

## Contents

# 1 Reading 9: Correlation and Regressions

## 1.1 Sample covar and sample correlation coefficients

Sample covariance:  $cov_{x,y} = \sum_i \frac{(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$

Sample correlation coeff:  $r_{x,y} = \frac{cov_{x,y}}{s_x s_y}$ , where  $s_x$  is the sample dev of X.

## 1.2 Limitations to correlations analysis

Outliers: The results will be affected by extreme data points.(outliers)

Spurious correlation: There might be some non-zero correlation coeff, but actually they have no correlation at all.

Nonlinear relationships: Correlation only describe the linear relations.

## 1.3 Hypothesis: determine if the population correlation coefficient is zero

Two-tailed hypothesis test:

$$H_0 : \rho = 0, H_a : \rho \neq 0$$

Assume that the two populations are **normally** distributed, then we can use t-test:

$$t = \frac{r\sqrt{n-2}}{1-r^2}$$

: Reject  $H_0$  if  $t > +t_{critical}$  or  $t < -t_{critical}$ . Here,  $r$  is the sample correlation. Remember, you need to check t-table to find the t-value.

## 1.4 Determine dependent/independent variables in a linear regression

**Simple linear regression:** Explain the variation in a dependent variable in terms of the variation in a single independent variable. **Independent variables** are called explanatory variable, the exogenous variable, or the predicting variable. **Dependent variable** is also called the explained variable, the endogenous variable, or the predicted variable.

## 1.5 Assumptions in linear regression and interpret regression coeff.

1. Assumptions of linear regression:

- (a) Linear relationship must exist.
- (b) The independent variable is uncorrelated with residuals.
- (c) Expected Residual term is value.  $E(\epsilon) = 0$
- (d) variance of the residual term is const.  $E(\epsilon_i^2) = \sigma_\epsilon^2$ . Otherwise, it will be "heteroskedastic"
- (e) The residual term is independently distributed. otherwise - "auto correlation"  $E(\epsilon_i \epsilon_j) = 0$
- (f) The residual term is normally distributed.

2. Simple Linear Regression Model

- (a) Model:  $Y_i = b_0 + b_1 X_i + \epsilon_i$ , where  $i = 1 \dots n$ , and  $Y_i$  is the actual observed data.
- (b) The fitted line, the line of best fit :  $\hat{Y} = \hat{b}_0 + \hat{b}_1 X_i$ . Where  $\hat{b}_0$  is the estimated parameter of the model.

- (c) How to choose the best fitted line? **Sum of squared errors** is minimum.

$$\hat{b}_1 = \frac{cov_{x,y}}{sigma_x^2}$$

where  $X$  is the independent variable.  $\hat{b}_1$  is "regression coefficient".

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

where  $\bar{X}, \bar{Y}$  are the mean.

3. Interpreting a regression coefficient: Similar to basic ideas of "slope". Keep in mind: any conclusion regarding this parameter needs the statistical significance of the slope coefficient.

### 1.6 Standard error of estimate, the coeff. of determination and a confidence interval for a regression coefficient.

1. Standard error of estimate (SEE): Standard deviation between  $Y_{estimate}$  and  $Y_{actual}$ . - Smaller: better
2. Coefficient of Determination ( $R^2$ ) The percentage of the total variance in the dependent variable that is predictable from the independent variable. - One independent variable:  $R^2 = r^2$ , where  $r^2$  is the square of correlation coefficient.
3. Regression Coefficient confidence interval

(a) Hypothesis:  $H_0 : b_1 = 0 \Leftrightarrow H_a : b_1 \neq 0$

(b) Confidence interval:  $\hat{b}_1 - (t_c s_{\hat{b}_1}) < b_1 < \hat{b}_1 + (t_c s_{\hat{b}_1})$   $s_{\hat{b}_1}$  is the standard error of the regression coeffi.

### 1.7 Hypothesis: Determine if $\hat{b}_1 = b_1$

1. t-test statistic:  $t_{b_1} = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}}$
2. Reject: if  $t > +t_{critical}$  or  $t < -t_{critical}$

### 1.8 Calculate the predicted value for the dependent variable

If an estimated regression model is known,  $\hat{Y} = \hat{b}_0 + \hat{b}_1 X_p$

### 1.9 Calculate and interpret a confidence interval for the predicted value of the dependent variable

1. Eq:  $\hat{Y} \pm (t_c s_f)$ , where  $s_f$  is the **std error of the forecast**.
2.  $s_f^2 = SEE^2 \left[ 1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_x^2} \right]$ 
  - (a)  $SEE^2$  = variance of the residuals
  - (b)  $s_x^2$  = variance of the independent variable
  - (c)  $X$  = value of the independent variable where the forecast was made.

### 1.10 ANOVA in regression. Interpret results, and calculate F-statistic

- Analysis of variance (ANOVA) is used to analyze the total variability of the dependent variable.
  - Total sum of squares(SST):  $SST = \sum_{i=1}^n (Y_i - \bar{Y})^2$   
SST is the total variation in the dependent variable.  $Variance = SST/(n - 1)$
  - Regression sum of squares(RSS):  $RSS = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2$   
RSS is the explained variation.
  - Sum of squared errors(SSE):  $SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$   
SSE is the unexplained variation.
  - $SST = RSS + SSE$  I cannot get this equation yet You need to know how to use these squares.
  - Degree of freedom: i) Regression(Explained):  $k = 1$ , since we only estimate one parameters. ii) Error(Unexplained)  $df = n - k - 1 = n - 2$  iii) Total variation  $df = n - 1$
- Calculating  $R^2$  and **SEE**
  - $R^2 = explainedvariation/totalvarn = RSS/SST$
  - $SEE = \sqrt{\frac{SSE}{n-2}}$  SEE is the std deviation of the regression error terms.
- The F-Statistic: used to explain whether *at least one* independent parameter can significantly explain the dependent parameter.
  - F-statistic eq:  $F = \frac{MSR}{MSE} = \frac{RSS/k}{SSE/n-k-1}$  where  $MSR$  = mean regression sum of squares.  $MSE$  = mean squared errors. Note: **One tailed test!**
- F-statistic with one independent variable.
  - Hypothesis:  $H_0 : b_1 = 0 \Leftrightarrow H_a : b_1 \neq 0$
  - degree of freedom:  $df_{rss} = k = 1, df_{sse} = n - k - 1$
  - Decision rule: reject  $H_0$  if  $F > F_c$

