 if $F(\text{test} - \text{statistic}) > F_c(\text{critical value})$

2.6 Distinguish between R^2 and adjusted R^2

1. coefficient of determination R^2 : used to test if a group of independent variable can explain the dependent variable:

$$R^2 = \frac{\text{total variation} - \text{unexplained variation}}{\text{total variation}} = \frac{SST - SSE}{SST} = \frac{RSS}{SST}$$

$$\text{Multiple } R = \sqrt{R^2}$$

2. Adjusted R^2

- (a) Note: R^2 : **Overestimating**: will increase as variables are added to the model. Even the marginal contribution of new variables are not statistically significant.

$$(b) \text{ Introduce } R_a^2: R_a^2 = 1 - \left[\left(\frac{n-1}{n-k-1} \right) \right] (1 - R^2)$$

2.7 Evaluate the quality of a regression model by analyzing the output of the equation/ANOVA table

1. ANOVA Tables, some important quantities

$$(a) R^2 = \frac{RSS}{SST}$$

$$(b) F = \frac{MSR}{MSE} \text{ with } k \text{ and } n - k - 1 \text{ df}$$

$$(c) \text{ Standard error of estimate: } SEE = \sqrt{MSE}$$

2.8 Formulate a multiple regression with dummy variables to represent qualitative factors

1. Def: Some value is quite qualitative. Using dummy values like 0 or 1 to describe their impacts.
2. Note: Pay attention to # of dummy variables. If n classes, we must use $n - 1$ dummy variables.
3. Interpreting the coefficients in a dummy variable regression. We can use F-statistics to test a group of parameters, or use t-test to test the individual slope coefficients.
4. Example of Regression application with dummy variables. See Notes directly.

2.9 Why multiple regression isn't as easy as it looks?

Pay attention to the assumptions that have been used. Violations like::

1. Heteroskedasticity
2. Serial correlation (auto-correlation)
3. Multicollinearity

Any violations on the assumptions will impact the estimation of SEE, and finally change the t-statistic and F-statistic, and change the conclusion of the hypothesis test.

2.10 Types of Heteroskedasticity, how heteroskedasticity and serial correlation affect inference

1. What is Heteroskedasticity?

Corresponding assumptions: Variance of the residuals is constant across observations. – Homoskedasticity Heteroskedasticity means the variance of the residuals is not equal.

- (a) Unconditional heter: Not related to the level of the independent variables. Will not systematically increase with changes in the value of the independent variables. **Usually will not cause major problems.**
 - (b) Conditional heter: Related to the level of the independent variables. Eg: Conditional heter exists if the variance of the residuals increase with the value of the independent variables increases. **Will cause big problems.**
2. Effect of Heteroskedasticity on Regression Analysis
- (a) Unreliable standard errors.
 - (b) The coefficient estimates aren't affected.
 - (c) Will change the t-statistic, and will change the conclusion.
 - (d) Unreliable F-test
3. Detect Heteroskedasticity
- (a) Scatter plot
 - (b) Breusch-pagan test: $BPtest = n \times R_{resid}^2$ with $df = k$. where n = the number of observations, $R_{resid}^2 = R^2$ from a second regression of the squared residuals from the first regression. k = the number of independent variables. If R^2 or BP-test are too large, something is wrong.
4. Correcting Heteroskedasticity
- (a) Calculate robust standard errors (White corrected std errors.). Use them for t-test.
 - (b) Generalized least squares.
5. What is serial correlations?
- (a) Def: auto-correlation, in which the residual terms are correlated. Common problem with time series data.
 - i. Positive serial correlation: a positive error in one time period will increase the possibility to observe a positive one next time.
 - ii. Negative serial correlation: Just opposite.
 - (b) Effect: positive serial correlation will get small coefficient std errors. Thus, too large t-statistics. therefore, too many Type I errors: reject the null hypothesis H_0 while it's actually true.
 - (c) Detection:
 - i. Residual plots
 - ii. Durbin-Watson statistics:

$$DW = \frac{\sum_{t=2}^T (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2}{\sum_{t=1}^T \hat{\epsilon}_t^2}$$

For large samples, $DW \approx 2(1 - r)$, where r is the correlation coefficient between residuals from one period and those from the previous period.

Results:

 - A. $DW = 2 \Rightarrow$ Homoskedastic and not serially correlated.
 - B. $DW < 2 \Rightarrow$ Positively serially correlated.
 - C. $DW > 2 \Rightarrow$ Negatively serially correlated.

Formulated hypothesis with DW-table, upper and lower critical values

 - A. Hypothesis: H_0 : the regression has **no** positive serial correlation.
 - B. $DW < d_l$: positive serially correlated. Reject null.
 - C. $d_l < DW < d_u$: inconclusive results.

D. $DW > d_u$: **There is no evidence that are positive correlated.**

- (d) Correcting serial correlation:
 - i. Adjust the coefficient std errors. **recommended.** Using Hansen method.
 - A. Serial correlation only: Hansen method.
 - B. Heteroskedasticity only: White-corrected stand errors.
 - C. Both: Hans methods.
 - ii. Imporoe the specification of the model.

2.11 Multicollinearity and its cause and effects in regression analysis

Multicollinearity: Independent variables or linear combinations of independent variables are highly correlated.

1. Effect of Multicollinearity on Regression Analysis: Will increase the std errors of the slope coefficients.
Type II Error: A variable is significant, while we conclude it's not.
2. Detecting: Common situation: t - statistic is not significant while F - test is significant. This tells us the independent variables are highly correlated.
A simple rule works if there are 2 independent variables: when the absolute value of the sample correlation betewen any two independent variables in the regression is greater than 0.7.
3. Correcting: omit one or more of the correlated independent variables. The problame is that it's hard to find the variables that result in the multicollinearity.

2.12 Model misspecification

1. Defination of **Regression model of specification**: decide which independent variables to be included in the model.
2. Types of misspecification
 - (a) The functional form can be misspecified: important variables are ommitted; variables should be transformed; data is improperly pooled.
 - (b) Explanatory variables are correlated with error term in time series model: A lagged dependent variable is used as an independent variable; a function of the dependent variable is used as an independent variable (forecasting the past); independent variables are measured with error.
 - (c) Other time-series misspecification.

2.13 Models with qualitative dependent variables

Include qualitative dependent variables, like default, bankcrupcy. Cannot use an ordinary regression model. Should use other models like **probit and logit models** or **discriminant models**.

1. Probit: normal distribution, give probability.
2. Logistic: logistic distribution.
3. Discriminant: result in an overall score or ranking.

3 Reading 11: Time-Series Analysis

3.1 Calculate/evaluate the predicted trend value for a time series given the estimated trend coefficients

1. Linear Trend Model and Log-linear Trend

- (a) Definition: $y_t = b_0 + b_1(t) + \epsilon_t$ Note: t is just time.
- (b) Coefficients is determined by OLS. Ordinary least squared regression.
 $\hat{y} = \hat{b}_0 + \hat{b}_1$
- (c) Log-linear Trend Models
- (d) Model: $y_t = \exp b_0 + b_1(t) \Rightarrow \ln y_t = b_0 + b_1(t)$

3.2 Factors that determine whether a linear or a log-linear model trend should be used

1. Factors that determine which model is best: plot data.
2. Limitations of trend models:
 - (a) residuals are uncorrelated with each other. Otherwise, it will cause auto correlation and we should not use the trend model.
 - (b) For log-linear model, it is not suitable for cases with serial correlations (autocorrelation).
 - (c) Detect auto correlation: Durbin Watson statistic. $DW = 2.0 \Rightarrow$ No auto correlation.

3.3 Autoregressive model, requirements for covariance stationary

1. Autoregressive model:
 - (a) Model: $x_t = b_0 + b_1x_{t-1} + \epsilon_t$
 - (b) Statistical inferences based on ordinary least squares estimates doesn't apply unless the time series is **covariance stationary**.
 - (c) Conditions for covariance stationary
 - i. Constant and finite expected value.
 - ii. Constant and finite variance.
 - iii. Constant and finite covariance between values at any given lag.

3.4 An autoregressive model of order p

1. Model(order p): $x_t = b_0 + b_1x_{t-1} + b_2x_{t-2} + \dots + b_px_{t-p} + \epsilon_t$
2. Forecasting with an autoregressive model:
 - (a) One-period-ahead forecast for $AR(1)$: $\hat{x}_{t+1} = \hat{b}_0 + \hat{b}_1x_t$
 - (b) Two-period-ahead forecast for $AR(1)$: $\hat{x}_{t+2} = \hat{b}_0 + \hat{b}_1\hat{x}_{t+1}$

3.5 How the residuals can be used to test the autoregressive model

1. The residual should have no *serial correlation* if an AR model is correct.
2. Steps
 - (a) Estimate: Start with $AR(1)$
 - (b) Calculate: the autocorrelations of the model residuals
 - (c) Test: whether the autocorrelations are significantly different from 0.
 The standard error is $\frac{1}{\sqrt{T}}$ for T observations. The t-test for each observation is $t = \frac{\rho_{\epsilon_t, \epsilon_{t-k}}}{1/\sqrt{T}}$, with $T - 2$ df.

3.6 Mean reversion and a mean-reverting level

1. Mean reversion: The time series tends to move toward its mean.
2. Mean-reverting level: $\hat{x}_{t+1} = x_t$, where \hat{x}_t is the predicted value.
3. All covariance stationary time series has finite mean-reverting level.