### 1.11 Limitations of regression analysis

- Parameter instability: the estimation eq may not be useful for other times.
- Limited usefulness: other participants may also use the same eq.
- Assumptions does not hold: i) Heteroskedastic, i.e., non-const variance of the error terms. ii) autocorrelation, i.e., error terms are not independent.

## 2 Reading 10: Multiple Regression and Issues in Regression Analysis

Some basic ides

- Model:  $Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \dots + b_kX_{ki} + \epsilon_i$
- Multiple regression methodology estimates the intercept and slope coefficients so that  $\sum_i^n \epsilon_i^2$  is minimized.

### 2.1 Interpret estimated regression coefficients and their p-values.

They are just simple linear functions with multiple parameters. Ignore.

## 2.2 Formulate a null/alternative hypothesis, do corresponding calculations

1. Hypothesis Testing of Regression coefficient. (Multi-parameters).  
Use t-statistics to determine if one parameter significantly contribute to the model.

$$t = \frac{\hat{b}_j - b_j}{s_{\hat{b}_j}}, df = n - k - 1$$

where  $k$  is the number of regression coefficients, and 1 corresponds to the intercept term, and  $s_{\hat{b}_j}$  is the coefficient standard error of  $b_j$

2. Determining statistical significance.  
“testing statistical significance”  $\Rightarrow H_0 : b_j = 0, H_a : b_j \neq 0$
3. Interpreting p-values.  
(a) Def: p-value is **the smallest level of significance for which the null hypothesis can be rejected**. If the p-value is less than significance level, the null
4. Other Tests of the Regression Coefficients:  $H_0 : a = \text{some value}$

## 2.3 Calculate and Interpret a confidence interval for the population value of a regression coefficient or a predicted value for the dependent variable if an estimated regression model.

1. Confidence intervals for a regress. coeff.:  $\hat{b}_j \pm (t_c \times s_{\hat{b}_j})$
2. predicting the dependent variable:  $\hat{Y}_i = \hat{b}_0 + \hat{b}_1 \hat{X}_{1i} + \dots + \hat{b}_k \hat{X}_{ki}$   
Even if you may conclude that some  $b_i$  are not statistically significantly, you cannot treat them as 0 and keep other parameters unchanged. You should use the original model, or you can throw  $\hat{b}_k$  away and make a new regression model.

## 2.4 Assumptions of a multiple regression model

1. Linear relationships exist.
2. The independent variables are not random, and there is no exact linear relation between independent variables.
3.  $E[\epsilon | X_1, \dots, X_k] = 0$
4. Variance of  $\epsilon = 0$ , i.e.  $E[\epsilon_i] = 0$
5.  $E(\epsilon_i \epsilon_j) = 0$
6.  $\epsilon$  is normally distributed.

## 2.5 Calculate and interpret F-statistic

F-test: whether at least **one** of the independent variables explains a significant portion of the variation of the dependent variable. F test is a one-tail test.

1.  $H_0 : b_1 = b_2 = b_3 = 0$  vs  $H_a : \text{at least one } b_j \neq 0$
2.  $F = \frac{MSR}{MSE} = \frac{RSS/k}{SSE/n-k-1}$
3. Degree of freedom:  $df_{\text{numerator}} = k, df_{\text{denominator}} = n - k - 1$
4. Rules: reject  $H_0$  if  $F(\text{test} - \text{statistic}) > F_c(\text{critical value})$

## 2.6 Distinguish between $R^2$ and adjusted $R^2$

1. coefficient of determination  $R^2$ : used to test if a group of independent variable can explain the dependent variable:

$$R^2 = \frac{\text{total variation} - \text{unexplained variation}}{\text{total variation}} = \frac{SST - SSE}{SST} = \frac{RSS}{SST}$$

$$\text{Multiple } R = \sqrt{R^2}$$

2. Adjusted  $R^2$

- (a) Note:  $R^2$ : **Overestimating**: will increase as variables are added to the model. Even the marginal contribution of new variables are not statistically significant.

$$(b) \text{ Introduce } R_a^2: R_a^2 = 1 - \left[ \left( \frac{n-1}{n-k-1} \right) \right] (1 - R^2)$$

## 2.7 Evaluate the quality of a regression model by analyzing the output of the equation/ANOVA table

1. ANOVA Tables, some important quantities

$$(a) R^2 = \frac{RSS}{SST}$$

$$(b) F = \frac{MSR}{MSE} \text{ with } k \text{ and } n - k - 1 \text{ df}$$

$$(c) \text{ Standard error of estimate: } SEE = \sqrt{MSE}$$

## 2.8 Formulate a multiple regression with dummy variables to represent qualitative factors

1. Def: Some value is quite qualitative. Using dummy values like 0 or 1 to describe their impacts.
2. Note: Pay attention to # of dummy variables. If  $n$  classes, we must use  $n - 1$  dummy variables.
3. Interpreting the coefficients in a dummy variable regression. We can use F-statistics to test a group of parameters, or use t-test to test the individual slope coefficients.
4. Example of Regression application with dummy variables. See Notes directly.

## 2.9 Why multiple regression isn't as easy as it looks?

Pay attention to the assumptions that have been used. Violations like::

1. Heteroskedasticity
2. Serial correlation (auto-correlation)
3. Multicollinearity

Any violations on the assumptions will impact the estimation of SEE, and finally change the t-statistic and F-statistic, and change the conclusion of the hypothesis test.

## 2.10 Types of Heteroskedasticity, how heteroskedasticity and serial correlation affect inference

1. What is Heteroskedasticity?

**Corresponding assumptions: Variance of the residuals is constant across observations. – Homoskedasticity** Heteroskedasticity means the variance of the residuals is not equal.

- (a) Unconditional heter: Not related to the level of the independent variables. Will not systematically increase with changes in the value of the independent variables. **Usually will not cause major problems.**
  - (b) Conditional heter: Related to the level of the independent variables. Eg: Conditional heter exists if the variance of the residuals increase with the value of the independent variables increases. **Will cause big problems.**
2. Effect of Heteroskedasticity on Regression Analysis
- (a) Unreliable standard errors.
  - (b) The coefficient estimates aren't affected.
  - (c) Will change the t-statistic, and will change the conclusion.
  - (d) Unreliable F-test
3. Detect Heteroskedasticity
- (a) Scatter plot
  - (b) Breusch-pagan test:  $BPtest = n \times R_{resid}^2$  with  $df = k$ . where  $n$  = the number of observations,  $R_{resid}^2 = R^2$  from a second regression of the squared residuals from the first regression.  $k$  = the number of independent variables. If  $R^2$  or BP-test are too large, something is wrong.
4. Correcting Heteroskedasticity
- (a) Calculate robust standard errors (White corrected std errors.). Use them for t-test.
  - (b) Generalized least squares.
5. What is serial correlations?
- (a) Def: auto-correlation, in which the residual terms are correlated. Common problem with time series data.
    - i. Positive serial correlation: a positive error in one time period will increase the possibility to observe a positive one next time.
    - ii. Negative serial correlation: Just opposite.
  - (b) Effect: positive serial correlation will get small coefficient std errors. Thus, too large t-statistics. therefore, too many Type I errors: reject the null hypothesis  $H_0$  while it's actually true.
  - (c) Detection:
    - i. Residual plots
    - ii. Durbin-Watson statistics:
 
$$DW = \frac{\sum_{t=2}^T (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2}{\sum_{t=1}^T \hat{\epsilon}_t^2}$$

For large samples,  $DW \approx 2(1 - r)$ , where  $r$  is the correlation coefficient between residuals from one period and those from the previous period.

Results:

      - A.  $DW = 2 \Rightarrow$  Homoskedastic and not serially correlated.
      - B.  $DW < 2 \Rightarrow$  Positively serially correlated.
      - C.  $DW > 2 \Rightarrow$  Negatively serially correlated.

Formulated hypothesis with DW-table, upper and lower critical values

      - A. Hypothesis:  $H_0$  : the regression has **no** positive serial correlation.
      - B.  $DW < d_l$ : positive serially correlated. Reject null.
      - C.  $d_l < DW < d_u$ : inconclusive results.

D.  $DW > d_u$ : **There is no evidence that are positive correlated.**

(d) Correcting serial correlation:

- i. Adjust the coefficient std errors. **recommended.** Using Hansen method.
  - A. Serial correlation only: Hansen method.
  - B. Heteroskedasticity only: White-corrected stand errors.
  - C. Both: Hans methods.
- ii. Imporoe the specification of the model.

## 2.11 Multicollinearity and its cause and effects in regression analysis

Multicollinearity: Independent variables or linear combinations of independent variables are highly correlated.

1. Effect of Multicollinearity on Regression Analysis: Will increase the std errors of the slope coefficients.  
**Type II Error: A variable is significant, while we conclude it's not.**
2. Detecting: Common situation:  $t$  - statistic is not significant while  $F$  - test is significant. This tells us the independent variables are highly correlated.  
A simple rule works if there are 2 independent variables: when the absolute value of the sample correlation betewen any two independent variables in the regression is greater than 0.7.
3. Correcting: omit one or more of the correlated independent variables. The problame is that it's hard to find the variables that result in the multicollinearity.

## 2.12 Model misspecification

1. Defination of **Regression model of specification**: decide which independent variables to be included in the model.
2. Types of misspecification
  - (a) The functional form can be misspecified: important variables are ommitted; variables should be transformed; data is improperly pooled.
  - (b) Explanatory variables are correlated with error term in time series model: A lagged dependent variable is used as an independent variable; a function of the dependent variable is used as an independent variable (forecasting the past); independent variables are measured with error.
  - (c) Other time-series misspecification.

## 2.13 Models with qualitative dependent variables

Include qualitative dependent variables, like default, bankcrupcy. Cannot use an ordinary regression model. Should use other models like **probit and logit models** or **discriminant models**.

1. Probit: normal distribution, give probability.
2. Logistic: logistic distribution.
3. Discriminant: result in an overall score or ranking.

## 3 Reading 11: Time-Series Analysis

### 3.1 Calculate/evaluate the predicted trend value for a time series given the estimated trend coefficients

1. Linear Trend Model and Log-linear Trend



- (a) Definition:  $y_t = b_0 + b_1(t) + \epsilon_t$  Note:  $t$  is just time.
- (b) Coefficients is determined by OLS. Ordinary least squared regression.  
 $\hat{y} = \hat{b}_0 + \hat{b}_1$
- (c) Log-linear Trend Models
- (d) Model:  $y_t = \exp b_0 + b_1(t) \Rightarrow \ln y_t = b_0 + b_1(t)$

### 3.2 Factors that determine whether a linear or a log-linear model trend should be used

1. Factors that determine which model is best: plot data.
2. Limitations of trend models:
  - (a) residuals are uncorrelated with each other. Otherwise, it will cause auto correlation and we should not use the trend model.
  - (b) For log-linear model, it is not suitable for cases with serial correlations (autocorrelation).
  - (c) Detect auto correlation: Durbin Watson statistic.  $DW = 2.0 \Rightarrow$  No auto correlation.