3.7 Contrast in-sample and out-of-sample forecasts and the forecasting accuracy of different time-series models based on the root mean squared error criterion.

1. in-sample, out-of-sample: determined by if the predicted data is in the range of the observations.
2. RMSE, root mean squared error: used to compare the accuracy. If the accuracy of out-of-sample is better, you should use it for future applications

3.8 Explain the instability of coefficients of time-series models

1. Instability or nonstationarity. Due to the dynamic economic conditions, model coefficients will change a lot from period to period.
2. Shorter time series are more stable, but longer time series are more reliable.

3.9 Random walk processes and their comparisons between covariance stationary processes

1. Random walk: $x_t = x_{t-1} + \varepsilon_t$
 - (a) $E(\varepsilon_t) = 0$: The expected value of each error is zero.
 - (b) $E(\varepsilon_t^2) = 0$: The variance of the error terms is constant.
 - (c) $E(\varepsilon_i, \varepsilon_j) = 0$: There is no serial correlation in the error terms.
2. Random walk with a Drift: $x_t = b_0 + b_1 x_{t-1} + \varepsilon_t$, where $b_1 \neq 0$
3. A random walk or a random walk with a drift have no finite mean-reverting level. Since $b_1 = 1$, $\frac{b_0}{1-b_1} = \frac{b_0}{0}$. Therefore, they are not covariance stationary.
4. $b_1 = 1$, they exhibit a unit root. Thus, **the least square regression that been used in AR(1) will not work unless we transform the data.**

3.10 Things about unit roots: when they will occur, how to test them, how to transform data to apply AR

1. Unit root testing for nonstationarity:
 - (a) run an AR model and check autocorrelations
 - (b) perform Dickey Fuller test.
 - i. Transform: $x_t = b_0 + b_1 x_{t-1} + \varepsilon \Rightarrow x_t - x_{t-1} = b_0 + (b_1 - 1)x_{t-1} + \varepsilon$
 - ii. Direct test if $b_1 - 1 = 0$ using a modified t-test.
2. First differencing
 - (a) For a random walk, transform the data $y_t = x_t - x_{t-1} \Rightarrow y_t = \varepsilon_t$ then start to use an AR model $y = b_0 + b_1 y_{t-1} + \varepsilon$, where $b_0 = b_1 = 0$
 - (b) y is covariance stationary.

3.11 How to test and correct for seasonality in a time-series model, and calculate and interpret a forecasted value using an AR model with a seasonal lag.

1. Detect: special autocorrelation exists for some seasonal lags.
2. Correction: Add an additional seasonal lag term.

3.12 Explain autoregressive conditional heteroskedasticity (ARCH) and describe how ARCH models can be applied to predict the variance of a time series

1. ARCH: the variance of the residuals in one period is dependent on the variance of the residuals in a previous period.
2. Using ARCH models:
 Example $ARCH(1)$: $\hat{\varepsilon}_t^2 = a_0 + a_1\hat{\varepsilon}_{t-1} + \mu_t$ if a_1 is significantly different from zero. $\hat{\varepsilon}_t^2$ is the squared residuals.
 Note: Things like generalized least squares should be used to correct heteroskedasticity. otherwise, the std errors of the coefficients will be wrong, leading to invalid conclusions.
3. Predicting the variance of a time series: using ARCH model to predict the variance of future periods: $\hat{\sigma}_{t+1}^2 = \hat{a}_0 + \hat{a}_1\hat{\varepsilon}_t^2$

3.13 Explain How time-series variables should be analyzed for nonstationarity and/or cointegration before use in a linear regression

1. Cointegration:
 - (a) Two time series are economically linked or follow the same trend and that relationship is not expected to change. – Error terms from regressing one on the other is covariance stationary and the t-test are reliable.
 - (b) How to test cointegration: regress y_t on x_t $y_t = b_0 + b_1x_t + \varepsilon$, y_t, x_t are two different time series. Then do a unit root test using the Dickey Fuller test with critical t-values calculated by Engle and Granger.
 If "A unit root" is rejected \Rightarrow covariance stationary, cointegrated.

4 Reading 12: Probabilistic Approaches: Scenario Analysis, Decision Trees, and Simulations

4.1 Describe steps in a simulation, Explain three ways to define the probability distributions for a simulation's variable, and describe how to treat correlation across variables in a simulation.

1. Steps in simulations:
 - (a) Determine the probabilistic variables
 - (b) Define probability distributions for these variables
 - i. Option 1: Historical data
 - ii. Option 2: Cross-sectional data: estimate the variable from similar companies.
 - iii. Option 3: Pick a distribution and estimate the parameters.