### 3.3 Autoregressive model, requirements for covariance stationary

1. Autoregressive model:
  - (a) Model:  $x_t = b_0 + b_1x_{t-1} + \epsilon_t$
  - (b) Statistical inferences based on ordinary least squares estimates doesn't apply unless the time series is **covariance stationary**.
  - (c) Conditions for covariance stationary
    - i. Constant and finite expected value.
    - ii. Constant and finite variance.
    - iii. Constant and finite covariance between values at any given lag.

### 3.4 An autoregressive model of order $p$

1. Model(order  $p$ ):  $x_t = b_0 + b_1x_{t-1} + b_2x_{t-2} + \dots + b_px_{t-p} + \epsilon_t$
2. Forecasting with an autoregressive model:
  - (a) One-period-ahead forecast for  $AR(1)$ :  $\hat{x}_{t+1} = \hat{b}_0 + \hat{b}_1x_t$
  - (b) Two-period-ahead forecast for  $AR(1)$ :  $\hat{x}_{t+2} = \hat{b}_0 + \hat{b}_1\hat{x}_{t+1}$

### 3.5 How the residuals can be used to test the autoregressive model

1. The residual should have no *serial correlation* if an AR model is correct.
2. Steps
  - (a) Estimate: Start with  $AR(1)$
  - (b) Calculate: the autocorrelations of the model residuals
  - (c) Test: whether the autocorrelations are significantly different from 0.  
 The standard error is  $\frac{1}{\sqrt{T}}$  for  $T$  observations. The t-test for each observation is  $t = \frac{\rho_{\epsilon_t, \epsilon_{t-k}}}{1/\sqrt{T}}$ , with  $T - 2$  df.

### 3.6 Mean reversion and a mean-reverting level

1. Mean reversion: The time series tends to move toward its mean.
2. Mean-reverting level:  $\hat{x}_{t+1} = x_t$ , where  $\hat{x}_t$  is the predicted value.
3. All covariance stationary time series has finite mean-reverting level.

### 3.7 Contrast in-sample and out-of-sample forecasts and the forecasting accuracy of different time-series models based on the root mean squared error criterion.

1. in-sample, out-of-sample: determined by if the predicted data is in the range of the observations.
2. RMSE, root mean squared error: used to compare the accuracy. If the accuracy of out-of-sample is better, you should use it for future applications

### 3.8 Explain the instability of coefficients of time-series models

1. Instability or nonstationarity. Due to the dynamic economic conditions, model coefficients will change a lot from period to period.
2. Shorter time series are more stable, but longer time series are more reliable.

### 3.9 Random walk processes and their comparisons between covariance stationary processes

1. Random walk:  $x_t = x_{t-1} + \varepsilon_t$ 
  - (a)  $E(\varepsilon_t) = 0$ : The expected value of each error is zero.
  - (b)  $E(\varepsilon_t^2) = 0$ : The variance of the error terms is constant.
  - (c)  $E(\varepsilon_i, \varepsilon_j) = 0$ : There is no serial correlation in the error terms.
2. Random walk with a Drift:  $x_t = b_0 + b_1 x_{t-1} + \varepsilon_t$ , where  $b_1 \neq 0$
3. A random walk or a random walk with a drift have no finite mean-reverting level. Since  $b_1 = 1$ ,  $\frac{b_0}{1-b_1} = \frac{b_0}{0}$ . Therefore, they are not covariance stationary.
4.  $b_1 = 1$ , they exhibit a unit root. Thus, **the least square regression that been used in AR(1) will not work unless we transform the data.**

### 3.10 Things about unit roots: when they will occur, how to test them, how to transform data to apply AR

1. Unit root testing for nonstationarity:
  - (a) run an AR model and check autocorrelations
  - (b) perform Dickey Fuller test.
    - i. Transform:  $x_t = b_0 + b_1 x_{t-1} + \varepsilon \Rightarrow x_t - x_{t-1} = b_0 + (b_1 - 1)x_{t-1} + \varepsilon$
    - ii. Direct test if  $b_1 - 1 = 0$  using a modified t-test.
2. First differencing
  - (a) For a random walk, transform the data  $y_t = x_t - x_{t-1} \Rightarrow y_t = \varepsilon_t$  then start to use an AR model  $y = b_0 + b_1 y_{t-1} + \varepsilon$ , where  $b_0 = b_1 = 0$
  - (b)  $y$  is covariance stationary.

### 3.11 How to test and correct for seasonality in a time-series model, and calculate and interpret a forecasted value using an AR model with a seasonal lag.

1. Detect: special autocorrelation exists for some seasonal lags.
2. Correction: Add an additional seasonal lag term.

### 3.12 Explain autoregressive conditional heteroskedasticity (ARCH) and describe how ARCH models can be applied to predict the variance of a time series

1. ARCH: the variance of the residuals in one period is dependent on the variance of the residuals in a previous period.
2. Using ARCH models:  
 Example  $ARCH(1)$ :  $\hat{\varepsilon}_t^2 = a_0 + a_1\hat{\varepsilon}_{t-1} + \mu_t$  if  $a_1$  is significantly different from zero.  $\hat{\varepsilon}_t^2$  is the squared residuals.  
 Note: Things like generalized least squares should be used to correct heteroskedasticity. otherwise, the std errors of the coefficients will be wrong, leading to invalid conclusions.
3. Predicting the variance of a time series: using ARCH model to predict the variance of future periods:  $\hat{\sigma}_{t+1}^2 = \hat{a}_0 + \hat{a}_1\hat{\varepsilon}_t^2$

### 3.13 Explain How time-series variables should be analyzed for nonstationarity and/or cointegration before use in a linear regression

1. Cointegration:
  - (a) Two time series are economically linked or follow the same trend and that relationship is not expected to change. – Error terms from regressing one on the other is covariance stationary and the t-test are reliable.
  - (b) How to test cointegration: regress  $y_t$  on  $x_t$   $y_t = b_0 + b_1x_t + \varepsilon$ ,  $y_t, x_t$  are two different time series. Then do a unit root test using the Dickey Fuller test with critical t-values calculated by Engle and Granger.  
 If "A unit root" is rejected  $\Rightarrow$  covariance stationary, cointegrated.

## 4 Reading 12: Probabilistic Approaches: Scenario Analysis, Decision Trees, and Simulations

### 4.1 Describe steps in a simulation, Explain three ways to define the probability distributions for a simulation's variable, and describe how to treat correlation across variables in a simulation.

1. Steps in simulations:
  - (a) Determine the probabilistic variables
  - (b) Define probability distributions for these variables
    - i. Option 1: Historical data
    - ii. Option 2: Cross-sectional data: estimate the variable from similar companies.
    - iii. Option 3: Pick a distribution and estimate the parameters.

- (c) Check for correlations among variables: Use historical data to determine whether any systematically related. Strong relations  $\Rightarrow$  1) Allow only one of the variables can be removed. Or 2) Build the rules of correlations into the simulation.
- (d) Run the simulation.

## 4.2 Describe advantages of using simulations in decision making

1. Advantages: 1) Better input quality 2) Provides a distribution of expected value rather than a point estimate.

## 4.3 Describe some common constraints introduced into simulations

1. Constraints: specific limits imposed by users of simulations.
2. Types of constraints
  - (a) Book value constraints:
    - i. Regulatory capital requirements: minimum capital requirements
    - ii. Negative equity
  - (b) Earnings and cash flow constraints: might be imposed to meet analyst expectations
  - (c) Market value constraints

## 4.4 Describe issues in using simulations in risk assessment

1. Limitations of using simulations
  - (a) Input quality: garbage in, garbage out
  - (b) Inappropriate statistical distributions
  - (c) Non-stationary distributions: parameters will change
  - (d) Dynamic correlations: correlations between input variables will change.
2. Risk-adjusted value: cash flows from simulations are not risk-adjusted. SHOULD NOT be discounted at risk-free rate.

## 4.5 Compare scenario analysis, decision trees, and simulations

1. Scenario analysis: computes the value of an investment under some specific cases. Total probability is less than 1.
2. Decision trees: good when risk is discrete and sequential. Sum of probability is 1

# 5 Reading 13: Currency Exchange Rates: Determination and Forecasting

## 5.1 Calculate and interpret the bid-ask spread

1. Exchange rates
  - (a) Important things: exchange rate, spot exchange rate, forward exchange rate.
  - (b) Bid/offer(ask) rates: //Bid: The price that bank will buy. Offer: The price that bank will sell.
  - (c) Foreign Exchange Spread. Unit: "1 pip" =  $1/10000 = 0.0001$ . Spread depend on:
    - i. Spread in the interbank market. (Currencies, time, market volatility)
    - ii. Size of transaction.
    - iii. Relationship between the dealer and client.

## 5.2 Identify a triangular arbitrage opportunity and calculate its profit

- Example: USD/AUD. USD is the price currency, and AUD is the base currency.
  - Buy the base currency at the ask  $\Rightarrow$  Sell the price currency at the ask
  - Sell the base currency at the bid  $\Rightarrow$  Buy the price currency at the bid
- For investors, Rule: *up-the-bid-and-multiply, down-the-ask-and-divide*
  - Convert USD into AUD: going down the quote – from USD on top to AUD on bottom. Use the ask price for the quote.
  - Convert AUD into USD: similar. But from bottom to top.
- Cross Rate: The exchange rate between two currencies with the help by a common third currency.
- Cross Rate with bid-ask spreads.
  - Rule 1:
 
$$\left(\frac{A}{C}\right)_{bid} = \left(\frac{A}{B}\right)_{bid} \times \left(\frac{B}{C}\right)_{bid}; \left(\frac{A}{C}\right)_{offer} = \left(\frac{A}{B}\right)_{offer} \times \left(\frac{B}{C}\right)_{offer}$$
  - Rule 2:
 
$$\left(\frac{B}{C}\right)_{bid} = \frac{1}{\left(\frac{C}{B}\right)_{offer}}; \left(\frac{B}{C}\right)_{offer} = \frac{1}{\left(\frac{C}{B}\right)_{bid}}$$
- Triangular Arbitrage: If the dealer's quote is different from the cross rate, arbitrage opportunities may exist. Check it with Notes.

## 5.3 Distinguish between spot and forward rates and calculate the forward premium/discount for a given currency

- Forward premium relative to a second currency: Forward price  $\hat{}$  Spot price. Forward premium =  $F - S_0$
- Calculate the market-to-market value of a forward contract

$$V_T = (FP_T - FP)(contractsize)$$

where:

- $V_T$  = value of the forward contract at time  $T$ , denominated in price currency
  - $T$  = maturity of the forward contract
  - $FP$  = forward price locked in at inception to buy base currency
  - $FP_T$  = forward price to **sell** the same currency at time  $T$
- Value prior to expiration.

$$V_t = \frac{(FP_t - FP)contractsize}{1 + R\left(\frac{days}{360}\right)}$$

where

- $V_t$  is the value of the forward price
- $FP_t$ : forward price at time  $t$
- $days$  number of days remaining
- $R$  interest rate

## 5.4 Explaining international parity relations (covered and uncovered interest rate parity, purchasing power parity, and the international Fisher effect)

1. Covered interest rate parity: “Covered” means bound by arbitrage. Investor should earn the same return using either currency.

$$F = \frac{1 + R_A(\frac{days}{360})}{1 + R_B(\frac{days}{360})} S_0$$

2. Uncovered interest rate parity: Forward currency contract is unavailable, which makes the interest rate not bound by arbitrage. For a quote A/B, the base currency is expected to appreciate

$$E(\% \Delta_S)_{(A/B)} = R_A - R_B$$

Uncovered interest rate parity can only **forecast** the future spot exchange rate.