- (c) Check for correlations among variables: Use historical data to determine whether any systematically related. Strong relations \Rightarrow 1) Allow only one of the variables can be removed. Or 2) Build the rules of correlations into the simulation.
- (d) Run the simulation.

4.2 Describe advantages of using simulations in decision making

1. Advantages: 1) Better input quality 2) Provides a distribution of expected value rather than a point estimate.

4.3 Describe some common constraints introduced into simulations

1. Constraints: specific limits imposed by users of simulations.
2. Types of constraints
 - (a) Book value constraints:
 - i. Regulatory capital requirements: minimum capital requirements
 - ii. Negative equity
 - (b) Earnings and cash flow constraints: might be imposed to meet analyst expectations
 - (c) Market value constraints

4.4 Describe issues in using simulations in risk assessment

1. Limitations of using simulations
 - (a) Input quality: garbage in, garbage out
 - (b) Inappropriate statistical distributions
 - (c) Non-stationary distributions: parameters will change
 - (d) Dynamic correlations: correlations between input variables will change.
2. Risk-adjusted value: cash flows from simulations are not risk-adjusted. SHOULD NOT be discounted at risk-free rate.

4.5 Compare scenario analysis, decision trees, and simulations

1. Scenario analysis: computes the value of an investment under some specific cases. Total probability is less than 1.
2. Decision trees: good when risk is discrete and sequential. Sum of probability is 1

5 Reading 13: Currency Exchange Rates: Determination and Forecasting

5.1 Calculate and interpret the bid-ask spread

1. Exchange rates
 - (a) Important things: exchange rate, spot exchange rate, forward exchange rate.
 - (b) Bid/offer(ask) rates: //Bid: The price that bank will buy. Offer: The price that bank will sell.
 - (c) Foreign Exchange Spread. Unit: "1 pip" = $1/10000 = 0.0001$. Spread depend on:
 - i. Spread in the interbank market. (Currencies, time, market volatility)
 - ii. Size of transaction.
 - iii. Relationship between the dealer and client.

5.2 Identify a triangular arbitrage opportunity and calculate its profit

- Example: USD/AUD. USD is the price currency, and AUD is the base currency.
 - Buy the base currency at the ask \Rightarrow Sell the price currency at the ask
 - Sell the base currency at the bid \Rightarrow Buy the price currency at the bid
- For investors, Rule: *up-the-bid-and-multiply, down-the-ask-and-divide*
 - Convert USD into AUD: going down the quote – from USD on top to AUD on bottom. Use the ask price for the quote.
 - Convert AUD into USD: similar. But from bottom to top.
- Cross Rate: The exchange rate between two currencies with the help by a common third currency.
- Cross Rate with bid-ask spreads.
 - Rule 1:

$$\left(\frac{A}{C}\right)_{bid} = \left(\frac{A}{B}\right)_{bid} \times \left(\frac{B}{C}\right)_{bid}; \left(\frac{A}{C}\right)_{offer} = \left(\frac{A}{B}\right)_{offer} \times \left(\frac{B}{C}\right)_{offer}$$
 - Rule 2:

$$\left(\frac{B}{C}\right)_{bid} = \frac{1}{\left(\frac{C}{B}\right)_{offer}}; \left(\frac{B}{C}\right)_{offer} = \frac{1}{\left(\frac{C}{B}\right)_{bid}}$$
- Triangular Arbitrage: If the dealer's quote is different from the cross rate, arbitrage opportunities may exist. Check it with Notes.

5.3 Distinguish between spot and forward rates and calculate the forward premium/discount for a given currency

- Forward premium relative to a second currency: Forward price $\hat{}$ Spot price. Forward premium = $F - S_0$
- Calculate the market-to-market value of a forward contract

$$V_T = (FP_T - FP)(contractsize)$$

where:

- V_T = value of the forward contract at time T , denominated in price currency
 - T = maturity of the forward contract
 - FP = forward price locked in at inception to buy base currency
 - FP_T = forward price to **sell** the same currency at time T
- Value prior to expiration.

$$V_t = \frac{(FP_t - FP)contractsize}{1 + R\left(\frac{days}{360}\right)}$$

where

- V_t is the value of the forward price
- FP_t : forward price at time t
- $days$ number of days remaining
- R interest rate

5.4 Explaining international parity relations (covered and uncovered interest rate parity, purchasing power parity, and the international Fisher effect)

1. Covered interest rate parity: “Covered” means bound by arbitrage. Investor should earn the same return using either currency.

$$F = \frac{1 + R_A(\frac{days}{360})}{1 + R_B(\frac{days}{360})} S_0$$

2. Uncovered interest rate parity: Forward currency contract is unavailable, which makes the interest rate not bound by arbitrage. For a quote A/B, the base currency is expected to appreciate

$$E(\% \Delta_S)_{(A/B)} = R_A - R_B$$