3. Comparing covered and uncovered interest parity:

- (a) Covered interest parity  $\Leftrightarrow$  No-arbitrage forward rate
- (b) Uncovered interest parity  $\Rightarrow$  **Expected** future spot rate

4. International Fisher Relation

- (a)  $R_{nominal} = R_{real} + E(inflation)$
- (b) Under real interest rate parity, the real interest rate are assumed to converge across different markets.

$$R_{nominalA} - R_{nominalB} = E(inflation_A) - E(inflation_B)$$

5. Purchasing Power Parity: Assumed by one price law.

- (a) Absolute purchasing power parity: The average price of a basket of consumption goods.

$$S(A/B) = CPI(A)/CPI(B)$$

May not hold due to different weights of consumptions.

- (b) Relative Purchasing Power Parity: Changes in exchange rates should exactly offset the price effects of any inflation differential between the two countries.

$$\% \Delta S(A/B) = Inflation_A - Inflation_B = change in spot price(A/B)$$

Not always held in short run.

- (c) Ex-Ante Version of Purchasing Power Parity: Similar to relative PPP, but Ex-Ante uses expected inflation instead of actual inflation.

## 5.5 Describe the relations among the international parity conditions

See Notes Page 263, Vol. 2.

## 5.6 Evaluate the use of the current spot rate, the forward rate, purchasing parity and uncovered interest parity to forecast future spot exchange rates

1. Real Exchange Rate =  $S_t \left[ \frac{CPI_B}{CPI_A} \right]$ ,  $S_t$  is the spot rate at time  $t$  given as A/B

## 5.7 Explain how flows in the balance of payment accounts affect currency exchange rates

1. Balance of Payments: accounting method to track transactions between a country and its international trading partners.
  - (a) Including government, consumer, and business transactions.
  - (b) current account + financial account + official reserve account = 0
  - (c)
    - i. Current account: Exchanges of goods/services, exchanges of investment income and unilateral transfers like gifts.
      - A. Surplus: we sell more to other countries, buy less from them
      - B. Deficit: we buy more from the rest, sell less to them
    - ii. Financial account/Capital account: Flows of funds for debt and equity investment into/out of a country. Surplus: Money is flowing into the country.
    - iii. Official reserve: those made from the reserves held by the government. Normally doesn't change from year to year.
2. Influence of BOP on Exchange Rates
  - (a) Current Account
    - i. Flow mechanism
      - A. Deficit: increase the supply of that currency in the market. Because exporters to our countries need to convert their revenue to their own currency.  $\Rightarrow$  Down on the exchange value.
      - B. Depreciation of the currency may rebalance the current account. Depending on **The initial deficit, the influence of exchange rates on import/export prices, price elasticity of traded goods**. See Notes P265 for details.
    - ii. Portfolio Composition mechanism. Countries with current account surpluses usually have capital account deficits, which typically take the form of investments in countries with current account deficits. As a result of these flows of capital, investor countries may find their portfolios' composition being dominated by few investee currencies. When investor countries decide to rebalance their investment portfolios, it can have a significant negative impact on the value of those investee country currencies.
    - iii. Debt sustainability mechanism: Current account deficit may be balanced by borrowing money from other countries. If the debt too high, lenders may question the security, leading to the depreciation of the borrower's currency.
  - (b) Capital Account Influences: Money flow in  $\Rightarrow$  Demand for my country's currency increases  $\Rightarrow$  Appreciation.
    - i. Good: can help to overcome a shortage of internal savings
    - ii. Bad: Too much money can be problematic for emerging markets.
      - A. Excessive appreciation of the domestic currency
      - B. Financial asset, real estate bubbles
      - C. Increase in external debt
      - D. Excessive consumption in the domestic market funded by credit
3. real exchange rate  $(A/B) = \text{equilibrium real exchange rate } (A/B) + (\text{real interest rate}_B - \text{real interest rate}_A) - (\text{risk premium}_B - \text{risk premium}_A)$   
 This equation is not precise. We cannot use it to calculate the rate.
4. Taylor Rule

$$R = r_n + \pi + \alpha(\pi - \pi^*) + \beta(y - y^*)$$

- (a)  $R$  = Central bank policy rate implied by the Taylor Rule

- (b)  $r_n$  = Neutral **real** policy interest rate
- (c)  $\pi$  = Current inflation rate
- (d)  $\pi^*$  = Central bank's target inflation rate
- (e)  $y$  = log of current level of output
- (f)  $y^*$  = log of central bank's target (sustainable) output
- (g)  $\alpha, \beta$  = policy response coefficients. (suggested value: 0.5 for both)

$$\text{Real interest rate} = r = R - \pi = r_n + \alpha(\pi - \pi^*) + \beta(y - y^*)$$

Substitute the real interest rate equation, we have

Real exchange rate (A/B) = equilibrium real exchange rate(A/B) + difference in neutral real policy interest rate(B-A) +  $\alpha$ [difference in inflation gap (B-A)] +  $\beta$ [difference in output gap(B-A)] - (risk premium<sub>B</sub> - risk premium<sub>A</sub>) // Where: Inflation gap = current inflation - target inflation, Output gap = current output - target output

## 5.8 Explain approaches to assessing the long-run fair value of an exchange rate

1. The ex-ante version of relative PPP holds  $\Rightarrow$  The real exchange rates constant. However, relative PPP does not necessarily hold over the short term. Over long term, PPP holds, and the real rate will be near its equilibrium level.
2. IMF assesses long-term equilibrium real exchanges rate based on
  - (a) Macroeconomic balance approach: if the Ex rates need to be adjusted to equalize the expected current account imbalance and the sustainable current account imbalance.
  - (b) External sustainability approach. How rates need to be adjust to force a country's external debt relative to GDP towards its sustainable level.
  - (c) Reduced-form econometric model approach.

## 5.9 Describe the carry trade and its relation to uncovered interest rate parity and calculate the profit from a carry trade.

1. FX carry trade: Invest in a higher yielding funding with the funds borrowed in a lower yielding currency. This is due to the uncovered interest rate parity may not hold.
2. Risk of the Carry Trade
  - (a) The exchange rate may change abruptly.
  - (b) The return distribution is not normal. Negative skewness and excess kurtosis (fat tails).  $\Rightarrow$  High probability of large loss
3. Risk Management in Carry Trades
  - (a) Volatility filter: if volatility  $\geq$  certain threshold, close the carry trade.
  - (b) Valuation filter: valuation band for each currency based on PPP. If the value of a currency falls below the band, we will increase its ratio.



**5.10 Describe the Mundell-Fleming model, the monetary approach and the asset market approach to exchange rate determination.**

**5.11 Forecast the direction of the expected change in an exchange rate based on balance of payment, Mundell-Fleming, monetary, and asset market approaches to exchange rate determination.**

**5.12 Explain the potential effects of monetary and fiscal policy on exchange rates.**

1. Mundell-Fleming Model: evaluate the impact of monetary and fiscal policies on interest rates, and therefore on exchange rates.

2. Flexible Exchange Rate Regimes: rate are determined by markets.

(a) High Capital Mobility: Expansionary M and F are likely to have opposite effects. Expansionary M will reduce the interest rate, reduce the inflow of capital investment, reduce the demand for domestic money, depreciation.

(b) Low Capital Mobility: Uncertain

(c) Summary:

Monetary/Fiscal	High Capital Mobility	Low Capital Mobility
Expan/Expan	Uncertain	Depreciation
Expan/Restr	Depreciation	Uncertain
Restr/Expan	Appre	Uncertain
Restr/Restr	Uncertain	Appreciation

(d) Fixed Ex rate regimes

i. If monetary expansionary (depreciation), governments need to buy money in the FX market, therefore will reverse the effect from monetary expansionary.

ii. Fiscal expansionary → Appreciation (More money needed) → Government need to sell money to keep Ex rate stable. → Fiscal effect on aggregate demand will be reinforced.

3. Monetary Approach to Exchange Rate determination

Inflation play no role in exchange rate in Mundell-Fleming model.

Assumptions: 1. Output is fixed.

(a) Method 1: Pure Monetary model. Assume: PPP holds, output is constant.

(b) Dornbusch overshooting model. Price are inflexible in short term. Expan Monetary → price increase, interest rate down → depreciation of currency. Therefore, in short term, price sticky, interest rate down too much. → depreciation is greater than PPP implies.

4. Portfolio Balance Approach to Exchange rate determination.

(a) It focuses on long-term implications of fiscal policy on currency values.

(b) Fiscal deficit → sell bonds → When investors think the country is safe, they will continue to buy bonds. If the investors refuse to fund the deficits → depreciation

5. In short term, with free capital flows, expan fiscal → appreciation

Long term → government has to reverse expan fiscal. Otherwise, investor will refuse to fund it, then the country have to monetize its debt (print money). → depreciation

**5.13 Objectives of central bank intervention and capital controls and describe the effectiveness of intervention and capital controls.**

See Notes P274. Old version.

### 5.14 Describe warning signs of currency crisis.

1. Terms of trade deteriorate
2. Foreign reserve down quickly
3. Real exchange rate is extremely higher than mean-reverting value.
4. Inflation increases.
5. Equity markets have a boom-bust cycle.
6. Money supply relative to bank reserves increases.
7. Nominal private credit grows.

### 5.15 Technical analysis

See Notes P275 Old version.

## 6 Economic Growth and the Investment Decision

### 6.1 Compare factors favoring and limiting economic growth in developed and developing economies

Two important factors. 1: GDP per capita. 2: Growth of GDP

1. Preconditions for Growth
  - (a) Saving and investment. Positively correlated with economic development.
  - (b) Financial markets and intermediaries. Help resources reallocation. However, it may increase leverage, risks.
  - (c) Political stability, rule of law and property rights.
  - (d) Investment in human capital. Worker's skills.
  - (e) Tax and regulatory systems. Lower tax burdens are good. Lower regulation levels are good.
  - (f) Free trade and unrestricted capital flows.

### 6.2 Describe the relation between the long-run rate of stock market appreciation and the sustainable growth rate of economy.

The growth in the price is related to earnings and GDP:  $\Delta P = \Delta_G DP + \Delta(E/GDP) + \Delta(P/E)$ . Over the long-term,  $\Delta(E/GDP) = 0, \Delta(P/E) = 0$ . Only GDP growth matters.

### 6.3 Explain why potential GDP and its growth rate matter for equity and fixed income investors.

Higher GDP growth  $\rightarrow$  Higher interest rates  $\rightarrow$  Higher real asset returns.

Higher GDP growth makes people think that future income is increasing, therefore increase consumptions and reduce savings. To encourage consumers save, higher interest rate is needed.

In short term, actual GDP in excess of potential GDP will result in rising prices  $\rightarrow$  inflationary pressure.

## 6.4 Distinguish between capital deepening investment and technological progress and explain how each affects economic growth and labor productivity

### 1. Factor input and Economic growth

- (a) Model: 2-factor aggregate production:  $Y = F(L, K)$  at a level of tech  $T$ . Output  $Y$  is a function of labor ( $L$ ) and capital.
- (b) Cobb-Douglas Production:  $Y = TK^\alpha L^{(1-\alpha)}$   
Dividing both sides by  $L$ , the output per worker is

$$Y/L = T(K/L)^\alpha$$

## 6.5 Forecast potential GDP based on growth accounting relations

### 1. Growth Accounting Relations

$$\Delta Y/Y = \Delta A/A + \alpha \times (\Delta K/K) + (1 - \alpha)(\Delta L/L)$$

i.e. growth rate in potential GDP = long-term growth of tech +  $\alpha$  long-term growth rate of capital +  $(1-\alpha)$ \*(long-term growth rate of labor)

The growth of technology is not observable. Can be estimated from previous data: ex-growth rate - ex-growth rate from  $L$  and  $K$

## 6.6 Explain how natural resources affect economic growth and evaluate the argument that limited availability of natural resources constrains economic growth

- 1. Access to natural resources does not require ownership of resources.
- 2. Another theory: ownership of natural resources may actually inhibit growth. → Dutch disease: global demand for natural resources drives up the country's currency, making all other exports more expensive and uncompetitive.

## 6.7 Explain how demographics, immigration, and labor force participation affect the rate and sustainability of economic growth

### 1. Labor Supply Factors

- (a) Demographics: A country's age distribution. Countries with younger age will have a higher potential growth.
- (b) Labor force participation.
- (c) Immigration: a potential source in developed countries ⇒ increase work force
- (d) Average hours worked

## 6.8 Explain how investment in physical capital, human capital, and technological development affects economic growth

- 1. Human capital: knowledge and skills that individuals possess. Can be enhanced via education.
- 2. Physical capital: infrastructure, computers, telecommunications (ICT) AND non-ICT capital (machinery, transportation and non-residential construction). More investment in physical capital ⇒ Good GDP growth.  
More investment may enhance the tech improvements.

3. Technological development. Investment in tech will increase the productivity.
4. Public infrastructure: like roads, bridges, and municipal facilities. This will enhance total productivity. Because the private investment will not invest these public things for their little returns.

## 6.9 Compare classical growth theory, neoclassical growth theory, and endogenous growth theory

1. Classical growth theory: In the long-term, population growth increases whenever there are increases in per capita income above subsistence level due to increase in capital or tech progress.  $\Rightarrow$  Growth in real GDP per capita is not permanent.  $\Rightarrow$  This is not supported by observed facts.
2. Neoclassical Growth theory:
  - (a) Estimate steady state growth rate. Equilibrium economy is when the output-to-capital ratio is constant. When the output-capital ratio is constant, the labor-to-capital ratio and output per capita also grow at the equilibrium rate. Check textbooks here.
  - (b) Based on Cobb-Douglas function,
    - i. Sustainable growth of output per capita:  $g^* = \frac{\theta}{1-\alpha}$ , where  $\theta$  is the growth rate in technology, and  $1 - \alpha$  is the labor's share of GDP.
    - ii. Sustainable growth rate:  $G^* = \frac{\theta}{1-\alpha} + \Delta_L$ , which is the growth rate of output per capita plus the growth of labor.
    - iii. Comments
      - A. Capital deepening will not affect the growth rate in the long run.
3. Endogenous Growth Theory
  - (a) Technological growth is a result of investment in physical and human capital. Returns to capital are constant.
  - (b) Private investments in R&D also benefits all economy.

## 6.10 Explain and evaluate convergence hypotheses

1. Absolute convergence: Less developed countries will achieve equal living standards overtime.
2. Conditional convergence: Convergence in living standards will only occur for countries with the same savings rates, population growth rates and production functions.
3. Club convergence: Countries may be part of a club. Poorer countries that are part of the club will catch up their richer peers. Institutional changes can help a country to join the club. Those are not in the club will never catch up.

## 6.11 Describe the economic rationale for governments to provide incentives to private investments in technology and knowledge.

R&D are risky. Governments support may provide incentives to private R&D, and therefore boosts the growth of the overall economy.

## 6.12 Describe the expected impact of removing trade barriers on capital investment and profits, employment and wages, and growth in the economies involved.

1. Increased investment from foreign savins

2. Allows focus on industries where the country has advantage.
3. Increased markets
4. Increased sharing of tech
5. Increased competition removes bad firms and relocating assets.

## **7 Reading 15: Economics of Regulation**

### **7.1 Describe classifications of regulations and regulators**

1. Regulations: i. Statues; ii. Administrative regulatinos; iii. judicial law
2. Regulators: government agencies/indepedent regulators/outside bodies. Independent regulators including self-regulating organizations that regulates and represents their members. Outside bodies will not regulate, but their products are referenced by regulators.

### **7.2 Describe uses of self-regulation in financial markets**

1. US: FINRA is an SRO recognized by SEC.
2. In civil-law countries, independent SROs are rare, and government agencies fulfill the role of SROs.
3. In common=law countries, independent SROs are historically good.

### **7.3 Describe the economic rationale for regulatory intervention**

1. Economic Rationale for Regulation. Regulations are needed when
  - (a) Information frictions. When information is not equally availabel or distributed.
  - (b) Externalities.

### **7.4 Describe regulatory interdependencies and their effects.**

1. Regulatory Interdependencies.
  - (a) Regulatory capture theory: Regulators will at some point in time be influenced or controlled byt the industiry that is being regulated. Because the regulators will be influenced by the industry, and the experience will sometimes lead to impartial conclusions.
  - (b) Regulatory competition: Regulatory difference between jurisdictions will lead to it. Regulators compete to provide the most business-friendly environment.
  - (c) Regulatory arbitrage: businesses find a coutry that best for itself.

### **7.5 Describe the tools**